**The Impact of Basel Accords on the Lender's Profitability under Different Pricing Decisions**

***Abstract***

In response to the deficiencies in financial regulation revealed by the global financial crisis a new capital regulatory standard, Basel 3, has been introduced. This builds on the previous regulations known as Basel 1 and Basel 2.

We look at how the interest rate charged to maximise a lender's profitability is affected by these three versions of the Basel Accord under three types of pricing- a fixed price model, a two price model and a variable risk based pricing model.  We investigate the result under two different scenarios. Firstly a fixed price of capital and secondly a fixed amount of equity capital available. We develop an iterative algorithm for solving the latter based on solution approaches to the former.

The riskiness of the portfolio has more significance then the Basel Accord requirements but the move from Basel I to Basel II has more impact than that from Basel II to Basel III.

Keywords:

consumer credit, pricing, Basel Accord regulations, Lagrange multiplier

# Introduction

Pricing consumer loans is implemented mainly through the interest rate charged, though in some cases there are also fees involved in setting up and operating the loans. In setting the interest rate for each loan the lender will consider the optimal trade off between interest income ( driven by the interest rates charges and the probability of take up) and the cost of the loan. Costs include the cost of sales, the cost of funding the loans, the cost of regulatory capital and the expected credit losses. The paper is particularly interested in the costs of capital which over the last 20 years have been driven by changing regulatory requirements.

Before 1988 regulatory capital did not need exceed some formulaic level. In the UK the regulation was that a bank had “sufficient regulatory capital ”but other countries had even weaker requirements. We analyse this situation, Basel 0 (B0)), by setting the capital requirement to be zero, since no specific figure was mentioned. Between 1988 and 2006, the regulations in the first Basel Accord ( B1) required that banks set aside a fixed percentage of equity capital to cover all their risks in lending (Basel 1). For most lending this was of the value of the loan. Kirstein (2002) pointed out this might result in adverse incentive influences. Although the cost of regulatory capital is the same for high risk and low risk borrowers, the bank will charge higher interest rates for more risky loans to compensate for the higher expected losses. So introducing such a regulatory capital requirement may encourage banks to replace low risk low profit customers with high risk high profit customers since they both require the same level of regulatory capital. In 2007 the Basel committee introduced a new capital requirement , denoted by Basel 2 (B2) where the capital requirements was sensitive to the credit risk inherent in bank loan portfolio. The Basel 2’s proposal required the bank to set different regulatory capital ratios for borrowers with different default risks. The third of the Basel Accords (Basel 3) was recently developed in a response to the deficiencies in financial regulation revealed by the [global financial crisis](http://en.wikipedia.org/wiki/Financial_crisis_of_2007%E2%80%932010). Basel 3 tightens up what is required as capital, introduces liquidity requirements, and increases the capital requirement by factoring up the capital requirements of Basel 2. The capital requirement in Basel 2 was taken as 8% of the risk weighted assets to have the same terminology as Basel 1, where the risk weighted assets were given by a function of the probability of default of the loans. Basel 3 requires the capital requirement to be again 8.0% of risk weighted assets but then adds both a capital conservation buffer and a counter cyclical buffer. The mandatory capital conservation buffer is 2.5% of risk weighted assets, and its objective is to ensure that banks maintain a buffer of capital thatcan be used to withstand future periods of financial and economic stress.The discretionary countercyclical buffer, allows regulators to impose further capital up to 2.5% of the risk weighted assets during periods of high credit growth. Thus we take the Basel 3 regulatory capital to be 13/8 ((8%+2.5%+2.5%)/8%) times the equivalent Basel 2 capital requirement. Table 1 shows the major differences between Basel 2 and 3.

The Basal Accords require the banks to set aside regulatory capital to cover unexpected losses on a loan, and this includes the Loss Given Default (LGD) factor, , which is the fraction of the defaulted amount that is actually lost. The minimum capital requirement (MCR) per unit of loan with a probability *p* of being good is defined as . We consider four different MCRs correspondiong to the four Basel sets of regulations:

 Basel 0: Describes the situation pre-1998 when there were no regulatory capital requirements so

 Basel 1: Describes the MCR under the first Basel Accord where

 Basel 2: Describes the MCR under the second Basel Accord where

, where (for credit cards), is the cumulative normal distribution and is the inverse cumulative normal distribution.

Basel 3: the MCR for the third of Basel Accord can be written as:

|  |  |
| --- | --- |
| Basel 2 | Basel3 |
| 1. Tier 1 Capital
 |  |
| Tier 1 capital ratio = 4%Core Tier 1 capital ratio = 2%The total capital requirement is 8.0%. | Tier 1 Capital Ratio = 6%Core Tier 1 Capital Ratio (Common Equity after deductions) = 4.5%The total capital requirement is 8.0%.  |
| 1. Capital Conservation Buffer
 |  |
| There no capital conservation buffer is required. | Banks will be required to hold a capital conservation buffer of 2.5% to absorb losses during periods of financial and economic stress.  |
| 1. Countercyclical Capital Buffer
 |  |
| There no Countercyclical Capital Buffer is required. | A countercyclical buffer within a range of 0% – 2.5% of common equity or other fully loss absorbing capital will be implemented. |

 Table 1: Differences between Basel 2 and Basel 3

Kashyap and Stein (2003) pointed out that there are many potential benefits to risk-based capital requirements, as compared to the “one-size-fits-all” approach embodied in the Basel 1 regulation. The objective of this paper is to understand the impact of different Basel regulatory capital requirements ( Basel 0, Basel 1, Basel 2 and Basel 3) on the lender’s profitability and pricing at the portfolio level under different pricing regimes. These are a fixed (one) price model, a two price model, and a variable pricing mode).

