

# 1                    **Modelling the optimal phosphate fertiliser and soil management strategy for crops**

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## 12    **Abstract:**

13    *Aims* The readily available global rock phosphate (P) reserves may be depleted within the next 50-  
14    130 years warranting careful use of this finite resource. We develop a model that allows us to assess  
15    a range of P fertiliser and soil management strategies for Barley in order to find which one  
16    maximises plant P uptake under certain climate conditions.

17    *Methods* Our model describes the development of the P and water profiles within the soil. Current  
18    cultivation techniques such as ploughing and reduced till gradient are simulated along with fertiliser  
19    options to feed the top soil or the soil right below the seed.

20    *Results* Our model was able to fit data from two barley field trials, achieving a good fit at early  
21    growth stages but a poor fit at late growth stages, where the model underestimated plant P uptake.  
22    A well-mixed soil (inverted and 25 cm ploughing) is important for optimal plant P uptake and  
23    provides the best environment for the root system.

24    *Conclusions* The model is sensitive to the initial state of P and its distribution within the soil profile;  
25    experimental parameters which are sparsely measured. The combination of modelling and  
26    experimental data provides useful agricultural predictions for site specific locations.

27

28    **Keywords** Mathematical modelling, phosphate, fertiliser strategy, barley field study, soil buffer  
29    power

## 30    **Introduction**

31    Within the agricultural industry, the management of soils and crops varies widely around the world  
32    (Jordan-Meille *et al.*, 2012), and slight adjustments to reduce costs and/or increase crop yields can  
33    make substantial differences on the global scale. The demand for food is increasing; from 1992 to  
34    2012 the production of cereals worldwide increased from 1.97 billion to 2.55 billion tonnes  
35    (<http://faostat.fao.org/>). In 2012 the UK alone produced 19.5 million tonnes of cereals, 5.52 million  
36    of which was barley. One of the most important nutrients for plant growth is phosphate (P), which is  
37    often the most limiting due to its low mobility in soils (Bucher 2007). The current world rate of P  
38    consumption for fertilisers is not sustainable, and there are warnings that readily available global

39 rock P reserves may be depleted within the next 50-130 years (Déry and Anderson, 2007; Cordell,  
40 Drangert and White, 2009; Vaccari D, 2009).

41 European governments (DEFRA, 2010 and Lalor *et al.*, 2013) are reducing the amount of P  
42 fertilisation in agricultural sites to reduce soil P content from a high Olsen P index 3 (26-45 mg l<sup>-1</sup>) to  
43 either index 2 (16-25 mg l<sup>-1</sup>) or index 1 (10-15 mg l<sup>-1</sup>), as an attempt to increase the sustainable use  
44 of P. However, lower P content soils can lead to reduced yields (Withers *et al.*, 2014). Therefore it is  
45 vital to identify optimal soil management strategies for more efficient use of P (Dungait *et al.*, 2012).  
46 However, optimal strategies can depend upon the current climate and the distribution of P within  
47 the soil. The distribution of P is a feature which is generally unknown for field situations, but is  
48 becoming more regularly sampled (Vu *et al.*, 2009 and Stutter *et al.*, 2012).

49 Farmers implement a range of phosphate fertiliser and soil management strategies based on  
50 information from a variety of sources. The fertiliser manual (RB209) published by the Department  
51 for Environmental, Food & Rural Affairs (DEFRA) provides a guide to farmers as to the amount of  
52 fertiliser to use for given soil types (DEFRA, 2009). Field-specific advice is also given by agronomists  
53 based on previous P use. Previous history of any specific site also remains an important factor as  
54 repeating cropping strategies for similar environments provides experience on which strategies  
55 perform best (Reijneveld *et al.*, 2010). The general guidelines in the RB209 manual for applying  
56 fertiliser are based on soil P concentrations, often taken from spot measurements. The amount of P  
57 application recommended is classified into different categories. However, this classification means  
58 that soils can have entirely different fertiliser recommendations if they have similar soil P  
59 concentrations, but lie across the boundaries of the classification. This leads to a varying selection of  
60 treatments on similar plots of land and makes it difficult to reduce the amount of P in soils, as a  
61 recent study in Ireland showed (Lalor *et al.*, 2012). Site-specific guidelines may provide a better basis  
62 to implement optimal fertiliser and soil cultivation strategies when it comes to cultivating crops. The  
63 aim is to more efficiently use applied P, not over-apply in cases where it is not needed or under  
64 apply it and not meet crop yield targets. Therefore, instead of having a table of discrete amounts of  
65 fertiliser to add, a simple linear or saturating continuously graded expression could govern how much  
66 P to add. Also, a better classification of soils is needed; much like the varied descriptions of soils in  
67 Scotland (Soil Survey of Scotland Staff, 1981).

68 Increasing information collected about soil type and characteristics will provide a better  
69 understanding of fertiliser placement and amount to apply, resulting in a more successful crop for a  
70 given season. However, collecting detailed data about soils is expensive. In addition, it is difficult to  
71 ascertain how much data is actually needed to give the best prediction for a successful strategy  
72 (Kamprath *et al.*, 2000). Mathematical models can provide the analysis needed to evaluate a large  
73 range of strategies that cannot all be tested at the field scale, due to time, money and location  
74 specific restrictions (Selmants and Hart, 2010; Jeuffroy *et al.*, 2012). Once optimal strategies are  
75 found, they can be tested and evaluated among other strategies to prove their validity, in the hope  
76 of better understanding when, where, and how much P to apply to soils.

77 It is expensive to experimentally determine the distribution and movement of water and P within  
78 the soil and the consequent uptake into the plant root system. The use of modelling in combination  
79 with experimental data allows us to predict optimal management strategies in agricultural systems.  
80 Many models exist that estimate water and P movement within soil. For example Dunbabin *et al.*

81 (2013) developed a model that predicts plant P uptake by estimating the distribution of P in 3D. The  
82 3D P information can be combined with other models, such as one that estimates the fractal  
83 geometry of simulated root systems in 1, 2 and 3D (Lynch *et al.*, 1997). Models used to describe the  
84 root system often consider a density of root mass for a given volume in soil. The root mass can be  
85 estimated from averaging a 3D growth approach (Lynch *et al.*, 1997; Chen *et al.*, 2013) or by  
86 considering a 3D growth model, for example L systems (Leitner *et al.*, 2010). However, due to  
87 memory and computational limitations, the above models are not appropriate for up-scaling to the  
88 field level as they can lead to numerical inaccuracies of up to 30% when compared to computed  
89 plant P uptake (Roose and Schnepf, 2008). Other models focus on the root architecture and the  
90 uptake of P by the root system (Ge *et al.*, 2000; Lynch and Brown, 2001; Grant and Robertson, 1997;  
91 Roose *et al.*, 2001). Roose *et al.* (2001) capture the P depletion zone along all roots and obtain an  
92 analytical solution; their model estimates plant P uptake per soil surface area which can be used to  
93 predict plant P uptake on a field scale. Roose and Fowler (2004b) advanced the model by tracking  
94 the movement of water and P spatially. In this paper, for the first time, we extend the model of  
95 Roose and Fowler (2004b) and Heppell *et al.*, (2015) by adding the effect of climate, via surface  
96 water flux and xylem pressures as in Heppell *et al.* (2014). This extension allows comparison of the  
97 model output, plant P uptake ( $\text{kg P ha}^{-1}$ ), against two sets of field trial data for barley, for different  
98 environmental conditions. In addition, we incorporate temperature-dependent root growth so that  
99 the model can be used for winter crops, as there is little or no root/plant growth at low  
100 temperatures. Following this, the extended model is used to predict the best fertiliser and soil  
101 cultivation strategy which maximises plant P uptake. As a result, the optimal strategy should also  
102 maximise 'P-use efficiency' within a low P environment. Our mathematical model is based on well-  
103 known equations governing P and water movement within the soil (Roose and Fowler, 2004b), and  
104 the aim of this work is to see if the model can explain variations in P uptake observed at field sites. If  
105 not, this indicates that further development of the model or model inputs is required.

106 In the Materials and Methods section we first describe details of the Roose and Fowler (2004b)  
107 model and then the adaptations made to it. We then describe how the data are collected and the  
108 values used for the model. Modelling results are described in the results section followed by a  
109 discussion section describing our findings and future avenues for work.

## 110 **Materials and Methods**

### 111 ***Roose and Fowler model***

112 Roose and Fowler (2004b) model water and P flow through soil to calculate uptake into a  
113 surrounding plant root system using a Richards Equation coupled to a diffusion-convection equation  
114 describing P movement in the soil. The model assumes that the soil is homogeneous and neglects  
115 horizontal movement of water and P, since at the field scale the differences in the horizontal  
116 variation for the root length density are negligible compared to the vertical variation (Roose and  
117 Fowler, 2004a). For model simplicity we assume that there is a concentration of P available to the  
118 root system (P in solution, 'available P') and a concentration sorbed to the soil particles (P sorbed,  
119 'non-available P'). Many new papers use the term 'available P' to represent this state of P in the soil,  
120 for example Johnson *et al.*, 2014. The Roose and Fowler model is described by the following two  
121 equations for water and P conservation, respectively,

122 
$$\phi \frac{\partial S}{\partial t} = \nabla \cdot [D_0 D(S) \nabla S - K_s k(S) \hat{\mathbf{k}}] - F_w(S, z, t),$$
 Eqn. 1

123 
$$\frac{\partial}{\partial t} [(b + \phi S)c] + \nabla \cdot [c\mathbf{u}] = \nabla \cdot [D_f \phi^d S^d \nabla c] - F(c, S, t),$$
 Eqn. 2

124 where the speed of water movement in the soil,  $\mathbf{u}$ , is given by Darcy's law,

125 
$$\mathbf{u} = -D_0 D(S) \nabla S + K_s k(S) \hat{\mathbf{k}}.$$
 Eqn. 3

126 In above equations  $S$  is the relative water saturation given by  $S = \phi_1 / \phi$ ,  $\phi_1$  is the volumetric water  
 127 content, and  $\phi$  is the porosity of the soil.  $D_0$  ( $\text{cm}^2 \text{day}^{-1}$ ) and  $K_s$  ( $\text{cm day}^{-1}$ ) are the parameters for  
 128 water 'diffusivity' and hydraulic conductivity, respectively.  $D(S)$  and  $k(S)$  characterize reduction in  
 129 water 'diffusivity' and hydraulic conductivity in response to the relative water saturation decrease,  
 130 where the functional forms for partially saturated soil are given by Van Genuchten (1980).  $\hat{\mathbf{k}}$  is the  
 131 vector pointing vertically downwards from the soil surface and  $F_w$  is the water uptake by the plant  
 132 root system per unit volume of soil as given by Roose and Fowler (2004a).

133 For the P mass conservation (Eqn. 2),  $c$  is the P concentration in pore water,  $b$  is the soil buffer  
 134 power,  $D_f$  is the P diffusivity in free water and  $d$  is an impedance factor given by the range  $1.5 \leq$   
 135  $d \leq 3$  (Barber, 1984; Nye and Tinker, 1977).  $F(c, S, t)$  describes the rate of P uptake by a  
 136 surrounding root branching structure as in Roose *et al.* (2001). Both  $F_w$  and  $F$  are affected by the  
 137 spatially and temporally evolving root structure. Water is only taken up by the main order roots  
 138 while P is taken up by all roots.

139 For the soil surface boundary condition, Roose and Fowler (2004b) apply a flux of water at the soil  
 140 surface denoted by  $W_{dim}$  ( $\text{cm s}^{-1}$ ), which is the volume flux of water per unit soil surface area per  
 141 unit time;

142 
$$-D_0 D(S) \frac{\partial S}{\partial z} + K_s k(S) = W_{dim} \quad \text{at } z = 0.$$
 Eqn. 4

143 The soil surface boundary condition for P, for a rate of fertilisation  $Q_{dim}$  ( $\mu\text{mol cm}^{-2} \text{s}^{-1}$ ), is given  
 144 by

145 
$$-D_f \phi^d S^d \frac{\partial c}{\partial z} + W_{dim} c = Q_{dim} \quad \text{at } z = 0.$$
 Eqn. 5

146 The boundary condition at the 'bottom' of the soil is assumed to be a zero flux boundary condition  
 147 at a given level  $l_w$ , for both water and P, respectively,

148 
$$-D_0 D(S) \frac{\partial S}{\partial z} + K_s k(S) = 0 \quad \text{at } z = l_w,$$
 Eqn. 6

149 
$$-D_f \phi^d S^d \frac{\partial c}{\partial z} = 0 \quad \text{at } z = l_w.$$
 Eqn. 7

150 Solving for relative water saturation ( $S$ ) and P concentration ( $c$ ) produces water and P profiles in  
 151 depth and time.

