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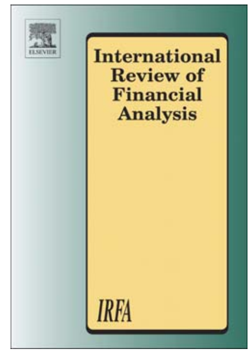

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ABSTRACT

Bubbles and crashes have long been an important area of research that has not yet led to a comprehensive theoretical or empirical understanding of how to define, measure, and compare such extreme market events. Highlights of the vast literature on bubbles, crashes, and volatility are surveyed and a promising direction for future research, based on a theory of short-side rationing, is described. The theory suggests that, especially in extreme market conditions, marginal quantities held or not held become transactionally more important than the prices paid or received. Our approach is empirically implemented by fitting monthly elasticity of returns variances to an exponential expression. From this there then follows a comparisons of changes in implied versus realized volatility, generation of an Extreme Events Line (EEL), and a crash intensity comparison metric. These methods open a new perspective from which it is possible to analyze bubble and crash events as applied to different time scales and asset classes that include bonds, real estate, foreign exchange, and commodities.

Key words:
Bubbles
Crashes
Elasticity of variance
Extreme events line
Herding
Tranquility zone
Volatility

JEL Classification: B41, B50, E32, G01, G1

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1. Introduction

Financial economists and the public at large have long been fascinated by the extreme events that are informally referred to as bubbles and crashes. This is not surprising given that such events usually have important economic policy and financial market effects and implications and allow for spectacular, newsworthy trading gains and losses.

As such extreme events unfold there is the risk that resources such as monetary and intellectual capital become increasingly misallocated, at least from a long-run perspective. A better understanding of bubbles and crashes is thus useful for experts and the general interested public.

In response, a voluminous literature on bubbles and crashes – especially on how to mathematically describe and test for them – has been developed over the last forty years. By and large, this is a literature based on the assumptions that rationality prevails and that markets are predominantly efficient. Existing models, however, still have difficulty in handling extreme market events and arguments and criticisms have not been satisfactorily addressed or resolved.

As a result, bubbles and crashes remain largely in the "you know one if you see one" stage of description, especially ex-post. And researchers have begun to realize that the definitional aspect is important given that the power of econometric testing in this area is known to be relatively weak. The colorful nomenclature in the literature – using inventive notions of bubbles that are rational, churning, collapsing, exploding, and intrinsic – reflects the grasping-at-straws nature of these attempts.

This paper surveys the key relevant literature in this area (which has been largely centered on equities) and considers how bubbles and crashes have been defined, measured, and analyzed. It is argued that estimates of elasticity of returns variances (EOV) provide a practical extension of the short-side rationing theory of supply and demand first developed in the works of Malinvaud (1985), Bénassy (1986), Muellbauer and Portes (1978), and Werner (2005). An EOV approach underscores the idea that – as extreme market events evolve – the investment focus shifts from emphasis on prices given or received to quantities held or not held.

Because the measurement of the market's *reaction* to news and fundamentals may be equally or more important than the news and fundamentals in and of themselves, the EOV methodology implicitly embeds these reactions (and also interest rates, because the elasticity with is taken respect to either an equity risk premium or a credit spread measure). In this view, the market action itself defines and reveals bubble and crash events that are expressed through excessive stock price variance that is *sui generis* and that can be empirically estimated by fitting the variance elasticities in sequential sample periods to an exponential (or mathematically similar parabolic) curve progressing over time toward infinity. Such an exponential trace of returns variance elasticity is a distinctive visual feature of all bubbles and crashes – including those that appear in real estate and property, foreign exchange, and also markets for bonds and commodities.

Section 2 provides a survey of the literature on volatility. Section 3 reviews the conventional theoretical approaches to modeling bubbles. The analytical device of the elasticity of variance is introduced in Section 4, with Section 5 applying this to volatility metrics for bubbles and crashes. Section 6 then summarizes and concludes the paper.

2. Volatility studies

Changes in market volatility, σ, will always affect the expected returns on all assets. Volatility itself has become not only a more substantive feature of capital markets but also an important tradable asset class in itself (Cole 2014). This is a direct outgrowth of the seminal Black-Scholes-Merton model of European call option pricing that was introduced in the early 1970s (Black and Scholes, 1973) and that led to creation of a vast options and derivatives trading business specialized in analysis of risk as it pertains to aspects of implied volatility (IV).

The classical Black-Scholes partial differential equation, which describes the price of the option over time, t, takes the form:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \qquad (1)$$

in which V represents the value of the option, S is the price of the underlying security, and r is the interest rate. By solving the equation using boundary and terminal conditions and rearranging terms, the price of an option can then be expressed as an implied volatility because risk can theoretically be eliminated through hedging via purchases and sales of a specific number of underlying asset and option units. Here it is assumed (among other things) that there are no transaction costs, that there are no arbitrage opportunities, that volatility is constant, and that the underlying distribution of continuously compounded returns is normally distributed with known mean and variance.

Nevertheless, further mathematical and empirical complications quickly arise once these basic assumptions are challenged. Rubinstein (1985) for instance, exposes biases, finding that call prices are actually higher than would be predicted by the Black-Scholes model. And Bates (1996) examines the observed inconsistency between the theoretical and observed option price distributions and the properties of the underlying asset prices.

In Bates-type models, jumps are expressed by a compound Poisson process with normally distributed jumps.

Most studies in this area, however, begin with the basic stochastic process assumptions that are expressed in the form of a generic geometric Brownian motion model shown by

$$\frac{dS_t}{S_t} = \mu \cdot dt + \sigma \cdot dW_t\,, \tag{2}$$

where $S(t)$ is the stock price at time t, μ is a measure of the average rate of growth of the asset price (a constant drift term), dt is the change in time, σ is the constant volatility, and dW_t, known as a Wiener process, is a normalized random variable with a mean of zero and unit standard deviation of dt. The aim here is to separate the expression into deterministic drift and stochastic components, making use of Itô's Lemma, which is the stochastic version of the Taylor series expansions used in standard calculus. The resulting structure allows a continuous martingale model to be formed.

To this, Cox, Ingersoll, and Ross (1985) then made an important realistic modification by also formulating a stochastic differential model for interest rates, *r*, as:

$$dr_t = a(b - r_t)\,dt + \sigma\sqrt{r_t}\,dW_t \tag{3}$$

in which a, b, and σ are parameters.

Heston (1993) then made further extensions to the basic stochastic process model by expressing the instantaneous variance as

$$d\nu_t = \kappa(\theta - \nu_t)\,dt + \xi\sqrt{\nu_t}\,dW_t^{\nu}\,, \tag{4}$$

- where ν_t, is the instantaneous variance,
- the dW term is a Wiener process (i.e., a random walk),
- μ is the rate of return of the asset.

- θ is the long variance, or long run average price variance (as t tends to infinity, the expected value of ν_t tends to θ).
- κ is the rate at which ν_t reverts to θ.
- ξ is the vol of vol, or volatility of the volatility;[1]

An important general extension to fixed-income markets that models the evolution of interest rate curves under the assumption that the volatility and drift of the instantaneous forward rate are deterministic was presented in the landmark paper by Heath-Jarrow-Morton (1992). Models that fall into the HJM framework may even be non-Markovian, and, as a class, will start with the relation between rates and bond prices, *P (t, T*), as:

$$P(t,T) = e^{-\int_t^T f(t,s)\,ds}. \qquad (5)$$

In this structure, forward rates *f (t, T)*, will again, for any time *T*, start with the familiar stochastic differential equation such as shown in (2) but modified as in (6). Banks now commonly use the HJM model approach for asset valuation and risk management because it (along with numerous subsequent offshoots) well-reflects the volatility of changes in the term structure of interest rates, provides reasonable price estimates for securities that are sensitive to changes in rates, and is also applicable to foreign exchange, commodities futures pricing, and arbitrage strategies.

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[1] All of these variations are direct descendants of an Ornstein-Uhlenbeck (OU) stochastic (stationary, Gaussian, and Markovian) process model. Though developed in the 1930s, the OU approach can be productively applied to the Brownian Motion–based financial models of today because OU describes velocity if friction is present, which it certainly is in real markets.

$$df\,(t,T) = \mu\,(t,T)\,dt + \xi\,(t,T)\,dW\,(t) \qquad (6)$$

Many such stochastic models in the literature on price jumps will also often begin with the probability theory assumptions known collectively as a Lévy process. Whether applied in discrete or continuous-time forms, a process of this type will have independent and stationary increments and continuity in probability. Applebaum (2004) reviews Lévy process ties to Brownian Motion models and Zaevski et al. (2014) shows how these can be applied to options pricing.

Three kinds of identifiable risk thus include: volatility itself, then the volatility of volatility, and then the standard error of volatility of volatility.[2] Barndorff-Nielsen and Veraart (2013) provide a sophisticated survey and model of such stochastic volatility of volatility (SVV) models and their explicit links to a variance risk premium (VRP), which reflects the fact that investors face not only uncertainty about the expected return possibilities (i.e., return variance) but also uncertainty about the return variance itself (as noted in Carr and Wu, 2009).

