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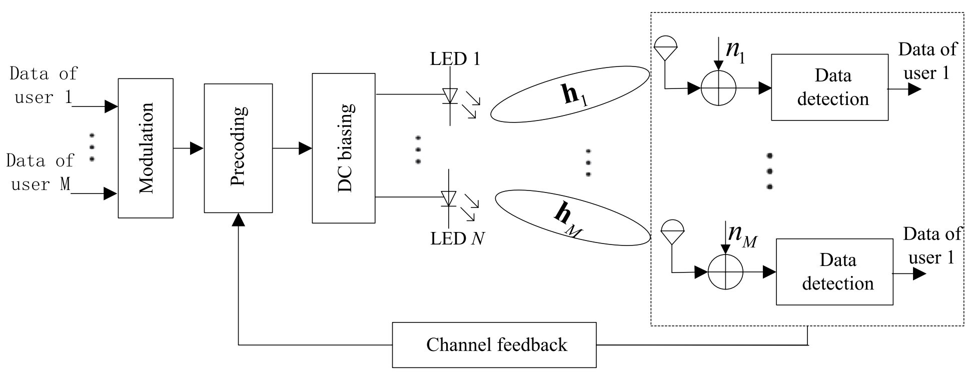

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Multiuser MISO Transceiver Design for Indoor Downlink Visible Light Communication Under Per-LED Optical Power Constraints

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Abstract: Light-emitting diode (LED)-based visible light communication (VLC), combining illumination and communication, is a promising technique for providing high-speed, low-cost indoor wireless services. In indoor environments, multiple LEDs routinely used as lighting sources may also be concomitantly invoked to support wireless services for multiple users, thus forming a multiuser multiple-input–single-output (MU-MISO) system. Since the user terminals detect all the light rays impinging from multiple LEDs, inter-user interference may severely degrade the attainable system performance. Hence, we conceive a transceiver design for indoor VLC MU-MISO systems to suppress the multiuser interference (MUI). In contrast to classic radio-frequency (RF) communication, in VLC, the signals transmitted from the LEDs are restricted by optical constraints, such as the real-valued nonnegativity of the optical signal, the maximum permissible optical intensity, and the constant brightness requirements of the LEDs. Given these practical constraints, we design the optimal transceiver relying on the objective function of minimizing the maximum mean square error (MMSE) between the legitimate transmitted and received signals of the users and show that it can be readily found by solving a convex second-order cone program. Then, we also propose a simplified transceiver design by incorporating zero-forcing (ZF) transmit precoding (TPC) and show that the TPC coefficients can be efficiently found by solving a linear program. The performance of both the optimal and the simplified transceiver is characterized by comprehensive numerical results under diverse practical VLC system setups.

Index Terms: Light-emitting diode (LED), visible light communication, transceiver design, multiuser multiple-input single-output (MU-MISO), mean square error (MSE).

1. Introduction

Light-emitting diodes (LEDs) are expected to displace existing conventional fluorescent and incandescent lighting devices and become the dominant lighting sources for indoor illumination owing to their higher power efficiency [1], [2]. As a further benefit, the optical intensity of LEDs can be modulated to transmit high-speed information with the aid of visible light communication (VLC) [3]–[5], which has attracted considerable interests owing to its compelling low cost, license-free operations, low electromagnetic interference, high security and so on. With the growing demand for higher speed data transmission and the saturation of the radio frequency (RF) spectrum, VLC is deemed to be a promising technique of realizing ubiquitous indoor wireless downlink coverage [6].

VLC systems often exploit multiple lighting sources for simultaneously supporting multiple users in indoor environments [7], which leads to the concept of multiuser MISO (MU-MISO) VLC systems. Since each user terminal receives light rays from multiple LEDs, inter-user interference arises, which may severely degrade the multiuser performance. Thus, the multiuser VLC system's transceiver has to suppress the multiuser interference (MUI). Although multiuser RF systems have been extensively studied, they cannot be directly applied to VLC systems owing to the following major aspects [8].

- The signals transmitted by LEDs must be non-negative real values instead of the bipolar complex values of RF communication.
- In VLC, the main functionalities of LEDs are to provide indoor illumination; hence, they are subject to optical power constraints for the sake of guaranteeing the indoor illumination, rather than to the electronic power constraints of RF communication.

Therefore, the above features must be taken into account in the design of VLC systems. An SVD-based transceiver architecture was adopted in [9] for maximizing the data rate, where the total optical power constraints were indeed considered, but the average optical intensity fluctuated as a function of the optical channel conditions, hence imposing lighting flickers. In [10], the block diagonalization (BD) based transmit precoding (TPC) strategy of RF communication was directly applied to multiuser VLC systems without considering the specific features of VLC. In [11] a pseudo-inverse based zero-forcing (ZF) TPC strategy conceived for MU-MISO broadcasting systems is considered. A locally optimal linear TPC method was proposed in [12] for MU-MISO VLC relying on the objective function (OF) of minimizing the total mean-squared error (MSE) between the received and legitimate transmitted signals of the users, which may impose unfairness among the users, because the users having a higher received signal power may monopolize the resources.

Against this background, we propose multiuser MISO indoor downlink VLC designs for suppressing the MUI, while providing stable illumination. Since in uplink transmission using VLC would lead to an unpleasant irradiance from the user devices, in practical uplink scenarios it is more apt to use either infrared (IR) or RF transmission [13], [14]. As a further benefit, IR or RF uplink transmission will not interfere with the downlink VLC and indoor illumination. Therefore, we focus our attention on downlink VLC design in this paper. The main contributions of the paper are summarized as follows.

1) Given our design objective, we impose a per-LED optical power constraint that takes into account both the non-negative nature of the optical signals, as well as the maximum affordable optical power, whilst maintaining stable brightness.
2) Our design goal is to optimize the entire system's performance, while maintaining throughput fairness for all users. Again, this is achieved by minimizing the OF constituted by the maximum MSE between the legitimate transmitted and received signals of all users. Given this OF, we first develop the globally optimal transceiver scheme under the above-mentioned optical power constraints and show that its minimum can be efficiently found by solving a convex second-order cone program (SOCP) [15].
3) Then, we propose a simplified ZF TPC, which outperforms the traditional pseudo-inverse based ZF TPC scheme of RF communication, and show that the weights of the simplified TPC can be more efficiently found by solving a linear problem. Finally, the performance of both the proposed globally optimal and of the simplified TPC designs is evaluated for different VLC setups.

The remainder of this paper is organized as follows. The model of the MU-MISO indoor VLC system is described in Section 2. Both the globally optimal and the simplified transceiver

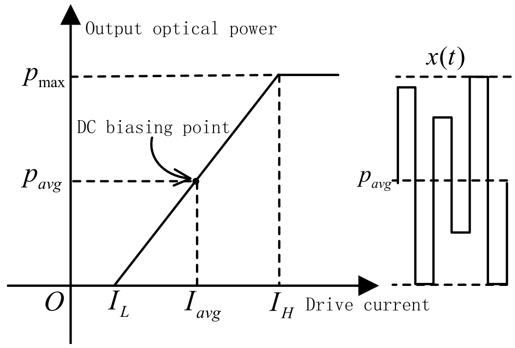

Fig. 1. Limited linear dynamic range of LEDs.

designs are developed in Section 3. Section 4 characterizes the numerical performance of the proposed transceiver designs. Finally, our conclusions are provided in Section 5.