152 The calculation of  $F$  and  $F_w$  depends on the plant root structure in the soil. The root growth rate  
 153 equation used in the Roose *et al.* (2001) model assumes that the rate of growth slows down over  
 154 time, i.e., the rate of growth is given by,

155 
$$\frac{\partial l_i}{\partial t} = r_i \left(1 - \frac{l_i}{K_i}\right),$$
 Eqn. 8

156 where  $l_i$  is the length of the order  $i$  root,  $r_i$  is the initial rate of growth of the order  $i$  root and  $K_i$  is  
 157 the maximum length of an order  $i$  root.

158 ***Adaptations to the Roose and Fowler Model***

159 To include climate effects within the Roose and Fowler (2004b) model, we let the flux of water into  
 160 the soil ( $W_{dim}$ ) be dependent upon rainfall, wind speed, temperature and humidity. This allows for a  
 161 more accurate calculation of the plant transpiration rate and the movement of water inside the soil  
 162 and within the plant. These adaptations are made in Heppell *et al.* (2014) and successfully capture  
 163 the movement of water within the soil profile and plant transpiration rate.

164 To model the water saturation levels in the soil, the flux of water into the soil ( $W_{dim}$ ) is estimated  
 165 from a combination of environmental factors. These include rainfall ( $R$ ), humidity ( $H$ ), wind speed  
 166 ( $WS$ ), temperature ( $T$ ) and a constant ( $E$ ), using a linear expression,

167 
$$W_{dim} = \delta R + \alpha H + \beta T + \gamma WS + E,$$
 Eqn. 9  
 168

169 where the parameters  $\delta, \alpha, \beta, \gamma$  and  $E$  are determined from the optimal fit to the soil water  
 170 saturation and climate data (Heppell *et al.*, 2014). The flux of water ( $W_{dim}$ ) can essentially be  
 171 considered as a Taylor expanded version of any other non-linear soil surface water permeation  
 172 relationship, for example the Penman-Monteith Equation (Beven, 1979). Therefore, the formulation  
 173 of Equation (9) allows for easy comparison with other models, such as Cropwat (Clarke *et al.*, 1998),  
 174 should this be necessary.

175  
 176  
 177 The driving pressure,  $P$  (Pa), inside the root is determined by the environmental conditions  
 178 (humidity and temperature) causing the stomata in the leaves to open and close (Tuzet et al, 2003).  
 179 When the air temperature is low and/or humidity is high, the plant opens its stomata to speed up  
 180 the loss of water and cause cooling. This leads to a decrease in the pressure of water inside the roots.  
 181 Thus the water pressure within the plant roots ( $P$ ) is given by,

182 
$$P = (p_r^0 + \lambda_3) + \lambda_1 T + \lambda_2 H,$$
 Eqn. 10

183 where  $p_r^0$  (Pa) is the baseline xylem pressure and  $\lambda_1$  (Pa/degC),  $\lambda_2$  (Pa/% humidity) and  $\lambda_3$  (Pa)  
 184 are determined by seeking the optimal fit to soil saturation data and are used to help calculate  $F_w$   
 185 (Heppell *et al.*, 2014). These parameters have been determined by Heppell *et al.* (2014) for a given  
 186 geographical monitoring site.

187 A new feature is added to the model to match the root growth over the cropping season (where  
 188 little growth is seen over the winter period) by making the rate of growth temperature dependent.  
 189 This transforms Equation (8) into,

190 
$$\frac{\partial l_i}{\partial t} = r(T(t)) \left(1 - \frac{l_i}{K_i}\right),$$
 Eqn. 11

191 where  $r(T(t))$  is taken from experimental data on temperature dependant root growth rates, Table  
 192 1.

193 In summary, the data needed for the adapted model to run includes: initial distributions of water  
194 and P concentrations in the soil, climate data for rainfall, humidity, wind speed and temperature  
195 values, fertiliser application and amount, soil cultivation strategy and temperature dependant root  
196 growth rates which are obtained from experimental data. Henceforth, when referring to the 'model'  
197 we mean the adapted model extended from the one by Roose and Fowler (2004b).

## 198 **Data collection**

### 199 **From the literature**

200 To run the adapted mathematical model a set of parameters were taken from Roose and Fowler  
201 (2004b), Heppell *et al.* (2014 and 2015) and Sylvester-Bradley *et al.* (1997), consisting of values for  
202 plant root dynamics and soil characteristics, Table 2.

### 203 **Pot trials**

204 To assess temperature effects on root growth rates in cereals *Triticum aestivum* seeds were soaked  
205 overnight in aerated de-ionised water to induce germination. They were then placed on filter papers,  
206 moistened with deionised water, put in parafilm sealed Petri dishes covered in aluminium foil and  
207 incubated at 20°C. After 48 hours the root lengths of each emerged seminal root were measured  
208 non-destructively using a ruler. The filter papers were re-moistened and the Petri dishes were  
209 grouped into different controlled temperatures, heating at 5, 10, 20 and 30°C. After another 24  
210 hours the lengths of the seminal roots were measured with WINRHIZO, and the differences in root  
211 length for each root were recorded as the average root growth rate per day.

212 Plant root growth rates increased from 5°C at which a zero growth rate was observed, Table 1. A  
213 straight line was fitted to the data such that the information could be translated into the  
214 mathematical model, for temperature  $T$  we set the growth rate  $R$  to be,

$$215 \quad R = \begin{cases} 0 & \text{for } T \leq 5^{\circ}\text{C} \\ 0.053(T - 5) & \text{for } T > 5^{\circ}\text{C} \end{cases} \quad \text{Eqn. 12}$$

216 Barley and wheat root structures are genetically different but phenotypically similar (Kutschera *et al.*,  
217 2009). The mathematical model uses the root morphology not its genetics and therefore we assume,  
218 consistent with Kutschera, that experimental data from wheat roots is a good first approximation for  
219 barley roots in this instance.

### 220 **Field trials**

221 Two data sets were taken from field scale trials, which consisted of a set of scenarios for different  
222 fertiliser application techniques and measurement of plant P uptake values (offtake); one winter  
223 barley and one spring barley. A decimal code system is used to measure the growth stages of barley  
224 based on description stages (Broad, 1987). The winter barley data includes values for P offtake at  
225 two different periods, growth stages 39 and 92; 232 and 313 days respectively. The winter barley  
226 variety was Winsome winter malting barley. Differing amounts of triple superphosphate (TSP) were  
227 incorporated (0, 15, 30, 60, 90 and 120 kg P ha<sup>-1</sup>) or banded (15 and 30 kg P ha<sup>-1</sup>) in the soil. The trial  
228 was on a clay soil with a low P index, based in Stetchworth, UK. The spring barley data includes  
229 values for P offtake at three different periods, growth stages 31, 45 and 91; 61, 77 and 151 days  
230 respectively. The spring barley variety used was Shuffle, being grown from seed, with typical farm

231 inputs used (e.g. fertiliser, herbicide, fungicide, etc.) except P which was imposed based on  
232 experimental requirements. Differing amounts of TSP were incorporated (0, 5, 10, 20, 30, 60 and 90  
233 kg P ha<sup>-1</sup>) or banded (10, 20 and 30 kg P ha<sup>-1</sup>) in the soil. The trial was on a sandy clay loam soil with a  
234 low P index, based near Aberdeen in Scotland, approximately 57°N. The trial was ploughed in  
235 January and ground power harrowed on the day of sowing (23-March-2011). The crop was rolled  
236 after sowing to consolidate the seedbed and reduce the risk of stone damage to harvesting  
237 equipment.

238 The field scale data only uses one Olsen P value for a given plot and there is no distinction  
239 concerning how P is distributed with depth. To provide a description of how plant-available P varies  
240 with soil depth, soil was collected from different depths within a spring wheat field trial located at  
241 Abergwyngregyn, North Wales. The soil there was classified as a free draining sand textured Eutric  
242 Cambisol. Samples were taken from four replicate plots (3 m x 12 m in size) at growth stage 39 at 10  
243 cm intervals down the soil profile to a depth of 1 m. Three sets of soils were tested, Olsen P index 5,  
244 3 and 2 (Fig. 1a). Plant-available P was determined by extracting the soil with 0.5 M acetic acid (1:5  
245 w/v) for 30 min, centrifuging the extracts (4000 g, 15 min) and colorimetric determination of P  
246 according to Murphy and Riley (1962).

247 In addition to the soil sites in North Wales, six sites within the UK with a sandy clay loam were  
248 sampled for Olsen P (Fig. 1b). Soil samples were taken at 0-30 cm, 30-60 cm and 60-90 cm. The sites  
249 varied from a high Olsen P index of 6 to a low index of 2. A similar P distribution was seen at 4 sites  
250 (Olsen P index 5 and 3) where P concentration decays exponentially with depth as in the free  
251 draining sand in Wales, whereas the other two sites (Olsen P index 6 and 2) had a constant P profile.  
252 To represent the P profile data a constant P profile with depth and an exponentially decaying P  
253 profile with depth will be compared in the model simulations.

#### 254 **Fertiliser strategies**

255 The model adapted in this paper is used to mimic field trials and to predict plant P uptake (kg P ha<sup>-1</sup>).  
256 In addition to the scenarios used in the field trial experimental data, we analyse the effects of  
257 different environmental conditions for a range of fertiliser and soil cultivation strategies.

258 We estimate that on average the ploughing depth is 25 cm. In Heppell *et al.* (2014) we used climate  
259 data (hourly values for temperature, humidity, wind speed and rainfall) to calibrate a plant water  
260 uptake model; the same parameters and data are used within the current extended model.

261 The amount of fertiliser applied in an average cropping season ranges from 0 to 120 kg P ha<sup>-1</sup>.  
262 Fertiliser, for example TSP, can be applied in two different ways, banded and broadcasted. The  
263 banded application involves injecting fertiliser pellets 5-10 cm below the soil and 5-10 cm away from  
264 the seed. This is represented in the model as fertiliser placed 9 cm below the seed. The aim of this  
265 strategy is to put fertiliser next to where most of the roots are likely to grow to try to maximise root  
266 P uptake. The broadcasted approach spreads fertiliser only on top of the soil.

267 The extended model predicts how different fertiliser strategies influence plant P uptake. The set of  
268 fertiliser strategies compared in the model are shown in Fig. 2. The soil is first cultivated and then  
269 fertiliser is applied. During the cultivation phase, different methods are used to mix P in the soil.  
270 Ploughing evenly mixes P to a specific depth between 10-25 cm, whereas a reduced till gradient

271 distributes P into bands; 0-5cm, 5-10cm and 10-15cm with a P concentration ratio of 1.5:1:0.5  
272 respectively; inverted plough inverts the P concentration between 0-15 cm; and lastly there is an  
273 option of no cultivation. We model top soil fertilisation, fertilisation applied at 9 cm below the seed  
274 or no fertilisation, and use climate data with or without an increased amount of rainfall. For each  
275 strategy the model predicts plant P uptake which is then compared to a control with no fertilisation  
276 or cultivation for a given soil type and climate data.

## 277 **Results**

278 Our adapted model in this paper is fitted against experimental field trial data to produce a site  
279 specific model. A selection of fertiliser strategies are then simulated using the model (Fig. 2), and  
280 values for plant P uptake are compared to predict which strategy might, under certain climate  
281 conditions, estimate the highest plant P uptake.