Here the most important and perhaps monetarily productive question to be answered is whether knowledge of implied volatility (IV) has any type of practical relationship to realized volatility (RV). Goyal and Saretto (2009), for instance, found that differences between historical realized volatility and at-the-money implied volatility produced significant average monthly returns. Realized volatility, however, is not usually considered to be independent of the time series from which it is calculated and implied volatility is not usually constant across different option exercise prices.

The literature, however, largely agrees that although financial asset returns are unpredictable, return volatility is highly predictable (Andersen et. al. 2001b, 2003). In

[2] This was expressed by stochastic modeling expert Peter Carr, but it is an offshoot of the Heston (1993), Bates (1996), and Derman-Kani (1994) models.

other words, there appears to be far greater probability of reversion to mean values of volatility than to mean values of metrics based on earnings, dividend, payouts, or cash flows.

The value of options on underlying securities is determined by the anticipated future volatility of the underlying security and this implied volatility reflects each option's price.[3] But because future volatility is unknown, historical and forecast volatilities are used to guess at the future realized volatility. Implied volatilities for options with different strike prices of the same maturities will typically also differ.

Implied volatility can be expected to rise as the market becomes more volatile and vice versa even though observation of an increasing spread between implied and realized volatility – the volatility premium – does not necessarily lead to the conclusion that a bubble is in formation. Importantly, however, Bollerslev et al. (2014) and (2009) find evidence that this premium is able to predict aggregate stock market returns. Gatheral (2006) further demonstrates that implied volatility can be expressed as a weighted average over all possible future scenarios of realized volatilities. And because implied volatility displays such a term structure there is, as with bonds and interest rate curves, a rate of decay as the implied moves closer in time toward becoming realized.

Indeed, Bollerslev and Zhou (2006) showed that the volatility series "exhibit pronounced own temporal dependencies" and Adrian and Rosenberg (2008) showed that investors require compensation when holding assets that depreciate as volatility rises.[4] The long-term average for implied volatility of a large-stocks index such as the S&P 500 is approximately 20 percent, with a normally strong correlation of around -0.75 between equity market returns and implied volatilities (and which reflects the incentive that options sellers require for liquidity protection).

[3] See Natenberg (1994, p.75).

[4] Jones and Wilson (2004) also compared the relative volatilities of bonds and stocks over many decades and found that volatility for bonds has been increasing relative to that for stocks.

The negative correlation can be seen in Figure 1 which compares percent changes in the CBOE's options-on-futures-based VIX to percent changes in the S&P 500 index.[5]

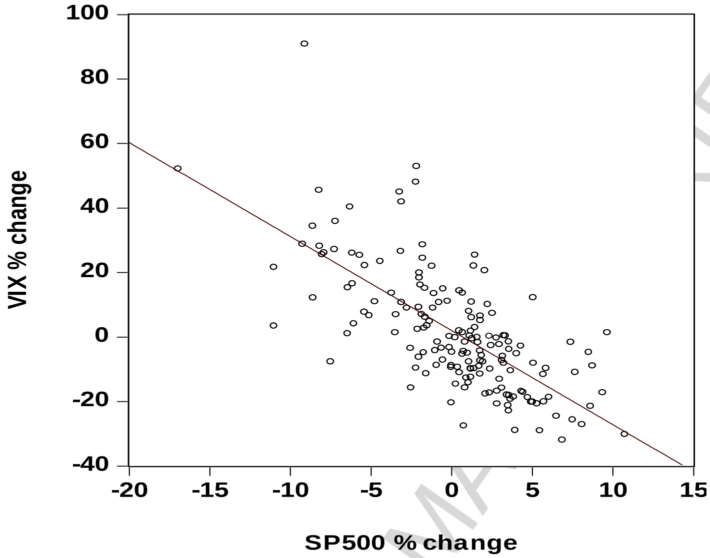

Fig. 1 Percent changes in the VIX relative to percent changes in the S&P 500 Index, monthly from 2000:01 to 2013:12.

Yet the classic article pertaining to volatility and expected returns is still the early one by French, Schwert, and Stambaugh (1987). This study presented evidence that the expected market risk premium is positively related to the predictable volatility of stock returns in a relationship that is expressed as

$$\mathrm{E}\left(R_{mt} - R_{ft}\right) \middle| \sigma_{mt}) = \alpha + B\sigma_{mt}^{p},\ p = 1, 2 \tag{7}$$

where R_{mt} is the return on a market portfolio, R_{ft} is the risk-free interest rate, σ_{mt} is an *ex ante* measure of the portfolio's standard deviation, and the square of σ_{mt} represents *ex ante* variance.

[5] VIX is an annualized implied volatility index also known as the "fear gauge" that translates to expected movements in the S&P 500 index over the upcoming 30-day period.

In summary, the literature suggests that realized volatility is the best estimator of volatility even though practical estimation problems often arise when dealing with bid-ask bounces and jumps and with various econometric testing issues. The volatility parameter, σ, can be estimated in several ways, including methods devised by Parkinson (1980), Garman and Klass (1980), and Rogers et al. (1994). The Parkinson number, for example, is an estimator of the volatility of returns presupposing that returns follow a geometric random walk. If this number is more than 1.67 times the estimated sample volatility, σ, it is an indicator of a high volatility, out-of-the-ordinary condition. [6] A related survey by Poon and Granger (2005) nevertheless found that forecasting volatility seems often more an art than a science.[7]

Given this state of affairs, what direction should future research in this area take? Few of these surveyed results appear to be directly relevant to an understanding of bubbles and crashes. That is because none of these studies are specifically designed for nor are they intended to analyze features of such extreme market events. The underlying data sets do not differentiate between stable trending markets and bubble and crash periods. Instead, the data used typically extend over (or are averaged over) all types of market periods including those containing bubbles and crashes and also those that do not. The following section briefly summarizes the theoretical approach that appears in most of the

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[6] All such measures are available on the standard Bloomberg terminal configurations (SPX Index>GV) and use information on daily trading ranges – the intraday high and low prices. Another source is the Market Data Express (MDX) of the Chicago Board Options Exchange (CBOE) which compiles option information for expiry months starting in 1990.

[7] Additional relevant studies include Canina and Figlewski (1993), Ball and Roma (1994), Mayhew (1995), Adjaoute et al. (1998), Andersen et al. (1998), Christensen and Prabhala (1998), Blair et al. (2001), Balaban et al. (2003), Hentschel (2003), Neely (2004), Brandt and Kinlay (2005), Chernov (2007), Clements and Collet (2008), Carr and Wu (2009), Wang (2009), Ederington and Guan (2010), Engstrom (2014), and Kenourgios (2014).

already extensive existing literature on bubbles and crashes and sets the stage for the new direction that we take.

3. A Survey of the Underlying Theories

Although economists have studied bubbles within the framework of business cycle analysis, they have most often attempted to place bubbles within the context of the by now well-developed random walk, efficient market hypothesis (EMH), and affiliated capital asset pricing model (CAPM) constructs. Most of the studies pertain to the market for stocks as reviewed in section 3.1. But bubbles in real estate, forex, and commodities have also been extensively analyzed and are covered in section 3.2. From the start, the inherent weakness always revolves around definitional issues: Just what is a bubble?

3.1. Stock market bubbles

Much of the theoretical literature rests primarily on the implicit supposition that investors at all times behave rationally and thus have rational expectations of underlying values. As a result, the RE approach is model-specific, with the main (random walk) assumption being that the *conditional* return expectation – and not merely the unconditional expectation – is zero. In this stochastic process, the current period's returns, X_t, are usually interpreted as the logarithm of the total payoffs including dividends and shown as $X_t = \mu + X_{t-1} + \varepsilon_t$, with $E[\varepsilon_t] = 0$ for all t and drift μ an arbitrary parameter. The conditional expectation is then shown as $E[\varepsilon_t \mid X_{t-1}] = 0$ rather than as $E[\varepsilon_t] = 0$.

Almost all of the traditional bubble models describe their structural approaches through the use of colorful adjectives such as "intrinsic," "collapsing," "exploding," "churning," and "rational" and employ an equation (or variant) of the form $P_t = E_t \sum_{i=1}^{\infty} \frac{D_{t+i}}{(1+R)^i}$ in which the expected discounted future price is assumed to have a limit of zero. If this assumption does not hold and future prices are expected to grow forever at the assumed rate of interest, a bubble term B_t that satisfies $B_t = E_t[B_{t+1}/(1+R)]$ is then added to the right-hand side of the previous equation. In a more abbreviated form, this can also be written as $B_t = \delta \cdot E_t[B_{t+1}]$, with δ the discount factor < 1. This is the core of the basic REH model for which there are an infinite number of solutions and both a deterministic and a stochastic term (even though the stochastic term is not a logical necessity).