Fixed-rate pricing was the dominant form of pricing of loans until the early 1990s. More recently the development of the internet and call centres as new channels for loan applications has made the offer process more private to each individual (Thomas 2009). Developments in credit scoring and response modelling have made banks more efficient in marketing their products,and in increasing the size of their portfolios of borrowers (Chakravoriti and To 2006). The banks are able to “price” their loan products at different interest rates by adopting methods such as channel pricing, group pricing, regional pricing, and product versioning. Variable pricing, therefore, can improve the profitability of the lender by individual bargaining and negotiation. We consider two variants of variable pricing; the situation where each loan is individually priced depending on the default risk of the borrower and the two price case where borrowers are split into two segments and a different price charged to each segment.

There is a limited literature on the impact of the Basel Accords on consumer loan pricing. Allen DeLong and Saunders (2004) outline the relationship between the Basel accord and credit scoring, and they observe how corporate credit models are modified to deal with small business lending. Ruthenberg and Landskroner (2008) analyze the possible effects of Basel 2 regulation on the pricing of bank loans related to the two approaches for capital requirements (internal and standardized). They indicate that big banks might attract good quality firms because of the reduction in interest rates produced by adopting the IRB approach. This is like moving from Basel1 to Basel 2.On the other hand lower quality firms will benefit by borrowing from small banks, which are more likely to adopt the standardized approach. Perli and Najda (2004) suggest an alternative approach to the Basel capital allocation. They offer a model for the profitability of a revolving loan. They use this to imply that the regulatory capital should be some percentile of the profitability distribution of the loan, but there is no reference to the effect on the operating decisions. Oliver and Thomas (2009) analysed the changes in the accept/reject decision for a fixed price loan because of the effects of the different Basel regulations imposed. Based on the model suggested in that paper, we analyze the impact of different Basel regulations on pricing decisions under the three pricing strategies. We do this both in the case when the lender has an agreed cost of equity for each unit of equity needed to cover the regulatory capital requirement and when the lender decides in advance how much of its equity capital can be set aside to cover the requirements of this loan portfolio, but ignores the cost of how the equity was originally acquired. Recent work (( Baker and Wurgler (2013) and papers cited there) show that the cost of capital to a bank varies according to its capitalization but for an individual bank the marginal cost of equity is approximately fixed which is what is assumed here.

This paper is organised as follows. Section 2 looks at the profitability model of different pricing decisions (fixed price, two prices and variable pricing) on consumer loans. Section 3 uses several numerical examples to investigate the impact of the Basel Accords on different pricing models associated with a portfolio of such loans. The objective each time is to maximise the expected profitability of the portfolio and we report the corrsponding optimal interest rates. This is under the case when there is an agreed cost of equity. InSection 4 we extend the models by assuming the bank decides in advance how much of its equity capital can be set aside to cover the required regulatory capital of a loan portfolio. Section 5 draws some conclusions from this work.

# Pricing Models at portfolio level

To consider the pricing models at the portfolio level one needs first to define the profit model for an individual loan. Consider a loan of one unit offered by a lender to a borrower whose probability of being good – that is of repaying the loan in full- is.Assume the rate charged on the loan is . If the loan defaults, it does so before there are any repayments and the fraction of the loan that is finally lost at the end of the collections process is . Let be the required return on equity capital, which must be achieved to satisfy shareholders and let be the risk free rate at which the lender can borrower the money that will be loaned to the consumers. So we assume all the money to finance the lending is borrowed. Let denote the sum of the cost of the regulatory capital and the risk free rate, which is the cost of lending 1 unit, so B=rF +rQ lDK(p) If the operating capital comes from equity rather than borrowed from the market then one replaces rF by rQ in the expression for B. If the lender offers loans at an interest rate r to a borrower whose probability of being Good and repaying is p, we assume the chance the borrower will take the loan – the take probability is . With these assumptions the expected profit from making an offer of a loan of 1 unit to an individual borrower is

 Eq.1

where .

This ignores the costs of acquisition which would subtract a fixed cost for each prospective borrower. This complicates the model by requiring the level of borrowing for each individual Similarly the amount spent on marketing costs could affect the take probability q(r,p) but choosing the appropriate level of marketing is not the thrust of this paper..

## Fixed (one) price model in portfolio level

For the fixed price model, we assume the lender only offers one interest rate to all potential borrowers but may not offer the loan to risky ustomers. Throughout the paper we compare the results by looking at a class of numerical examples In these we assume that the take rate or response rate function is the linear function :

 , for

 Eq. 2

This means the borrower’s response rate is dependent on their riskiness as well as on the interest rate charged.. In the numerical examples, we assume and . This implies that an increase in interest rate drops the take probability by while if the default probability of the borrower goes up by the take probability goes up by . For borrower with a default rate of , of them would take a loan of rate , while only of them would take a loan with interest rate . To maximise the profit over a portfolio, the lender must accept the loans that have positive expected profit, but reject all the loans that are unprofitable. This defines a cut-off probability (or cut off point ) from (1) which is the probability of being Good at which the expected profit from the borrower is zero. Thus the lender should accept all the customers with probability of being good above the cut off point, and reject all the applications with probability of being good or lower. If the lenders are more risk averse and want the profits to be x times the default costs, then the cut off would be . However we will concentrate on profit maximising strategies. In our numerical examples, we assume the probability of the borrowers being good has a uniform distribution on [a.1]. The probability density function is , so it means no one with probability of being good less than is in the potential borrowers’ population. Therefore, the cut off point for a portfolio actually is .

If we define to be the expected profit from a portfolio with cut off point, we find

 where Eqn(3)

## Two Price Model at portfolio level

For this model, we assume the lender has two different rates that can be offered to potential borrowers. Suppose the rates are and , . The lender’s strategy is given by two values (a segmentation point and a cut off point ). Rate is offered to the borrowers whose probability of being good is ; rate is offered to those who probability of being good is , where again it is assumed the probability of the borrowers being good in the portfolio has a uniform distribution over [a,1].