282 By looking at the experimental data we find that the initial P distribution in the soil has a high  
283 concentration at the top of the soil and then the concentration decays with depth; at 1 m there is  
284 very little P left (Fig. 1). This decay is much stronger for higher initial P concentrations, whereas at P  
285 index 1 there are almost indistinguishable changes in the P distribution (no decay). To assess the  
286 difference at P index 1 between a constant and an exponentially decaying P profile, we will model  
287 both profiles. In each case (constant and exponentially decaying P profiles) the total P down to 0.55  
288 m is kept identical to represent similar amounts of P being available to the root system. The P  
289 profiles for a constant and exponentially decaying distribution are represented in Fig. 5a and 5c,  
290 respectively, for time = 0 days.

291 The model fits the winter barley data better at growth stage 39 (GS 39) compared with growth stage  
292 92 (GS 92). At GS 39 the model predictions are within the error bars with the exception of the 30 kg  
293 P ha<sup>-1</sup> placed scenario (Fig. 3a). At GS 92 the model under predicts on all scenarios, but follows the  
294 trend of increasing plant P uptake values for increasing amounts of TSP applied (Fig. 3b). The main  
295 reason for the under prediction stems from the unknown parameters, which include soil buffer  
296 power and the initial P profile in the soil. Other plant factors, such as root exudates or mycorrhizae,  
297 could also have increased P availability and hence plant P uptake but are not taken into account  
298 within the model since they were not monitored and quantified in the experiments. Wheat is not  
299 thought to be strongly mycorrhizal (Li *et al.*, 2005) and conceivably neither is barley. The initial P  
300 profile, at index 1, is depleted before the end of harvest and the final total plant P uptake is  
301 therefore capped. This depletion effect is also seen when modelling the spring barley data (Fig. 4b,  
302 4c), and in addition at GS 31 the model fails to capture the effects between small and large amounts  
303 of TSP applied, fitting well at 0-20 kg ha<sup>-1</sup>, but not at 30-90 kg ha<sup>-1</sup> (Fig. 4a). In regards to the spring  
304 barley crop, GS 31 is only a short time of 61 days and this is perhaps why little effects are seen  
305 between modelling different amounts of applied TSP. The amount of available P is unaffected by an  
306 additional supply as there is only a small root system generated by GS 31. The plant P uptake  
307 estimate from the model, on average decreases from a constant P distribution to an exponentially  
308 decaying P distribution. There is a decrease of 4.7% (GS 39) and 18.3% (GS 92) for winter barley, and  
309 -10.5% (GS 31), -12.3% (GS 45) and 5% (GS 91) for spring barley. The reason for a negative value (i.e.  
310 decrease as opposed to increase in plant P uptake) for spring barley at GS 31 and 45 is because the  
311 root system is small, and as a consequence the P deeper in the soil profile has not been utilised.

312 The depletion of P for different initial P profiles can be seen in Fig. 5. In a low P content soil (P1) with  
313 an exponentially decaying initial P distribution there is a reduction in the plant P uptake rate after  
314 147 days. This is because the majority of the available P is taken up at an early growth stage. This  
315 effect is not seen with a constant initial P distribution as P is spread out more evenly with depth;  
316 however the available P is still all taken up by the end of the simulation (GS 92, 313 days). For a high  
317 P content soil (P3) there is no decrease in the plant P uptake rate and most of the available P is taken  
318 up by the root system.

319 We tested the sensitivity of the model output, plant P uptake, for two different parameters (soil  
320 buffer power and initial volumetric soil water content) to see if unknown or badly measured  
321 parameters would have an effect. We compared four different soil buffer power values 20, 23.28, 30  
322 and 40 and found that plant P uptake is very sensitive to the soil buffer power value (Fig. 6a). Plant P  
323 uptake values at GS39 ranged between 8-12 kg P ha<sup>-1</sup>, a large difference for only a small change in  
324 realistically measured soil buffer power values.

325 We also changed the initial volumetric soil water content, however little differences of 1% are seen  
326 between starting values of 0.1 to 0.5 (Fig.6b). Thus, the initial volumetric soil water content has little  
327 effect on plant P uptake. Instead, the climate conditions throughout the cropping season affect plant  
328 P uptake as discussed below.

329 We run the model for a range of fertiliser and soil cultivation strategies under wet and normal  
330 climate conditions at GS 92, for an initial low P Olsen index soil (P1 – 20 mg l<sup>-1</sup> P 'decay'; Fig. 7a-  
331 normal, Fig. 7b-wet) and a high P Olsen index soil (P3 – 60 mg l<sup>-1</sup> P 'decay'; Fig. 7c-normal, Fig. 7d-  
332 wet). Instead of considering different amounts of applied fertiliser, six cultivation techniques are  
333 simulated (mix 25, 20 and 10 cm, inverted plough, minimum tillage and no cultivation) alongside 3  
334 fertiliser treatments (placed 90 kg P ha<sup>-1</sup>, incorporated 90 kg P ha<sup>-1</sup> and no fertiliser). At GS 92 the  
335 highest plant P uptake is achieved from an inverted plough down to 15 cm and placing 90 kg P ha<sup>-1</sup>,  
336 followed by mixing the soil to 25cm and placing 90 kg P ha<sup>-1</sup>. Under a wet climate, plant P uptake  
337 values are increased on average by 2% across all fertiliser and soil cultivation strategies; the highest  
338 increase of 5% was seen when broadcasting fertiliser. When broadcasting fertiliser the increased  
339 water helped diffuse the top soil P and allowed more to be taken up by the plant. It should be noted  
340 that under field conditions, an increase in soil water content can lead to greater root growth which  
341 would increase plant P uptake more so than just via P diffusion. However, for this study the  
342 modelled root system is only temperature dependent, future studies may include additional root  
343 growth effects. In a high P index soil (P3) there is almost no response to plant P uptake values when  
344 adding P fertiliser, which is to be expected. For a low P index soil, plant P uptake is limited due to a  
345 lack of available P (depletion of P as seen in Fig. 5) and this results in little distinction between  
346 ploughing techniques. Root chemotropism (stimulation of root growth by the added P) was not  
347 considered within this model since it was not possible to quantify this in the experiments.

348 In summary, applying P near the rooting zone (inverted plough and mixing at 25 cm while placing  
349 fertiliser) provides the best chance for maximising plant P uptake; under certain conditions placing  
350 fertiliser (banding) rather than broadcasting can result in an 11% increase to plant P uptake.

## 351 Discussion

352 To determine the optimal strategy for maximising plant P uptake, a set of fertiliser and soil  
353 cultivation strategies are simulated in the model. The difference if broadcasting and banding  
354 fertiliser is chosen depends upon price, accessibility, soil cultivation etc. (Mahler, 2001). For example,  
355 applying fertiliser 20 cm away from the plant and at a depth of 10 cm in the soil gave optimal  
356 conditions for a certain Maize plant study (Owusu-Gyimah *et al.*, 2013), and Randall and Hoefl (1988)  
357 found placing (banding) P better than broadcasting because of the enhanced P concentration within  
358 the rooting zone. However, similar yields were seen between applying large amounts of P fertiliser  
359 via broadcasting or banding, and it was effects from starter P with rates as low as 10 kg P ha<sup>-1</sup> that  
360 dramatically increased corn yields (Sultenfuss and Doyle, 1999). The model predicted that in a single  
361 harvest the ability to mix P in the rooting zone (inverted plough and mix at 25 cm) is highly desired  
362 over a minimum tillage gradient. In addition, placing fertiliser (banding) below the seed, rather than  
363 broadcasting, gave a sizeable increase of 11% to plant P uptake (6% for a wet climate). The effect of  
364 a heavy rainfall throughout the cropping season slightly increased average plant P uptake by 2%  
365 across all scenarios. The additional water enhances diffusion of P in the soil, and hence increases  
366 plant P uptake.

367 The field trial data only had one Olsen P index to characterise the amount of available P in the soil.  
368 To represent this in the model, we let the P concentration in the soil have either an exponentially  
369 decaying or constant distribution with depth. By only knowing sparse information about the initial P  
370 concentration in the soil, a number of problems can arise. Firstly, if the concentration of P found in  
371 the soil is near a boundary (between Olsen P index 2 and 3, for example) then it is treated as an  
372 average in that category. Set amounts of fertiliser are prescribed to such soils and in certain cases  
373 this can cause a waste of resources (Hooda *et al.*, 2001). In countries such as Ireland, there are  
374 stricter rules to the amount of applied P added to soils. Obtaining only one soil test for a field site  
375 can be misrepresentative and allow for more fertiliser to be added where perhaps it is not necessary.  
376 Secondly, there is a range between each Olsen P index and modelling a particular indexed soil can be  
377 ambiguous. For example, the model estimates that in a P index 1 soil, using an initial constant P  
378 distribution of 10mg P l<sup>-1</sup> will give a lower plant P uptake than 15mg P l<sup>-1</sup> by 33%. Perhaps further  
379 classification is needed when characterising soils, to more accurately prescribe an optimal amount of  
380 fertiliser to add. This is the case in Scotland, where soils are given extra classification (namely  
381 descriptive features including, colour, texture, structure, consistence, organic matter, roots, stones,  
382 moisture, mottles and thickness of the horizon) to help use fertiliser more efficiently (Soil Survey of  
383 Scotland Staff, 1981).

384 Current methods for calculating available P in soil are not consistent across Europe, with a wide  
385 range of techniques, each with their own methods, causing similar soils to have uncorrelated results  
386 (Neyroud and Lischer, 2002; Jordan-Meille *et al.*, 2012). This provides confirmation that due to these  
387 current methods, site specific treatments are needed and one method cannot be used on all soils.  
388 However, new methods are being developed that calculate the amount of available P within the soil,  
389 that use more advanced methods compared to the very sensitive approach of Olsen P for example  
390 (Van Rotterdam *et al.*, 2009). One method, Diffusive Gradients in Thin films (DGT) measures the  
391 diffusion of P taken from a soil sample to calculate the available P (Tandy *et al.*, 2011). These new  
392 approaches are attempting to develop a robust method for all soils and if successful could result in a  
393 breakthrough and a better understanding of P dynamics within the soil. The more accurate soil  
394 measurements are the better estimates models can provide.

395 Within some field sites there is little notion of how available P is distributed within the soil profile,  
396 with respect to depth. The idea that the majority of P added through fertilisers is given to the crop is  
397 partly true, as a set amount is sorbed to the soil particles. However, from the modelling work  
398 presented in this paper we can conclude that the distribution of initial P within the soil profile  
399 significantly affects total plant P uptake. There was an increase in plant P uptake, from a constant P  
400 distribution to an exponentially decaying P distribution, of 18.3% (GS 92) for winter barley, and 5.0%  
401 (GS 91) for spring barley. The field data for the distribution of P with depth, showed an exponential  
402 decay of available P, with the majority of P situated within the top 30 cm. The steepness of the  
403 decay differs from P index to P index, decreasing with lower P content. In addition, it has been  
404 shown that the steepness of decay for similar P content soils also differs from site to site (Jobbágy  
405 and Jackson, 2001) and this could alter the optimal fertiliser strategy. Data concerning the state and  
406 distribution of P within the soil is now becoming more available, as it can be used to save on  
407 fertiliser costs (Yang *et al.*, 2013).

408 The soil buffer power value, a term used to describe the relationship between available and non-  
409 available P (in equilibrium), is very sensitive within the model. The higher the soil buffer power value  
410 the greater amount of P is sorbed to the soil compared to being in solution (Van Rees *et al.*, 1990).  
411 Small changes to the soil buffer power value cause plant P uptake values to vary by 50% (for soil  
412 buffer power values of 40 and 23.28, Fig. 5). Field trial data generally has at best one value for the  
413 soil buffer power for a plot of land, despite the fact that there is evidence to show that this values  
414 changes within plots, and even with depth (Bhadoria *et al.*, 1991). The soil buffer power is not a  
415 single soil property but a combination of soil properties and P fertiliser history. Therefore, to  
416 accurately model the available P within the soil, the soil buffer power value should be validated for  
417 site specific data and this could affect the optimal fertiliser and soil cultivation strategy. Deriving  
418 relationships between soil P values and buffer power can help estimate fertiliser requirements to  
419 maximise crop yield and/or increase fertiliser efficiency (Moody, 2007). Improving P fertiliser  
420 recommendations follows from a better prediction of P availability in the soil. This can be achieved  
421 from just two soils tests (P-AL and P-CaCl<sub>2</sub>) which approximate P intensity, P quantity and buffering  
422 capacity. The combination of these three values yields higher quality estimations of the soil P supply  
423 potential to an artificial P sink (Van Rotterdam *et al.*, 2014).

