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An Investigation into Composites Size Effects using Statistically Designed Experiments

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ABSTRACT

FACULTIES OF ENGINEERING AND APPLIED SCIENCE,
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In this study, the problem of scaling design data from test to ship scales has been outlined. The meaning of the term 'size effect' has been clarified through the consideration of dimensional analysis and engineering modelling techniques. The mathematical models used to quantify the strength 'size" effect for fibrous composites have been described, and the effect of the 'scale' of production on the material properties has also been considered.

A review of the literature on composites size effects indicated that the problem concerns the possible joint effects of a number of variables on the mechanical properties of the material. Also, the measurement of these properties is known to give considerable experimental scatter. Such problems benefit from the methods of statistically designed experimentation, and an introduction to these techniques has been given.

The methods of experimental design were used to plan, execute and statistically analyse the results of a test programme involving over 400 specimens. Both tensile and flexural four-point bending tests were used in three distinct test series; one concerning vacuum assisted hand laid-up unidirectional laminates, and two concerning Woven-Roving (WR) E-glass / polyester laminates typically used in maritime applications. The effects of coupon type on the tensile tests and of interlaminar shear stress on the flexural tests were also investigated for the WR laminates.

The experimental design techniques used were non-standard, involving both nested factor and split-plot design structures. A distinction was made between the strength variation amongst separate laminates and the variation in strength of coupons cut from the same panel. The use of experimental design methods proved to be extremely efficient, and allowed the interaction between variables to be investigated. Simple designs, with sufficient specimen replications, are advocated for the study of composites such as those considered here.

Engineering interpretations of the statistical analyses allowed a number of conclusions to be reached. Most importantly, for the WR laminates, 'size effects' in the classical sense were not distinguishable above the experimental variation. However, the effects of manufacturing variables were seen to be important, and hence the effect of the scale of production on the material properties may produce a 'scale' effect. For the unidirectional flexural tests, effects of length and thickness were seen, as reported in some of the literature.

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Nomenclature

Chapter 2»

b	Breadth
d	Depth
E	Young's Modulus
F	Force primary quantity
1	Support span
L	Length primary quantity
m	Indicates model
M	Mass primary quantity
P	Indicates prototype
P	Load
T	Time primary quantity
x	Secondary physical quantity
a	Power ensuring dimensional homogeneity
P	Power ensuring dimensional homogeneity
e	Strain
Y	Power ensuring dimensional homogeneity
X	Scaling factor
n	Pi Term
a	Stress

Chapters 3 and 4

a	Half crack length
b	Breadth
C	Ratio of maximum to nominal stresses in over-stressed fibre
C.V.	Coefficient of variation
d	Depth
E	Young's Modulus
F(CT)	Probability of failure
F,,(a)	Probability of failure of a chain of n elements
i	ith value in an ordered set of strength values
K	Stress intensity factor
K_c	Critical stress intensity factor

K_s	Stress distribution factor
1	Length
L	Length of fibres
m	Weibull shape parameter or modulus
m_b	Weibull modulus in breadth direction
m_d	Weibull modulus in depth direction
m,	Weibull modulus in length direction
n	Number of elements
n_i	Number of fibres around each i-plet
N	Number of fibres
N	Number of values in an ordered set of strength values
P	Load
q_i	Number of i-plets present
Q,(CT)	Number of i-plets created
r	Distance between crack tip and elemental volume
*	x-axis extent of plastic zone
S(o)	Probability of survival
U.D.L	Uniformly distributed load
V	Volume
x	Cartesian co-ordinate
y	Cartesian co-ordinate
z	Cartesian co-ordinate
5	Ineffective length
s	Strain
q>(c)	(Weibull) Distribution function
λ	Effective length
n	Pi Term
e	Angle between crack and line joining crack tip and elemental volume
a	Stress
σ_c	Stress corresponding to critical stress intensity factor
Of	Stress at which first i-plet appears
σ_{iu}	Stress at which i-plets become unstable
$^{CT}{}_{Max}$	Maximum stress in over-stressed fibre
a,	Reference stress

σ_u	Weibull threshold stress
a^{ult}	Ultimate strength
a_{ys}	Yield stress
σ_0	Weibull scale parameter
x	Shear stress

Chapter 5

a	Coded value of factor A
ab	Coded interaction between A and B
abc	Coded interaction between A, B and C
ac	Coded interaction between A and C
b	Coded value of factor B
be	Coded interaction between B and C
c	Coded value of factor C
y	Response variable
a	Unknown parameter
p	Unknown parameter
8	Unknown parameter
8	Random error
Y	Unknown parameter
Tl	Unknown parameter
ξ	Unknown parameter

Chapters 6, 7and 8

B	Butts factor
CW	Coded width factor
CL	Coded length factor
1	Indicates linear effect
m_d	Weibull modulus based on depth
m,	Weibull modulus based on length
m_v	Weibull modulus based on volume
M	Woven roving manufacturer factor
Mod	Initial Modulus
q	Indicates quadratic effect
R	Reinforcement factor
RD	Roller diameter factor

S	Skew factor
T	Thickness factor
V_f	Fibre volume fraction
X	Hypothetical factor
Y	Hypothetical response variable
a	Hypothetical Y-axis intercept
α_i	Coefficient corresponding to effect of ith factor
α_0	Intercept
p	Hypothetical effect of X
£	Sub-plot error
E	Hypothetical Random error
E	Whole-plot error
^lens.ult	Ultimate tensile stress
^tens.ult	Ultimate tensile strain
°flex.ult	Ultimate flexural stress
^flex.ult	Ultimate flexural strain

Appendix A

a_n	Exponents in physical equation
b_n	Exponents in physical equation
i	Number of Pi terms
k	Number of primary physical quantities
M	Coefficient in physical equation
N	Dimensionless number
Pi	ith secondary physical quantity
Q_k	kth primary physical quantity
Qn	nth physical quantity
r	Ratio of similar physical quantities
x,	Exponents of ith Pi term
n,	ith Pi term

Appendix D

f_a	Pooled degrees of freedom
F	F-ratio
H_o	Null hypothesis
H,	Alternative hypothesis

n_R	Number of experimental replications
N	Total number of responses
Sj^2	Variance of high and low averages about the mean
s_R^2	Variance for Rth factor level
S^2	Pooled variance
SS_O	pooled sum of squares
$\ddot{y}$	Average response for experiment
$\ddot{y}_R$	Average response for Rth factor level
$y_{R,}i$	Response variable for ith replication of Rth factor level
a	Significance level
fi	Average response
u,	F-ratio degree of freedom
u_2	F-ratio degree of freedom

1. Introduction

1.1 Composite Materials

A composite structural material consists of more than one phase at a macroscopic scale. The mechanical properties of the composite are designed to be superior to that of the constituent materials considered independently. Usually, two phases are present, a stiff, strong *reinforcement* set in a less stiff and weaker *matrix.* The characterisation of these materials ranges from low performance composite materials, where the reinforcement usually consists of short fibres or particles and the matrix is the main load bearing constituent, to high performance composites, where continuous fibres carry most of the load in the direction of their alignment. The matrix of a high performance composite supports and protects the fragile and unstable fibres, and also serves to transfer load to and between the fibres. The interface between the two phases can be very important in controlling the failure of the composite. The materials considered in this study are fibre reinforced plastic composites, or F.R.P. , where the fibres are set in a synthetic polymer.

The first fibreglass boat was made in 1942 by the US Navy, this was followed by larger craft, and 1979 saw the completion of the first 60m, all F.R.P. vessel, HMS Brecon. These materials are now common throughout the marine industry, and are used in the construction of windsurfers, dinghies, canoes, speedboats, yachts, workboats, lifeboats, submarines, offshore platforms and Navy patrol boats and minehunters. The vast majority of marine composites use E-glass fibres, most commonly in the form of a balanced woven roving, in a cold-curing polyester resin matrix. Usually such composites are fabricated using contact moulding in an open female mould using 'spray-up' or 'hand lay-up' methods. Spray-up entails the incorporation of short glass fibre rovings into a stream of resin which is sprayed onto the mould. Hand lay-up involves the successive laying down of reinforcement material plies onto a liberally applied layer of resin, followed by wetting out and consolidation by rolling or brushing into this wet resin. Some automation of the lay-up process has been achieved, but unless constant production is required, inactive periods require uneconomical cleaning of the equipment between production runs. These techniques lead to composites with generally lower and more variable mechanical properties than those commonly used in the aerospace industry, where the fibres are pre-impregnated with resin in a controlled mechanised process.

1.2 Seating Problem

The relatively recent introduction of FRP composite technology, together with the large range of materials used and being introduced, mean that a broad design base, such as that available for many metals, has not yet been compiled for FRP materials. Hence much testing of composite components has to be carried out either on full scale prototypes, or, in order to save both time and expense, on small scale models using the principles of dimensional analysis. It follows therefore, that any discrepancies encountered whilst scaling from model to full size (i.e. any size effects) should be both identified and understood. Similarly, much of the design of composite components is based on material properties derived from small laboratory scale coupons, often leading to a trial and error approach if the properties obtained in the laboratory tests do not correctly predict the component behaviour.

It has been thought for some time that a strength 'size effect' may exist for some composites, which is usually (but not exclusively) detrimental with increasing size. This is thought to be due to the increased probability of a larger specimen containing a flaw large enough to lead to failure. However, an accurate quantitative description of such effects, or even concrete evidence of their existence, has proved elusive. These problems may be compounded by the fact that a separately manufactured specimen will not necessarily have the same properties as a comparable specimen cut from the full scale structure. This latter effect is due more to the scale of production than to the actual size of the composite laminate considered. Hence it may be helpful to think of this as a 'scale effect' rather than a 'size effect'.

The scaling problem is especially complex for composites due to the intricate nature of their micro-structure. Difficulties arise when it becomes either very difficult, or impossible to scale, for example, such elements as fibre diameter and fibre / matrix interface. The 'custom-made' nature of composites introduces further complications, as there are many possible material variables to be considered, such as manufacturing route, manufacturing conditions, fibre and matrix materials, stacking sequence, fibre volume fraction and void content.

Similarly, the measured mechanical properties of composites are dependent on testing variables, such as test method, environmental conditions, test set-up, specimen geometry, failure mode and loading rate. Although some features of the test set-up are considered in this report, the problem of testing standards comprises a whole field of research on its own and is considered to be outside the scope of this study.

Other features of composites testing are the high degree of experimental scatter obtained in the mechanical properties obtained, and also the difficulty encountered in repeating such results. This is especially relevant for hand laid up marine composites, whose mechanical properties are much more variable than those of aerospace pre-preg laminates.

1.3 Aims

The aim of this study has been to investigate the existence of 'size effects' which affect the strength of composites used in the marine industry. A test programme at the coupon scale has been completed in order both to identify the relevant important factors and also the manner in which they interact. Statistical experimental design methods have been used to plan and analyse the results of this test programme, and the sources of the variability commonly seen in composites empirical strength data are investigated. The materials covered in the study include E-glass and carbon reinforcements set in epoxy and polyester matrices. The reinforcements include uni-directional (UD) and woven roving (WR) cloths. The effects of production technique and fabrication factors are also considered.

2. Engineering Experimental Modelling

2.1 Background

There are many texts concerning experimental modelling and the field of dimensional analysis. David and Nolle (1982) provide a very thorough yet clear explanation from an engineering point of view, and a similar approach is also taken by Murphy (1950) and Emori and Schuring (1977). Langhaar (1951) provides an account of the theory of models with respect to dimensional analysis. Texts mainly concerning dimensional analysis include Isaacson and Isaacson (1975), Barenblatt (1987), Taylor (1974), and Bridgman (1922).

The use of scale models by engineers in order to predict the performance of the full scale article has been an integral part of the design process for centuries. The first "modern" scientist to do so was Cauchy, who investigated the vibration behaviour of model plates and rods in 1829. In 1869 W. Froude made the first models for use in a water-basin in order to predict full scale ship performance, and in 1883 O. Reynolds carried out his classic model experiments on the motion of fluids in pipes. The beginning of the century saw the Wright brothers successfully complete the first heavier than air powered flight in an aircraft designed using scale wing models in a wind tunnel.

There are many fields of application of scale models including architecture, aerospace engineering, hydrology, meteorology and geophysics, but the oldest and best known is that of naval architecture. An important part of the design of any ship involves model testing; frictional and wave-making resistance, propeller performance, manoeuvrability, ship bending and vibration and sea-keeping are all investigated using this technique. The design of large and complex structures also uses model prediction extensively, although this is being replaced by computational methods such as finite element analysis. However, the use of such computational methods with composite materials is limited by factors such as the lack of material data available and the complexity and inherent variability of the material system. Hence model construction, testing, re-designing and re-testing, often many times, is still an integral part of composite component design.

The use of experimental modelling may well reveal factors or behaviour that would not have been envisaged by theoretical studies alone. Emori and Schuring (1977) state, "Despite the current strong tendency toward computerisation, the researcher in physical and engineering sciences relies more than ever on experimentation. More theory demands more testing, not less; otherwise, the theorist would drift toward playing mere games." Further, the limitations of experimental data must also be remembered; the data obtained through model tests is empirical in nature and does not necessarily disclose any underlying physical laws. Hence care must be taken when extrapolating outside the ranges of the variables considered.

There are, broadly speaking, three methods of predicting the performance of an engineering structure (Murphy (1950));

(i) The physical laws concerning the system are used to give a mathematical model and the system performance is predicted using this analysis,

(ii) The structure itself is fabricated and tested before use,

(iii) Tests are conducted on a scale model.

The first method becomes impractical for complex structures concerning many variables and the designer must be confident that the theories used are not only correct but also complete. The second method has the obvious drawback of being both expensive and time consuming unless small, easily fabricated items are in question. This often leaves model experiments as the most reliable yet cost effective route to final product design.

The purpose of an experimental engineering model is often to gain information about the behaviour of the full sale component or *prototype* through tests conducted on an easier to manage *model.* By "easier to manage" we may mean that the model is smaller, for example when considering a ship, or that the process takes a much shorter time at model scale, for example river sedimentation problems. It must be noted that these cases are only examples; the model may be larger than the prototype if the latter is too small to be easily manipulated. However, in the field of composite structure design the models are usually smaller than the prototype for reasons of economy. Experimental modelling may also be used for purposes other than the design of an engineering product, such as the investigation of physical processes. This is particularly true for those phenomena where the effect of scale is thought to play an important role.

In order for the use of experimental model scale testing to be successful it is imperative that the unique relationship between the behaviour of the model and that of the prototype is well understood. It must be known that the model and prototype obey the same physical laws and that all the relevant features are correspondent if the model data is to be extrapolated to full scale. The unique relationship between model and prototype is broadly referred to as *similarity* and the conditions required to ensure similarity are developed using a technique known as dimensional analysis which is based on our concepts and conventions of measurements and observations.

2.2 Dimensional Analysis

The use of dimensional analysis is directed towards finding pertinent non-dimensional combinations of variables for the physical system. These terms are subsequently employed in order to ascertain the required relationships between model and prototype. There is no one set of algorithms that the experimenter may follow in order to come to the correct relationship; there is no one correct method. Much is left up to the experimenters knowledge of the field and ingenuity. In fact Emori and Schilling (1977) describe scale modelling as an art rather than a technique and points out that each problem must be treated anew.

In the field of mechanics it is usual to describe systems using three "basic", or *primary quantities*, mass (M), length (L) and time (T). Mass is often replaced with force (F) by engineers (through the use of Newton's second law), since this more directly corresponds both to the measurements taken and to the results required. The choice of primary quantities is purely arbitrary in much the same way as is the measuring system we have chosen to adopt. The primary quantities must, however, satisfy three conditions;

(i) They must be mutually independent,

(ii) They must be measurable,

(iii) They must be sufficient in number to define the dimensions of every pertinent variable.

The methods of dimensional analysis are based upon two principles inherent to the conventions of measurement;

(i) The ratio of the magnitudes of two like quantities is independent of the units used, if the same units are used for both quantities,

(ii) General relationships may be established between two quantities only when they have the same dimensions.

From these axioms follows the concept of an *homogenous* equation which is defined as an expression which remains valid for any measuring system used throughout the equation in question (David and Nolle (1982)).

Any variables pertinent to the system considered may be expressed as functions of the primary quantities, examples of such *secondary* quantities include stress and velocity. Using the concept of homogenous equations it can be shown (David and Nolle (1982)) that the dimensions of any such physical quantity (x) can be expressed as the product of powers of the primary quantities of the system;

$$[x] = \left[M^{\alpha} L^{\beta} T^{\gamma}\right] \qquad (2\text{-}1)$$

where a, p and y are powers ensuring dimensional homogeneity.

The basis of the mechanics of dimensional analysis is the use of non-dimensional groups or *Pi terms* after the "Pi-theorem" as set out by Buckingham (1914). The use of such groups not only enables similarity conditions between model and prototype to be established but also reduces the number of variables that has to be considered. This may save the experimental effort required considerably. There are three main methods of obtaining non-dimensional groups;

(i) The system equation approach,

(ii) Rayleiglvs method,

(iii) The Buckingham-Pi method.

The first method is dependent upon a thorough knowledge of the system concerned and a complete mathematical analysis to yield the system equations. These equations are then normalised to give non-dimensional groups. The method has the advantages that, once the mathematical model has been established and solved, the extraction of the non-dimensional groups is relatively simple and that these groups may represent physical interactions.

However, the mathematical model may be unknown, too complex, too time consuming or even insoluble, requiring alternative methods for obtaining Pi terms. The Rayleigh and Buckingham-Pi methods use dimensional analysis to identify Pi terms. The reader is directed to David and Nolle (1982) for an explanation of the former and to Appendix A for the background and theory of the latter.

The results of both dimensional analysis approaches are more easily interpreted and utilised if the non-dimensional groups relevant to the problem are selected, i.e. terms which have some physical meaning are advantageous. To do this requires an understanding of both the physical processes and the relevant variables of the system. It is hence also important that initially all the pertinent variables are selected. Omission of any variables involved in the system processes will result in errors in the prediction of prototype behaviour from model experiments. Conversely, selection of unnecessary or uninvolved variables may not affect the validity of the model, but may entail unnecessary complexity, possibly obscuring the physical relevance of the non-dimensional groups developed.

As previously mentioned, there is no one set of algorithms which provides a single path to the dimensional analysis, and so it is easiest to provide a simple example to illustrate how the technique is applied. Consider the example of the four-point bending of a thin beam as shown in Figure 2.1;

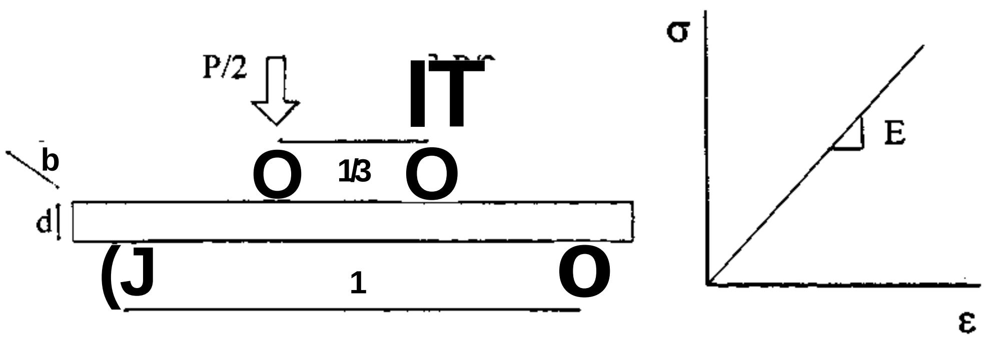


Figure 2.1: Four-point bending

Firstly the pertinent variables are selected. For this example it is initially assumed that the only relationship known to apply to the variables of the system is that shown on the right hand side of Figure 2.1. i.e. that the specimen material behaves in a linearly elastic manner;

$$G = Ee \quad (2.2)$$

where E is the Younii's modulus of the material.

The pertinent variables and their respective dimensions are given in Table 2-1.

Variable	Definition	Primary Quantity
1	Support length	L
P	Load	F
o	Stress	FL^{-2}
8	Strain	[I]
E	Young's Modulus	FL^{-2}
b	Specimen width	L
d	Specimen depth	L

Table 2-1: Pertinent Variables

The number of independent Pi terms required is equal to the difference between the number of system variables and the number of primary quantities involved. In this case there will be 5 Pi terms.

Once the relevant variables have been identified the Buckingham Pi theory is applied. Taking 1 and P to be the fundamental units;

$$n_i = I^{a_i} P^{\prime n} tr \qquad (2.3)$$

This equation must be dimensionally homogenous, i.e.;

$$[\Pi] = [I] = L^0 F^0 \qquad (2.4)$$

Hence;

$$L^{al} F^{fil} FL^{\sim 2} = L^\circ F^\circ \qquad (2.5)$$

Equating indices gives;

$$a_x = 2 \qquad (2.6)$$
$$\beta_1 = -1$$

i.e.

$$\Pi_1 = \frac{/V}{P} \qquad (2.7)$$

Similar procedures give;

$$U_2 = £ \qquad (2.8)$$

$$\Pi_3 = \frac{1^2 E}{P} \qquad (2.9)$$

$$\Pi_4 = \frac{*}{l} \qquad (2.10)$$

$$\Pi_5 = \frac{d}{l} \qquad (2.11)$$

It is perhaps not surprising that strain appears as a Pi term, since it is already defined as a non-dimensional quantity.

2.3 The Theory of Models

Once a set of Pi terms has been identified the next step is to use them to give the relationship between model and prototype behavior. It is convenient to consider two types of Pi terms, the test parameter and the design parameters or conditions. The test parameter is the Pi-term containing the variable which is to be predicted and the design conditions are those containing the other system variables. As is normal engineering practice the system or prototype is expressed as a relationship between the parameter to be predicted and those which can be measured;

$$n_1=F(n_2,n,,n_4,...n_5.) \qquad (2.12)$$

where ITi is the test parameter and II2 to Ils are the design conditions.

For the four point bending example considered above, the stress in the beam could be of interest. If this is the case then the Pi term containing this variable (TIi) would be taken as the test parameter;

$$\frac{I^2 a}{P} \quad J \quad b \quad d\backslash \qquad (2.13)$$

Since equation (2.12) is entirely general it also applies to the model system since the latter is a function of the same variables;

$$n_{lm} = F\{n_{2m}, n_{3a}, n_{4m}, \ldots n_{Sm}) \qquad (2.14)$$

where the subscript m denotes model.

Therefore the relationship between prototype and model may be expressed as;

$$\frac{n,}{n_{1m}} = \frac{F(n_2,n_3,n_4,.,.n,,)}{F(n_{2m},n_{3m},n_{4mJ}\ldots n_{sm})} \qquad (2.15)$$

Take the case where that all the test parameters are equivalent between prototype and model; i.e.

$$n_2 = n_{2/H} \qquad (2.16)$$

$$n, = n,,,,$$

etc.

Since the function F is the same for model and prototype, it follows that;

$$n, = n_{1m} \qquad (2.17)$$

The prediction equation (2.17) is valid if all the design conditions are satisfied, i.e. if equations (2.16) are true. In this case there is *complete similarity* and all of the relevant aspects of the prototype are faithfully reproduced in the model.

Complete similarity can be further subdivided into distinct types of similarity relating to specific Pi terms. For example if a Pi-term contains only geometric variables, such as length, width, depth or angles, and the design condition concerning this Pi-term is satisfied then *geometrical similarity* is said to exist. This is the most obvious interpretation of the term similarity, but many types may be defined including those concerning;

(i) Geometry,

(ii) Material properties,

(iii) Forces, moments etc.,

(iv) Mass distribution,

(v) Timing,

In the case of complete similarity there is said to be a *true model* and predictions using this model will be accurate, if all the pertinent variables were included initially.

The design conditions must now be met through manipulation of the system variables. In order to do this the ratio of a variable for the prototype to its value for the model is defined as a scaling factor. For example, for variable X;;

$$\lambda_{xi} = \frac{*-,}{*>,} \qquad (2\text{-}18>$$

From this definition it follows that for similarity the corresponding scaling factor for a Pi-term should have a value of one;

i.e. $$\lambda_{\Pi_i} = \frac{\Pi_{i,p}}{\Pi_{i,m}} = 1 \qquad (2.19)$$

Expressing each design condition or Pi-term as a function of the relevant variables, x_i;

i.e. $$\Pi_i = f(x_i) \qquad (2.20)$$

Hence, for complete similarity;

$$\lambda_{\Pi_i} = f(\lambda_{x_i}) = 1 \qquad (2.21)$$

The equations (2.21) can then be used to give the required scaling factors. The number of design scaling factors which may be independently varied is limited to the difference between the number of variables and the number of Pi terms. For the previous example of four point bending of a beam, this would amount to only two variables. Returning to this example will illustrate how the model variable values may be derived from the equivalent prototype values and the Pi terms resultant from the dimensional analysis. Firstly, restating the test parameter and including its scale factor form;

$$\Pi_1 = \frac{l^2 \sigma}{T} \quad ; \quad \lambda_{\Pi_1} = \frac{\%K}{\sim\sim I7} \qquad (2\text{-}22)$$

The corresponding equations for the design conditions are;

$$\Pi_2 = e \quad ; \quad \lambda_m = \lambda_\varepsilon \qquad (2.23)$$

$$\Pi_3 = \frac{tE}{P} \quad ; \quad \lambda_m = \frac{\lambda_t \lambda_E}{\lambda_P} \qquad (2.24)$$

$$\Pi_4 = \frac{*}{l} \quad ; \quad \lambda_{\Pi_4} = \frac{\wedge}{\lambda_l} \qquad (2.25)$$

$$\Pi_5 = \frac{ff}{l} \quad ; \quad \lambda_{\Pi_5} = \frac{\lambda_d}{\lambda_l} \qquad (2.26)$$

Suppose, not unrealistically, that the purpose of the example is to achieve the same stress state in the model as would exist in the prototype but that for reasons of economy the model is to be one tenth the length of the prototype. This sets the values of two scaling factors;

$$X_a = l \quad ; \quad A, = 10 \qquad (2.27)$$

This means that the two variables which may be independently varied have been set and the equations (2.22) to (2.26) provide the values of the other five;

$$\lambda_P = 100 \quad ; \quad A, = 1 \qquad (2.28)$$
$$X_b = 10 \quad ; \quad k_d = 10$$
$$\lambda_E = 1$$

Hence, if the model is scaled geometrically by a factor of 1/10 then in order to achieve the same stress strain state as in the prototype a load of 1/100 that at full scale must be applied.

To summarise the procedure for the use of a completely similar model;

(i) List all the relevant system variables,

(ii) Employ dimensional analysis techniques to give a set of non-dimensional Pi terms,

(iii) Set the values of the required scaling factors

(iv) Use the requirement that all *Xm* must be unity to give the other scaling factors,

(v) Use the scaling factors together with the prototype variable values to give the corresponding model values.

Murphy (1950) devotes a whole chapter to the application of completely similar experimental models to structural problems.

2.4 Distorted models and Scale Effects

In the above example only two scaling factors were required to be set and hence it was possible to have a completely similar model by keeping all Pi terms constant between model and prototype. What would the repercussions be if for some reason the width of the model was set at one fifth that of the prototype (i.e. *Xb* is set at 5)? In this case the value of II3 would differ between model and prototype scales and the model would be said to be *distorted* with respect to this Pi term. Since this term is purely geometrical the model would be said to be geometrically distorted and this would be readily apparent from the fact that the model would be a different shape to the prototype. Geometric distortion is the most common in static structure modelling but others include loading and material properties distortion. Loading distortion is usually simply that the loads applied to the model are either too large or too small to ensure similarity, but these forces can also be applied at non-commensurate positions. Material properties distortion may occur because, for example, the Young's modulus or Poisson's ratio of the material used to fabricate the model is not the same as that of the prototype. It may also be because of different stress strain behaviour of the model and prototype, especially if non-linear elastic or plastic behaviour occurs.

A distorted model may arise either because it is not possible to set all the variables in order to achieve complete similarity, or because it is not desired to. Using the previous example to illustrate this, it may not be practical to produce a beam of one tenth of the width of the prototype to the required precision. It could also be that the way in which the width of the beam affects the strength independently of the length is of interest. In either case some recourse must be taken to reconstitute the relationship between model and prototype. Usually additional knowledge in the form of an equation not used in the dimensional analysis performs this task.

Returning once more to the beam bending example, if the width of the model is twice the value for complete similarity, then what is the required load to achieve equivalent stress states? This may be found from the equation developed from simple beam theory linking the maximum stress in the beam to its geometry and the load applied;

$$\sigma = \frac{Pl}{bd^2} \qquad (2.28)$$

From this it can be seen that the applied load must be twice that for the completely similar model, i.e. the value of λ_P must be 50. Murphy (1950) devotes a whole chapter to the subject of distorted structural models.

It is convenient here to illustrate how using the system equations produces the relationship between model and prototype variables by a more direct route than does dimensional analysis. Expressing equation (2.28) in terms of scaling factors gives;

$$\lambda_\sigma = \frac{\lambda_P \lambda_l}{\lambda_b \lambda_d^2} \qquad (2.29)$$

It is then relatively easy to produce a set of the scaling factors on the right hand side of equation (2.29) to give a value of X_a of unity.

Once the analysis has been completed the prototype behaviour may be predicted from the experiments performed on the model. It is often the case that on construction of the prototype these predictions are found to be inaccurate to some degree. This is often referred as a *"scale effect"*, since the scale of the system appears to affect its behaviour. However this is simply a name given to an unknown phenomenon which has not been included in the initial appraisal of the system and hen?e the scaling relationship used. In other words the model is still distorted since we have not allowed for all the features of the system considered. David and Nolle (1982) suggest some reasons for such differences between model and prototype behaviour;

(i) Some effect may be insignificant at prototype size but significant at model size or vice versa. For example the effect of surface tension on storm waves is slight whereas this force will have a large effect upon the waves produced in a laboratory scale hydrological model of an estuary.

(ii) There may be a change in behaviour. For example the flow regime may change from laminar to turbulent.

(iii) Measurement or construction accuracy may be different for different scales. For example it may be easy to fabricate a one metre prototype to a tolerance of one millimetre, but impossible to obtain the equivalent tolerance for a one centimetre model.

(iv) The material properties are affected by scale. This may be due to differences in fabrication methods, for example.

Some of these points will be addressed with respect to marine composite structures in Chapters.

From a design point of view such scale effects may be allowed for using previous model and prototype scale experimentation experience to give "engineering" factors. A more scientific approach is to carry out experiments on a range of scaled systems with the view to use the results to formulate and verify a theoretical explanation for the model distortion. As described in Chapter 4. it is thought that such a scale effect may occur for certain composite materials. In order to try to quantify and explain any such effects the second approach is taken here through a series of experiments at different scales.

3. Analysis of Size Effects

In order to enable accurate design, any scale effects concerning the material properties of fibre reinforced plastic composites (see Section 1.2 and Chapter 2) need to be identified, understood and then compensated for. Here, the methods used to describe such effects, both quantitatively and qualitatively, are outlined. In section 2.4 four possible causes for scale effects were suggested. When considering the strength and stiffness of a structural material such as fibre reinforced composites it is possible to rationalise these into two main categories;

(i) Differences due solely to the amount of material considered,

(ii) Differences between model and prototype due to construction techniques and conditions, difficulties in scaling certain variables and other distortions.

The first category has been the main preoccupation of the literature on the subject of material strength scaling (see Chapter 4). The underlying principle here is that the strength of brittle materials is controlled by the presence of defects, or flaws in the material and that a larger amount of material will contain more of these flaws. This approach is often referred to as statistical strength theory and is considered in sections 3.1 and 3.2.

The second category is far more wide ranging, but generally concerns the different implications of constructing structures at small scale and at large scale. For example, the small scale process may take place under carefully controlled laboratory conditions whereas the large scale process may take place in a workshop environment. It may also be difficult to scale exactly between prototype and model due to physical constraints on the system. Section 3.3 considers these types of effect. A possible distortion of the similarity across scale is suggested by linear elastic fracture mechanics (L.E.F.M) and the background to this theory is given in section 3.4.

3.1 Weakest Link Theory

Statistical strength theory or statistical weakest link theory has formed the basis of conventional brittle fracture study for many years. The concept of the weakest link was first used by Pierce (1926) to investigate the strengths of long lengths of cotton yarns by considering them to made up of shorter lengths linked together. Shortly after this study, Tucker (1927) applied the same concept to concrete. Weibull (1939) made great advances in the subject, giving his name to the most widely used distribution used in weakest link theory and showing that the theory could be applied to many brittle materials. More research in the field by Epstein (Epstein (1948a), Epstein (1948b)) recognised the close relationship between weakest link theory and the statistical theory of extreme values.

Weakest link theory is based on the assumption that the material is made up of smaller elements linked together and that, like the links in a chain, failure of the material as a whole occurs when any one of these elements or "links" fail. The probability of failure of each link subjected to a stress increase from 0 to a is described by the distribution function F(a). The probability of survival of that link is then given by;

$$S(\sigma) = 1 - F(\sigma) \qquad (3.1)$$

It is also assumed that F(c) describes the strength distribution for every element and that each F(a) is an independent randomly distributed variable. The probability of survival of n elements in series is then given by;

$$S» = [1-(F)\sigma]'' \qquad (3.2)$$

Hence the probability of failure of a chain of n elements is given by;

$$F_n(\sigma) = \backslash-[\backslash-F(*)]' \qquad (3.3)$$

Equation (3.3) forms the basis of statistical weakest link theory. The equation shows that, since F(a) must be positive and less than one, for a given stress level a, as the number of elements increases so does the probability of failure. In other words as the chain becomes larger it becomes weaker and a scale effect has been described.

The function F(a) may be described generally as;

$$F(o-) = l-\exp[-\wedge(c7)] \qquad (3>4)$$

The function cp(o) must be positive, non-decreasing and tend to zero at a specified value of a, a_u say, in order to give a form of F(a) comparable with material strength behaviour. Hence to describe the probability of failure of a chain of elements the function cp(a) must be specified.

A specific form of cp(cr) was put forward by Weibull (1939) and is still used widely today. He describes it as the simplest mathematical function which is positive, non-decreasing and tends to zero at a value of a of a_u. This function has become known as the "Weibull distribution" and is given by;

$$\varphi(\sigma) = \left(\frac{\sigma - \sigma_u}{\sigma_0}\right)^m \qquad (3.5)$$

i.e.

$$F(\sigma) = 1 - \exp\left[-\left(\frac{\sigma - \sigma_u}{\sigma_0}\right)^m\right] \qquad (3.6)$$

Where a,, is the threshold stress below which failure does not occur and a_0 and m are called the *scale parameter* and the *shape parameter* respectively. This form is termed the three parameter distribution.

Weibull does not argue any theoretical reasoning behind the derivation of this function, but simply states that it fits many real data sets and is also simple to use. This leads to a probability of failure of n elements in series of;

$$F_n(\sigma) = 1 - \exp\left[-n\left(\frac{\sigma - \sigma_u}{\sigma_0}\right)^m\right] \qquad (3.7)$$

For the strength of a material subjected to an increasing load from zero to *a* there is zero chance of failure only when there is no applied load. This means that a_u is usually taken to be zero and hence the two parameter form is used;

$$F_n(\sigma) = 1 - \exp\left[-n\left(\frac{\sigma}{\sigma_0}\right)^m\right] \qquad (3.8)$$

Considering a volume of material comprising of small elemental volumes, 8V, instead of a chain of "links" gives;

$$F_V(\sigma) = 1 - \exp\left[-\int\left(\frac{\sigma}{\sigma_0}\right)^m dV\right] \qquad (3.9)$$

For tensile testing the stress o is uniformly distributed through the material volume. Initially assuming this simple case gives;

$$F_{V}(\sigma) = 1 - \exp\left[-V\left(\frac{\sigma}{\sigma_0}\right)^m\right] \quad \textbf{(3.10)}$$

In order to use this equation to describe experimental data it is convenient to express equation (3.10) in a linear form. Rearranging and taking logarithms twice gives;

$$\ln\left[\ln\left(\frac{1}{1-F_{V}(\sigma)}\right)\right] = m\ln(cr) - m\ln(cr_0) + \ln(V) \quad \textbf{(3.11)}$$

Hence a plot of ln(a) versus the left hand side of this equation for N replications of an experimental strength test for a given volume of material, V, will give a linear relationship if the material strength variability is described by the Weibull distribution. From the slope and y-axis intercept of this line both the shape and scale parameter respectively may be estimated. Varying the volume of material translates the line vertically as shown in Figure 3.1 .

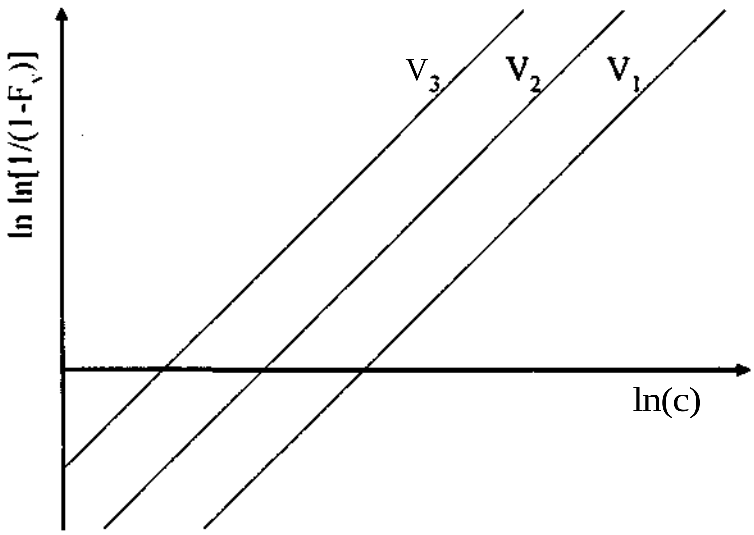


Figure 3.1: Logarithmic Plot of Weibull Strength Data

The calculation of $F_v(o)$ for a set of experimentally obtained stresses, *a,* can be carried out using a statistical approximation technique as described by Weibull (1939). Here the N values of a obtained for a given volume, V, are arranged in ascending order, and for the i^ value ;

$${}_i r_{\gamma}(\mathrm{CT}) = \frac{\mathrm{L}}{N+1} \quad \textbf{(3.12)}$$

Alternatively, the shape parameter m may be approximated from the coefficient of variation (C.V.) of the data set. as stated by Madsen and Buchanan (1986);

$$m\wedge C.V.^{-1.22} \qquad \textbf{(3.13)}$$

Similarly, Hitchon and Phillips (1978) quote;

$$m w \frac{12}{C.V.} \qquad \textbf{(3.14)}$$

Considering the more general case of a varying stress field through the material volume. In Cartesian co-ordinates;

$$a = cx\{x,y,z) \qquad (3.15)$$

Hence, from equation (3.9);

$$F_{r}(\sigma) = 1 - \exp\left[-\int\left(\frac{\sigma(x,y,z)}{\sigma_0}\right)^{m} dV\right] \qquad \textbf{(3.16)}$$

On integration we can express this generally as;

$$F_{V}(\sigma) = 1 - \exp\left[-VK_{s}\left(\frac{\sigma_r}{\sigma_0}\right)^{m}\right] \qquad \textbf{(3.17)}$$

Where K_s is a factor dependent upon the stress distribution and a_r is a reference stress at a specific point in the material.

For tensile tests the stress distribution is uniform and hence K, has a value of one. The derivation of K_s for four-point bending is given in Appendix F and results in;

$$K_s = \frac{m+2}{(m+1)^2} \qquad \textbf{(3.18)}$$

If the strength distribution of a material is described by Weibull theory then it is possible to correlate the strengths of specimens or components of differing size. An assumption is made that the values of the shape and scale parameters m and CT_0 are material constants, independent of the size of the specimen and its stress field. Considering two specimens of different size but identical stress distributions and considering equation (3.10) gives;

$$F_{r1} = 1 - \exp\left[-V_1\left(\frac{\sigma_1}{\sigma_0}\right)^m\right] \qquad (3.19)$$

$$F_{r2} = 1 - \exp\left[-V_2\left(\frac{\sigma_2}{\sigma_0}\right)^m\right]$$

Assuming that we are interested in the same probability of failure at each size, for example the mean strength with a probability of failure of 0.5, gives;

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{V_1}{V_2}\right)^{\frac{1}{m}} \qquad (3.20)$$

This equation directly links strength to volume and hence quantifies the size effect. A logarithmic plot of stress versus volume gives a straight line relationship of slope -1/m, as shown in Figure 3.2 .

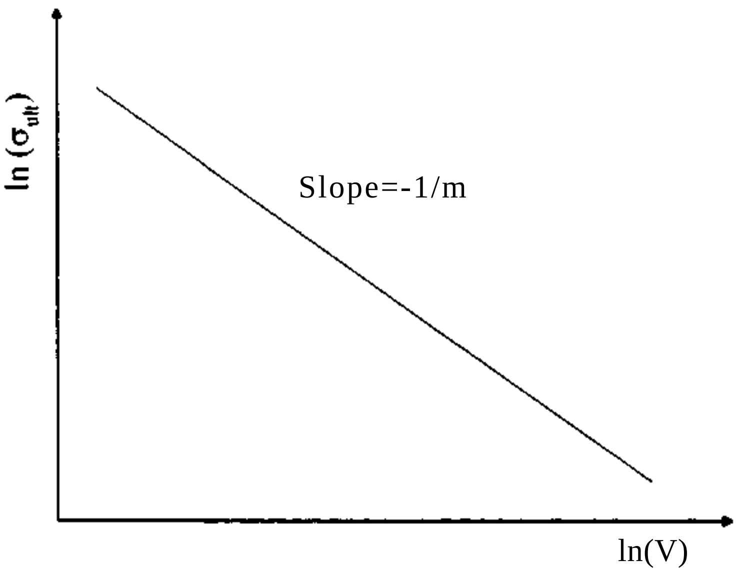


Figure 3.2: Logarithmic Plot of a Strength Size Effect

We can extend this approach to include the effect of differing stress distributions by using equation (3.17) to give;

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{K_{s1}V_1}{K_{s2}V_2}\right)^{\frac{1}{m}} \qquad \mathbf{(3.21)}$$

This equation again quantifies the size effect but also includes the influence of differing stress distributions.

For anisotropic materials such as fibre composites both the strength distributions and the effects of flaws in differing directions may not be the same. Integrating equation (3.9) over length instead of volume, for example gives;

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{l_1}{l_2}\right)^{\frac{1}{m_l}} \qquad (3.22)$$

Similarly for breadth and depth;

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{b_1}{b_2}\right)^{\frac{1}{m_b}} \qquad (3.23)$$

S^Jd^ _ o_x [dj (3.24)

Where m_l, m_b and m_d are not necessarily equal.

The theory as applied to anisotropic materials is referred to as *modified weakest* link theory.

Although almost all of the literature on composite materials concerns the Weibull distribution many others are available. Other distributions which have been considered include the rectangular and Laplace distributions (Epstein (1948a)).

3.2 Extensions of Weakest Link Theory for Composite Materials

Weakest link theory accurately describes the failure of brittle materials. However, although most composite materials fail at very low tensile strains, final failure generally occurs after some damage accumulation. There is much available literature suggesting the constituent fibres of many composites do behave as brittle materials (Moreton (1969), Metcalf and Schmitz (1964), Watson and Smith (1985), Padgett et al. (1995)). This is one of the assumptions of the theory first put forward by Rosen (1964) and Zweben and Rosen (1970), but damage accumulation is also taken into account. The solution of the damage accumulation problem proffered by Zweben and Rosen involves many simplifications and assumptions. Further work by Harlow and Phoenix (1978a) and Smith (1980) gave comprehensive exact mathematical solutions. However these more complicated models also require the estimation of often difficult to measure parameters. Also, other sources of variation are likely to be comparatively large and so the simpler and more easily interpreted theory of Zweben and Rosen is outlined here, as described in more detail by Batdorf (1990). The bundle of fibres model, as first suggested by Daniels (1945), is in fact an extension of simple Weibull theory. A chain of elements is again considered, but in this case the elements are assumed to consist of many fibres, as illustrated in Figure 3.3.

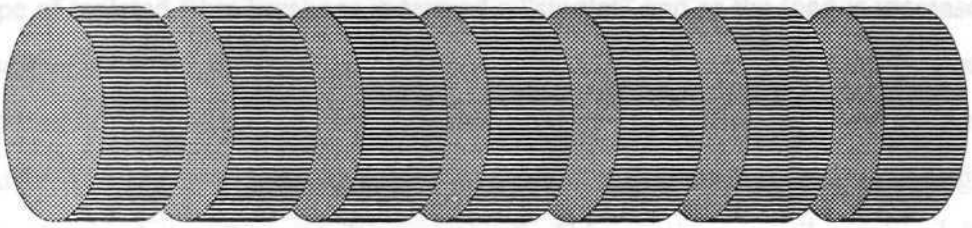


Figure 3.3: Bundle of Fibres Elements

Daniels assumed that for a loose bundle of fibres when an individual fibre failed, the bundle as a whole did not fail due to redistribution of the load equally among the other fibres. However, a fibre reinforced plastic composite material consists of high stiffness and high strength fibres embedded in a relatively low modulus weak polymer matrix. The fibres in an undamaged unidirectional uniaxially loaded composite carry the majority of the load. In a similar manner to Daniels, Zweben and Rosen hypothesised that when the first of these fibres failed the composite as a whole did not fail, because of load transfer by the matrix. However, they supposed that the load previously taken by the broken fibre is now transferred via the matrix only to the *adjacent* fibres, around the break and then back to the original fibre. A consequence of this shear transfer is that, for a certain length either side of the break, the failed fibre carries less load whilst those adjacent carry more. This is shown in Figure 3.4.

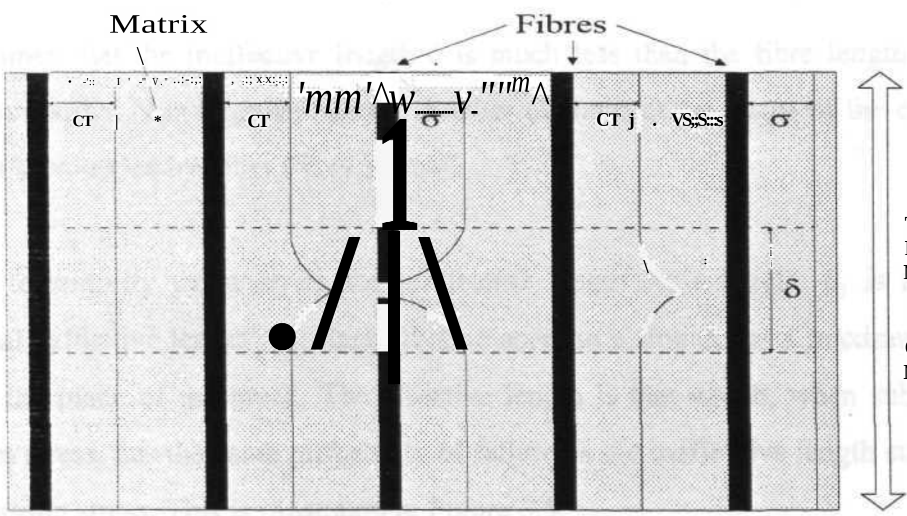


Figure 3.4: Illustration of Fibre Load Sharing

The length over which this shear transfer occurs is known as the "ineffective length" denoted by 5. Despite the fact that the adjacent fibres now carry more load, since 8 is small the probability of a critical flaw occurring in this length is also small and hence failure is unlikely. Also the excess load is shared amongst all neighbouring fibres and so the load increase is not large.

This type of isolated fibre breakage is termed a "singlet" and as the load is increased more of these will appear. As the load is further increased it becomes more likely that the over stressed parts of the adjacent fibres should themselves fail. When this occurs there are two adjacent broken fibres and this is termed a "doublet". Still further loading will give rise to more singlets and doublets and then "triplets". This continues until a critical "multiplet" occurs and the process becomes unstable, resulting in the failure of the composite as a whole.

Considering the two parameter Weibull distribution;

$$\varphi(\sigma) = \left(\frac{a)}{\sigma_0}\right) \qquad (3.25)$$

For the fracture of a fibre this may be interpreted as the number of defects unable to sustain a stress a per unit length of fibre. Hence the number of singlets formed in N fibres of length L is;

$$Q_1(\sigma) = 4 \wedge 1'' \left(\sigma_0\right) \qquad (3J6)$$

This assumes that the ineffective length 5 is much less than the fibre length L. A further assumption is that N is large so that the number of flaws at the edges of the composite and hence not surrounded by other fibres is small.

In order to simplify the analysis the ineffective length for a singlet 5, is replaced by a conceptual "'effective length" A.,. Each fibre adjacent to a singlet has a maximum increase in stress in the plane of the break. The effective length is that which, when subjected to this maximum stress, has the same probability of failure as the ineffective length subjected to the actual varying stress. This is illustrated in Figure 3.5.

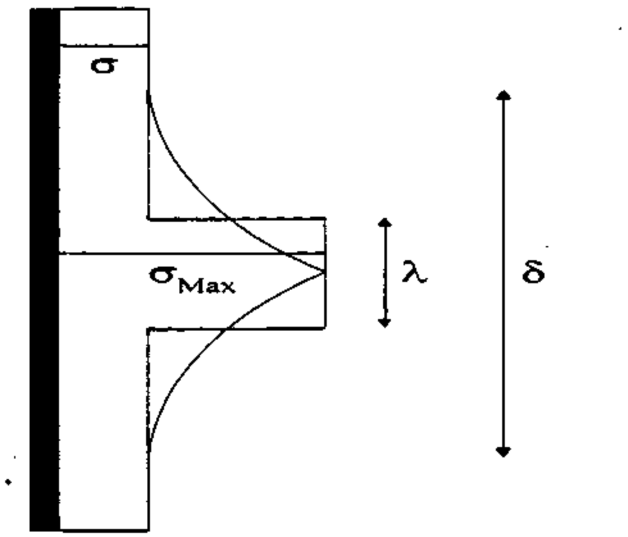


Figure 3.5: Ineffective Length (8) and Effective Length *(k)*

The total length of overloaded fibres surrounding the Q, singlets is hence Qin,^, where n, is the number of fibres around each singlet. Defining the ratio of a_{Max} to a as C, gives the number of failures expected in this length as;

$$n \quad nr,A\left(\frac{C\wedge Y}{\sigma_0}\right) \qquad (3{,}27)$$

This is the number of singlets converted to doublets at stress *a* and may be generalised to higher order multiplets;

$$Q_M = Q_i n_i \lambda_i \left(\frac{C_i a}{\sigma_0}\right)^{m} \qquad \textbf{(3.28)}$$

This is not generally equal to the number of i-plets present at load a since some will have been converted to higher order multiplets, the actual number is hence;

$$q_i = Q_i - Q_{i+1} \qquad \textbf{(3.29)}$$

It can be seen from equation (3.28) that a logarithmic plot of Q< against a for the **ith** multiplet will yield a linear graph of slope $m_{(}$. This is illustrated for i = 1 to 4 in Figure 3.6 .