In the two price model, the optimal cut off point depends on the higher rate , so since that rate gives higher profit than the other one. In this case is ‘first round of screening’ to determine whether or not to accept the borrower. The expected profit to the lender of the lower rate is always equal to or less than the expected profit to the lender of the higher rate , but the chance of borrowers accepting such a loan is always higher. So the probability of being good , at which the lender start to offer rather than, can be achieved from the equality

 Eq. 4

Using the take function given in (2) this results in

 Eq. 5

So is the segmentation point which divides those who passed the lender’s ‘first round of screening’ into the two groups to be offered different interest rates.

The lender has to consider whether the rate offered is attractive to some borrowers. This lead to two constraints –one for each interest rate- that says there is an upper limit on the goodness of the borrowers so that at least some of them will want to accept loans at that interest rate. That is when:

,which results into

and

, which leads to

So if one offers only the borrowers who probability of being good is below will accept it, and only those whose probability of being good is below will accept .

Hence the expected value of lender’s total profit is showed by following equation,

 Eq.6

where

## Variable Pricing Model in portfolio level

Variable pricing (risk based pricing) means that the interest rate charged on a loan to a potential borrower depends on the lender’s view of the borrower’s default risk and is specific to that borrower..

If the lender believes the borrower has a probability of being Good, then the lender believes the expected profit if a rate is charged to be

 Eq.7

where

In order to find the optimal interest rate for a certain probability of being Good, we differentiate the integrand with respect to  and set the derivate to zero, to find when the profit is optimised. This gives a risk based interest rate of

 Eq.8

This calculations of risk based interest rate can be found in the book by Thomas (2009). Note in this case there is no cut off probability of being Good below which one will not take an applicant. Instead one offers such applicants so high interest rates that it is highly unlikely that they will accept the offer. This occurs at probability levels of *p* where *q(r\*(p),p) =0*. Note that the model assumes that both lender and borrower are rational and honest. There are clearly some borrowers who will misrepresent their situation to assure themselves a loan which they are then not in a position to repay.

# The impact of the Basel Accords on the different pricing models at portfolio level

We calculate the various business measures such as the expected profit to the lender, optimal interest rate and optimal cut off on numerical examples under the different Basel Accords. This allows us to see the impact of the changes in the Accord on profit and on who is likely to get loans.

## Example for one price model

Consider the situation where , and so that equity holders expect a return of 5% and the rate at which the lender can borrow money to subsequently lend out is also 5%. This is the extreme situation where equity holders are not being compensated for the risk they take. It means the Basel accord regulations have the lowest impact on the costs in the model.

We assume that for loans that default 50% of the original amount lent will be recovered by the end of the collections process. With this, regulatory capital for each Basel rule leads respectively to:.

 Basel 0: since, we have .

Basel 1: , hence we have B.

Basel 2: since, where (credit cards), then we have .

 Basel 3: we assume , we get

This is the Basel 3 requirement on the total capital and we do not distinguish between the different tiers of capital.

We now consider the portfolio where 1 unit is available to be lent over the whole portfolio. The portfolio has a distribution over the riskiness of the potential borrowers so that their chance of each being good is uniform over the region [a,1] where a can be 0.6,0.7,0.8, 0.9 or 1.0. The last means none of the portfolio will default. The resultant optimal interest rates and the expected profits achieved under different Basel Accords are obtained by solving (2) and are shown in Figure 1 and Table 2.

We describe the profits relative to each other where the profit under Basel 0 when the portfolio has a=1 and so no one defaults, is set at 100. The profit in this case, 0.0950625, is almost the highest possible profit because there is no Basel capital requirement and the portfolio is default free.

**Figure 1: Relative profits under one fixed price**

Figure 1 describes the relative optimal profits when everyone accepted is offered the same rate. It is noticeable here and in almost all subsequent examples that the change in profit due to the different Basel regulations ( 5% maximum) is less than the changes in profit due to the riskiness of the portfolio , where there is a 14% change as a goes from 0.7 to 0.8. Obviously Basel1 always leads to smaller profits than Basel 0 and Basel 3 to even smaller profits. Basel 2 profits are less than Basel 1 profits except for the low risk population when a=0.9. This is because even risk free borrowers incur capital requirements under B1, as can be seen when a=1.0.

**Table 2: Optimal interest rates under one fixed price**



We can see from Table 2 that optimal interest rates for Basel 1, Basel 2 and Basel 3 are always higher than optimal interest rates achieved under Basel 0. This is because the costs of regulatory capital need to be covered. For this problem one accepts all the potential borrowers in the portfolio because the cut-off probabilities are lower than the minimum a value in each portfolio. It is also the case that Basel 3 requires the highest interest rate except that the rate for B1 overtakes it when the portfolio is essentially risk free ( a=1.0) The optimal interest rates charged drop as the potential borrower portfolio become less risky because one needs to offer competitive interest rates to get less risky customers to take the offer. Again the difference in interest rate offered under the different Basel regulations, never more than 1.7%, is much smaller than the impact of the riskiness of the potential portfolio.

## Example for two prices model

In this example we take the same borrowing rates and the LGD as in Example 3.1, namely, , and and we keep the same linear response rate function with

In the two price case it is difficult to calculate the portfolio profit under Basel 2 and Basel 3 regulations because to calculate the regulatory capital required it is necessary to integrate the expression over the whole portfolio. We take two approximations to ease this calculation. In the first conservative approximation we set the probability of the borrowers being good in expression to equal if they take the higher interest rate *r2* and if they take the lower interest rate *r1* . With this approximation the regulatory capital for all borrowers is always larger than the true regulatory capital required. We also build a model assuming that for the group who have the higher interest rate they all have regulatory capital corresponding to a probability of being good which is the average value between and . For the higher quality group we assume the regulatory capital is taken as if they all have the average good probability between and min{1, We found there is no significant difference in these two approximations so we report just one set of results.