424 Plant access to sorbed P depends on both the soil and the plant, where the strength of the soil P  
425 sorption site can greatly affect the amount of P released to the available P pool during a growing  
426 season. A plant root structure with significant numbers of root hairs can deplete solution P to low  
427 levels at the root surface, which can result in a greater P diffusion in response to the steeper  
428 concentration gradient. For a high soil buffer power value there is a lower chance of adding P and  
429 getting a response in plant P uptake. In addition, when P levels in soil are high, possibly due to over-  
430 fertilisation (Borda *et al.*, 2011), there is an increased loss of P to surface water resulting in  
431 eutrophication (Hartikainen, Rasa & Withers, 2010).

432 The idea to draw down sites from a high P index 3, to 1 and 2 is achievable, but happening at a much  
433 slower rate due to over fertilisation where it is not necessarily needed (Lalor *et al.*, 2012). It is  
434 therefore important to study which processes can help improve crop yields in low P content soils  
435 and perhaps more information is needed in this area. For example, field tests and the collection of  
436 more data in conjunction with models are necessary for the future.

437 When a low-P soil is first fertilised, appreciable amounts of sorbed P penetrate the particle. That  
438 fertiliser therefore becomes less effective, and there is a sorption/desorption hysteresis (Okajima *et*  
439 *al.*, 1983). However as a consequence of the penetration, the negative charge on the reacting  
440 particles increases and the buffering capacity therefore decreases. Consequently the effectiveness of  
441 further applications increases until the pathways become saturated. When this happens, the  
442 adsorption/desorption hysteresis disappears and P becomes more effective (Bolland and Baker,  
443 1998). This hysteresis effect is of importance for models which simulate long term fertiliser  
444 strategies. However, for short term models (up to one crop season) it is adequate to use the buffer  
445 power to approximate the amount of added P that becomes sorbed to the soil.

446 Within this paper we have studied the effect of plant P uptake in barley for different fertiliser and  
447 soil cultivation strategies given certain initial conditions. However in reality, these initial conditions  
448 change from year to year and the best strategy in one year is not necessarily best in the following  
449 year. A sustainable strategy is needed as well as a way of estimating how this will affect the soil 5 or  
450 10 years from now. As long term field trials are expensive, models provide the ability to simulate the  
451 effects of different strategies *in silico*. This work has given us a better understanding of the  
452 important factors concerning cultivation methods and fertiliser treatments, with the aim to guide  
453 future field studies on potential optimal strategies which can improve P efficiency in crops.

#### 454 **Acknowledgements**

455 We would like to thank the BBSRC and DEFRA (BB/I024283/1) for funding S.P. and The Royal Society  
456 University Research Fellowship for funding T.R. K.C.Z. was partially funded by Award No. KUK-C1-  
457 013-04 of the King Abdullah University of Science and Technology (KAUST); J.F. by EPSRC and  
458 CORMSIS; J.H. by EPSRC Complexity DTC; and S.P., P.T., D.L., R.S-B., R.W., D.L.J. and T.R. by DEFRA,  
459 BBSRC, Scottish Government, AHDB, and other industry partners through Sustainable Arable LINK  
460 Project LK09136.

#### 461 **References**

- 462 **Barber S (1984)** Soil nutrient bioavailability: a mechanistic approach. Wiley-Interscience.
- 463 **Beven, K. 1979.** A sensitivity analysis of the Penman-Monteith actual evapotranspiration estimates.  
464 *Journal of Hydrology* **44**(3-4): 169-190.
- 465 **Bhadoria P, Kaselowsky J, Claassen N, Jungk A (1991)** Soil Phosphate Diffusion Coefficients: Their  
466 Dependence on Phosphorus Concentration and Buffer Power. *Soil Science of America Journal* **55**(1):  
467 56-60.
- 468 **Bolland M, Baker M (1998)** Phosphate applied to soil increases the effectiveness of subsequent  
469 applications of phosphate for growing wheat shoots. *Australian Journal of Experimental Agriculture*  
470 **38**(8): 865-869.
- 471 **Borda T, Celi L, Zavattaro L, Sacco D, Barberis E (2011)** Effect of agronomic management on risk of  
472 suspended solids and phosphorus losses from soil waters. *Journal of Soils and Sediments* **11**(3): 440-  
473 451.

474 **Broad H (1987)** The decimal code for the growth stages of cereals, with illustrations. *Annals of*  
475 *Applied Biology*. **110**: 441-454.

476 **Bucher M (2007)** Functional biology of plant phosphate uptake at root and mycorrhiza interfaces.  
477 *New Phytologist*. **173** (1): 11-26

478 **Chen Y, Dunbabin V, Postma J, Diggle A, Siddique K, Rengel Z (2013)** Modelling root plasticity and  
479 response of narrow-leaved lupin to heterogeneous phosphorus supply. *Plant and soil*. **372**(1-2): 319-  
480 337.

481 **Clarke D, Smith M, El-askari K (1998)** New software for crop water requirements and irrigation  
482 scheduling. *ICID Bulletin of the International Commission on Irrigation and Drainage*. **47**(2):45-48.

483 **Cordell D, Drangert J, White S (2009)** The story of phosphorus: Global food security and food for  
484 thought. *Global Environmental Change*. **19**(2): 292-305.

485 **Department for Environment, Food and Rural Affairs (DEFRA) (2010)** Fertiliser Manual (RB209). The  
486 Stationery Office. ISBN 978 0 11 243286 9.

487 **Déry P, Anderson B (2007)** Peak phosphorus. In: Energy Bulletin, 08/13/2007. Post Carbon Institute.  
488 Available: [energubulletin.net/node/33164](http://energubulletin.net/node/33164).

489 **Dunbabin V, Postma J, Schnepf A, Pages L, Javaux M, Wu L, Leitner D, Chen Y, Rengel Z, Diggle A**  
490 **(2013)** Modelling root-soil interactions using three-dimensional models of root growth, architecture  
491 and function. *Plant and soil*. **372**(1-2): 93-124. DOI 10.1007/s11104-013-1769-y.

492 **Dungait J, Cardenas L, Blackwell M, Wu L, Wither P, Whitmore A, Murray P, Chadwick D, Bol R,**  
493 **Macdonald A, Goulding K (2012)** Advances in the understanding of nutrient dynamics and  
494 management in UK agriculture. *Science of the Total Environment*. **434**: 39-50.

495 **Ge Z, Rubio G, Lynch J (2000)** The importance of root gravitropism for inter-root competition and  
496 phosphorus acquisition efficiency: results from a geometric simulation model. *Plant and Soil* **218**(1-  
497 2): 159-171.

498 **Grant R, Robertson J (1997)** Phosphorus uptake by root systems: mathematical modelling in ecosys.  
499 *Plant and Soil* **118**(2): 279-297.

500 **Hartikainen H, Rasa K, Withers P (2010)** Phosphorus exchange properties of European soils and  
501 sediments derived from them. *European Journal of Soil Science* **61**(6): 1033-1042.

502 **Heppell J, Payvandi S, Zygalakis K, Smethurst J, Fliege J, Roose T (2014)** Validation of a spatial-  
503 temporal soil water movement and plant water uptake model. *Geotechnique*. **64**(7): 526-539.

504 **Heppell J, Talboys P, Payvandi S, Zygalakis K, Fliege J, Withers P, Jones D, Roose T (2015)** How  
505 changing root system architecture can help tackle a reduction in soil phosphate (P) levels for better  
506 plant P acquisition. *Plant, Cell & Environment*. **38**:118-128

507 **Hooda P, Truesdale V, Edwards A, Withers P, Aitken M, Miller A, Rendell A (2001)** Manuring and  
508 fertilization effects on phosphorus accumulation in soils and potential environmental implications.  
509 *Advances in Environmental Research* **5**(1): 13-21.

- 510 **Jeuffroy M, Vocanson A, Roger-Estrade J, Meynard J (2012)** The use of models at field and farm  
511 levels for the ex ante assessment of new pea genotypes. *European Journal of Agronomy* **42**(October),  
512 68-78.
- 513 **Jobbágy E, Jackson R (2001)** The distribution of soil nutrients with depth: Global patterns and the  
514 imprints of plants. *Biogeochemistry* **53**(1): 51-77.
- 515 **Johnson J, Fixen P, Poulton P (2014)** The efficient Use of Phosphorus in Agriculture. *Better Crops*  
516 **98**(4): 22-24.
- 517 **Jordan-Meille L, Rubæk G, Ehlert P, Genot V, Hofman G, Goulding K, Recknagel J, Provolo G,**  
518 **Barraclough P (2012)** An overview of fertilizer – P recommendations in Europe: soil testing,  
519 calibration and fertilizer recommendations. *Soil Use and Management* **28**(4): 419-435.
- 520 **Kamprath E, Beegle D, Fixen P, Hodges S, Joern B, Mallarino A, Miller R, Sims J, Ward R, Wolf A**  
521 **(2000)** Relevance of Soil Testing to Agriculture and the Environment. *Council for Agricultural Science*  
522 *and Technology*. Issue paper, No. 15.
- 523 **Kutschera L, Lichtenegger E, Sobotik M (2009)** Wurzelatlas der kulturpflanzen gemabigter Gebiete  
524 mit Arten des Feldemusebaues. *DLG-Verlag* 201-229. ISBN: 978-3-7690-0708-4.
- 525 **Lalor S, Wall D, Plunkett M (2013)** Maintaining optimum soil fertility – focus on offtake. Proceedings  
526 of Spring Scientific Meeting 2013, *The fertilizer association of Ireland*. No. 48.
- 527 **Leitner D, Klepsch S, Bodner G, Schnepf A (2010)** A dynamic root system growth model based on L-  
528 systems. Tropisms and coupling to nutrient uptake from soil. *Plant and Soil* **332**(1-2): 117-192.
- 529 **Li H, Zhu Y, Marschner P, Smith F, Smith S (2005)** Wheat responses to arbuscular mycorrhizal fungi  
530 in a highly calcareous soil differ from those of clover, and change with plant development and P  
531 supply. *Plant and Soil* **277**: 221-232.
- 532 **Lynch J, Nielsen K, Davis R, Jabllokow A (1997)** SimRoot: Modelling and visualization of root systems.  
533 *Plant and Soil* **188**(1): 139-151.
- 534 **Lynch J, Brown K (2001)** Topsoil foraging – an architectural adaptation of plants to low phosphorus  
535 availability. *Plant and Soil* **237**(2): 225-237.
- 536 **Mahler R (2001)** Fertilizer Placement. CIS 757. Soil Scientist, Department of plant, Soil, and  
537 Entomological Sciences, University of Idoha.
- 538 **Moody P (2007)** Interpretation of a single-point P buffering index for adjusting critical levels of the  
539 Colwell soil P test. *Soil Research* **45**(1): 55-62.
- 540 **Murphy T, Riley J (1962)** A modified single solution method for the determination of phosphate in  
541 natural waters. *Analytica Chimica Acta* **27**:31-36.
- 542 **Neyroud J, Lischer P (2002)** Do different methods used to estimate soil phosphorus availability  
543 across Europe give comparable results? *Journal of Plant Nutrition Soil Science* **166**(4): 422-431.
- 544 **Nye P, Tinker P (1977)** Solute movement in the soil-root system. Blackwell science publishers.