As McQueen and Thorley (1994) write, "any price of the form, $p_t = p_t^* + b_t$, where $E_t[b_{t+1}] = (1+r_{t+1})b_t$, is a solution to the equilibrium condition…Thus, the market price can deviate from the fundamental value by a rational speculative bubble factor, b_t, if, on average, the factor grows at the required rate of return."

One problem is that the rational expectations (RE), random-walk, and CAPM literature was never originally developed for the explicit purpose of analyzing bubbles or crashes: It sprang from the optimization and equilibrium-seeking approaches that typify mainstream economic thinking. RE-based models attempt to define the environment in which there would be no bubbles and in which arbitrage – strictly defined as being a riskless and costless way to achieve profits that exceed the risk-free rate of return – can be readily implemented. [8]

[8] Shleifer and Vishny (1997) treated this subject in their classic work on the limits of arbitrage. Cochrane (2005) presents important examples of when arbitrage could not be implemented. Earlier bubble literature includes Diba and Grossman (1988a, b), Flood and Hodrick (1990), and Evans (1991). And Grossman and Stiglitz, 1980) note that markets cannot be perfectly arbitraged when information is costly and information is not perfectly reflected in prices.

Real business cycle theory, in addition, suggests that extreme market events might alternatively be explained by adjusting existing macroeconomic models by adding terms relating to stochastic random shocks. Everything from changing weather patterns to earthquakes and sunspots and to political upheavals and realignments may provide unpredictable random shocks that can be included in such models. The results will often be able to explain macroeconomic fluctuations under carefully considered hypothetical conditions even though variables for population growth rates and compositions (young/old/ethnic/cultural mix, educational attainments, etc.) and technological changes might not be included. Table 1 provides a representative list of studies on bubbles, crashes, and tests.

Malinvaud (1985) and Bénassy (1986) have alternatively provided the theoretical underpinnings for what we propose. Both take issue with the Walrasian notion of general equilibrium in which it is assumed that all exchanges occur at a single point in time, that each individual confronts the same known set of prices, and that there are no price spreads. According to Malinvaud: "[P]urchase (or sale) is the quantity actually traded, whereas demand (or supply) is the quantity that the individual would like to trade in this market."

Werner (2005, pp. 27 and 326) further explains that "[R]ationed markets are determined by quantities not prices, according to the 'short-side' principle: Whichever quantity of demand or supply is smaller will determine the outcome…there is no guarantee that equilibrium will be obtained. It would be pure chance if demand equaled supply."

Table 1. Representative studies of bubbles, crashes, and tests[a]

Models	**Approach**	**Type/comment**
	Bubbles	
Adam and Szafarz (1992)	RE	Survey
Allen, Morris, and Postlewaite (1993)	RE	Asymmetric information
Asako (1991)	RE with OLG model	Explores Japanese land bubble

		and concludes it was not always rational
Blanchard and Watson (1982)	RE with periodically collapsing bubbles	Launched series of bubble articles
Campbell (2000)		Survey
Diba and Grossman (1988)	Rational bubble component	Explosive conditional expectations
Froot and Obstfeld (1991)	Intrinsic – i.e., modified RE bubbles driven by fundamentals	Separates present value and bubble components of stock prices
Hardouvelis (1988)	Bubble premium	It's rational to stay in market if it seems highly probable that a bubble will grow
Lux and Sornette (2002)	REH	Fat tails and power laws suggest bubbles are a special case of multiplicative stochastic processes
Meese (1986)	"fundamentals are just part of the story"	Finds bubbles in fx markets
Santos and Woodford (1997)	Intertemporal competitive equilibrium	Bubbles only under special circumstances
Shiller (1981)	Price and dividend series do not grow by the same amounts – variance bounds violated	Began new branch of investigation
Tirole (1982)	RE equilibrium and OLGs	No bubbles in infinite-horizon models with finite number of agents, but bubbles with infinite number of agents (OLG)
	Crashes	
Brunnerrmeier (2001)	REH	Four different crash category models
Frankel (2008)	Adaptive expectations	Rational and naïve investors interact
Lee (1998)	Information avalanches	Agents learn from the action of others
Shiller et al. (1996)	Survey of changes in expectations	Nikkei crash due to changes in price expectations
	Tests	
Caginalp et al. (2000)	Price momentum	Behavioral finance/experimental
Evans (1991)	Econometric weaknesses	Survey/tests unable to detect a class of rational bubbles
Flood and Hodrick (1990)	Self-fulfilling prophecy (i.e., sunspots) within an RE framework	Survey/ tests are misspecified
McQueen and Thorley (1994)	Rational speculative bubbles and duration of runs	High probability of returns compensates for probability of a crash
West (1987)	Hausman spec test compares expected PDV parameters	Bubbles exist
West (1988)	Standard models do not explain volatility	Volatility survey

[a] A similar but more detailed table appears in the appendix to Taipalus (2012).

Source: Vogel (2010).

As a result, some agents experience rationing. And thus, in addition to price signals, there are quantity signals. Demands and supplies must then be substantially accommodated and modified on the basis of such signals.

In practice this suggests that in bubbles the rationed buyer readily meets asking prices but remains unfulfilled. Then, the only way to obtain the full quantity desired is to bid prices even higher, enticing potential sellers with a better deal. In so doing, the buyer thereby extends prices beyond the previously established boundaries and thus also increases the variance of prices. In crashes the variance of prices and returns is similarly expanded, only now with returns to the downside as rationed sellers hit every bid and then sell more at even lower prices so as to trade the quantity, à la Malinvaud, that they would like to trade.

From this perspective, the primary motivating force in both bubbles and crashes is not the price paid (or in crashes price received on sale) but instead the need – on a time-sensitive basis – to adjust quantity held. These needs will in the real world be behaviorally rooted in portfolio manager concerns that relative underperformance as compared to that of the peer group will result in withdrawal of assets managed, in reductions of fees and bonuses, and ultimately in diminishment of job security. Plainly, as bubbles or crashes evolve, it is this behaviorally rational response – not at all captured by traditional economic pricing models – that makes prices diverge (sometimes greatly) from so-called "fundamental" asset values.

Under such circumstances, considerations of price come to be of secondary concern relative to those of quantities presently held or not held. Indeed, the more urgent is the transactionally–expressed need to adjust quantity, the greater is the resulting price concession (cost or loss). The degree of urgency in execution may then often also be reflected in price discontinuities (or jumps) that appear on a chart as gaps – with, say, a trade at $12.60 followed the next instant by a trade at $14.10 (or, in a crash, $9.75).[9]

At many interim prices there may thus be no trades at all, which begs questions such as: What are the unseen transaction volumes and how deep is the market? What kind of "equilibrium" can this be if it is only a trade that clears the market of a thousand shares though a fund has, hovering in the background, another ten million for sale that are not known to the buyer of the first thousand, or where another investor has been trying to execute a fill-or-kill limit purchase order and failed to do so?[10]

3.2. Bubbles in other asset classes

Bubbles have throughout history appeared in a wide variety of assets not limited to only share prices. There of course was a famously speculative run on tulips beginning as far back as 1634, and bubbles have also appeared quite frequently in real estate and commodities. Garber (1989 and 2000) and White (1990) provide excellent starting points and Kindleberger (1989, 1996) is the classic survey of the field. Allen and Gale (2003) examine the interlinkages between asset price bubbles and stock markets. Other important asset classes also include fixed income and foreign exchange.

Real estate bubbles, which arguably touch more lives than do bubbles of any other type, have notably occurred in the mid-1920s in Florida, in Japan in the 1980s, and in several major countries between 1997 and 2007 (and accelerating noticeably in 2001).[11] As in stocks, these bubbles are also always fueled by the lowering of credit standards in times when monetary policies are relatively loose. Malpezzi and Wachter (2005), for

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[9] See Zaevski, Kim, and Fabozzi (2014).

[10] See Vogel (2010, p. 191).

[11] For 2001 to 2004, housing prices increased in the United States by 29%, the U.K., by 50%, Canada by 31%, France by 48%, Spain by 63%, and Hong Kong, 27%, according to BIS data appearing in Hilsenrath and Barta (2005).

example, developed a model of real estate cycles. And Herring and Wachter (2003) discuss real estate market bubbles more generally with regard to the role of banks.

In these types of models the interest rate discount factor is usually expressed in the form of cap (or capitalization) rates, which move directionally through the cycle in the same way that the equity risk premium does for stocks.[12] Some further examples: Garino and Sarno (2004) find evidence for the existence of bubbles in U.K. house prices by using an overlapping-generations model. And Gan (2007) provides evidence concerning the adverse effect on lending channels when banks have high real estate asset exposure going into a crash.

Bubbles in foreign exchange have also attracted much research attention. Meese (1986) and Woo (1987), for instance, found mixed evidence of forex bubbles, whereas Wu (1995) found no significant evidence of such bubbles.