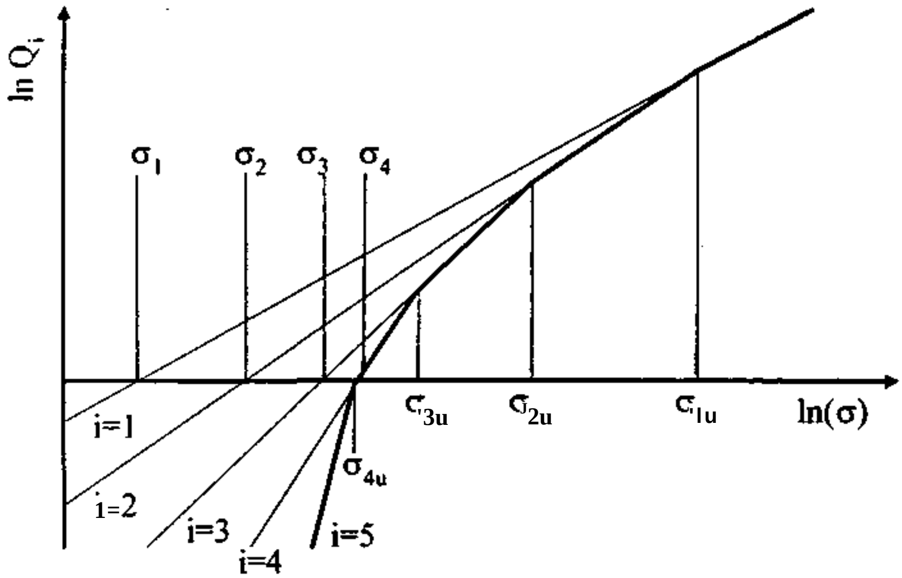


Figure 3.6: Logarithmic Plot of the Bundle of Fibres Model

Using this graph it is possible to predict the failure behaviour of the material as the stress is increased. As the stress is increased the first fibre failure occurs at a, where the singlet (i = 1) line cuts the In Q, = 0 line, i.e. where the number of singlet failures is one. Simple Weibull theory, whereby the initial failure is said to lead to catastrophic failure would predict failure of the material as a whole at this point and the i = 1 line does in fact correspond to this theory. However failure does not occur in this case and further increase of stress results in more singlets and at a, the first doublets are formed. Since all doublets are formed from singlets there can never be more doublets than singlets. Hence at CT_{1U} the i = 2 line follows that for i = 1. Above this stress any singlets formed are unstable and are immediately converted into doublets. From this it is apparent that failure of the material occurs when the stress at which a certain multiplet first appears is equal to or greater than that at which it is unstable.

ie - $a_{r} * o *$ (3.30)

This occurs when the envelope of the $Q_{\{}$ lines indicated in bold in Figure 3.6 cuts the ln(cr) axis. For the case in Figure 3.6 further increase in stress from a_2 results in more singlets and doublets until, at a:, the first triplets are formed. Further stress produces more singlets, doublets and triplets. At a_4 the first quadruplet is formed, but this is unstable and failure of the material results.

From equations (3.26) to (3.28) it is apparent that the number of multiplets is proportional to NL. which in a uniform unidirectional composite, is proportional to the composite volume,

V. Hence a change in V translates the lines in Figure 3.6 vertically, changing the values of the intercepts with the ln(a) axis. This relationship may be represented on a logarithmic plot of failure stress against NL, as shown in Figure 3.7. This plot is analogous to that for simple Weibull theory (see Figure 3.2).

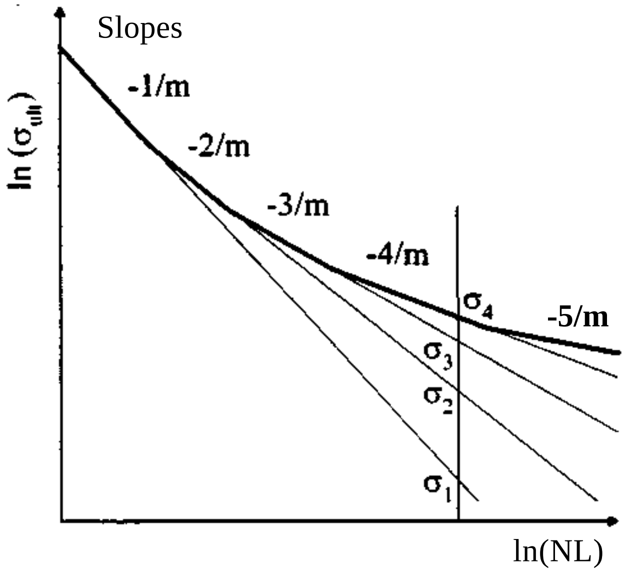


Figure 3.7: Logarithmic Plot of Bundle of Fibres Model

The failure line is again the bold line, the dashed line indicating the situation shown in Figure 3.6 where failure occurs when the first quadruplet is formed. As the material volume is increased the failure stress again decreases, i.e. a strength size effect is present. The order of the critical multiplet is seen to increase with composite volume. Also, the dependence of strength on volume decreases as larger amounts of material are considered.

The example above is only an illustration, the exact form of the curve will change with the values of the parameters m, a_n, rij, k, and Q . The predictions made using the model are thus highly dependent upon the estimation of these parameters.

3.3 Scaling of FRP

The most obvious reason for differences between the material properties of the full scale prototype and small scale models or test specimens is that the material itself is not the same at both scales. This possibility is often overlooked in favour of precise mathematical models such as those described in sections 3.1, 3.2 and 3.4. However, the properties of fibre reinforced plastics are generally sensitive to the methods and conditions of the production stage. Johnson (1979) discusses the influence of fabrication and environment on the properties of FRP and some of the sources of material property discrepancies between model and prototype construction are suggested below.

The model and prototype may be constructed in two quite different environments. The model may involve laboratory preparation of test specimens using small amounts of material under very controlled conditions. Conversely, the full scale marine article will probably be constructed on a factory floor using large quantities of raw materials. Working practices are likely to differ betweea technically trained laboratory staff and workshop laminators and it is probable that the material variability in ten 25cm test specimens is less than that in a 50m ship. Will the same volume fraction, for example, be achieved using a small hand roller on a flat, level desk as with a bucket of resin and a broom sized roller high up the side of a ships mould? Levels of accuracy may also differ. Geometrically, construction to a tolerance of one millimetre for the model is not comparable to the same tolerance for the much larger prototype. Factors which may be harder to control in the ship's mould may include 'waviness' of the weave, the inclusion voids and foreign material and orthogonality of warp and weft among others. Control of the working temperature, humidity etc. in a large hangar may be difficult whereas test specimens may be prepared under closely controlled conditions. These are all questions of scale not only in the sense of absolute size, but also in a sense analogous to the differences between baking a cake in the kitchen and a continuously operating chemical engineering plant producing confectionery on an industrial scale.

The properties of a composite will also depend upon the method of construction. For example, if the prototype is constructed using an automated lay-up procedure then test specimens laid up by hand may not behave in the same manner. The model may be constructed using one sheet of reinforcement whereas the ship may require joins between the available sizes of cloth. Procedures may be followed to mirror full scale practices as far as possible at model scale but some effects of scale may be unavoidable. Murphy (1950) points out that the cooling rate of an eighth of an inch diameter cast iron model of a four inch prototype will lead to different material properties even if they are cast from the same ladle of molten metal. Similarly thick composites may be more prone to the effects of exotherms than thin lay ups. As another example the percolation of voids to the surface may not be as complete for the prototype laminate as for the test specimen.

The discrete microstructure of fibre reinforced composites leads to further scaling complications. David and Nolle (1982) describe a similar problem for concrete where exact scaling down of the aggregates in concrete would lead to using sand in the model and hence making meaningless comparisons between mortar and concrete. From a similarity viewpoint is it valid to compare the behaviour of a five ply composite with that of a twenty five ply one? This is acceptable if the effects of each ply may be assumed to be sufficiently smeared through the thickness but, for example, failure of one ply in a test specimen may lead to total failure whereas for the prototype accumulation of more damage may be required to initiate collapse of the structure. One solution would be to scale the fibre dimensions, but this would be complicated and expensive. There would also be no guarantee that the scaled down reinforcements would have properties comparable to those of the full scale. Intermediate solutions to the problem have been suggested, including ply level and sub-laminate level scaling (Jackson et al. (1992)). The former replicates each ply n times in blocks of similar plies as shown in Figure 3.8.

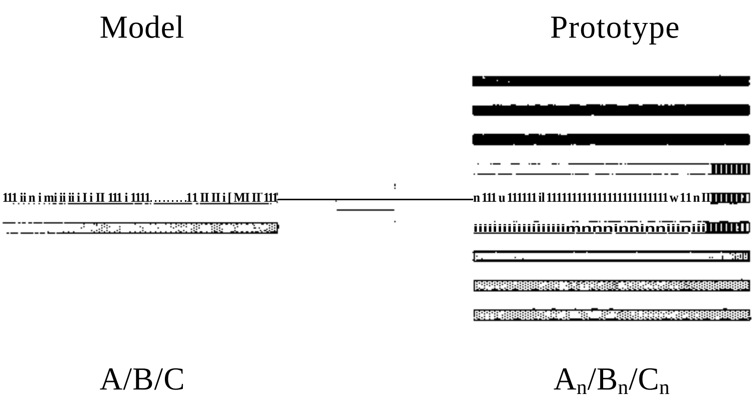


Figure 3.8: Ply-Level Scaling

The later uses blocks or sub-laminates of the original group of plies which are combined in a building block manner as shown in Figure 3.9. For laminates where each ply is identical both of these methods simplify to the case of increasing the number of plies.

The aim of specimen or model testing is to predict the behaviour of the prototype. Hence preparation of the specimens or models should involve procedures as similar as possible to those used to construct the prototype. One way to ensure this is to use material taken from the full size component itself for example from the material removed when cutting hatchways.

However, information using this technique may only be used in retrospect or in a quality control capacity since it requires that the component is already under construction.

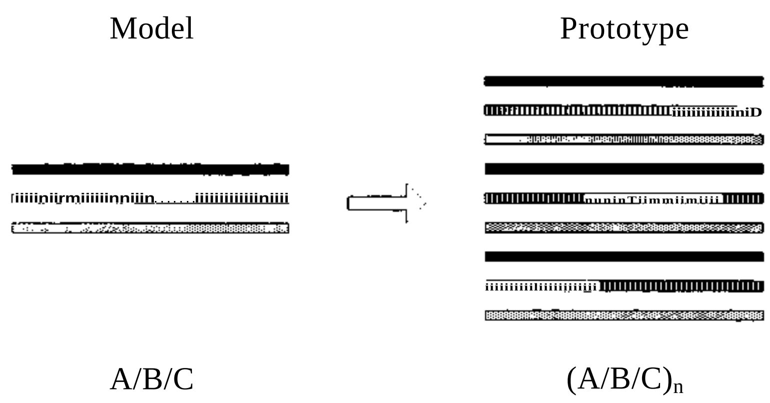


Figure 3.9: Sub-Laminate Level Scaling

3.4 Linear Elastic Fracture Mechanics

There follows a basic introduction to the concepts of linear elastic fracture mechanics (L.E.F.M.). For more in depth information the reader is directed to Broek (1974), Knott (1973) and Hayes (1972) as examples of the literature on this subject.

Considering the simplest case of a crack of length 2a in an infinite plate subjected to a uniform stress a as shown in Figure 3.10.

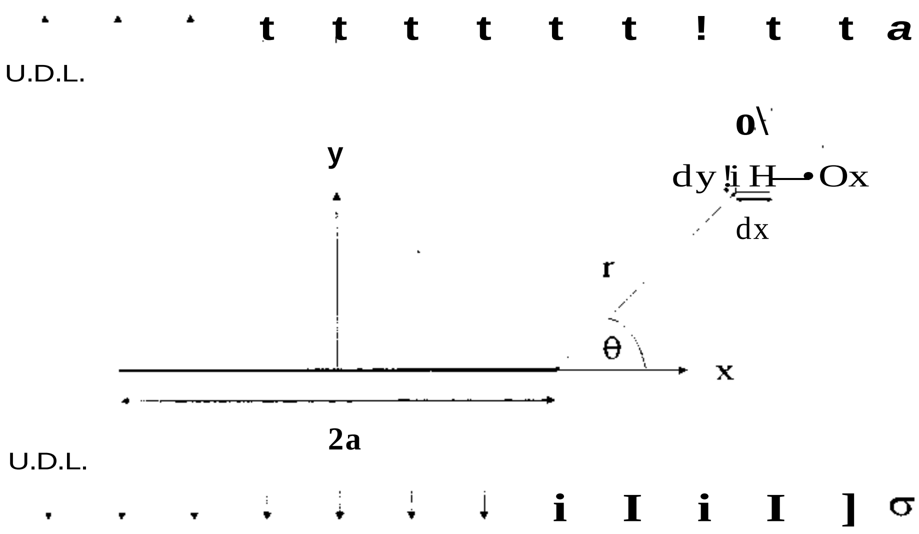


Figure 3.10: Stressed Crack

It can be shown (9) that the stresses on the elemental volume dx.dy at a distance r from the crack tip at an angle of 9 to the x-axis are;

$$\sigma_x = \sigma\sqrt{\frac{a}{2r}}\cdot Cos\left(\frac{\theta}{2}\right)\left[1 - Sin\left(\frac{\theta}{2}\right)Sin\left(\frac{3\theta}{2}\right)\right] \qquad (3.31)$$

$$\sigma_y = \sigma\sqrt{\frac{a}{2r}}\cdot Cos\left(\frac{\theta}{2}\right)\left[1 + Sin\left(\frac{\theta}{2}\right)Sin\left(\frac{3\theta}{2}\right)\right]$$

$$\tau_{x,y} = \sigma\sqrt{\frac{a}{2r}}\cdot Sin\left(\frac{\theta}{2}\right)Cos\left(\frac{\theta}{2}\right)Cos\left(\frac{3\theta}{2}\right)$$

In fact these equations are approximations and only apply for small r as indicated by the fact that as r tends to infinity a_y tends to zero instead of a. All three equations may be written;

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\vartheta) \qquad (3.32)$$

Where;

$$K = \sigma\sqrt{\pi a} \qquad (3.33)$$

K is known as the stress intensity factor and failure of the plate occurs when this reaches a critical value. K_c giving a corresponding stress value o_c;

$$K_c = \sigma_c\sqrt{\pi a} \qquad (3.34)$$

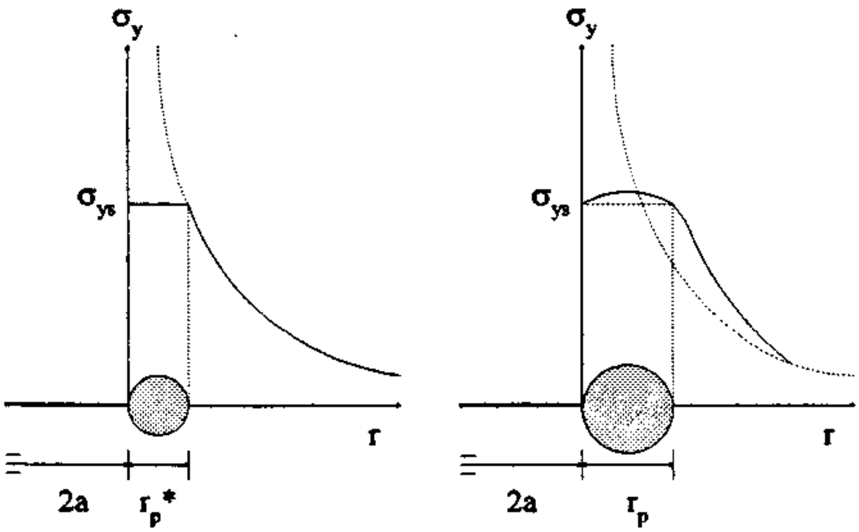
This critical stress intensity factor is normally assumed to be a material property and is found experimentally by methods such as the tensile testing to destruction of a sample with a crack of known length.

From inspection of equation (3.32) it would appear that the stresses at the crack tip, where r tends to zero, become infinite. In reality this is not the case and plastic deformation at the crack tip keep the stresses here finite. An estimate of the extent of the plastic zone may be obtained by substituting the material yield stress for o_y in equation (3.32) to give the distance along the x-axis (0 = 0) where this stress occurs. This distance is denoted r_p*;

$$r_p^* = \frac{K^2}{2\pi\sigma_{ys}^2} = \frac{\sigma^2 a}{2\sigma_{ys}^2} \qquad (3.35)$$

Where r_p* is the distance along the x-axis to which the plastic zone is assumed to extend, and a_{ys} is the material yield stress.

The real plastic zone is slightly larger than the assumed one and this is shown in Figure 3.11.



a) Assumed stress distribution

b) Approximate stress distribution

Figure 3.11: Plastic zone at the Crack Tip

Since it is assumed that the crack propagates when the stress intensity factor reaches its critical value then it would appear to be pertinent to include it in the dimensional analysis of the system. Returning to the example of dimensional analysis given in section 2.1 .

Firstly, restating the test parameter and including its scale factor form;

$$\Pi_1 = \frac{l^2 a}{P} \quad ; \quad \lambda_{\Pi 1} = \frac{\lambda_l^2 \lambda_\sigma}{X_p} \qquad (3.36)$$

The corresponding equations for the design conditions are;

$$\Pi_2 = \varepsilon \quad ; \quad \lambda_{\Pi 2} = \lambda_\varepsilon \qquad (3.37)$$

$$\Pi_3 = \frac{l^2 E}{P} \quad ; \quad \lambda_{\Pi 3} = \frac{\lambda_l^2 \lambda_E}{X_p} \qquad (3^8)$$

$$\Pi_4 = \frac{h}{l} \quad ; \quad \lambda_{\Pi 4} = \frac{\lambda_h}{\lambda_l} \qquad (3.39)$$

$$\Pi_5 = \frac{d}{l} \quad ; \quad \lambda_{\Pi 5} = \frac{\lambda_d}{\lambda_l} \qquad (3.40)$$

Remembering that 1 and P were taken as the fundamental units and introducing the stress intensity factor as given in equation (3.33) gives;

$$\Pi_6 = \frac{l^{3/2}K}{P} \quad ; \quad \lambda_{\Pi 6} = \frac{\lambda_l^{3/2}\lambda_K}{\lambda_P} \qquad (3.41)$$

Both the number of variables and the number of Pi terms have increased by one and hence it is possible to set two of the Pi terms and still have a completely similar model. Before, the same stress-strain behaviour was stipulated, now the criteria is that the critical stress intensity factor should scale as unity. Again *K* and XE will be unity to satisfy Hz and II3. Also FL* and FI_5 are conditions of geometric similarity. Equation (3.41) together with the condition that XK = 1 gives;

$$\frac{\text{if}}{\lambda_P} = 1 \qquad (3.42)$$

And hence from equation (3.36);

$$k_o = \wedge \qquad (3.43)$$

This equation states that failure occurs at the same value of the critical stress intensity factor for both model and prototype then the failure stress will scale as one over the square root of length if there is geometric similarity. Thus strength decreases with scale and a scale effect has been described.

The system equation approach using equation (3.33) gives;

$$\frac{\lambda_a \lambda_a^{1/2}}{\lambda_K} = 1 \qquad (3.44)$$

From this it is evident that if K is to remain constant and if equation (3.43) applies then the crack size, a, should scale as length. This would be expected from the condition of geometric similarity. However, this is a false assumption for most cases. For an advanced composite material, the critical crack or flaw is generally within the fibres. As is usually the case, if the same reinforcement is used for both model and prototype then the crack size is the same at both scales. This would also be true if the critical flaws were within the matrix phase, in the form of microcracks, for example. This point is further discussed in section 4.4.

4. Strength Size Effects Literature Review

As a background to the problem of composites size effects, a brief history of the theories commonly used, and examples of the application of these theories to brittle materials, is given. The literature concerning some brittle fibres commonly used as the reinforcement phase of polymer matrix composites is discussed, and then a review of the composites strength size effects literature is presented. Finally, an overview of the areas, results and shortcomings of this literature, and the methodology adopted by this study in order to add to the current knowledge of the field is given. A similar review is given in Sutherland (1997a).

4.1 History of Statistical Strength Theory

The earliest connections made between the decrease in material strength with size and the variability of this strength is often attributed to the work of Pierce (1926) on the strengths of cotton yarns which is based on Griffith's theory of flaws (Griffith (1920)). Lieblein (1954) pointed out that this is preceded by some 35 years by the study of the strengths of long and short bars by Chaplin (1880). However, the first mention of a material strength size effect dates back 400 years. Hertzberg (1976) reproduces a sketch and a passage from the notebook of Leonardo da Vinci detailing an experiment exploring the variation of the tensile strength of iron wires with length. Tucker (1927) considered the same concepts as Pierce to study concrete. Probably the most important work in the field was published by Weibull (1939). The impact of Weibull's work led to the naming of the most commonly used weakest link theory, described in section 3.1, as Weibull theory (Weibull (1951)). Epstein (Epstein (1948a), Epstein (1948b), Epstein (I960)) further investigated the subject, highlighting the close relationship between weakest link theory and the statistical theory of extreme values. The extension of statistical fracture theory to cover loose bundles of fibres originated with Daniels (1945). The model for unidirectional composites, summarised in section 3.2, originates from that outlined by Batdorf (Batdorf (1990), Batdorf (1989)) and Batdorf and Ghaffarian (1984) and is based on a conceptual model pioneered by Rosen (1964) and Zweben and Rosen (1970).

4.2 Brittle Materials Size Effects

The weakest link theory has been used in the design of ceramics for some time and examples of the literature on this subject are numerous. An early example which discusses the theory in some detail is that by Weil and Daniel (1964). Davies (1973) gave an easily interpreted overview of the theories used and the mechanics of how to apply them. The same methods had been applied to the brittle strength of steel in an earlier paper by Davidenkov et al. (1947).

The strength size effect of a natural anisotropic fibrous composite, wood, is analogous to that for man-made fibre reinforced plastics. This problem has also been approached using Weibull weakest link theory as early as 1966 by Bohannan (1966). Simple Weibull theory was applied to the decrease in strength with volume by Barrett (1974) and Madsen and Nielson (1976). This theory was extended to allow for the anisotropic nature of wood using a modified Weibull theory in which the effects of length, width and thickness are considered separately (Bohannan (1966), Buchanan (1983), Madsen and Buchanan (1986)). Buchanan (1983) also suggested that the variability in wood properties may be important, and this is analogous to the concepts discussed in section 3.3.

The analysis of the size effect commonly seen in the strength of concrete is usually fracture mechanics based (Litle and Paparoni (1966), Carpinteri and Bocca (1987), Carpinteri (1989), Saouma (1991), Urano et al. (1996)). The microstructural concepts of scaling this material (Litle and Paparoni (1966), Saouma (1991), Urano et al. (1996)) are comparable to the problems encountered in fibre composite plastics and, as mentioned in section 3.3, this is described by David and Nolle (1982).

4.3 Brittle Fibres Size Effects

The consensus of work concerning individual filaments and bundles of fibres (Zweben (1994), Watson and Smith (1985), Moreton (1969), Metcalf and Schmitz (1964), Herring (1966), Diendorf and Tokarsky (1975), Bader and Priest (1982)) does indicate that there is a decrease in strength as length is increased, and also as the number of filaments increases. However, the description of these effects by the statistical theories used is by no means conclusive. For example, Moreton (1969) describes only as "fair" the fit of the weakest link model he used to describe the strength of carbon fibres.

Further extensions of these theories appears to have increased not only the complexity of the statistics involved but also of the interpretation of the results obtained. Watson and Smith (1985) used the carbon fibre data of Bader and Priest (1982) to fit more complicated models, but still admitted that the theory is not apparently satisfied for bundles. Despite their mathematically elaborate model, phrases such as "...but this is just a guess and major source of uncertainty." describing the estimation of the relevant parameters raises doubts as to the need for such detailed models. Padgett et al. (1995) used a further development of Weibull theory only to obtain "reasonable" fits to the same data. The mathematical simulations of Karbhari and Wilkins (1991) used experimental data only to give estimates of certain parameters and there is no experimental verification of their predictions. This pattern of more and more complicated statistical analyses with either very little experimental back-up or inconclusive correlation is also seen in the work concerning aramid fibres (Wagner et al. (1984), Wagner (1987), Wagner (1989), Knoff (1987), Schwartz and Sembach (1986), Penning (1994)).

4.4 Composites Size Effects

In his article entitled "Is There a Size Effect in Composites?" Zweben (1994) stated that the question of the existence of a size effect in composites has been around since the 1960s. The fact that much work is still currently underway in the subject shows that conclusive evidence has not yet arisen to answer the question. However, several viewpoints have been established and to quote Zweben, "...arguments on the subject have taken on a religious quality, with each person believing his or her creed dogmatically. The intensity of feeling is exceeded only by questions relating to test methods and failure criteria." The issues of test methods and failure criteria are, of course, also an integral part of the composite size effects problem. This article is recommended as an unbiased overview of the subject of composite size effects.

4.4.1 Statistical Strength Theories

Various statistical aspects of the fracture of composites were discussed by Argon (1974) and Kelly and MacMillan (1986). Batdorf (1990) provided an excellent overview of both simple weakest link and bundles of fibres models for fibrous composites. He took the view that, although the simplifications made by Zweben and Rosen (1970) lead to a non-exact mathematical solution, the increase in accuracy achieved by such an exact solution is small when compared to the errors inherent in estimating the model parameters such as the ineffective length (see section 3.2). He also recognised the advantages given to engineers of a model easily reconcilable with physical quantities over abstract statistical models. Exact mathematical results for the chain of bundles model were first obtained by Harlow and Phoenix (1978a). Harlow and Phoenix (1978b), and later, Smith (1980) developed asymptotic approximations. Phoenix and Smith gave further refinements to, and reviews of, statistical fracture theories in papers authored individually, together and with various co-workers (see Phoenix (1974), Smith (1976), Phoenix (1981), Phoenix and Smith (1983), Phoenix (1995), Beyerlein and Phoenix (1996)).

Not all the literature concerning the statistical fracture theories as applied to aligned composites attempts to verify the models derived and, as for fibres, conclusive evidence either for or against the model is not gained from the experimental data. The models appear to give adequate qualitative predictions but are quantitatively inaccurate. Model simplifications (such as fibre arrangement, load sharing and diameter assumptions) as well as the difficulties in estimating parameters (for example ineffective length and stress concentration factor) have been suggested as possible reasons for this. Another supposition is that the theory does not allow for the fact that sources of variation, other than those due to flaws, are inevitably present (see section 3.3). Rosen (1964) verified qualitatively the progressive and random nature of fibre fractures before final failure of a single layer glass laminate by experimental observation. Zweben and Rosen (1970) analysed Rosen's strength data and found that their theory predictions correlated well with the data for small specimens. However, they questioned whether these results could be extrapolated to larger volumes such as those found in structures. Batdorf and Ghaffarian (1984) re-analysed the data of Bullock (1974) and found that their model fitted only when the estimate of the ineffective length parameter was unrealistically large. Bader and Priest (1982) were unable to reach any firm conclusions about the agreement with theory for data from single carbon fibres and impregnated bundles. Reanalyses of these data by Smith (1976) also failed to assimilate theory and experiment exactly, suggesting that this may be due to variations in fibre diameter. In order to simplify the problem, Beyerlein and Phoenix (1996) considered carbon fibres and simple micro-composites of four fibres in an epoxy matrix. However this approach, together with the inclusion of some measure of the variations in fibre diameter, still gave only a partial fit to the experimental data. Manders and Chou (1983) concluded that it is unrealistic to expect better agreement between theory and practice, in view of the approximations in the models and in the chosen values of parameters such as ineffective length. Good predictions between the strengths of carbon fibres, minitows and tows were achieved by Batdorf (1990) but the variation of strengths was much greater than expected. He suggested that this is due to sources of variation other than those due to flaws.

4.4.2 Carbon Composites Size Effects

Most of the studies of composite size effects has been carried out in the aerospace field using pre-preg carbon-epoxy laminates and mainly use simple Weibull theory to explain any size effects seen.

A frequently quoted paper is that by Bullock (1974) which described a study of two graphite-epoxy systems. Simple Weibull analysis was used to compare the strengths of single strand tows, tensile coupons and flexural three point bending specimens. The effect of volume between the similarly stressed tows and coupons was predicted very accurately by a Weibull theory that uses both volumes and stresses based on the fibres alone. Similarly, the theory predicts very well the differences in strength observed between the tensile and flexural coupons of equal volume by considering the different stress distributions present. Similar values for the Weibull shape parameter were obtained for tows, tensile coupons and flexural specimens (for 36, 27 and 13 specimens respectively) for one of the material systems, and an average value of 24 was taken. However, for the other system, more variability in the strengths was observed and a value of 18 was taken for the shape parameter. Bullock suggested that sufficient specimens must be tested in order to estimate adequately the specific value of the shape parameter for the material considered.

The effect of fabrication route on the strength of CFRP was investigated by Hitchon and Phillips (1978) who considered two types of both fibre and matrix. The specimens produced using pultrusion, hand lay-up, filament winding and pre-preg techniques were most conveniently tested using different test methods. Tensile, hoop-burst and three point flexural tests were carried out. Here Weibull theory only partially explained strength changes; tensile and hoop-burst results were well explained but flexural and tensile strengths were not reconciled. Variations in the shape parameter m (between 10.3 and 38.4) were cited as a possible reason for this; the Weibull analysis being carried out using an average value of 20. Changes in material properties between fabrication routes and failure mode differences between test methods were thought to be responsible for the observed variation in the shape parameter. Also the small number of specimens tested (between 4 and 8) was noted with reference to the confidence in the parameter estimates obtained. This statistical aspect was further investigated in the second part of the study which sought to resolve these problems by comparing the strengths of different sizes of filament wound hoop-burst specimens. No significant strength decrease ($p < 0.01$) was seen, but it was noted that, for the small number of observations made, the size effects expected for the strength variability seen would be too small to discern. Hitchon and Phillips postulated that Weibull theory may be more applicable when volume is changed through stressed fibre length rather than composite cross-sectional area, and they concluded that further investigation was required.

Tensile and flexural beam column tests of carbon-epoxy were the subject of a series of studies by Kellas and Moreton (1990),Jackson et al. (1992) and Jackson and Kellas (1995). Various lay-ups were tested, initially using ply-level scaling, and a size effect noted for flexural and tensile testing which depended on the lay-up. No effect of size is seen on the initial stiffness. Both Weibull and LEFM theories were applied, the former giving better, but variable, correlation between theory and experiment. No attempt was made to estimate the Weibull shape parameter from the variation of the data, instead this was estimated using equation (4.1) and the strengths of two specimen sizes;

$$\frac{\sigma_2^{ult}}{\sigma_1^{ult}} = \left(\frac{V_1}{V_2}\right)^{1/m} \qquad \textbf{(4.1)}$$

where σ^{ult} is the specimen strength, V is the specimen volume, m is the shape parameter and 1 and 2 indicate the two different sizes of specimen considered.

Large variations of m (7.22 to 156 for the tensile tests, and 8.5 to 18.3 for the flexural) were observed across the different lay-ups. The fracture mechanics model was thought to be inappropriate due to the complex damage modes exhibited by the composite laminates. Failure mode transitions were noted as the size increased for both tensile and flexural coupons, and again this was dependent on lay-up. Sub-ply level scaling, whereby the number of fibres in each laminate is varied, was then applied to the same flexural beam column arrangement. This was found not to alleviate the strength scale effect and, in fact, the effect was amplified compared with the earlier ply-level scaled specimens.

Wisnom (1991a) observed a size effect for unidirectional carbon epoxy for four-point bending and pinned-end buckling tests. A change in failure mode from tensile to compressive was seen with increasing size. Wisnom postulated that a greater size effect in compression than in tension caused the larger specimens to have lower compressive strengths than tensile strengths. Importantly, it was noted that the cure used heated plates for thin specimens, whereas an autoclave was used for the thicker specimens. Although visual inspection showed no difference in material quality, the different manufacture processes could provide a possible explanation for the size effects seen. A Weibull shape parameter of 25 was estimated from Wisnom's data using equation (4.1), but less scatter in the results than this suggests was seen. In order to explain this behaviour, a model of the composite between the extremes of a brittle solid and a loose bundle of fibres was postulated. This model predicted a size effect more dependent upon length than upon the other dimensions. Hence a second study of specimens of the same cross section, but with varying lengths using three-point bending tests was completed Wisnom (1991b). Here a Weibull model with both volume and length terms was fitted to the data and found to account for the lower than expected variation. Wisnom draws attention to the caution required when comparing relatively small differences in strength based on observations from a small number of specimen tests. In a further paper Wisnom (Wisnom (1993)) reports a size effect for interlaminar tensile strength measured using curved beam four point bending tests.

Grothaus et al. (1995) compared three and four point bending of carbon fibre reinforced plastic using Weibull theory. The stress concentrations at the loading rollers were found to influence failure mode, with steel rollers producing compressive failure and plastic rollers giving tensile failure. By considering tensile and compressive failures separately, Weibull theory was used to explain strength differences.

An investigation into scale effects for fatigue by Chou and Croman (1979) also concerns graphite-epoxy. Here in-line holes drilled in the specimens were used to represent the "links" in weakest link theory. Application of Weibull theory then allowed reasonable predictions to be made. Grimes (1995) reviewed selected literature on the static and fatigue scale effects of graphite epoxy bonded and bolted joints.

Some work has been carried out on the scaling of the impact of carbon reinforced plastics (Morton (1988), Morton (1995), Qian et al. (1990), Swanson (1995), Poe and Jackson (1995)). The scaling of CFRP notched strength has also been studied by Shahid et al. (1995).

Publications which consider the effects of scaling at a microstructural level include a study of defects by Wang (1995) and studies into the relationships between ply thickness and damage accumulation by Crossman and Wang (1980), Crossman et al. (1983) and Lagace et al. (1989).

4.4.3 Glass Composites Size Effects

There is less information available concerning glass reinforced plastic composites, which are of far greater interest to the marine engineer. Camponeschi (Camponeschi (1989), Camponeschi (1995)) evaluated the effect of size on compression strength of carbon and glass composites for large naval structures. He postulated that although strength was observed to decrease with thickness, this could be attributed to fixture restraint effects. A size effect for the strength of glass fibres was found by Kies (1964) but he stated that, with good design, a correspondingly large size effect is not seen in the strength of filament wound pressure vessels. He also states that, "The rather large discrepancy between virgin filament strength and strength in structure should not be regarded as due to fiber degradation but rather associated with unequal tensioning limitations due to resin, surface finishes, and design factors not yet optimised". Elliot and Sumpter (1993) considered a material common in the marine industry, woven roving glass-polyester. In this case no change in compression strength with size was found. However, a change was found for tensile tests, and again this was attributed to fixture effects. Interestingly, compressive strength was seen to vary through the thickness of the parent laminate from which the specimens were cut. Wisnom (1996)) reports a size effect for the interlaminar shear strength of glass epoxy measured using short beam shear tests .The behaviour of woven roving glass-polyester was characterised in tension, compression, shear and flexure by Zhou and Davies (Zhou and Davies (1995a), Zhou and Davies (1995b)). LEFM and simple Weibull theory were used to explain strength variations with size. The latter method was found to give better predictions of experimental results, although the lack of statistical analysis of small differences in strength obtained from small numbers of observations does not instil confidence in the conclusions proffered.

Crowther and Starkey M. S. (1988) found a size effect in the fatigue of unidirectional glass reinforced epoxy and used Weibull statistics to explain this. They recognised that, "The success of this will depend on how sensitive fatigue life is to the difference in the manufacturing routes used to make small specimens and large components".

4.4.4 Effects of'Scale'

One reason for the concentration of the literature on size effects due purely to the amount of material could be that most of the work has been carried out in the aerospace field using pre-preg carbon-epoxy laminates. Here the material is fairly consistent and perhaps the effects of manufacturing are thought to be unimportant.

The significance of any size effects due to the scale of production rather than solely due to the size of the artefact, as detailed in Section 1.2, are mentioned only briefly in very little of the literature on composites size effects. This issue is most comprehensively discussed by Zweben (Zweben (1994), Zweben C. (1995)) and Batdorf (1990). Most of the literature on the subject appears to be more interested in obtaining rigorous mathematical and empirical solutions to the theories discussed in sections 3.1 ,3.2 and 3.4 than in the original impetus for examining the possibility of a size effect, i.e. how do we scale from low cost models or coupons to full scale structures? In fact, the other type of possible explanations for size effects, outlined in section 3.3, are mentioned only rarely. Zweben recognised them as important but then ignores them. Hitchon and Phillips (1978) suggested that the reason for differences between their data and that of Bullock (1974) is due to differences in the materials used and also note quality differences in batches of their own material, despite efforts to ensure that the manufacturing processes were identical. Crowther and Starkey M. S. (1988) suggested that manufacturing differences may be important and that it would be useful to investigate these effects. A rare example of a study of the real life problem of scaling coupon data to full scale testing is the rather qualitative and specific work of Lowe and Satterly (1994) concerning filament wound glass-polyester spars for wind turbine blades. A more quality-control orientated paper by Karbhari et al. (1991) suggested that the effect of the volume of production and material processing should be considered as well as geometric scale, but no specific details are mentioned. Scaling effects in nature were discussed by Wilkins (1995) that might be relevant to composites, but again no direct comparisons are made.

4.4.5 Other Distribution Functions

The basis of most of the statistical fracture theories in the literature is the Weibull two parameter distribution but others have also been considered. Moreton (1969) used the normal distribution, as did Hwang and Han (1987) and Term (1981), the latter two also using the log normal distribution. Term also considered the three parameter Weibull distribution. The Birnbaum-Saunders distribution was fitted to data by Jerina (1977). Durham and Padgett (1996) also employed this form, and also the inverse Gausian distribution. Phoenix (1974) applied the discrete Poisson distribution to model the strength of composites.

4.5 Synopsis of the Current Situation

Chapter 2 uses dimensional analysis and the theory of models to explain what is meant by the term 'size effect' as applied to the properties of a structural material. In Chapter 3 the mathematical models developed to describe this effect have been presented. In this chapter the work carried out thus far to try to reconcile the reality of experimental observation with the theories postulated has been reviewed. An overview of the areas, results and shortcomings of this literature is presented here.

The statistical fracture theories used are well developed mathematically and are usually based upon the weakest link theory also known as 'Weibull theory'. This theory assumes brittle behaviour and has been found to satisfactorily describe the size effects seen for the strength of both ceramics and single constituent fibres of composite materials (such as those of Carbon and Glass). For the failure of bundles of fibres the simple fracture theories have been developed to give reasonable descriptions of the progressive failure mechanisms encountered.

However, when fibre reinforced plastic composites are considered the agreement between experiment and theory is by no means clear, and the literature is often contradictory. In some cases the theories have only been presented as a mathematical exercise without any reference to experimental data. The complexity of some of the models becomes redundant when their parameters become difficult or impossible to estimate, and gross approximations are required. Also, the advantages of further refining the mathematical model must be balanced against the assumptions made in the derivation of the theory (especially concerning the uniformity of the microstructure).

The LEFM model, although appearing in a number of publications, does not appear to describe the experimental data to which it is applied at all well. One reason for this could be that the dimensional analysis which produces this theory requires that any cracks in the composite are geometrically scaled with the specimen or component (see Section 3.4). Since the strength of the fibre reinforced plastic composites concerned is mainly fibre controlled, this is not plausible. There is no reason why the flaws in a larger laminate should be any different from those in a smaller one, if the same reinforcement is used in both. The theory also does not allow for the complex failure modes often seen in FRP's.

The studies in the literature, as a group, do not follow any particular pattern. The testing of composites is a complex subject on its own even before the issue of size effects is raised (see Section 1.2). Often the phrase, "Composites size effects" is used to cover the entire spectrum of material properties, test methods, test parameters and material systems considered in all of the literature. For example, one study may consider the tensile testing of hand laid-up, 0°/90° carbon / epoxy laminates (Kellas and Moreton (1990)), whereas another may concern four-point flexural testing of pre-preg unidirectional carbon / epoxy (Wisnom (1991a)). Moreover, comparisons have been made within individual studies between composites manufactured using completely different processes such as filament winding, pultrusion, hand lay-up and pre-preg (Hitchon and Phillips (1978)). Considering these points, it is perhaps not unexpected that the results are often contradictory. As will be described in Section 5.3, it is neither effective nor efficient to approach a problem involving a large number of variables by considering each in turn and in isolation.

Despite the lack of a general consensus of opinion on the nature of strength size effects for fibre reinforced plastics (or even on their existence), a number of authors have come to the conclusion that the phenomenon exists and that statistical strength theory may be used to quantify it. However, despite the fact that this theory is statistical in nature, very little statistical analyses of the results and trends are reported. Much of the literature simply fits the model to the data to obtain the appropriate parameters and then uses this to predict specific data values (Hitchon and Phillips (1978), Bullock (1974), Kellas and Moreton (1990), Jackson et al. (1992), Wisnom (1991a), Wisnom (1991b), Wisnom (1993), Zhou and Davies (1995b)). This may appear, on face value, to be acceptable. For the data here, however, there is a degree of scatter and some indication of the confidence with which these predictions are made should be given. Hitchon and Phillips (1978) do carry out some statistical significance tests but come to the conclusion that, for the small strength variations seen, the number of samples considered is too small too enable these effects to be distinguished from the experimental variability. This is also pointed out by Wisnom (1991b), and some average values are based on around five or even one observation (Kellas and Moreton (1990), Zhou and Davies (1995b)), with no indication as to how this affects the confidence with which the predictions may be accepted.

Hitchon and Phillips (1978) also highlight the fact that the statistical theories do not allow for variations in composite quality such as those discussed in Section 3.3. The literature appears to have lost sight of the original impetus for the study of size effects - the problem of scaling up test data to full-scale. If such effects as manufacturing / processing variables, quality and volume of production could be important, then why have they been ignored? This could be because many of the studies were concerned with high quality, aerospace type composites. Those that were not appear to have directly translated the methods already used for such materials to those with much more variable mechanical properties. In fact, Hitchon and Phillips do try to explore this issue, but do so in a non-systematic way and hence are unable to interpret their results easily. As described in Section 1.2, for the more variable marine composites of this study, it is important that such sources of variation should be investigated.

4.6 Methodology

In summary, an investigation of possible composite materials strength size effects concerns both a number of pertinent variables, and also experimental data subject to considerable scatter. This type of problem requires an efficient experimental programme and statistical analysis techniques in order to separately estimate the effects of each variable, and also to distinguish these effects from the random variation in the experimental data. The methods of statistically designed experimentation (as described in the next chapter) have been developed to benefit exactly this type of problem, and hence it is advantageous to use them here (Sutherland (1997b)). This represents a new approach to the question of strength size effects for composite materials, and hence this study was designed to facilitate the accumulation of experience in the application of such methods to the study of composite materials mechanical properties.

As indicated in Section 4.4, the majority of the existing work in the field concerns high quality pre-preg carbon / epoxy laminates for use in the aerospace industries. This thesis, however, is concerned with much more variable, ship-building quality marine composites. It is not appropriate to simply apply the same rationale as is used for the study of the former category of composites to that of the latter. Since the processes involved in the fabrication of marine composites are not subject to the same degree of control as those for aerospace materials, production factors may be important here, concerning effects of scale as described in Section 3.3.

The full test programme involved three distinct stages. Firstly, testing of unidirectional laminates (Sutherland (1997b)) and two series of tests on woven roving glass-polyester shipbuilding quality material (Sutherland (1997c)). The aims of the unidirectional tests were two-fold; firstly to gain initial experience of experimental design techniques applied to destructive composites testing, and secondly, to provide a link between the existing work on aerospace type materials and the planned investigation regarding more variable marine type composites. This series of tests trials are referred to as the 'Uni-DirectionaF tests and are described in detail in Chapter 6.

For the next series of tests the emphasis of the programme moved to marine fibre reinforced plastics. A "'work-horse" composite material used throughout the marine industry is E-glass woven roving / polyester. This material is used by Vosper Thornycroft (UK) Ltd. to construct mine-hunters, and 'in-house' testing of such laminates raised questions about the determination of the mechanical properties of shipbuilding GRP, and the transfer of such data to the ship scale. A production related variable, in the form of variations in the reinforcement, was introduced here through the use of woven rovings produced by three different manufactures to the same nominal specifications. A more elaborate experimental design was also used in order to add to the existing knowledge of such methods. This series of tests are referred to as the "Woven Roving Manufacturer' tests and are described in Chapter 7.

The investigation of possible size effects in the strength of shipbuilding quality E-glass woven roving / polyester GRP was extended in the final, largest and most comprehensive test series. Laminates were again produced under shop-floor conditions by Vosper Thornycroft, and the experience of this shipyard in the commercial production of such materials was used to identify further relevant production related factors. The more extensive experimental design used here enabled predictions to be made with greater statistical confidence, and also required the use of fairly non-standard statistical methods to analyse the results obtained. The use of such methods made it possible to distinguish between different sources of the experimental variation. This series of trials are referred to as the 'Final Woven Roving' tests and are described in Chapter 8.

5. Experimental Design

Experimental design has been defined as the 'purposeful changes of the inputs (factors) to a process in order to observe the corresponding changes in the outputs (responses)' (Shmidt and Launsby (1992)). The subject of experimental design and analysis is well documented (see section 5.2). An introduction to the subject is given in this chapter with particular emphasis on concepts and techniques employed in the experimental work described in Chapters 6, 7 and 8. More detailed and wider-ranging information is available in the literature, for example. Grove and Davis (1992)and Box et al. (1978)).

5.7 *History*

Although statistically designed industrial experiments were investigated in the 1930's (Tippet (1935)), work in the 1940's on improving yields in agriculture (Fisher (1951)) and in the 1950's in chemical engineering studies, e.g. for I.C.I. (Davies (1956)), gave most of the early grounding in the subject. The need for carefully designed agricultural experiments arose from the problem that many nuisance factors such as weather conditions, type of soil, and the position in the field where the crop is grown might mask any changes in yield between different crop varieties. The statistical methods developed to overcome these problems have become important in industry, particularly in the field of quality improvement. In fact, the remarkable upturn of Japanese industry, which changed from one with a reputation for making 'junk' in the 50's and 60's to one renowned for its high quality products at a low price in the 70's and 80's, has been attributed in part to its willingness to accept and use statistical methods such as experimental design (Shmidt and Launsby (1992)). The work of two American statisticians Deming (1986) and Juran (1964) was instrumental in this Japanese upturn, but it was not until the early 1980's when the Japanese engineer and statistician Genichi Taguchi became widely known in the West that the methods of experimental design were widely embraced in Western manufacturing industry. The main contribution of Taguchi (1986) was the ethos that the engineering design of the product, the manufacturing process and the operating environment should be investigated in carefully planned experiments. The information gained from these experiments is then used to reduce the variability in the product through improved design or manufacturing changes. In fact the term 'Taguchi Methods' is now often, and inaccurately, applied loosely to any statistically designed industrial experiment.

5.2 Overview

There is much literature concerning experimental design, and the statistical methods used to analyse the resultant data is even more extensive. Hence this section is not intended to be a comprehensive review of this literature, but aims to direct the reader to accessible accounts of the subject which are of relevance to the engineer.

Basic introductions to statistical concepts and experimental design are provided for the technologist by Mendenhall and Sincich (1995) and Chatfield (1989). An excellent and comprehensive treatment of experimental design, written in the language of the engineer, rather than that of the statistician, is given by Grove and Davis (1992). This text arose as a result of the success of a Ford Motor Company engineer's training programme. Shmidt and Launsby (1992) have also approached the subject from an engineers' viewpoint. Both texts use numerous industrial examples to illustrate the methods. Once a working knowledge of the subject has been gained, the more statistically orientated works (Box, Hunter and Hunter (1978), Cochran and Cox (1957), Cox (1958), John (1971), Kempthorne (1952), John and Quenouille (1977). Anderson and Mclean (1974)) can give a deeper and wider understanding. Of these, Box, Hunter and Hunter (1978) is perhaps the most comprehensive and comprehensible for the engineer.

A large number of studies utilising the methods of statistical experimental design reflect the history of the subject. Agricultural and biological examples are common in the literature (Fisher (1951). John and Quenouille (1977)), as are manufacturing process and product improvement exercises (Grove and Davis (1992), Box, Hunter and Hunter (1978)). Investigations into material properties are also present. For example, Anderson and Mclean (1974) use metallurgy data to illustrate experimental design techniques and Basheer et al. (1994) use very simple experimental designs to investigate the durability of concrete. Despite the applicability of statistical experiments to composite material properties problems (see Section 4.4), far less work has been published in this field. Production related literature includes investigations into the effects of machining graphite / epoxy composites (Ramulu and Arola (1994), Kohkoner et al. (1991)) and also studies of the processing effects on thermoplastic composites (Giles and Reinhard (1992), Strong et al. (1990), Madenjian (1985)). The mechanical properties of thermoplastic composites have been explored using experimental design techniques (Madenjian (1985), Ramasamy et al. (1996), Weil (1994), Khunova et al. (1993), Sain and Kokta B. V. (1993), Adams (1991)), as have those of polyethylene reinforced concrete (Soroushian (1993)). Such studies of carbon composites are very rare: Linyuan et al. (1991) investigated carbon / carbon composites; Moncman (1995) considered the thermal properties of carbon / epoxy composites, and Knight (1989) completed a study into the residual strength of impact damaged carbon / epoxy pressure vessels. No such studies concerning composite size effects, glass reinforced composites or marine composites have been found.

A characteristic of many statistically designed experiments is that the effect on the performance of a system of more than one variable is investigated. This is achieved by changing the values of the variables simultaneously. This is better than the alternative of looking at each variable individually in a series of separate experiments since a more complete understanding of the features of the system and how they interact may be gained.

The overall aim for designed experiments is to understand how one or more measured output variables are affected through varying a variety of input factors. This is achieved by constructing an approximating statistical model for the measured variables in terms of the factors which are thought to affect it. An example of such a model is given in Section 5.3. In much of the literature this type of model is rarely mentioned. Nevertheless it underpins the methods of analysis employed. It would not be unreasonable to assume that the initial selection of the appropriate model would be crucial to the success of the experiment. Ansell and Phillips (1994) state, however, that, "Whilst in some specific cases this may be the case, generally the desire is to identify factors or variables which are having an effect on the component's or system's performance. In such cases most of the models will at least be able to detect the effects of factors which are strongly associated with lifetimes,...".
The basic terms used in experimental design will now be defined in order to simplify further descriptions of this subject;

Response variable: This is the output or measurement of interest from the system under consideration, for example the strength of a composite coupon.

Factors: Factors are those features of the experimental system which are controllable and whose influence on the response is of interest to the experimenter, for example specimen size and fibre reinforcement material. Factors may fall into one of two groups;

1. *Quantitative,* factors whose values can be arranged in order of increasing magnitude, for example length,
2. *Qualitative,* factors whose values cannot be arranged in order of increasing magnitude, for example fibre reinforcement material.

Factor levels: The values taken by a factor in an experiment are called the levels of the factor. The experimenters, together with as many persons familiar with the system as possible, should set the factor levels on the basis of existing knowledge to encompass the normal regions of operation. Quantitative factors may take any number of levels within reason, for example the length of a specimen may be set at any number of values. Qualitative factors are generally more restricted, for example there may only be a finite number of fibre reinforcement materials available. However most designed experiments use only 2 or 3, or occasionally as many as 4 or 5, levels for their factors in order to keep the size of the experiment practicable.

Treatment combination: A treatment combination, is a specified combination of factor levels at which an experimental run is made in order to make an observation.

Full factorial experiment: In such an experiment, a response is measured at every possible treatment combination.

Fractional factorial experiment: Here only a subset of all possible treatment combinations is used in the experiment.

Experimental unit: This is the object on which a particular trial is carried out, for example specimens tested to destruction to obtain strength data.

Effects: The change in the response variable caused by changing the level taken by a factor is referred to as the '*main effect'* of that factor. Any variation of the effect of a factor with the level taken by another factor is described by an '*interaction effect'*. Both of these terms will be explained further in Section 5.3.

The planned experiment should be thought of as part of an overall engineering cycle of design, testing, redesign and so on. This cycle has been split up into four main activities, the Plan, Do, Check and Act stages forming the Deming, Shewart or PDCA cycle (Deming (1986)). The importance of the Planning stage should not be underestimated; engineers should carry out all the necessary analysis and research to avoid wasting resources on a badly planned experiment. If an inapplicable design is used it may well be difficult or even impossible to obtain valid conclusions from any data gathered. Due to the cyclic nature of the procedure it is also inadvisable to try to make the initial experimentation too far-reaching, since there is a necessity to hold back resources for follow up experiments. The 'Do' and 'Check' steps consist of the experimentation itself and the analysis of the results. Often the 'Act' stage is a decision to use accumulated knowledge from previous cycles to plan another experiment, or it may be a final decision such as on which changes to make to the design or which conclusions may be made about the behaviour of the system.

Grove and Davis (1992) suggest the following ten point plan which further breaks down the PDCA cycle into smaller steps;

(i) Decide on the objective (or objectives) of the programme of experimentation,
(ii) Bring the right team of people together to plan, run and learn from the experiment,
(iii) Choose the response or responses to measure,
(iv) Verify the capability of the process of measurement,
(v) Choose the factors that are believed to have an effect,
(vi) Choose the number of levels of the factors, and the values of the levels,
(vii) Choose the experimental plan, which may involve cutting out some of the factors initially listed, and decide on the run order,
(viii) Conduct the experiment,
(ix) Analyse the results,
(x) Take action on the findings.

If step (x) includes planning another experiment, as it usually will, it may be necessary to go through the sequence again from as far back as step (iii), step (ii) or even step (i). As with any experimentation it is important that any existing knowledge, together with engineering common sense, be used in conjunction with any results obtained from steps (viii) and (ix).