545 **Okajima H, Kubota H, Sakuma T (1983)** Hysteresis in the phosphorus sorption and desorption  
546 processes of soils. *Soil Science and Plant Nutrition* **29**(3): 271-283

547 **Owusu-Gyimah V, Nyatuame M, Ampaw F, Ampadu P (2013)** Effect of depth and Placement  
548 Distance of Fertilizer NPK 15-15-15 on the Performance and Yield of Maize Plant. *International*  
549 *Journal of Agronomy and Plant Production*. **4**(12): 3197-3204.

550 **Randall G, Hoefst R (1988)** Placement Methods for Improved Efficiency of P and K Fertilizers: A  
551 Review. *Journal of Production Agriculture*. **1**(1): 70-79.

552 **Reijneveld J, Ehlert P, Termorshuizen A, Oenema O (2010)** Changes in the soil phosphorus status of  
553 agricultural land in the Netherlands during the 20th century. *Soil Use and Management* **26**(4): 399-  
554 411.

555 **Roose T, Fowler A, Darrah P (2001)** A mathematical model of plant nutrient uptake. *Mathematical*  
556 *biology*, **42**(4): 347-360.

557  
558 **Roose T, Fowler A (2004a)** A model for water uptake by plant roots. *Journal of theoretical*  
559 *biology*, **288**(2): 155-171.

560  
561 **Roose T, Fowler A (2004b)** A mathematical model for water and nutrient uptake by plant root  
562 systems. *Journal of theoretical Biology*, **288**(2): 173-184.

563  
564 **Roose T, Schnepf A (2008)** Mathematical models of plant-soil interaction. *Philosophical transactions.*  
565 *Series A, Mathematical, physical, and engineering sciences* **366**(1885): 4597-611.

566  
567 **Selmants P, Hart S (2010)** Phosphorus and soil development: Does the Walker and Syers model  
568 apply to semiarid ecosystems? *Ecology*, **91**(2): 474-484.

569  
570 **Soil survey of Scotland Staff (1981)** Soil maps of Scotland at a scale of 1:250 000. Macaulay Institute  
571 for Soil Research, Aberdeen.

572  
573 **Stutter M, Shand C, George T, Blackwell M, Bol R, Mackay R, Richardson A, Condron L, Turner B,**  
574 **Haygrath P (2012)** Recovering phosphorus from soil – A root solution? *Environmental Science and*  
575 *Technology*. **46**(4): 1997-1978.

576  
577 **Sultenfuss J, Doyle W (1999)** Better Crops with Plant Food. Phosphorus Fertiliser Placement. A  
578 *Publication of the International Plant Nutrition Institute (IPNI)*. **83**(1): 34-39.

579  
580 **Sylvester-Bradley R, Scott R, Clare R (1997)** The wheat growth guide. London: Home Grown Cereals  
581 Authority, <http://www.hgca.com/media/185687/g39-the-wheat-growth-guide.pdf>, last accessed  
582 12/09/2014.

583  
584 **Tandy S, Mundus S, Yngvesson J, de Bang T, Lombi E, Schjoerring J, Husted S (2011)** The use of DGT  
585 for prediction of plant available copper, zinc and phosphorus in agricultural soils. *Plant and Soil*  
586 **346**(1-2): 167-180.

587  
588 **Tuzet A, Perrier A, Leuning R (2003)** A coupled model of stomatal conductance, photosynthesis and  
589 transpiration. *Plant, Cell & Environment* **26**(7): 1097-1116.

590  
591 **Vaccari D (2009)** Phosphorus: a looming crisis. *Scientific American* **300**(6): 54-49.

592  
593  
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637  
638  
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640  
641  
642

**Van Genuchten M (1980)** A closed-form equation for predicting the hydraulic conductivity of unsaturated soil. *Soil Science Society of America Journal*, **44**(5): 892-898.

**Van Rees K, Comerford N, Rao P (1990)** Defining soil buffer power: implications for ion diffusion and nutrient uptake modelling. *Soil Science Society of America Journal* **54**(5):1505-1507.

**Van Rotterdam A, Temminghoff E, Schenkeveld W, Hiemstra T, Riemsdijk W (2009)** Phosphorus removal from soil using Fe oxide-impregnated paper: Processes and applications. *Geoderma* **151**(3-4): 282-289.

**Van Rotterdam A, Bussink D, Reijneveld J (2014)** Improved Phosphorus Fertilisation Based on Better Prediction of Availability in Soil. International Fertiliser Society, Proceeding 755, ISBN 978-0-85310-392-9.

**Vu D, Tang C, Armstrong R (2009)** Tillage system affects phosphorus form and depth distribution in three contrasting Victorian soils. *Australian Journal of Soil Research*. **47**(1): 33-45.

**Withers P, Sylvester-Bradley R, Jones D, Healey J, Talboys P (2014)** Feed the crop not the soil: rethinking phosphorus management in the food chain. *Environmental Science & Technology* **48**(12): 6523-6530.

**Yang X, Post W, Thornton P, Jain A (2013)** The distribution of soil phosphorus for global biogeochemical modeling. *Biogeosciences Discussions*. **9**(11): 16347-16380.

643 **List of Figures**

644 **Figure 1:** The concentration of P with depth in the soil profile, a) taken at intervals of 10 cm down to  
645 1 m, for three different sites (Olsen P index 2, 3 and 5), b) taken at intervals 0-30 cm, 30-60 cm and  
646 60-90 cm for six different sites (Olsen P index 2, 3, 5 and 6).

647 **Figure 2:** A set of scenarios to test the mathematical model; ploughing at 25, 20 or 10 cm, an  
648 inverted plough or using the reduced till gradient, top soil fertilisation, no fertilisation or fertiliser  
649 applied at 5cm below and to the side of the seed, and finally using climate data with or without an  
650 additional constant heavy rainfall.

651 **Figure 3:** Experimental data and model predictions for winter barley at growth stages 39 (a) and 92  
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653 'decay'.

654 **Figure 4:** Experimental data and model predictions for spring barley at growth stages 31 (a), 45 (b)  
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657 **Figure 5:** Model predictions for winter barley P uptake and P concentration against depth at five  
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660 of 20 mg P l<sup>-1</sup> 'decay' (P3-low) and d) an initial P concentration of 60 mg P l<sup>-1</sup> 'decay' (P3-high).

661 **Figure 6:** Model estimates for winter barley P uptake by the root system at growth stage 39 for a)  
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664 **Figure 7:** Model predictions for the set of scenarios described in Fig. 2, for 6 cultivation strategies  
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666 placement options (90 kg P ha<sup>-1</sup> incorporated (broadcast) or placed (banded) and no fertiliser), for a)  
667 and b) an initial P concentration of 20 mg P l<sup>-1</sup> 'decay' (P1-low) for a normal and wet climate  
668 respectively, and c) and d) an initial P concentration of 60 mg P l<sup>-1</sup> 'decay' (P3-high) for a normal and  
669 wet climate respectively.

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678 **List of Tables**

679 **Table 1:** Wheat root growth rates at four different temperatures, 5, 10, 20 and 30°C measured by  
 680 WINRHIZO after 24 hours.

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Temperature (°C)	5	10	20	30
Average root growth rate (cm day <sup>-1</sup> )	0	0.2340	0.8234	1.299
Standard deviation / number of samples	0	0.0175	0.0150	0.0129

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685 **Table 2:** Types of data used in the modelling and where it is sourced.\* General strategies used on  
 686 fields across the UK were provided by Argii.

Type of data	Parameter	Value	Units	Source
Model parameter	$D_f$	10 <sup>-5</sup>	cm <sup>2</sup> day <sup>-1</sup>	Roose and Fowler 2004b
Model parameter	$d$	3	-	Roose and Fowler 2004b
Model parameter	$l_w$	200	cm	Roose and Fowler 2004b
Model parameter	$k_0$	150	cm	Sylvester-Bradley <i>et al.</i> , 1997
Model parameter	$k_1$	7.9	cm	Heppell <i>et al.</i> , 2015
Model parameter	$b$	23.28	-	Heppell <i>et al.</i> , 2015
Model parameter	$p_r^0$	1	Pa	Heppell <i>et al.</i> , 2014
Model parameter	$D_0$	10 to 900	cm <sup>2</sup> day <sup>-1</sup>	Heppell <i>et al.</i> , 2014
Model parameter	$K_s$	0.05 to 0.5	cm day <sup>-1</sup>	Heppell <i>et al.</i> , 2014
Model parameter	$m$	0.1 to 0.5	-	Heppell <i>et al.</i> , 2014
Model parameter	$\delta$	2.69*10 <sup>-2</sup>	-	Heppell <i>et al.</i> , 2014
Model parameter	$\alpha$	1.2*10 <sup>-6</sup>	m s <sup>-1</sup> of water	Heppell <i>et al.</i> , 2014
Model parameter	$\beta$	2.22*10 <sup>-6</sup>	m s <sup>-1</sup> of water/degC	Heppell <i>et al.</i> , 2014
Model parameter	$\gamma$	5.35*10 <sup>-4</sup>	m s <sup>-1</sup> of water/ m s <sup>-1</sup> of air	Heppell <i>et al.</i> , 2014
Model parameter	$E$	5*10 <sup>-4</sup>	m s <sup>-1</sup> of water	Heppell <i>et al.</i> , 2014
Model parameter	$\lambda_1$	2.7*10 <sup>-3</sup>	Pa/ degC	Heppell <i>et al.</i> , 2014
Model parameter	$\lambda_2$	8.46*10 <sup>-4</sup>	Pa/% humidity	Heppell <i>et al.</i> , 2014
Model parameter	$\lambda_3$	7.9*10 <sup>-2</sup>	Pa	Heppell <i>et al.</i> , 2014
Model input	$\phi$	0.3	-	Roose and Fowler 2004b
Type of data	Description			Source
Model parameter	Temperature dependant root growth as in Table 1			Bangor pot experiment
Model input	Fertiliser strategies, Fig.2			Agrii*
Model input	Cultivation methods, Fig. 2			Agrii*
Model input	Climate values for rainfall, wind speed, temperature and humidity			Heppell <i>et al.</i> , 2014
Model input	P concentrations at different depths, Fig. 1			Bangor field experiment
Model output comparison	P uptake(kg P ha <sup>-1</sup> ) at GS39 and GS92 for Barley, Fig. 3			ADAS field experiment
Model output comparison	P uptake(kg P ha <sup>-1</sup> ) at GS31, GS45 and GS91 for Barley, Fig. 4			SRUC field experiment

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Figure2  
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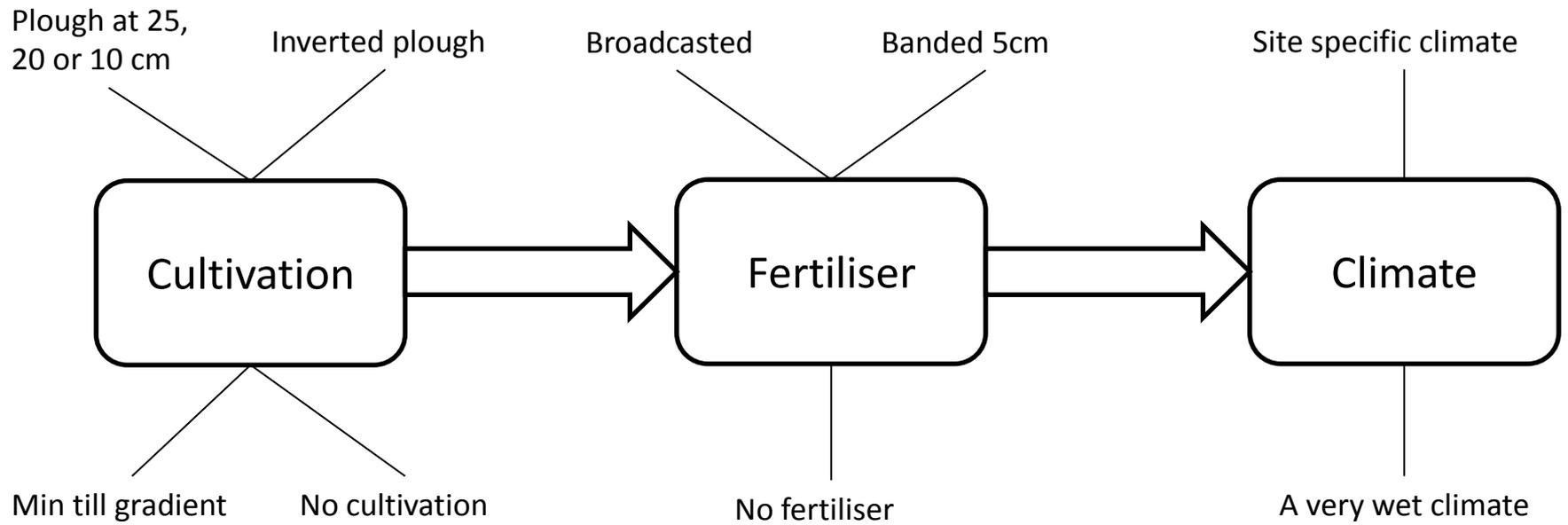