**Figure 2: Relative profits under two price model**



**Table 3: Optimal interest rates and lower cutoffs under two price model**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| a |   | 0.6 | 0.7 | 0.8 | 0.9 |
|   |   | r2 | r1 | r2 | r1 | r2 | r1 | r2 | r1 |
|   | max(a,Pc) | p\* | max(a,Pc) | p\* | max(a,Pc) | p\* | max(a,Pc) | p\* |
| B0 | Rate | 48.56 | 31.73 | 41.57 | 29.80 | 35.37 | 27.96 | 29.73 | 26.21 |
| lower cutoff | 0.60 | 0.79 | 0.70 | 0.85 | 0.80 | 0.90 | 0.90 | 0.95 |
| B1 | Rate | 48.85 | 31.96 | 41.84 | 30.02 | 35.62 | 28.18 | 29.95 | 26.41 |
|  |  |  |  |  |  |  |  |  |
| lower cutoff | 0.6 | 0.79 | 0.7 | 0.85 | 0.8 | 0.9 | 0.9 | 0.95 |
| B2 | Rate | 48.89 | 31.96 | 41.86 | 29.97 | 35.59 | 28.08 | 29.85 | 26.55 |
| lower cutoff | 0.6 | 0.79 | 0.7 | 0.85 | 0.8 | 0.9 | 0.9 | 0.95 |
| B3 | Rate | 49.1 | 32.1 | 42.04 | 30.01 | 35.73 | 28.16 | 29.94 | 26.29 |
| lower cutoff | 0.6 | 0.79 | 0.7 | 0.85 | 0.8 | 0.9 | 0.9 | 0.95 |

Figure 2 and Table 3 show that when the minimum good rate increases so that the borrowers become less risky, the expected portfolio profits increase as well, but the optimal interest rates ( charged decrease. Note that in all four portfolios we make offers to all the potential borrowers and make the offer of the lower interest rate to just over half of them. The expected profit increases as the portfolio becomes less risky but the interest rates decrease in order to be able to attract sufficient of the higher quality applicants.

 Figure 2 shows the profits in the two price case are slightly higher than the fixed price case of Figure 1. Again though the difference in profits between the different capital requirements is no more than the difference when the minimum good rate in the portfolio is increased by 0.05. The profits under B1 are just above the profits under B2 until the portfolio has improved sufficiently so the minimum good probability is 0.9..

Table 3 shows that there is hardly any difference in the cut-off points for the different interest rates under the different Basel requirements. There is slightly more difference in the rates charged but these are minimum compared with the differences in rates a riskier portfolio would lead to. Again the B0 rates are always less than the B2 rates which in turn are less than the B3 rates. These higher rates are needed to pay for the capital requirements The B1 rate are very similar to the B2 rates until the riskiness of the portfolio is low (a=).8) when they rise above them.

## Example for variable pricing model

We assume there is no adverse selection in this variable pricing model. Adverse selection (Huang and Thomas (2013)) assumes that the fact the borrower accepts the loan affects the estimate of the borrower being Good. We use the cost structure of the pervious example with , and , and assume again the parameters for the linear response rate function are , , . In Figures 3a and 3b we define the relative profit of an individual borrower as a function of their probability of being Good. This is profit relative to the profit of a borrower with p=1 under the B0 regulations being set at 100. The optimal interest rates offered by the lender under the various Basel regulations Accords are shown in Table 4.

Figure 3a: relative profit per applicant with variable pricing

With these values, the lender only accepts borrower’s where the probability of being Good is at least 0.35. So there are zero profits from borrowers whose Good probability is below this value. The differences in profits between the different regulations is small compared with the profit difference of customers with different risks.

Figure 3b shows the relative profits among low risk applicants , where p goes from 0.9 to 1.0. Obviously for all cases B0 gives higher profits than B2 which gives higher profits than B3. One can see that B1 gives higher profits than B2 if p is less than 0.89 and greater than B3 provided p is less than 0.95. Surprisingly the lender makes more profit from borrowers whose probability of being Good is around 0.96 than those whose probability of being Good is 1. This is because one can charge the former a higher rate to get the same take probability as the latter. The corresponding extra profit more than compensates for the possible default losses

Figure 3b; relative profit per applicant for low risk applicants under variable pricing

The optimal interest rate to charge are shown in Table 4. They drop as the risk of the borrower drops both because there are less expeted default losses and because the borrowers are less likely to take high rates. The differences in rates under the different Basel regulations are never more than 1.2% .Again the B0 rate is lower than the B2 rate which in turn is below the B3 rate. The B1 rate goes above the B2 rate when p=0.9 and above the B3 rate when p=0.95

Table 4: Optimal rates in variable pricing model



# How the Basel Accords affects the different pricing models when there is a predetermined amount of equity capital

The previous model looked at what impact the Basel Accord requirements for regulatory capital have on the performances of lender’s different pricing decisions assuming a known required rate of ROE at the portfolio level. An alternative model is to consider the portfolio has a predetermined amount of equity capital available and so there is no known acceptable ROE. Instead the lender decides in advance how much of its equity capital can be used to cover the minimum capital requirements of a particular loan portfolio.

We assume that the funding of the portfolio loans is raised by borrowing from the market at the risk free rate and loss given default on any loan is irrespective of the rate charged. Ignoring the regulatory capital, the profit from a loan is modified from Eq.1 to

 Eq. 9

 where the linear take probability function is

 .

We retain the previous assumption that the probability of these portfolio loans being good also has a uniform distribution in the range [a,1]. If we define to be the expected profit from a portfolio where the pricing regime is to charge an interest rate r(p) and the potential portfolio only has borrowers with an a chance of being good, then

 Eq.10

The regulatory capital required for such a portfolio cannot exceed the equity capital Q set aside. Thus, given the limit on the equity capital provided, we need to solve the following constrained optimization problem in order to maximize the expected profit from the portfolio.