5.3 Factorial Experimentation

A common method of scientific experimentation entails keeping all factors constant at a 'base' level for the first trial, using this response as a reference value, and then varying each factor individually, i.e. *a* 'one-at -a-time' experimental design. The incentive for this approach is a desire to carry out as few trials as possible, which on face value seems to be satisfied by this design. Also, practitioners will often claim that this type of experiment gives results which are "easier to understand". However this is a false logic for 'one-at-a-time' testing is seriously flawed, as will be shown in this section. A far more efficient method is that of factorial experimentation, where several factors are changed before each run. The changes are not made in an arbitrary manner, but in such a way that the overall plan enables the effects of each factor to be retrieved from the data. In fact, not only the experimental plan but also the whole approach to statistically designed experiments should be systematic rather than 'ad-hoc' so that the data can be interpreted sensibly.

To illustrate the increased efficiency obtained through the use of a factorial experiment consider the simple case where there are three factors each at two levels. The full factorial two-level, three-factor experimental plan is termed the $L_8(2^7)$ design by Taguchi (1986): L_8 because it consists of eight runs, 2^7 because it has seven factors each being considered at two levels. As there are eight observations, it is possible to examine seven comparisons in the data. These may be chosen to be the estimates of the main and interaction effects, as follows. It is convenient to represent the lower level of each factor ('lower' will be arbitrary for a qualitative factor) with a '-' and the upper level with a '+'. The full factorial design is shown in Table 5.1 and the equivalent 'one-at-a-time' experiment in Table 5.2, where each row represents a treatment combination or experimental trial, for example a test on an individual specimen. The meaning of the word 'equivalent' as used here will become apparent.

Run Number	Factor A	Factor B	Factor C
1	.	.	.
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+

Table 5.1: L_8 (2^7) Experimental design

Run Number	Factor A	Factor B	Factor C
1	.	.	.
2	.	.	.
3	.	.	.
4	.	.	.
5	+	-	-
6	+	-	-
7	+	-	-
8	+	-	-
9	-	+	-
10	-	+	-
11	-	+	-
12	-	+	-
13	-	-	+
14	-	-	+
15	-	-	+
16	-	-	+

Table 5.2: 'One-at-a-time' Experiment

An important property of the array in Table 5.1 is that of ***Orthogonality;*** each column corresponding to a factor has an equal number of '+' and '-' entries and is said to be orthogonal with respect to the main effects. This property gives the design a useful 'balanced' pattern ; for example on inspection it can be seen that for the four runs with A at the '-' level two are with B at the "+" level and two are with B at the '-' level, and the same pattern occurs for the four runs with A at the '+' level. This 'balance' is evident when any pair of factors is considered. At this point it is helpful to introduce a simple statistical model for the response which includes only main effects;

$$y = \alpha a + \beta b + \gamma c + Const. + e$$

where a, b and c are coded -1 and +1 for low and high levels of factors A, B and C respectively, a. p and y are unknown parameters and 8 is a random error term.

Hence, for the treatments (runs) with A at the low level in Table 5.1 we have;

$$y_x = -a - p - y + Const. + e_i \qquad (5.2)$$
$$y_3 = -a + P - y + Const. + £)$$
$$y_s = -a \sim P + y + Const. + s_5$$
$$y_j = -a + P + y + Const. + e_1$$

It can now be see that an average value for the response variable with A at the low level may be obtained through the summation of y_1, y_3, y_5, and y_7 and subsequent division by four, and that this value is independent of P and y ;

$$y_{A-} = -a + Const. + s_{A-} \qquad (5.3)$$

Similarly for A at the high level;

$$y_{A+} = +a + Const. + e_{A+} \qquad (5.4)$$

The orthogonal factorial design gives three main advantages;

(i) The effects of each factor may be easily separated through an averaging process,
(ii) The effect of altering a factor level is seen at more than one level of the other factors,
(iii) The experimental error may be estimated with greater precision,

The first advantage is important as it means that the statistical precision of the designed experiment is maximised. Replications of the experimental runs are not required to give estimates of the experimental error, much more of the data than individual runs are available for this purpose. For example the $L_8(2^7)$ design will effectively have four runs available for calculating the response for each factor at each level, since the effects of changing the levels of the other factors between these runs are 'balanced' and hence cancel. Because of this the $L_8(2^7)$ design needs only 8 runs to give 4 replications for each level of each factor whereas the equivalent "one-at-a-time' experiment requires 16 trials for the same precision.

The second advantage makes possible an investigation of the joint action of factors on the response through the examination of *interactions.* An interaction between two factors occurs when the effect of changing one factor is not constant but depends on the value taken by the other factor. This is most easily illustrated graphically as in Figure 5.1.

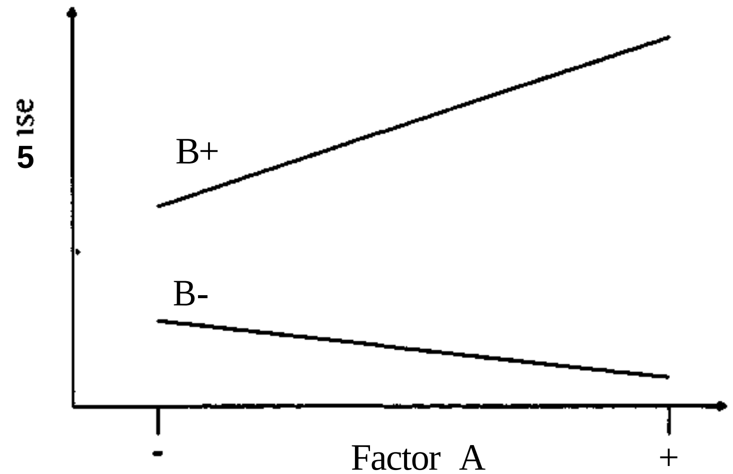
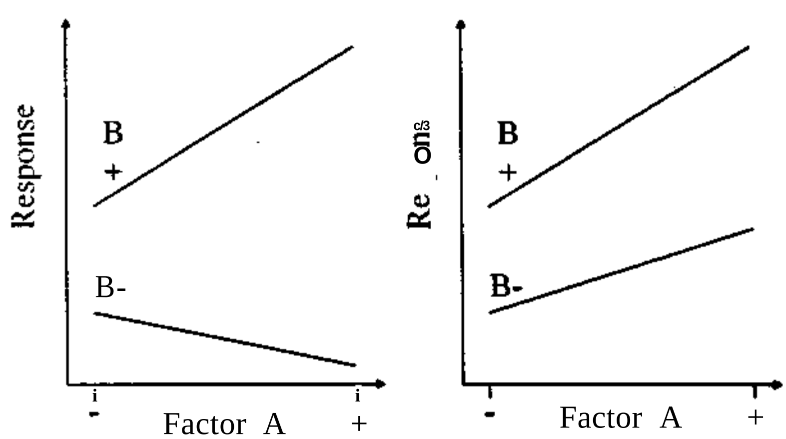


Figure 5.1: Illustration of Interaction

In this case, when B is at the '+' level, changing A from '-' to '+' gives an increase in the response whereas the same change in A when B is at the '-' level causes the response to decrease. This is only one specific example, interaction will be present for any case where the two lines are not parallel, they may cross, diverge, or converge. The interaction shown in Figure 5.1 is denoted by AB or AxB and is called *a two-way interaction* since it involves two factors. If this interaction is itself dependent upon the level of another factor, C say, then we have what is termed a *three-way interaction,* denoted by ABC or AxBxC. An example of this type of interaction is illustrated in Figure 5.2 .



(i) At C-

(ii) At C+

Figure 5.2: Illustration of a three-way interaction

This concept can be extended to any number of factors, giving higher order interactions between any number of factors. In physical systems three-way and higher order interactions are often small enough to be neglected. In most industrial or engineering applications this is often taken as an initial assumption, unless previous knowledge indicates otherwise. Such an assumption, if wrong, would lead to a poor fit of the statistical model to the data. Assuming that the higher order interactions are negligible does not mean that the comparisons amongst the observations that could be used to estimate them have not been used. The sizes of these comparisons are assumed to be due solely to experimental variation and are often used to estimate the experimental error, thereby increasing the precision with which the main effects and two-factor interactions can be estimated. This means that, not only the data from any duplication of experimental runs may be used to estimate the variation in the results, but that other comparisons within the data set can be used to give a more accurate estimate. This is one of the great advantages of the use of experimental design methods over conventional methods of experimentation.

The *main effects* are virtually meaningless in isolation if there are any important ***interaction*** *effects.* This can be seen when we introduce interactions into the previous statistical model for the response;

$$y = \alpha a + \beta b + \gamma c + \delta ab + \varepsilon ac + \eta bc + \lambda abc + Const. + \varepsilon \qquad (5.5)$$

where the products ab, ac and bc represent two-way interactions, abc is a three-way interaction and α, β ... λ are unknown constants.

We can now see the importance of the second advantage of orthogonal designs given above; viewing the effect of a factor at differing values of other factors enables any interactions to be calculated. Compare this to the one-at-a-time experiment where, for example 'A+' only, occurs when B and C are at the '-' level, rendering it impossible to calculate any interaction effects.

If it is known a-priori that some or all of the interactions between the factors are negligible then the number of experimental runs required to give the required information may be reduced through the use of fractional factorial designs. However, this also reduces the precision with which the experimental error can be estimated and a compromise in the design of the experiment must be reached. Such a design may be chosen to be orthogonal and its treatments are often a subset of those which form a full factorial experiment. For example the design shown in Table 5.3 can be seen to be derived from the full factorial $L_g(2^7)$ experiment of Table 5.1.

Run Number	Factor A	Factor B	Factor C
1	-	-	+
2	+	-	-
3	-	+	-
4	+	+	+

Table 5.3 : $L_4(2^3)$ Fractional Factorial Design

The examples used so far have all concerned only two levels for each factor. However if more information is required on the form of the factor effect then three, four or even five levels may be used. This complicates the statistical analysis, however, and care must be taken when interpreting the results of such an analysis. The design and data analysis techniques used for experiments with factors at more than two levels are explained further in Grove and Davis (1992) and Box, Hunter and Hunter (1978).

The nature of the system studied may require a more sophisticated design structure. If the levels taken by one factor are required to be different for each level of another factor then a *nested* design structure is present and the former factor is said to be nested under the latter. For example, flexural testing may require the length to depth ratio to be within a certain range and this may lead to lower length values for thinner specimens than for thicker ones. Here length would be said to be nested under thickness. Also the nature of the system may require the experimental runs to be conducted in discrete sections. For example, in an agricultural context, crops may have to be grown under different conditions in separate areas or plots. This is where the term *split-plot* design originates, there may be certain variations between different plots (across the *whole-plot),* and different variations within each plots *(sub-plots).* Another, more relevant, example is the fabrication of composite specimens for mechanical testing whereby specimens of different thickness' must originate from different panels due to the nature of the fabrication process. Nested and split-plot designs are further explained in the literature (Cochran and Cox (1957), Kempthorne (1952), Anderson and Mclean (1974)).

5.4 Statistical Analysis Techniques

Initially the relevant main and interaction effects for the factors must be calculated. This may be achieved using various statistical techniques including normal plots, the method of contrasts, analysis of variance (ANOVA) and multiple regression (Mendenhall and Sincich (1995)). Grove and Davis (1992) recommend the use of the method of contrasts for the engineer in industry since it is easily implemented using simple spreadsheet software. This method also provides a useful introduction to the concepts of experimental design and so Appendix C contains a simple worked example.

Further analysis of the results of the experiment must determine whether or not an observed effect is due to changes made between treatments ('real') or solely due to the random residual variation from uncontrolled noise factors. Simple statistical methods, especially graphical methods, are advocated for this task by Grove and Davis (1992), after which the relevant effects and interactions may be included in an analysis such as linear regression or ANOVA to give a prediction equation. However, especially when one factor effect is much larger than the others, this may lead to the case where important factors are overlooked. Hence the approach used in this study is to include all pertinent main and two-way interaction effects in an ANOVA or regression analysis, and then to interpret the forms of the important effects graphically.

It should be stressed that, although the statistical methods used are more rigorous than simply accepting some effects to be 'real' because they are much 'larger', care must be taken to ensure that the analysis used takes into account the structure of the design used. The basis of the methods commonly used is to compare the size of an effect with the estimated error and then to decide whether, at a specified significance level, this effect is 'real' or just due to the random noise of the system. If we choose a significance level of, say, 5 percent then in order to accept that an effect is real we need to assume that the probability of the observed effect being due solely to noise is less than 5%, i.e. that we are 95% sure that the effect is 'real'. The significance level is set somewhat subjectively on the basis of the consequences of an incorrect prediction. For example, we would want to be, perhaps, 95% sure that our car would start in the morning, but would be far happier with a 99.999% chance that it would stop each time we applied the brakes. Hence it is important to remember that the results of these statistical methods must depend on the assumptions made which should be realistic and pertinent.

ANOVA and regression statistical methods using 'MINITAB' and 'SAS' software packages were employed in the analysis of the data obtained in this thesis. A simple example of an analysis of variance is included in Appendix D. For more detailed and wide-ranging information of the mechanics of these methods the reader is directed to the literature (Grove and Davis (1992). Chatfield (1989), Box, Hunter and Hunter (1978), Mendenhall and Sincich (1995)). The specific statistical analyses carried out in this study are, however, detailed in Chapters 6, 7 and 8.

6. Uni-Directional Tests

6.1 Test Programme Development

The main aim of this series of tests was to gain experience in applying the techniques of statistical experimental design and analysis to the materials property testing of composite materials in order to explore possible size effects. Previous work by Shenoi et al. (1994) had developed manufacturing procedures using unidirectional glass-epoxy laminates and so, in order to use this experience, the same methods and materials (see Section 6.3) were used here. This earlier study was designed to follow on from the work of other authors (see Chapter 4) and so as to strengthen this link from mainly aerospace orientated work to the planned future studies of marine quality composites, unidirectional carbon-epoxy was also included.

The tensile response and strength of composite materials is important from a design point of view, and so the tensile testing method was selected. An easier, more economical and often used method is that of flexural testing. Problems have been encountered in relating the results from flexural to tensile tests and this has been attributed to the statistical distribution of flaws within the material (see Section 3.1). In order to further investigate this issue flexural tests were also considered. The use of these test methods also allows comparisons with much of the literature outlined in Chapter 4. The details of these methods are detailed in Sections 6.2 and 6.4.

As described in Chapter 4, various authors have postulated that any size effect may depend upon the way in which the volume of the composite material is increased. Hence the length, width and thickness of the test specimens were varied separately. In order to allow greater variation between the levels of width for the tensile tests, and length for the flexural tests, a nested structure of length and width under thickness (number of plies) was formulated. Width and length were varied within each of three thicknesses. For the flexural tests the influence of the diameter of the loading and support rollers was also investigated. The lists of factors for the two test methods are thus;

Tensile:

(i) Thickness,

(ii)Reinforcement,

(iii)Length,

(iv)Width.

Flexural:

(i) Thickness,

(ii)Reinforcement,

(iii)Length,

(iv)Width,

(v)Roller Diameter.

Since the series of experiments was to be an introduction to the methods of experimental design, a simple two level full factorial design for the three factor tensile tests was selected. A single replicate of each specimen led to $2 \times 2^3 = 16$ runs (specimens) at each thickness, i.e. a total of 48 specimens. The flexural tests included an extra factor in that of roller diameter. In order both to achieve the same number of specimens as the tensile tests, and also to broaden the experience with experimental design methods, a half-replicate design was selected for these tests. The coded designs are shown in Table 6.1 and Table 6.2 and a schematic of the structure of the unidirectional test programme is given in Figure 6.1.

Specimen	**Reinforcement**	**Length**	**Width**
1	Glass	Low	Low
2	Glass	Low	High
3	Glass	High	Low
4	Glass	High	High
5	Carbon	Low	Low
6	Carbon	Low	High
7	Carbon	High	Low
8	Carbon	High	High

Table 6.1: Experimental design of unidirectional tensile tests

Specimen	**Reinforcement**	**Length**	**Width**	**Roller** 0
1	Glass	Low	Low	Low
2	Glass	Low	High	High
3	Glass	High	Low	High
4	Glass	High	High	Low
5	Carbon	Low	Low	High
6	Carbon	Low	High	Low
7	Carbon	High	Low	Low
8	Carbon	High	High	High

Table 6.2: Experimental design of unidirectional flexural tests

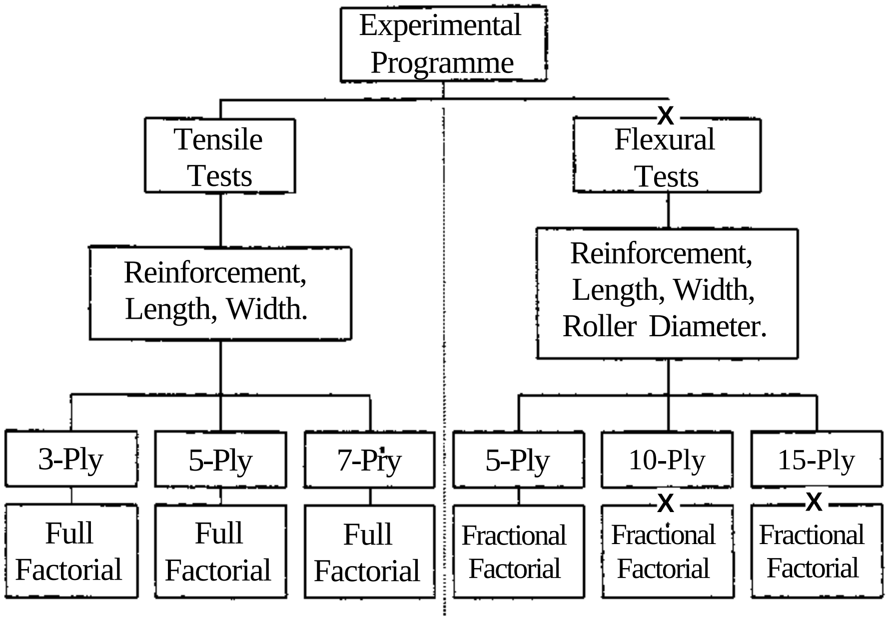


Figure 6.1: Schematic of unidirectional test programme

Each group of 16 specimens of the same thickness were scaled geometrically across thickness, for example the values of the low length and widths of the 10 ply flexural specimens were twice those for the 5 ply specimens. As described in Chapter 2, although the use of the system equation for stress can allow for geometric dissimilarity, any effect outside the scope of the basis of this equation may be altered by this model distortion. Keeping the groups of specimens geometrically similar between thickness minimised the model distortions between specimens. For example, this ensured that the corresponding flexural specimens at each thickness had the same length to depth ratio, an important factor in this type of test.

6.2 Specimen Sizing

Testing was carried out using the American Society for the Testing of Materials (ASTM) standard methods as guidelines. For the flexural tests the relevant standard is ASTM D790M: "Standard Test Methods for Flexural Properties of Unreinforced and Reinforced Plastics and Electrical Insulating Materials" (ASTM (1986)). The equivalent for the tensile tests is ASTM D3039: "Standard Test Method for Tensile Properties of Fiber-Resin Composites" (ASTM (1989)). However, since the experimental design techniques used necessitated independent variation of the specimen dimensions, these standards were used mainly as a starting point for specimen sizing. The values of the specimen dimensions were then set around those stipulated to give as large a variation in specimen dimensions within specific restraints.

The starting point for the flexural specimens was the thickness and values of 5, 10 and 15 plies were decided upon in order to coincide with those of an earlier study (Shenoi et al. (1994)). The fibre volume fraction was set at 40%, again following on from this earlier study. Also, the marine industry is likely to produce more matrix dominated materials. The unidirectional E-glass was already available and so was used for this study. The Glass was of weight 450 g/m^2 and, together with the material densities and fibre volume fractions, gave the required ply thickness for the glass specimens of 0.45. Hence the thicknesses for the 5, 10 and 15 ply specimens were set. The cloth weight of carbon required to give equivalent thickness for the same fibre volume fraction was calculated and the nearest available to this value was found to be 300 g/m^2. This gave a ply thickness of 0.412mm and thus the carbon specimen thicknesses. The rig set-up enabled the factor levels of load and support roller diameter to be set at 6 and 12 mm.

The values of the 5 ply length and width factors were then set using the standard as a guideline but endeavouring to provide as much variation of each factor to enable any possible effects to be more easily discerned. The values for the 10 and 15 ply coupons were then scaled up from these values. The length to depth ratios selected were 33 and 66 in order to ensure that the carbon specimens did not fail in shear (ASTM (1986)). However, when testing of the 5 ply coupons commenced it was found that the longer glass specimen central deflection became too large and failure was not seen. Hence the length factor levels were adjusted to give lower length to depth ratios and the 5-ply specimens were remade. The final factor levels for the unidirectional flexural specimens are shown in Table 6.3.

Units: mm	**Depth**	**Level**	**Length**	**Width**	**Roller ∅**
5 Ply	**Glass: 2.25 / Carbon: 2.05**	Low	**54**	**12.5**	**6**
		High	**75**	**25**	**12**
10 Ply	**Glass: 4.5/Carbon: 4.1**	Low	**108**	**25**	**6**
		High	**150**	**50**	**12**
15 Ply	**Glass: 6.75/Carbon: 6.15**	Low	**162**	**37.5**	**6**
		High	**225**	75	**12**

Table 6.3: Unidirectional Flexural Test Factor Levels

The ASTM standard was used to size the tensile 5 ply specimens. However, when these dimensions were scaled up for the corresponding 15 ply coupons, the expected load exceeded the maximum load of 10 tonnes attainable by the test rig. Hence the three levels of thickness selected for the tensile tests were 3, 5 and 7 plies. Tabs were bonded to the gripped portions of the specimens using epoxy resin (again Ampreg 20) and these were scaled between the thicknesses. The tabs became detached from the coupons on testing, however, and after several failed attempts to use adhesives with higher shear strengths their use was discontinued. The final factor levels for the unidirectional tensile specimens are shown in Table 6.4.

Units: mm	Depth	**Gripped Length**	**Level**	**Length**	**Width**
3 Ply	Glass: 1.35/ Carbon: 1.25	25	**Low** **High**	52.5 105	3.75 7.5
5 Ply	Glass: 2.25 / Carbon: 2.05	**40**	**Low** **High**	87.5 175	6.25 12.5
7 Ply	Glass: 3.15/ Carbon: 2.90	**56**	**Low** **High**	122.5 245	8.75 17.5

Table 6.4: Unidirectional Tensile Test Factor Levels

6.3 Specimen Manufacture

The composites for the unidirectional series of tests were produced using a vacuum assisted hand lay-up method. The materials used were Vetrotex 450 g/m^2 uni-directional E-glass, S.P. Systems 300 g/m^2 uni-directional carbon (UC300/350A) and S.P. Systems Ampreg 20 epoxy with standard hardener. The laminating process and the subsequent production of coupons from the laminate are described in the steps below.

1. The laminates were laid-up by hand using a serrated roller,
2. The composite was placed in a vacuum bag lined with non-porous release layers and a vacuum applied,
3. The vacuum bag and its contents were transferred to a hydraulic heated press, still under vacuum,
4. The resin was allowed to gel for 40 minutes at room temperature,
5. The appropriate stops for the required laminate thickness were fitted and the hydraulic press operated,
6. The first part of the cure for 16 hours at room temperature was completed,
7. The second part of the cure for 5 hours at 80°C was completed,

9. Oversize specimens were cut from the laminate using a diamond edged saw and then the required coupon dimensions were obtained using emery paper and a micrometer.
10. The centre of one face was prepared for the application of strain gauges by abrading the surface with fine grade emery paper and then cleaning thoroughly with acetone
11. Strain gauges were fitted using cyanoacrylate adhesive, applying pressure to the gauge for 2 or 3 minutes.

6.4 Experimental Details

Both tensile and flexural tests were carried out using an Instron 3664 used at constant cross-head speed. A load length of one third the support length was used for the flexural tests and the load and support rollers were clamped in place in order to avoid slippage. An even distribution of the load was ensured by allowing the loading block and hence the two load rollers to pivot with respect to the cross-head. To avoid damage due to the load rollers, especially for the smaller specimens, the strain gauges were fitted in the centre of the lower (i.e. tensile) face. The flexural test method used is shown in Figure 6.2.

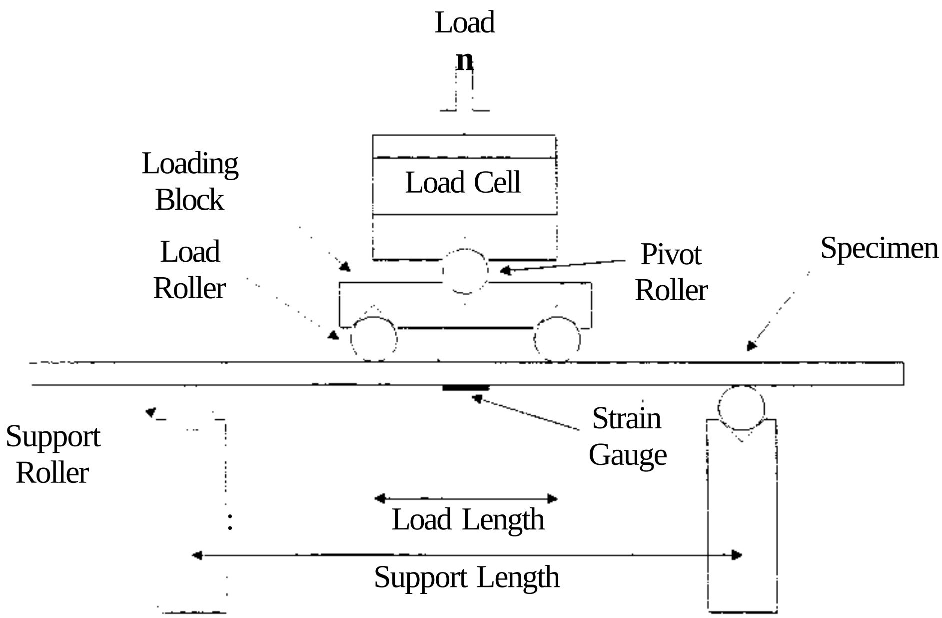


Figure 6.2: Flexural Test Configuration

The tensile specimens were secured at each end using self tightening 'wedge' type grips, one of which was attached to the rig via a universal joint. The strain gauges were attached to the centre of one face. The tensile test method used is shown Figure 6.3

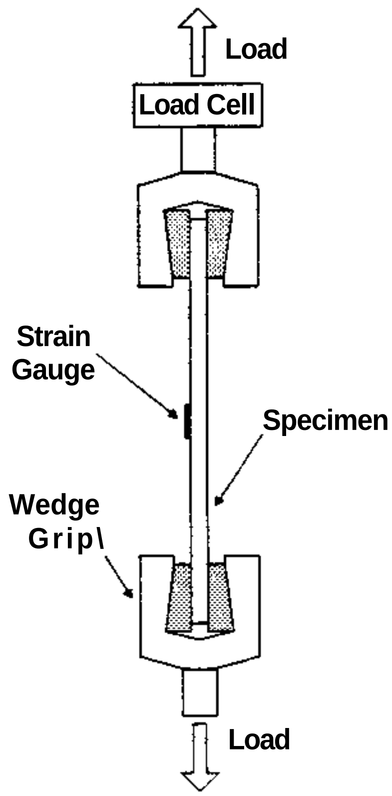


Figure 6.3: Tensile Test Configuration

For both types of test, the width and depth of each specimen was measured at five positions along the length using a micrometer before commencing the test. Any significant features of the specimen such as voids were also recorded at this stage. After starting the test, the load at which the first cracks or clicks were heard was noted and then the failure sequence was described quantitatively.

Analogue plots of load from the Instron load cell versus cross-head deflection were obtained for each test. Graphs of load versus strain from the strain gauges were also produced (see Appendix L). The appropriate cross-head speeds were obtained from the relevant standards (see Section 6.2) for the five ply specimens for each test configuration. These values were then scaled with respect to the number of plies for the other specimen thicknesses. In some cases, due to the limited number of speeds attainable on the Instron, nearest values had to be taken. The cross-head speeds, under deflection control, used for the tests are given in Table 6.5.

Test Method	**Thickness (Number of Plies)**	**Cross-Head Speed (mm min*^{1})**
Tensile	**3**	**0.5**
Tensile	**5**	**2**
Tensile	**7**	**2**
Flexural	**5**	**5**
Flexural	**10**	**10**
Flexural	**15**	**10**

Table 6.5: Instron Cross-Head Speeds

6.5 Data

The data obtained from the unidirectional test programme can be found in Appendix B, and typical failure modes in Appendix L. The specimen codes start with the letters 'UNI' to denote the fact that they are part of the unidirectional test programme. This is followed by a letter indicating the test method used, T for tensile and F for flexural. The middle part of the code consists of the number of plies and then a 'G' for glass specimens or a 'C for carbon specimens. Next a number relates to the length and width of the specimen as shown in Figure 6.4.

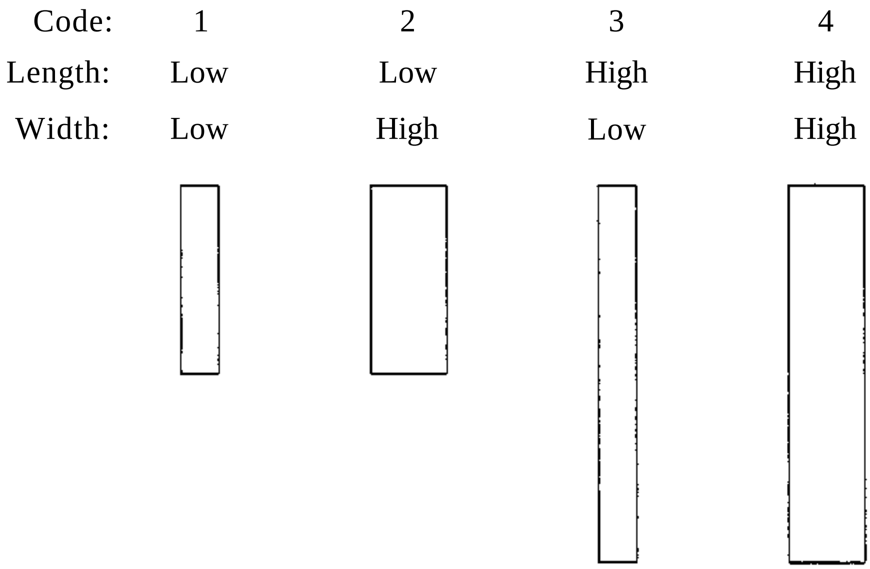


Figure 6.4: Specimen Coding

The final part of the code refers to the replication of each specimen, two of each were tested, 'A' and 'B'. Hence, as an example, the first unidirectional tensile three ply glass specimen with low length and width would have the code 'UNIT/3G/1 A'.

6.6 Method of Analysis ***of Tensile Data***

The analysis described in this Section was applied using both failure stress and failure strain as the response. A summary of the results obtained is included in Appendix E. During the description of the analysis, an explanation of the underlying statistical assumptions and how these may be checked is given. These general points apply to the later analyses in the chapter.

The concept of a "split-plot' design was introduced in Section 5.4, and this design structure is present in this test series. In this case a number of specimens are cut from each of a number of separate panels. Separate panels must be constructed for each thickness and also for each fibre material (glass and carbon) due to the nature of the laminating process. Hence, in this case, there are six panels, one of carbon and one of glass, for each of the three thicknesses. From each of these panels 8 specimens were cut for both tensile and flexural tests, a total of 16 specimens from each laminate. The terminology used is that the panels form the whole-plots, and that the specimens cut from within each laminate form the sub-plots. This structure may be used to classify the factors into whole-plot and sub-plot variables. Here the thickness and fibre reinforcement factors are whole-plot factors and those found in each panel (length and width) are defined as sub-plot factors. Interactions between factors may be similarly categorised.

In physical systems three-way and higher order interactions are often small. In most industrial or engineering applications this is taken as an initial assumption, unless previous knowledge indicates otherwise. In this case there was no information indicating that any higher order interactions were important and hence, for the purposes of this analysis, three way interactions and above were considered to be negligible. Such an assumption, if erroneous, would lead to a poor fit of the statistical model to the data. Assuming that the higher order interactions are negligible does not mean that they have not been used. These effects are assumed to be due solely to experimental variation and are thus used to increase the precision with which the experimental error is estimated. This means that, in this case, not only the data from the two duplicates of each specimen is used to estimate the variation in the results, but that the data set as a whole may be used to give a much more accurate estimate. This is one of the great advantages of the use of experimental design methods.

The main effects and interactions considered and their respective classifications into whole- and sub-plot terms are given in Table 6.6

Whole-Plot	**Sub-Plot**
Thickness	Length
Reinforcement Material	Width
Thickness x Reinforcement Material	Length x Width
	Thickness x Length
	Thickness x Width
	Reinforcement Material x Length
	Reinforcement Material x Width

Table 6.6: Classification of Factors and Interactions

The basis of the statistical analyses performed in this study consist of the postulation of a statistical model, such as that described in Section 5.3, and the subsequent comparison of the effects of the terms in this model with the experimental error or variation. It is quite possible that the variation in properties between panels (the whole-plot variability) may differ from that between specimens cut from the same panel (the sub-plot variability). Hence, in this case, the statistical model includes the factors and interactions in Table 6.6 together with error terms both for the whole-plot variation and the sub-plot variation. It is necessary to use the pertinent sources of variation for the relevant statistical tests, i.e. the whole-plot terms must be compared with the whole-plot error and the sub-plot terms must be compared with the sub-plot error. However, the simple design used for the tensile tests does not give enough degrees of freedom to estimate the whole-plot error after the whole-plot terms in the model have been fitted to the data.

Since the whole-plot variance was not estimable, to give some indication of the relative importance of whole- and sub-plot variance, simple qualitative comparisons were made between the whole-plot terms and the sub-plot terms. Here, the 'sums of squares' (SS) for the whole- and sub-plot terms expressed as percentages of the 'total sums of squares' were used. The meaning of the "sums of squares' of a term is further explained in Appendix C and Mendenhall and Sincich (1995). However, it is sufficient here to know that the 'sums of squares' of the terms describe the amount of variation in the response variable attributed to each term. Here it must be emphasised that these percentages may be used in a purely comparative sense only, and that no statistical inferences may be made from them.

Also introduced in Section 5.3 was the design structure of nested factors. Here the length and width factors are nested under thickness. That is to say that, for example, the low length factor level for the 3-ply specimens is not the same as the low length factor level for the 5-ply specimens, which in turn differs from that for the 7-ply specimens (see Table 6.4). A similar statement can be made concerning the length high factor levels and the width low and high levels. This can make the interpretation of the analyses' results less straightforward. The interaction of. for example, length with thickness is no longer meaningful, since the same lengths have not been considered at each thickness. A modified main effect is defined as, in this case, length *within* thickness, and similarly for width. Montgomery (1984) points out that this nested effect is in fact the sum of the main effect (length or width) and the interaction effect (length x thickness or width x thickness). For the case where the interaction term is small the interpretation is simplified since the length effects at each thickness are equal even though they apply to different ranges of the length factor.

The software package "MINITAB' was used to find the correlation matrix for the factors and responses in order to give a 'feel' for the data. The same software was then used to fit a statistical model consisting of the terms given in Table 6.6. The 'General Linear Model' (GLM) procedure was employed (MINITAB (1994)) which carries out an analysis of variance (see Appendix D) using a least squares approach (Mendenhall and Sincich (1995)). The length and width factor levels were coded as 1 and 2 for low and high respectively in order to facilitate the analysis of variance, and also to reduce computational errors. These coded variables are referred to as 'Clength' and CWidth'. The variation in the actual length and width values was considered to be small enough to allow this coding. Similarly, reinforcement was coded as 1 for glass and 2 for carbon. The numbers of plies (5, 7 and 9) were used for the three levels of thickness. The nested length and width effects were then calculated in a spreadsheet format. Main effects plots, where the response at each level of the factor (averaged over the values of the other factors) are plotted, were also produced using MINITAB. These aid interpretation of the statistical analyses and an illustration of such a plot is given in Figure 6.5.

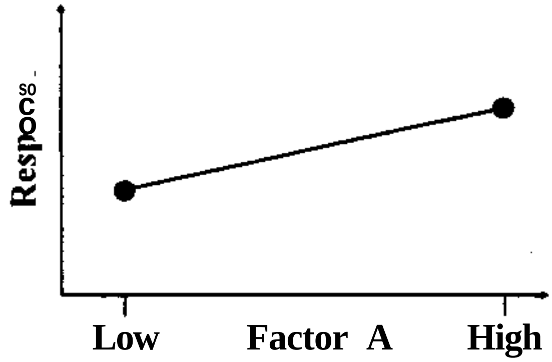


Figure 6.5: Illustration of Main Effects Plot

Interaction plots consisting of the response of one factor for each level of another factor (averaged across the values of any other factor) are illustrated in Figure 6.6, and were again produced using MINITAB.

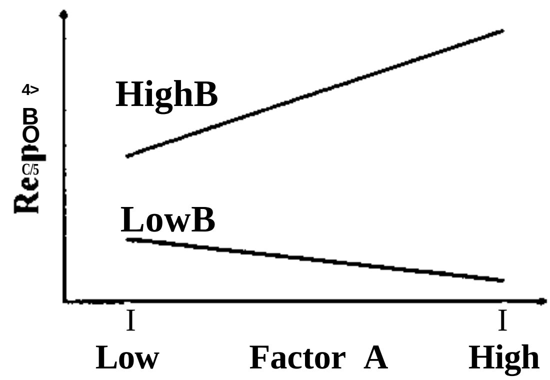


Figure 6.6: Illustration of Interaction Plot

To check the validity of a statistical model, a graphical analysis of the residuals (errors) of the model should be carried out. Consider the hypothetical data on a response variable Y obtained from a single factor X, where Y is modelled as a simple linear function of X, together with a random error.

(6.1)

where a is the Y-axis intercept, P is the effect of X and s is the random error.

The fitted line, obtained by estimating a and P in equation (6.1), together with the experimental data, is shown in Figure 6.7.

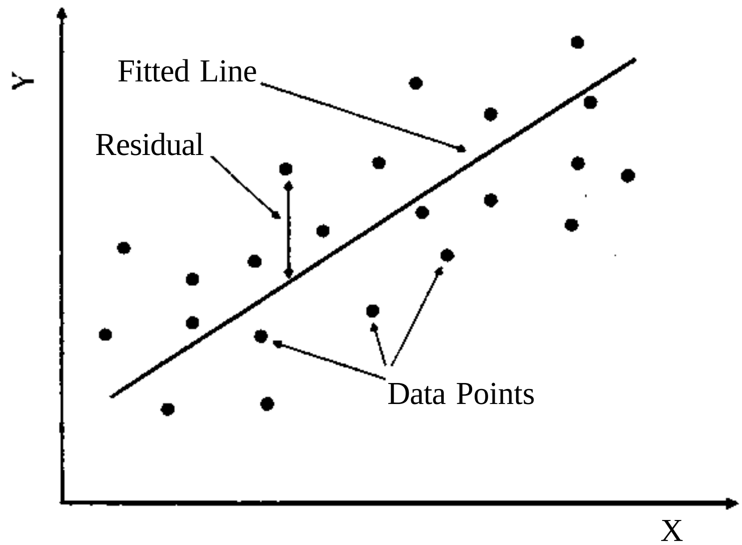


Figure 6.7: Hypothetical Data and Fitted Line Illustrating Residuals

The random errors are assumed to be independent and normally distributed with a mean of zero. Also, their variance is assumed not to vary with the settings of the factors, or with any other aspects of the experiment (for example the run order). The residuals are the deviations of the experimental data from the fitted line, and are used as estimates of the errors, s in equation (6.1).

Hence, the assumptions about the errors may be checked using plots of the residuals in a 'residual analysis". The normality of the residuals may be checked using a normal probability plot since it is possible to calculate the values expected if the residuals were normally distributed (Grove and Davis (1992)). If a plot of these calculated expected values against the residuals from the data yields a straight line with a slope of unity, then the residuals are normally distributed. The symmetry of the distribution of residuals may be assessed using a frequency histogram, although the selection of the histogram intervals can distort this information. The independence of the residuals is examined using plots both against observation number and also the fitted values (i.e. the points on the fitted line). In both cases there should be no patterns to the data. More information can be found concerning residual analysis in Mendenhall and Sincich (1995). Examples of these plots for a normally distributed independent set of residuals are given in Figure 6.8.

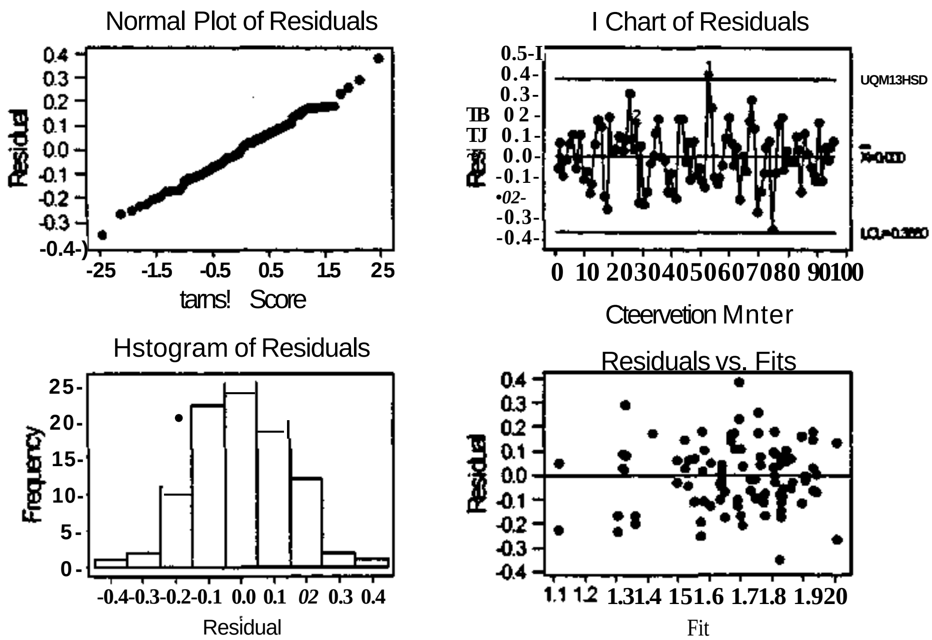


Figure 6.8: Residual Plots

The three levels of thickness considered enable two independent comparisons to be made, i.e. two degrees of freedom are allocated to this factor in the model. The factor effect is subdivided into a linear term and a quadratic term as shown in Figure 6.9.

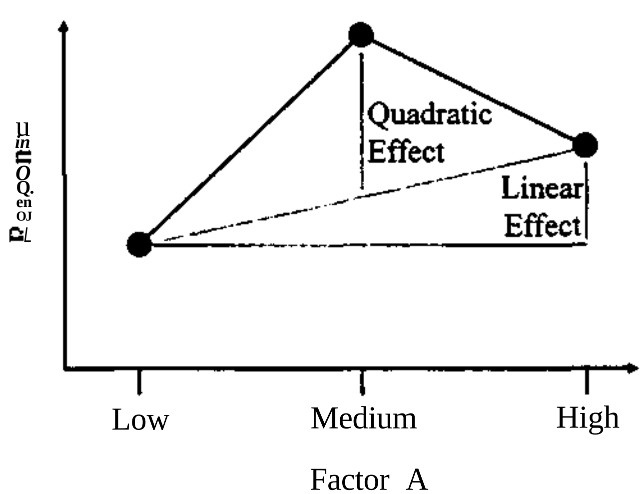


Figure 6.9: Definition of Linear and Quadratic Effects

Hence the linear effect is the difference between the average response at the low factor level and that at the high factor level. The quadratic effect is the difference between the average response at the medium factor level and the mean of the two averages which define the linear effect. The quadratic effect can be thought of as an indication of the 'curvature' of the line when the response is plotted against the factor level.

The 'SAS' software was used to further subdivide the thickness factor and its interactions into linear and quadratic components. Again a GLM procedure was used (Cary (1989)), giving a cross-check with the MINITAB analyses. Also the mean values used to plot the main and interaction effects were tabulated together with the corresponding standard errors (standard deviations of the means).

It was thought that possible indications of the quality of the specimens could be given by the initial Young"s modulus and so this measurement was included in the model as a possible covariate. However, this was not found to have a significant effect in the analyses for stress or strain and so was dropped from the model in both cases.

The summary of the statistical analyses in Appendix E comprises correlation matrices, tabulated mean values and standard errors and analysis of variance tables. The standard errors of the means may be used to give simple estimates of the confidence with which the means may be quoted. A margin of two standard errors either side of the experimental mean gives a 95% certainty of containing the 'real' value (i.e. that which would be obtained if there were no experimental error). The mean values are used to give the main and interaction plots in Section 6.8. The concepts behind the analysis of variance are outlined in Appendix D, but to interpret the ANOVA's given in Appendix E requires a working knowledge of just a few points. The 'P-value' quoted for each term in the proposed model is the probability that, in truth, the factor has no effect and the value observed is due solely to random error or scatter, i.e. 'by chance". Hence if a P-value is very low then the effect does not occur just 'by chance' and is considered to be important. For example, a P-value of 0.05 indicates a 5% probability that the effect is due to scatter or a 95% chance that it is a 'real' effect. The P-value at which we decide to accept an effect will depend on the implications of an erroneous inference, but in engineering 0.05 is often taken as 'statistically significant' and 0.01 as 'highly significant'. The value of "R-Square" quantifies the 'goodness of fit' of the model; if the fitted line passes through all of the experimental data points then $R^2 = 100\%$.

As indicated earlier in this section, the lack of an estimate of the whole-plot variance means that the appropriate F-Ratios and P-values for the whole-plot terms cannot be calculated and hence do not appear in Appendix E. However, to identify the important whole-plot terms, their respective sums of squares were 'normalised' with respect to the sub-plot error estimate. Again, it must be emphasised that these ratios may be used in a purely comparative sense only, and that no statistical inferences may be made from them.

6.7 Method ofAnalysis ***ofFlexural Data***

For the flexurai tests the same statistical analysis as described in Section 6.6 was completed with an additional sub-plot factor in load / support roller diameter. This factor's interactions with length, width, thickness and reinforcement fibre material were also considered. Table 6.7 shows the whole- and sub-plot terms for the flexural model.

Whole-Plot	**Sub-Plot**
Thickness	Length
Reinforcement Material	Width
Thickness x Reinforcement Material	Roller Diameter
	Length x Width
	Length x Roller Diameter
	Width x Roller Diameter
	Thickness x Roller Diameter
	Thickness x Length
	Thickness x Width
	Reinforcement Material x Length
	Reinforcement Material x Roller Diameter
	Reinforcement Material x Width

Table 6.7: Classification of Factors and Interactions

The inclusion of the initial Young's modulus as a covariate was again not found to be statistically significant for failure stress and failure strain and hence this term was not included in the model. Summaries of the statistical analyses of failure stress and failure strain are also included in Appendix E.

6.8 ***Findings*** *of the Data* ***Analyses***

In this section the results of the statistical analyses detailed in Sections 6.6 and 6.7 are reviewed, and important factors and trends are highlighted. A summary of these analyses is included in Appendix E.

6.8.1 Model Fitting

As a preliminary and simple examination of possible linear relationships between the response variables, treatment factors and possible covariates, consider the correlation matrices given in Appendix E. It should be noted here that the estimates of these correlation between two variables do not take into account the variations in the other variables, and hence should only be used to get an initial 'feel' for the data.

The matrix for the tensile tests, given in Table E.1, shows that initial modulus and reinforcement material are extremely highly correlated (0.991). This explains why the inclusion of modulus as a covariate is not required here. The correlation between reinforcement material and both failure stress and strain is high (0.965 and -0.985 respectively), indicating that the effect of the reinforcement factor on the responses is important. The negative sign of the latter coefficient indicates a linear relationship with a negative slope.

The corresponding matrix for the flexural tests, Table E.2, also shows that modulus is highly correlated with reinforcement material (0.949), and hence both factors together are not necessary in any linear model used to describe the data. Reinforcement is again highly correlated with strain (-0.952), but in this case there is no evidence of a correlation with stress (a correlation of 0.265). The correlation coefficients of failure stress with thickness (-0.583), length (-0.406) and roller diameter (0.417), although only moderately high, suggest that these factors may be important.

The statistical models postulated for both tensile stress and tensile strain may be represented by;

$$Strength = a_0 + a_x T_t + a_2 T_q + a_3 R + a_4 T_{\{} xR + a_5 T_q xR + E + a_6 CL + a_7 CW + a_{\%} CLxCW + a_9 RxCL + a_w RxCW + e \quad (6.2)$$

where the a's are the coefficients corresponding to the effects, Ti is linear thickness, T_q is quadratic thickness, R is reinforcement, E is the whole-plot error, CL is coded length, CW is coded width and e is the sub-plot error.

Similarly, for the flexural tests;

$$Strength = a_0 + a_x T_{\}} + a_2 T_q + a_3 R + aJ_t xR + aJ_q xR + E + \alpha_6 CL + \alpha_7 CW + \alpha_8 RD + \alpha_9 CLxCW + \alpha_{10} CLxRD + \alpha_{11} CWxRD + \alpha_{12} T_{l+q} xRD + \alpha_{13} RxCL + \alpha_{14} RxCW + \alpha_{15} RxRD + \varepsilon \quad (6.3)$$

where RD is the load / support roller diameter.

As will be described in Section 6.8.3, the reinforcement material has a very large effect on all but one of the responses considered. To enable a simple comparison of the whole-plot and sub-plot variation without the overwhelming influence of the reinforcement material it is appropriate to remove the reinforcement sums of squares values from the whole-plot totals. Similarly, to compare only material variations, as opposed to testing variations, it is appropriate to remove the roller diameter values from the flexural test sub-plot totals. This results in the values given in Table 6.8.

	Whole-Plot SS (%)	**Sub-Plot SS** (%)
Tensile Stress	3.4	1.6
Tensile Strain	0.1	1.0
Flexural Stress	39.1	25.5
Flexural Strain	7.6	0.9

Table 6.8: Modified Whole-Plot and Sub-Plot Sums of Squares Comparisons

The whole-plot sums of squares can be seen to be greater than the sub-plot sums of squares for both of the flexural test responses. For the tensile tests this is also the case for the stress measurements. For the tensile strain measurements, however, the whole-plot value, at 0.1%, is much less than that for the sub-plot, and also much lower than for any of the other values in the table.

The sub-plot coefficients of variation (the standard deviation normalised by the mean) are shown in Table 6.9.

	Tensile Stress	**Tensile Strain**	**Flexural Stress**	**Flexural Strain**
Sub-Plot CV	0.0398	0.0570	0.0607	0.0302

Table 6.9: Sub-Plot Coefficients of Variation

The coefficients for the tensile test stress and strain measurements are comparable, at approximately 4% and 5.5% respectively. The flexural values are of a similar size, but there is some indication that the stress value, at approximately 6%, is greater than the strain value, at approximately 3%.

6.8.2 Checking the Adequacy ofthe Model

The R-Square values given in Table 6.10 indicate that the models postulated fit the experimental data very well. The lowest values for both tensile analyses and the flexural strain analysis is 98%. The flexural stress value is slightly lower at 89%, but this still indicates a good tit to the data.

	Tensile **Stress**	**Tensile Strain**	**Flexural Stress**	**Flexural Strain**
R-Square	98.00%	98.10%	89.2%	99.7%

Table 6.10: R-Square Values

The MINITAB software outputs denote any data points with a residual of greater than two standard deviations as "unusual observations'. As approximately 95% of the values from a normal distribution lie within this range, only when substantially more than five percent of the data points exceeded these bounds, or when exceptionally large residuals are obtained, would doubts be raised as to the fit of the model considered. The Tables E.4, E.6, E.8 and E.10 show that this is not the case for any of the four analyses.

An inspection of the residuals for the tensile tests using Figure 6.10 and Figure 6.11 shows both stress and strain normal plots to be roughly linear, and also that the histogram of strain is not skewed. The small degree of asymmetry for the failure stress histogram can be seen, on closer inspection, to be mainly due to the choice of intervals plotted.