Commodity bubbles such as those experienced most recently in gold and oil are of great further interest. Blanchard (1979), though among the first to present a gold bubble model in a rational expectations framework, was also among the first to admit that detecting a bubble's presence or rejecting its existence was difficult. Johansen and Sornette (1999) found speculative bubbles in gold that led up to crashes. And Miller and Ratti (2009) write about the links between oil and stock market bubbles.

For the important commodity price spike of 2007-2008, Gutierrez (2013), for example, found evidence of bubbles in some major agricultural goods. And Etienne et al

[12] Cap rates are defined as the cash flow rate of return on investment property based on the expected income that the property will generate (i.e., annual net operating income divided by cost or value)..

(2014) found that over the long term, agricultural markets experienced episodes of bubble behavior, but for only relatively brief periods.

4. Elasticity of variance measures

The stumbling block in differentiating extreme event periods from other quieter periods always appears when attempting to define and measure bubbles and crashes via the highly model-sensitive rational expectations approach. Assuming that variance bounds can be statistically specified, it might then be argued that any violations of such bounds are caused by unexpected large deviations (i.e., shocks) from the RE-model conditions that preclude bubbles. Such deviations are, from the RE perspective, usually macroeconomic in nature – and the resulting violations are indicators or symptoms of something gone greatly awry and far-strayed from neoclassical theoretical prescriptions.

We instead propose that the aforementioned notions of short-side rationing can lead to the development of a new method for defining and then identifying, measuring, and analyzing bubbles and crashes. In our approach, these extreme events are always characterized by increasingly frequent and relatively large violations of historical price variance bounds that are consistent with the time-scale of the event being studied.[13] In our view, those violations provide (and are) the signature features – i.e., exponential (and/or parabolic) price-hugging curves – that are seen in all bubbles and crashes.

In normally trending markets, there would rarely be a need for variance bounds to be broken as "the trend is your friend" and the standard price and dividend discount models predominate in decisions to buy or sell at or near the most recent prices. The few

[13] Variance bounds have been explored in LeRoy and Porter (1981), Shiller (1981), Kleidon (1986), Smant (2003), Engel (2005), and Lansing (2010) among others – and with different authors modeling the variance of a stock's price, $\sigma^2 = Var\ (p_t - p_{t-1})$ and coping with issues of series stationarity differently.

instances in which bounds might be broken are so infrequent and/or of such small magnitude that the effect on a period's average variance would be minimal and hardly noticeable. Under such relatively serene conditions, investors have plenty of time to be "rational" and carefully analytical, to fully weigh potential risks, to meticulously study balance sheets and income statements and growth prospects, to plow through dense analysts' reports and call on company managements and industry experts, and to earnestly deliberate with portfolio selection committees. Even so, traders will still need to adjust quantities not only relative to their own prior positions (and to align with their better-performing peer group's portfolios), but also in reaction to the *market's reaction* to news relative to prior expectations.[14] It is rational to do so and irresponsible from a fiduciary standpoint not to do so.

The situation is different, however, in the case of bubbles and crashes. In these there is by definition an element of pressure and urgency for the desired quantities to be accumulated or disposed and there is much less time available for deep analysis and lengthy deliberations. As a result, previous price bounds *must* be broken as a growing proportion of investors and speculators would likely become progressively less price-sensitive and decide to add to quantities held in bubbles and to reduce holdings and/or add to negative quantities (i.e., short positions) in crashes. In a bubble, the rush is to convert from cash (risk-on and flight from quality) just as in a crash the impulse is to convert to cash (risk-off and flight to quality). In either circumstance, herd psychology and emotionally-driven environmental considerations – including comparative performance rankings and the relationship to asset management fees, job security, bonuses, and bragging rights – are the motivating factors.

An illustration of how variances and price percent changes in the S&P 500 relate in terms of gains and losses is shown in Figure 2. It can be visually determined that larger percent gains or losses are associated with larger variances.

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[14] See Roll (1984, 1988), Cutler et al. (1989), and Joulin et al. (2008).

One way this relationship can be portrayed is as an elasticity of variance with respect to equity risk premium (ERP) estimates (or, with near zero interest rates, perhaps a measure of yield spreads or credit default swap prices). The study of ERPs is itself a large topic, but for general purposes it can be taken as being the expected or required return above the contemporaneous risk-free rate on treasury bills. The elasticity estimates are arithmetically determined in the usual way, which is:

$$\varepsilon_{erp} = \frac{\frac{\Delta\,\text{var}}{\text{var}}}{\frac{\Delta\,\text{erp}}{\text{erp}}} = \frac{\text{erp}}{\text{var}} * \frac{\Delta\,\text{var}}{\Delta\,\text{erp}} \tag{8}$$

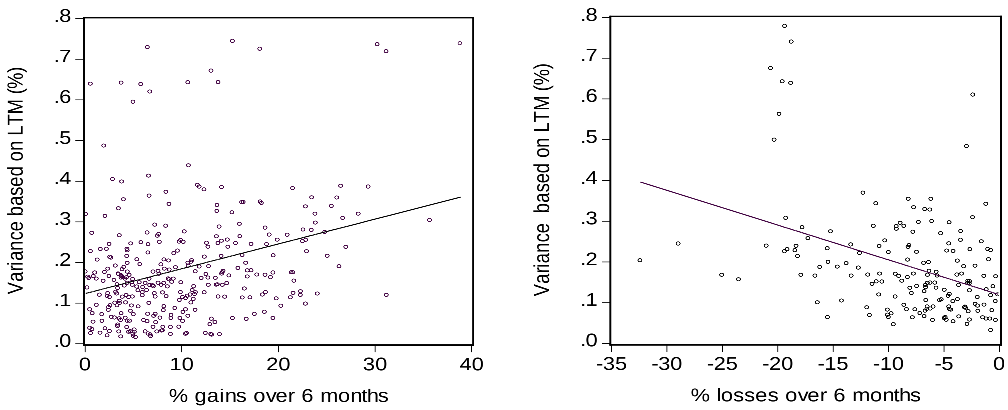
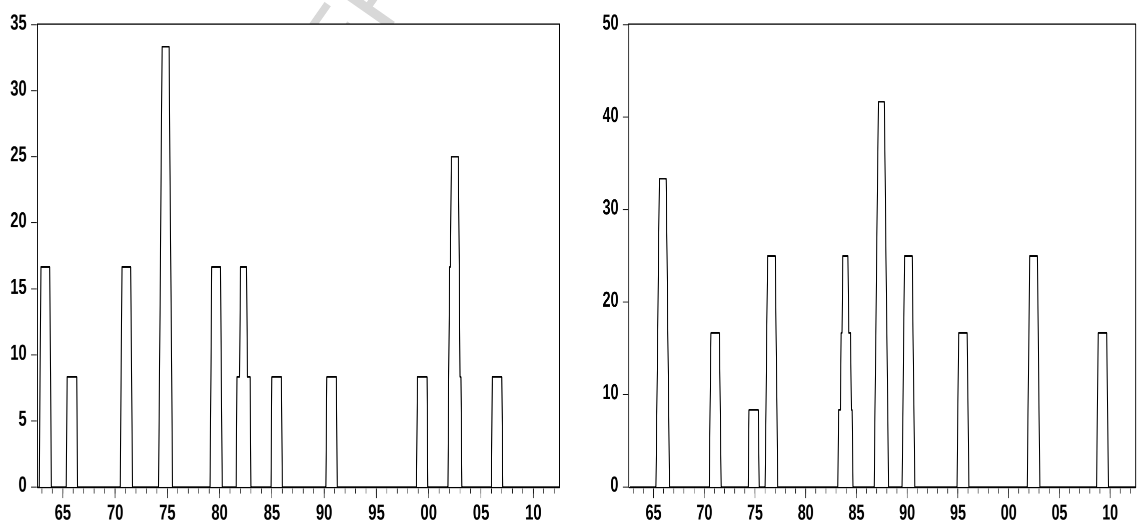


Fig. 2. Variance versus price change percentages: An example. The panels show that variance is related to the size of gains or losses as measured in percentage changes over six months. Gains (left) and Losses, S&P 500 Index, 1960:01 to 2005:12, Monthly Rolling Index Percentage Change Measured Over Closing Prices Six Months Prior, With Estimated Variance In Percent Based On Rolling Last Twelve Months Data. *Source:* Vogel (2010, p. xxi).

The key aspect is that large *price* changes – up in bubbles and down in crashes – are necessary but not sufficient conditions for defining the extreme events. The EOV definition of a bubble or crash hinges instead on the *elasticity* of variance approaching the limit of infinity. The main reason for this is that if price variance remains within recent boundaries, there is no way for the quantities that are respectively desired and demanded in a bubble or not desired and undemanded or supplied in a crash to be rapidly accumulated or distributed within typical performance measurement and benchmarking time periods. Extreme event periods are instead characterized by the forced compression of decision-making time horizons for every market participant and asset class.