 Eq. 11

subject to

 Eq. 12

This is equivalent (Thomas 2009) to solving an unconstrained optimisation problem with a Lagrangian function that must be maximised namely

 Eq 13

Under the Basel 2 and Basel 3 regulatory requirements where

it is not feasible to get a closed form solution to the integration in (12) in any of the fixed price, two price and variable price cases. So the integral (12) is approximated by a summation. We describe this numerical approximation for the fixed price case. Given that we have a uniform distribution of borrowers over the probability of being good and that for any r we are only interested in a region where the take probability of the borrowers is larger than zero ( ), Eq.12 can be rewritten into

 Eq 14

 If we define

 Then (14) becomes

We choose a set of points, a+kh, k=0,1,2, n where 1=a+nh and then approximate *I[r,a]* by

,

For example, if an interest rate is being charged to borrowers whose probability of being good is not less than , the equity capital to be set aside is given by the following approximate calculation:

 ;

Unlike the earlier part of this paper there is no market price of equity. We first check if the unconstrained solution to (11) satisfies the constraint (12). If so, that solution, which is the solution under Basel 0, will also be the solution under the relevant Basel requirement. If it does not satisfy the constraint then there is a positive shadow price of equity capital. and we need to solve (13). However for a fixed shadow price finding the optimal rate r in (13) is the same as the problems solved in section 3 where there was a fixed cost of equity. So one starts with a particular value for the shadow price and uses this as rQ in the analysis in section 3 to find the optimal rates. One then checks whether with these optimal rates the constraint is exactly satisfied and if not whether the required equity is greater or less than Q. One can then apply the bisection method or other iterative routines to the shadow prices until one finds the shadow price and the corresponding optimal related interest rates that exactly satisfies constraint (12).

## Numerical Examples

We use numerical examples to explore the impact of the various Basel Accord requirements on different pricing models when there is a predetermined amount of equity capital set aside to cover the regulatory requirements of the portfolio, We compare the optimal interest rate, optimal cut off and expected portfolio profits under the different Basel regulations.

### Example for one price model

We use the cost structure of the examples from earlier part of this paper withand , and the parameters for the linear response rate function are We also assume the loan portfolio is of credit cards and so under the Basel 1 requirements the regulatory capital is while the correlation in the Basel 2 and Basel 3 capital requirement is We now solve Eq. 11 and Eq. 12 for different values of Q under the three regulatory regimes (Basel 1, Basel 2, Basel3) and in the unconstrained case corresponding to B0..

Table 5 relative profits for fixed rate pricing with limited capital

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | a | 0.6 | 0.7 | 0.8 | 0.9 |
| 0.01 | B1 | 33.66 | 36.09 | 38.90 | 42.22 |
|   | B2 | 36.15 | 40.42 | 47.90 | 67.41 |
|   | B3 | 27.26 | 31.51 | 35.41 | 56.79 |
| 0.02 | B1 | 52.63 | 59.33 | 67.33 | 73.67 |
|   | B2 | 50.01 | 59.14 | 63.97 | 92.77 |
|   | B3 | 40.01 | 47.46 | 55.53 | 75.22 |
| 0.03 | B1 | 62.37 | 74.44 | 85.81 | 92.63 |
|   | B2 | 58.84 | 72.21 | 85.62 | B0 |
|   | B3 | 48.61 | 57.62 | 68.47 | 89.63 |
| 0.04 | B1 | 65.10 | 81.63 | 93.13 | 99.09 |
|   | B2 | 63.68 | 78.09 | 93.21 | B0 |
|   | B3 | 54.24 | 64.36 | 74.02 | 98.62 |
| all values | B0 | 78.97 | 89.50 | 96.44 | 99.87 |

Table 5 describes the relative profitability of the one rate policy for portfolios where the probability of

the borrowers being a Good is uniformly distributed over [a,1] . Each row corresponds to a different

Basel requirement ( B0, B1, B2, B3) and a different amount of capital available to cover the

 portfolio’s Basel requirement, Q,where Q=0.1,0.2,0.3,0.4. The profits are described relative to the

 profit of a risk free portfolio,, ( a=1), under Basel 0, B0. This profit is set at 100.

In all case the profitability increases as a increases so the potential borrowers become less risky. The

Profitability also increases as the capital available, Q, increases. When there is sufficient capital that

equation (12) becomes inactive we have the unconstrained solution which corresponds to the solution

 under the B0 requirements. This is because the capital requirements do not affect the solution in any

 way which is equivalent to there being no capital requirements. This occurs under Basel 2, B2, when

 a=0.9 and Q ≥ 0.3 and is represented in the Table as B0. The impact of Q on the profit is far greater,

 especially when Q is small, than the impact of the riskiness of the portfolio(a), or which Basel

 regulation is in force..B0 always has the highest profits and B2 obviously has higher profits than B3.

Pportfolios under B3 are more profitable than those under B1 if the capital is low ( Q≤0.2) and the

 quality high ( a≥0.8) . The relation between B1 and B2 are less obvious but B2 is more profitable if

 a=0.9 and for the large and small values of Q if a=0.8.