The following notations are used. Boldface upper case letters denote matrices, boldface lower case letters represent column vectors, while standard lower case letters denote scalars. Furthermore, $\mathbf{I}_N$ denotes the identity matrix of size N, $\mathbf{e}_i$ is the all-zero vector having a single one in the ith element, $\mathbf{1}$ denotes the vector of all ones. The operators vec$(\cdot)$, $(\cdot)^{\mathrm{T}}$, and $(\cdot)^{-1}$ represent the vectorization, transpose and inverse of a matrix, respectively. The operator diag$\{\mathbf{b}\}$ denotes the diagonal matrix with the element $\mathbf{b}$, whilst $\|\cdot\|_1$ and $\|\cdot\|_2$ are the L_1 and L_2 norm of vectors, respectively. The Kronecker product of matrix $\mathbf{A}$ and $\mathbf{B}$ is denoted as $\mathbf{A} \otimes \mathbf{B}$.

2. Multiuser MISO Indoor VLC Systems

2.1. Optical Power Constraints

In VLC, the LEDs are the key component used for both indoor illumination and communication. The transfer characteristic of LEDs leads to the unique optical power constraints imposed on the transmitted signals. Firstly, the LEDs exhibit a limited linear dynamic range. As seen in Fig. 1, we can linearize the non-linear input-output characteristic with the aid of predistortion over the limited dynamic range of $[I_L, I_H]$, which linearly corresponds to the optical power range of $[0, p_{\max}]$ [16], [17], where I_L denotes the turn-on current, while I_H is considered to be the maximum tolerable AC current. Hence the tuplet $[I_L, I_H]$ allows us to avoid both the overheating of the LED chips and the potential light intensity reduction. The signals transmitted by LEDs will be clipped beyond this dynamic range. Therefore, based on the direct linear relationship between the radiated optical power and drive current, the constraint imposed on the transmitted signal x in terms of its optical power may be expressed as $0 \leq x \leq p_{\max}$. The constraint ensures the non-negativity of the signals as well as the normal operation of the LEDs. Additionally, since the human eye is insensitive to optical intensity fluctuations beyond the fusion frequency of the human eyes, but only responds to the average optical intensity of LEDs, the average optical intensity should remain constant over time to provide stable brightness, which is formulated as $\mathrm{E}[x] = p_{\mathrm{avg}}$, where p_{avg} depends on the illumination requirements [18].

In conclusion, the above pair of optical power constraints should be satisfied simultaneously to prevent signal distortion and to retain the primary functionality of VLC as well as stable brightness. Explicitly, the above optical power constraints of VLC are different from the electronic power constraints widely used in RF communication.

2.2. Transmitter Model

The MU-MISO indoor VLC system considered consists of several LED transmitters and user terminals, as shown in Fig. 2. Since the user terminals receive light rays from multiple LED transmitters, the light signals transmitted to a specific user will interfere with these of the other

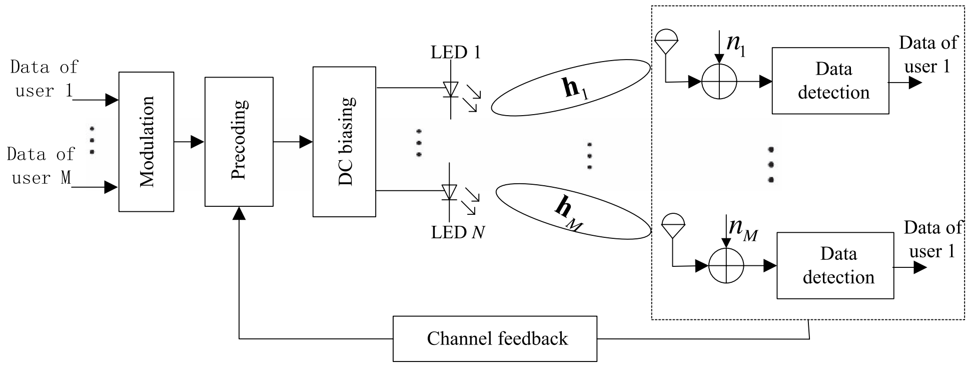

Fig. 2. Schematic of TPC-aided MU-MISO indoor VLC systems.

users and, thus, severely degrade their communication performance. Therefore, the transmitted data symbols have to be preprocessed based on the knowledge of the channel state information (CSI) to suppress the MUI.

Let s_k represent the pulse-amplitude modulation (PAM) symbol transmitted to the kth user with a zero mean, i.e., $\mathrm{E}[s_k] = 0$, which is normalized for ensuring that $s_k \in [-1, 1]$. Let furthermore u^2 denote the covariance of s_k, yielding $\mathrm{E}[s_k^2] = u^2$, and $\mathbf{s} = [s_1, \ldots, s_M]^{\mathrm{T}}$ represent the symbol vector of all the M users. The information symbols transmitted to different users are independent of each other, i.e., we have $\mathrm{E}[\mathbf{s}\mathbf{s}^{\mathrm{T}}] = u^2\mathbf{I}_M$. After modulation, the TPC relying on the knowledge of the CSI is invoked for suppressing the MUI and a direct current (DC) component is added to guarantee the non-negativity of the signal. After the TPC and DC biasing operations, the signal vector $\mathbf{x} = [x_1, \ldots, x_N]^{\mathrm{T}}$ transmitted by the LED array is given by

$$\mathbf{x} = \mathbf{W}\mathbf{s} + \mathbf{p} = \sum_{k=1}^{M} \mathbf{w}_k s_k + \mathbf{p} \tag{1}$$

where $\mathbf{W} \in \mathbf{R}^{N\times M}$ is the TPC matrix, $\mathbf{w}_k \in \mathbf{R}^N$ is the kth column of $\mathbf{W}$, which denotes the TPC vector of the kth user, and $\mathbf{p} = [p_1, \ldots, p_N]^{\mathrm{T}}$ is the DC offset added to the signals. The signal x_n transmitted by the nth LED can be expressed as

$$x_n = \sum_{k=1}^{M} w_{nk} s_k + p_n \tag{2}$$

where w_{nk} is the element of $\mathbf{W}$ in the nth row and kth column. Therefore, the average optical intensity of the nth LED is expressed as

$$\mathrm{E}[x_n] = \mathrm{E}\left[\sum_{k=1}^{M} w_{nk} s_k\right] + p_n = p_n. \tag{3}$$

We find from the above equation that the average optical intensity of the LED will not be influenced by the PAM symbols transmitted to the users and it is uniquely determined by p_n. Therefore, p_n should remain constant over time for maintaining the brightness of the LEDs. Since we have $s_k \in [-1, 1]$, the dynamic range of x_n becomes

$$-\sum_{k=1}^{M} |w_{nk}| + p_n \leqslant x_n \leqslant \sum_{k=1}^{M} |w_{nk}| + p_n, \qquad n = 1, \ldots, N. \tag{4}$$

To ensure that the LEDs operate within their dynamic range of $[0, p_{\max}]$, the TPC should satisfy

$$\begin{cases} -\sum_{k=1}^{M} |w_{nk}| + p_n \geq 0 \\ \sum_{k=1}^{M} |w_{nk}| + p_n \leq p_{\max}. \end{cases} \tag{5}$$

Rearranging the above inequalities, the optical power constraints imposed on the TPC matrix $\mathbf{W}$ are given by

$$\sum_{k=1}^{M} |w_{nk}| = \left\|\mathbf{W}^{\mathrm{T}}\mathbf{e}_n\right\|_1 \leqslant p'_n, \qquad n = 1, \ldots, N \tag{6}$$

where $p'_n = \min\{p_n, p_{\max} - p_n\}$, $\mathbf{e}_n$ denotes a zero vector with the nth element being one, and $\| \|_1$ is the L_1 norm of vectors (note that traditional electronic power constraints are often based on the L_2 norm).