Such EOV-defined events can be econometrically estimated by fitting the variance elasticities for sequential sample periods to a simple exponential curve, $y = e^{kt+u_t}$, that extends toward infinity. In this simple uncluttered model, t is time and the u_t represent the regression's error terms. A bubble or crash will then be discernable whenever the estimated parameter, k, is significantly different from the null hypothesis of zero at the chosen significance or p-value.[15]

All of this links directly to and reflects the aforementioned short-side rationed markets approach. The results of this curve-fitting, as calculated using EViews programming is shown in Figure 3, with bubbles on the left-hand panel and crashes in the other.



[15] As the elasticities will rise toward infinity in both types of events, the distinguishing characteristic of a bubble is that the net arithmetic sign of the sequential sample elasticity is negative (i.e., a positive change divided by a negative change has a net negative sign).

Bubbles **Crashes**

Fig. 3. EOV bubbles and crashes, (*p*-values at 0.15), 1962:08 to 2014:06.

Monthly data for ERPs prior to 2006 is based on Ibbotson's *Stocks, Bills, Bonds, & Inflation* annual publications. Later ERPs use S&P 500 monthly returns minus 3-month treasury bill prices. Monthly variances are based on same month's average daily returns, usually 20 or 21 days per month. Although the Crash of '87 was exceptionally brief in duration, its intensity is still prominently seen in the right-hand panel.

5. Volatility metrics

Classical volatility estimators, for example, a high Parkinson number, can provide important numerical indicators of markets that are fast-moving, risky, and likely to be in a bubble or crash phase. Such high volatility is always a marker of extreme market conditions because short-side rationed investors are forced to reduce their investment decision time horizons even while overall investor diversity of opinion disappears: Both decision-making time horizons and diversity of opinion approach a limit of zero with the time horizon correlation approaching 1.0.

5.1 New views

The traditionally estimated historical volatility of the S&P 500 Index is sequentially compared in Figure 4 to estimated implied volatility for this index taken over 12 months (left) and 3 months (right) for at-the-money call options of the same periods respectively.[16]

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[16] To keep the exposition simple, these same option characteristics are used in all of the material presented below.

The series start in 2004:09 because that is the furthest into the past that the Bloomberg-generated data for implied volatility extends.[17] The drawback of having no data on implied volatility going back into the 1990s is that none of the significant market history of that time, including the technology-stock boom, the meltdown of the Long Term Capital Management firm, and the financial stresses created by the Russian and Asian currency crises are captured. Similar data can be generated for put options and/or for options in or out of the money, or for volatilities estimated using models other than the classical. All such variations are fodder for future research.

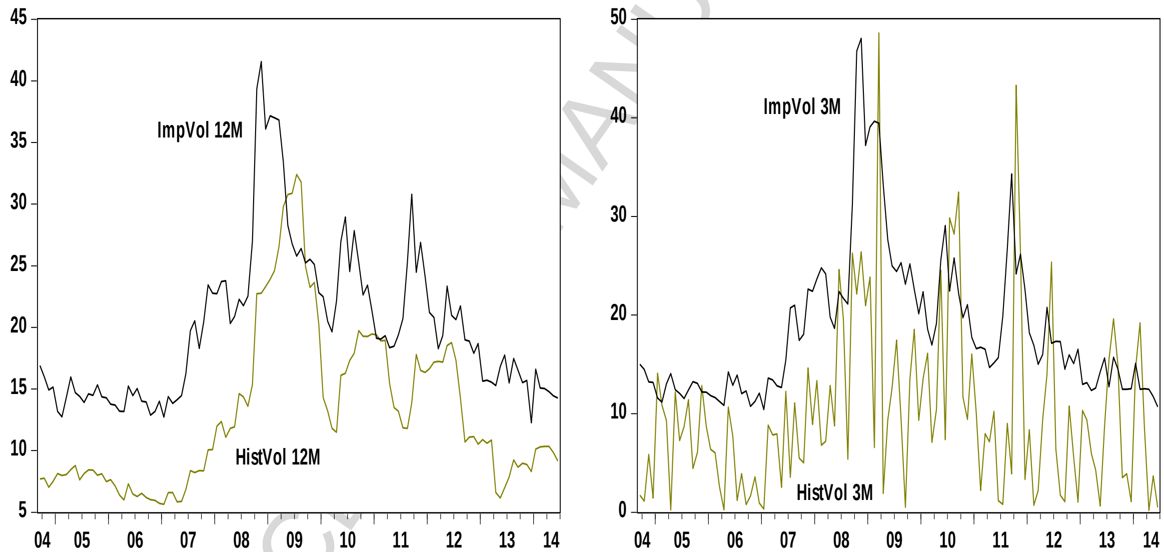

Fig. 4. Estimated historical backwards realized and implied at-the-money call option volatility time series, 12 months (left) and 3 months, 2004:09 to 2014:06.

Still, there already is enough material to see that important turning points first appeared in 2007 when the potential for systemic financial failures began to be recognized by the market (and previously established variance bounds began to be exceeded). As the market's liquidity-related problems subsequently intensified, it is evident that implied volatility was anticipatory, soaring in both the left and right panels of

[17] Bloomberg indicates that it may eventually provide data prior to 2004:09.

Figure 4 to peaks in mid-2008. This indicates that implied volatility tends to rise earlier and faster than historic volatility and that it tends to overshoot the historic series.

Figure 5 provides another perspective of the same basic data, dividing implied by historical to derive a ratio of one series to another. In dividing one series by the other, a "normalized" relationship is extracted from the raw data. This makes it not only easier to interpret the underlying relationships but it also generates a pure number that allows comparisons no matter what the prevailing interest rates and/or S&P 500 p/e ratios are at any point in time. The mean ratio for this sample period is 1.71 for the 12 months' comparisons (left) and 6.04 for the 3 months' (right). The ratio for 3 months, by dint of lesser smoothing in its underlying calculation, is clearly much more sensitive and volatile, even though the story it tells is much the same for most of the period covered.

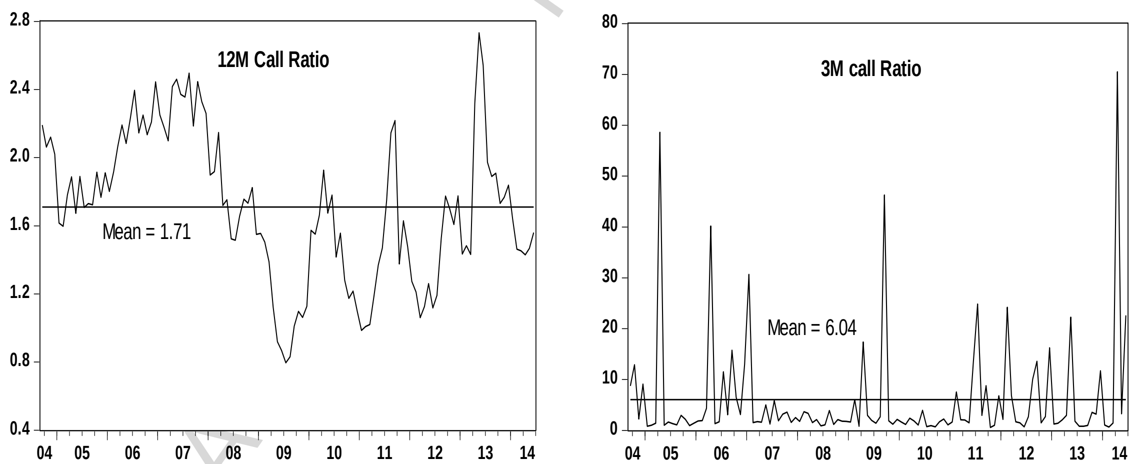

Fig. 5. Estimated historical and implied at-the-money call option volatility time series, implied divided by historical, 12 months (left) and 3 months, 2004:09 to 2014:06.

The basic revealed characteristic features – substantial but relatively brief variations that travel well above and below a mean value – that are evident in Figures 4 and 5 are in support of the empirical notion that underlying contracts tend toward a mean volatility.

Although there might be many ways to statistically parse such data so as to further validate this statement, the most straightforward is to count the number of times that the volatility ratio has been above or below the mean. For the 12-month comparison (left-hand panel of Figure 5), the ratio has been above the mean of 1.71 for a total of 60 out of 118 months, or about half the time (50.8 percent). But for the jagged 3-month comparison, the ratio has only been above its mean of 6.04 for 28 out of 118 months, or 23.7 percent of the time.

While this type of measurement obviously does not provide irrefutable evidence that an underlying contract is likely to have a long-term mean volatility to which there is reversion, it might still be contrarily argued that the 3-month volatility ratio simply does not cover a period that is sufficiently long for such reversion to occur. In this respect, the 12-month ratio is to be preferred. Porterba and Summers (1988) suggested that stock returns are indeed mean-reverting, and so it appears that volatility is too.

There are at least two probable reasons for this. The first is that most options trading volume is for contracts that are within two years of expiration. This time-limitation factor is not present in the multi-year long-horizon common stock investing strategies and structures used by pension plans and also by many mutual funds and individual investors. Price-based metrics can wander widely depending on the popularity of stocks versus bonds versus commodities versus real estate versus cash. There have been long stretches in history when stocks were either highly in or out of favor (the 1990s and the 1950s, respectively) with investors.