The residual versus fits plot for tensile stress shows no trends, but the equivalent strain plot indicates a higher variability in the residuals for the higher fitted values. Further analyses of these residuals showed that this was due to slightly different variances between the results corresponding to each of the two reinforcements concerned (carbon and glass). Since a basic assumption of the statistical methods used is that the variance is constant, this point required further investigation. Where the variance is seen to vary with the fit of the model, a procedure known as "transformation" of the data may be applied to rectify this situation. The procedure is comprehensively discussed by Box et al. (1978), and essentially consists of applying a mathematical transformation to the data before the statistical analyses is carried out. The most common transformations are; ln(x), x"^{2}, 1/x and l/$x^{1/2}$ Each of these are tried and that which produces the most uniform variance is used. Care must be taken here to ensure that the interpretation of such a transformation has some physical meaning.. In this case, a 1/x transformation produced a uniform variance. The statistical analysis of the transformed data, however, produced the same trends as those seen using the raw data. Also, such a transformation did not have any physical justification. Hence, the influence of the non-uniform variance was thought to be unimportant. Further analysis methods, not considered in this study but which may be applicable here, are considered in Chapter 11.

The flexural residual plots in Figure 6.12 and Figure 6.13 give very similar results to those of the tensile tests. Only the residual versus fits plot for the failure strain measurements gives cause for concern, as for the tensile tests, the scatter is greater for higher fits, but only marginally so in this case and so was again thought to be unimportant. Also, the apparent skew of the histogram is mainly due to the intervals plotted.

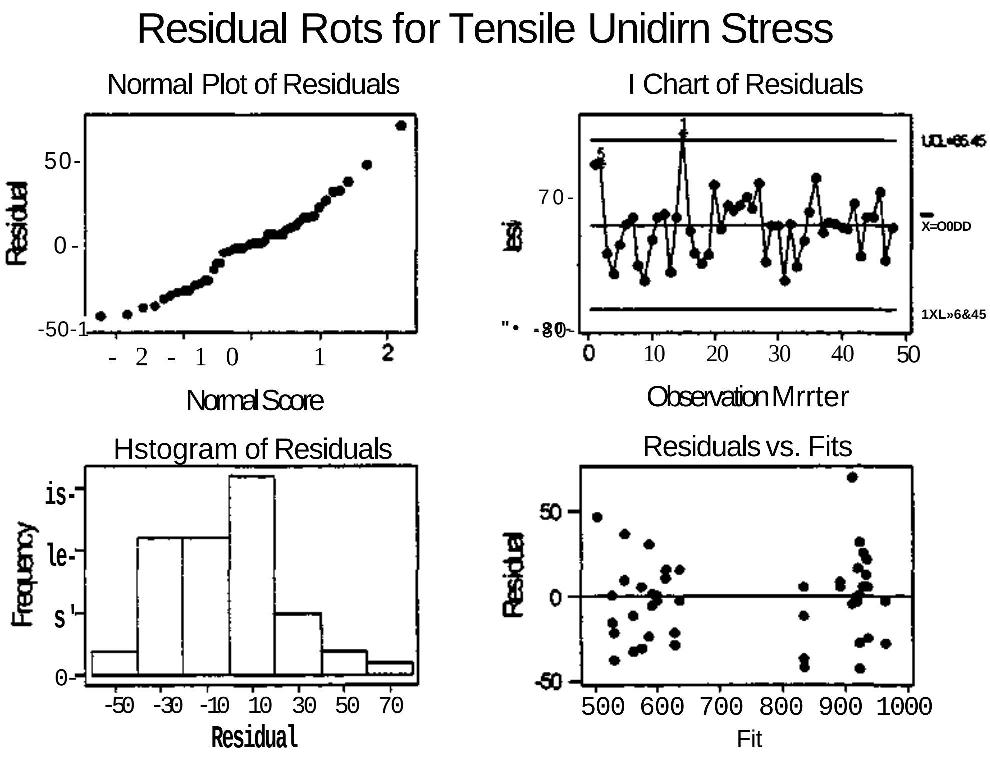


Figure 6.10: Tensile Stress Residual Plots

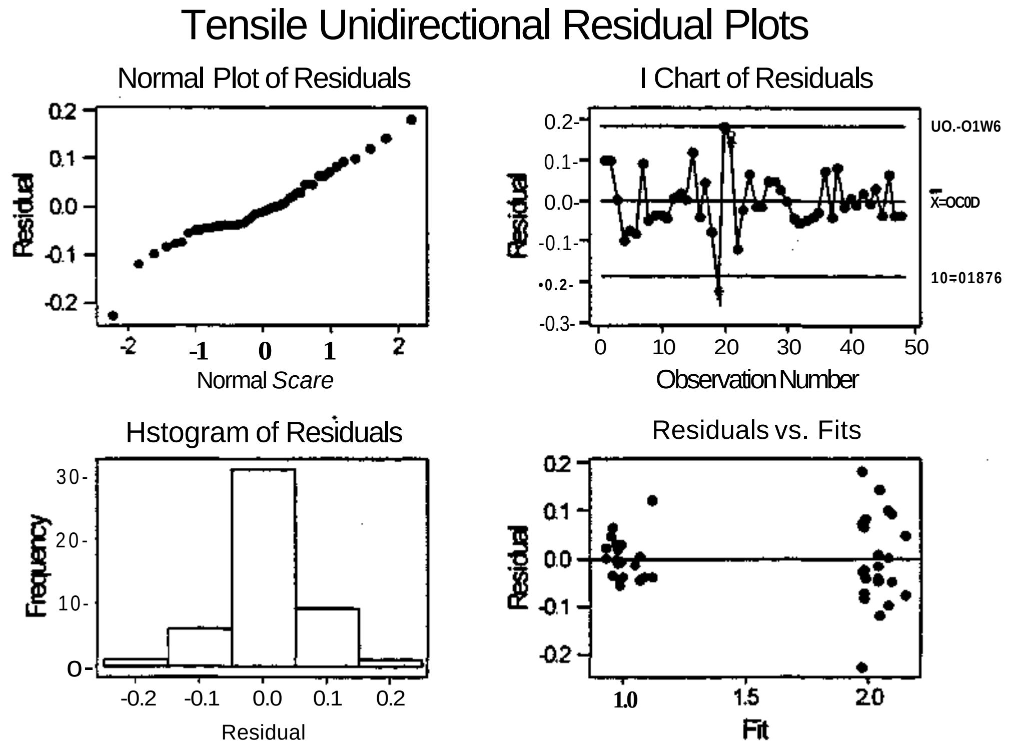


Figure 6.11: Tensile Strain Residual Plots

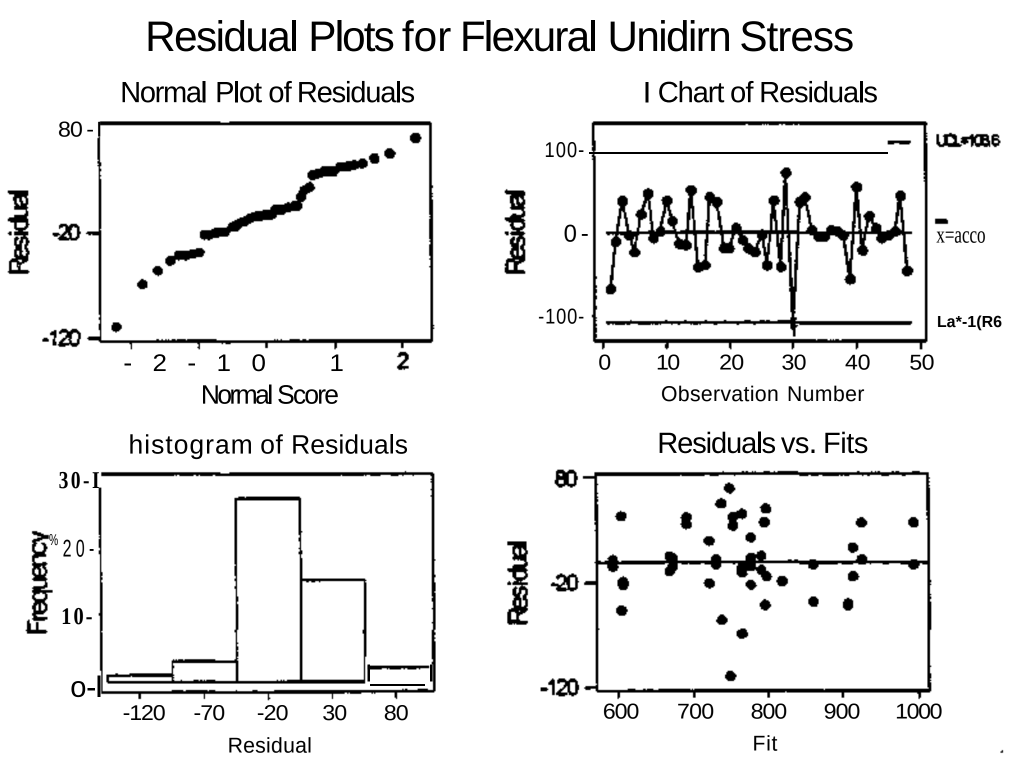


Figure 6.12: Flexural Stress Residual Plots

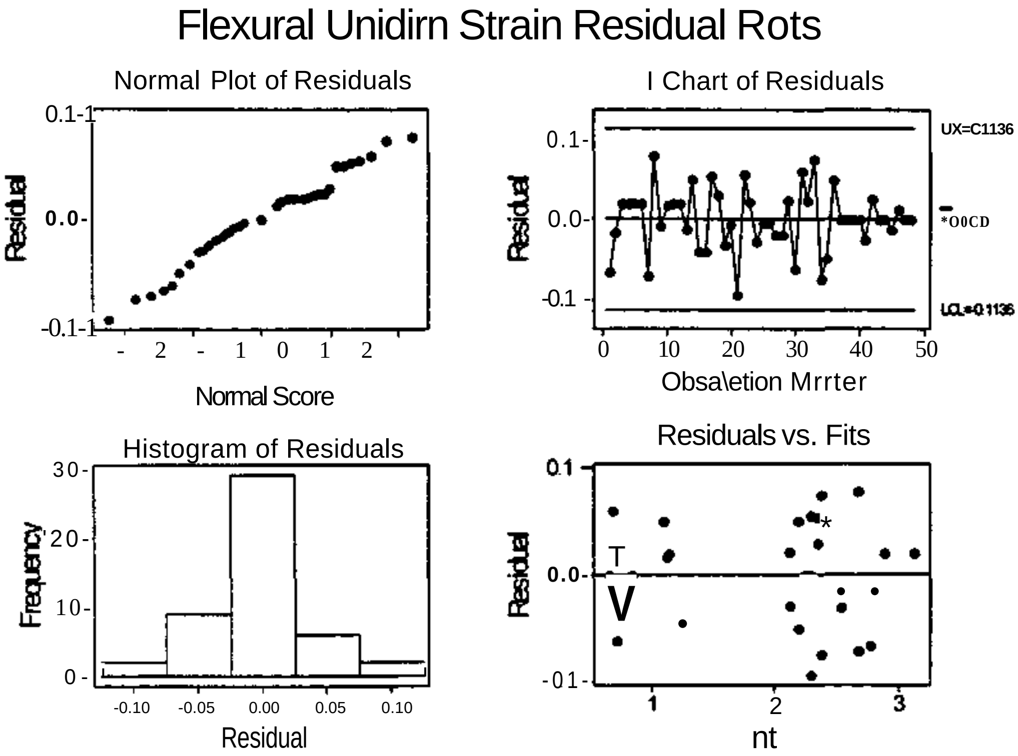


Figure 6.13: Flexural Strain Residual Plots

6.8.3 Conclusions on the Importance ofthe Factors

The sums of squares normalised with respect to the sub-plot error in Table 6.11 are now used to compare the relative importance of the whole-plot factors. Here it must be stressed that these ratios are purely comparative, and that no statistical inferences should be made from them. The important values in the table are underlined. The trends may be seen in the appropriate main effects and interaction plots, which are included in the text.

	Tensile		**Flexural**	
Source	**Stress**	**Strain**	**Stress**	**Strain**
Thick				
(Linear)	**27**	1	**85**	745
(Quadratic)	**21**	0	**6**	239
Reinforc	1559	1688	**17**	12000
Thick*Reinforc				
(Linear)	2	0	**2**	25
(Quadratic)	5	0	**4**	0

Table 6.11: Whole-Plot Sums of Squares Normalised with respect to Sub-Plot Error

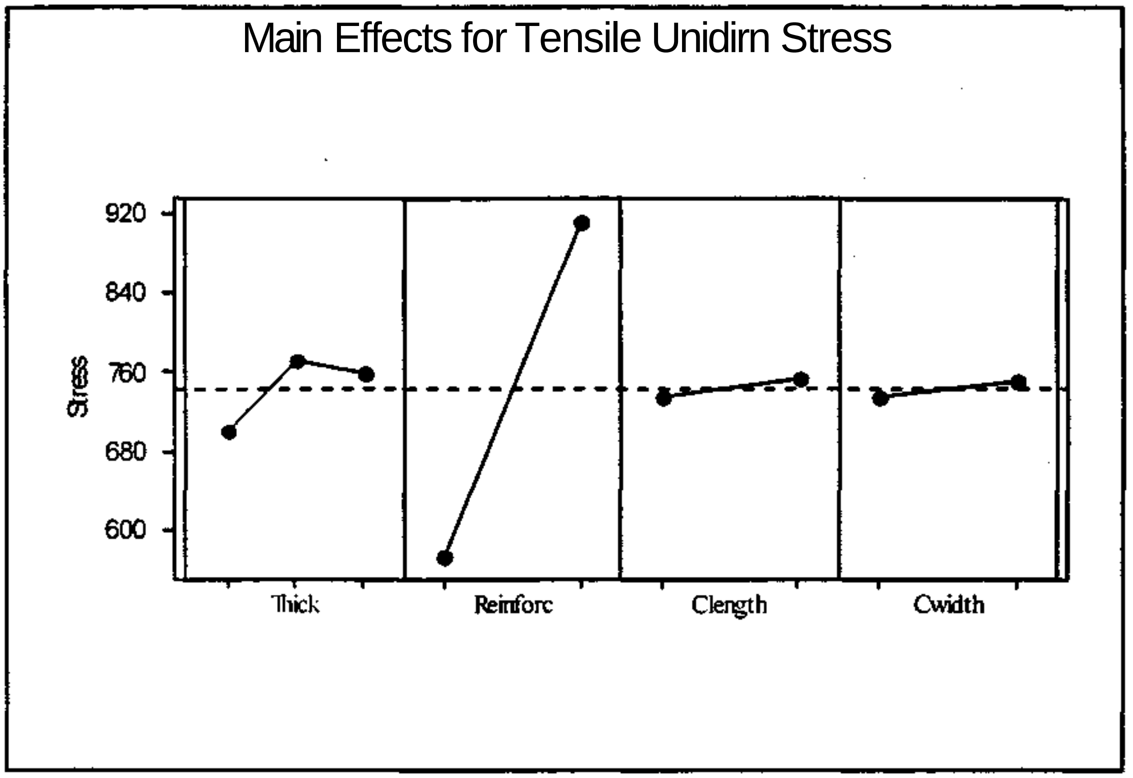


Figure **6.14:** Tensile Stress Main Effects Plots

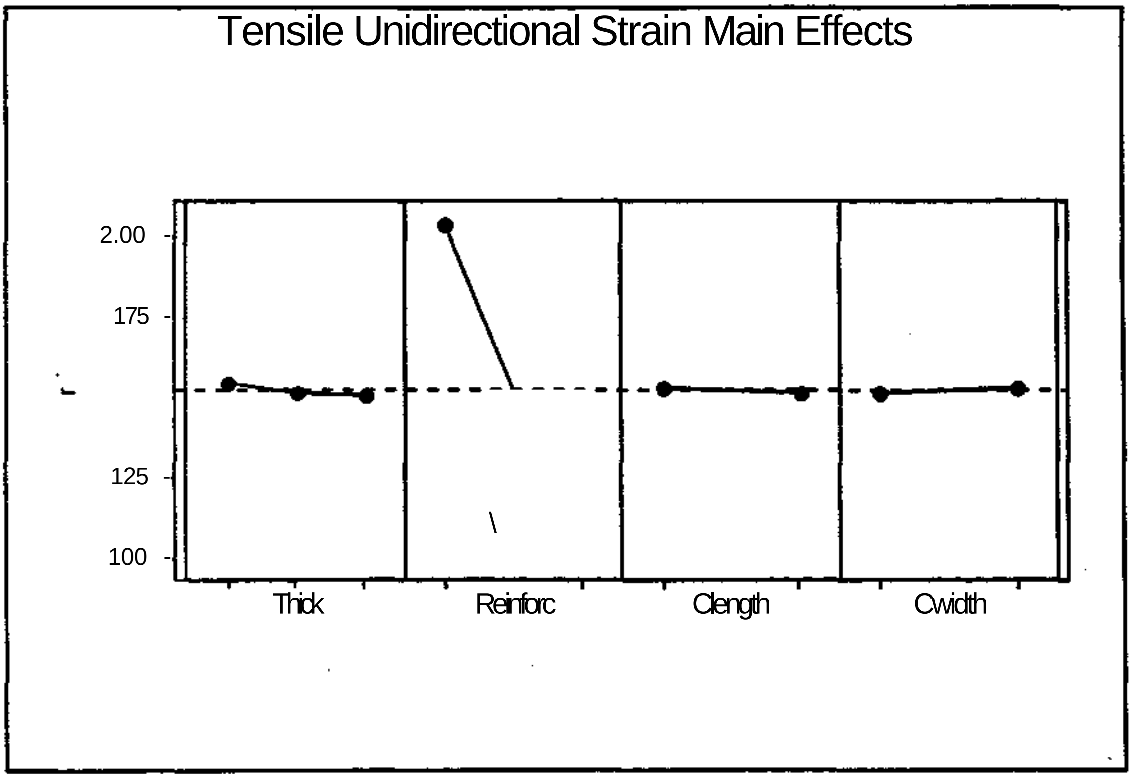


Figure 6.15: Tensile Strain Main Effects Plot

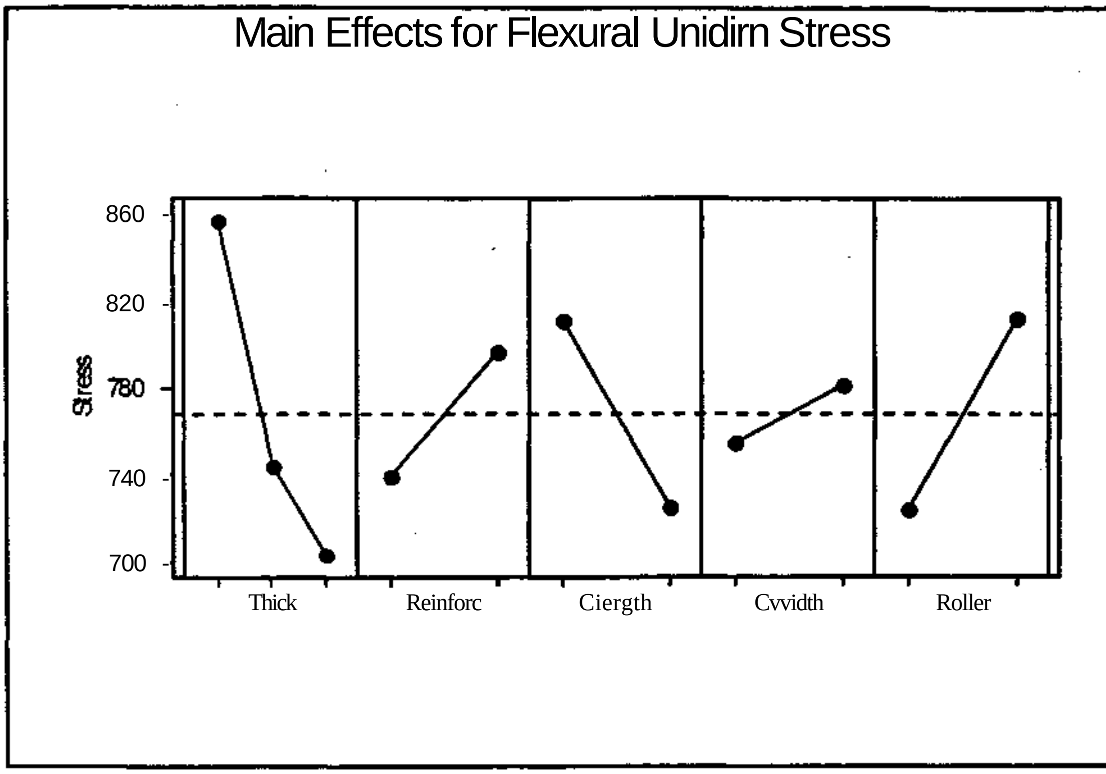


Figure 6.16: Flexural Stress Main Effects Plots

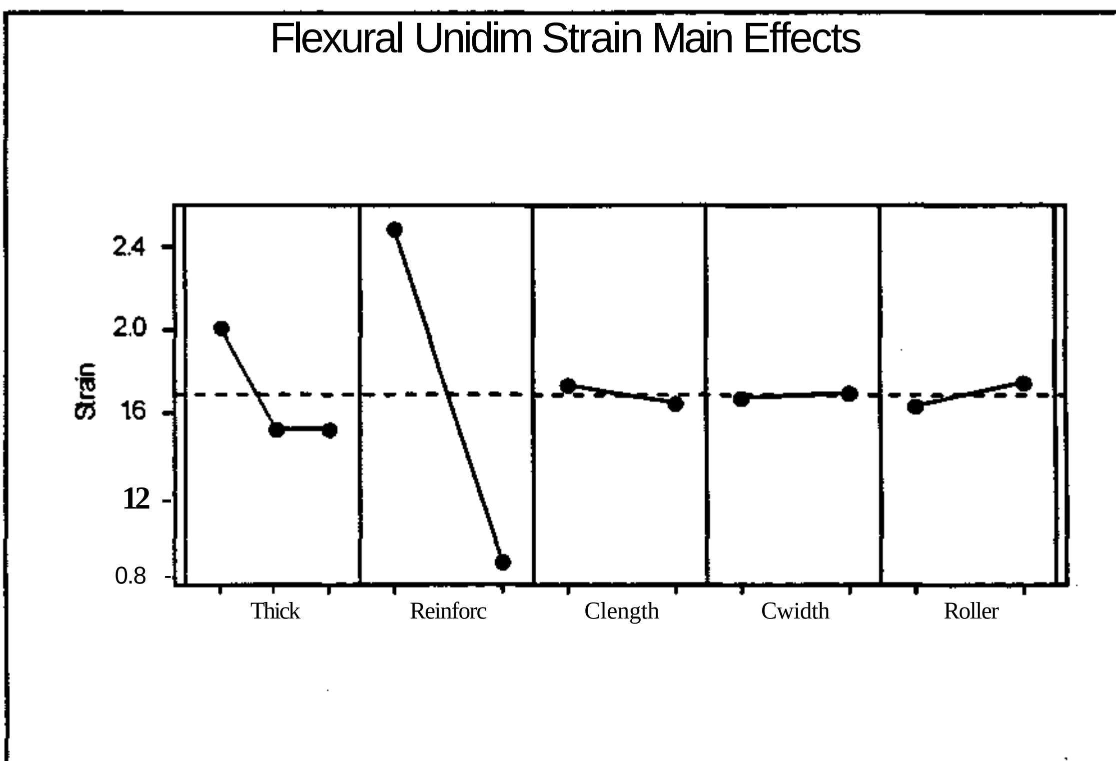


Figure 6.17: Flexural Strain Main Effects Plot

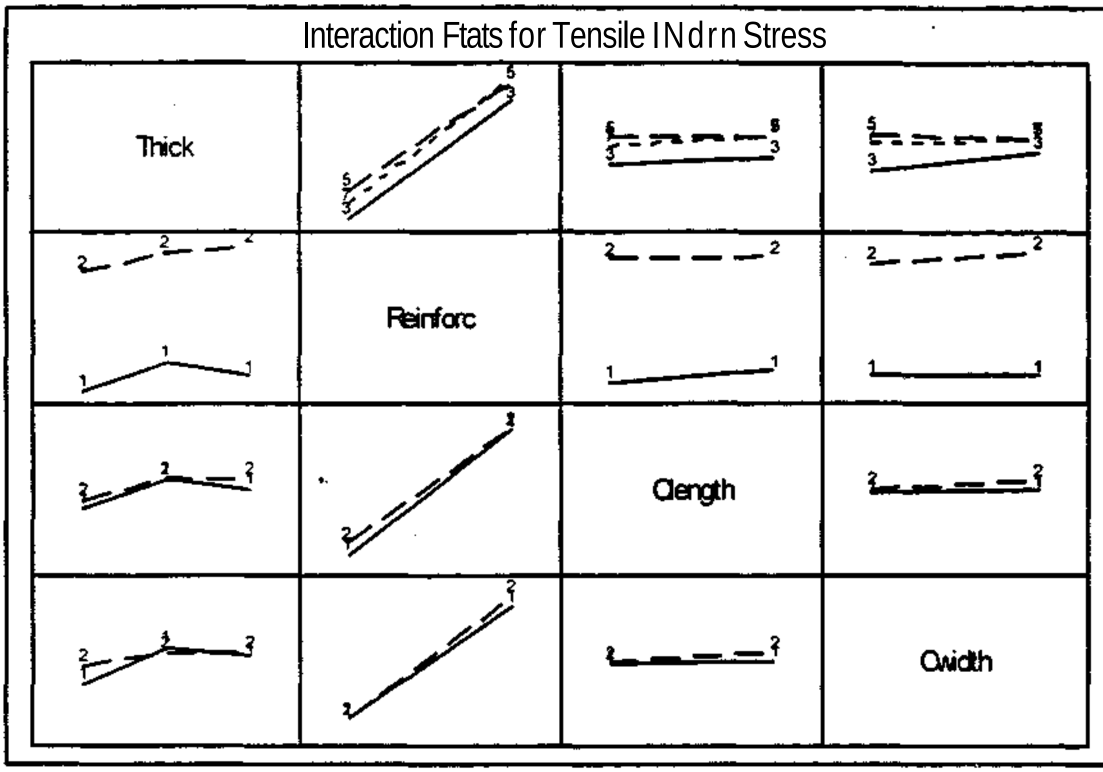


Figure 6.18: Tensile Stress Interactions Plot

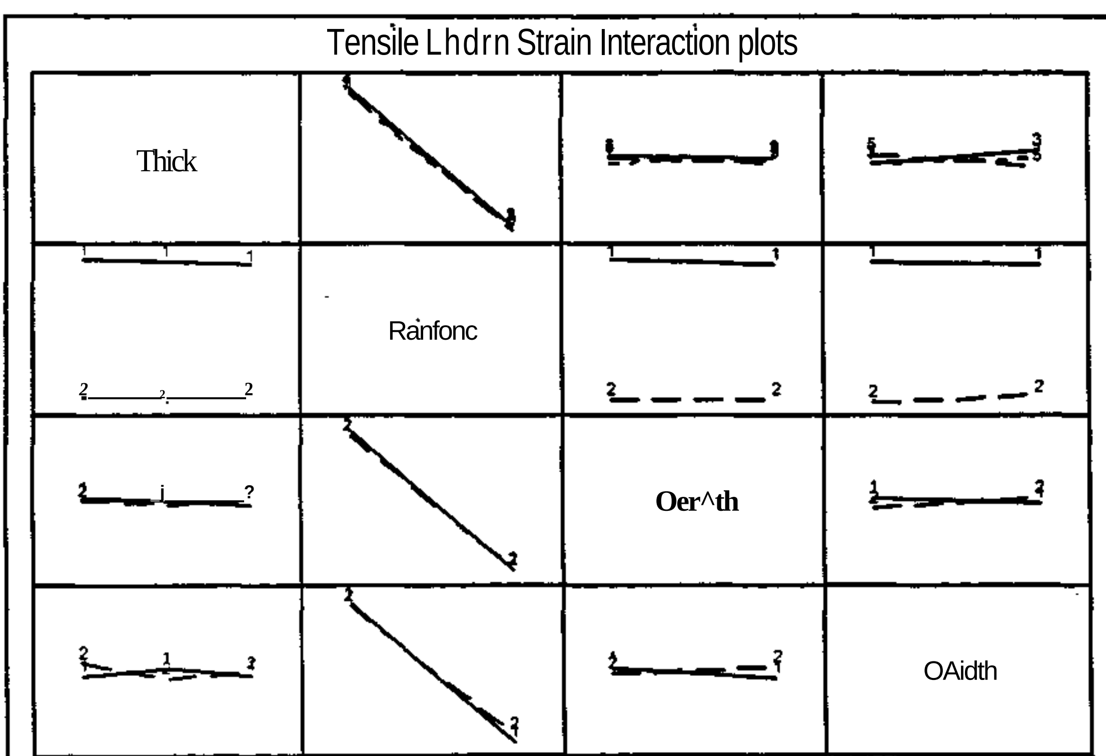


Figure 6.19: Tensile Strain Interactions Plot

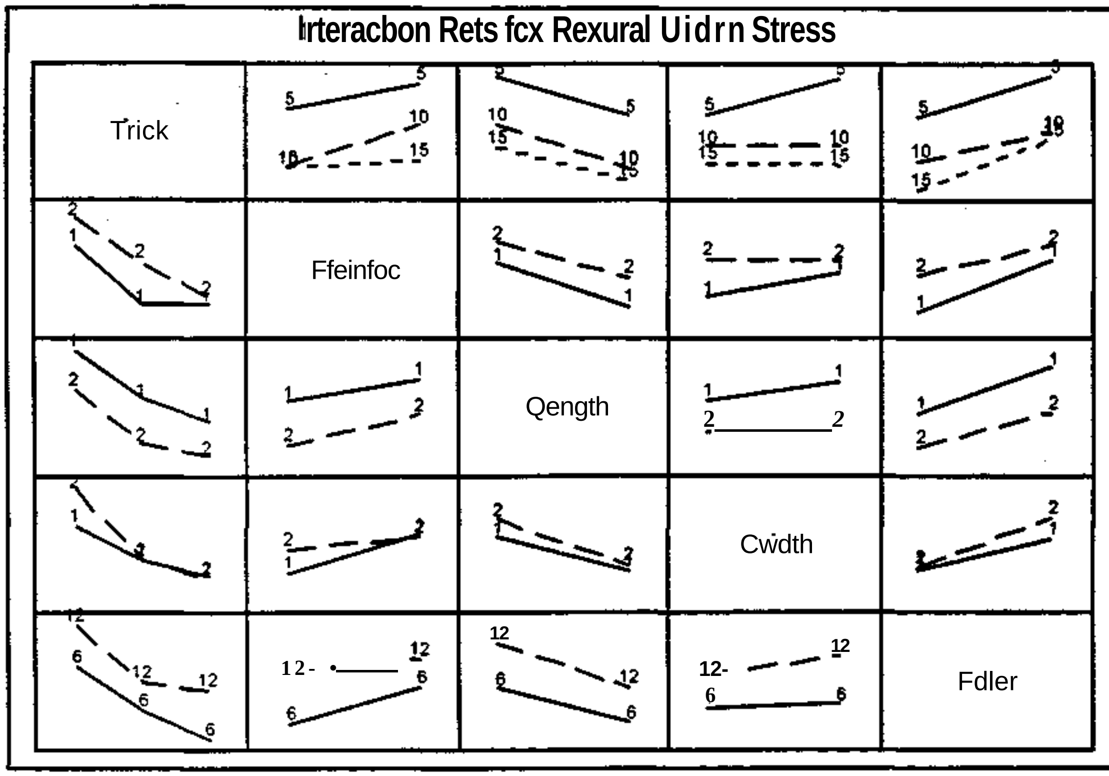


Figure 6.20: Flexural Stress Interactions Plots

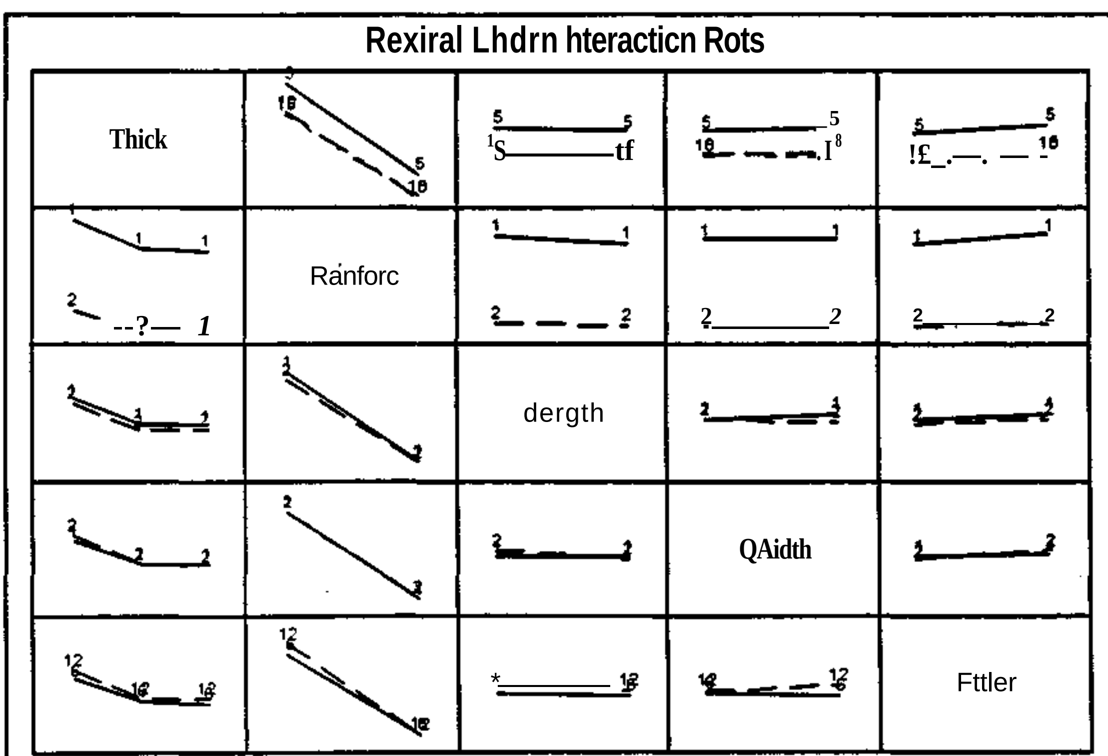


Figure 6.21: Flexural Strain Interactions Plot

The effect of thickness for stress measured from the tensile tests is relatively large, with a large quadratic component as seen in Figure 6.14. The corresponding effect, shown in Figure

6.15. for tensile strain is very small. A large decrease in flexural failure stress with thickness, mainly linear, is indicated in Figure 6.16. The effect of thickness on the flexural failure strain in Figure 6.17 is similar, but in this case, has a prominent quadratic component.

Table 6.11 shows that the reinforcement material (carbon or glass) affects the strength of the composite strongly for tensile stress and strain and also flexural strain. However, for the flexural tests using failure stress as the response, the reinforcement factor has a much weaker effect. Figure 6.15 and Figure 6.17 show that the tensile and flexural failure strains are higher for glass than for carbon. A higher tensile failure stress is seen for carbon than for glass in Figure 6.14. Although not a large effect, the same trend is seen for flexural failure stress in Figure 6.16.

The interaction plots are arranged so that the lines of each graph correspond to the different levels of the factor indicated on that row of plots. For example, in Figure 6.18, the three labelled lines in the top right plot correspond to the response averages at the three levels of the thickness factor, as indicated at the left of this row of plots. Interactions between reinforcement and thickness are not important for either strength response in each test method. That is. the reinforcement material used does not influence the way in which the laminate thickness affects the tensile or flexural failure stresses and strains, or vice versa.

The P-values in Table 6.12 are now used to identify important sub-plot factors. The important values in the table are underlined. The trends may be seen in the main effects and interaction plots already included in the text.

No effect of length on either tensile test response could be distinguished, as can be seen from the relatively high P-values in Table 6.12 and the horizontal lines in Figure 6.14 and in Figure 6.15. The effect of length is significant at the 1% level for both flexural stress and strain, the trends in both Figure 6.16 and in Figure 6.17 showing a decrease in strength with increasing length. Since length is nested within thickness, the estimates for this factor include any interaction with thickness. However, the ANOVA Tables in Appendix E and the parallel lines of the length and thickness interaction plots of Figure 6.20 and Figure 6.21 indicate that these interactions do not make a substantial contribution to these estimates.

Source	Tensile Stress	Tensile Strain	Flexural Stress	Flexural Strain
CLength	0.109	0.722	0.000	0.000
CWidth	0.007	0.048	0.023	0.021
Roller	~	~	0.000	0.000
CLength* CWidth	0.295	0.030	0.267	0.000
CLength*Roller	~	~	0.387	0.542
CWidth*Roller	~	~	0.175	0.000
Thick* Roller	~	~	0.360	0.014
Reinforc*Clength	0.155	0.600	0.825	0.119
Reinforc*C width	0.094	0.132	0.064	0.283
Reinforc* Roller	~	~	0.136	0.000
R-Squared	98.00%	98.10%	89.20%	99.70%
Sub-Plot C.V.	0.0398	0.0570	0.0607	0.0302

Table 6.12: Sub-Plot P-Values for Unidirectional Tests

A significant increase (at the 1% level) in tensile failure stress is seen with increasing width. This is also the case for tensile failure strain, but the effect is only just significant at the 5% level. For both responses this nested effect is seen (from the ANOVA's in Appendix E) to be due mainly to the interaction with thickness. More specifically, Figure 6.18 and Figure 6.19 show that the increase in response with increasing width at the three ply thickness is mainly responsible for this nested effect. The same trends are seen for flexural stress and strain, the significant (at the 2% level in both cases) effect is mainly due to an increase in response with increasing width at the 5 ply thickness level (see Figure 6.20 and Figure 6.21).

There is a significant interaction between length and width for the failure strain measurements from both tensile (at the 5% level) and flexural tests (at the 1% level). Figure 6.19 shows that, for the tensile tests, at the low level of length, an increase in width causes a fall in the failure strain, whereas at the high level of length, an increase in width causes a rise in the failure strain. The other plot of this interaction in this figure shows a different interpretation of this interaction; that, at the low level of width, an increase in length causes a fall in the failure strain, whereas at the high level of width, an increase in length causes a rise in the failure strain. One interpretation of the flexural failure strain interaction between length and width, shown in Figure 6.21, is that the response rises with increasing width for the low level of length, and falls with an increase in width for the high level of length. Alternatively, there is very little effect of length on failure strain for the low widths, and a drop in the response as length is raised for the high widths.

The diameter of the load and support rollers of the flexural experiments is seen to have a strong effect (at the 1 % level) for both failure measurements. An increase in diameter leads to a higher response value in both cases (see Figure 6.16 and Figure 6.17). The interactions of the roller diameter with thickness, reinforcement material and width, all using failure strain as the response, are significant (at the 5%, 1% and 1% levels respectively). The forms of these interactions are shown in Figure 6.21. None of the equivalent interactions are significant for failure stress. The trends in the roller diameter and reinforcement interaction for stress (see Figure 6.20). although not statistically significant (with a P-value of 0.136), do mirror those seen for the strain measurements.

The statistically significant trends indicated by the statistical analyses conducted have been detailed in this section. In Section 6.9 explanations and hypothesis as to the physical interpretation of these effects will be postulated.

6.9 Engineering Interpretation

The comparison of the whole-plot and sub-plot percentages of the total sums of squares in Table 6.8 shows that the whole-plot effects are more important than the sub-plot terms. These values should be used with caution, however, since the absence of any replication of laminates does not allow an examination of the whole-plot variation. The reinforcement sums of squares have been removed from these comparisons so as to try to compare between laminate and within laminate sources of variability. Hence the values reflect the thickness terms in the model. Any differences between the properties of laminates produced with different thicknesses could be directly due to the effect of thickness, or could be as a result of differences in the manufacture of the separate panels. It is not possible to distinguish between these two effects as the panels were not replicated (in experimental design terminology the two effects are said to be *confounded).* The exception to this trend is seen in the analysis of the tensile failure strain measurements. Here the whole-plot sums of squares are an order of magnitude less than the sub-plot value. It is thought that this results from the lack in precision in these comparisons as described above. This conforms with the philosophy that "Statistics is not a substitute for engineering knowledge." (Grove and Davis (1992)).

The coefficients of variation of around 5% in Table 6.9 are relatively low for marine composites, and reflect the comparatively controlled environment under which the specimens were fabricated. The coefficient for flexural stress is higher than that for flexural strain, and this suggests that the measurement of strain is more accurate than that of stress for these tests. Strain is measured directly by the strain gauges used, whereas stress is obtained indirectly through calculations involving other parameters. Additionally, the calculation of flexural stress involves an approximation of the effect of large central deflections of the specimen.

The models postulated fit the data well and this shows that the appropriate terms have been included. This is. in part, due to the relatively high quality, laboratory methods used to produce specimens. The variation of other parameters that could affect the strength of the coupons, such as void content and misalignment of the fibres, for example, have been well controlled within each laminated panel.

The lack of a definite trend in the effect of thickness on the tensile strengths suggests that these effects are mainly due to production variability. The flexural strengths do show a trend, that of a loss in strength with increasing thickness. This could be a direct result of the size of the specimen, i.e. a "size-effect' as described in Sections 3.1 and 3.2, and this is discussed further later in this section. The effect may also be due to the confounding between thickness and manufacturing effects, i.e. a lower quality specimen is obtained when a thicker laminate is produced. A plausible explanation for this would be that, when there are more wet, and hence unstable, plies, the fibres of the top lamina (where the failure of the flexural specimens occurred) are more difficult to keep straight as they are 'wet-out'. Since fibre waviness is far more critical for compressive failure, this would explain why a similar trend was not seen for the tensile tests.

The effect of the reinforcement used is as expected from the published strength data of unidirectional E-glass / epoxy and high strength carbon / epoxy (see Chapter 9). The trends in the experimental data correspond with those expected, the glass failing at a lower stress than carbon, but at a higher strain value, for both tensile and flexural tests. However, the average flexural failure stress of the carbon coupons is only marginally greater than that of the glass specimens. This is reflected in the results of the statistical analysis, which showed the effect of reinforcement to be less important for the flexural failure stress. This is because the flexural specimens failed in compression, since the compressive failure stress of carbon / epoxy (900-1050 MPa. from Hancox and Mayer (1993), and again using the rule of mixtures) is lower than the tensile value, and is close to the flexural strength of E-glass / epoxy laminates (see Chapter 9). This is substantiated by the observations of the failure mode made during the tests.

There is a decrease in both failure stress and strain with increasing length for the flexural tests. This could be due to a 'size effect' dependent on length such as that described in Sections 3.1 and 3.2, and may be related to a similar effect of thickness mentioned above. Both of these effects were very similar for both carbon and E-glass composites and so both materials are treated as one group. The mean values from the tables of means in Appendix E are given in Table 6.13 and Table 6.14. Logarithmic plots of these data are made in Figure 6.22 to Figure 6.25.

Thickness (Plies)	Failure Stress (MPa)	Failure Strain (%)
5	856	2.01
10	743	1.52
15	704	1.52

Table 6.13: Unidirectional Flexural Strengths w.r.t. Thickness

Thickness (Plies)	Length (mm)	Failure Stress (MPa)	Failure Strain (%)
5	54	900	2.05
	75	812	1.97
10	108	792	1.59
	150	695	1.46
15	162	741	1.55
	225	667	1.49

Table 6.14: Unidirectional Flexural Strengths w.r.t. Length

As described in Section 3.1, brittle material weakest link behaviour is indicated by linear relationships between the logarithms of strength and volume with a slope equal to minus the reciprocal of the Weibull modulus, m. If the modified theory is used, thickness or length may be substituted for volume to give the Weibull moduli ma and mi respectively. Most of the published data assumes the size effect to be due to a change in volume, and considers geometrically similar coupons where volume is proportional to length and thickness cubed. To allow comparisons to be made with such data the equivalent Weibull moduli based on volume are three times those based on thickness and length. Simple linear regression (Mendenhall and

Sincich (1995)) was used to estimate these Weibull moduli from the data in Table 6.13 and Table 6.14, and these are presented in Table 6.15. Since the effects of length at all three thickness were very similar the logarithmic slope could be determined across the whole range of lengths.

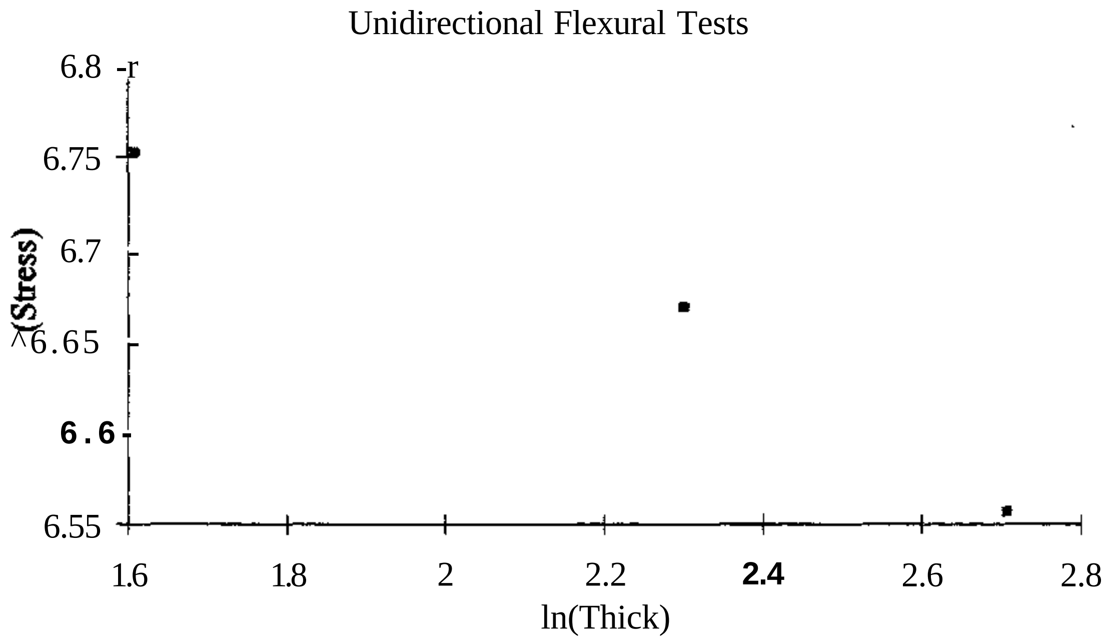


Figure 6.22: Unidirectional Flexural Failure Stress

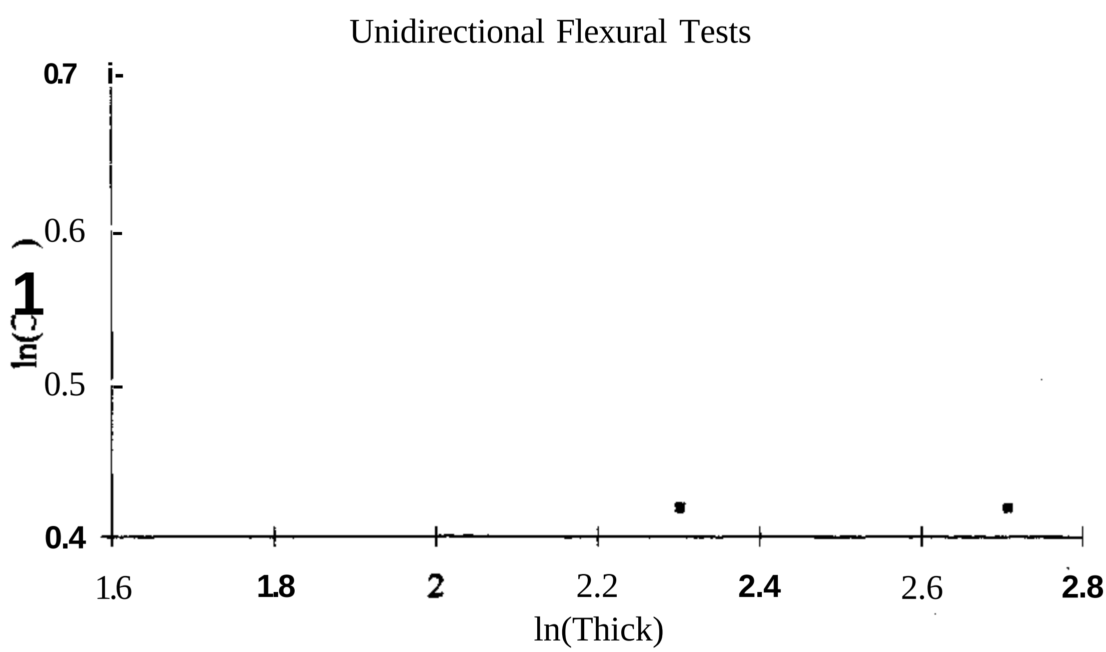


Figure 6.23: Unidirectional Flexural Failure Strain

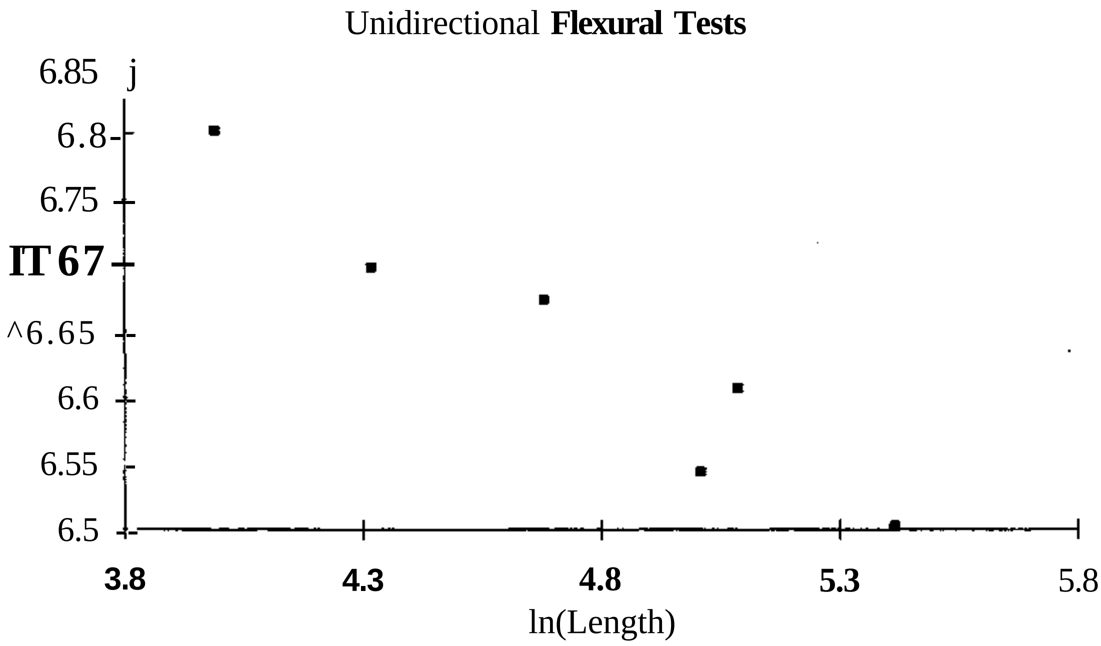


Figure 6.24: Unidirectional Flexural Failure Stress

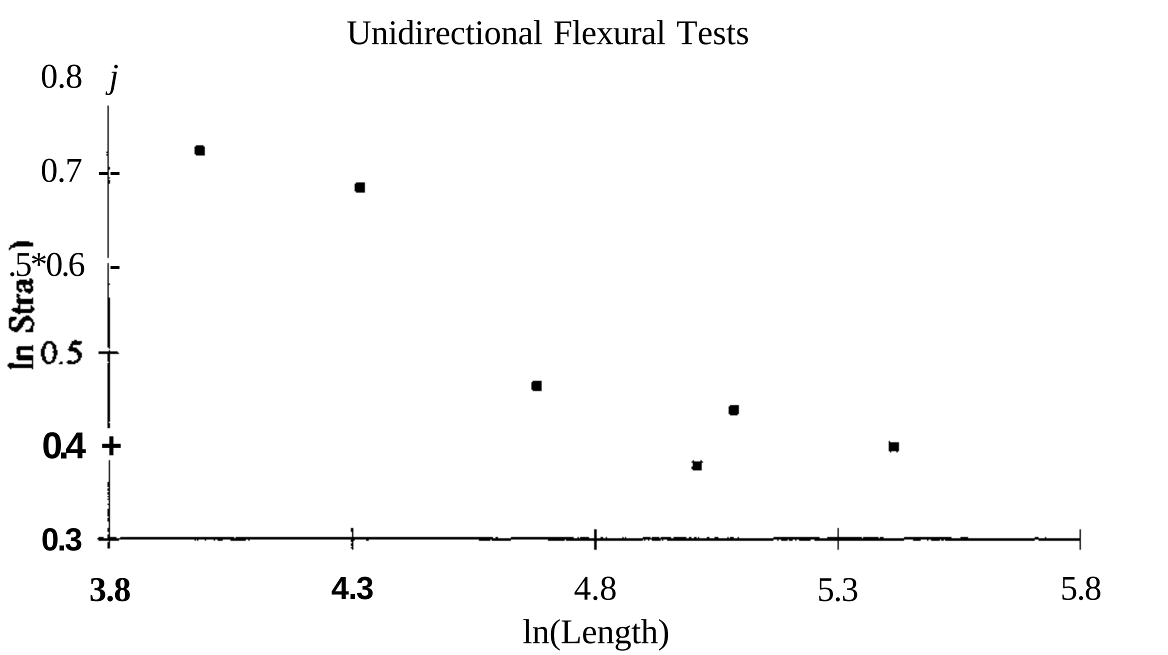


Figure 6.25: Unidirectional Flexural Failure Strain

	Thickness		Length	
	Did	IDv	mi	D9v
Failure Stress	6	**18**	5	**15**
Failure Strain	4	**12**	**4**	**12**

Table 6.15: Unidirectional Flexural Weibull Moduli Estimates

Here it must be noted that the plots are only approximately linear (especially that of strain against thickness) and that the points used to plot the four graphs are themselves subject to variation. Hence the confidence with which these estimates of Weibull modulus may be quoted is limited.