2.3. Receiver Model

The light rays received by the users are converted into electronic signals by the photo diode (PD) at the receiver. In general, the VLC signals received by PD are constituted by two components, namely the line of sight (LOS) component that propagates directly from the LED to the receiver and the diffuse component arriving via reflections. As shown in [19], [20], the electrical power of the strongest diffuse component in the room is still at least 7 dB lower than that of the weakest LOS component received. In [21], it is revealed that the LOS component represents more than 95% of the total received optical power; hence, the diffuse component is negligible compared to the LOS component. Therefore, it is reasonable to focus on the LOS component in indoor VLC system designs. With only the LOS component considered, the channel attenuation between the nth LED and the kth user is formulated as

$$h_{kn} = \begin{cases} \frac{\rho A(m+1)n_c^2}{2\pi l^2 \sin^2\psi_c} \cos^m(\phi) T_s(\psi)\cos(\psi), & \psi \leq \psi_c \\ 0, & \psi > \psi_c \end{cases} \tag{7}$$

where ρ is the detector's responsivity, A is the area of the PD, n_c is the refractive index of the optical concentrator, l is the distance between the LED and the user, m is related to the half power semi-angle $\Phi_{1/2}$ via $m = -\ln 2/\ln(\cos(\Phi_{1/2}))$, ϕ is the angle of irradiance, ψ_c is the receiver's field of view (FOV), ψ is the angle of incidence, and $T_s(\psi)$ is the gain of an optical filter.

After removing the transmitter's DC offset at the receiver, the useful received signal of the kth user is expressed as

$$y_k = \mathbf{h}_k^{\mathrm{T}}\mathbf{W}\mathbf{s} + n_k = \mathbf{h}_k^{\mathrm{T}}\mathbf{w}_k s_k + \sum_{j \neq k} \mathbf{h}_k^{\mathrm{T}}\mathbf{w}_j s_j + n_k \tag{8}$$

where $\mathbf{h}_k = [h_{k1}, \ldots, h_{kN}]^{\mathrm{T}}$ is the channel vector spanning from the LED array to the kth user, and n_k is the additive Gaussian noise constituted by the shot noise plus thermal noise in VLC. It is the second term of (8), which represents the MUI that has to be suppressed. The variance of the noise, and the relevant parameters can be found in [21]. Consequently, the mean square error (MSE) between the received signal and the original legitimate transmitted signal of the kth user is given by

$$\begin{aligned} \mathrm{MSE}_k &= \mathrm{E}\left\{(c_k y_k - s_k)^2\right\} = \mathrm{E}\left[\left(c_k \mathbf{h}_k^{\mathrm{T}}\mathbf{w}_k - 1\right)^2 s_k^2\right] + \sum_{j \neq k} \mathrm{E}\left[\left(c_k \mathbf{h}_k^{\mathrm{T}}\mathbf{w}_j\right)^2 s_j^2\right] + \mathrm{E}\left[c_k^2 n_k^2\right] \\ &= c_k^2\left[\sum_{j=1}^{M} u^2\left(\mathbf{h}_k^T\mathbf{w}_j\right)^2 + \sigma_k^2\right] - 2c_k u^2 \mathbf{h}_k^T \mathbf{w}_k + u^2 \end{aligned} \tag{9}$$

where c_k is the receiver coefficient of the kth user, which offers an additional degree of freedom that can be used for system optimization, and σ_k^2 is the variance of the noise. Note that the MSE is a compelling performance metric that strikes a trade-off between taking into account both the effects of the channel and the noise, which will, hence, be adopted in this paper. The objective of the paper is to develop efficient transceivers (including both the TPC and the receiver coefficients) for our MU-MISO VLC system by minimizing the MSE between the transmitted and received signals of the users in the presence of the per-LED optical power constraints.

3. Transceiver Designs for MU-MISO Indoor VLC

In this section, our MU-MISO VLC transceiver is designed under specific per-LED optical power constraints for suppressing the multiuser interference, while maintaining stable illumination. Again, the coefficients of both the TPC and of the receiver are found by minimizing the maximum MSE between the received and legitimate transmitted signals of the users. We first develop the optimal transceiver scheme and then propose a simplified design relying on ZF TPC.

3.1. Optimal Transceiver Design

Let us now formulate the VLC MU-MISO transceiver design as the following optimization problem that minimizes the maximum MSE between the received and legitimate transmitted user signals under per-LED optical power constraints as

$$\min_{\mathbf{W},\{c_k\}} \max_{k=1,\ldots,M} \mathrm{MSE}_k \quad \text{s.t.} \quad \|\mathbf{W}^T\mathbf{e}_n\|_1 \leqslant p'_n, \quad n=1,\ldots,N \tag{10}$$

where the MSE_k is calculated according to (9), and $p'_n = \min\{p_n, p_{\max} - p_n\}$. Explicitly, the TPC matrix $\mathbf{W}$ and the receiver coefficient c_k have to be jointly optimized. Therefore, we will perform the optimization in two stages, firstly we optimize c_k for a fixed TPC $\mathbf{W}$ and then optimize $\mathbf{W}$. For a fixed $\mathbf{W}$, the optimal receiver coefficient c_k satisfying (10) is given by

$$c_k^* = \arg\min_{c_k} \mathrm{MSE}_k = \arg\min_{c_k} c_k^2\left(\sum_{j=1}^{M} u^2\left(\mathbf{h}_k^{\mathrm{T}}\mathbf{w}_j\right)^2 + \sigma_k^2\right) - 2c_k u^2 \mathbf{h}_k^{\mathrm{T}}\mathbf{w}_k + u^2. \tag{11}$$

The above equation accrues from the fact that c_k can only influence the MSE of the kth user. Therefore, the optimal receiver coefficient c_k^* can be calculated as

$$c_k^* = \frac{u^2\mathbf{h}_k^T\mathbf{w}_k}{\sum_{j=1}^{M} u^2\left(\mathbf{h}_k^T\mathbf{w}_j\right)^2 + \sigma_k^2}. \tag{12}$$

Substituting (12) into (9) yields the following MSE expression, where only the TPC matrix $\mathbf{W}$ is involved:

$$\mathrm{MSE}'_k = u^2 - \frac{u^4\left(\mathbf{h}_k^{\mathrm{T}}\mathbf{w}_k\right)^2}{\sum_{j=1}^{M} u^2\left(\mathbf{h}_k^{\mathrm{T}}\mathbf{w}_j\right)^2 + \sigma_k^2}. \tag{13}$$