But the most important reason is that were volatility to remain very high for an extended period of more than a few months, the market would begin to falter and then – as in the crash of October 1987, the Long-Term Capital Management debacle of 1998, or the ‘flash’ crash of May 2010 – veer towards dysfunctionality: The wheels would begin to come off of the truck. The market would eventually have to shut down.

Sustained high volatility imposes such high costs and penalties – for hedging, for incorrectly positioning against the increasingly violent directional movements, and for the toll it takes on psychology and confidence and capital resources – that it must subside (i.e., revert to a mean) sooner than later.[18] Yet that is not the case with p/e ratios or the prices paid for $1 of dividends or other such measures (Prechter and Parker, 2007). High volatility – which always accompanies bubbles and crashes – thus contains the seeds of its own destruction.

5.2 *The Extreme Events Line***.**

Engle (1982) first found that volatility tends to cluster, which is another way of saying that volatility also tends to exhibit serial correlation and that volatility in the present period tends to depend on the previous period's volatility. This suggests that it might be useful to examine clusters of volatility and to see if such clusters can provide additional insights about what happens in bubbles and crashes.

A scatter plot of implied versus past realized historical volatility appears in Figure 6, which introduces the notion of an extreme events line (EEL). The EEL provides a new and readily comprehensible display of clusters that illustrates relationships between implied and historical volatilities as they specifically pertain to bubbles and crashes. Although it is merely a variation on the volatility ratios discussed earlier and uses the same underlying data, the EEL functions as a simple market X-ray that allows volatility relationships to be compared in both extreme and non-extreme market periods. It is also minimally arbitrary in terms of procedural implementation.

[18] Capital resources are stretched because margin requirements are generally raised and spreading distrust of financial counterparty viability necessitates that loans are only extended if more collateral is placed at risk.

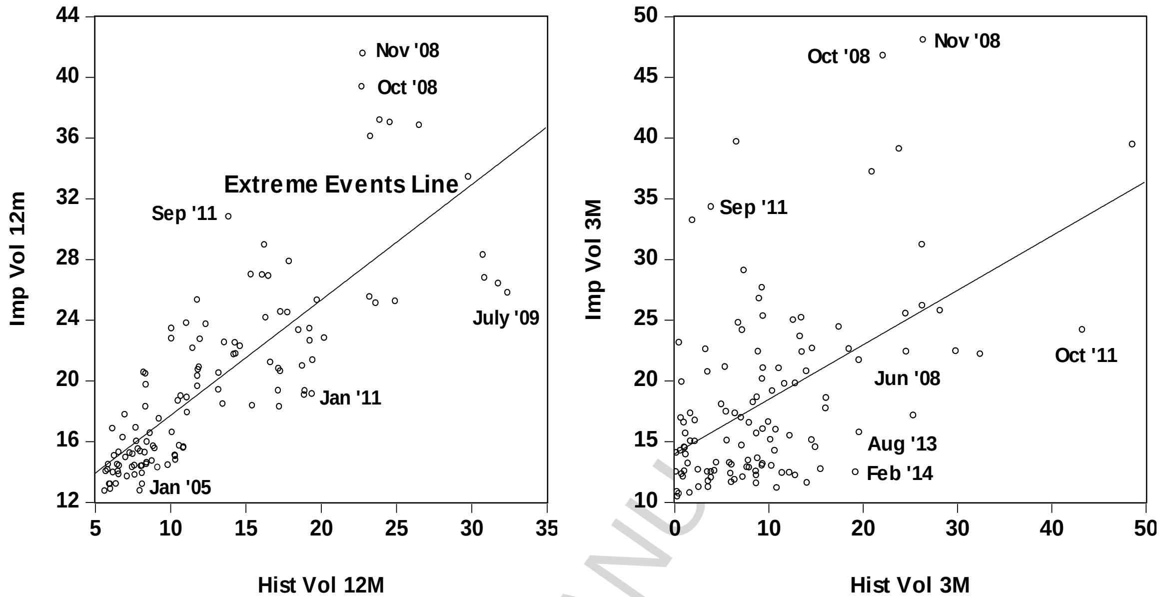

Fig. 6. Implied volatility versus past realized historical volatility, 12 months (left) and 3 months, 2004:09 to 2014:06. The tranquility zone is the area below the EEL, which is the boundary between extreme market event volatility and non-extreme market volatility. For the 12 months data, the estimated OLS equation (including 118 observations) is IV= 10.13 + 0.76HV (and *p*-values of 0.0), and for 3 months data it is IV=14.01 + 0.45HV (and *p*-values of 0.0). On the left-hand panel, October and November 2008 and September 2011 are more than 2 standard errors above the EEL. The s.e. of regression is 3.79 for the left panel and 6.53 for the right.

Although various option times to expiry and different types of options (e.g., puts versus calls, European versus American, out-of-the-money versus in-the-money) will generate somewhat different EEL lines, none of this changes the basic concept; in bubble and crash episodes, the contemporaneous ratio of implied versus historical volatility spikes dramatically above the ordinary least squares regression line – the EEL – that has been estimated by inclusion of data points extending over all types of (extreme and non-extreme) market environments.

Above the EEL are observations that can only be found when markets are abnormally volatile. For instance, with the 12-month data, the two highest observations at implied volatility of just above and below 40 are for October and November 2008 – times of deep-seated anxiety as a result of the late September collapse of the Lehman investment bank and of frozen credit markets in which even high-grade commercial paper (e.g., General Electric's) could not be rolled over. The observation for September 2011 also recalls a highly volatile month.

But the much less volatile-than-average month of July of 2009 (left-panel) appears well below the line. Similarly, on the 3-month right-panel data, the relatively tranquil December 2010 also appears below the EEL. The space below the EEL thus depicts a zone of tranquility, with the extreme events known as bubbles and crashes appearing only above the EEL.

Why are most data points above the EEL crash-related? The reason is that bubbles can gestate, grow, or extend over long periods of time, sometimes over many years. Following a long down period, it typically takes a while before the fears of investors and speculators have been sufficiently dissipated and abated and confidence is restored. A bubble might then eventually follow if monetary conditions are easy, important new technologies are introduced, and other positive social-mood factors (e.g., no wars) are present and thereby enable the revival of speculative interest. Carried too far, this interest can then turn into bravado and, finally, into overwhelming exuberance. Bubbles therefore usually evolve much more slowly than crashes and the variance increases that are most noticeable appear closer to a bubble's end than at its beginning.

But crashes are relatively rapid affairs because fear of losses crystallizes much faster than does the confidence that must gel before bubbles begin to attract widespread attention. Accordingly, more dots above the EEL should be expected for crashes than for bubbles, although the degree of difference would depend on many factors, the most important of which is the time scale used for measurement.

If extreme events are represented by dots above the EEL, then there are at least two remaining issues that need to be addressed. The first one is how far above the EEL is considered extreme? This will be a matter for further research, but for now we suggest – while assuming an approximately normal distribution – that any observations be defined as extreme if they are above the EEL by more than two standard errors.

Another heretofore unexplored issue for volatility studies pertaining to bubbles and crashes is that, especially in crashes, correlations go to one. Because implied correlation (Buraschi et al. 2010), is closely related to implied volatility, a special program (in EViews) was written for illustration in Figure 7.[19] From this it can be seen that in rising markets, positive return correlations (UPCORR) rise along with the volatility and that returns in a crash also begin to correlate on the downside (DOWNCORR). As Wilmott (2001, p. 340) explains, "a crash isn't just a rise in volatility…During a crash, all assets fall together. There is no such thing as a crash where half the stocks fall and the rest stay put."[20]

[19] In July 2009 the Chicago Board Options Exchange began disseminating daily values for the CBOE S&P 500 Implied Correlation Index, with historical values back to 2007.

[20] The implication for both bubbles and crashes, as Muellbauer and Portes (1978) recognize, is that "[A]n agent who is rationed as a buyer or seller on one market and cannot transact his notional excess demand…will in general alter his behaviour on other markets."