For both test methods, the effects of width seen for failure stress and strain used are restricted to the smallest specimens, where an increase in width gives a higher failure strength. For the tensile tests the lower width (3.75mm) is approaching the fibre bundle width of the unidirectional tapes used (2-3mm and 4-5mm for the carbon and glass reinforcements respectively). Hence a significant proportion of the bundles may be cut or damaged at the edges of the narrowest coupons, weakening them. This is borne out by the fact that the three ply specimens are weaker than the five or seven ply coupons. However, the smallest flexural coupons were 12.5mm wide, so it would imply that flexural testing is far more sensitive to fibre bundles edge damage, if the same explanation applies to these tests.

The interactions between the effects of length and width on the failure strains are not easily explained. Opposite trends are seen for the tensile and flexural tests, and the equivalent effects for both failure stresses are not significant. This is further discussed in Chapter 9.

The fact that a larger support roller diameter is seen to delay the failure of the flexural specimens correlates with the fact that the observed compressive failure of the specimens initiated at the loading rollers. The statistical analysis of the data shows that this is a very strong effect for both failure stress and strain. Initial separate statistical analyses of the E-glass and carbon data failed to recognise this effect. Similarly, an earlier study using 24 specimens of the same E-glass / epoxy material system (Shenoi et al. (1994)) concluded that this factor did not affect the strength, although this deduction was not statistically based. This highlights the danger of using small numbers of coupons to investigate the behaviour of composite materials without due considerations of the statistical implications.

The interaction of the effect of roller diameter with width on the flexural failure strain is statistically very significant. The effect on the failure strain of an increase in roller diameter is greater for a wider specimen. This is further discussed in Chapter 9.

The effect of roller diameter on the flexural fâilure strain is observed to be greatest for the five ply specimens. It would be intuitive to assume that, as the thickness and hence loads increased, the stress concentrations at the rollers would become more important. This could be the case, but at the higher loads the 12mm diameter rollers may not be large enough to reduce these concentrations as effectively as they do for the five ply coupons. Alternatively, the damage to the top lamina may be critical in a five ply specimen, but not so for a ten or fifteen ply coupon, because of the differences in the percentage of plies damaged.

The failure strain of the E-glass / epoxy specimens is more strongly affected by the roller diameter than that of the carbon / epoxy specimens. The failure strain and impact resistance of carbon are much lower than those for E-glass (Hancox and Mayer (1993)). Hence, it is plausible that this effect occurs because the 12mm rollers do not reduce the stress concentrations in the carbon coupons sufficiently to affect the failure strain. In this case this trend is also seen in the failure stress, although it is not statistically highly significant.

In this section explanations for the behaviour of the unidirectional specimens have been forwarded. In Chapter 9 these explanations are correlated with those concerning the Woven Roving Manufacturer specimens (Section 7.9) and those for the Final Woven Roving Tests (Section 8.9).

7. Woven Roving Manufacturer Tests

7.1 Test Programme Development

The experience gained in the application of experimental design techniques as applied to the strength testing of composite materials in the unidirectional test programme was both employed, and built upon, in this series of tests. The laminates supplied by Vosper Thornycroft originated from an in-house test programme with the aim of evaluating the relative suitability of three different suppliers of the E-glass woven roving for the possible construction of mine-hunters for the Australian Navy. Panels were selected from these laminates so that three different woven roving manufacturers were represented. For the same reasons as outlined in section 6.1, and in order to give continuity in the test programme, four-point flexural and tensile tests were considered. Again, specimen length, width and thickness were varied independently. A nested structure of width and length within thickness was adopted for the flexural tests. However, as detailed in section 7.2, the length and width level values for both thicknesses of tensile specimens were identical. The list of factors considered for both test methods is thus;

(i) Thickness.
(ii) Woven Roving Manufacturer (Reinforcement),
(iii) Length,
(iv) Width.

Laminate panels were supplied in two thicknesses, four and eight plies, and, as mentioned above, three manufacturers of woven roving were considered. To further explore the use of experimental design methods in this context, and to give more detailed information of any possible size related factor effects, the length and width factors were assigned three levels.

Three factors at three levels gives a full factorial experiment of 27 runs (i.e. specimens) at each thickness, a total of 108 specimens, for this stage of the test programme. This was thought to be excessive and, together with a desire to gain more experience of the use of experimental design methods, this led to the selection of a fractional factorial design. For the flexural tests a 15 run central composite design as described by Grove and Davis (1992) was selected for each thickness. This design is shown schematically in Figure 7.1, where each filled circle represents a test coupon in this case. Since the three woven rovings used were nominally identical it was expected that interactions with the reinforcement factors would be small. Hence it was thought that the design would be sufficient to give the required factor effects, and adequate estimation of the experimental variation, without the need for replication of the specimens.

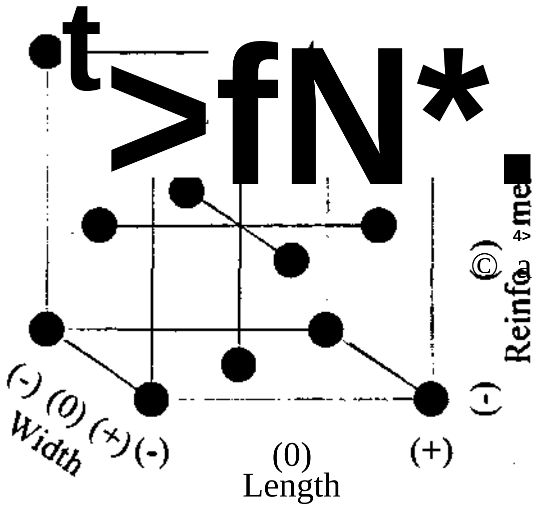


Figure 7.1: 15-run, Central Composite Design

Since the tensile test length and width factors were assigned the same level values at both thicknesses, a design without the nesting of length and width under thickness was considered for these tests. The method of 'column collapsing' (Grove and Davis (1992)) was used to give a 27 run design. This used as its starting point a four factor, three level full factorial, and then one factor was 'collapsed' into a two level factor. The three-level factors were reinforcement, length and width, and the factor at two levels was thickness. However, this design introduces a high degree of non-orthogonality, both complicating the statistical analysis of the data and the interpretation of this analysis, in return for a very small reduction in the number of runs (from 30 to 27). To avoid this complication, and so as to give homogeneity to the test programme, a 15 run central composite design was selected for each of the two thicknesses for the tensile tests.

A schematic of the structure of the woven roving manufacturer test series is shown in Figure 7.2 and the coded design of the 15-run central composite plan is given in Table 7.1.

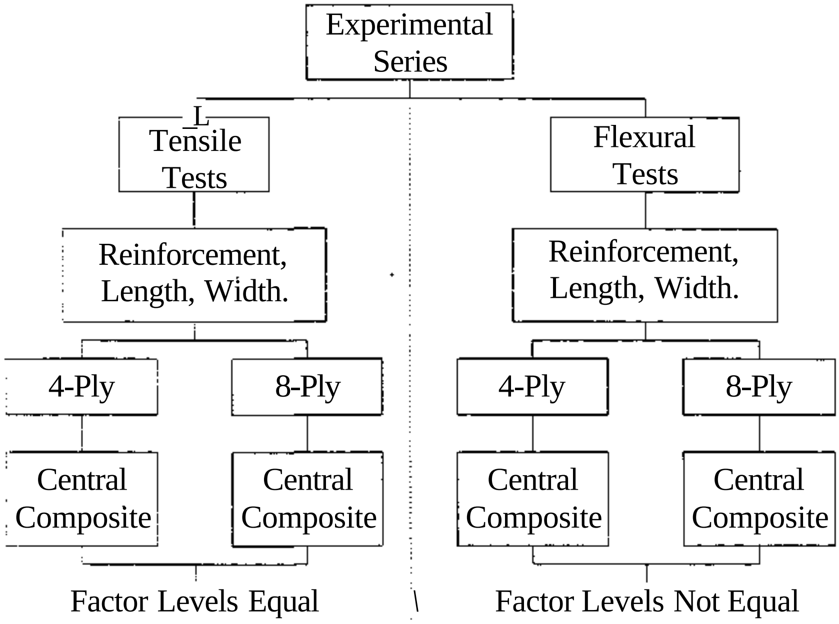


Figure 7.2: W.R. Manufacturer Experimental Series

Specimen	Reinforcement	Length	Width
1	Low	Low	Low
2	High	Low	Low
3	Low	High	Low
4	High	High	Low
5	Low	Low	High
6	High	Low	High
7	Low	High	High
8	High	High	High
9	Medium	Medium	Medium
10	Low	Medium	Medium
11	High	Medium	Medium
12	Medium	Low	Medium
13	Medium	High	Medium
14	Medium	Medium	Low
15	Medium	Medium	High

Table 7.1: 15-Run, Central Composite Design

7.2 Specimen Sizing

As for the unidirectional tests, the ASTM standards for flexural and tensile testing of fibre reinforced plastics (ASTM (1986), ASTM (1989)) were used here as guidelines. The levels of the reinforcement factor consisted of three different suppliers of nominally identical E-glass woven roving of weight 780 gm^{-2}. The three levels are coded 'AUS 8', 'AUS 11' and 'AUS

12' after the original Vosper Thornycroft notation. Panels fabricated using each of these reinforcements were available at thicknesses of four and eight plies.

The parameter limiting the size of the flexural specimens was the length to depth ratio. It is desirable to obtain as large a range between the length levels, but this range is limited by possible shear effects at low lengths and excessive deflections at high lengths. The ASTM standard gives a minimum value of approximately 14 for the length to depth ratio for glass reinforced plastics, and this value was used to set the low length levels at each thickness. Due to problems previously encountered whilst using these standards to estimate the maximum permissible length to depth ratio (see section 6.1), a pilot study using four ply coupons was undertaken to experimentally determine this maximum value. Four specimens with length to depth ratios of 30. 35. 40 and 45 were tested. No large deflection problems were encountered for any of the specimens. Hence a length to depth ratio of 45 was used to give the high length levels at each thickness. However, the pilot test specimens were cut out using a jig-saw and were tested on a rudimentary test rig (see section 7.4), and so duplicates of the long length flexural specimens were produced and tested first as an extra precaution. These specimens produced excessive deflections and hence the high length factor level values were decreased accordingly. The high width levels were set to give a reasonably high aspect ratio at the corresponding low length levels (i.e. to avoid approaching square plan shape specimens). The low width level for the four ply specimens was limited by the roving width and the 8 ply level was scaled up from this. Medium factor levels were set so as to bisect the range between high and low levels. 12 mm load and support rollers were used throughout. The factor levels for the flexural tests are given in Table 7.2.

Thickness	Depth (mm)	Level	Length (mm)	Width (mm)
4-Ply	3	Low	50	10
		Medium	75	20
		High	100	30
8-Ply	6	Low	100	20
		Medium	150	40
		High	200	60

Table 7.2: Flexural Test Geometric Factor Levels

The tensile tests were limited by the maximum load attainable by the test rig of 10 tonnes. This was used to give a maximum cross sectional area based on Vosper Thornycroft strength data for W.R. / Polyester laminates, and hence the high width level for the (strongest) 8 ply specimens. The low width level was set so as to give as large a range of widths as possible. If these width levels were to be scaled down to give the corresponding 4 ply values then this would have led to one of two cases; either the low 4 ply width level would be less than the roving width, or the range of widths considered would have to be reduced significantly. Hence, as mentioned in the previous section, the same width values for both thicknesses were used. The length values were selected for the 4 ply specimens considering the practical difficulties involved in the cutting out of very long, narrow coupons. For this reason the same values were taken for the eight ply specimens. Again, medium factor levels were set so as to bisect the range between high and low levels. The factor levels for the tensile tests are given in Table 7.3. The total coupon lengths were larger than those quoted below, including an additional grip length of 40 mm at each end

Thickness	Depth (mm)	**Level**	**Length**	**Width**
4-Ply	3	Low	50	10
8-Ply	6	**Medium**	100	17.5
		High	150	25

Table 7.3: Tensile Test Geometric Factor Levels

7.3 Specimen Manufacture

The panels were all laminated horizontally by hand under factory floor conditions at Vosper Thornycroffs Woolston shipyard. The resin system used was DSM Synolite 73-2785, a medium reactivity isopthalic polyester resin described by the manufacturers as "suitable for the construction of large marine structures". 0.75% NL 49 P accelerator and 1.5% Butanox M50 catalyst were used to cure the resin. A nominal fibre weight fraction of 50% was stipulated, along with instructions to the laminator to ensure that the laminate was wetted out thoroughly, but did not have excess resin on the surface. The four ply laminates were fabricated to be one cloth width square and then four panels were cut from this laminate. The eight pi)' panels were fabricated to be two cloth widths square, requiring a system of butting together two cloth widths, and then eight panels were cut from this laminate. The laminates were post-cured for 24 to 72 hours at a temperature not exceeding 25°C, followed by 16 hours at 40°C ± 2°C. Details of the lay up procedure and resin system used are included, together with details of the three woven rovings used, in Appendix K.

The specimens were then cut from the panels using a diamond edged saw. The moulding surface produced a smooth and flat surface on one side of the panels but the coarseness of the weave meant that the other surface was very uneven. Hence the strain gauges were attached to the moulded surface. The same method as described in section 6.3 were used to adhere the gauges to the coupons.

7.4 Experimental Details

The experimental set up of this series of flexural tests was identical to that described in section 6.4. The specimens were again all tested with the strain gauge on the tension side, i.e. with the moulded face in tension. For the pilot study to find a suitable length to depth ratio, however, the Instron machine was not used and the specimens were not strain gauged. Here load was applied in increments by adding weights to a bar suspended from the loading block. The central deflection was measured using a dial type extensometer, and readings noted immediately after each weight had been added. The tensile test set up was identical to that set out in section 6.4.

The experimental procedure and the measurements taken for both types of tests were also as described in section 6.4. The cross head speeds (under deflection control) used in this case are shown in Table 7.4.

Test Method	**Thickness** (Plies)	**Cross-Head Speed** (mm $\mathbf{min^{-1}}$)
Flcxural	**4**	**5**
	8	**10**
Tensile	**4**	**5**
	8	**5**

Table 7.4: Cross-Head Speeds

The resin "burn-off method described in ASTM standard D2584 (ASTM (1985a)) was used to obtain fibre volume fractions for each specimen. On completion of the tests, small samples were cut from an undamaged section of each coupon and the volume fractions of fibre and matrix determined. Each specimen was weighed before the matrix was burnt off in an oven for 3 hours at 625°C. After cooling, the fibres were then weighed, enabling calculation of the weight fractions. The volume fractions were then calculated using the known resin and glass densities.

7.5 Data

The results obtained from the woven roving manufacturer test programme can be found in Appendix G, and typical failure modes in Appendix L. The specimen codes start with the reinforcement woven roving used in the laminate. These are as used by Vosper Thornycroft and all start with the letters 'AUS' indicating the connection with the Australian Navy. This is followed by a number indicating the specific reinforcement;

8 : Colan W.R. / Chinese Roving,

11 : Asahi Fiber Glass Co.,

12 : Key Trading Corp..

The specifications of these woven rovings are given in Appendix K. A letter then follows indicating the test method, 'F' for Flexural or 'T' for tensile. The length and width levels are then indicated by a letter followed by a number respectively. For length low is 'A', medium is 'B' and high is 'C\ and for width low is T, medium is '2' and high is '3'. After a V the number of plies, 4 or 8, is indicated. As an example "AUS11FB3/8 is a flexural specimen of medium length and high width containing eight plies of Asahi woven roving.

7.6 Method ofAnalysis of Tensile Data

The analysis described in this section used both failure stress and failure strain as the response. A summary of the results of this analysis is included in Appendix H. During the description of the analysis, an explanation of the underlying statistical assumptions and how these may be checked is given. These general points apply to the later analyses in the chapter.

As for the unidirectional tests, this series of tests has a split-plot structure as described in Section 6.6. In this case, thickness has only two levels and hence no quadratic effects (also as described in Section 6.6) may be estimated. However, the reinforcement manufacturer, length and width factors are each assigned three levels, and hence quadratic effects may be estimated for these factors and their interactions. The factors and interactions considered in the statistical model proposed, and their subdivision into whole- and sub-plot terms for the W.R. manufacturer tests are given in Table 7.5. As for the unidirectional tests, three-way and higher interactions were assumed to be negligible. Again h should be stressed that this does not mean that the contrasts corresponding to these interactions have been ignored, they have been used to give a more accurate estimation of the experimental variation.

If the linear and quadratic manufacturer interactions with thickness were assumed to be negligible then their estimates would be solely due to the whole-plot variation. In this case they could have been used as an estimate of the whole-plot error. However, there was little confidence in the assumption that these two-way interactions were negligible, and hence no estimate of the whole-plot error was made. As for the unidirectional tests (see Section 6.6), simple qualitative comparisons were made between the whole-plot terms and the sub-plot terms using the whole- and sub-plot sums of squares expressed as a percentage of the total sums of squares. To identify the important whole-plot terms, their respective sums of squares were 'normalised' with respect to the sub-plot error estimate. Again, it must be emphasised that these ratios may be used in a purely comparative sense only, and that no statistical inferences may be made from them.

Whole-Plot	Sub-Plot-
Linear Manufacturer	Linear Length
Quadratic Manufacturer	Quadratic Length
Thickness	Linear Width
Linear Manufacturer x Thickness	Quadratic Width
Quadratic Manufacturer x Thickness	Linear Manufacturer x Linear Length
	Linear Manufacturer x Linear Width
	Quadratic Manufacturer x Linear Length
	Quadratic Manufacturer x Linear Width
	Linear Length x Linear Width
	Thickness x Linear Length
	Thickness x Linear Width

Table 7.5: Classification of Factors and Interactions

For these tensile tests the length and width factors were not nested under thickness; all factors had identical level settings at each of the other factor levels. The software package 'MINITAB" gave the correlation matrix for the factors and responses in order to give a 'feel' for the data. MINITAB did not enable the quadratic parameter estimates to be made with ease and hence the "SAS" software was used to perform the analyses, using the general linear model, or "GLM". procedure (Cary (1989)). For this experimental design, the software was not able to analyse the data using an analysis of variance method, and the GLM procedure was used to perform a regression analysis. This required the input of coded quadratic as well as linear factor levels. For thickness 4 plies was coded as -1 and 8 plies as +1. The thickness term is referred to as "Thick' in the analysis. The three-level factor linear levels were coded as -1, 0 and +1 for low, medium and high settings, respectively. The coded linear reinforcement manufacturer, length and width terms are referred to as 'Man', 'CLength' and 'CWidth', respectively. In order to give the correct comparisons, the three-level factor quadratic levels were coded as 1,-2, 1 for low, medium and high settings, respectively. The coded quadratic reinforcement manufacturer, length and width terms are referred to as 'QMan'. 'QLength" and "QWidtiY, respectively.

A feature of the 15 run central composite design is that it is not fully orthogonal with respect to the second order model (Grove and Davis (1992)). This means that the estimates of the quadratic terms are not fully independent of each other. In practical terms the result of this is that the estimates of the quadratic terms will vary according to which order they are fitted in the statistical model. Hence, in order to allow meaningful comparisons to be made, the 'type III' sums of squares are used to give the P-values for each term. Each of these is calculated after all the other terms have already been fitted to the model. This may be interpreted as how much extra each term adds to the fit of the model to the data, over and above the other terms in the model.

The SAS output produced estimates of the parameters in the model together with their type III sums of squares and associated p-values. These have been used to give the ANOVA tables in Appendix H. The standard errors of the parameter estimates were also calculated. As described in Section 6.6, the standard errors of the parameters may be used to give simple estimates of the confidence with which theses parameters may be quoted. Tables of the parameter estimates and their standard errors are also included in Appendix H. The fitted values and residuals from the SAS analysis output were used with the MINITAB software to give the residual plots found in Section 7.8.2. MINITAB was also used to give the main and interaction plots included in Section 7.8.3. Explanations of residual, main and interaction plots and their use may be found in Section 6.6. As indicated earlier in this section, the lack of an estimate of the whole-plot variance means that the appropriate F-Ratios and P-values for the whole-plot terms cannot be calculated, and hence do not appear in Appendix H.

It was thought that possible indications of the quality of the specimens could be given by the initial Young's modulus and the fibre volume fraction. Hence, these measurements were included in the model as possible covariates. Both covariates were found to be just statistically significant at the 5% level for the tensile stress measurements, and hence were kept in the model. However, these variables were not found to be important for the tensile strain, flexural stress or flexural strain analyses, and hence were not included in these models.

It should be stated here that, initially, one of the tensile specimens failed at an extremely low stress and strain. This was noted at the time, but there was no obvious explanation for this behaviour. The analysis above was completed before a replacement specimen was fabricated and tested. The strength of the replicate specimen was comparable with that of the rest of the data set, and hence this strength replaced the earlier, much lower, value. When the statistical analysis was repeated with the new data set, although drastically different results were not obtained, there were some differences in the relative importance of some of the factors. This shows that, although more economical, fractional factorial designs with relatively few experimental runs may be sensitive to small numbers of erroneous results.

7.7 *Method of Analysis of Flexural Data*

The same statistical analysis as described in Section 7.6 was carried out for the flexural tests of the series, with one additional consideration. In this case the length and width factors are nested under the thickness factor. This structure and how it is accommodated in the analysis are described in Section 6.6. The nested factor estimates of length and width were calculated as the sum of the main effect and the interaction with thickness.

The inclusion of initial Young's modulus and fibre volume fraction as covariates was not found to be statistically significant for failure stress or failure strain, and hence these terms were not included in either flexural model. Summaries of the statistical analyses of failure stress and failure strain for the flexural tests are also included in Appendix H.

7.8 Findings of the Data Analyses

In this section the results of the statistical analyses detailed in Sections 7.6 and 7.7 are reviewed, and important factors and trends are highlighted.

7.8.1 Model Fitting

As a preliminary and simple examination of possible linear relationships between the response variables, treatment factors and possible covariates, the correlation matrices given in Appendix H were considered. It should be noted here that the estimates of the correlation between two variables do not take into account the variations in the other variables, and hence should only be used to get an initial 'feel' for the data.

The tensile tests correlation matrix, given in Table H.I shows fibre volume fraction to be fairly well correlated with failure stress, with a correlation coefficient of 0.743. Modulus is weakly correlated with fibre volume fraction (0.550) and failure stress (0.577). The matrix for the flexural tests shows a correlation coefficient of -0.678 between strain and thickness. A smaller coefficient of -0.475 is seen between stress and thickness. Modulus is weakly associated with fibre volume fraction (correlation coefficient 0.465).

The statistical model postulated for the failure strain of the tensile tests and the failure stress and strain of the flexural tests is given by;

$$
\begin{aligned}
Strength = {} & a_0 + a_j M, + a_2 M_q + a\hat{} T + a_4 M,xT + a_5 M_q xT + \mathrm{E} \qquad (7.1) \\
& + a_{(,} A4_l xCL_l + a\text{-}jM\hat{} xCL, + a_{\%} CL_t + a_9 CL_{t/} \\
& + a_{u,} M,xCW_l + a_u M_q xCW, + a_n CW, + a_n CW_q \\
& + a_u ('L/xCH'] + a_{l5} TxCL, + cc\hat{} TxCW, + e
\end{aligned}
$$

where the ot\s are the parameters corresponding to the effects, M is manufacturer, T is thickness. E is the whole-plot error, CL is coded length, CW is coded width, e is the sub-plot error, the subscripts / and *q* refer to linear and quadratic effects respectively.

The three levels of the woven roving manufacturer factor, M, were arbitrarily assigned the coded values 1, 2 and 3 for the Colan W.R. / Chinese Roving (AUS 8), the Asahi Fiber Glass Co. (AUS 11) and the Key Trading Corp. (AUS 12) reinforcements, respectively.

The model for the tensile failure stress differed slightly in that the covariates initial modulus and fibre volume fraction were also included;

$$Strength = a_0 + a_tM, + a_2M_q + a_3T + a_4M,xT + a_5M_qxT + E \qquad (7.2)$$
$$+a_{<i}M,xCL_l + a\text{-},M_qxCL, + a_{\&}CL, + a_9CL_{(l}$$
$$+a_{l0}M,xCW, + a_uM_{tl}xCW, + a_nCW, + a_uCW_q$$
$$+a_uCL_lxCW_j + a_{l5}TxCL, + a\hat{}TxCW, + a_nMod + a_{ls}V_f + s$$

where Mod is the initial modulus and V_f is the fibre volume fraction.

Table 7.6 shows the relative importance of the whole-plot and sub-plot terms using the respective sums of squares expressed as a percentage of the total sums of squares.

	Whole-Plot SS **(%)**	**Sub-Plot SS** (%)
Tensile Stress	57.7	22.8
Tensile Stress with mod. & V_f	22.5	40.2
Tensile Strain	24.6	20.8
Flexural Stress	55.7	19.1
Flexural Strain	52.6	20.2

Table 7.6: Whole-Plot and Sub-Plot Sums of Squares Comparisons

The whole-plot values are seen to be larger than the sub-plot values for all failure measurements, except for that of tensile stress (where the covariates initial modulus and fibre volume fraction are included).

The sub-plot coefficients of variation are given in Table 7.7.

	Tensile Stress	**Tensile** Stress (Mod. & V_f)	**Tensile Strain**	**Flexural Stress**	**Flexural Strain**
Sub-Plot C.V.	0.0685	0.0523	0.0930	0.1330	0.1504

Table 7.7: Sub-Plot Coefficients of Variation

The tensile stress coefficient is lower than that for tensile strain, and is reduced further by the inclusion of the covariates initial modulus and fibre volume fraction. Both of the flexural coefficients are higher than those for the tensile failure measurements, with the strain value again higher than that of stress.

7.8.2 Checking the Adequacy of the Model

The R-Square values given in Table 7.8 show that the model postulated for the tensile failure stress measurements fits the data well, and that the model is improved further through the inclusion of the two covariates. However, the fit of the tensile failure strain model is poor, with an R squared value less than 50%. Such a poor fit means that the results of the statistical analysis are unreliable and hence further deductions are not made from this analysis of the tensile failure strain response. Both models of the failure measurements for the flexural tests show reasonable fits to the experimental data with R-squared values of around 75%.

	Tensile Stress	**Tensile Stress** (Mod. **& V_f**)	**Tensile Strain**	**Flexural Stress**	**Flexural Strain**
R-Square	82.4.%	91.3%	47.5%	77.3%	72.7%

Table 7.8: R-Square Values

The use of residual plots for a residual analysis has been explained in Section 6.6. The plots of residuals versus observation number in Figure 7.3, Figure 7.4 and Figure 7.5 show that there are no excessively large residuals. There are also no strong trends in the plots of residuals against fitted values, and no excessive skew is noted in the residual histograms. The normal plot for tensile stress, shown in Figure 7.3, is reasonably linear, although there are discernible sections to the line. Both normal plots for flexural stress and strain, in Figure 7.4 and Figure 7.5. are again reasonably linear with discernible sections to the line.

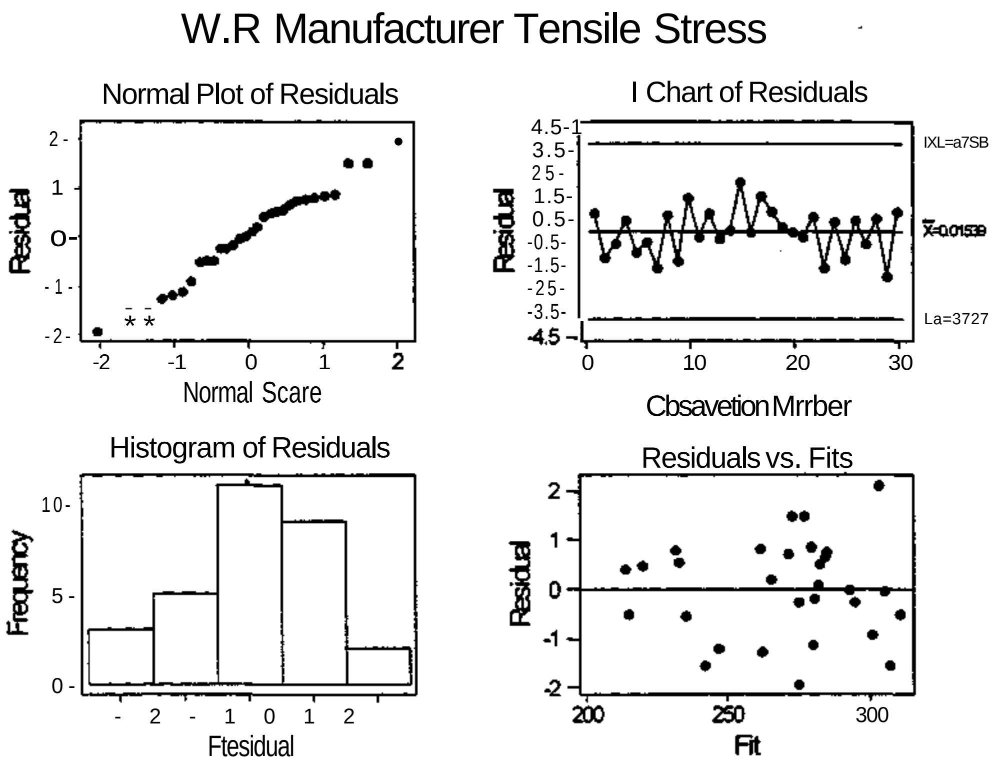


Figure 7.3: Tensile Stress Residual Plots

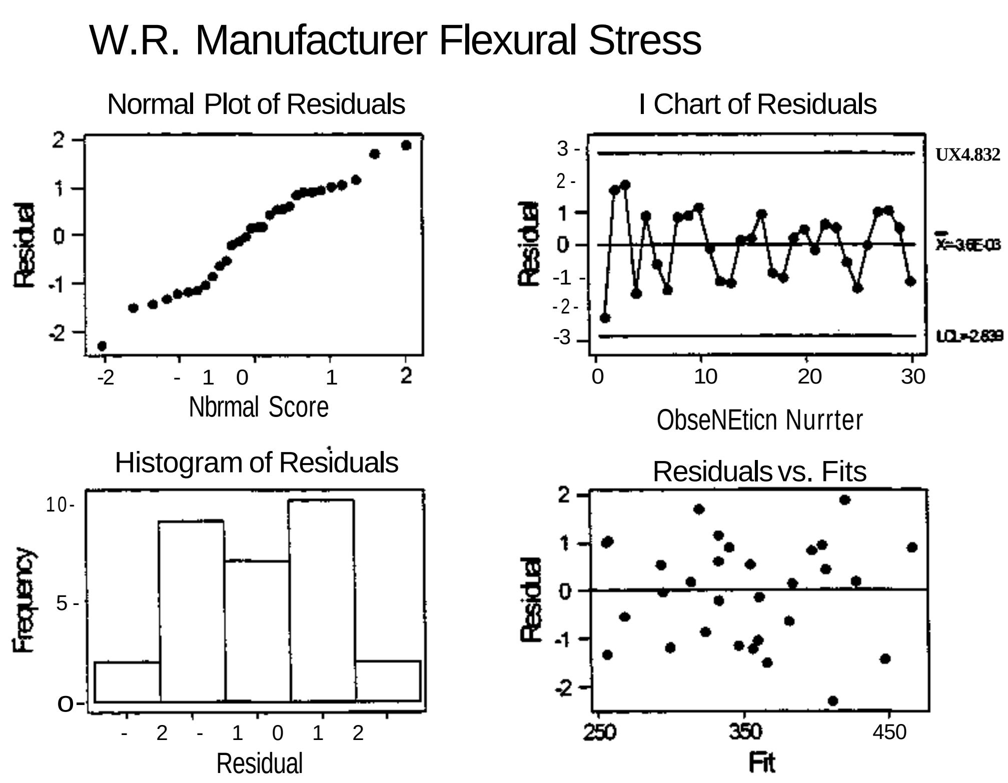


Figure 7.4: Flexural Stress Residual Plots

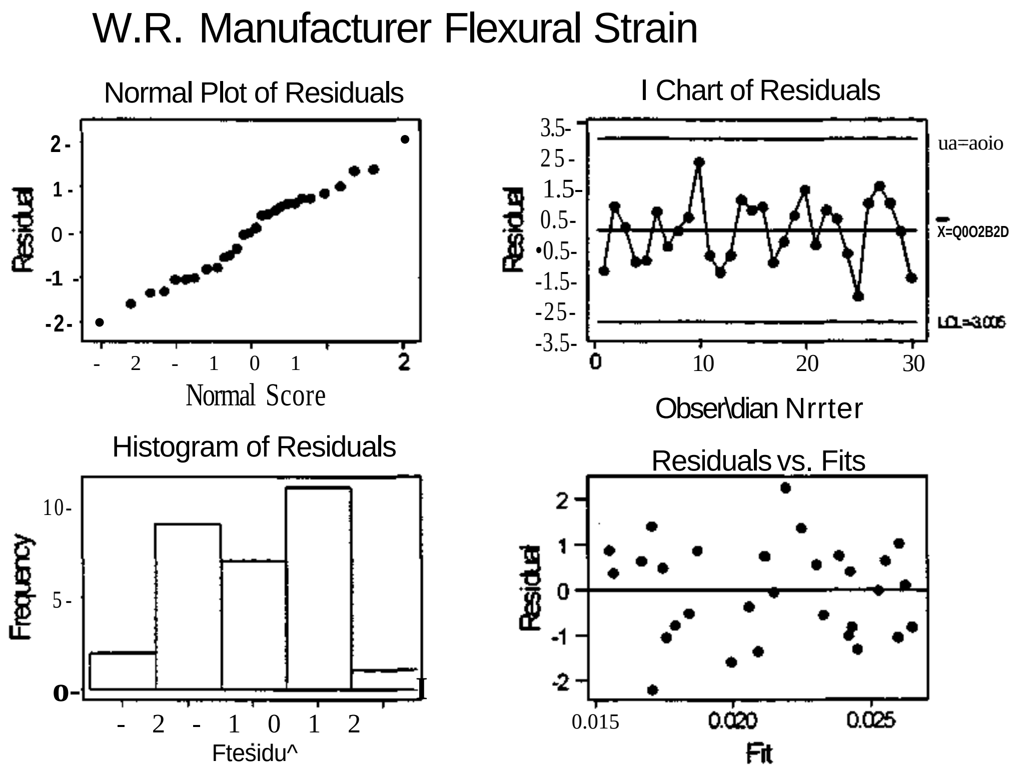


Figure 7.5: Flexural Strain Residual Plots

7.8.3 Conclusions on the Importance of the Factors

The sums of squares normalised with respect to the sub-plot error given in Table 7.9 are now used to compare the relative importance of the whole-plot factors. Here it should be stressed that these ratios are purely comparative and that no statistical inferences should be made from them. The trends may be seen in the appropriate main effects and interaction plots, which are included in the text. The table shows that the quadratic effect of the manufacturer factor is important for the tensile failure stress measurements.

Source	Tensile Stress	Tensile Stress (Mod & Vf)	Flexural Stress	Flexural Strain
Man.	4	0	1	1
QMan.	21	1	*11*	2
Thick	7	*11*	*11*	22
Man*Thick	1	0	2	0
QMan*Thick	5	0	0	0

Table 7.9: Whole-plot Sums of Squares Normalised with respect to Sub-Plot Error

Figure 7.6 shows that the medium level W.R. fails at a lower stress than either of the other two. When initial modulus and fibre volume fraction are included in the model, however, the quadratic effect of manufacturer becomes very small. A relatively large effect of thickness on tensile failure stress is seen, both with and without the inclusion of the covariates in the model. Figure 7.6 shows a decrease in strength with increasing thickness. The large quadratic manufacturer effect for flexural stress is seen, from Figure 7.7, to be of the same form as that for tensile stress; the medium level W.R. is weaker than the other two. Large decreases in strength with thickness are also seen in Figure 7.7 and Figure 7.8 for both flexural failure criteria.

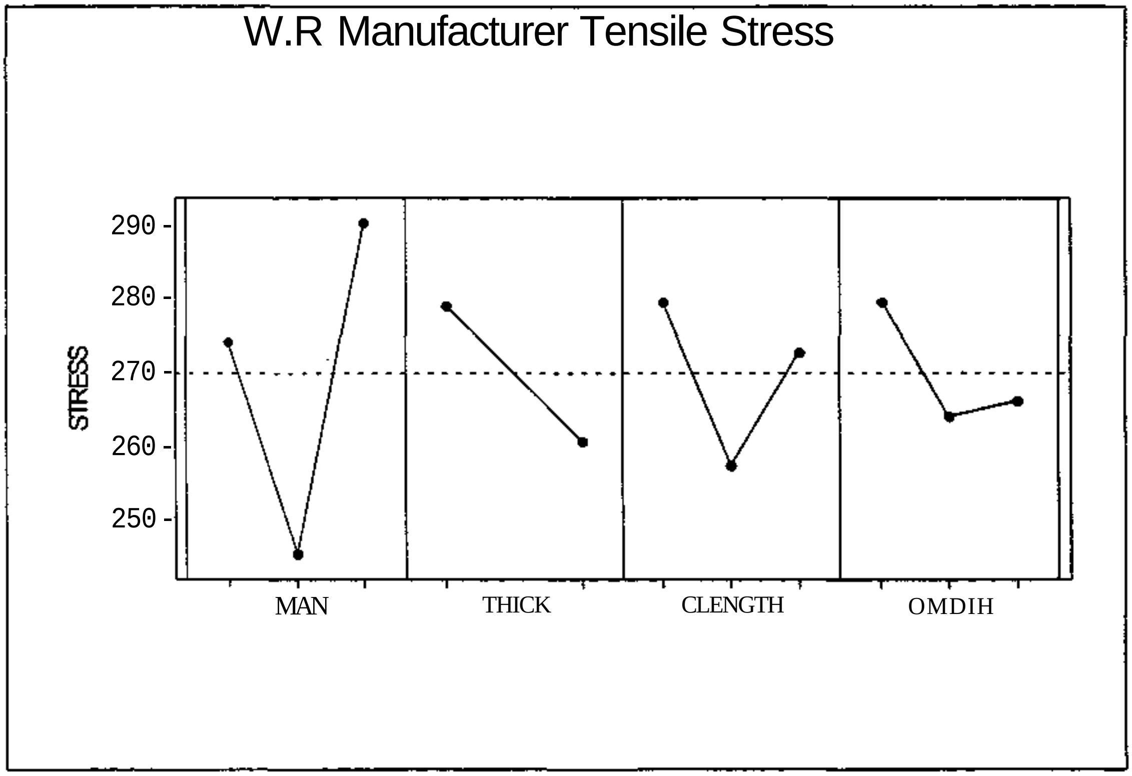


Figure 7.6: Tensile Stress Main Effects Plots

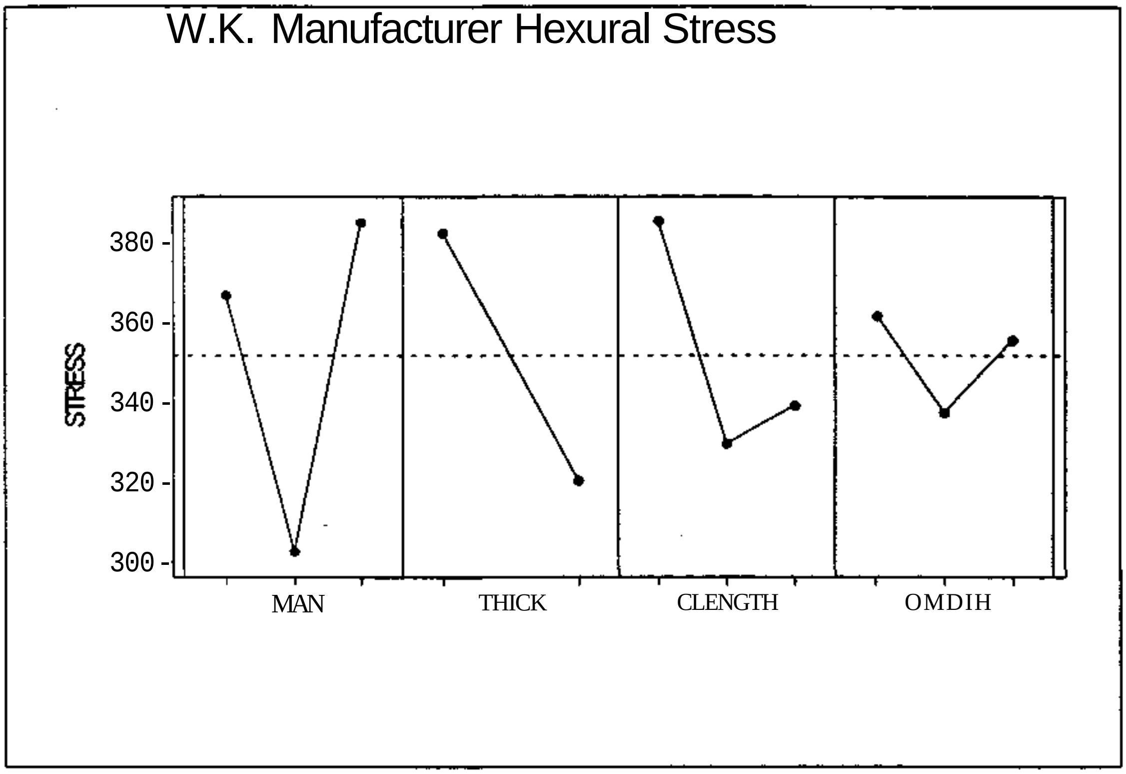


Figure 7.7: Flexural Stress Main Effects Plots

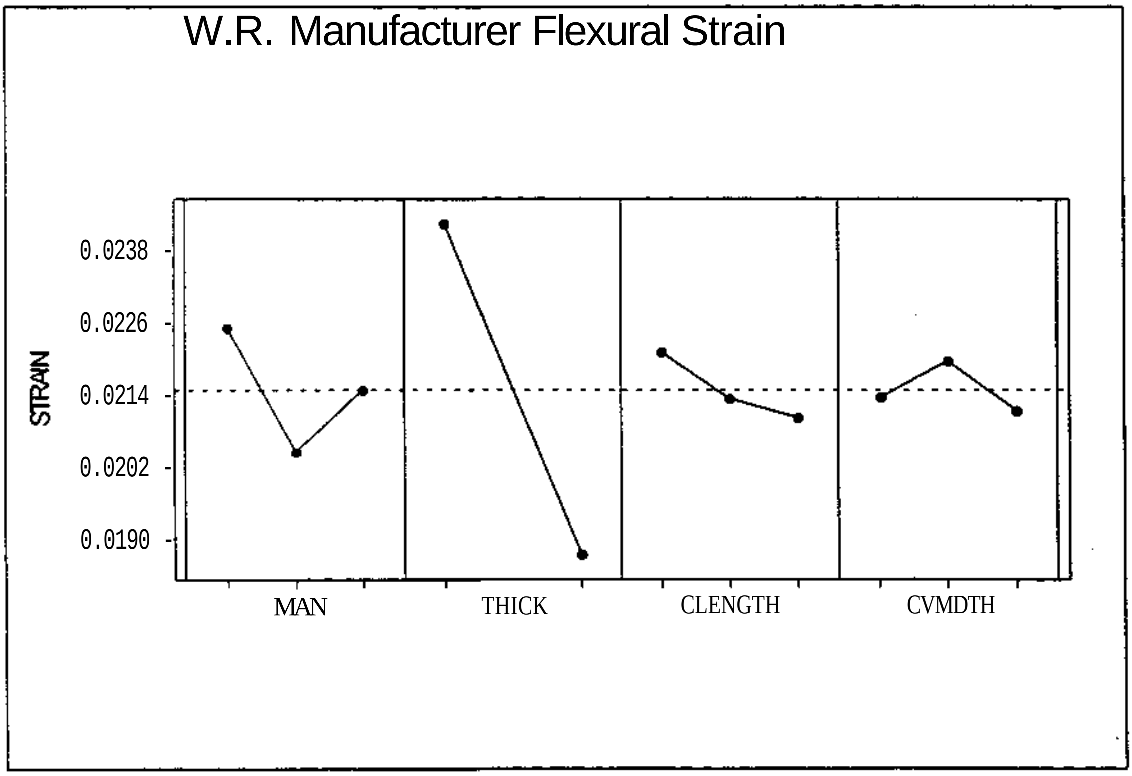


Figure 7.8: Flexural Strain Main Effects Plots

The sub-plot P-values in Table 7.10 are now used to identify important sub-plot factors. The trends may be inspected using the main and interaction plots included in the text. The interaction plots (Figure 7.9 to Figure 7.10) are arranged so that the lines of each plot correspond to the different levels of the factor indicated on that row of plots.

Source	Tensile Stress	Tensile Stress (Mod & Vf)	Flexural Stress	Flexural Strain
Man*CLength	0.164	0.653	0.908	0.508
QMan*CLength	0.873	0.313	0.080	0.168
CLength	0.420	0.829	0.352	0.584
QLength	0.388	0.147	0.734	0.997
Man*CWidth	0.822	0.785	0.525	0.436
QMan*CWidth	<u>0.018</u>	<u>0.005</u>	0.412	0.700
CWidth	0.497	0.374	0.433	0.265
QWidth	0.291	0.472	0.574	0.280
CLength*CWidth	0.947	0.618	0.461	0.355
Thick*CLength	0.184	0.060	~	~
Thick*CWidth	0.473	0.285	~	~
Covariates:				
Mod	~	0.044	~	~
Vf	~	0.042	~	~
R-Squared	82.4%	**91.3%**	**77.3%**	**72.7%**
Sub-Plot C.V.	0.0685	0.0523	0.1330	0.1504

Table 7.10: Sub-Plot P-Values for W.R. Manufacturer Tests

The only sub-plot effect which is seen to be statistically significant (at a 5% level) for this experiment is the interaction term between quadratic manufacturer and width for the tensile failure stress. Referring to the interaction plot in Figure 7.9, the form of this interaction can be described as; an increase in width decreases the failure stress for the low and high levels of W.R. manufacturer, but for the medium level of manufacturer the opposite is true. The meaning of this, in terms of the behaviour of the three reinforcements manufactured by different manufacturers is explained in Section 7.9

The two covariates. initial modulus and fibre volume fraction, are seen to be significant at a 5% level for tensile failure stress, and hence have been included in the model.