In the next step, we will use the MSE expression in (13) to develop the optimal TPC matrix $\mathbf{W}$. The optimization problem of (10) may be rewritten as

$$\min_{\mathbf{W}} \max_{k=1,\ldots,M} \mathrm{MSE}'_k \quad \text{s.t.} \quad \|\mathbf{W}^T\mathbf{e}_n\|_1 \leqslant p'_n, \quad n=1,\ldots,N. \tag{14}$$

The L_1 norm in (14) is equivalent to the following set of linear inequalities:

$$-\mathbf{d}_n\mathbf{W}^{\mathrm{T}}\mathbf{e}_n \leqslant \mathbf{d}_n, \quad \mathbf{1}^{\mathrm{T}}\mathbf{d}_n \leqslant p'_n, \quad n=1,\ldots,N \tag{15}$$

where $\mathbf{d}_n \in \mathbf{R}^M$ is a newly introduced optimization variable, and $\mathbf{1} \in \mathbf{R}^M$ is an all-one vector. Let us introduce $\mathbf{w} = \text{vec}(\mathbf{W}^\mathsf{T}) = [\mathbf{e}_1^\mathsf{T}\mathbf{W}, \ldots, \mathbf{e}_N^\mathsf{T}\mathbf{W}]^\mathsf{T}$, and $\mathbf{d} = [\mathbf{d}_1^\mathsf{T}, \ldots, \mathbf{d}_N^\mathsf{T}]^\mathsf{T}$. The optical power constraints of all LEDs may be reformulated in the following concise expressions:

$$-\mathbf{d} \leq \mathbf{w} \leq \mathbf{d}, \qquad \mathbf{V}\mathbf{d} \leq \mathbf{p}' \tag{16}$$

where we have $\mathbf{p}' = [p_1', \ldots, p_N']^\mathsf{T}$, $\mathbf{V} = \mathbf{I}_N \otimes \mathbf{1}^\mathsf{T}$, $\mathbf{I}_N$ is the unity matrix of size N and $\otimes$ is the Kronecker product. Consequently, the optical power constraints in (14) are transformed to the linear constraints seen in (16). Based on (13), for the fixed maximum of ε, the formulation $\text{MSE}_k' \leqslant \varepsilon$ will lead to

$$\sum_{j=1}^{M} u^2 \left(\mathbf{h}_k^\mathsf{T}\mathbf{w}_j\right)^2 + \sigma_k^2 \leq \frac{u^4}{u^2 - \varepsilon}\left(\mathbf{h}_k^T\mathbf{w}_k\right)^2, \qquad k = 1, \ldots, M. \tag{17}$$

Rewriting the left hand side of (17) using the L_2 norm, we arrive at the equivalent expression of

$$\left\| \begin{matrix} u\mathbf{W}^\mathsf{T}\mathbf{h}_k \\ \sigma_k \end{matrix} \right\|_2 \leqslant \sqrt{\frac{u^4}{u^2 - \varepsilon}}\left(\mathbf{h}_k^T\mathbf{w}_k\right), \qquad k = 1, \ldots, M. \tag{18}$$

Since we have $\mathbf{w} = \text{vec}(\mathbf{W}^\mathsf{T})$ and $\mathbf{w}_k = \mathbf{W}\mathbf{e}_k$, it follows that

$$\begin{aligned} \mathbf{w}_k &= \text{vec}\left(\mathbf{w}_k^\mathsf{T}\right) = \text{vec}\left(\mathbf{e}_k^\mathsf{T}\mathbf{W}^\mathsf{T}\right) = \left(\mathbf{I}_N \otimes \mathbf{e}_k^\mathsf{T}\right)\text{vec}(\mathbf{W}^\mathsf{T}) = \mathbf{T}_k\mathbf{w} \\ \mathbf{W}^\mathsf{T}\mathbf{h}_k &= \text{vec}(\mathbf{W}^\mathsf{T}\mathbf{h}_k) = \left(\mathbf{h}_k^\mathsf{T} \otimes \mathbf{I}_M\right)\text{vec}(\mathbf{W}^\mathsf{T}) = \left(\mathbf{h}_k^\mathsf{T} \otimes \mathbf{I}_M\right)\mathbf{w} \end{aligned} \tag{19}$$

where we have $\mathbf{T}_k = \mathbf{I}_N \otimes \mathbf{e}_k^\mathsf{T}$. Therefore, we can reformulate (18) as

$$\|\mathbf{A}_k\mathbf{w} + \boldsymbol{\sigma}_k\|_2 \leqslant \sqrt{\frac{u^4}{u^2 - \varepsilon}}\left(\mathbf{h}_k^T\mathbf{T}_k\mathbf{w}\right), \qquad k = 1, \ldots, M \tag{20}$$

with $\mathbf{A}_k \in \mathbf{R}^{(M+1)\times MN}$ and $\boldsymbol{\sigma}_k \in \mathbf{R}^{M+1}$ are defined as

$$\mathbf{A}_k = \begin{bmatrix} \mathbf{h}_k^\mathsf{T} \otimes \mathbf{I}_M \\ \mathbf{0}_{1\times MN} \end{bmatrix}, \quad \boldsymbol{\sigma}_k = [0, \ldots, 0, \sigma_k]^\mathsf{T} \tag{21}$$

where $\mathbf{0}_{1\times MN}$ is an all-zero row-vector of size MN. The feasible set defined by (20) is a second-order cone, which is convex [22]. Hence, the optimization problem of (14) is a quasi-convex problem, which can be efficiently solved by the bisection methods of [23]. The corresponding feasibility problem, which is used in the bisection method to find our TPC, is given by

$$\begin{aligned} \text{find} \quad & \mathbf{w} \\ \text{s.t.} \quad & \|\mathbf{A}_k\mathbf{w} + \boldsymbol{\sigma}_k\|_2 \leqslant \sqrt{\frac{u^4}{u^2 - \varepsilon}}\left(\mathbf{h}_k^T\mathbf{T}_k\mathbf{w}\right), \qquad k = 1, \ldots, M \\ & -\mathbf{d} \leq \mathbf{w} \leq \mathbf{d}; \quad \mathbf{V}\mathbf{d} \leq \mathbf{p}'. \end{aligned} \tag{22}$$

Again, the above problem is a convex SOCP, which can be efficiently solved by standard convex optimization methods, such as the interior-point method [15]. The procedure of obtaining the globally optimal precoder is formally described in Algorithm 1. Given the optimization precision η, there are exactly $\lceil \log_2((\varepsilon_{\max} - \varepsilon_{\min})/\eta) \rceil$ number of iterations in Algorithm 1 [23].

Algorithm 1 Optimal TPC design algorithm

1: Let $[\epsilon_1, \epsilon_2]$ $(\epsilon_2 < u^2)$ be the range of MSE, where ϵ_1 is sufficiently small so that (22) is infeasible when $\varepsilon = \epsilon_1$, and ϵ_2 is large enough so that (22) is feasible when $\varepsilon = \epsilon_2$. Let $\eta > 0$ be the desired precision. Initialize $\varepsilon_{\text{min}} = \epsilon_1$ and $\varepsilon_{\text{max}} = \epsilon_2$
2: **while** $\varepsilon_{\text{max}} - \varepsilon_{\text{min}} > \eta$ **do**
3: Set $\varepsilon = (\varepsilon_{\text{max}} + \varepsilon_{\text{min}})/2$.
4: Solve the feasibility problem in (22).
5: If the problem is feasible, then set $\varepsilon_{\text{max}} = \varepsilon$. Otherwise, set $\varepsilon_{\text{min}} = \varepsilon$.
6: **end while**
7: The optimal precoder vector for the kth user is $\mathbf{w}_k^* = \mathbf{T}_k \mathbf{w}'$, where $\mathbf{w}'$ is the last feasible solution to (22).