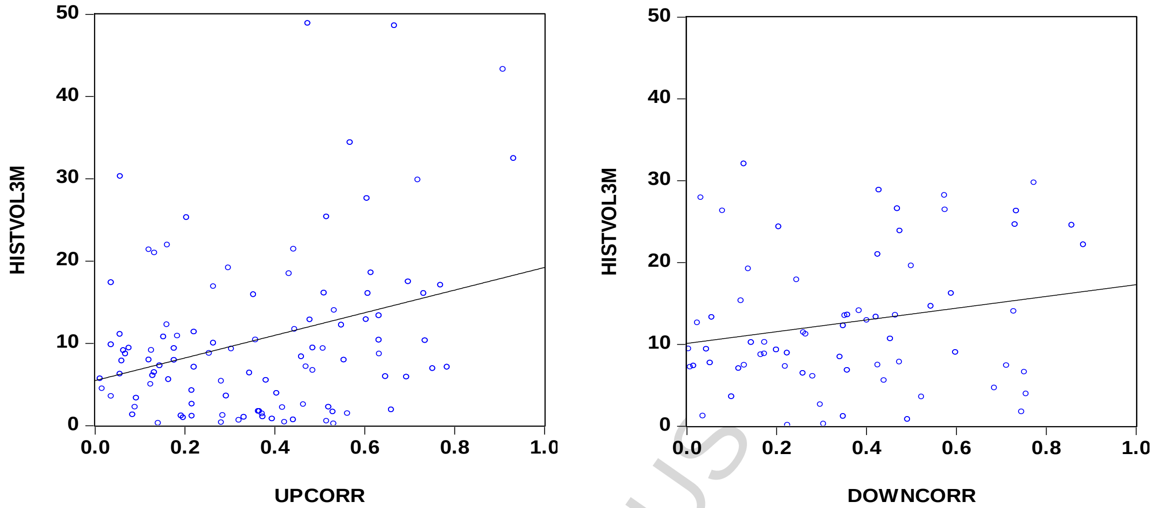

Fig. 7. Historical volatility, based on 3 month at-the-money call options for the S&P 500 related to positive and negative return correlations within the 500 stocks in the index, 1999:01 to 2014:06. The higher the historical volatility, the higher the correlation for both bubbles (left panel) and crashes (right panel),

In other words, in a crash there is nowhere to hide, a crash characteristic that becomes evident in Figure 8. Here it can be seen that even on a monthly basis, at the lows of 2008, the percentage of all S&P 500 stocks up was just below 40%. At intra-month crash lows it is indeed not unusual for the percentage of stocks up to approach zero.

5.3 *Storm cats*

Because crashes tend to be much more distinctive in their speed and intensity and

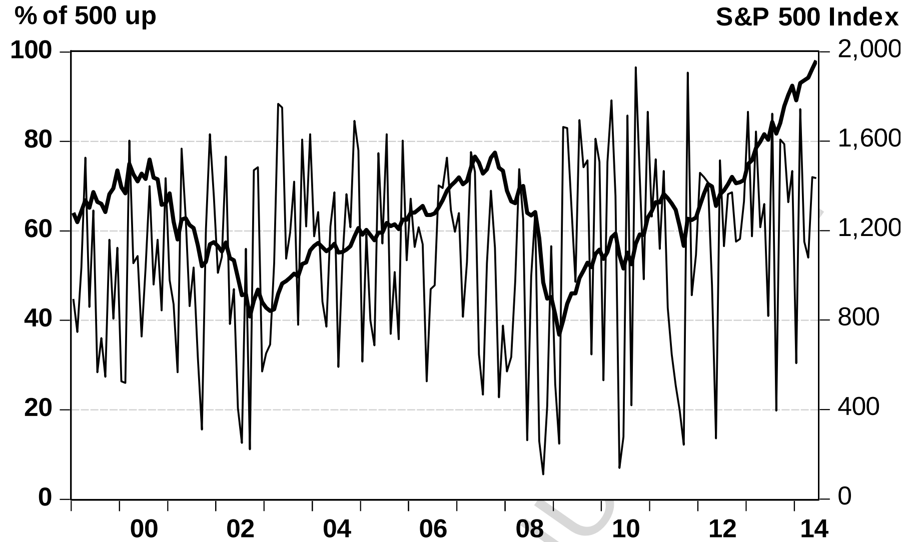

Fig. 8. In a bear phase, there's no place to hide: Percentage of S&P 500 stocks rising in the month (unsmoothed raw data) and monthly S&P 500 index (right-hand scale), January 1999 to June 2014. *Source data*: Standard & Poor's.

beginnings and endings than are bubbles, it is also possible to introduce a crash scale that is somewhat akin to the Saffir-Simpson hurricane classification system which evaluates storms on a weakest to strongest scale of 1 to 5 based on wind speeds, potential property damage, and flooding. [21]

In the last fifty years, the most important crash episode for a broad-based index began at the S&P peak at 336.77 of August 25, 1987 and ended on October 19, 1987 – 38 trading days later – while generating an average per day loss of 2.95 points and 0.87 percent. The most dramatic and climactic part of this episode occurred on October 19, 1987, when the S&P 500 fell by a record 22.61 percent in one day – the largest price

[21] The scale introduced here is arithmetic not logarithmic as is the Richter-Gutenberg scale used to assess the strength of earthquakes (and in which a 5.0 quake is ten times more powerful than a 4.0 quake).

change over the briefest time on record.[22] With the subsequent implementation of stock exchange “circuit breaker” rules the likelihood of this ever happening again is remote.

The very intensity and brevity of this crash, however, allows it to be used as a benchmark reference against which to arithmetically scale other crash episodes. Ten crashes in the S&P 500 and six in the NASDAQ are compared in Figure 9 on the basis of average percentage losses per day taken as a percent of the S&P 500 average loss per day (0.87 percent) seen in 1987. The underlying base data appear in Table 2

In this representation, crashes broadly fall into three categories. Several S&P episodes are below 25 percent of the intensity of the Crash of ’87, and these might be labeled as category 1 storms. But given that shares of companies in the NASDAQ are generally less seasoned and financially secure than those in the S&P, it is not surprising to find several NASDAQ episodes that are above 25 percent and up to around 60 percent of the intensity of the 1987 S&P experience. These might be labeled as category 2 events. Any crashes that are above 60 percent of the intensity of the Crash of ’87 might be labeled as category 3s. They are so potentially catastrophic (“cat3cats” or C3Cs) as to be able to not only wreck a market’s infrastructure but also a nation’s economy.

[22] In the crash of 1929, the Dow-Jones Industrial average fell 11.73 percent on October 29, and 12.82 percent on October 28, which was the largest daily percentage price decline on record for a major index until October 19, 1987.

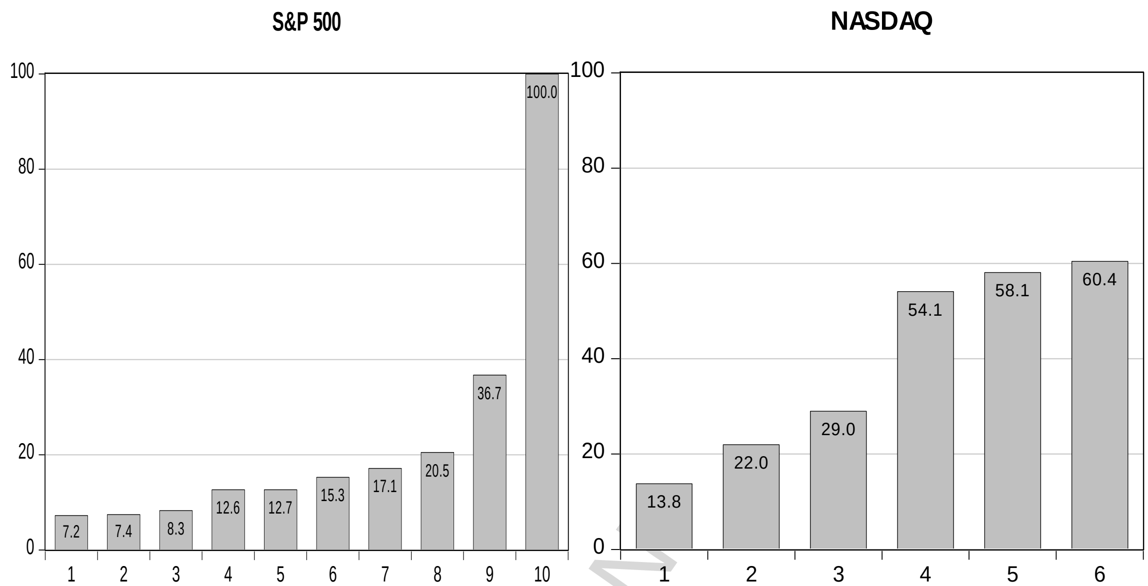

Fig. 9. Crash intensity comparisons for the S&P 500 and NASDAQ, 1962 -2014. Bars show the relative average daily percent decline in each episode as a percent of the average daily percent decline in the Crash of ’87. Bar number 10 in the S&P (left panel) is the Crash of ’87. For the S&P, bars 1 through 9 began: 1) 28-Nov-80; 2) 31-Dec-76; 3) 10-Oct-83; 4) 11-Jan-73; 5) 24-Mar-00; 6) 14-May-69; 7) 9-Feb-66; 8) 9-Oct-07; 9) 16-Jul-90. For the NASDAQ, bars 1 through 6 began: 1) 10-Mar-00; 2)27-Apr-94; 3) 31-Oct-07; 4) 17-Jul-90; 5) 27-Aug-87; 6) 21-Jul-98. Underlying data appear in Table 2.

Table 2.

Crash or collapse? Important peak-to-trough moves (>10%), daily closing prices, S&P 500, 1962-2010 and NASDAQ, 1984 – 2010.