Table 6: Pricing decisions for fixed rate prices with limited capital

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Q | a | 0.6 | 0.7 |   | 0.8 | 0.9 |
|   |   | rate |  max(a, Pc) | rate |  max(a, Pc) | rate |  max(a, Pc) | rate |  max(a, Pc) |
| 0.01 | B1 | 40.19 | 0.817 | 40.74 | 0.841 | 41.41 | 0.87 | 43 | 0.9 |
| B2 | 37.46 | 0.827 | 37 | 0.854 | 36.32 | 0.885 | 35.37 | 0.919 |
| B3 | 36.7 | 0.869 | 36.29 | 0.886 | 35.49 | 0.915 | 34.94 | 0.932 |
| 0.02 | B1 | 38.67 | 0.74 | 39.32 | 0.776 | 40.19 | 0.819 | 38 | 0.9 |
| B2 | 38 | 0.788 | 37.87 | 0.788 | 37.44 | 0.829 | 32.94 | 0.9 |
| B3 | 37.69 | 0.809 | 37.42 | 0.83 | 36.75 | 0.867 | 35.65 | 0.909 |
| 0.03 | B1 | 37.9 | 0.679 | 38.42 | 0.724 | 37 | 0.8 | 33 | 0.9 |
| B2 | 38 | 0.708 | 38.02 | 0.745 | 37.07 | 0.8 | B0 | B0 |
| B3 | 37.99 | 0.7666 | 37.83 | 0.793 | 37.32 | 0.836 | 34.07 | 0.9 |
| 0.04 | B1 | 37.68 | 0.627 | 36 | 0.7 | 32 | 0.8 | 28 | 0.9 |
| B2 | 37.77 | 0.665 | 38 | 0.71 | 31.53 | 0.8 | B0 | B0 |
| B3 | 38.02 | 0.737 | 37.98 | 0.768 | 37.52 | 0.823 | 29.22 | 0.9 |
| B0  | B0 | 37.68 | 0.6 | 35 | 0.7 | 31.4 | 0.8 | 27.91 | 0.9 |

Table 6 describes the optimal lending rate and optimal accept/reject cut-offs on the probabilities of

customers not defaulting. It shows that in order to maximise the portfolios profitability offers should

 only be made to all customers if there is ample capital available and the portfolio is low risk. For

 B3 this is a=0.9, Q≥0.03; for B2 it is a=0.9, Q≥0.02; and for B1, a=0.9,Q≥0.1, a=0.8,Q≥0.03,

 a=0.7,Q≥0.04. The cut-off values drop and so riskier customers are taken as the capital available

 increases. This happens under all Basel regulations and all levels of minimum portfolio risk.

The Basel 1 interest rates drop as more equity becomes available. Under the Basel 2 and Basel 3 regulations the optimal interest rate increases as the equity available increases when there is very little equity available, However interest rates then decrease as the equity increases when there is more equity available. This is because increasing the interest rate has two counterbalancing effects. It increases the profit on each of the borrowers who accept the loan but it diminishes the chance an individual borrower will take the loan. When there is very little equity available, that equity constraint means one cannot take a large number of borrowers anyway and so it is more profitable to increase the interest rate. When the amount of equity is still constraining but quite large then one wants to make sure one has enough borrowers to “use” all the equity and hence one starts to decrease the interest rate .

### Example for two prices model

For the two price model, the customers are segmented into two groups by the lender. Thus the expected profit from the portfolio given the limit on the equity capital provided is modified from Eqs 11 and 12 to ,

 Eq. 15

subject to

 Eq. 16

Where  , and p\* is defined by equation (4)

This is again a constrained optimisation problem and so we can solve using the Lagrangian approach namely to optimise

Note again that this is equivalent to the problem in section 3 if we assume the cost of equity is λ and so for any λ we can solve to find the optimal rates that maximise the Lagrangian given that λ. So we solve the unconstrained problem which is equivalent to solving the problem under Basel 0. If this does not satisfy constraint (16) then we use a bisection approach to find the λ that when we solve to find the corresponding optimal rates r1 and r2 the constraint is exactly satisfied.

We take the same values used in the one price model example so that the risk free rate is 0.05 and the loss given default value is set as 0.5. The response rate function is also the same as in that example with In this model, the calculations applied in Eqs 15 and 16 give the following results under the different regulatory regimes.

Table 7 describes the profits under the optimal two price strategies when the profit from a risk free portfolio under the Basel 0 regulations is set at 100. Comparing table 7 with table 5 one can see that in all cases the profits from the two prices model are always greater than those in the one fixed price model.as one would expect. The difference is greater the riskier the portfolio. As in the fixed price case the profits increase both as the portfolio becomes less risky ( a increases) and when more capital is available (Q increases). In all cases if there is enough equity then one ends up with the profit under the B0 Basel case where there is no regulatory capital. That has happened in Table 7 at q=0.04 for all the B1 cases and some of the B2 cases. The portfolio profits are close to being convex in the equity capital Q that is available

Table 7: relative profits for two rate pricing with limited capital



Obviously B2 is always more profitable than B3. Basel 2 and Basel 3 require less regulatory capital for very good borrowers than B1 but more capital for risky borrowers.. So with very limited capital both B2 and B3 are more profitable than B1. However when there is plenty of capital, so one is taking risky borrowers B1 is more profitable than B2 and B3.

Table 8: Pricing decisions for two price model with limited capital



In the two price model the interaction between the two rates and the two cut-off points means the trends in the relationships are not always simple. In some cases the two rates come down as more capital is available ( B1 and B2 with less risky portfolios). In all other cases the rates go up if Q is small but go back down again if Q is large. This is the phenomenon seem in the single rate case caused by the two counterbalancing effects of increasing the interest rates. . Doing so increases the profit on each of the borrowers who accept the loan but diminishes the chance an individual borrower will take the loan. With little equity and so plenty of borrowers the first effect is dominant. With lots of equity and so a difficulty in using up all the equity it is the second effect which is dominant.