3.2. Simplified Transceiver Design

In this subsection, we propose a simplified transceiver design imposing a lower computational complexity than the maximum mean square error (MMSE)-optimal one by incorporating the optimum ZF TPC. The ZF TPC is also an attractive transmission strategy, because it strikes a compelling tradeoff between the complexity imposed and the performance obtained. The pseudo-inverse based ZF TPC has been widely used in classic RF communication and it has been shown to be optimal in the sense of the ZF criterion under the total electronic power constraints [24]. However, under the specific optical power constraints, the pseudo-inverse based ZF TPC has been observed to be suboptimal and indeed, it is outperformed by other ZF TPC strategies. At the time of writing, the optimal ZF TPC of VLC MU-MISO systems is unknown. Hence in this subsection, we will solve this open problem and derive the optimal ZF TPC strategy under specific per-LED optical power constraints.

The ZF TPC eliminates the MUI by preprocessing the transmitted signals upon exploiting the knowledge of the CSI at the transmitter. Consequently, the multiuser channels are transformed into independent parallel subchannels. Thus, the matrix product of the TPC and of the channel matrix leads to the diagonal matrix of

$$\mathbf{HW} = \text{diag}\{\gamma\}. \tag{23}$$

Here, $\gamma = [\gamma_1, \ldots, \gamma_M]^{\text{T}}$ is the vector of equivalent channel gains of the parallel subchannels and its elements are all real and non-negative. Assuming that $\mathbf{H}$ is a matrix of full row rank, the traditional pseudo-inverse based ZF TPC of classic RF communication is given by

$$\mathbf{W}_{\text{P}} = \mathbf{H}^{\dagger} \text{diag}\{\gamma\} \tag{24}$$

where $\mathbf{H}^{\dagger} = \mathbf{H}^T (\mathbf{HH}^T)^{-1}$ is the pseudo-inverse of $\mathbf{H}$. Note that according to linear algebra [25], the pseudo-inverse based ZF TPC is merely a special case of the entire set of all ZF TPC satisfying (23). The general structure of the ZF TPC is actually given by

$$\mathbf{W}_{\text{ZF}} = \mathbf{H}^{\dagger} \text{diag}\{\gamma\} + (\mathbf{I}_N - \mathbf{H}^{\dagger}\mathbf{H})\mathbf{B} \tag{25}$$

where $\mathbf{B}$ is an arbitrary real-valued matrix of size $(N \times M)$. We can observe that the pseudo-inverse based ZF TPC is obtained from (25) by setting $\mathbf{B}$ to $\mathbf{0}$.

Taking the ZF condition into consideration, we can formulate the ZF TPC design problem as

$$\min_{\mathbf{W}, \{\gamma_k\}} \max_{k=1,\ldots,M} \text{MSE}'_k \quad \text{s.t.} \quad \|\mathbf{W}^T \mathbf{e}_n\|_1 \leqslant p'_n, \quad n = 1, \ldots, N; \quad \mathbf{HW} = \text{diag}\{\gamma\} \tag{26}$$

where MSE'_k is given in (13). Note that the receiver coefficient c_k is still given by (12), but with the precoder replaced by the ZF TPC obtained by solving (26). In contrast to the design in (14), the above precoder design is suboptimal, since extra ZF constraints are imposed. Nevertheless, we will show that the ZF design of (26) will lead to a low-complexity TPC exhibiting a comparable performance. Note that the problem in (26) can also be solved by Algorithm 1 in an iterative manner. However, in the following, we will show that the optimal ZF TPC can actually be more efficiently obtained by solving a linear problem.

Given that the MUI has been totally eliminated by the ZF TPC, the MSE in (13) can be simplified to

$$\mathrm{MSE}'_k = \frac{u^2\sigma_k^2}{u^2\gamma_k^2+\sigma_k^2}. \tag{27}$$

Note that the optimal solution to the problem produces the same MSE between the received and transmitted signals for all users, i.e., at the optimal point, we have

$$\mathrm{MSE}'_k = \mathrm{MSE}'_l \qquad \forall k, l. \tag{28}$$

The detailed proof is given in Appendix. Consequently, we can focus our attention on the situation, where all users' MSE is equal to a common value.

Then, from (27) and (28), we obtain

$$\frac{\gamma_k}{\sigma_k} = \frac{\gamma_l}{\sigma_l} = \hat{\gamma} \qquad \forall k, l \tag{29}$$

where $\hat{\gamma}$ is a new variable. In this case, we have $\boldsymbol{\gamma} = \hat{\gamma}\boldsymbol{\sigma}$, where $\boldsymbol{\sigma} = [\sigma_1, \ldots, \sigma_M]^{\mathrm{T}}$. Additionally, the MSE can be expressed as

$$\mathrm{MSE}'_k = \frac{u^2}{u^2\hat{\gamma}^2+1}, \qquad k = 1, \ldots, M. \tag{30}$$

Based on (25), the general structure of the ZF TPC can be expressed as

$$\mathbf{W}_{\mathrm{ZF}} = \hat{\gamma}\mathbf{H}^{\dagger}\mathrm{diag}\{\boldsymbol{\sigma}\} + (\mathbf{I}_N - \mathbf{H}^{\dagger}\mathbf{H})\mathbf{B}. \tag{31}$$

Consequently, we can transform the original optimization problem of (26) into

$$\min_{\mathbf{B},\hat{\gamma}} \quad \frac{u^2}{u^2\hat{\gamma}^2+1} \qquad \text{s.t.} \quad \left\| \left(\hat{\gamma}\mathbf{H}^{\dagger}\mathrm{diag}\{\boldsymbol{\sigma}\} + (\mathbf{I}_N - \mathbf{H}^{\dagger}\mathbf{H})\mathbf{B}\right)^{\mathrm{T}}\mathbf{e}_n \right\|_1 \leqslant p'_n, \qquad n = 1, \ldots, N. \tag{32}$$

Now, the minimax ZF TPC problem reduces to a single minimization problem over the new variables $\hat{\gamma}$ and $\mathbf{B}$. Since we have $\hat{\gamma} > 0$ and the objective function in (32) is monotonically decreasing with $\hat{\gamma}$, (32) is equivalent to

$$\max_{\mathbf{B},\hat{\gamma}} \quad \hat{\gamma} \qquad \text{s.t.} \quad \left\| \left(\hat{\gamma}\mathbf{H}^{\dagger}\mathrm{diag}\{\boldsymbol{\sigma}\} + (\mathbf{I}_N - \mathbf{H}^{\dagger}\mathbf{H})\mathbf{B}\right)^{\mathrm{T}}\mathbf{e}_n \right\|_1 \leqslant p'_n, \qquad n = 1, \ldots, N. \tag{33}$$