				Total		# trading days	Avg loss per day	
	Peak		Trough	Points lost	% lost		points	%
S&P 500								
9-Feb-66	94.10	29-Aug-66	74.53	19.57	20.8	139	0.14	0.15
14-May-69	106.16	26-May-70	69.29	36.87	34.7	260	0.14	0.13
11-Jan-73	120.24	3-Oct-74	62.28	57.96	48.2	436	0.13	0.11
31-Dec-76	107.46	6-Mar-78	86.90	20.56	19.1	296	0.07	0.06
28-Nov-80	140.52	12-Aug-82	102.42	38.10	27.1	430	0.09	0.06
10-Oct-83	172.65	24-Jul-84	147.82	24.83	14.4	199	0.12	0.07
25-Aug-87	336.77	19-Oct-87	224.80	111.97	33.2	38	2.95	0.87
16-Jul-90	368.95	11-Oct-90	295.50	73.45	19.9	62	1.18	0.32
24-Mar-00	1527.46	4-Apr-01[a]	1103.25	424.21	27.8	251	1.69	0.11
9-Oct-07	1565.15	10-Mar-08	1273.37	291.78	18.6	104	2.81	0.18
				avg[b]	**26.4**	**222**	**0.93**	**0.21**
Alternatives:								
24-Mar-00	1527.46	11-Mar-03	800.73	726.73	47.6	741	0.98	0.06
19-May-08	1426.63	20-Nov-08	752.44	674.19	47.3	130	5.19	0.36
19-May-08	1426.63	9-Mar-09	676.53	750.1	52.6	202	3.71	0.26
NASDAQ								
27-Aug-87	455.8	7-Dec-87	293.70	162.10	35.6	70	2.32	0.51
17-Jul-90	469.5	17-Oct-90	325.10	144.40	30.8	65	2.22	0.47
27-Apr-94	800.39	27-Jun-94	694.16	106.23	13.3	69	1.54	0.19
21-Jul-98	2018.46	8-Oct-98	1420.94	597.52	29.6	56	10.67	0.53
10-Mar-00	5048.62	9-Oct-02	1114.11	3934.51	77.9	647	6.08	0.12
31-Oct-07	2859.12	17-Mar-08	2177.01	682.11	23.9	94	7.26	0.25
				avg[b]	**35.2**	**167**	**5.01**	**0.35**
Alternatives:								
5-Jun-08	2549.94	20-Nov-08	1316.12	1233.82	48.4	118	10.46	0.41
5-Jun-08	2549.94	9-Mar-09	1268.64	1281.3	50.2	190	6.74	0.26

[a] The entire decline might also be measured to the post-terrorist attack low of 965.80 on 21 September 2001, which would be 36.8%. Thus the alternative row below

[b] This average excludes the last row alternatives.

Source: Calculations based on Yahoo.com data and Vogel (2010, p. 245).

6. Conclusions

Development of numerous derivatives-based securities products has expedited and energized the emergence of volatility as a distinct asset class in itself and made volatility an increasingly important feature of modern algorithmically-traded markets. It is now widely accepted that volatility tends to cluster and to mean-revert and that implied volatility deviates from the realized. Significant volatility changes also tend to have relatively brief life spans measured in months or weeks and days but not necessarily in years.

Much of the literature on volatility has been devoted to examination of econometric issues that include the estimation of biases and the viability of using historical volatility as a predictor of implied future volatility. Studies have also suggested that idiosyncratic volatility (a measure of market efficiency) has over the last decades been rising even though volatility – at least leading up to the financial meltdown that began in 2007 and prior to the widespread implementation of high-frequency trading – has probably remained about the same for the overall market as taken for periods of more than a year or two.[23] The rise of idiosyncratic volatility implies that correlation among securities within a broad-based index such as the S&P 500 is higher than previously and that more securities and/or asset classes are now needed to properly diversify portfolios according to the tenets of the modern portfolio theory and its numerous variants.

However, little of the literature appears to have either implicitly or explicitly studied volatility as it relates directly to bubbles and crashes.[24] This is not surprising given the absence of a commonly accepted statistical definition of the extreme events that are rather informally and typically after-the-fact described as having been bubbles and crashes. It is also an indirect artifact of the thus far ineffectual attempts to shoehorn, contort, and otherwise mold extreme event characteristics into the efficient market and CAPM structures. These structures were never expressly designed or intended to model bubbles and crashes and they are theoretically incompatible with the notion that bubbles and crashes even exist.

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[23] Bekaert et al. (2012).

[24] Topol (1991) and Jarrow et al. (2011) are the exceptions.

The approach presented is based on the short-side rationed principle and the idea that bubbles and crashes can be statistically defined, modeled, and measured on any time scale by looking for the elasticity of variance with respect to equity risk premiums (and/or credit spreads or cap rates in real estate) to move exponentially toward infinity and thereby break previously established variance bounds. The method is applicable to all asset classes including real properties, bonds, forex, and commodities because for bubbles and crashes in those classes, credit spreads and cap rates respond directionally in the same way that equity risk premiums do in stock market bubbles and crashes.

A rising or falling price is only a necessary but not sufficient condition for an extreme event to occur. The EOV approach reflects the quantity-related urgency to fulfill rationed demand or supply and it thereby captures the essence of asset price behavior in bubbles and crashes: The empiricist needs only to specify the probability level (significance as measured by the *p*-value) at which the analysis is being made. With this, there is then no guessing as to which variables and which type of specific model might potentially describe the best fit. Here, the *market itself tells us directly* when something unusual is probably happening.

Simple comparisons between historical and implied volatility time series suggest that implied leads historical and that a rise in the implied foreshadows a bubble or crash condition. There is reason to expect that, in using the proposed short-side rationed-EOV approach – in which considerations of price take a backseat to considerations of quantities held or not held – a relationship of this kind can be found for and applied to extreme market events across different time scales and different asset classes, including bonds, commodities, and real estate.

It has, in addition, been demonstrated that as such extreme market events evolve, directional correlation – be it either up or down – among the individual components of a broadly-based stock index such as the S&P 500 increases notably and that the correlation of returns rises along with the volatility. A plausible reason for this is that increasing volatility leads to more emotionally-based decision-making which then heightens the appeal of and the need for herding (and rationed short-sided) behavior.

Under such time-pressured circumstances, opinions become increasingly undiversified and there is a contemporaneous collective reduction in decision-making time horizons. An extreme event line (EEL) and also a zone of "tranquility" provide another way to conveniently represent these ideas. But, overall, because fear crystallizes and is initially acted upon much faster than enthusiasm and exuberance – which will usually build over longer periods of time (and be most noticeable near the end) – the EEL will likely tend to be more useful for illustrating crashes than bubbles.[25] Because of their distinctive speed and intensity, a method of comparing and categorizing crashes on an arithmetic scale into three categories has also been introduced.

Upon further analysis and testing, these notions ought to have the potential to evolve into a new set of volatility-related metrics that are apart from the traditional, RE- and equilibrium-based approaches. Moreover, these notions are fully compatible with recent indications that central banks have begun to take volatility more seriously into account when formulating policy decisions.[26]

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[25] As the time scale over which the metrics are applied is reduced, the ratio of identifiable bubbles to crashes would be expected to rise and vice versa. Development of such a bubble-to-crash (B/C) ratio taken over relatively short periods would, for instance, provide another tool for portfolio managers, investment strategists, and market technicians.

[26] Both the Fed and the Bank for International Settlements (BIS *84th Annual Report*, p. 9) have recently commented on volatility. From the Fed Open Market Committee minutes of June 17-18. 2014: "The VIX, an index of option-implied volatility for one-month returns on the S&P 500 index, continued to decline and ended the period near its historical lows. Measures of uncertainty in other financial markets also declined; results from the Desk's primary dealer survey suggested this development might have reflected low realized volatilities, generally favorable economic news, less uncertainty for the path of monetary policy, and complacency on the part of market

In statistical terms, bubbles and crashes are, even after many years of research, still not mathematically well-defined, modeled, and understood. Human nature is such that these extreme events and their sizable and often long-lasting social and political effects on the distribution of income and wealth will never disappear entirely.

But with the further development of volatility metrics such as those proposed, central bankers and governments might begin to assess economic and market conditions from new perspectives. This should enable better-informed policy formulation and execution and, hopefully, mitigation of the most pernicious aspects of bubbles and crashes.

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participants about potential risks." From the BIS: "Volatility in equity, fixed income and foreign exchange markets has sagged to historical lows. Obviously, market participants are pricing in hardly any risks." See also Engstrom (2014) who discusses the Option Implied Probability (OIP) of crashes and *Barron's* article by Sears (2014), "Fed Gets Hip to the VIX."

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Highlights

- Surveys previous significant studies on bubbles, crashes, and volatility.
- Why price becomes secondary to quantity via the short-side rationed principle.
- Adds new perspectives on defining and measuring bubbles, crashes, and volatility.
- Introduces an extreme events line (EEL) and a crash intensity indicator.