### Example for variable pricing model

Variable pricing means that the interest rate charged on a loan to potential borrower depends on the default risk of individuals. Thus in this case the expected profit is

 Eq. 17

 Subject to Eqn 18

Using the Lagrangian approach this is equivalent to finding the that maximises where is chosen so that the constraint is satisfied. So initially one solves the unconstrained problem, which gives the solution under B0, where there is no Basel requirements. The variable rate function is where is the interest rate in (8) when there is no Basel requirement and when . If there is insufficient equity available for this solution to hold we know that the shadow price of equity is positive and that the equity constraint is exactly satisfied. We find the solution by adjusting the shadow price of equity until the constraint is exactly satisfied. Solving the problem to find the optimal interest rates for a given λ is equivalent to solving the variable pricing problem in the previous section with the cost of equity being λ. So initially if there is no equity available obviously the lender accepts no one. This is done by making very large which leads to extremely high interest rates, which no one will accept. As equity increases and so starts to fall we make lower interest rate offers to all borrowers. Eventually we will start making the optimal interest rate offer under Basel 0 to some of the potential borrowers. Since this is the one that maximises the profit from this group of borrowers we do not make lower interest rate offers to those even if has been further reduced. The point at which this happens for borrowers whose probability of being good is is when their expected profitability per unit of regulatory capital is equal to where is the ratio

 Eq. 19

Further discussion on the use of this marginal return on equity (ROE) can be found in Thomas (2009).

Using the cost structure of all the previous examples in this section, we show the results when and .

In this problem we describe the profits when 1 unit is leant to an individual borrower of a given probability of being Good. It one wanted to find the portfolio profit for the portfolios with uniform Good distribution over [a,1]one would need to integrate ( or average ) the individual profits shown here over the appropriate range. To be consistent with the previous work we describe the profitability relative to 100 for a risk free borrower under Basel 0. Note that with this individual based pricing we make offers to every borrower but expected the offers to some risky borrowers to be so high that no one will take up the offer. Hence the zeros in the top left corner of Table 9. Note though this phenomena occurs under B3 even when Q reaches 0.03. It is noticeable again that under B0 or if there is enough capital under the other regulations, the maximum profit occur with those whose probability of being Good is in the range -.96. to 0.97. In most levels of capital though we see the profit increases monotonically with the drop in default risk under all Basel regimes. Notice that when q=0.04, Basel 1 has enough capital to cover the unconstrained Basel 0 solutions .Thus it gives the maximum profit as the Basel 2 and Basel 3 solutions are still constrained by the lack of capital. If the capital is lower the optimal solutions are capital constrained under all Basel regulations In that case Basel 1 leads to the highest profit from risky borrowers, while it gives the lowest profits from those whose probability of being Good is above 0.9 at Q=0.01 or above 0.96 if Q=0.03.

Table 9: relative expected individual profits under variable pricing with limited capital



Table 10:Pricing decisions for variable pricing with limited capital

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Q | A | 0.6 | 0.7 | 0.8 | 0.9 | 0.95 | 0.96 | 0.97 | 0.98 | 0.99 |
| 0.01 | B1 | 77.83 | 64.57 | 53.63 | 44.22 | 39.95 | 39.12 | 38.30 | 37.49 | 36.69 |
|   | B2 | 82.58 | 68.05 | 54.61 | 41.19 | 33.90 | 32.32 | 30.67 | 28.91 | 26.98 |
|   | B3 | 87.46 | 72.12 | 57.75 | 43.17 | 27.95 | 27.25 | 26.55 | 25.86 | 25.18 |
| 0.02 | B1 | 71.33 | 59.00 | 48.75 | 39.89 | 35.84 | 35.06 | 34.28 | 33.51 | 32.75 |
|   | B2 | 74.92 | 61.68 | 49.70 | 38.08 | 31.98 | 30.68 | 29.34 | 27.93 | 26.40 |
|   | B3 | 80.16 | 66.04 | 53.06 | 40.21 | 28.47 | 27.69 | 26.91 | 26.13 | 25.33 |
| 0.03 | B1 | 65.00 | 53.57 | 44.00 | 35.67 | 31.84 | 31.10 | 30.36 | 29.64 | 28.92 |
|   | B2 | 68.79 | 56.58 | 45.77 | 35.59 | 30.44 | 29.37 | 28.28 | 27.14 | 25.93 |
|   | B3 | 76.26 | 62.79 | 50.56 | 38.62 | 29.40 | 28.49 | 27.56 | 26.61 | 25.62 |
| 0.04 | B1 | 58.83 | 48.29 | 39.38 | 31.56 | 27.95 | 27.25 | 26.55 | 25.86 | 25.18 |
|   | B2 | 62.87 | 51.64 | 41.96 | 33.19 | 28.96 | 28.11 | 27.25 | 26.38 | 25.48 |
|   | B3 | 72.28 | 59.48 | 48.00 | 37.01 | 30.38 | 29.32 | 28.23 | 27.11 | 25.91 |
|   | B0 | 58.83 | 48.29 | 39.38 | 31.56 | 27.95 | 27.25 | 26.55 | 25.86 | 25.18 |

Table 10 describes the optimal interests to be charges to individuals when there is limited capital available. The interest rate charged under B1,B2 and B3 is always above that charged under B0 until the equity capital is sufficient for the B0 solution to be optimal . This is the case under B1 when Q=0.04.The rates always decrease as the probability of the borrower being Good increases. This ensures that enough Good borrowers are taking the loans. One might also expect that the rate would drop as more capital became available. This is almost always the case but not under B3 for very good borrowers with p≥ 0.95. This is because it is so difficult to attract these customers compared with higher risk ones, it is more profitable to use the capital on more risky customers. This is related to the fact that under B3 one is charging risky customers such high rates that no one accepts the loan until Q is at least 0.4

# Conclusion

This paper introduces a simple model that relates the profit on a loan to the interest rate charged. This is then used to model the profit under three different pricing strategies for portfolio of such loans under different Basel capital requirements. We analysed the results of these models by using numerical examples based on a common set of portfolio parameters. This allowes us to explore how the different Basel Accords regimes should affect various operational and business measures such as expected profit, optimal interest rate, and optimal cut off.