Figure 1

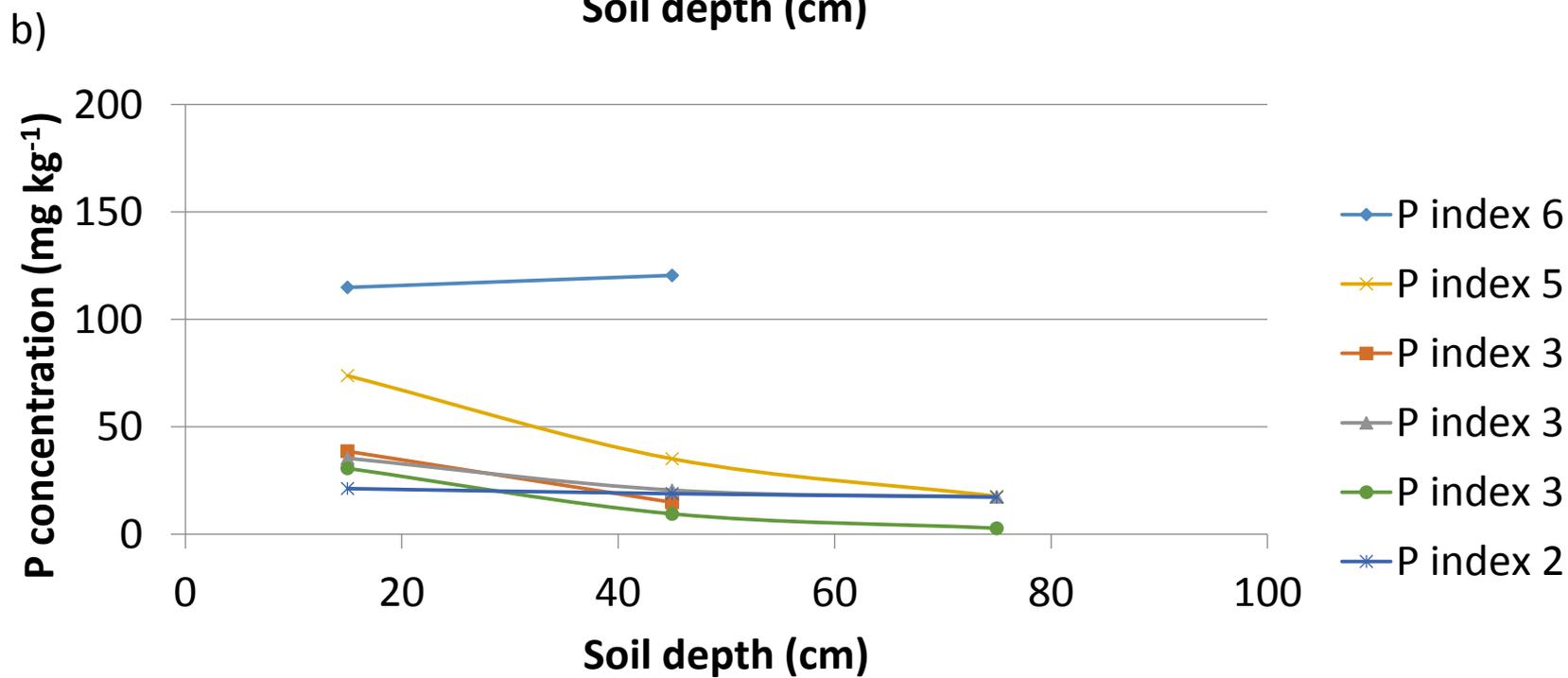
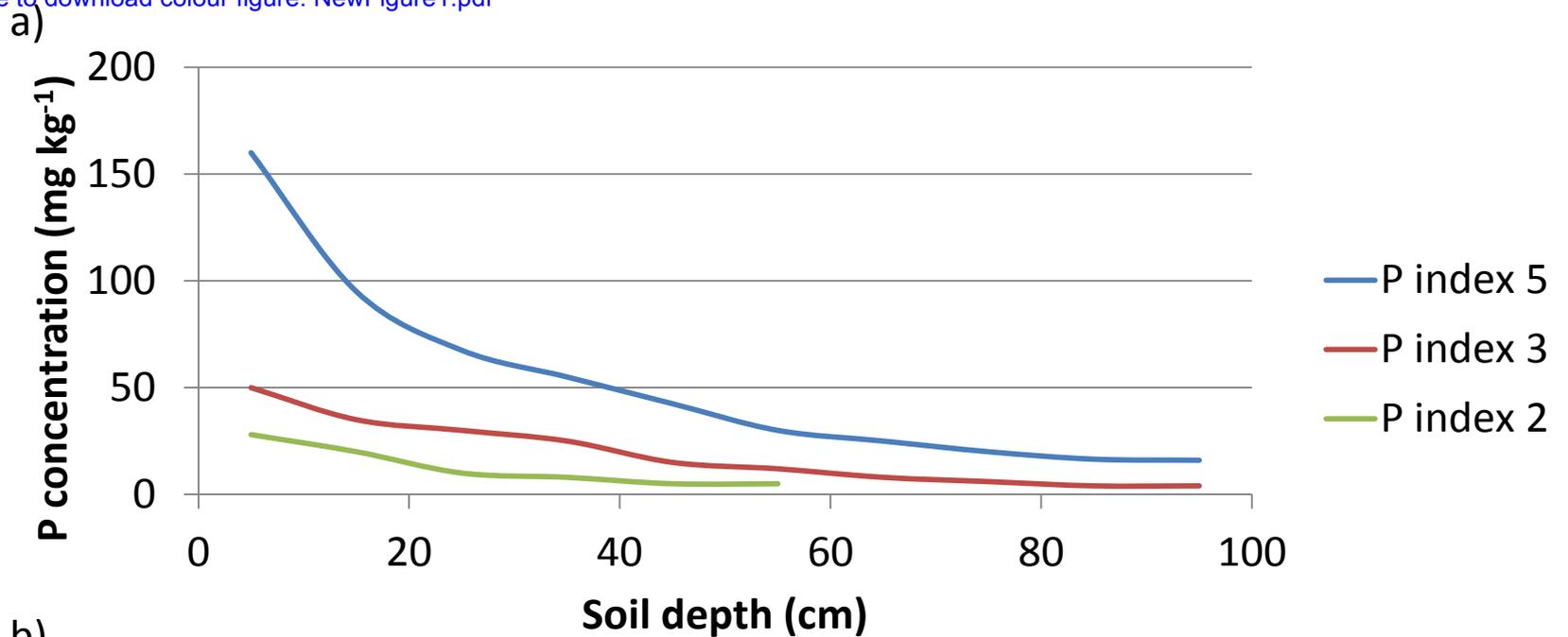
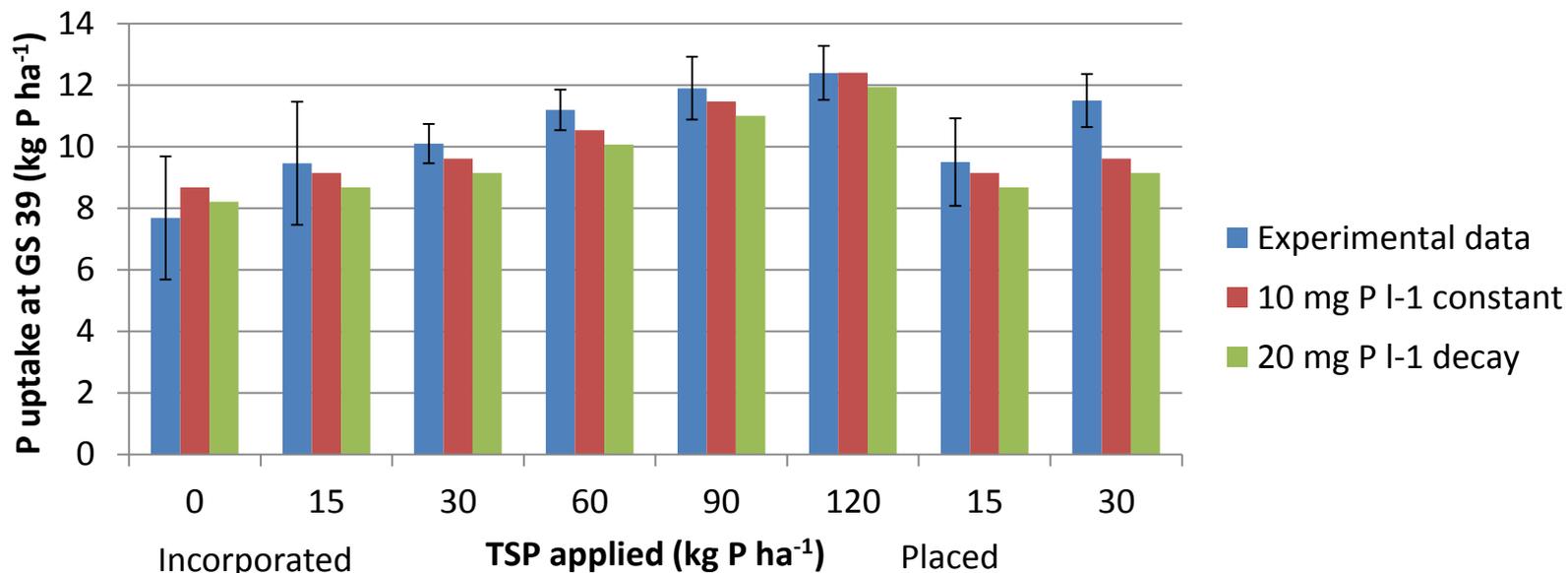
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Figure3

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b)