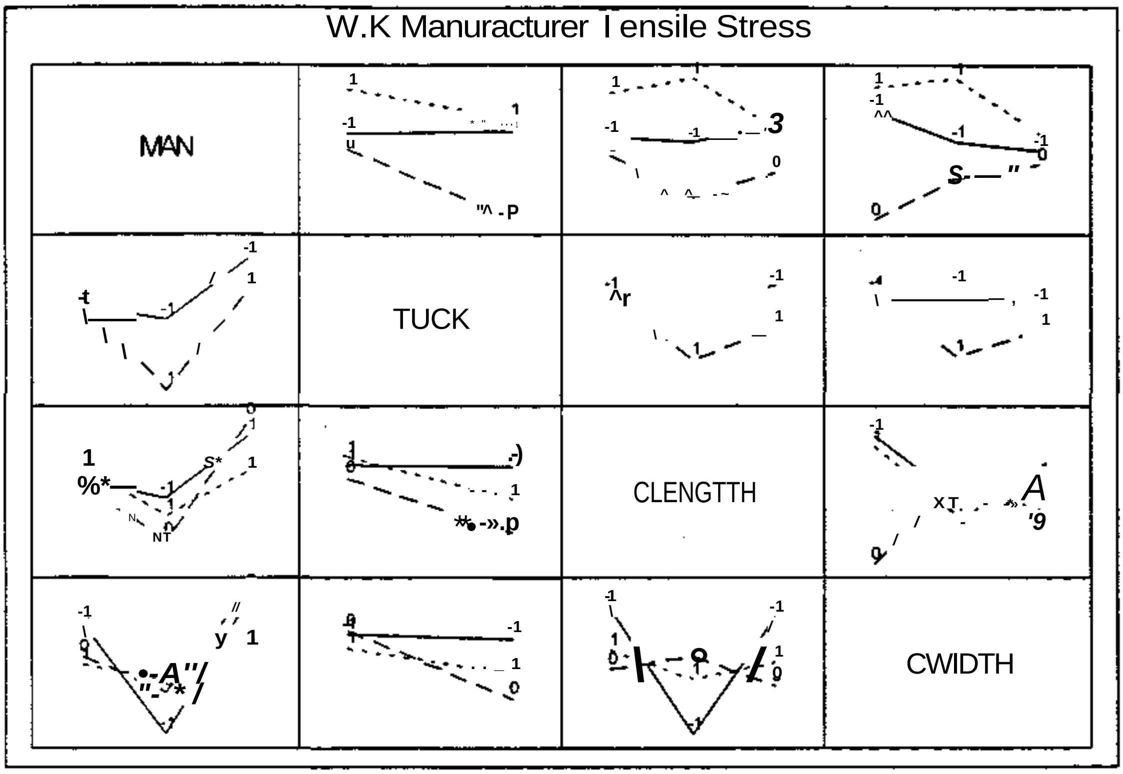


Figure 7.9: Tensile Stress Interactions Plots

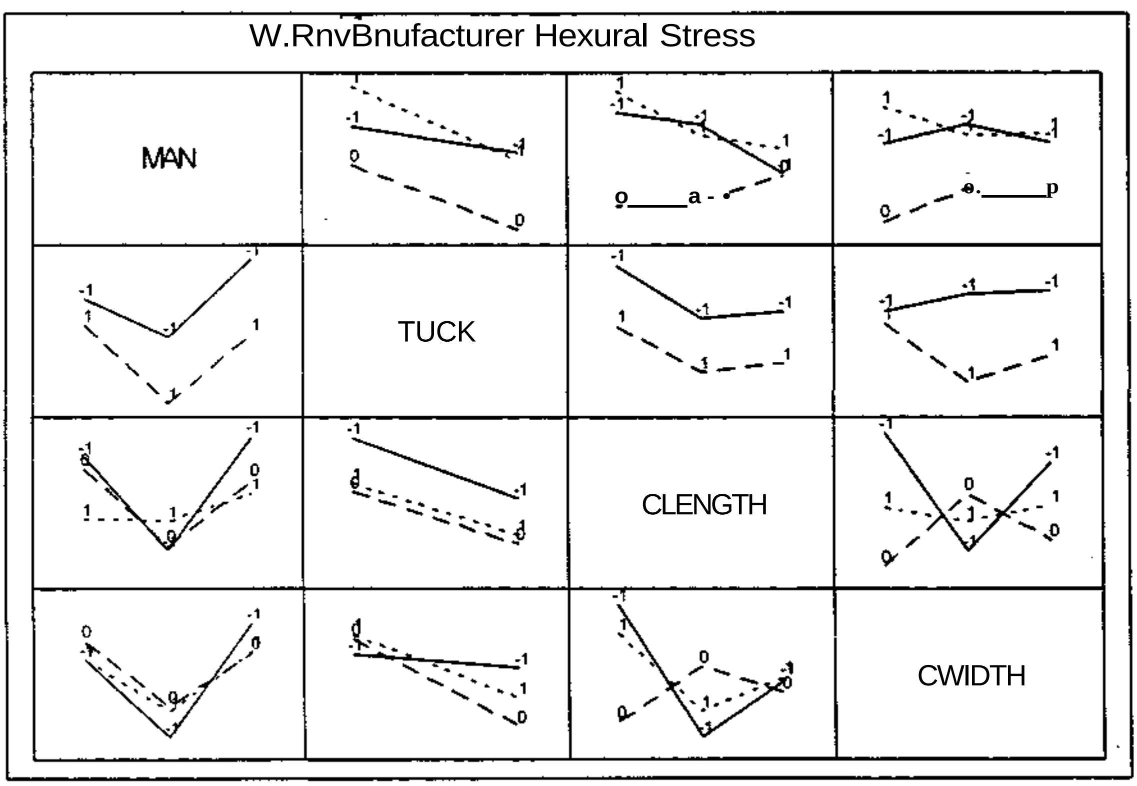


Figure 7.10: Flexural Stress Interactions Plots

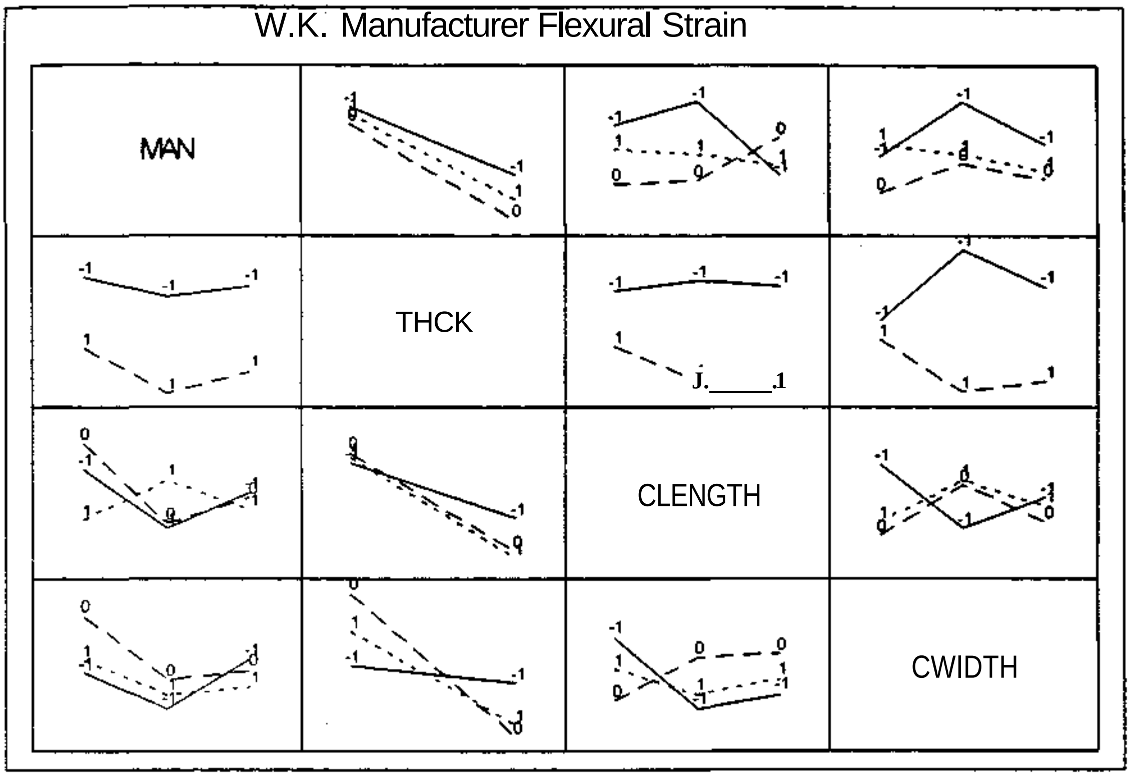


Figure 7.11: Flexural Strain Interactions Plots

7.9 Engineering Interpretation

Inclusion of the initial Young's modulus and fibre volume fraction of the tensile specimens in the statistical model gives an improved description of the failure stresses observed. The fibre volume fraction gives a measure of the 'quality' of the specimen in terms of the ratio of fibre to matrix materials within the composite. Variations from this specified value will contribute to the experimental variation, since the strength of a composite material is strongly dependent on this ratio. Through consideration of the fibre volume fraction, some of the experimental variation has been accounted for. Similarly, the initial Young's modulus can give an indirect measure of other variations in specimen 'quality'. Imperfections in the laminate, such as fibre misalignment, fibre crimp and void inclusion, will lead to changes in the initial stiffness. Hence inclusion of this covariate further accounts for some of the experimental variation. The fact that these covariates are not statistically significant for the flexural failure stress is puzzling considering the known sensitivity of compressive strength to such imperfections as described above. However, the fact that the woven roving manufacturer term has already been included in the statistical model may account for this.

The load versus tensile strain plots obtained using the strain gauges did not show smooth traces. Sections of the graphs showed an increase in strain with very little or no increase in load. It was often necessary to estimate the failure strain by ignoring these sections and / or extrapolating the line to the failure load in order to give the failure strain. This is an obvious explanation for the poor fit of the statistical model to the tensile failure strain data. The length of the strain gauges, 10mm, is of a comparable dimension to that of the weave of the roving reinforcement. This means that the local positioning of the gauges with respect to the weave becomes important. The gauge may or may not be situated, for example, at a position on the specimen surface where warp and weft crossover. The gauge may hence be subject to local strain variations in the coupon. For example, during tensile loading, an increase in load may cause the crimp in the warp at, or near to, the gauge to be straightened. This would lead to the observed case where an increase in strain is recorded by the gauge for little or no increase in load. Provision for this behaviour could not be included in the statistical model proposed, and hence a poor fit to the data was obtained. This means that, for the tensile specimens, there was little confidence in the measurements of failure strain, and no further inferences were made from this data set.

The flexural failure stress values were, by necessity, obtained indirectly via a calculation involving the failure load and test dimensions. Also, an approximation was made to allow for the effects of the central deflections of the specimens seen. Both of these procedures introduce errors into the stress values. This accounts for the fact that the fit of the model to the flexural stress values is slightly lower than that for the tensile tests.

The flexural strain measurements were also measured using the same 10mm foil type gauges as used on the tensile specimens. However, in this case, the gauges were not adhered to the compressive failure surface. Although the local positioning of the gauges with respect to the weave may again be important, the flexural strain traces did not show any of the anomalies seen for the tensile tests, and a reasonable fit to the data was achieved.

The whole-plot factors are seen to be more influential than the sub-plot factors, indicating that the manufacturing related effects are important. The only exception to this is when initial modulus and volume fraction are included in the analysis, and account for a large proportion of the variation in the failure strain previously attributed to the whole-plot factors. Since laminates of differing reinforcement and thickness must be laminated as separate panels, and since there was no replication of each laminated panel, it is impossible to distinguish between the effects of the whole-plot factors and the effects of panel to panel variation.

The coefficient of variation of the experimental error for the tensile stress measurements, at 7%, is comparable to the values obtained by Vosper Thornycroft (5-8%) in their in-house study (Vosper Thornycroft (1996)), and slightly less than the value of 12% quoted by Smith (1990). Inclusion of the covariates reduces the coefficient to 5%, again since they provide some indication of the laminate quality.

The inaccuracies associated with the calculations of flexural stress and the localised flexural strain measurement are reflected in the flexural failure stress and strain coefficients of 13 and 15% respectively. These values are comparable to Vosper Thornycroft's results of around 10%, but are about half the figure quoted by Smith of 29%. However, the data used by Smith is from many sources and hence material and test set-up differences would explain why his coefficients are relatively large.

The importance of the whole-plot quadratic woven roving manufacturer for both tensile and flexural failure stress simply means that the laminates fabricated using the 'AUS 11' woven roving were weaker than those constructed using either of the other woven rovings. This could be due to a number of reasons such as difficulty in wetting out when laminating, poor stacking of laminates, inferior bonding between matrix and fibre and increased fibre damage incurred by the weave process used. The first two examples would be reflected in a variation in the fibre volume fraction, and this hypothesis is reinforced by the removal of the importance of the quadratic manufacturer effect when fibre volume fraction is considered as a covariate for the tensile tests.

The strengths of the 8 ply specimens are lower than those of the 4 ply specimens for both test methods and for both stress and strain measurements. The 4 ply panels were laminated one continuous sheet of woven roving in each ply. However, the eight ply panels were fabricated with more than one sheet of woven roving in each ply using the 'butting system' described in Appendix K. This gives discontinuities in the reinforcement and would account for the decrease in material strength.

Another explanation for this trend is that a lower quality specimen is obtained when a thicker laminate is produced. A plausible explanation for this would be that, when there are more wet, and hence unstable, plies, the fibres of the top lamina are more difficult to keep straight as they are wetted-out. Compressive failure of the flexural specimens was initiated in the top laminate and since fibre waviness is critical for compressive failure, this would explain why the thicker laminates were weaker. A similar trend was seen for the tensile tests, where the straightening of crimped fibres may lead to localised matrix failure and subsequent failure of the specimen.

The tensile stress interaction between the quadratic manufacturer and width terms is mainly due to the low strength obtained for the low width (10mm wide) AUS 11 reinforced specimens. An explanation for this behaviour may be derived from the values for the number of ends (of warp rovings) per 100mm quoted in Table K.2 in Appendix K; 20 for AUS 8 and 12 reinforcements, and 17.8 for the AUS 11 reinforcement. Hence for a 10mm wide specimen there would be. on average, 2 warp rovings in the AUS 8 and 12 coupons but only 1.78 rovings in the AUS 11 coupons. This would lead to a higher number of cut or damaged warp rovings in the AUS 1110 mm wide specimens and would account for their reduced strength.

8. Final Woven Roving Tests

8.1 Test Programme Development

One of the objectives of this series of tests was to further investigate the behaviour of marine E-glass woven roving polyester composites, using the experience already gained in the application of experimental design techniques to the mechanical testing of such materials. Vosper Thornycroft's own in-house testing of the laminates considered in Chapter 7 had raised questions concerning the interpretation of such test data. The shipyard conducts mechanical tests on coupons taken from panels laminated under realistic shop-floor conditions to evaluate the material properties for design purposes. This testing creates a large amount of data which has a considerable amount of scatter and is crudely analysed statistically by taking a design value of the mean minus two standard deviations. It is desirable to further understand the sources and implications of this scatter with respect to how these coupon test results relate to the pertinent design material properties. The two main points raised were;

1. Defects and imperfections may have a strong influence upon small coupons but are probably less important at full-scale.
2. Coupon testing is practical and economical but the number of specimens required and the interpretation of the data are open to question.

Hence it was decided to conduct a test programme to try to identify and understand the factors which are responsible for the variation in the material properties of shipbuilding quality GRP with respect to the fundamental problem of correlating coupon to full-scale properties.

Initially consultations with Mr. Alan Dodkin and Mr John Holness from the structures group at Vosper Thornycroft produced a list of the factors which it was thought could affect the material properties of woven roving GRP, and the variability in these properties;

(i) Fibre orientation,
(ii) Fibre waviness,
(iii) Fibre crimp,
(iv) Cloth type,
(v) Cloth weight,
(vi) Cloth-butt staggering,
(vii) Resin system,
(viii) Resin gel time,
(ix) Resin thixotropy,
(x) Resin ratio,
(xi) Void content,
(xii) Panel position of coupon,
(xiii) Production Method,
(xiv) Panel Thickness,
(xv) Laminae misalignment.

Further discussions led to a smaller, more pertinent, list whose investigation required fewer experimental resources. Items (viii), (ix), (xiii) and (xv) were thought to be relatively unimportant and problems were envisaged in controlling and quantifying (ii), (iii), (v), (x) and (xi). Since woven roving is the preferred choice for the construction of large scale marine structures, and different types of woven roving had already been considered in the W.R. manufacturer tests, cloth type (iv) was not considered further. Similarly the resin system preferred is that of polyester, and so (vii) was also not investigated. However a different polyester resin system to that in the first stage tests was employed, enabling comparisons between the two alternatives to be made. In order to both simplify the production of the panels and to reduce the number of panels required, only the hand lay-up process was considered.

The remaining factors were fibre orientation, cloth-butt staggering and panel thickness. In addition to these factors, the effects of specimen length and width were also investigated. The analysis of the W.R. manufacturer data (see sections 8.6 and 8.7) did not suggest any strong dependence of coupon strength on length or width, and so two levels for each of these factors was thought to be sufficient.

The action of laminating by hand often produces distortions in the orthogonal structure of the warp and weft of a woven roving composite. A similar effect is seen when the cloth has to be worked into a three dimensional shape. This was modelled by skewing the warp and weft in the woven roving to give the levels of a 'fibre orientation' factor. Two levels were allocated to this factor. 'orthogonal" warp and weft and 'skewed' warp. The angle of skew had to be large enough to be practically achieved and controlled. An angle of 15° was considered to be appropriate. The two levels of this factor are illustrated in Figure 8.1.

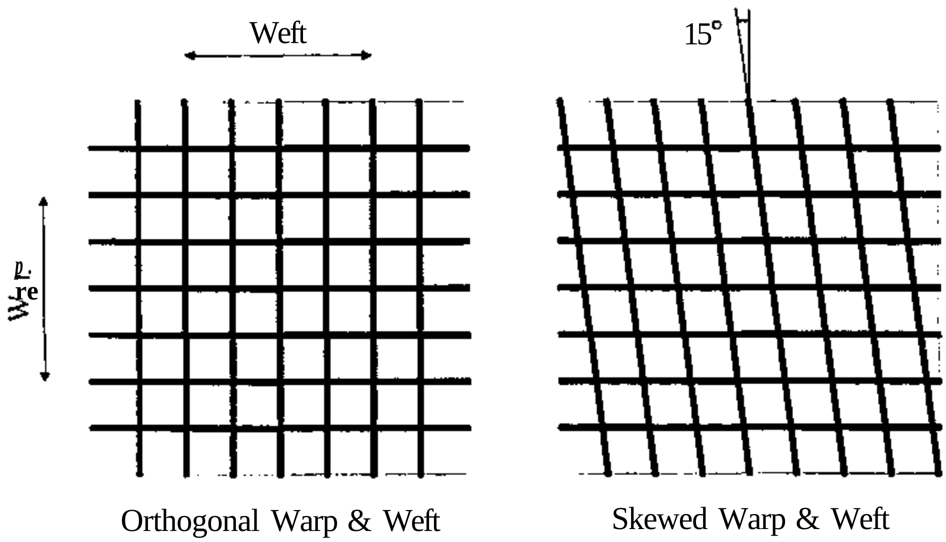


Figure 8.1: Skewed Warp and Weft

In practice woven roving cloth is usually supplied in rolls of a given width. Hence to fabricate large structures these widths of cloth are laid up side by side, introducing a 'butt' where the two cloths meet. Vosper Thornycroft employ a system where these butts are staggered through the laminate thickness. The butt in each successive ply is offset by 150 mm, giving a diagonal "stagger' as shown in Appendix K. This would have been inapplicable to coupons of the size to be considered here and so a system of butts was introduced to simulate that used in practice. A butt was introduced into every fourth ply, as shown in Figure 8.2. Normally the butts would run parallel with the warp, but since testing was only to be conducted in the warp direction, the butts were positioned perpendicular to the warp along the centre-line of each panel fabricated. The two levels of this factor consisted of the absence and the presence of this 'butting system'.

Three levels of panel thickness were selected: 4 and 8 plies to correspond to those in the W.R. manufacturer tests, and also 12 plies to try to induce any effects associated with thicker laminates. To give continuity in the test programme as a whole, the tensile and four point flexural test methods were again employed.

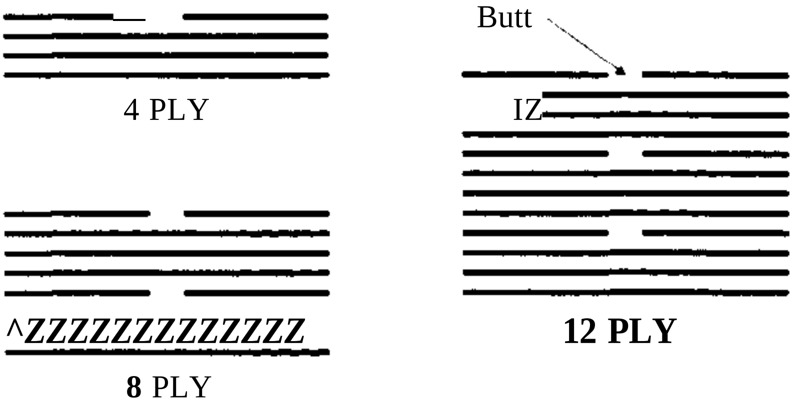


Figure 8.2: 'Butting System'

The large amount of scatter in the results observed for the results of the W.R. manufacturer tests led to a fairly conservative approach to the design of the experimental plan for the final test programme. That is. the loss in statistical precision associated with the use of fractional factorial designs was not thought to be beneficial for the degree of experimental variation expected. This also meant that the statistical analysis of the data in this case would be fairly straightforward. A full factorial design was selected for the factors thickness, 'butts' and 'skew', leading to the fabrication of the 12 panels described in Table 8.1. The full factorial combination of the length and width factor levels, with one replication of each coupon gave 16 specimens from each panel (8 for each of the two test methods). This gave a total of 192 coupons for this test series. The structure of the test programme is shown in Figure 8.3, where the panel numbers refer to the last digit of the panel code given in Table 8.1.

Panel Code	Thickness (Plies)	Butts	Skewed Warp
VOS/41	4	No	No
VOS/42	4	No	Yes
VOS/43	4	Yes	No
VOS/44	4	Yes	Yes
VOS/81	8	No	No
VOS/82	8	No	Yes
VOS/83	8	Yes	No
VOS/84	8	Yes	Yes
VOS/121	12	No	No
VOS/122	12	No	Yes
VOS/123	12	Yes	No
VOS/124	12	Yes	Yes

Table 8.1: Panel Codes

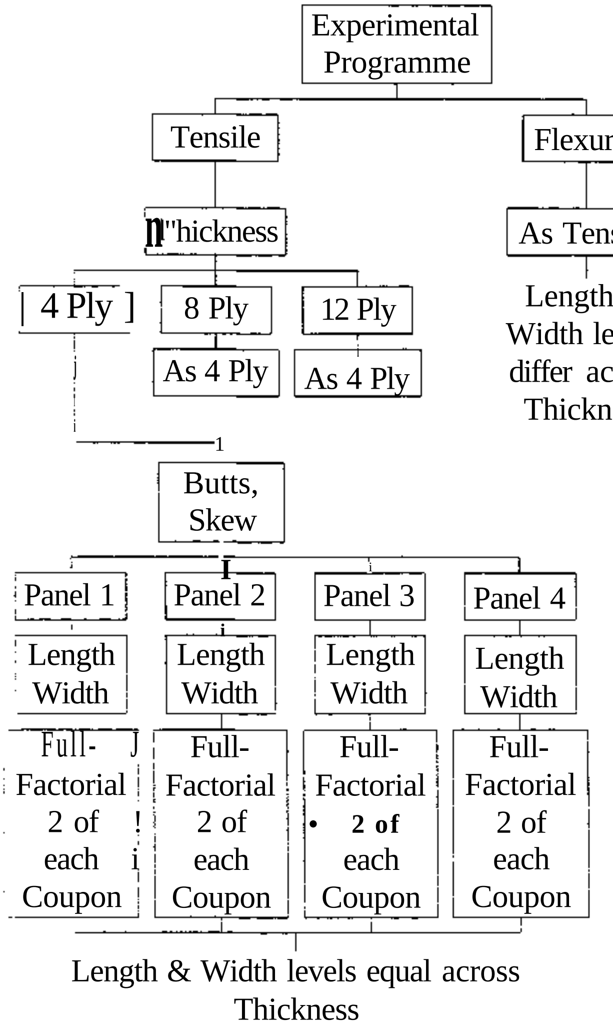


Figure 8.3: Final W.R. Test Series Structure

The sometimes unexpected behaviour shown by the composites of the previous test series led to the inception of preliminary tests for both test methods in this programme. The main objective of these tests were to provide prior information about the behaviour of the specific materials considered before fabricating the specimens for the test programme itself. This ensured that any alterations to the specimens and test set up could be made without the need to re-manufacture large numbers of specimens. The specimens for the preliminary tests were cut from 4 ply panels. Those with orthogonal warp and weft, despite coming from panel VOS / 42, were extracted from areas without butts.

Two small additional test series were also conducted. The interlaminar shear strength of the composite was estimated using short beam shear tests, enabling verification that shear would not affect the flexural failures. The effects of the end gripping conditions on the results of the tensile tests were explored by a series of tests on rectangular specimens (as used in the main programme), "waisted" specimens and specimens with bonded tabs. Details of the 'tensile geometry' and 'ILSS' tests are given in Section 8.10 and Section 8.11 respectively.

8.2 Specimen Sizing

The experience gained from the W.R. manufacturer test programme was used to set the specimen dimensions given in Table 8.2 and Table 8.3. Additionally, an extensometer was used for the tensile tests, and this dictated that the minimum specimen length be 100 mm. A grip length of 50 mm at each end of the tensile specimens was introduced to minimise the likelihood of slippage.

Units: mm	Depth (Approx.)	Level	Length	Width
4-Ply	4	Low	54	10
		High	100	30
8-Ply	7	Low	105	20
		High	200	60
12-Ply	11	Low	162	30
		High	300	90

Table 8.2: Flexural Test Factor Levels

Units: mm	Depth (Approx.)	Level	Length	Width
4-Ply	4	Low	100	10
		High	300	25
8-Ply	7	Low	100	10
		High	300	25
12-Ply	11	Low	100	10
		High	300	25

Table 8.3: Tensile Test Factor Levels

8.3 Specimen Manufacture

As for the W.R. manufacturer tests, all panels were laminated on a horizontal surface by hand, under factory floor conditions at Vosper Thornycroft's Woolston shipyard. The panels were of dimensions 1m by 1m. The reinforcement consisted of Fothergill Engineering Fabrics YO 530 plain weave, woven roving E-glass of weight 780 gm^{-2}. The matrix phase was Scott Bader Crystic 489 PA isophthalic boat-building polyester resin with 1.5% Butanox M50 catalyst. A nominal fibre weight fraction of 50% (equivalent to a fibre volume fraction of approximately 35%) was stipulated and the laminates were cured at room temperature for 2 months. Further details of the reinforcement, resin system and the instructions given to the laminators are given in Appendix K.

The specimens were then cut from the panels using a diamond-edged saw, with the warp in the length direction. Those specimens with butts were cut out of the panels so as to position the butt halfway along the length of the specimen. Strain gauges were adhered to the centre of the moulded face of each flexural specimen using the method described in section 6.3 .

8.4 Experimental Details

The experimental set-up of the flexural tests was identical to that described in section 6.4. The flexural specimens were again all tested with the strain gauge on the tension side, with the moulded face in tension. The experimental procedure and the measurements taken for these flexural tests were as described in section 6.4. The tensile tests here differed from the previous tests only in that the specimens were not strain gauged. An extensometer was instead used to give strain measurements. The cross head speeds (under deflection control) for these tests are given in Table 8.4.

Test Method	Thickness (Number of Plies)	Cross-Head Speed (mm min^{-1}) Low Length	High Length
Flexural	4	5	10
	8	10	10
	12	10	20
Tensile	4	2	2
	8	2	2
	12	2	2

Table 8.4: Final W.R. Tests Cross-head Speeds

The method described in ASTM standard D2584 (ASTM (1985a)) was used to obtain the fibre volume fractions of each specimen.

8.5 Data

The experimental data obtained from the final woven roving test programme can be found in Appendix I and typical failure modes in Appendix L. The specimen codes start with either 'VT for tensile tests or 'VF' for flexural tests. This is followed by the panel number from which the coupon was cut (see Table 8.1). Finally the same system as detailed in Section 7.5 is used to indicate the length and width levels and the replication identification.

8. 6 ***Method*** *of Analysis of* ***Tensile Data***

As for the previous test series, the experimental design used here has a split plot structure. The main and interaction effects considered in the model, and the division between whole-plot and sub-plot terms are shown in Table 8.5.

Whole-Plot	**Sub-Plot**
Thickness	Length
Butts	Width
Skew	Length x Width
Thickness x Butts	Thickness x Length
Thickness x Skew	Thickness x Width
Butts x Skew	Butts x Length
Thickness x Butts x Skew	Butts x Width
	Skew x Length
	Skew x Width

Table 8.5: Classification of Tensile Factors and Interactions

In order to retain the orthogonality of the design and to reduce computational errors, coded values for length and width of 1 and 2 for the high and low levels respectively were used in the statistical analyses ('CLength' and 'CWidth'). Coded values of 1 and 2 were also used to indicate the absence or presence respectively of butts and skewed warp and weft. The number of plies (4, 8 or 12) gave the coded thickness values. The analysis was conducted using both failure stress and failure strain as the response. The results of these analyses are summarised in Appendix J.

he ANOVA based 'General Linear Model' available in the MINITAB software allowed incorporation of the split-plot structure of the design into the statistical analysis of the data. Initially the sub-plot analysis of variance was produced using the 'GLM' command (MINITAB (1994)). The whole-plot three-way interaction term between thickness, butts and skew was then specified as an estimate of the whole-plot error and this was used as the basis for a whole-plot analysis of variance. Residual plots and main and interaction effects plots were then produced.

In order to further subdivide the analysis of the thickness main and interaction effects into linear and quadratic effects (see Section 8.6) the SAS software was used (Cary (1989)). Again a 'General Linear Model' procedure was used to give a split-plot analysis of variance and then contrasts were used to give ANOVA linear and quadratic thickness main and interaction effects. Whole-plot tests were again performed using the interaction between thickness, butts and skew as an estimate of the whole-plot error. Standard errors were also calculated for the whole- and sub-plot main and interaction effects using the relevant estimates of error.

Since the designs used were fully orthogonal, the type I and type III sums of squares were identical. However, this orthogonality was disturbed by the consideration of both fibre volume fraction and initial modulus as covariates. Hence the type III sums of squares were used to allow useful comparisons to be made between the factor effects. The only statistically significant covariate for the tensile test results was found to be initial modulus for failure stress.

8.7 Method of Analysis of Flexural **Data**

A similar split-plot structure to that described in Section 8.6 was also present for the flexural tests. Additionally the length and width factors are nested within the thickness factor. As described in Section 6.6, a nested factor effect may be found by summing its main and interaction sum of squares. The relevant whole-plot and sub-plot terms are as given in Table 8.6.

Whole-Plot	**Sub-Plot**
Thickness	Nested Length
Butts	Nested Width
Skew	Length x Width
Thickness x Butts	Butts x Length
Thickness x Skew	Butts x Width
Butts x Skew	Skew x Length
Thickness x Butts x Skew	Skew x Width

Table 8.6: Classification of Flexural Factors and Interactions

Again coded values for the levels of length, width, butts and skewed warp and weft were used, and the number of plies taken as the thickness levels. The same statistical analyses described in Section 8.6 were then applied to the flexural data. The nested sums of squares were then obtained through addition of the appropriate values from the MINITAB and SAS outputs. Summaries of these analyses are given in Appendix J. The inclusion of the initial modulus covariate was found to be statistically significant for flexural failure stress. Additionally, initial modulus was shown to be weakly significant for the flexural failure strain analysis (P-value 0.09), but due to the poor fit of the statistical model in this case, the covariate was not considered further.

8.8 Findings of the Data Analyses

In this section the results of the statistical analyses described in Sections 8.6 and 8.7 are reviewed, and important factors and trends are highlighted. A summary of these analyses are included in Appendix J.

8.8.1 Model Fitting

As a preliminary and simple examination of possible linear relationships between the response variables, treatment factors and possible covariates, the correlation matrices given in Appendix J were considered. As pointed out in earlier chapters, the estimates of these correlations between pairs of variables do not take into account the variations in the other variables.

The matrix for the tensile tests, given in Table J.I, shows that the initial modulus and the failure stress are correlated with a coefficient of 0.821. This indicates that the initial modulus may be included as a covariate, as described in Section 8.6. Skew is seen to be only moderately correlated with stress (correlation coefficient -0.591), and also with modulus (correlation coefficient -0.560). Thickness is weakly correlated with failure strain (correlation coefficient 0.471). The corresponding matrix for the flexural tests, given in Table J.2, shows that modulus varies with thickness to some degree (correlation coefficient 0.608), and also to failure strain (correlation coefficient -0.629). Skew is negatively associated with modulus (correlation coefficient -0.539), and failure stress (correlation coefficient -0.578).

The statistical models postulated for the tensile and flexural failure strain is represented by;

$$\begin{aligned} \textit{Failure Strain} = {} & a_{()} + a_x T_t + a_2 T_q + a\text{^}B + a_4 S + a_5 T,xB \\ & + a_h T_{tj} xB + a_7 T,xS + a_{\%} T_{tl} xS + a_9 BxS + \mathrm{E} \\ & + \alpha_{10} CL + \alpha_{11} T_t xCL + \alpha_{12} T_q xCL + \alpha_{13} CW \\ & + \alpha_{14} T_t xCW + a\text{^}xCW + a_{lb} CLxCW \\ & + \alpha_{17} BxCL + a_n BxCW + a\text{^}SxCL + a_{20} SxCW + s \end{aligned} \qquad (81.)$$

where the a's are the coefficients corresponding to the effects, T_l is linear thickness, T_q is quadratic thickness. B is the coded value of butts, S is the coded value of skew, E is the whole-plot error. CL is the coded length, CW is the coded width and e is the sub-plot error.

For the tensile and flexural failure stress, the initial modulus was included and the models may be represented as:

$$
\begin{aligned}
Failure\ Stress = {} & a_0 + a_1 T_l + a_2 T_q + a_3 B + a_4 S + a_5 T_l xB \\
& + a_6 T_q xB + a_7 T_l xS + a_8 T_q xS + a_9 BxS + E \\
& + a_{10} CL + a_{11} T_l xCL + a_{12} T_q xCL + a_{13} CW \\
& + a_{14} T_l xCW + a_{15} T_q xCW + a_{16} CLxCW \\
& + a_{17} BxCL + a_{18} BxCW + a_{19} SxCL + a_{20} SxCW \\
& + Mod + \varepsilon
\end{aligned}
\qquad (8.2)
$$

where Mod is the initial modulus.

As mentioned in Section 8.6, the three-way interaction between thickness, skew and butts was used as an estimate of the whole-plot error. This was used as an estimate of the whole-plot coefficients of variation, which are presented, together with the sub-plot values, in Table 8.7.

	Whole-Plot C.V. (%)	**Sub-Plot C.V. (%)**
Tensile Stress	12.5	9.5
Tensile Stress with Modulus	12	9.5
Tensile Strain	10	9
Flexural Stress	20.5	12
Flexural Stress with Modulus	20	11
Flexural Strain	11	12.5

Table 8.7: Whole-Plot and Sub-Plot Coefficient of Variation Comparisons

This table shows that the variation between specimens cut from a single panel is less than that between separate panels, for all measurements except for flexural failure strain. In this case the whole-plot C.V. is marginally less than that for the sub-plot. However, as will be shown in the next section, the model postulated for this flexural failure strain measurements does not fit the data well. This means that any statistical inferences made about these data should be viewed with caution.

For the tensile tests, the whole-plot coefficients of variation are between 10 and 12%, whilst those for the sub-plot are around 9%. For the flexural stress failure criteria the whole-plot variation is much higher at around 20%, and the sub-plot C.V. is slightly higher than the corresponding tensile value at 12%.

8.8.2 Checking the Adequacy of the model

The R-Square values presented in Table 8.8 show that the model postulated for the tensile failure stress tit the experimental data very well. The fits of the models for tensile strain and flexural stress are not as good, but still reasonable, with R-Square values of around 70%. However, The R-Square value of under 60% for the flexural failure strain data indicates a poorer fit to the data, and hence care must be taken when interpreting the statistical analysis for these data.

	R-Square (%)
Tensile Stress	95
Tensile Stress with Modulus	95
Tensile Strain	71
Flexural Stress	72
Flexural Stress with Modulus	72
Flexural Strain	58

Table 8.8: R-Square values

The use of residual plots to check the assumptions of the statistical analysis methods is described in Section 6.8.2. The residual plots for this test series are given in Figure 8.4 to Figure 8.7.

No discernible pattern to the data is apparent from the tensile strain versus fits plot in Figure 8.5. A slight "funnelling", or increase in scatter with fitted value, is seen in Figure 8.6 and Figure 8.7 for the tlexural tests, but this effect is almost entirely due to four data points. This was assumed to be a purely statistical occurrence, and transformation of the data was not considered necessary in this case.

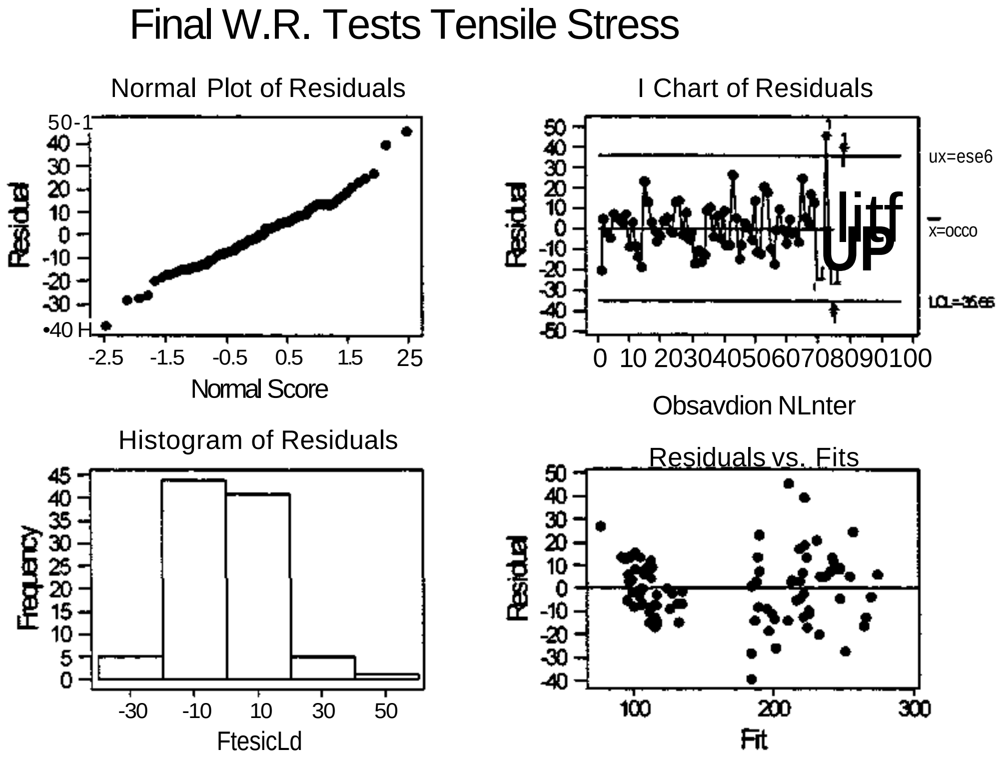


Figure 8.4: Tensile Stress Residual Plots

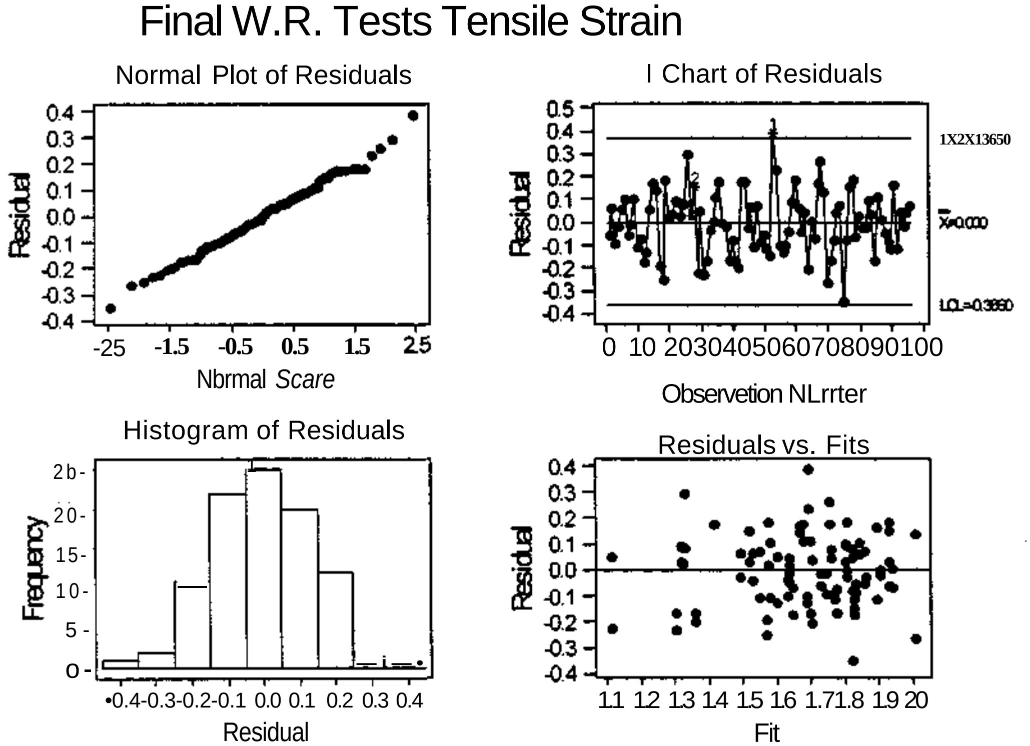


Figure 8.5: Tensile Strain Residual Plots

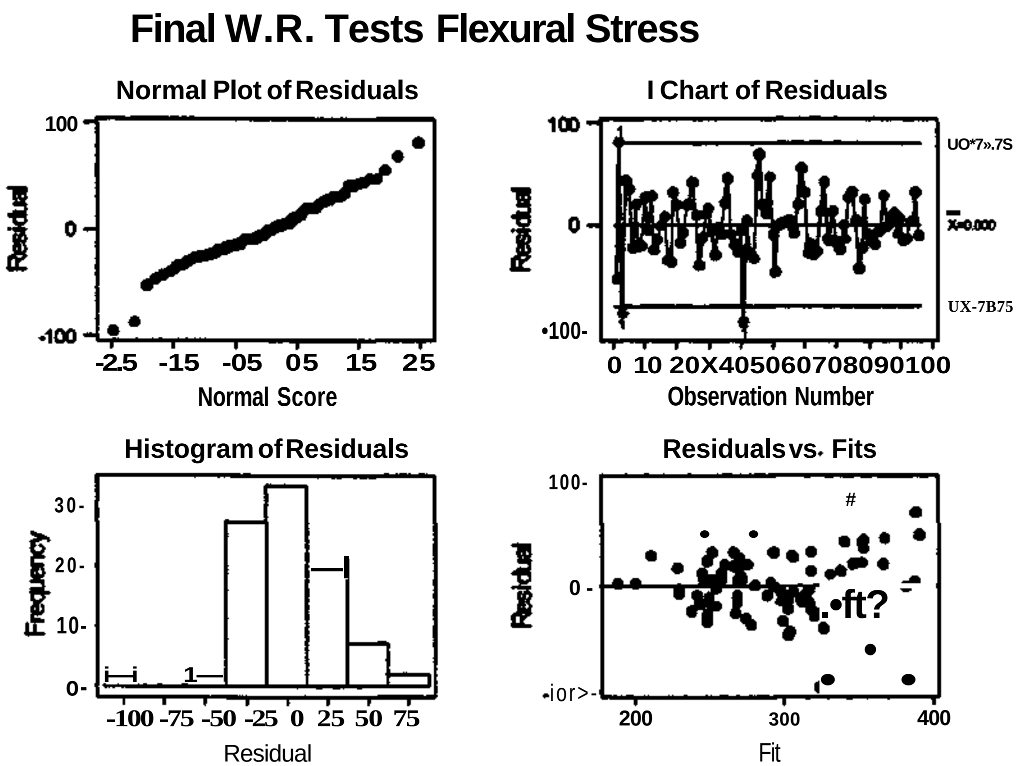


Figure 8.6: Flexural Stress Residual Plots

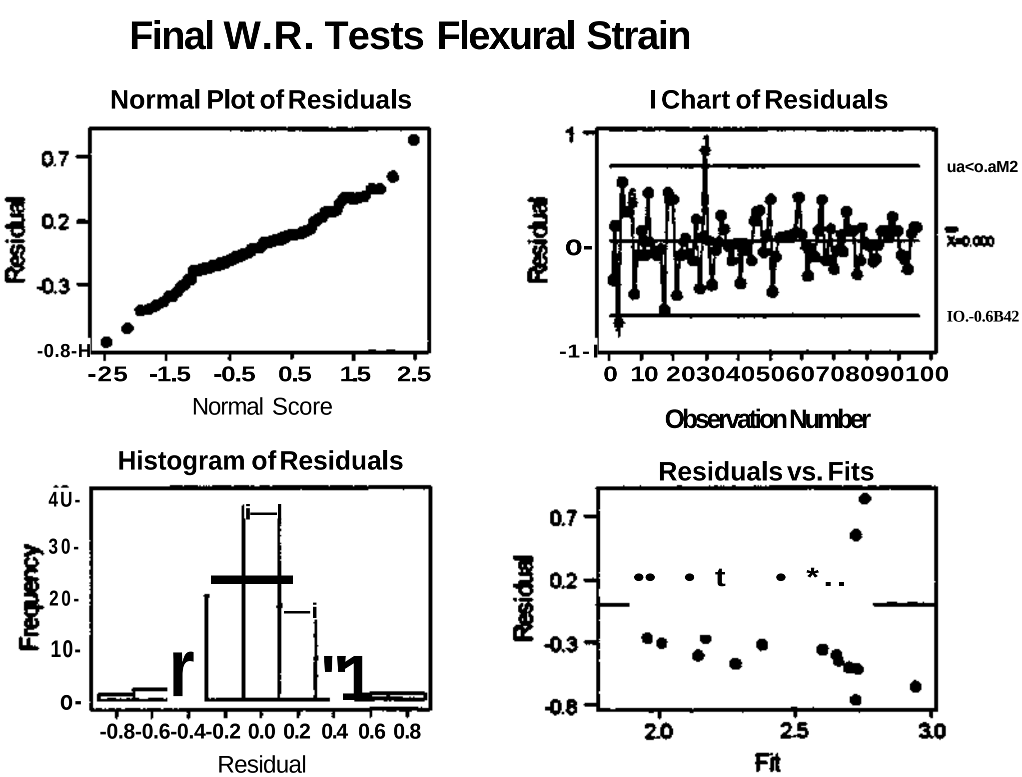


Figure 8.7: Flexural Strain Residual Plots

The roughly linear normal plots give no indication that it is unreasonable to describe the estimates of the errors using a normal distribution in all cases. There is no obvious skew in any of the residual histograms, and no strong patterns in the residual 'I charts'. The increased scatter for the higher fits in the tensile stress residuals versus fits plot in Figure 8.4 can, on closer inspection of the data, be seen to correspond to the 12-ply specimens with orthogonal warp and weft.

Since a basic assumption of the statistical methods used is that the variance is constant, this point required further investigation. Where the variance is seen to vary with the fit of the model, a procedure known as 'transformation' of the data may be applied to rectify this situation. The procedure is comprehensively discussed by Box et al. (1978), and essentially consists of applying a mathematical transformation to the data before the statistical analyses is carried out. The most common transformations are; ln(x), $x^{1/2}$,1/x and $1/x^{1/2}$. Each of these are tried and that which produces the most uniform variance is used. Care must be taken here to ensure that the interpretation of such a transformation has some physical meaning. In this case, a 1/x transformation produced a uniform variance. The statistical analysis of the transformed data, however, produced almost identical results to those obtained using the raw data. Also, such a transformation did not have any physical justification. Hence, the non-uniform variance was thought to be unimportant. Further analysis methods, not considered in this study but which may be applicable here, are considered in Chapter 11.

8.8.3 Conclusions on the Importance of the Factors

The summary of the P-values produced by the statistical analyses given in Table 8.9 are now used to highlight the important factors and interactions. The important values are underlined. The trends may be seen in the appropriate main and interaction effects plots, in Figure 8.8 to Figure 8.15.

The whole-plot terms are considered initially. There is a significant linear increase in tensile failure strain with increasing thickness (see Figure 8.9). The effects of butts and skew on the tensile failure stress are also significant at a *2%* level, the presence of either reducing the failure stress, as can be seen in Figure 8.8. Although not as strongly significant (with P-values of around 0.08), these factors have the same effect on the tensile failure strain, as shown in Figure 8.9. An interaction between butts and skew is just significant at the 5% level for tensile failure stress and Figure 8.12 indicates that the effect of either feature is reduced by the presence of the other. The quadratic interaction effect of thickness with butts on tensile stress, also shown in Figure 8.12, can be interpreted as a much greater effect of the inclusion of butts on the failure stress for the 8 ply coupons than for the 4 and 12 ply coupons. The opposite trend is seen in this figure for the interaction between thickness and skew; The effect of skewed warp and weft on the failure stress is much greater for the 4 and 12 ply specimens than for the 8 ply specimens. A similar interaction is also present for the tensile failure strain measurements, but is not as strongly significant (P-value of 0.05).