Similar to the optimal TPC design of Section 3.1, we can equivalently rewrite the optical power constraints as

$$-\mathbf{d} \le \mathrm{vec}\left(\left(\hat{\gamma}\mathbf{H}^{\dagger}\mathrm{diag}\{\boldsymbol{\sigma}\} + (\mathbf{I}_N - \mathbf{H}^{\dagger}\mathbf{H})\mathbf{B}\right)^{\mathrm{T}}\right) \le \mathbf{d}, \quad \mathbf{V}\mathbf{d} \le \mathbf{p}' \tag{34}$$

where the auxiliary variable $\mathbf{d}$ is a real-valued vector with size MN and $\mathbf{V} = \mathbf{I}_N \otimes \mathbf{1}^{\mathrm{T}}$. Since we have

$$\begin{aligned} \mathrm{vec}\left(\left(\hat{\gamma}\mathbf{H}^{\dagger}\mathrm{diag}\{\boldsymbol{\sigma}\}\right)^{\mathrm{T}}\right) &= \hat{\gamma}\mathrm{vec}\left(\left(\mathbf{H}^{\dagger}\mathrm{diag}\{\boldsymbol{\sigma}\}\right)^{\mathrm{T}}\right) \\ \mathrm{vec}\left(\left((\mathbf{I}_N - \mathbf{H}^{\dagger}\mathbf{H})\mathbf{B}\right)^{\mathrm{T}}\right) &= \left((\mathbf{I}_N - \mathbf{H}^{\dagger}\mathbf{H}) \otimes \mathbf{I}_M\right)\mathrm{vec}(\mathbf{B}^{\mathrm{T}}) \end{aligned} \tag{35}$$

the optical power constraints can be reformulated as

$$-\mathbf{d} \le \hat{\gamma}\mathbf{f} + \mathbf{U}\mathbf{b} \le \mathbf{d}, \quad \mathbf{V}\mathbf{d} \le \mathbf{p}' \tag{36}$$

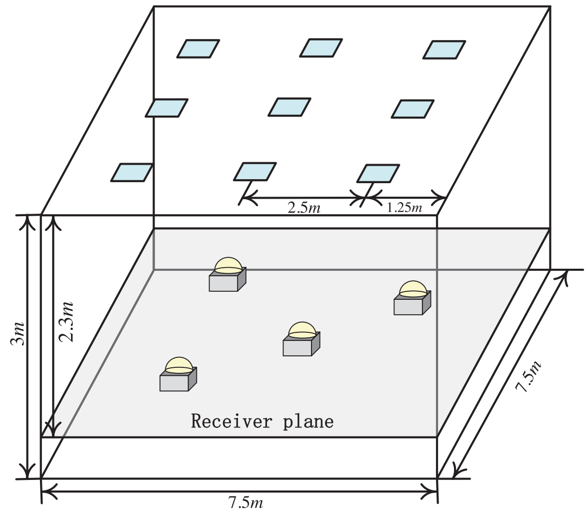

Fig. 3. Multiuser MISO indoor VLC system.

where $\mathbf{b} = \text{vec}(\mathbf{B}^T)$, whereas $\mathbf{f}$ and $\mathbf{U}$ are defined as

$$\mathbf{f} = \text{vec}\Big(\big(\mathbf{H}^\dagger \text{diag}\{\boldsymbol{\sigma}\}\big)^T\Big), \quad \mathbf{U} = \big(\mathbf{I}_N - \mathbf{H}^\dagger\mathbf{H}\big) \otimes \mathbf{I}_M. \tag{37}$$

Therefore, the ZF TPC design problem can be restated as

$$\max_{\mathbf{b},\mathbf{d},\hat{\gamma}} \quad \hat{\gamma} \qquad \text{s.t.} \quad -\mathbf{d} \le \hat{\gamma}\mathbf{f} + \mathbf{U}\mathbf{b} \le \mathbf{d}; \quad \mathbf{V}\mathbf{d} \le \mathbf{p}'. \tag{38}$$

The above problem is a linear programming problem, which can be efficiently solved by diverse mature methods, such as the simplex algorithm of [26]. Let $(\gamma^*, \mathbf{b}^*)$ represent the optimal solution of (38). Consequently, the optimal ZF TPC, which is denoted by $\mathbf{W}^*$, is given by

$$\mathbf{W}^*_{\text{ZF}} = \hat{\gamma}^*\mathbf{H}^\dagger \text{diag}\{\boldsymbol{\sigma}\} + (\mathbf{I} - \mathbf{H}^\dagger\mathbf{H})\mathbf{B}^* \tag{39}$$

where $\mathbf{B}^*$ is related to $\mathbf{b}^*$ through

$$\mathbf{B}^* = [\mathbf{T}_1\mathbf{b}^*, \ldots, \mathbf{T}_k\mathbf{b}^*, \ldots, \mathbf{T}_N\mathbf{b}^*] \tag{40}$$

and $\mathbf{T}_k = \mathbf{I}_N \otimes \mathbf{e}_k^T$. In contrast to the optimal TPC design of Section 3.1, the simplified ZF TPC design only has to solve a linear problem instead of solving a sequence of nonlinear convex problems, thus imposing a substantially reduced complexity.

4. Numerical Results and Discussion

Our numerical results are presented in this section for characterizing the performance of the proposed VLC designs. Note that in contrast to the mobile RF fading channels, the VLC channels are often static, as long as the positions of the users are fixed on the receiver plane. Therefore, we commence by considering the scenario in which the user terminals are stationary. The MU-MISO indoor VLC system shown in Fig. 3 consists of nine LED arrays installed uniformly on the ceiling and a number of user terminals on the receiver plane, which is 2.3 m away from the ceiling. In order to provide uniform brightness in the room, the average optical power of each LED array is set to be identical as $p_n = p \ \forall n$, and the range of p in our numerical results is 5 $\sim$ 15 dBW, which corresponds to 3.16 $\sim$ 31.6 W. In this case, a single LED of the LED array will operate in the range of 0.88 $\sim$ 8.8 mW, which can be readily achieved by the existing LED products, such as GaN-based LEDs [27], [28]. Furthermore, without loss of generality, we assume that the maximum affordable optical power $p_{\max}$ is sufficiently high for ensuring that

TABLE 1

Parameters of the MU-MISO indoor VLC system

Parameter	Value	Parameter	Value
Room size $(W \times L \times H)$	$7.5m \times 7.5m \times 3m$	Number of LED transmitters	9
Data Rate	$100Mbit/s$	Modulation	2PAM
Number of LEDs per array	$3600\,(60 \times 60)$	LED pitch	$0.01m$
Size of LED array	$0.59m \times 0.59m$	Transmitter semi-angle	$70deg.$
Receiver FOV	$60\ deg.$	Detector responsivity	$0.53A/W$
Physical area of the detector	$1cm^2$	Gain of optical filter	1.0
Refractive index	1.5		

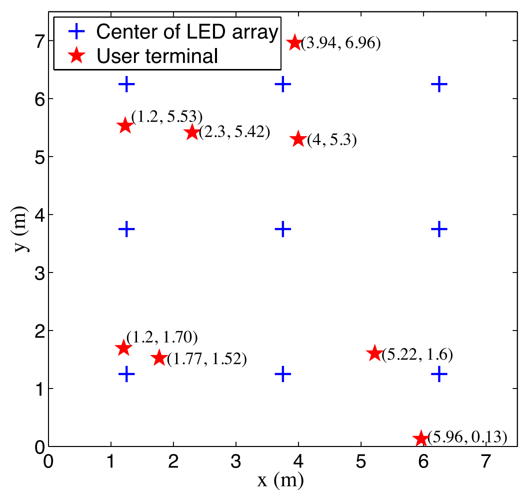

Fig. 4. Distribution of user terminals on the receiver plane.