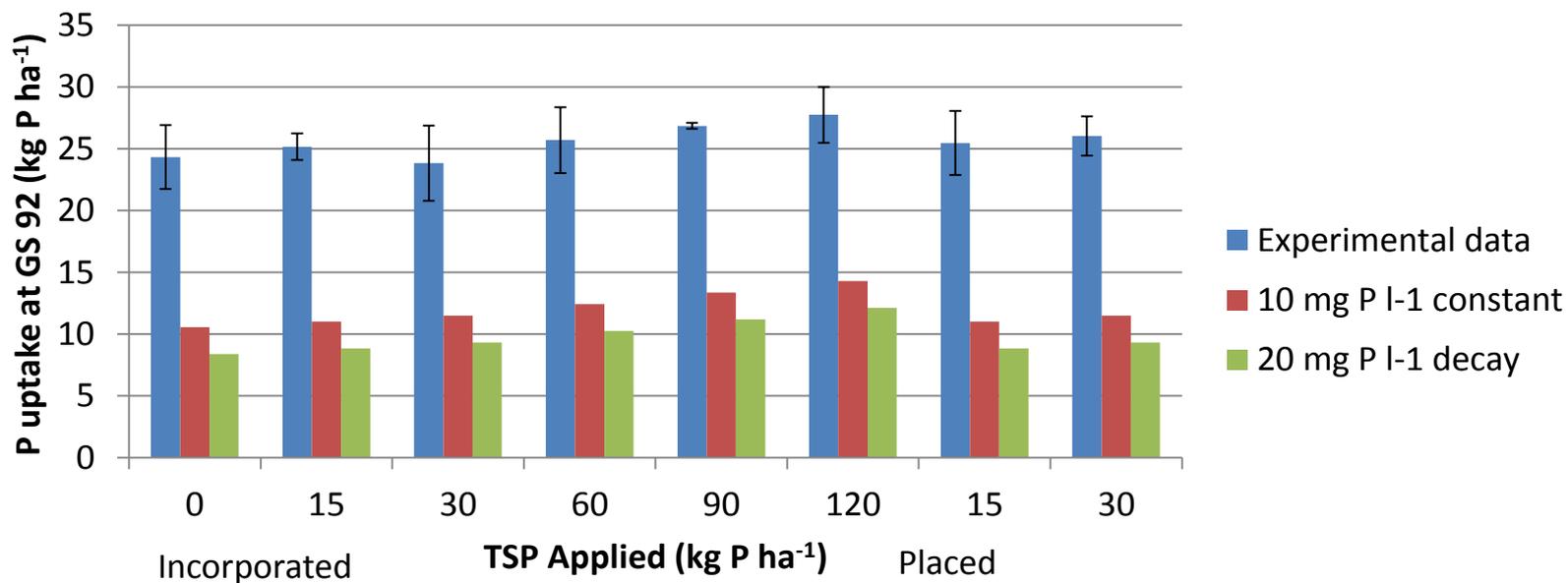
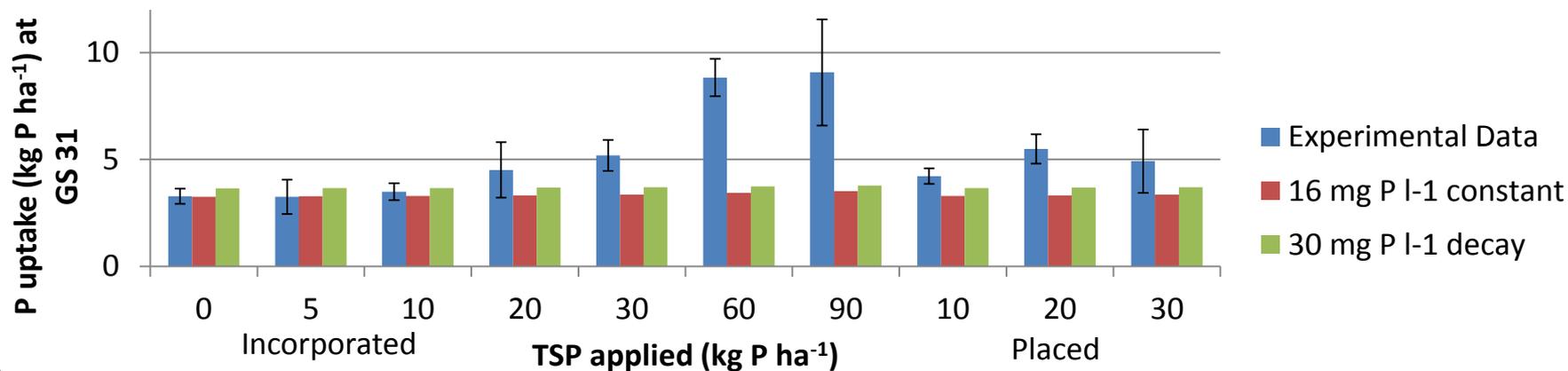


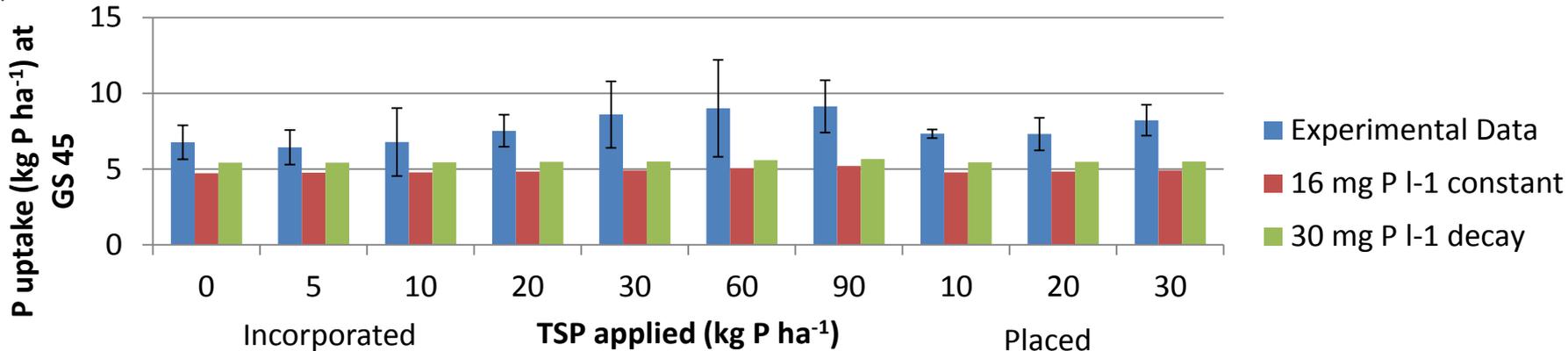
Figure 4

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a)



b)



c)

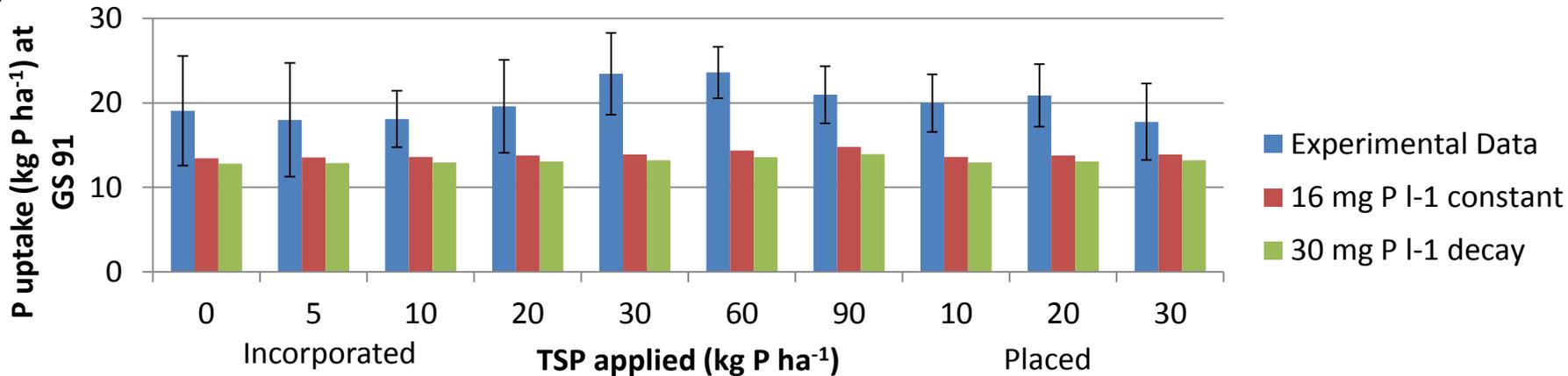


Figure 5

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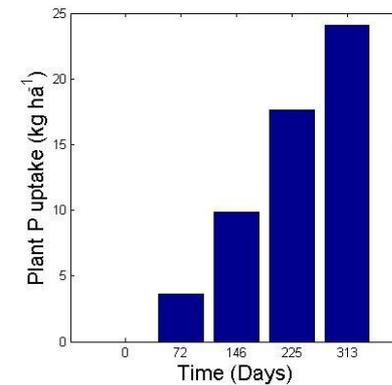
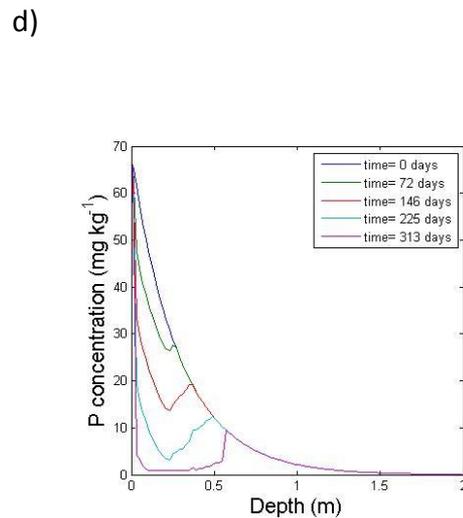
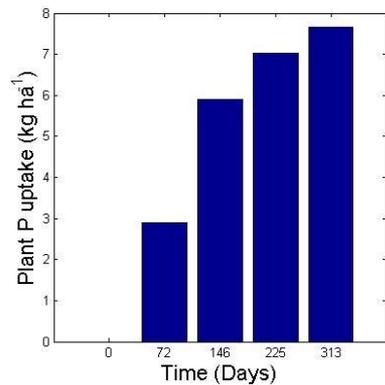
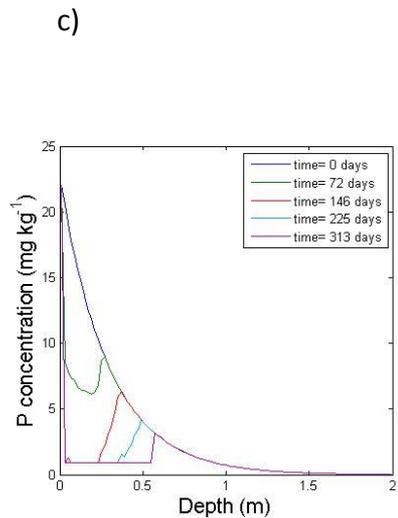
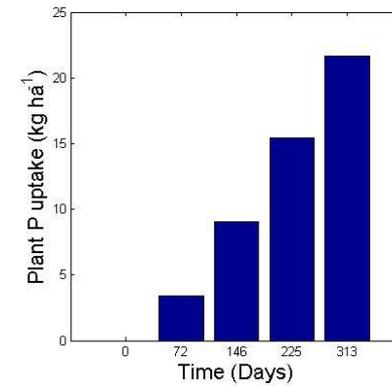
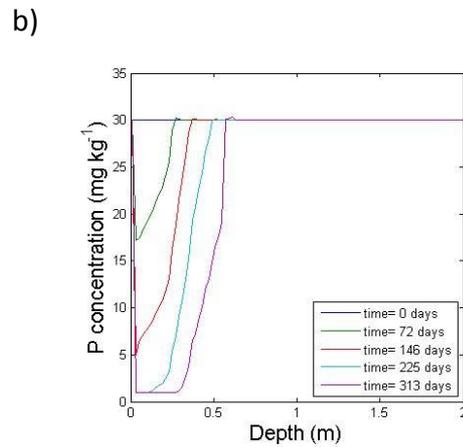
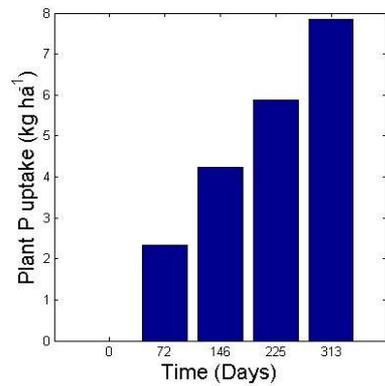
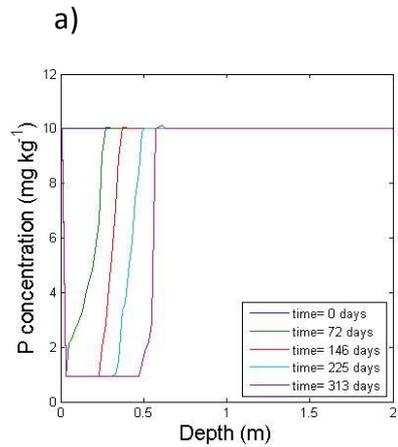