	Tensile			Flexural		
Source	Stress	Stress (With Mod.)	**Strain**	Stress	Stress (With Mod.)	**Strain**
Whole-Plot:						
Thick						
Linear	0.277	0.289	0.024	0.607	0.107	0.014
Quadratic	0.066	0.291	0.770	0.467	0.751	0.267
Butts	0.006	0.018	0.076	0.517	0.303	0.188
Skew	0.003	0.017	0.077	0.036	0.456	0.166
Thick*Butts						
Linear	0.902	0.560	0.210	0.994	0.609	0.794
Quadratic	0.006	0.013	0.201	0.511	0.767	0.892
Thick*Skew						
Linear	0.777	0.896	0.139	0.829	0.716	0.974
Quadratic	0.008	0.019	0.051	0.242	0.282	0.213
Butts*Skew	0.045	0.048	0.288	0.133	0.277	0.376
Sub-Plot:						
CLength	0.764	0.698	0.108	0.056	0.399	0.586
Thick*CLength						
Linear	0.352	0.876	0.247	~	~	~
Quadratic	0.150	0.724	0.273	~	~	~
CWidth	0.004	0.005	0.721	0.398	0.922	0.426
Thick*CWidth						
Linear	0.005	0.001	0.366	~	~	~
Quadratic	0.305	0.218	0.741	~	~	~
CLength*CWidth	0.442	0.262	0.016	0.668	0.526	0.999
Butts*CLength	0.148	0.282	0.541	0.698	0.645	0.089
Butts*CWidth	0.101	0.070	0.003	0.202	0.022	0.471
Skew*CLength	0.771	0.916	0.000	0.121	0.231	0.124
Skew*CWidth	0.000	0.000	0.044	0.031	0.434	0.277
Covariates:						
Initial Modulus	~	0.0307	~	~	0.000	~

Table 8.9: Summary of P-Values for Final W.R. Tests

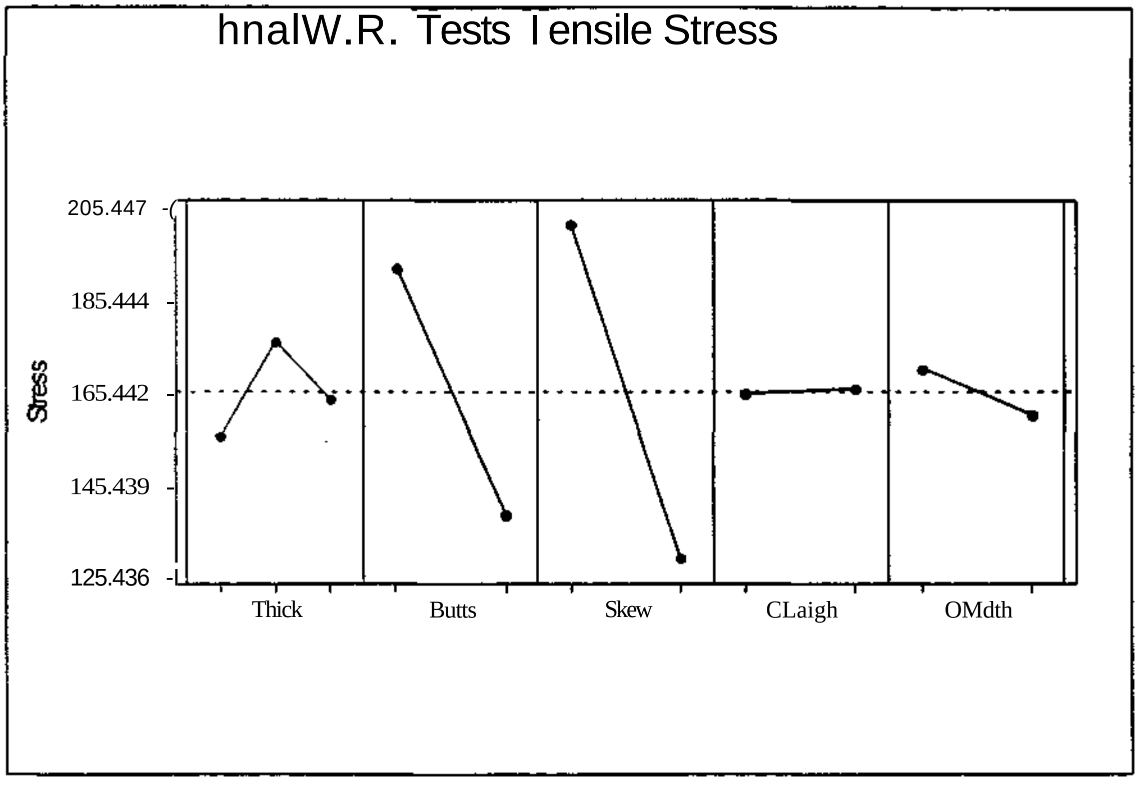


Figure 8.8: Tensile Stress Main Effects Plots

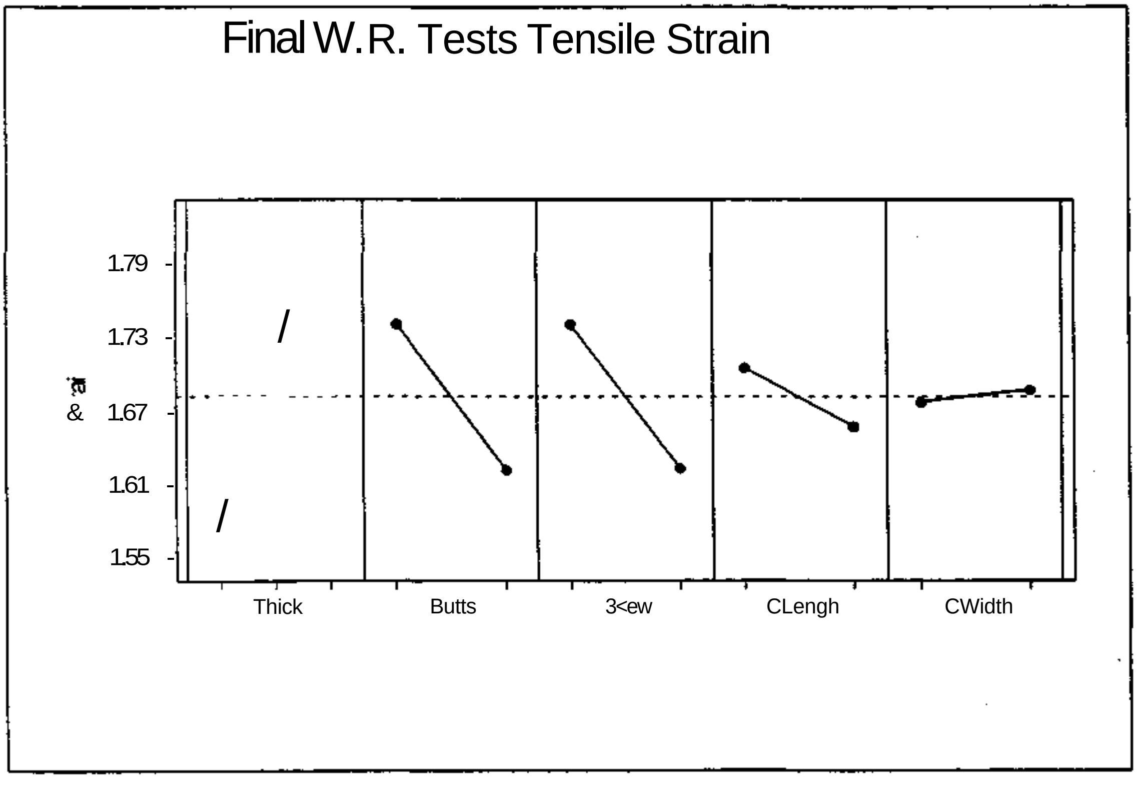


Figure 8.9: Tensile Strain Main Effects Plots

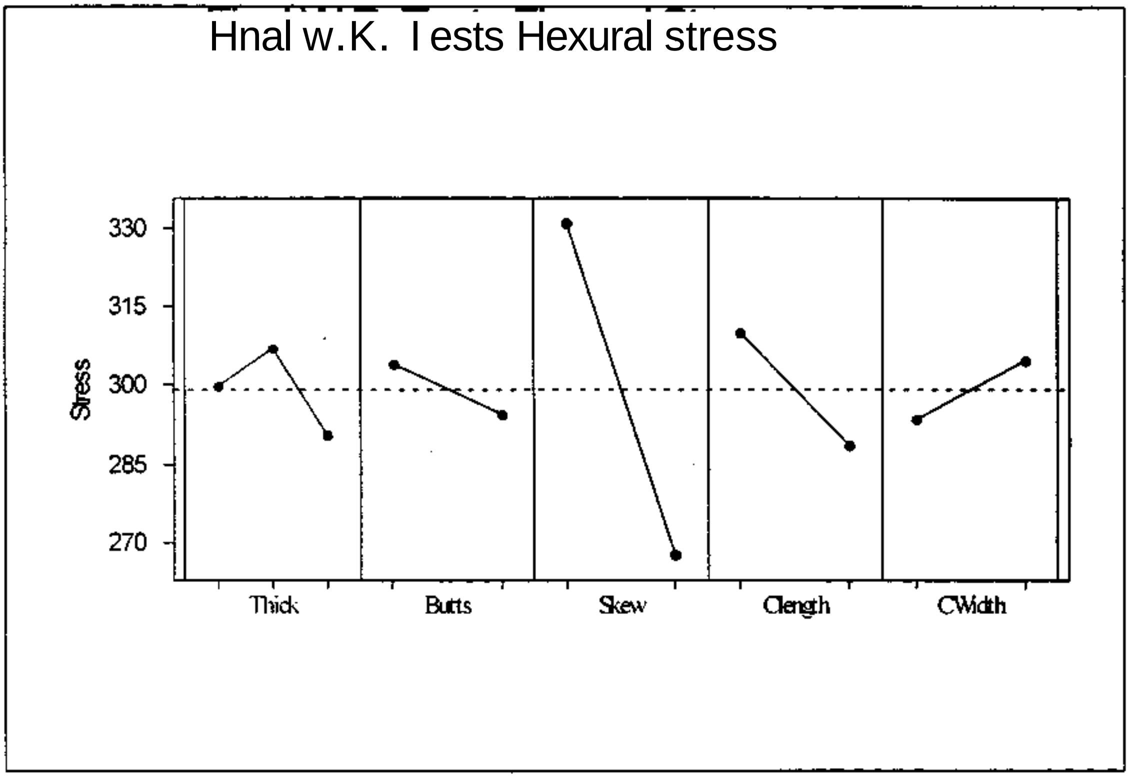


Figure 8.10: Flexural Stress Main Effects Plots

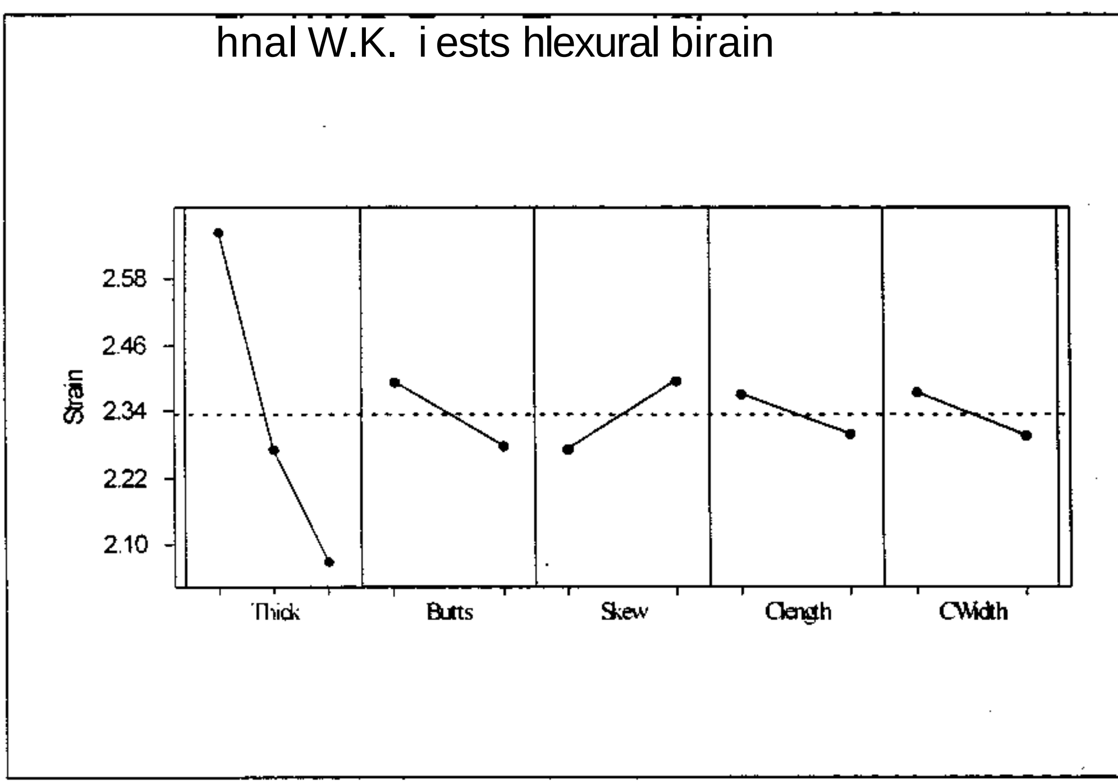


Figure 8.11: Flexural Strain Main Effects Plots

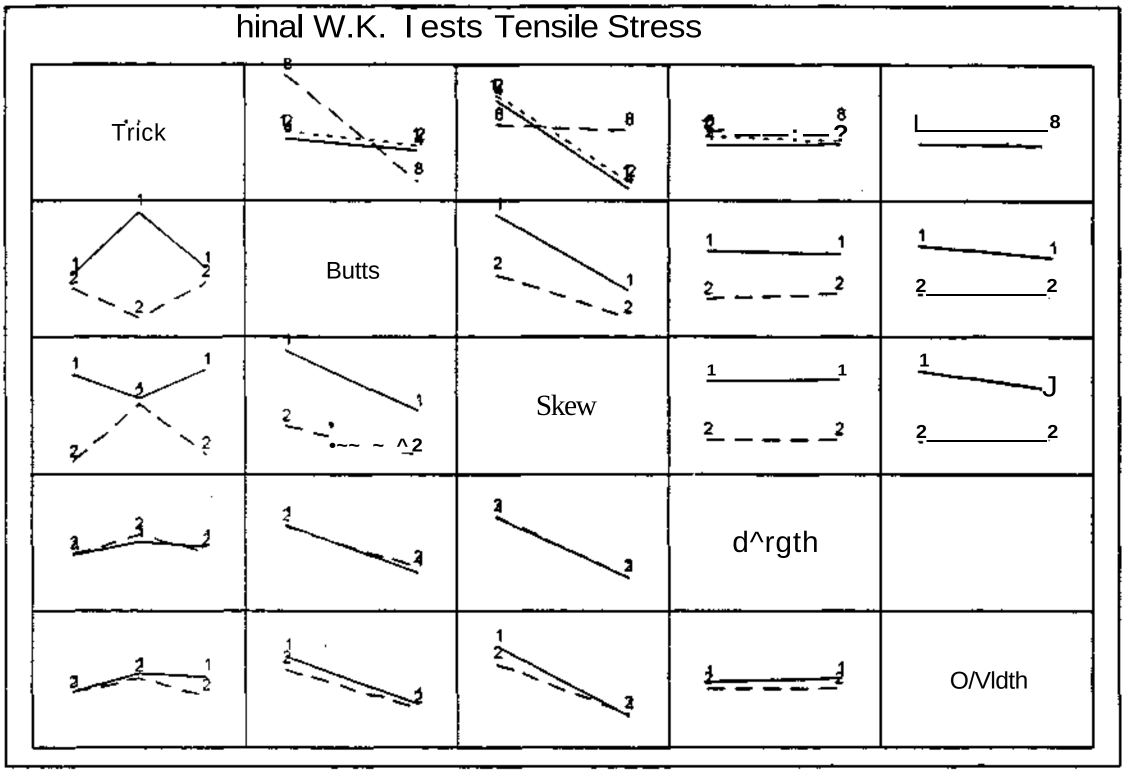


Figure 8.12: Tensile Stress Interactions Plots

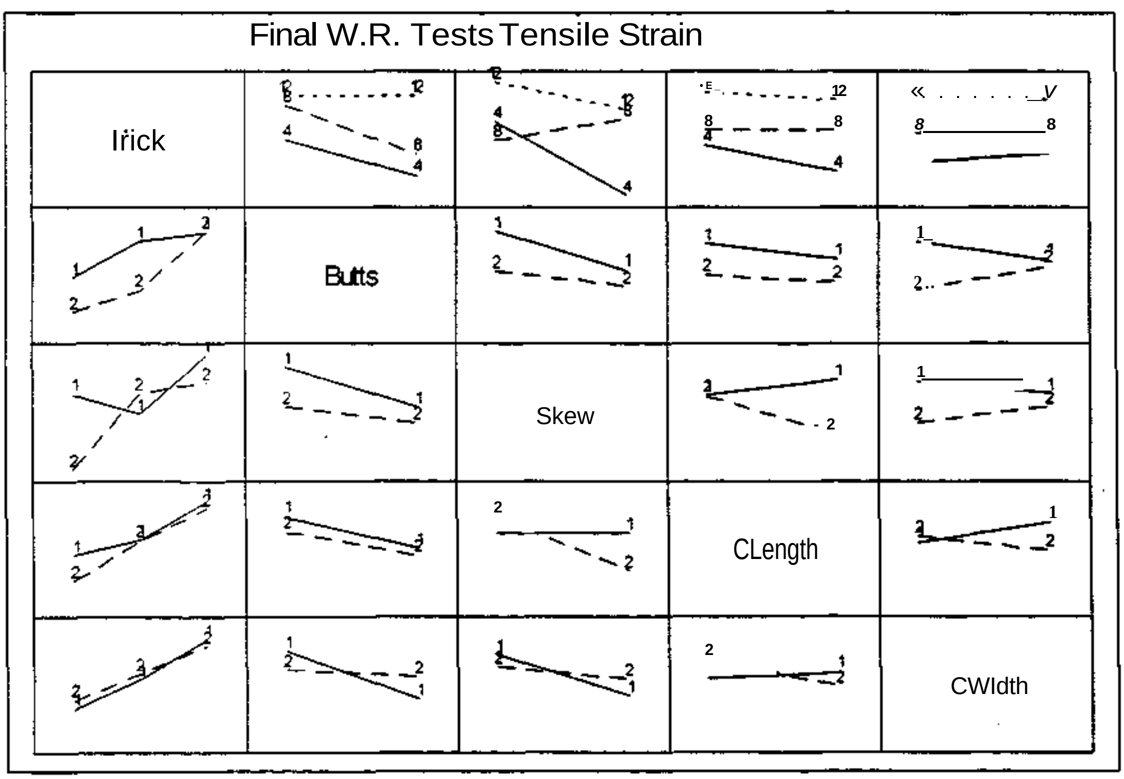


Figure 8.13: Tensile Strain Interactions Plots

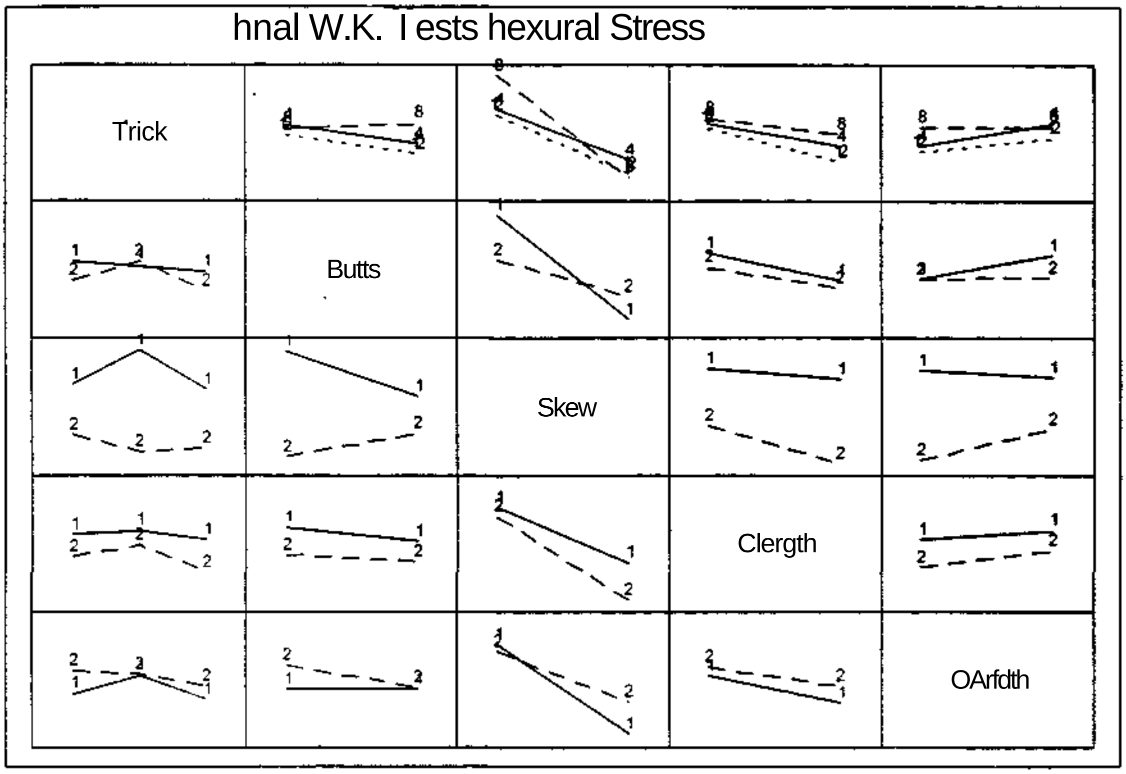


Figure 8.14: Flexural Stress Interactions Plots

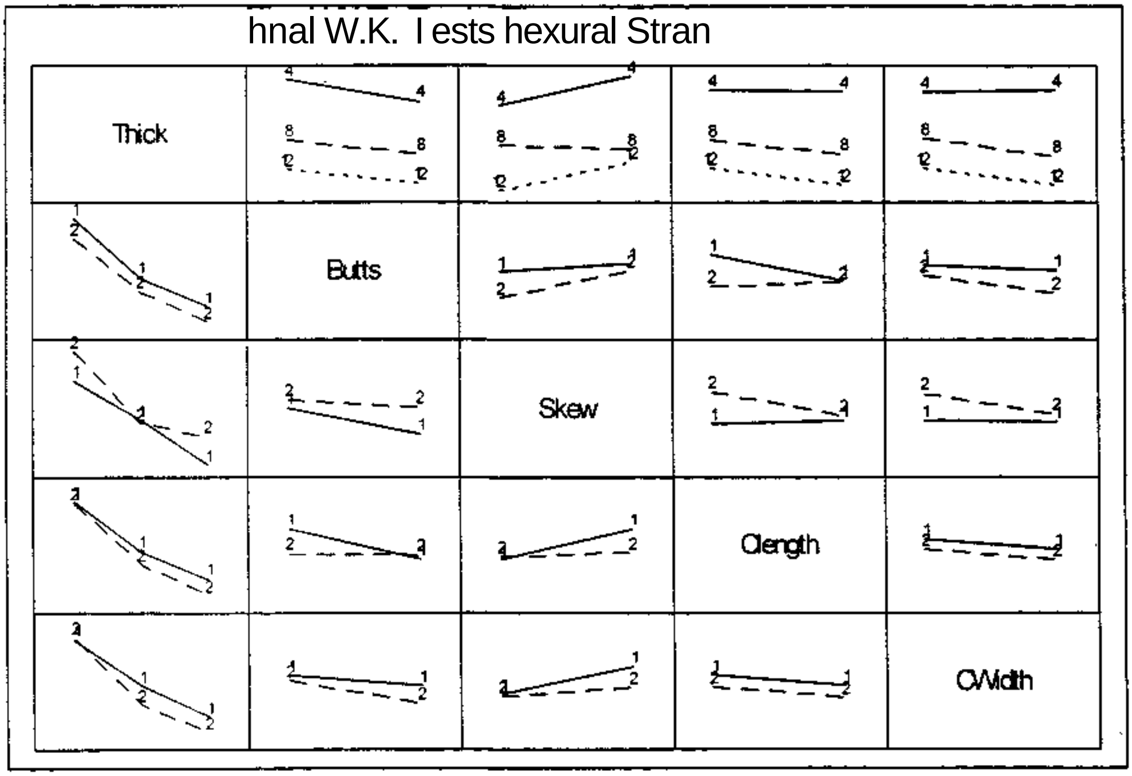


Figure 8.15: Flexural Strain Interactions Plots

There is also a strong linear effect of thickness on the flexural failure strain, but in this case Figure 8.11 shows this to be an decrease in strength with increasing thickness. Skew is seen to be important for the flexural stress measurements, again lowering the strength of the coupons (see Figure 8.10). However, when the initial modulus is included as a covariate in the model, the skew factor adds nothing extra to the fit of the model to the data.

Now the sub-plot main effects and interactions are considered. The tensile failure stress is seen from Figure 8.8 to be significantly reduced by an increase in the width of the coupons. Figure 8.12 shows this to be mainly due to the strong interaction between thickness and width, the large influence of width occurs only for the 12 ply coupons. There is a strong interaction between width and skew for tensile stress in Figure 8.12, the weakening effect of skew is greater for the narrower specimens.
The initial modulus is seen to be statistically significant at a 3% level for tensile failure stress (see Table 8.9). and hence is included in the model.

Plots of the four statistically significant two-way interactions for the tensile strain measurements are shown in Figure 8.13, together with the other interaction plots. The length and width interaction shows a greater effect of length for wider specimens. For the specimens without butts, the wider specimens are marginally weaker than the narrower specimens. However, for the specimens with butts, the wider specimens are marginally stronger than the narrower specimens. For the longer specimens, skewing of the warp and weft decreases the tensile failure strain, but for the shorter specimens, little influence of skew on strain is seen. The reduction in tensile failure strain caused by the skewing of the warp and weft fibres is greatest for the narrower specimens

For the flexural failure stress analysis (see Table 8.9), the initial modulus is seen to be statistically highly significant, with a P-value of less than 0.001, and is therefore included in the model.

A weakly significant decrease in flexural failure stress with increasing length (P-value 0.056) is seen in Figure 8.10. but is no longer important when the initial modulus is included in the statistical model as a covariate. A significant interaction between width and skew is seen in Figure 8.14 for ilexural failure stress, of the same form as that seen for tensile failure stress and strain (Figure 8.12 and Figure 8.13), but is no longer important when modulus is included in the model as a covariate. Conversely, the flexural stress interaction between butts and width is significant only when the modulus covariate has been included in the model.

None of the sub-plot effects or interactions for the flexural failure strain in Table 8.9 are statistically significant at a 5% level for this study.

8.9 Engineering Interpretation

The high R-squared value of 95% obtained for the tensile failure stress analysis shows that the model proposed accounts for most of the experimental variation. It also indicates that the ultimate tensile stress is a pertinent measurement of the tensile strength of these woven roving composites. The R-squared value for the ultimate tensile strains (71%) suggests that the use of an extensometer provides a reasonable measure of this material property.

The flexural failure stress values were, by necessity, obtained indirectly via a calculation involving the failure load and test dimensions. Also, an approximation was made to allow for the consequences of the central deflections of the specimens seen. Both of these procedures introduce errors into the stress values. This accounts for the fact that the fit of the model to the flexural stress values is lower than that for the tensile tests, although at 72% this is still reasonable.

The fit to the flexural failure strain data, collected using 10mm long foil type strain gauges, is not good. This is because localised strain variations in the coupon may affect the readings due to the positioning of the gauges with respect to the relatively coarse weave. This is exacerbated by the presence of butts. Also, for the skewed specimens, the strain gauge was aligned with the principal direction of the specimens, which is not aligned with the warp.

The higher experimental error variation in the data for the 12-ply coupons without skew could be due to the differences in failure mode, and this cange in variation could be investigated using further statistical techniques (see Chapter 11).

The whole-plot coefficients of variation show that the experimental error variation between panels is important, especially for the flexural tests. This indicates that variability in the manufacture of the specimens is influential. The compressive failure of the flexural specimens would be more sensitive to such variability. The sub-plot values are lower, again especially for the flexural tests, implying that the variation in specimen strengths from the same panel is less than that of specimens from different panels.

The significance of the modulus covariate for the failure stresses shows that this material property may give an indication of the 'quality' of the laminate, as described in section 7.9. For the flexural tests, effects concerning skew become much less significant when initial modulus is included in the statistical model. This is because the skewing of the warp directly affects the stifihess of the specimen.

The statistically significant whole-plot main and interaction effects are now considered. The strength of the tensile test specimens is lowered for both stress and strain by the skewing of the warp. This is due to the observed change in failure mode from fibre-dominated tensile failure to one of matrix and fibre-matrix interface dominated shear failure.

The thicker tensile specimens have a higher strain to failure, and Figure 8.13 shows this to be mainly due to the behaviour of the skewed specimens. The shear failure of the skewed tensile coupons is possibly more damage tolerant for thicker specimens, producing more matrix yielding before final failure.

The lower tensile failure stress and strain for the coupons containing butts is due to the discontinuity of a quarter of the reinforcing plies. The interaction between the butt and skew factors for both tensile failure measurements shows that the effects of these two factors are not additive, the weakest failure mode is due to the skew and is the dominant of the two. The indication, from Figure 8.13, that the reduction in the tensile failure strain with the inclusion of butts is greatest for 8 ply specimens (and not the 4 or 12 ply specimens) suggests that this interaction effect between thickness and butts is not directly related to the thickness, per se. It is more probable that the butts in the 8 ply panels were of a lower quality than those in the 4 or 12 ply equivalents.

Similarly, the interaction between thickness and skew for both failure criteria is thought to be due to a lower skew angle in the 8 ply specimens. This is not unfeasible since this factor is difficult to quantify and control, both between panels and within panels.

The reduction of flexural failure stress with the skewing of the warp is also due to the change to a mixed failure mode including shear as well as compressive failure. The flexural failure strain is lower for a thicker laminate. An explanation for this trend is that a lower quality specimen is obtained when a thicker laminate is produced. A plausible reason for this would be that, when there are more wet, and hence unstable, plies, the fibres of the top lamina are more difficult to keep straight as they are wetted-out. Compressive failure of the flexural specimens was initiated in the top laminate and since fibre waviness is critical for compressive failure, this would explain why the thicker laminates were weaker. The apparent contradiction of this trend to that seen in the tensile tests is due to the different failure modes in each case.

The dominant sub-plot main and interaction effects are now considered. The effect of width on the tensile failure stress is mainly due to a reduction in strength with increasing width for the 12 ply specimens only. The width of the wider 12-ply coupons is close to that of the jaws, and hence the possibility of the edge of the jaws affecting the failure is greatest here.

For both tensile failure stress and strain, the weakening effect of skew is greatest for the narrow specimens. For a given angle of skew, the wider a coupon, the greater number of warp fibres which will traverse the whole coupon, leading to a reduction in strength. Similarly, the longer a coupon, the greater number of warp fibres which will traverse the width of the coupon, leading to a reduction in strength and explaining the similar interaction of skew with length. Extending this concept, the interaction between length and width may be understood. There will be a small number of shear failure planes in the wide, short specimens compared to the other three plan shapes, leading to a greater effect of length for the wider coupons.

The effect of the presence of butts on the tensile failure strain is greater for the narrower specimens. The stress concentrations at the edges of the butt would affect a greater proportion of the coupon width for the narrow specimens. This trend is also weakly significant for the tensile failure stress.

The weak effect of length on the flexural failure stress could be a purely statistical occurrence. Alternatively, for the skewed specimens, the longer specimens would have more warp fibres crossing the width of the specimen, thus reducing the strength. Similarly the flexural stress interaction between skew and width can be explained as described above for the tensile specimens.

The strengthening effect on the flexural failure stress of an increase in width seen for the specimens with no butts is removed by the inclusion of such joins. This could be because the butts provide an initiation site for compressive delamination which is independent of the coupon width.

8.10 TensHe Geometry Tests

In order to transfer the force required to give failure of the tensile specimens, the coupons must be gripped at either end. This may lead to damage at the 'gripping jaws' and possible premature failure. The two most common methods of reducing this effect are the bonding of protective 'tabs' to the coupon where it will be gripped, and 'waisting' the central section of the coupon. Details of the use of these methods are given in the relevant ASTM (ASTM (1989)) and BSI (BSI (1976)) standards. However, these methods have their own disadvantages. For example, ensuring a strong enough adhesion between coupon and tab may be difficult, and the waisting of fibre reinforced composite specimens may lead to fibre damage worse than that which may be induced at the jaws. Hence, simple, and thus far more economical, rectangular specimens, such as those used in this study, are often used.

To investigate the relative merits of the alternative tensile specimen types to those used here, a small comparative experiment was completed. The ASTM (ASTM (1989)) and BSI (BSI (1976)) tensile test standards for composite materials were used to design the rectangular, tabbed and waisted coupons shown in Figure 8.16. In order to give an estimate of the variation in properties estimated using the different coupons, 6 of each type were manufactured.

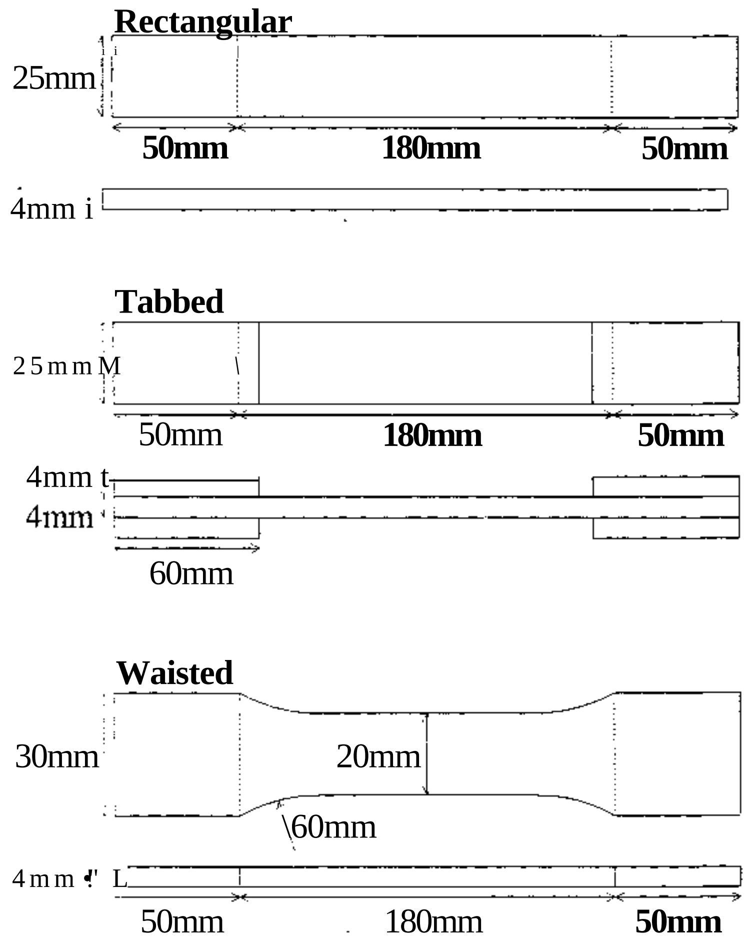


Figure 8.16: Tensile Geometry Test Coupons

In this figure the dotted lines indicate the position of the end of the gripping jaws. The maximum shear stress in the coupon / tab interface of the tabbed specimens was estimated to be 10 MPa. The lowest lap-joint shear strength for Areldite 2011 quoted by the manufacturers was 17 MPa, and so this adhesive was used to bond on the tabs.

The test procedure described in Section 8.4 was used, with an extensometer again measuring the tensile strain. The data from these tests are given in Appendix I, and summarised in Figure 8.17, Figure 8.18, Figure 8.19 and Table 8.10.

A statistical analysis comparing the means and variances of the initial moduli, failure stresses and failure strains was completed. The results of this analysis are included in Appendix J (Tables J. 15 and J. 16 and Figures J.I, J.2 and J.3), and the P-values obtained are summarised in Table 8.11. A low P-value here indicates a large difference in mean or variance. At the 5% level, the only significant difference is that the mean failure strain of the rectangular specimens is lower than that of the other two specimen types.

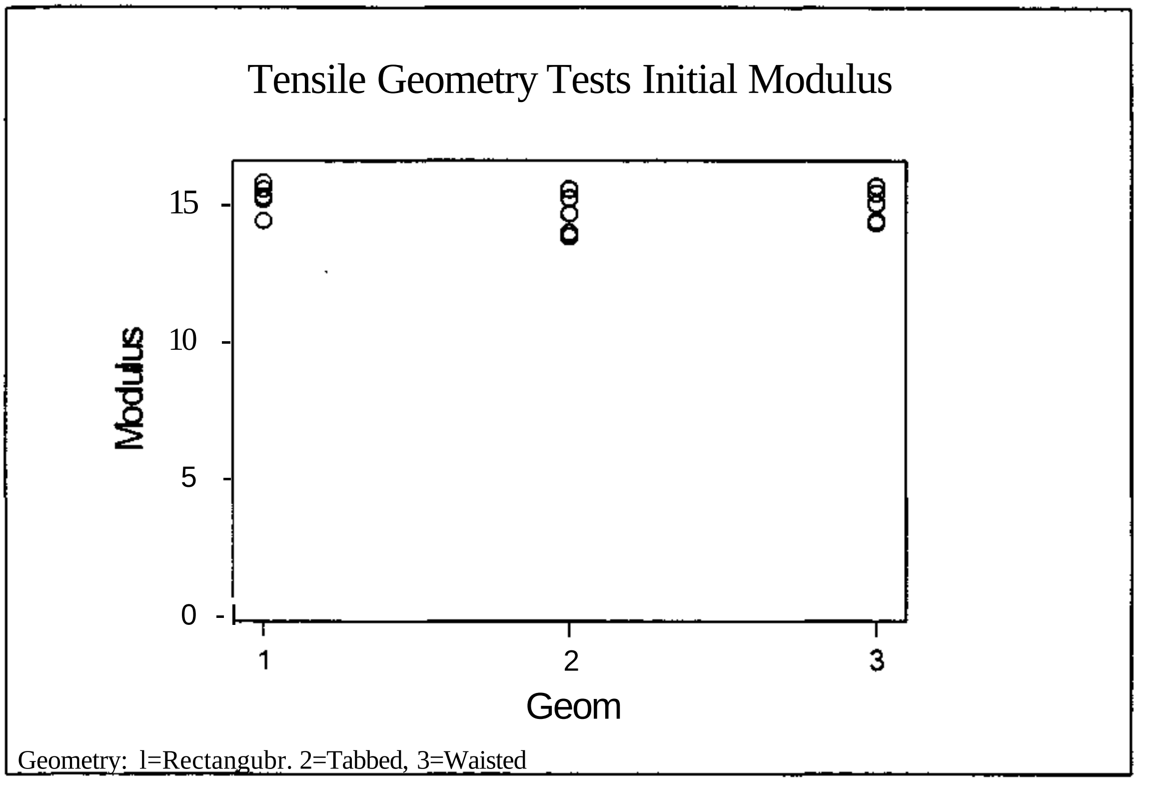


Figure 8.17: Variation of Initial Modulus with Specimen Type

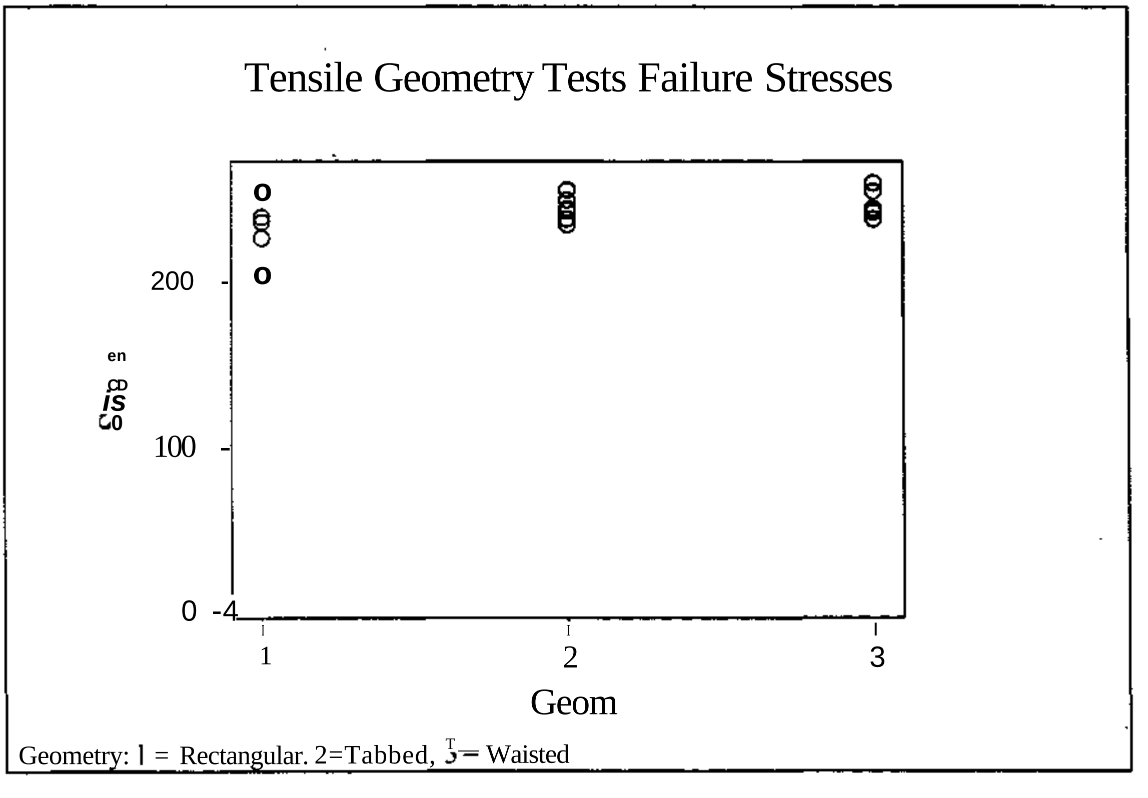


Figure 8.18: Variation of Failure Stress with Specimen Type

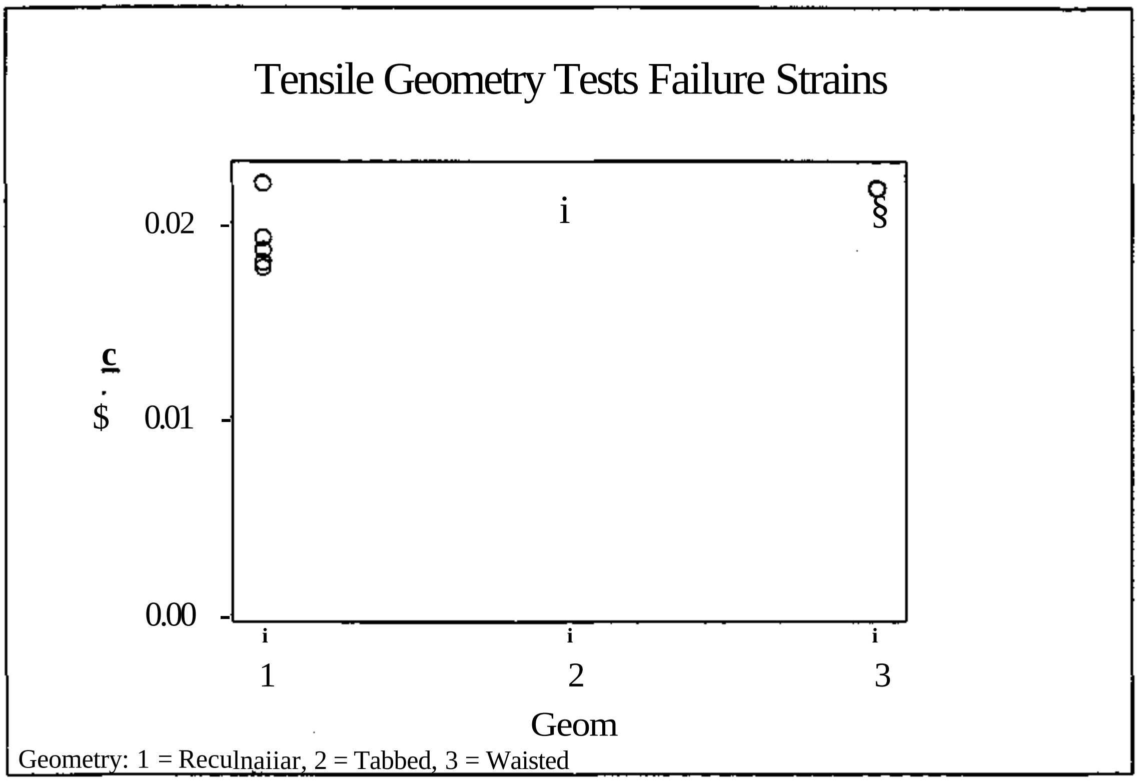


Figure **8.19:** Variation of Failure Strain with Specimen Type

Variable	Geometry	Mean	StDev
Modulus	Rectangular	15.128	0.484
	Tabbed	14.668	0.765
	Waisted	14.848	0.524
Stress	Rectangular	234.83	16.56
	Tabbed	244.58	7.49
	Waisted	247.37	8.30
Strain	Rectangular	0.01922	0.00153
	Tabbed	0.02144	0.00037
	Waisted	0.02086	0.00073

Table 8.10: Tensile Test Geometry Results

Statistical Test	Modulus	Stress	Strain
Equal Means:			
Rectangular vs. Tabs	0.25	0.24	0.018
Rectangular vs. Waisted	0.36	0.14	0.05
Tabs vs. Waisted	0.65	0.56	0.12
Equal Variances:			
Any combination	0.213	0.442	0.221

Table 8.11: P-values for Identical Means

8.11 Interlaminar Shear Strength Tests

The lengths of the short flexural specimens tested were set, according to the guidelines in the ASTM standard (ASTM (1986)), in order to avoid the influence of shear. If the length to depth ratio of a four point bending specimen is too low, then interlaminar shear failure may occur. To check that this was not the case here, a series of short beam shear tests was conducted to estimate the interlaminar shear strength (ILSS) of the composite material. The coupons were designed using the ASTM standard for short beam shear tests (ASTM (1985b)). Twelve 12-ply, 11mm thick specimens were tested in three point bending at a support length of 50mm, giving a length to depth ratio of approximately 4.5. The widths of the specimens were 10, 25 and 30mm, and four coupons of each width were tested. The data obtained from these tests are given in Appendix I.

The average ILSS measured was approximately 35 MPa, with a standard deviation of 2.70 MPa. Using this value, even for the shortest flexural specimens, the maximum shear stress at failure was less than half this value of ILSS. Hence it is concluded that the effects of shear were negligible.

9. Discussion

The composites strengths measured in this study are first compared with published values in order to validate the experimental data. The experimental trends and effects observed and the physical explanations for this behaviour are then discussed with respect to **the** question of size effects in marine composites. Finally, the experience gained in the application of statistically designed experimentation techniques to destructive fibre reinforced composite testing is considered.

9.1 Strength Data Comparisons

Initially comparisons are made between the experimental strength values determined in this study and published data. The ranges of properties quoted by Hancox and Mayer (1993) for unidirectional composites are for a fibre volume fraction of 0.6. These have been adjusted using the rule of mixtures (Smith (1990)), assuming the fibre strength to be much greater than that of the matrix, to calculate the values corresponding to a fibre volume fraction of 0.4 given in Table 9.1. Here it should be noted that the rule of mixtures is not accurate for strength values as it is for stiffness predictions, but it was thought that, for comparative purposes, its use was justified. The corresponding average experimental values obtained in this study can be found in the tables of means and standard errors of Appendix E, and are also included in Table 9.1.

	Unidirectional E-glass / Epoxy		**Unidirectional H.S. Carbon / Epoxy**	
Property	**Experiment**	**Published**	**Experimentt**	**Published**
Tensile Failure Stress (MPa)	574	800-1060	910	1000-1420
Tensile Failure Strain (%)	2.04	1.85*	1.01	0.75 - 0.85
Flexural Failure Stress (MPa)	740	800	796	1130-1195
Flexural Failure Strain (%)	2.50	Not Found	0.88	(0.8)**

* From Daniel and Ishai (1994)
** Value not found; compressive failure strain given.

Table 9.1: Comparison of Experimental and Published Data

Comparing the experimental and published values, the experimental values are generally lower than those published. This may be explained through the consideration of three points. Firstly, the published data is not that for exactly the same fibre / matrix systems as used in this study. This is important, as can be seen from the large ranges of published data in Table

9.1 which encompass the properties of equivalent material systems produced by different manufacturers. Secondly, the fibre volume fraction of the published data is 60%, not 40% as for the experimental data. Although the rule of mixtures has been used to try to compensate for this fact, the method is known not to be accurate for predictions of strength. Finally, the published data generally concerns pre-preg materials which will have sustained less damage to the fibres, because there is no need for wetting out by hand, and hence are liable to be stronger. This last point can be extended to include any other differences between the production methods used.

The average strengths for laminates fabricated using the three woven rovings supplied by different manufacturers obtained in this study can be found in the tables of means in Appendix H. Again it must be stressed that the tensile strain values should be treated with caution. These are compared with the data from Vosper Thornycroft's (V.T.'s) study (Vosper Thornycroft (1996)) in Table 9.2 and in Figure 9.1. No strain measurements were made by Vosper Thornycroft.

W.R.	Source	cW,UK **(MPa)**	e,**,,,,., (%)	**Onex,oK** (MPa)	**8flex,ult (%)**
AUS8	Experiment	274	(1.77)	366	2.25
	V.T.	260	~	416	~
AUS11	Experiment	245	(1.85)	302	2.05
	V.T.	259	~	301	~
AUS12	Experiment	290	(1.73)	384	2.15
	V.T.	356	~	409	~
Average	Experiment	270	(1.78)	351	2.15
	V.T.	292	~	375	~

Table 9.2: Comparison of Experimenta1 and Published Data

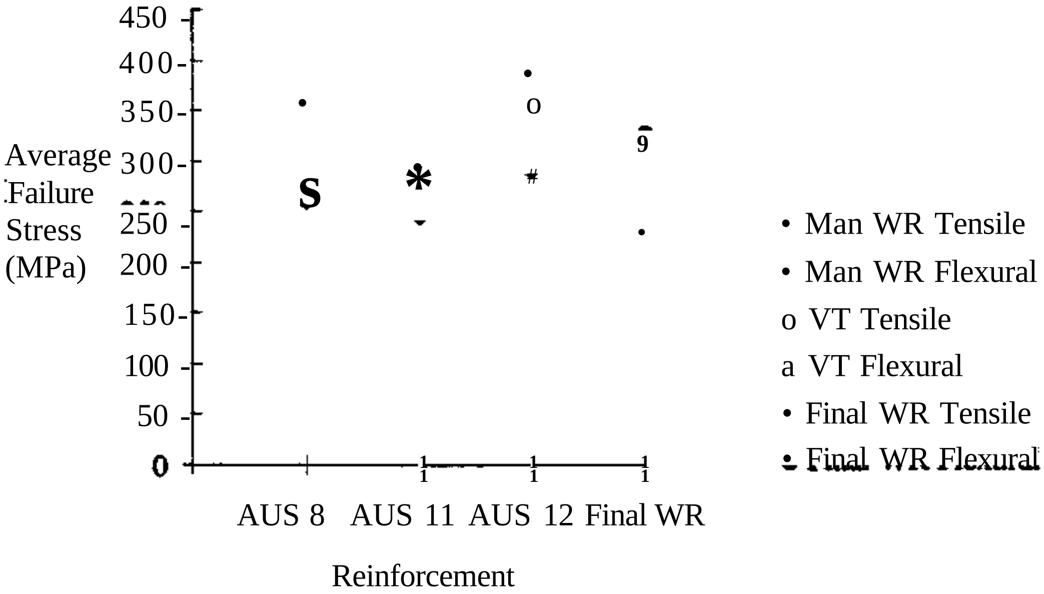


Figure 9.1 Comparison of Experimental and Published Data

Considering that the coefficients of variation of the tensile tests from both sources are between 5 to 8% and using the rule of thumb that a 95% confidence interval is approximately two standard deviations either side of the mean, the tensile failure stresses for the AUS 8 and AUS 11 W.R. specimens are comparable. However, the value obtained by Vosper Thornycroft for the AUS 12 W.R. specimens is substantially higher than that determined in this study, and is also much higher than their figures for the other two reinforcements.

Before comparing the flexural strengths, it is noted that this study utilised a one third span four point bending test geometry, whilst the Vosper Thornycroft tests were of the three point bending configuration. All three pairs of measured flexural failure stresses are comparable, the higher coefficients of variation in this case (10 to 15%) accounting for the relatively large discrepancy between the two AUS 8 strengths. Neither values for the flexural strength of the AUS 12 reinforced coupons are conspicuously high, casting doubt on the accuracy of the Vosper Thornycroft determined tensile failure stress of the same material.

The average strengths show the Vosper Thornycroft figures to be the larger for both tensile and flexural stress, though within the bounds of experimental variation. The data quoted by Smith (1990) originates from a variety of sources, and the associated material differences are reflected in the wide spread of strength values, as shown in Table 9.3. Both the experimental and the V.T. values fall close to the mean values of Smith's data.

	Tensile Strength (MPa)	Flexural Strength (MPa)
Minimum	250	150
Mean	300	325
Maximum	360	500

Table 9.3: W.R. E-Glass / Polyester Strengths (11)

The flexural strengths of the 'AUS' W.R. reinforced materials are approximately 30% greater than the tensile strengths for both data sets. This could be attributed to the differing stress distributions in the specimens as described by the weakest link theory (see Section 3.1). However, the lack of any evidence of a consistent size effect does not reinforce this explanation. The discrepancy is much more likely to be due to the differences in failure modes between the tests.

The average experimental strengths obtained for the final woven roving tests, taken from Appendix J, are summarised in Table 9.4.

	Tensile		Flexural	
	Stress (MPa)	Strain (%)	Stress (MPa)	Strain (%)
Orthog. / No Butts	228	1.82	326	2.36
Orthog. / Butts	164	1.65	290	2.18
Skew/ No Butts	149	1.65	289	2.42
Skew/ Butts	122	1.58	291	2.37

Table 9.4: Average Strengths

The only values comparable with the Vosper Thornycroft and the W.R. Manufacturer data are those for the orthogonal warp laminates with no butts. This comparison is shown in Figure 9.1 above. The flexural strength is within the range of flexural strengths for the similar W.R. manufacturer laminates obtained both by this study and by Vosper Thornycroft. The tensile strength is also comparable to the corresponding values for the W.R. manufacturer panels (except for the conspicuously high V.T. AUS 12 result), but is marginally lower than these values. This difference could be explained by the variation in the data of around 10%, but could also be due to the different failure mode exhibited by the final W.R. tensile specimens. This could be due to the different materials used for these specimens (see Appendix K), or the different cures used (see sections 7.3 and 8.3).

The investigation into the effect of the different tensile test coupon tests for W.R composites showed that there was little difference in the results obtained. The tensile failure strain of the rectangular specimens was found to be slightly lower than that of the tabbed specimens and only marginally lower than that of the waisted coupons. Little difference was seen for the modulus or failure stress measured by the three types of coupons. The variability in the stiffness and strength results was also very similar for each type of specimen used. This shows that the far less expensive rectangular specimens used in this study were sufficient.

The interlaminar shear stress of the W.R. composites obtained (35 MPa) compares well with published data, Hancox and Mayer, for example, also quote a figure of 35 MPa. The scatter in this property, with a coefficient of variation of approximately 8%, is again high. Calculations using the ILSS showed that the effect of shear in the flexural tests was minimal.

9.2 Marine Composites Size Effects

Previous work in the field of composites size effects has either made no statistical inferences about any apparent size effects, or has made simple statistical tests assuming the experimental variance to be constant throughout the experiment. Here the experimental error has been split into two pertinent parts, that due to the variation between the properties of separate panels (the 'whole-plot' variation), and that due to the variation between coupons cut from the same panel (the 'sub-plot' variation). For all three test series the variation between panel properties is seen to be important, and is generally greater than that within a panel. The implications of this are that the laminate properties are sensitive to variations at the fabrication stage, and hence that size effects due to the scale of production are important (see Section 3.3).

If this large panel-to-panel variation is not recognised, then the design of the experiment will not take this into account. Hence the effects of the factors which change between panels will be inseparable from this variation. This may lead to overestimation of these effects, and 'false' trends. Any predictions of the effects of size made from such deductions will thus be inaccurate or even unnecessary. In this study such between panel variation was recognised, and the final series of tests was designed to use this variation as part of the statistical analyses.

Although the unidirectional laminates were laid-up by hand and were of relatively low fibre volume fraction, these tests showed both lower experimental variation and better fits of the statistical models to the data than for the two woven roving test series. The unidirectional tests were designed to be a 'stop-gap' between the published work concerning aerospace quality laminates and the proposed work on marine type laminates. This shows that the question of size effects for such marine type materials is not the same as that already considered in the literature, and is complicated by the more variable quality of the laminates. The difference in experimental variation also reflects the differences between the laboratory and workshop production conditions and practices. Again this indicates that size effects due to the scale of production are important (see Section 3.3).

The trends seen are not always consistent between the tensile and flexural test methods used. This is because the different failure modes present in each method make direct comparisons meaningless. Even considering the same test method, different configurations, in this case flexural roller diameter, have lead to different material behaviour. Similarly, varying behaviour was seen for the different materials considered, even between the three nominally identical woven roving laminates as considered in the W.R. manufacturer tests. Extending this, different results are seen depending on the failure criteria considered (stress or strain in this case) and on the method of its measurement. Hence it is not sufficient to describe a 'composites size effect' or many other effects in such a generic manner; the behaviour may well depend upon the specific material, methods and failure criteria used.

The analysis of the data obtained using the different forms of failure strength measurement have given information on the most 'reliable' methods in different cases. For the unidirectional coupons the measurement of failure strain through the use of foil type gauges gives very consistent readings for both flexural and tensile methods. For the same materials the failure stress obtained through the maximum load was also very consistent for the tensile tests. However, the equivalent flexural measurement of failure stress required a less direct calculation and also an approximation due to the central deflection of the specimen. Hence, this measurement, although still a good indication of failure, was not as consistent as the others.

For the woven roving reinforced laminates the use of strain gauges was not as successful. Tensile measurements were poor due to localised strain effects, and the use of an extensometer was found to be more applicable. However, a robust construction of such an instrument is required if it is to survive the failures of the stronger specimens. The flexural strain gauges were also influenced by localised strain gradients. However, for the more uniform specimens without butts or skewed warp, the placement of these on the non-failure tensile face, avoided much of such local effects. This last point must be moderated by the fact that the gauges are not directly recording the compressive failure strain.

By far the best indication of the tensile failure of the W.R. laminates was given by the stress calculated from the failure load and coupon cross-sectional area. The equivalent flexural stress measurements were also the best option for these tests, although the less direct calculations and the central deflection approximation reduce the accuracy of this failure measurement.