$p_{\max} - p_n \gg p_n$. Thus, $p'_n = \min\{p_n, p_{\max} - p_n\} = p_n = p$. The per-LED optical power constraints become $\|\mathbf{W}^T \mathbf{e}_n\|_1 \leqslant p'_n = p$, $n = 1, \ldots, N$. The variance of the noise, and the relevant parameters are set according to [21]. The other parameters used in the VLC system are specified in Table 1.

4.1. Performance Analysis for Stationary Locations

We first consider $M = 8$ stationary user terminals distributed across the receiver plane, whose coordinates and relative horizontal locations with respect to the LED arrays are presented in Fig. 4. The proposed transceiver designs and the traditional pseudo-inverse based ZF transceiver (including a pseudo-inverse based ZF TPC and the corresponding receiver coefficient) techniques are used in this multiuser VLC system. We characterize their MSE and BER performance in Fig. 5, where we observe that the optimal transceiver design achieves the best performance. The simplified transceiver design relying on the optimal ZF TPC provides a better performance than the suboptimal pseudo-inverse based ZF transceiver, which indicates that the traditional pseudo-inverse based ZF TPC directly transplanted from classic RF communication is unsuitable for VLC systems.

4.2. Performance Analysis for Random Locations

Next, we assume that the users are randomly and uniformly distributed across the receiver plane and investigate the average performance of different transceiver designs. Due to the intensity modulation employed, the VLC channels will be highly correlated if two users are located

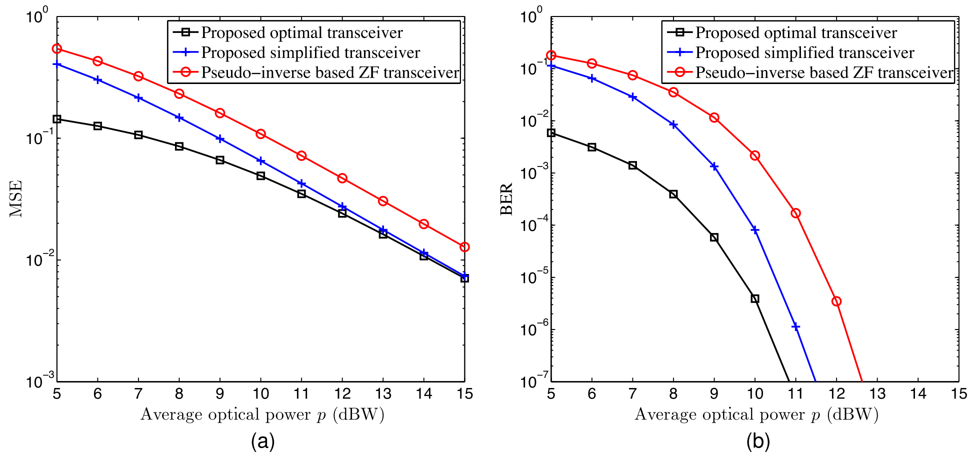

Fig. 5. Performance of the MU-MISO VLC system supporting $M = 8$ user terminals in fixed positions. (a) MSE at different optical power p. (b) BER at different optical power p.

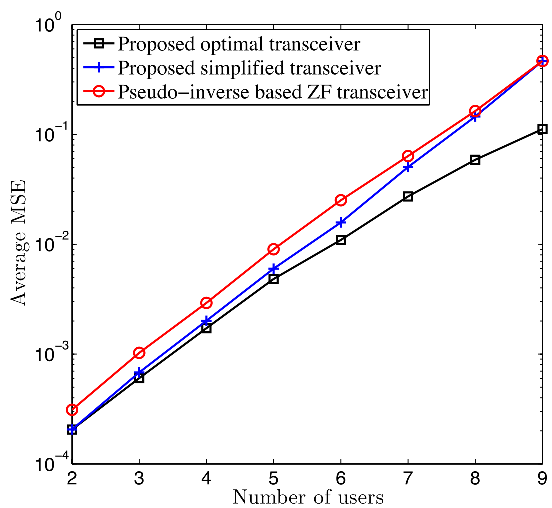

Fig. 6. Average MSE versus the number of users.

close to each other. We assume that the distance between two users is no less than a minimum distance of d, say $d = 0.5$ m. This assumption is reasonable in a multiuser scenario, because the users are generally dispersed across the room. Moreover, appropriate scheduling algorithms can be used for selecting the active user group satisfying the minimum distance requirement [29], [30]. We will first consider the influence of the number of users at a given d and then study the impact of the minimum user-distance d.

Fig. 6 displays the relationship between the average MSE and the number of users associated with $p = 10$ W and $d = 0.5$ m. Observe in Fig. 6 that the proposed simplified transceiver still outperforms the traditional pseudo-inverse based ZF transceiver, especially for $M < 7$ users. The proposed simplified transceiver tends to approach the optimal performance, as the number of users decreases. On the other hand, the performance of the ZF TPC gradually deviates from the optimal performance upon increasing the number of users, which indicates that the transceiver schemes incorporating the ZF TPC perform noticeably worse than the optimal transceiver for $M > 5$ users.

In Fig. 7, we portray the average BER at different average optical powers for $M = 3$ and 7 users. We can clearly observe for $M = 3$ users that the proposed simplified transceiver exhibits

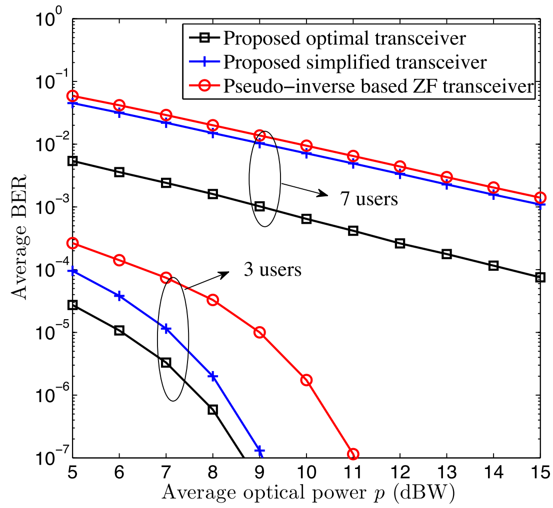

Fig. 7. Average BER performance with $M = 3$ and 7 users.