Figure 6a  
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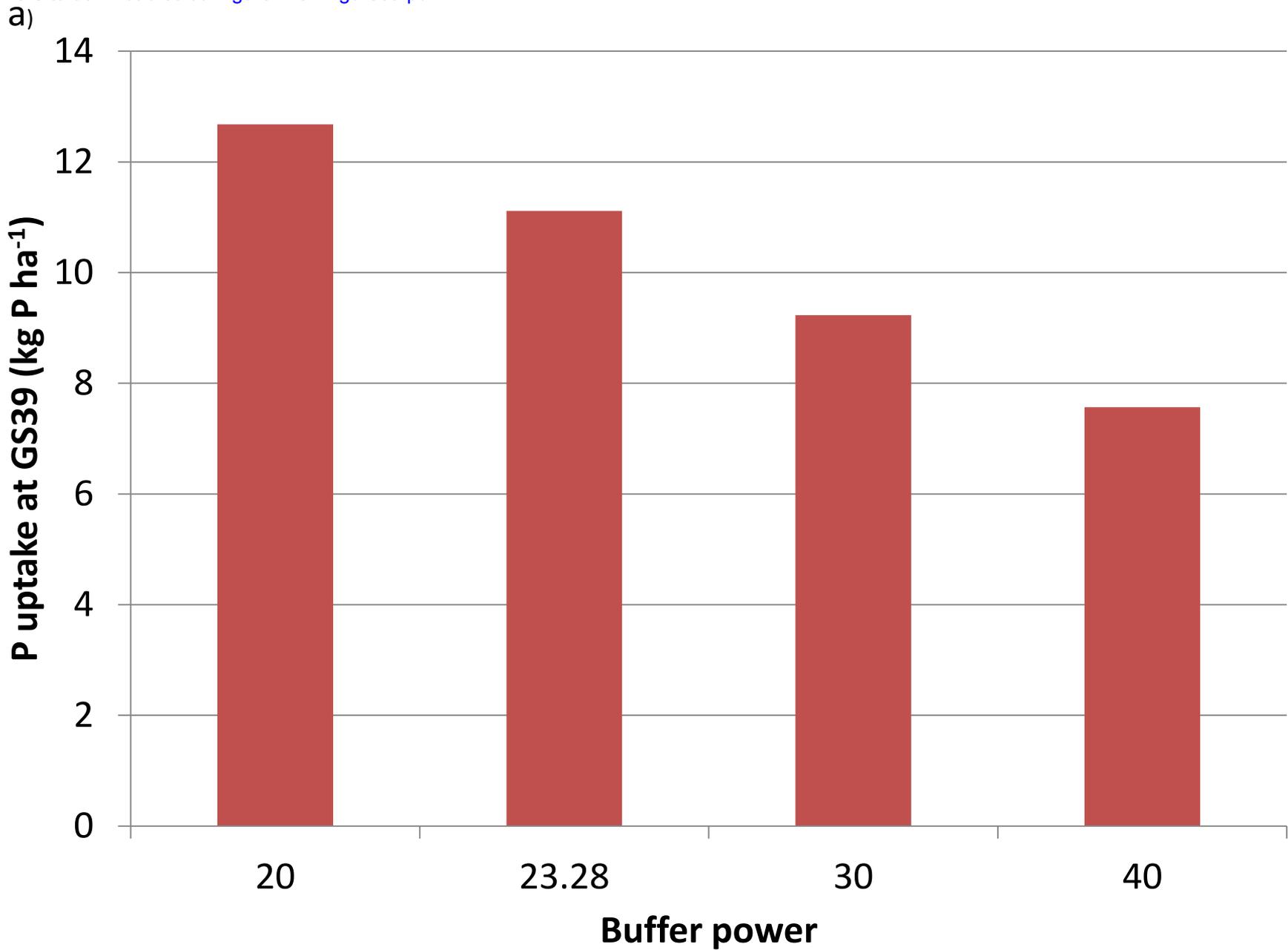


Figure 6b  
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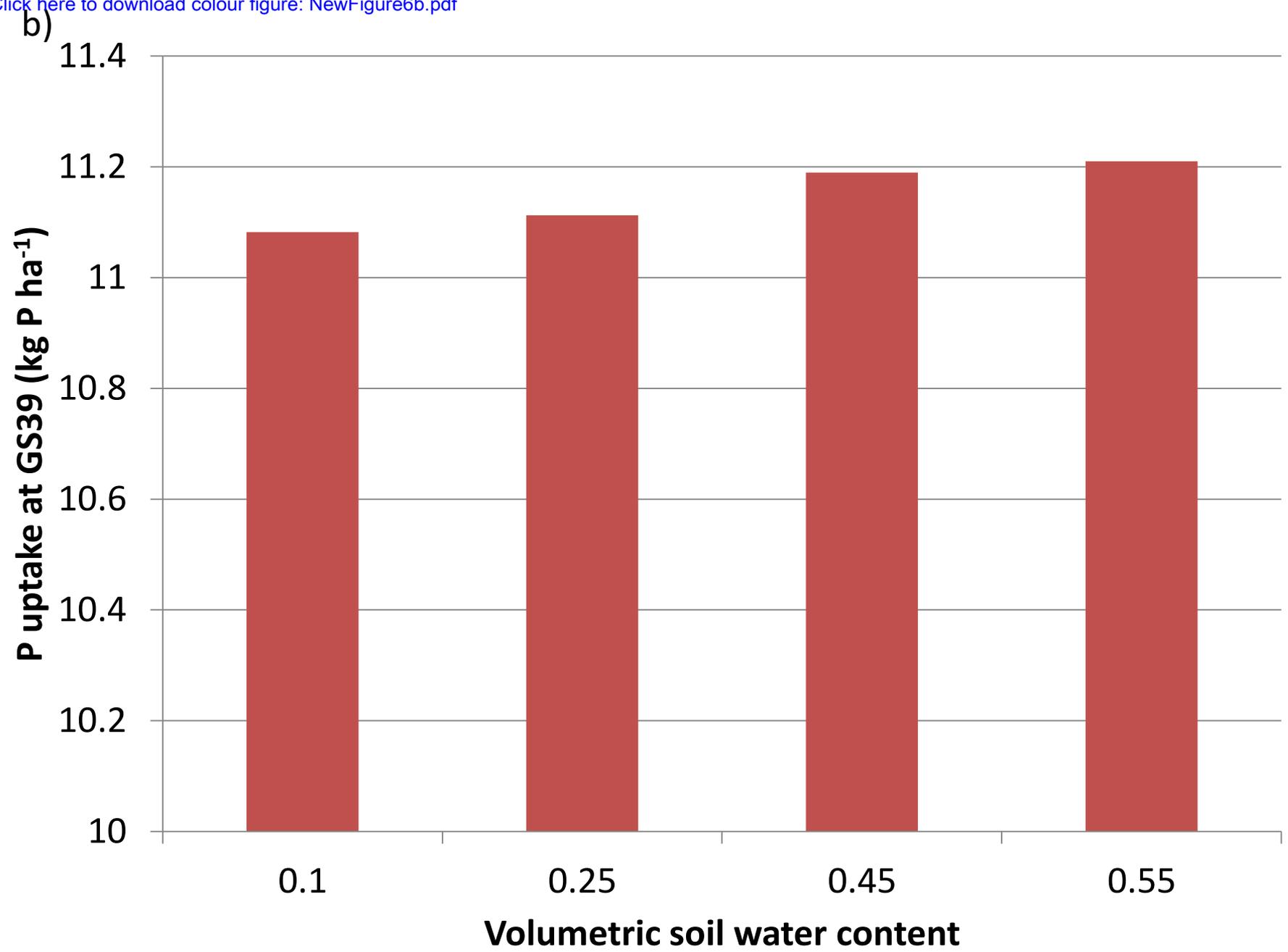
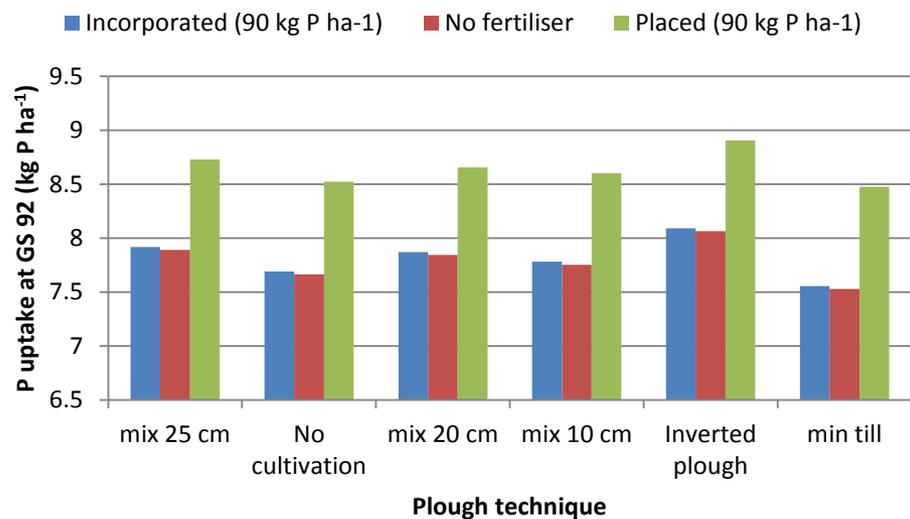


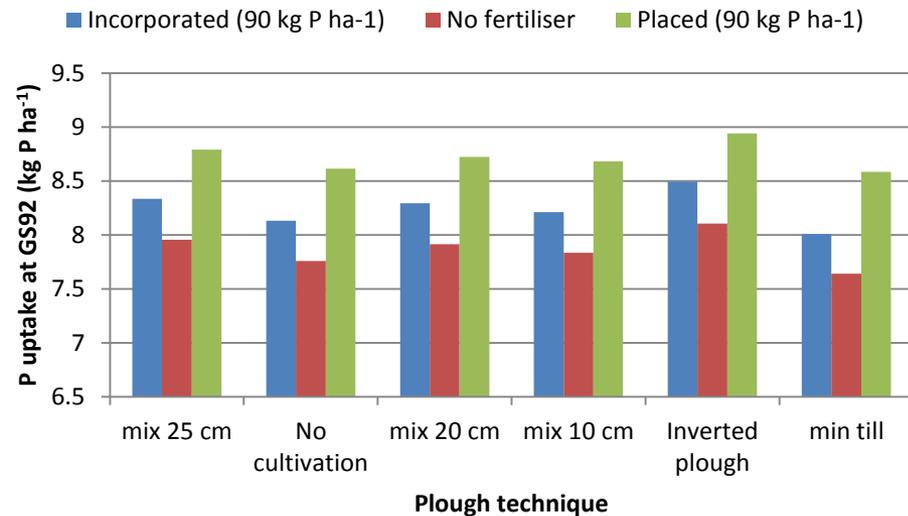
Figure 7

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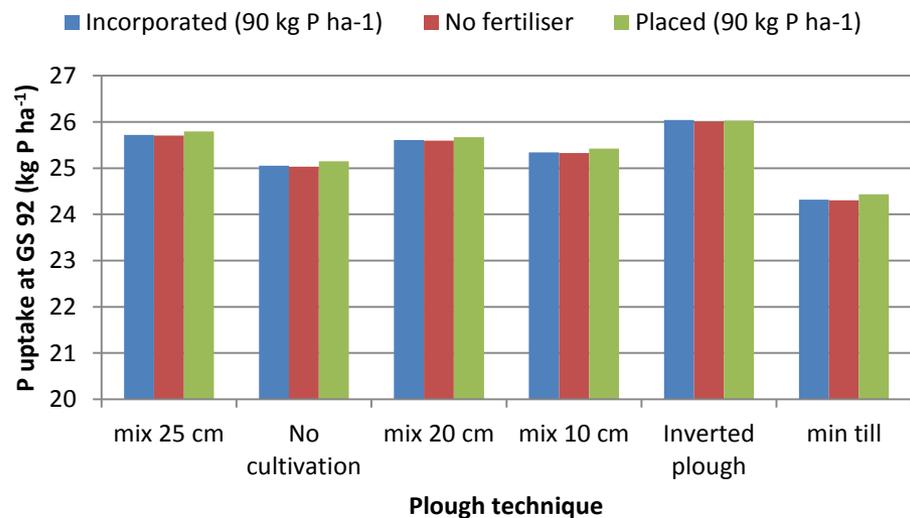
a)



b)



c)



d)