The numerical results confirm the obvious facts that using variable pricing produces higher profits than two rate pricing, which in turn produces higher profits than fixed rate pricing. It also confirms the obvious fact that profits under Basel 0 are higher than those when the other Basel restrictions are introduced. Similarly Basel 2 leads to higher profits than Basel 3. Although most of the time Basel 1 gives higher profits than Basel 2 this is not the case if the portfolio of prospective borrowers has low default risks. However the higher requirements of Basel 3 capital means it leads to lower profits than Basel 1 unless the portfolio is essentially risk free. It is interesting to note that the quality of the portfolio (the “a” value) affects the optimal interest rate and the expected profits far more than the changes in the Basel regulations.

When the cost of equity is fixed then the optimal rates to charge decrease and the expected profits increase as the quality of the portfolio (the a value) increases under all Basel conditions. This is because one needs low interest rates to attract the high quality borrowers. This is true in the fixed price and two price cases. The latter is representative of the multi-rate pricing policies which are used by many lenders. The examples suggest that in the two rate case , the difference in Basel requirements affect the rates charged more than the cut-off values. However the rates change much more because of the quality of the portfolio then because of changes in which Basel regulation is in force. The variable pricing model is less widely used in practice but is an aspiration of many lenders. Obviously it gives higher profits then the multi-rate policies and the rate charged under it drops as the quality of the borowers increases. Interestingly in the examples in this paper where the highest profit comes from borrowers who have a small but non-negligible probability of default rather than from the risk free borrowers. This is because one can increase in interest rate for such borrowers compared with the risk free borrowers and keep the take rate the same. The extra profit this increased interest rate brings more than compensates for the increase in the default rate. This phenomenon is seen under the Basel 0,1 and 2 regimes but the extra capital such borrowers incur under Basel 3 mean it is not the case there.

One can solve the problem when there is a fixed amount of equity available by a Lagrange multiplier approach, inserting a price of equity into the models of section 3 which ensures the correct amount of equity capital is used Obviously profitability increases as the equity capital available increases and when there is enough capital one arrives at the optimal solutions under Basel 0 no matter what the basel regime.The difference in profit between the different regimes is much more marked in this problem, especially when there is very limited capial available. The optimal rates under the different \Basel regimes tend to be very similar to one another , especially when there is lots of capital available and so they are all converging to the optimal B0 solutions. In the two rate case the interactions between the rates and the cut-off values is such that the rates are often not monotone functions of the capital available. This is because increasing the rate introduces two conflicting effects – it increases the profits on the borrowers who accept but drops the percentage of borrowers who will accept the loan. Which of these is the more important depends on how much capital is available. One finds in many cases the optimal rate tends to increase as the capital increases when there is little capital available. As capital increases further, this is reversed and rates start to drop as the capital increases.

Under the fixed capital costs used in this paper it is optimal to make offers to everyone in the portfolio. In the variable pricing case one makes offers to all borrowers but only those whose probability of being Good is only at least 0.4 will take the offer, since the others are offered very high interest rates.. With the fixed equity capital case, one tends not to make offers to all the borrowers until the equity capital starts to approach the levels where one can almost use the optimal rates in the Basel 0 case. So restricted equity capital has more of an effect on the percentage of the population who are made an offer rather than on the rates they are offered.

The most important current issue is what the change from Basel 2 to Basel 3 mean in terms of business and operational measures. It appears that whatever pricing regime is used, provided reasonable amounts of capital are set aside against the portfolios Basel requirements, the profitability is only slightly lower (<5%) under Basel III than Basel II. So loan availability should be only slightly changed since loans that are profitable now will still be broadly profitable under Basel III while in most cases the increase in interest rate charged is quite small.

One could undertake further numerical exploration using other take functions such as the logistic take function (Phillips 2005). This paper gives the formulation of how to calculate the optimal pricing strategies for all such problems and the results using a linear take function give insights into the general trends in the pricing policies.

# Reference

Allen, L. DeLong, G.and Saunders, A. (2004). *Issues in the credit risk modeling of retail markets*, Journal of Banking and Finance 28, 727-752

Baker M., Wurglwe J., (2013). *Do strict capital requirements raise the cost of capital? Banking regulations and the low risk anomaly*, National Bureau of Economic Research Working Paper 19018, Cambridge MA.

Basel Committee on Banking Supervision(2001). *The new Basel Capital Accord*, Consultative Document, Bank of International Settlements.

Basel Committee on Banking Supervision (2004). *International convergence of capital measurement and capital standards – a revised framework*, Bank for International Settlements.

Basel Committee on Banking Supervision (2010). *Basel III: A global regulatory framework for more resilient banks and banking systems*, Bank for International Settlements.

Chakravoriti, S. and To, T. (2006). *A theory of credit cards.* International Journal of Industrial Organization, 25, 583-595.

Huang, B., & Thomas, L.C. (2013). Credit card pricing and impact of adverse selection. Journal of the Operational Research Society doi10.1057/jors 20.12.173

Kirstein, R (2002). *The New Basel Accord, Internal Ratings, and the Incentives of Banks,* International Review of Law and Economics 21, 393-412.

Kashyap, A. and Stein,J. (2003). *Cyclical Implications of the Basel II Capital Standards*, University of Chicago, working paper.

Oliver, R.W. and Thomas , L. (2009) *Optimal Score Cutoffs and Pricing of Regulatory Capital In Retail Credit Portfolios,* WDiscussion paper in Centre of Risk Research CRR-09-01.

Perli, R.and Nayda, W. (2004). *Economic and Regulatory Capital Allocation for Revolving Retail Exposures*, Discussion Paper, Federal Reserve Board, Washington.

Phillips R.L., (2005), Pricing and Revenue Optimization, Stanford Business Books, Stanford, California

Ruthenberg , D. and Landskroner, Y. (2008). *Loan pricing under Basel II in an imperfectly competitive banking market,* Journal of Banking and Finance 32: 2725–2733.

Thomas, L. (2009). *Consumer credit models: pricing, profit, and portfolios*. Oxford: Oxford University Press.