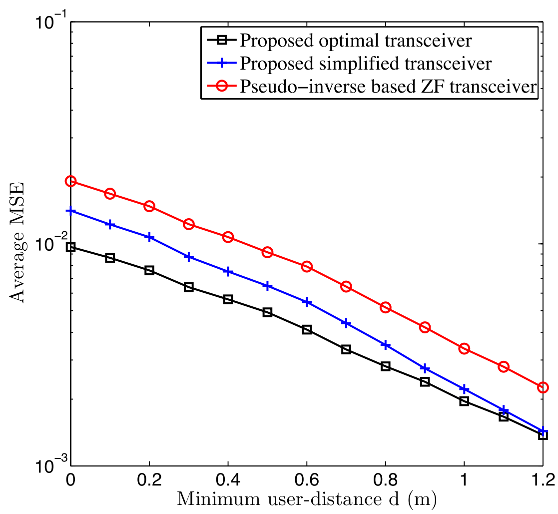

Fig. 8. Average MSE as a function of d for $M = 5$ user terminals and $p = 10$ W.

a performance loss compared to the optimal transceiver but provides a much better performance than the pseudo-inverse based ZF transceiver. However, when there are $M = 7$ users, the performance gap between the optimal and the ZF transceivers becomes significant, which is in line with the above conclusions of Fig. 6.

Furthermore, in Fig. 8 we will investigate the impact of the minimum user-distance d on the average performance of multiuser systems. As mentioned above, the VLC channel's correlation is directly related to the distance between user terminals. In addition, since the LED arrays are generally spread uniformly across the ceiling to provide uniform indoor illumination, the more widely the user terminals are spread across the room, the higher the spatial multiplexing gain becomes. Naturally, the minimum user-distance d reflects the distance distribution of user terminals on the receiver plane. Therefore, we can anticipate that a better performance will be achieved for multiuser systems associated with a larger d.

The average MSE versus d recorded for $M = 5$ users and $p = 10$ W is shown in Fig. 8, where we observe that the average MSE of all the transceiver schemes is reduced upon increasing the minimum user-distance d, which is consistent with the above analysis. We can also observe

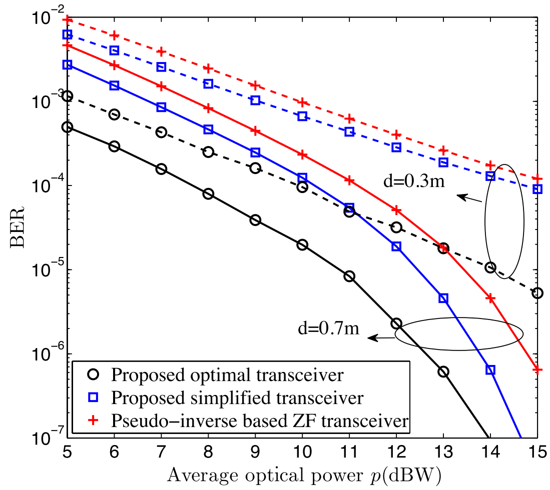

Fig. 9. Average BER as a function of p for $d = 0.3$ m and $d = 0.7$ m.

from Fig. 8 that the performance of the proposed simplified transceiver tends to approach the performance of the optimal transceiver, as the minimum user-distance d increases.

We present the average BER of our multiuser system versus d in Fig. 9, which is significantly reduced upon increasing d from 0.3 m to 0.7 m. At a BER of 10^{-4}, the proposed simplified transceiver requires 5 dB more optical power than the optimal transceiver for $d = 0.3$ m, but only 2.5 dB more for $d = 0.7$ m. Thus, the performance gap between the optimal and the simplified transceivers has been substantially decreased upon increasing the minimum user-distance d.

5. Conclusion

New MU-MISO VLC systems were designed for suppressing the MUI, while offering stable illumination. Under specific per-LED optical power constraints, we first conceived the optimal transceiver, then proposed a simplified transceiver relying on a ZF TPC. Our results have shown that as expected, the optimal transceiver achieves the best performance. The proposed simplified transceiver outperforms the traditional pseudo-inverse based ZF TPC transplanted from RF communication, which indicates that the pseudo-inverse based TPC is suboptimal under the per-LED optical power constraints of VLC. When the number of users decreases or when they are more widely dispersed across the receiver plane, the performance of the simplified transceiver tends to approach that of the optimal transceiver.

Appendix
Proof of the Same Received MSE

In order prove this, let us assume that there is a ZF TPC $\overline{\mathbf{W}} = [\overline{\mathbf{w}}_1, \ldots \overline{\mathbf{w}}_M]$ satisfying the constraints in (26), where we have $\mathbf{H}\overline{\mathbf{W}} = \mathrm{diag}\{\overline{\gamma}_1, \ldots, \overline{\gamma}_M\}$, with $\overline{\mathrm{MSE}}_k$ representing the MSE of the kth user relying on the precoder $\overline{\mathbf{W}}$ and $\overline{\mathrm{MSE}}_{\max} = \max\{\overline{\mathrm{MSE}}_k,\ k = 1, \ldots, M\}$. Then, we have the following properties.

- If the optical power constraints are non-identical, i.e., we have $\|\overline{\mathbf{W}}\mathbf{e}_n\|_1 < p_n\ \forall n$, we can always construct a new TPC $\delta\overline{\mathbf{W}}$ associated with $\delta = \max_n\{p_n/\|\overline{\mathbf{W}}\mathbf{e}_n\|_1\} > 1$, in order to achieve a lower MSE than $\overline{\mathbf{W}}$.
- If there exists a user whose MSE is not equal to $\overline{\mathrm{MSE}}_{\max}$, we can always find another TPC that outperforms $\overline{\mathbf{W}}$. Without loss of generality, let us assume $\overline{\mathrm{MSE}}_m = \overline{\mathrm{MSE}}_{\max}$. Consider the TPC $\overline{\mathbf{W}}' = [\nu_1\overline{\mathbf{w}}_1, \ldots, \nu_k\overline{\mathbf{w}}_k, \ldots \nu_M\overline{\mathbf{w}}_M]$ with $\nu_k = (\overline{\gamma}_m\sigma_k)/(\sigma_m\overline{\gamma}_k)$. In this case, we have $\mathbf{H}\overline{\mathbf{W}}' = \mathrm{diag}\{\overline{\gamma}_m\sigma_1/\sigma_m, \ldots \overline{\gamma}_m\sigma_k/\sigma_m, \ldots \overline{\gamma}_m\sigma_M/\sigma_m\}$. Hence, $\overline{\mathrm{MSE}}'_1 \ldots = \overline{\mathrm{MSE}}'_k = \ldots = \overline{\mathrm{MSE}}'_M = \overline{\mathrm{MSE}}_m = \overline{\mathrm{MSE}}_{\max}$, where

$\overline{\text{MSE}}_k'$ denotes the MSE associated with the TPC $\overline{\mathbf{W}}'$. Then, it follows from (27) that we have $\nu_k \leq 1$ and the equality holds if and only if $\overline{\text{MSE}}_k = \overline{\text{MSE}}_{\max}$. Thus, if $\overline{\text{MSE}}_k < \overline{\text{MSE}}_{\max}$, then $\nu_k < 1$, implying that $\|\overline{\mathbf{W}}'\mathbf{e}_n\|_1 < \|\overline{\mathbf{W}}\mathbf{e}_n\|_1 \leq p_n\ \forall n$. In this case, the optical power constraints are strictly unequal for $\overline{\mathbf{W}}'$, and $\overline{\mathbf{W}}'$ can be scaled proportionally to obtain a lower MSE than that of $\overline{\mathbf{W}}$.

From the above properties, we can conclude that the optimal solution to the problem of (26) produces the same MSE between the legitimate transmitted and received signals of the users.

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