One of the reasons for the greater variability of the strength of the W.R. reinforced materials considered, compared to those considered in the bulk of the literature, is that the production methods and the nature of the materials lead to laminates of more variable 'quality'. By this it is meant that 'defects' such as air bubbles ('voids'), mis-alignment of the fibres ('waviness' and 'skew') are more prevalent in such laminates, and, since they are not easily controlled, lead to greater variation in the material properties. These variables are often reflected in variations in the stiffness of the material, and the consideration of the initial modulus for the stress measurements, especially for the tensile tests, reduced such variation and improved the description of the data. Similarly, the consideration of the fibre volume fraction of the coupons also improved the description of the data, but only for the flexural W.R. manufacturer tests. These points again show the importance of keeping the fabrication methods at the test scale as similar to those at the ship scale as possible. It is also indicated that the consideration of the initial stiffness and perhaps the fibre volume fraction may be beneficial in this light.

The 'skew' and 'butts' factors considered in the final W.R. tests were designed to reproduce more macroscopic defects introduced in the laminating of an actual ship. These were found to be important, but the behaviour at the coupon scale was found to be dependent on other factors including the length and width of the specimen. This indicates that the influence of such features may be different at the coupon and ship scales, although the accuracy of such an extrapolation is obviously limited by the range of coupon sizes considered in this study. The strength variability of the laminates is also seen to be dependent on these factors. It has also been shown that they are difficult to control, and that, as described above, the initial stiffness of the specimens may be used to allow for variations to a certain extent.

Narrower specimens have been found to be influenced by the cutting of the fibre rovings at the edges. This is thought to be exacerbated by the skew or waviness of these rovings. This means that a suitable lower limit to the coupon will depend on both the width of the rovings and their alignment. It also means that whilst this may be important at the coupon scale, for a ship with few such edges it may not. This is another effect of the 'scale' of the material considered.

The use of statistical experimental design methods has enabled an investigation into the effects of a number of variables and, importantly, their joint effects. The influences of individual variables have been seen, in many cases, to be dependant upon others. This 'broader view" of the problem shows how consideration of individual variables in isolation can give different "slices' of the real behaviour of the system with dissimilar or even conflicting trends. This would explain the different conclusions reached in the literature concerning composites size effects; the different materials, conditions, test methods and specifications etc. lead to different results.

Although the unidirectional specimens of this study differ slightly from the majority of those in the literature, a trend reported in some of this literature was also seen here. A 'size effect', in the sense that the strength of the material varies with the dimensions of the composite coupon, was found to be distinguishable, but only for the thickness and length of the flexural test specimens. In Section 6.9 simple Weibull theory was applied to the mean strength values to give estimates of the Weibull modulus based on volume of between 12 and 18. These values are of a comparable order to those quoted in the literature of around 20, but are slightly lower, corresponding to a larger size effect.

However, the data are subject to considerable variation and the confidence limits to these values will be correspondingly wide. Also the presence of a linear trend is not certain, especially in the case of thickness for failure strain. This is due to the manufacturing related variation which is confounded with the thickness effect, and may be responsible for some or even all of the apparent effect of thickness. This is also seen in the different sections of the line in the length plots. The coefficients of variation as predicted by equation (3.14) using the Weibull moduli calculated do not agree with the sub-plot values. This could be because the coefficients of variation estimates do not allow for other sources of variation. The interaction of the tlexural failure strain length effect with width shows that the effects seen here are not as straightforward as is often assumed in the literature.

The alternative explanation offered for the effect of thickness is that thicker specimens are harder to lay-up by hand and hence are weaker. This is thought to be the main reason for the effect seen for the W.R. materials considered., since no corresponding effect of length or width is seen. This also corresponds to a 'size effect', but in the sense of 'scale of production' as described in section 3.3.

A small number of the interaction effects found to be statistically significant offer no immediate engineering interpretation. This could be due to the statistical nature of the data, no matter how statistically significant an effect is there is always a chance that it is in fact due to the experimental scatter. For the levels of confidence used this is unlikely, but not impossible. The study has achieved its aim of identifying the important factors, but in certain cases, closer inspection of specific areas may be required (see Chapter 11).

9.3 Experimental Design Experience

Experience in the application of statistically designed experimental design techniques to the destructive testing of fibre reinforced plastic composites has been gained. The effects of a number of variables on the failure stress and strain for flexural and tensile test methods have been investigated. The use of the statistical methods described in Chapters 5 and 6 enabled the important individual and interaction effects to be distinguished from the experimental error variation in a quantitative manner. This gives a broader picture of the system considered than would be obtained if each variable were considered in turn in separate experiments.

A number of trends have been indicated by the statistical analyses, which are often inter-dependent. This may make the physical interpretation of these results more complicated, but this is a better alternative than ignoring the interactions between variables simply because this makes the interpretation easier. It must be remembered that the statistical methods are used as a tool to identify engineering phenomena, and that the use of engineering knowledge to interpret statistical inferences is of primary importance.

The need to rigorously stick to an experimental plan once it has been designed may cause problems. It is difficult to persuade those with no previous experience of statistically designed experimentation that to change aspects of the system part way through the experiment will probably lead to the loss of useful information. This also constrains the experimenter's ability to overcome problems which may be experienced during testing

The "poor communication between statisticians and engineers" to quote Grove and Davis (1992), arising from different terminology and approaches, can also present problems. Engineers often shy away from the very word 'statistics' and very rarely use any such quantification of how their model fits the data. This is in part due to unwillingness on the part of the engineer, but is also attributable to the often rather abstract manner in which the statistical approaches are described. Few texts recognise that those who would benefit most from these methods, i.e. engineers and industrialists, do not generally have an in-depth knowledge of statistical methods and terminology. Statisticians may have a much better grasp of the intricacies of experimental design, but will probably not be familiar with the engineering realities of the problem at hand. This may lead to unrealistic experimental requirements, which could lead to unplanned deviations from the design due to the 'ingenuity' of the engineer. Obviously, sustained communication between statistician and engineer will alleviate these problems, but this process is made much easier if the areas of knowledge of both parties share common ground.

The use of the statistical methods used here allows the fit of a model to experimental data to be quantified. However, the appropriate analyses will depend on the structure of the experimental design and the assumptions made through engineering knowledge. Hence, care must be taken to ensure that any assumptions are pertinent and that the interpretation of the results of the statistics takes the design structure into account. The correct statistical analyses in this study were arrived at using an iterative process. Hypotheses were made, for example, does a covariate affect the response?, and then tested. The results of these tests were then used to further refine the statistical model. The use of the software packages 'SAS' and 'MINITAET made this process easier, but as with any computer package, this also makes the production of erroneous information easier. Care must be taken to check which methods have been used and which assumptions have been made in the calculations. However, once the analysis has been formalised, the statistics may be performed using different response variables with ease (provided that they are of the same type, i.e. continuous variables in this case).

Hence it is initially advisable to keep the experimental designs as simple as possible in order to keep the statistical analysis and interpretation correspondingly simple. This is especially true for the experimenter with limited knowledge of experimental design, if mistaken conclusions are not to be drawn from the data or important effects not identified. The use of graphical representations such as effects and interaction plots are useful to the engineer for interpreting the statistical results. However, the use of these initially followed by reference to the statistical results, as advocated by Grove and Davis (1992) is not always the most direct route to the identification of the important effects. If, as is the case for the unidirectional tests, one factor has a very strong effect on the response, then the software used will adjust the scales of the plots accordingly. This may mean that, compared to the large effect other, still strongly significant, effects would be overlooked in initial observations of the graphs. It would only be when the P-values are inspected that their importance would be recognised. Hence, the method used here was to consult the P-values first to identify the influential effects and then to use the plots to interpret them.

The large amount of scatter in the experimental data from composites testing means that introductory designs should be fairly 'conservative'. That is, the design should not try to obtain too much information about too many factors using too few experimental runs. The use of full factorial designs and coupon replications is advised. In addition to giving increased statistical precision, if two of each specimen are available, if the test of one is unsuccessful for some reason (which is not uncommon in composites testing), then there is another experimental result from which to estimate the missing response. There is relatively little extra work involved in fabricating a larger laminated panel and then cutting the duplicate specimens from it.

The use of more 'conservative' designs also reduces the depth of experimental design methods required. The more complex design used for the W.R. manufacturer tests required fairly involved statistical analyses, and the interpretation of the results of these analyses was not straightforward. For the engineer using such methods the possibility of mis-interpretation of the results is much greater if the analyses is complex. For the study of composite materials where the scatter in measured properties is high, the advantages of the use of more complex designs is outweighed by the correspondingly complex interpretation of the results.

The lamination process of fibre composites, and the batch effects associated with this, means that it is both easier and sensible to produce the coupons initially and all at once. However this means that if, on testing, if it is found that factor levels concerning the nature of some or all of these coupons must be changed, then the specimens may have to be remade. The batch effects may also require that all, and not just those affected, are remade. This can be time consuming and expensive, as was the case here when problems with the length to depth ratio of the unidirectional flexural specimens were encountered. Similarly, the fact that the tests are destructive means that the repetition of tests requires the re-fabrication of specimens, which can be time consuming, and again batch effects may become important.

An experimental technique often used is that of 'run-randomisation'. This requires that the experimental tests are run in a random order, and is designed to avoid serial effects on the response. For example, if the response is affected by the environmental temperature and all the tests with a factor at the low value are run on a cold day and all those at the high factor level on a subsequent hot day, then the factor may appear to be affecting the response when in fact it may not. However, the conditions under which these tests were well controlled and no such effects were thought to be important. Also, the test set up means that such a randomisation would entail adjustments to the rig such as the repositioning of the support and load rollers after most of the runs. This would lead to an unnecessarily large amount of work and would also lead to more sources of error.

The simultaneous variation of the length, width and thickness between specimens investigated here makes adherence to the testing standards (ASTM (1986), ASTM (1989)) impossible. For example, a specimen of given length and thickness may be as designated by a given test standard, but, if another specimen is produced with the same length but twice the thickness, then this second specimen will probably not adhere to the test standard. However, the fact that composite materials are a family of materials requires that the stipulations of the test standards are defined as bounds, rather than exact values. Hence the standards were used as a guide wherever possible. In fact, often when the standards were used as a guide, large adjustments to the specimen design would have to be made before the required behaviour of the material system considered here was obtained.

Physical restraints on the factor levels may impose on the design, limiting the range of factor levels available and leading to a more intricate design structure. For example, in this study the excessive tlexural deflections resulting from large length to depth ratios restricts the range of lengths available at a given structure and also means that the appropriate ranges of length are different at each thickness.

10. Conclusions

In this study the driving question has been, "how do we relate the material properties obtained using small scale coupons to those of the laminates in a full-scale ship". To answer this question in full would entail testing at both the coupon and ship scales. Obviously this was not an option for this study. Instead, the importance of factors thought to influence the laminate strength at both scales was investigated through a series of experiments at the coupon scale. The sources of the variability commonly seen in composite material properties were also investigated.

An objective approach has been taken to ascertain, within the limits of the size of the experiment, which variables do in fact influence the strength of the composites studied. This is an important point for the highly variable nature of composites strength data, where apparent effects may be due solely to this scatter. The statistical methods used enable the data set as a whole, and not just the repetitions of each coupon, to be used to decide which trends are in fact Meal". This is important as it greatly reduces the number of coupons which must be fabricated and then tested. The variation in the composite strengths has not only been allowed for and utilised, but the sources of such variability have also been investigated. The use of fairly simple experimental designs with adequate replication of specimens is advocated for composites testing. The large variability in the measured material properties of composites means that the use of such designs is advantageous.

The effects seen were not straightforward. Not only was there considerable interaction between some of the variables considered, but the trends observed were dependent on the test method, the test set-up, the materials considered and the measurements taken. Even the use of nominally identical, or very similar materials, gave differing results. Hence it is thought that the generic term 'composites size effects' is too generalised to be of any use.

An effect of length and thickness on the strength of the unidirectional flexural coupons was seen. These effects have been interpreted in terms of the simple Weibull theory used in the literature. This has shown that a 'size effect' may also exist for the unidirectional composites studied here, which are of a lower fibre volume fraction than those of the literature. However, it was also shown that other effects related to the manufacturing process are also important.

In terms of the impetus for this investigation, the term 'composites size effects' requires further definition. Discrepancies between the material properties obtained through small coupon scale tests and those present at the ship scale may differ because of two chief effects;

1. 'Size Effect': Due to the statistically distributed flaws present in the material, a larger body of material will be weaker than a smaller body.
2. 'Scale Effect': The manufacturing related variables and processes used are different, or have differing effects, on the material properties at the coupon and ship scales.

For the shipbuilding-grade, woven roving reinforced marine composites studied here, the effects of factors related to manufacturing were found to be important. These effects include variations between the properties of nominally identical panels, the use of nominally identical woven rovings from different manufacturers, warp-weft distortions and the inclusion of butts in the reinforcement layers. These effects were often seen to depend on other factors, including the size of the test coupon. In terms of an influence of 'scale', this shows that discrepancies between coupon and ship scale material properties are influenced by the effects of the 'scale" of production, as described in Section 3.3, and that the manufacturing processes used for the coupons should be as close to those used at the ship scale. The 'size effects' reported for the mainly aerospace grade composites of the literature were not observed for the W.R. composites studied here and often used in the marine industry. The unidirectional tests also showed that the variability in the marine composite properties was greater than that in the carefully prepared unidirectional composites often studied in the literature. In terms of an effect of "scale" this shows that the manufacturing processes used for the coupon should not only be as close lo. but should also be as variable as, those used at the ship scale. For the woven roving E-glass / polyester composite materials considered here and often used in the marine industry, the 'scale effect' is of far more concern than is the 'size effect'.

The use of experimental design methods has proven invaluable in investigating the question of 'size effects" in marine composites. The complicated nature of the problem has been illustrated, a number of important factors and their interaction have been identified, and a deeper understanding of the sources of the empirical variability has been obtained. In addition to this, information has been gained concerning the appropriate measurement techniques for different tests and materials, the influence of different tensile specimen geometries and the interlaminar shear strength of shipbuilding, woven roving laminates. The experimental programme completed was large in comparison to other studies, and in terms of the information gained, it was extremely efficient.

11. Further Work

The conclusions made from this study have further defined the significant areas of research relevant to future investigation into the behaviour of marine composites, and more specifically into the variation with scale of the material properties.

Information concerning the effect of the manufacture related variables, both on the material properties and on the variability of these properties, is severely lacking for marine type composites. This is especially relevant here where there is a high degree of variability in the measured properties, and difficulties in controlling the production process. Any discrepancies between coupon and ship scale material properties has been found to be due mainly because of differences in such variables and their effects at the two scales. Hence, identification of the relevant factors would enable care to be taken to keep such effects to a minimum, or to account for them. Identification of the relevant factors would enable effort to be focused in controlling these variables, reducing the variability in the measured properties and hence increasing confidence in the design calculations.

Experimental design techniques would again be highly beneficial for this type of study. An analogy can be drawn with chemical engineering production processes, where the use of such techniques has long been proven. An additional statistical technique, known as 'graphical modelling' would also enable closer inspection of the sources of the variability. Here, not only is the effect of each factor on the measured response estimated, but also the effects on the variability of this response. This method, however, requires fairly large data sets. The data set obtained through this study may be large in terms of composites testing, but from the point of view of a statistician, the amount of data limits the analyses possible. Hence, graphical modelling methods were researched here, but for the amount of data available from this study its use was not thought to be beneficial.

It would be a relatively simple task to fabricate and test the W.R. manufacturer coupons necessary to give all combinations of the factors. This would simplify the structure of the experimental design and hence the statistical analysis, increase the statistical precision of the analyses and hence the information available. Also, further analyses using the initial modulus and the fibre volume fractions as the measured response would give valuable data for no extra experimental effort, and would need very little modification to the statistical analyses used here.

Finally, further investigation is required to investigate the interpretation of a small number of relationships between the effects of some of the variables considered here.

Appendix A Buckingham Pi Theory

Consider the most general form of a physical equation with a number of physical quantities of n different kinds, with similar quantities denoted by their ratios to one of these similar quantities. Indicating each different kind of quantity as Q,....Q_n and each ratio of similar quantities as r', r'', etc. Gives the general physical equation,

$$f(Q_{,}, \ Q_2, \ .... \ Q_H, \ r\backslash \ r'') = 0 \qquad (A.I)$$

Assuming, initially, that the ratios r are constant for the area of application of the equation enables equation (A.I) to be written as,

$$\boldsymbol{f(Q_{,}, Q_3, ... Q,J = o} \qquad \textbf{(A.2)}$$

If no quantities have been omitted then the equation will be a complete one, i.e. its coefficients will be dimensionless numbers that will not depend upon the units chosen to measure the quantities Q. Equation (A.2) may be expressed in the more specific form,

IM g*'; £# .... $Qfc = 0$ (A.3)

A physical equation must be dimensionally homogonous and so dividing equation (A.3) through by any single term gives,

$$ZN \ Q\backslash > \ Q^1; \ .... \ Ql: \ + 1 = 0 \qquad (A\text{-}^4)$$

where the N's are dimensionless numbers.

Since equation (A.4) must show dimensional homogeneity the exponents a,, a_2,.... a„ must be such that each term is dimensionless, i.e.

$$[QT \ \ QV \ \bullet\bullet\bullet\bullet \ \ Qh''J \ = \ [I] \ \ (for\ each\ term) \qquad (A.5)$$

We now define the Pi terms as dimensionless products of the form,

$$n \ = \ Q\backslash' \ \ Q?.... \ \ Qi; \qquad (A\text{-}^6)$$

Hence equation (A.4) may be written as,

$$\Sigma N \Pi \ +1 \ = \ 0 \qquad (A.7)$$

Since n is dimensionless. so too is IT , and by the same reasoning so is a product of the form El,"[1]]!/[2] ....ni" . Hence if II, , n$_2$, ... Oi are all the possible *separate independent* dimensionless Pi terms then (A.7) may be written as,

$$\Sigma N \; \Pi vm,.... \; nr + i = o \qquad \textbf{(A.8)}$$

Here the N merely represents some unknown function of the independent Pi terms and equation (A.8) may be simplified to,

$$\psi(\Pi/.rb, \; -.W = o \qquad (A.9)$$

The value of i is the maximum number of separate independent Pi terms that may be made by combining the n quantities Q, , Q_2,.... Q_n in different ways.

Let k be the number of primary quantities required to define the system. These primary quantities may be the conventional 'fundamental dimensions' (e.g. M, L, T) but can also be any k of the quantities Q which contain all k 'fundamental dimensions' between them. The remaining (n-k) units may be derived from this set of k primary quantities and on inspection it can be seen that the set of equations (A.5) are in reality equivalent to these (n-k) equations of derivation.

Hence. $$i = n-k \qquad (A.10)$$

Furthermore, if [Q,],[Q_2], •••• [QJ are k of the n quantities which have been taken as primary, then the i equations (A.5) may be written as,

$$[n,] = [QV \; Q^{p}{}_2'.... \; QVPi] = [i] \qquad (A-n)$$

$$fr_3] = [Q"r \; `Q^{p}y` .... \; QV?_2] = [I]$$

. . . .

$$[nj = [QT \; Q^{l}l> .... \; Q1PJ - [IJ$$

Where the P's represent Q_{k}, ... Q_n (the secondary quantities).

Any one of the i equations (A.I 1) may be utilised through substitution of the k conventional primary quantities (M. L. T etc.) for the Q and P terms, and since the exponents of each dimension must be equal to zero this gives us k equations in k unknowns, and hence the Pi term in question. This can then be repeated to give all i Pi terms. It should be noted here that although there are only i separate independent Pi terms they are by no means unique and different sets may be derived through different choices of the k primary quantities.

Reintroducing the ratios of similar quantities, r', r'', etc. to give the general form of equation (A.9) gives,

$$\psi(\Pi i,\ n_3,\ ....\ Tli,\ r',\ r''....) = 0 \qquad (A.12)$$

As before if [Q,].[Q_2],.... [Q_k] are any k independent quantities taken as primary and the remaining units are denoted by [P,],[P_2],.... [P,], each of the i Pi terms may be found from equation (A. 13) through substitution of the primary quantities for the [Q] and [P] terms

$$[\Pi] = [Q_1^{\alpha}\ Q_2^{\beta}\ ....\ Q_k^{\kappa} P] = [1] \qquad (A.13)$$

Appendix B Unidirectional Test Data

Specimen Code	Length (mm)	**Width (mm)**	**Depth (mm)**	**Initial Modulus (GPa)**	**Deflection at Failure (mm)**	**Failure Stress (MPa)**	**Failure Strain (%)**
UNIT/3G/1A	52.5	3.69	1.53	25.4	3.2	549	2.180
UNIT/3G/1B	52.5	3.69	1.53	25.4	3.2	549	2.180
UNIT/3G/2A	52.5	7.49	1.53	25.5	3.0	508	2.080
UNIT/3G/2B	52.5	7.49	1.52	24.9	2.9	492	1.980
UNIT/3G/3A	105	3.71	1.42	29.2	4.3	513	1.910
UNIT/3G/3B	105	3.71	1.39	27.5	3.5	528	1.900
UNIT/3G/4A	105	7.49	1.41	26.5	4.3	580	2.190
UNIT/3G/4B	105	7.50	1.40	26.7	3.9	543	2.050
UNIT/3C/1A	52.5	3.73	1.26	82.1	2.2	793	0.960
UNIT/3C/1B	52.5	3.71	1.26	85.1	2.3	824	0.960
UNIT/3C/2A	52.5	7.49	1.26	88.3	3.0	898	1.030
UNIT/3C/2B	52.5	7.46	1.25	84.0	2.6	901	1.080
UNIT/3C/3A	105	3.76	1.33	84.0	2.9	800	0.950
UNIT/3C/3B	105	3.75	1.33	92.2	3.4	842	0.930
UNIT/3C/4A	105	7.50	1.34	79.3	3.9	983	1.240
UNIT/3C/4B	105	7.52	1.34	85.2	3.4	908	1.080
UNIT/5G/1A	87.5	6.18	2.19	28.2	4.8	606	2.200
UNIT/5G/1B	87.5	6.25	2.14	29.5	4.5	598	2.075
UNIT/5G/2A	87.5	12.51	2.20	32.7	5.1	563	1.750
UNIT/5G/2B	87.5	12.36	2.23	29.5	5.6	617	2.160
UNIT/5G/3A	175	6.25	2.18	32.4	7.2	633	2.190
UNIT/5G/3B	175	6.21	2.19	34.6	6.8	651	1.925
UNIT/5G/4A	175	12.47	2.25	31.5	7.0	624	1.960
UNIT/5G/4B	175	12.49	2.21	31.6	7.2	629	2.050
UNIT/5C/1A	87.5	6.26	2.12	93.1	4.3	957	1.038
UNIT/5C/1B	87.5	6.27	2.12	91.4	4.2	948	1.038
UNIT/5C/2A	87.5	12.44	2.06	94.8	4.9	956	1.000
UNIT/5C/2B	87.5	12.42	2.10	97.8	5.3	896	1.000
UNIT/5C/3A	175	6.20	2.09	90.8	4.6	918	1.000
UNIT/5C/3B	175	6.18	2.08	95.3	4.6	918	0.968
UNIT/5C/4A	175	12.48	2.11	94.6	5.4	883	0.941
UNIT/5C/4B	175	12.49	2.13	100.4	5.2	926	0.928

Specimen Code	Length (mm)	Width (mm)	Depth (mm)	Initial Modulus (GPa)	Deflection at Failure (mm)	Failure Stress (MPa)	Failure Strain (%)
UNIT/7G/1A	122.5	8.51	3.18	28.3	6.5	527	1.990
UNIT/7G/1B	122.5	8.59	3.10	28.5	6.7	548	2.000
UNIT/7G/2A	122.5	17.46	3.09	28.9	6.7	558	1.950
UNIT/7G/2B	122.5	17.41	3.12	29.5	7.1	585	2.050
UNIT/7G/3A	245	8.66	3.07	30.1	8.6	587	1.950
UNIT/7G/3B	245	8.70	3.09	29.3	8.6	595	2.075
UNIT/7G/4A	245	17.48	3.10	29.5	9.6	600	2.025
UNIT/7G/4B	245	17.44	3.12	29.2	9.6	597	2.050
UNIT/7C/1A	122.5	8.71	3.00	94.5	5.0	918	0.970
UNIT/7C/1B	122.5	8.59	2.98	94.3	5.2	938	1.000
UNIT/7C/2A	122.5	17.29	2.98	90.8	6.1	915	0.988
UNIT/7C/2B	122.5	17.48	3.02	91.7	6.4	945	1.025
UNIT/7C/3A	245	8.68	2.98	100.7	6.4	935	0.925
UNIT/7C/3B	245	8.72	2.97	93.4	6.7	956	1.025
UNIT/7C/4A	245	17.41	2.97	89.5	7.6	938	1.050
UNIT/7C/4B	245	17.45	2.99	91.8	7.6	963	1.050

Table B.I: Uni-Directional Tensile Tests Results

Specimen Code	Length (mm)	Width (mm)	Depth (mm)	Roller Diam. (mm)	Initial Mod. (GPa)	Deflect.n at Fail (mm)	Failure Stress (MPa)	Failure Strain (%)
UNIF/5G/1A	54	12.50	2.37	**6**	26.0	6.9	698	2.700
UNIF/5G/1B	54	12.44	2.34	**6**	27.7	7.2	756	2.750
UNIF/5G/2A	54	24.98	2.33	12	29.3	7.8	1034	3.150
UNIF/5G/2B	54	25.01	2.35	12	30.5	8.1	994	3.150
UNIF/5G/3A	75	12.48	2.3	12	27.0	14.2	756	2.900
UNIF/5G/3B	75	12.46	2.3	12	28.2	14.8	801	2.900
UNIF/5G/4A	75	24.68	2.16	**6**	35.0	15.8	813	2.600
UNIF/5G/4B	75	24.76	2.14	**6**	32.7	14.0	760	2.750
UNIF/5C/1A	54	12.49	2.21	12	82.0	3.1	928	1.125
UNIF/5C/1B	54	12.41	2.21	12	82.5	3.0	965	1.150
UNIF/5C/2A	54	24.55	2.2	**6**	77.7	3.6	928	1.175
UNIF/5C/2B	54	24.91	2.17	**6**	74.6	3.6	900	1.175
UNIF/5C/3A	75	12.50	2.17	**6**	71.2	5.9	784	1.088
UNIF/5C/3B	75	12.46	2.19	**6**	73.4	6.0	849	1.150
UNIF/5C/4A	75	25.05	2.31	12	70.1	6.2	865	1.200
UNIF/5C/4B	75	24.99	2.29	12	71.5	6.5	867	1.200
UNIF/10G/1A	108	25.00	4.46	**6**	31.8	13.6	736	2.400
UNIF/10G/1B	108	24.84	4.49	**6**	32.6	13.6	730	2.375
UNIF/10G/2A	108	49.83	4.46	12	33.9	14.0	800	2.500
UNIF/10G/2B	108	50.03	4.49	12	33.3	14.0	800	2.525
UNIF/10G/3A	150	24.65	4.03	12	32.5	27.0	676	2.200
UNIF/10G/3B	150	24.43	4.03	12	28.9	28.8	661	2.350
UNIF/10G/4A	150	49.85	4.06	**6**	32.2	25.6	588	2.150
UNIF/10G/4B	150	49.70	4.06	**6**	32.6	27.0	585	2.100
UNIF/10C/1A	108	25.07	3.62	12	117.4	4.6	857	0.700
UNIF/10C/1B	108	25.04	3.70	12	112.5	4.6	821	0.700
UNIF/10C/2A	108	50.05	3.44	**6**	111.2	6.2	836	0.750
UNIF/10C/2B	108	49.96	3.55	**6**	102.9	6.0	756	0.750
UNIF/10C/3A	150	24.64	3.45	**6**	101.9	10.2	821	0.750
UNIF/1OC/3B	150	25.57	3.50	**6**	93.8	10.2	641	0.665
UNIF/10C/4A	150	49.84	3.73	12	101.7	9.8	791	0.750
UNIF/10C/4B	150	49.88	3.57	12	110.2	10.4	799	0.715

Specimen Code	Length (mm)	Width (mm)	**Depth (mm)**	**Roller Diam. (mm)**	Initial Mod. (GPa)	**Deflect.n at Fail (mm)**	**Failure Stress (MPa)**	**Failure Strain (%)**
UNIF/15G/1A	162	37.51	6.67	12	33.7	21.8	782	2.450
UNIF/15G/1B	162	37.46	6.68	12	34.2	21.4	774	2.300
UNIF/15G/2A	162	74.88	6.69	**6**	32.7	21.0	669	2.150
UNIF/15G/2B	162	74.95	6.68	**6**	32.0	20.6	677	2.250
UNIF/15G/3A	225	37.43	6.59	**6**	30.5	38.4	595	2.250
UNIF/15G/3B	225	37.51	6.59	**6**	29.6	37.0	**589**	2.250
UNIF/15G/4A	225	75.01	6.67	12	32.2	38.4	683	2.300
UNIF/15G/4B	225	75.00	6.19	12	37.4	38.4	795	2.300
UNIF/15C/1A	162	37.53	6.07	**6**	91.0	8.5	702	0.750
UNIF/15C/1B	162	37.53	6.14	**6**	92.7	7.6	743	0.800
UNIF/15C/2A	162	74.48	6.13	12	92.6	8.8	798	0.850
UNIF/15C/2B	162	75.00	6.16	12	91.1	8.6	785	0.850
UNIF/15C/3A	225	37.52	5.88	12	97.4	14.0	729	0.725
UNIF/15C/3B	225	37.55	6.01	12	97.5	13.6	734	0.750
UNIF/15C/4A	225	75.04	5.98	**6**	96.4	7.7	650	0.665
UNIF/15C/4B	225	75.04	6.15	**6**	83.9	7.0	559	0.665

Table B.2: Uni-Directional Flexural Tests Results

Appendix C Method of Contrasts

The method of contrasts is a technique which may be used to analyse the results of a designed experiment. The data for a hypothetical simple two-level, three-factor, full factorial experimental design, as shown in Table C.I, is used here to illustrate the method.

Run	Factor A	Factor B	Factor C	Response
1	+	+	+	19.0
2	-	+	+	14.0
3	+	-	+	16.9
4	-	-	+	16.1
5	+	+	-	24.0
6	-	+	-	18.0
7	+	-	-	22.1
8	-	-	-	20.9

Table C.I: Hypothetical results of an Experimental design

Initially the main effect of factor A is calculated as the difference between the average response values for A at the '-' and '+' levels. It is usual to subtract the '-' level average from the '+' level average;

$$A = \left(\frac{19.0 + 16.9 + 24.0 + 22.1}{4}\right) - \left(\frac{14.0 + 16.1 + 18.0 + 20.9}{4}\right) = 3.25 \qquad (C.1)$$

To show that for an orthogonal design the effect of a factor is not altered by any change in the effect of the other factors, 10 units are arbitrarily added to each response where factor B is at the '+' level. Rewriting (C.I) gives;

$$A = \left(\frac{29.0 + 16.9 + 34.0 + 22.1}{4}\right) - \left(\frac{24.0 + 16.1 + 28.0 + 20.9}{4}\right) = 3.25 \qquad (C.2)$$

Which is the same value as given by (C.I).

In order to show the convenience of expressing the levels of the factors as '+' and '-', rearranging(C.I);

$$A = \frac{(19.0 - 14.0 + 16.9 - 16.1 + 24.0 - 18.0 + 22.1 - 20.9)}{\;} \qquad (C.3)$$

On inspection of this equation it can be seen that the values in the brackets are the experimental responses and that the sign of each of these values corresponds to the level taken by A in the design for that run. Also, it is evident that the divisor of the whole expression is equal to half the number of experimental runs. This is because in this example there is an equal number of treatments with A at each of the two levels. Hence the effect of A with its levels expressed in this manner may be calculated through cross multiplication of the column of the levels taken by A with the column of the response values. This can also be seen to be true for the other factors. The list of '+' and '-' signs (or of +1's and -1's) that make up the column for a given factor in the design is called a *contrast.* The coefficients of the contrast add up to zero and *contrast* the results according to the factor level. The main effects of A. B and C are shown in the *response table,* Table C.2, and also in the form of an *effects plot* in Figure C.I. The ends of each line of the effects plot represents the response average at the "+" and '-' level of the relevant factor.

	Factor A	**Factor B**	**Factor C**
'+' **Average**	20.50	18.75	16.50
'-' **Average**	17.25	19.00	21.25
Effect	3.25	-0.25	-4.75

Table C.2: Response Table

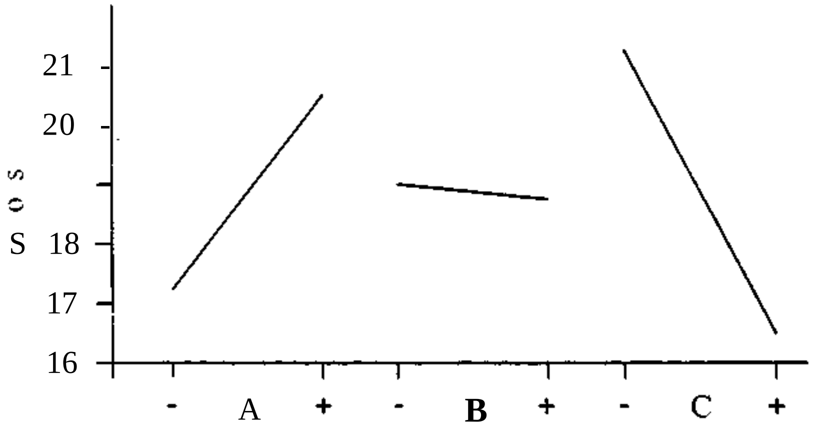


Figure C.I: Main Effects Plot

Figure C.I shows that the effects of A and C are far larger than that of B, and a 'common sense" approach would lead to the conclusion that the effect of B is negligible. However, it is more objective to use simple statistical methods to decide which effects are to be considered 'real' (see Appendix D).

In calculating the main effects no account has been taken of any interaction between the factors, i.e. that the influence of a factor on the response may be altered by changing the level of another factor. The interaction effect between two factors, A and B say, can be defined as half the difference between the effect of A for B at the '+' level and the effect of A for B at the '-' level. Again this interaction will not be altered by the effects of the other factor C in our example due to the orthogonal nature of the design, allowing average values to be taken. Hence;

$$AB = \frac{1}{2}\left\{\underbrace{\left[\left(\frac{19.0+24.0}{2}\right)-\left(\frac{14.0+18.0}{2}\right)\right]}_{Effect\ of\ A\ for\ B+}-\underbrace{\left[\left(\frac{16.9+22.1}{2}\right)-\left(\frac{16.1+20.9}{2}\right)\right]}_{Effect\ of\ A\ for\ B-}\right\} \qquad (C.4)$$

$$= \frac{1}{2}[(21.5-16.0)-(19.5-18.5)] \quad =2.25$$

This may be shown graphically as in Figure C.2, the interaction showing as the difference between the slopes of the two lines.

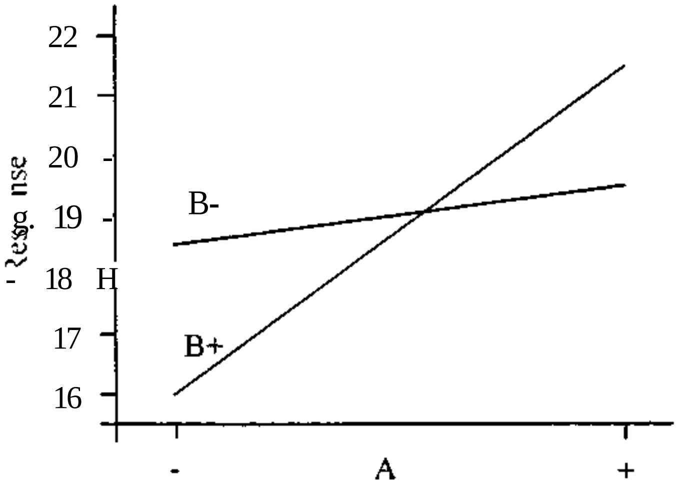


Figure C.2: AxB Interaction

The interaction AB may be thought of as measuring the effect of B on the effect of A as above, or equivalently as the effect of A on the effect of B. The latter approach leads to the definition of AB as half the difference between the effect of B for A at the '+' level and the effect of B for A at the "-" level as in equation (C.5).

$$AB = \frac{1}{2}\left\{\underbrace{\left[\left(\frac{19.0+24.0}{2}\right)-\left(\frac{16.9+22.1}{2}\right)\right]}_{Effect\ of\ B\ for\ A+}-\underbrace{\left[\left(\frac{14.0+18.0}{2}\right)-\left(\frac{16.1+20.9}{2}\right)\right]}_{Effect\ of\ B\ for\ A-}\right\} \qquad (C.5)$$

$$= \frac{1}{2}[(21.5-19.5)-(16.0-18.5)] \quad =2.25$$

This is again illustrated by an effects plot as in Figure C.3.

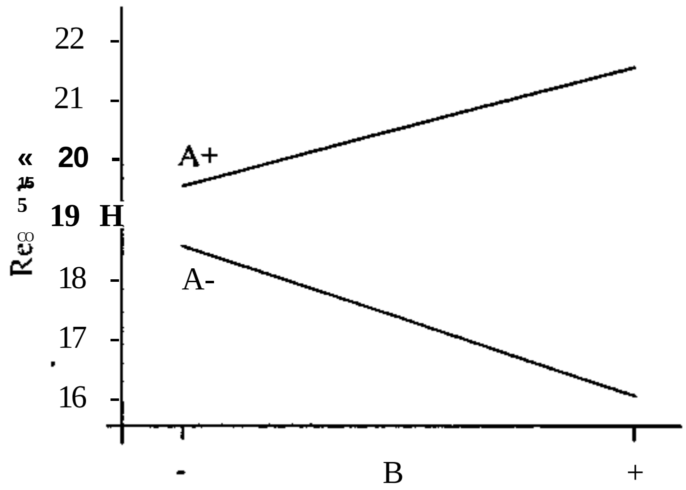


Figure C.3: BxA Interaction

Although Figure C.2 and Figure C.3 are not the same plots, the difference between the slopes of the two lines in both cases are identical. Hence the interaction between A and B may be thought of in either way: AxB or BxA. This idea can be reinforced through the expression of both (C.4) and (C.5) using (C.6):

$$AB = \left(\frac{19.0 + 14.0 - 16.9 + 16.1 + 24.0 - 18.0 - 22.1 - 20.9}{4}\right) = 2.25$$

This equation shows another advantage of representing the levels of the factors with the labels '+* and "-". The signs taken by the response values in (C.6) are obtainable by cross multiplication of the A and B contrasts to give a corresponding contrast for the AxB interaction. This idea may be extended to give the other contrasts AxC, BxC and also AxBxC. as shown in Table C.3.

Any further cross multiplication of the columns in Table C.3 results in a contrast that is already present, i.e. there are only seven possible orthogonal contrasts for this particular design. In general a set of n results can be used to generate (n-1) orthogonal contrasts. From Table C.3 it is evident that the only important interaction is that between A and B, although it is again worth stressing that this conclusion should be statistically tested. Hence ,the important terms to be used in any prediction equation would be A, C and AxB. However, this highlights the inadequacy of the use of main effects without the consideration of any important interactions. The main effect of B appears to be insignificant only because of the nature of its interaction with factor A. Looking at Figure C.3 we can see that the two effects of B for the two levels of A have different signs, so averaging these effects leads to a misleadingly small main effect. Hence the important effects to be used in the prediction equation are A. C. AxB and also B.

Run	A	B	AxB	C	AxC	BxC	AxBxC	Response
1	+1	+1	+1	+1	+1	+1	+1	19.0
2	-1	+1	-j	+1	-1	+1	-1	14.0
3	+1	-1	-1	+1	+1	-1	-1	16.9
4	-1	-1	+1	+1	-1	-1	+1	16.1
5	+1	+1	+1	-1	-1	-1	-1	24.0
6	-1	+1	-1	-1	+1	-1	+1	18.0
7	+1	-1	-1	-1	-1	+1	+1	22.1
8	-1	-1	+1	-1	+1	+1	-1	20.9
Sum	13	-1	9	-19	-1.4	1	-0.6	Average:
Divisor	4	4	4	4	4	4	4	ȳ = 18.875
Contrast Value	3.25	-0.25	2.25	-4.75	-0.35	0.25	-0.15	

Table C.3: Contrast Values

In order to predict future responses at specific treatment combinations, a procedure based on the theory of least squares is used. The important effects are first multiplied by the level of their corresponding factor or interaction in the treatment combination considered. The sum of these values is then halved and added to the average of all of the responses. For example, to predict the response at A+. B-, C-;

	A	B	C	AxB	Sum
Level	+1	-1	-1	-1	
Effect	3.25	-0.25	-4.75	2.25	
Product	3.25	0.25	4.75	-2.25	6.0

$$\text{Prediction} = y + Sum/2 \qquad \textbf{(C.7)}$$

$$= 21.875$$

This is close to the value already obtained, but differs slightly due to the fact that only four of the possible seven terms have been used in the prediction equation. This does not mean that the remaining three terms have been ignored, but that they have been used to estimate the random noise in the response variable. The prediction is an estimate of the average response value expected for many repeated trials at the treatment combination in question.

Appendix D Analysis of Variance (ANOVA) Example

In this appendix ANOVA is illustrated, initially considering a very simple one factor, two level experiment with five replications (i.e. $n_R = 5$), see Table D.I.

Level	Responses					Averages
	y_1	y_2	y_3	y_4	y_5	$\bar{y}_R$
'HIGH' (+)	10	11	12	7	13	10.6
'LOW (-)	4	7	1	5	3	4.0
					$\bar{\bar{y}}$:	7.3

Table D.I: One factor, two level experiment with five replications

The contrast value for this example is 6.6. The object of the statistical method is to see which effects are significant, i.e. can be treated as 'real' over and above the level of random noise. Hence a sensible first step would be to estimate the amount of random noise present in the experiment. This is described by the estimated variance s^2 and may be calculated for each factor level in turn from ;

$$s_R^2 = \frac{\sum_{i=1}^{n_R} (y_{R,i} - \bar{y}_R)^2}{n_R - 1} \qquad \text{(D.I)}$$

where R indicates each level ('-' or '+') so that, for example, $y_{+,1} = y_{1,1} = 10$, and n_R is the number of replications of each level. For this example;

$$n_R = 5$$
$$s_+^2 = 5.3$$
$$s_-^2 = 5.0$$

The next step is to "pool" these values to obtain an estimate of the random noise for the whole experiment of 10 treatments or runs. Define the pooled sum of squares as

$$SS_R = \sum (n_R - 1) s_R^2 \qquad \text{(D.2)}$$

Also define the pooled degrees of freedom as

$$f_R = \sum (n_R - 1) \qquad \text{(D.3.)}$$

Then the pooled estimate of the variance for the experiment is given by,

$$_5 ' = \quad S\S IL \qquad \text{(D.4)}$$

For this example;

V

$$s2 = \frac{(4x5.3) + (4x5.0)}{f4 + 4J}$$

$$= 5.75$$

The statistical notion of a Hypothesis' is now introduced whereby two complementary situations are considered. In this example the appropriate hypotheses are that the factor has no effect on the response or alternatively that it does have an effect on the response.

$$Ho\text{-}H, \quad = \quad M. \qquad \text{(D.5)}$$

$$Hi : M_+ \quad * \quad M.$$

Where Ho is the 'null hypothesis', Hi is the 'alternative hypothesis', u+ is the true average response value at the '+' level (estimated by *J+*) and u_ is the true average response value at the '-' level (estimated by $\bar{y}$.)

It is then asked whether Ho is true, or whether Ho should be rejected in favour of Hi. A decision cannot be made with certainty because we only have a finite sample of data points which we hope is representative of the actual system. Hence a decision is made as to whether Ho is accepted or rejected at a 'significance level of a'. This means that the probability that Ho is rejected when it is, in fact, true is (a x 100)%. It then follows that the probability of a correctly accepting Ho is ((1-a) x 100)%. A value of a is set and then statistical tests carried out to see if the null hypothesis is should be accepted at this level. The value of a chosen depends upon the consequences of an erroneous conclusion, but for most industrial experiments a 5% significance level is adopted.

If we assume Ho to be true, i.e. that the factor does not change the response, then any variation of the 'HIGH' and 'LOW' responses around the overall mean (estimated by $\bar{y}$) will be due solely to random noise. If this is the case then;

$$y \quad n_R$$

Where Sy^2 is the estimated variance of each of the high and low averages about the mean, S^2 is the pooled random variation estimate and IIR is the number of responses observed at each factor level.

For a two level experiment;

$$
\begin{aligned}
{}^{,}_{*} &= \frac{(f-\bar{y}j + (M-\bar{y}.f}{TTl} \qquad (D7)\\
&= 2(\mu-\bar{y}_+)^2\\
&= 2(ii \;\bar{}-yj\\
&= \frac{Contrast^2}{2},
\end{aligned}
$$

Also $n_R = \#/2$, (D.g)

where N is the total number of responses.

Hence, from (D.6), (D.7) and (D.8),

$$\frac{Contrast^2}{2} = \frac{S^2}{N/2}$$

ie. (D.9)

ram

$$S^2 = \text{l}\frac{4}{\ }Contrast^2$$

$$= Contrast\,Contribution$$

We note that this equation is based on the initial assumption that Ho : $\mu_+ = \mu_-$ is true.

Inthiscase; (D.10)

$$
\begin{aligned}
S^2 &= \frac{8}{4}x<5.6^2\\
&= 87.12
\end{aligned}
$$

So if there is no real effect on the response of changing the factor level then the values for S^2 given by equations (D.4) and (D.9) will estimate the same quantity, namely the true (unknown) variance of the population of all observed responses on the system a^2. However, if the expression given by equation (D.9) is larger than that obtained using equation (D.4), then this indicates that the response could have been affected by the change of the factor level. It must be decided whether any discrepancies between the two estimates of a^2 is due to a factor or is due solely to random variation. That is, is the value of S^2 from equation (D.9) larger purely *by chance*? To do this it is helpful to introduce a 'test statistic' in the form of an F ratio, defined as;

$$r = \frac{S^2 \text{ using equation (D.9)}}{S \text{ using equation (DA)}}$$

i.e.

$$F = \frac{Contrast Contribution}{(SS. \; / \; /_{„})} \qquad (D.11)$$

For the example;

$$F = 87.12/5.15$$
$$= 16.92$$

Since equations (D.4) and (D.9) give independent estimates of the 'real' variance o^2, the F ratio has an F distribution on vi and V2 degrees of freedom. The probability density function is illustrated in Figure D.I.

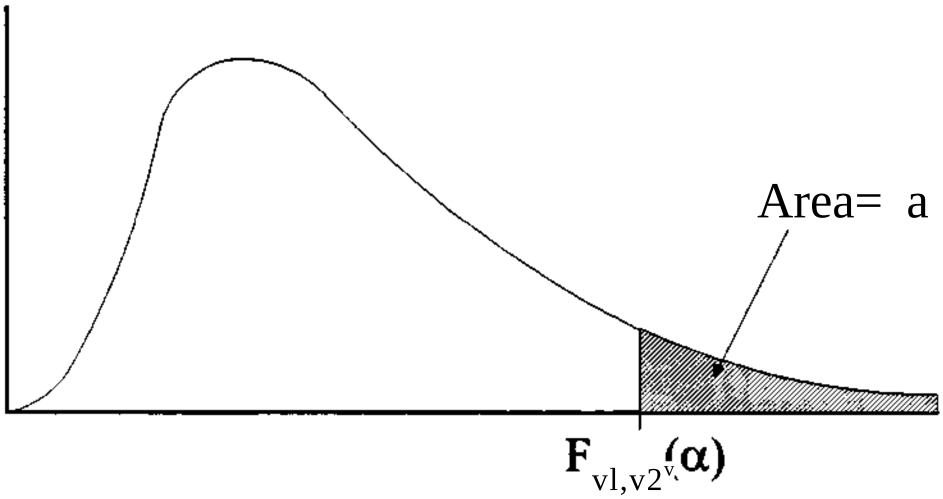


Figure D.I: F density function

For this application Ui is one less than the number of factor levels, i.e. 1, and u_2 is the pooled degrees of freedom £,.

The critical value of the test ($F_{vi,v2}$ (a)) is found for the chosen significance level a (either from tables or using statistical software such as 'MINITAB' or 'SAS'). Then the value calculated from the data using equation (D.11) is compared with this critical value. For example, if a value of 5% has been chosen for a and the value of the test statistic F for the effect is greater than the value of $F_{vi,v2}$ (5%) then the null hypothesis is rejected at the 5% significance level.

In summary, if the calculated value of the test statistic F exceeds that given in statistical tables or software for the specified value of a then we conclude that there is a measurable effect of the factor on the response at that significance level.

In summary, if the calculated value of the test statistic F exceeds that given in statistical tables or software for the specified value of a then we conclude that there is a measurable effect of the factor on the response at that significance level.

Using MINITAB gives;

$$F_{K6}(0.05) = 5.99$$

$$F_{1<6}(0.025) = 8.81$$

$$F_{,.6}(0.01) = 13.75$$

Since the value of the test statistic F for the example here is 16.92, we conclude that the effect of the factor level upon the response is 'real', and that the observed value of F does not occur merely by chance.

This method can be extended to factorial experiments with more than one factor simply by calculating the contrast contribution of each factor main effect and interaction using equation (D.9) and calculating the F value for each in turn. The variation (measured by the sum of squares in the response) in the experiment has then been 'partitioned' into contributions from each factor plus a contribution from the 'random' sources. This is the basis for the construction of an analysis of variance or 'ANOVA' table. Consider the experiment shown in Table D.2 as an example.

Factors			Responses					
A	B	C	y_1	y_2	y_3	$\bar{y}_R$	s^2	d.f.
1	1	1	18.6	19.0	19.4	19.0	0.16	2
-1	1	1	15.2	12.8	14.0	14.0	1.44	2
1	-1	1	15.0	16.9	18.8	16.9	3.61	2
-1	-1	1	17.3	16.0	15	16.1	1.33	2
1	1	-1	25.2	21.6	25.2	24.0	4.32	2
-1	1	-1	18.6	16.3	19.1	18.0	2.23	2
1	-1	-1	21.9	20.4	24.0	22.1	3.27	2
-1	-1	-1	22.0	21.2	19.5	20.9	1.63	2
					$\bar{y}$:	18.9	**Total d.f:**	16

Table D.2: Example Experiment

From these values.

$$\frac{SS_\sigma}{f_\sigma} = \frac{(2X0.16)+(2X1.44)+\ K}{(8X2)}$$

$$= 2.249$$

Using the method of contrasts as outlined in Appendix F, and equation (D.I 1) the following analysis of variance table may be constructed.

Effect	**Contrast**	**Contrast Contribution**	**F Ratio**
A	3.25	63.4	28.2
B	-0.3	0.38	0.17
C	2.25	30.4	13.5
A x B	-4.8	135.0	60.2
A x C	-0.35	0.735	0.327
B x C	0.25	0.38	0.17
A x B x C	-0.15	0.14	0.06

Table D.3: ANOVA Example

Using MINITAB gives:

$F_{U6}(0.05) = 4.49$

$F_{U6}(0.025) = 6.12$

$F_{U6}(0.01) = 8.53$

From these values we can see that the effects C, A, AxB and hence B would be considered statistically significant even at the 1% significance level, but that the other effects are probably due to random variation alone.

The total sum of squares is defined as;

$$ssr = \sum_{i=1} (y_i - \bar{y}) \qquad (D.12)$$

For the example in Table D.2 this gives SS_T = 266.725, which is the sum of all the contrast contributions plus SS_a . Hence the total sum of squares has been partitioned into contributions from each effect. These values have then been compared, each in turn, with the sum of squares attributed to random variation divided by the degrees of freedom, in order to decide which are statistically significant at a given level. This is the general procedure for the construction of an analysis of variance or ANOVA table.

Now consider the case where no replications of experimental runs were made. There are now no replications to enable the estimation of SS_a , so how are the appropriate F ratios calculated? This is often achieved by assuming that some of the contrasts are solely due to random variation and then these values are used to estimate SS_O . In many practical applications it can be assumed at the start of the experiment that either all interactions or all three-factor and higher interactions, for example, are negligible and thus that their contrast values are due solely to experimental error. The choice of interactions may be justified through existing knowledge of the system considered or the results of similar work or previous experiments. It is quite common to use three-factor and higher interactions for estimating the error variance in industrial or engineering problems. The contrasts which may be used to estimate SS_O can be found using a simple graphical method using a normal plot of all the contrasts and selecting those readings lie "off of the line'.

Appendix E Unidirectional Data Analysis

Correlations (Pearson) for Tensile Tests:

	Thick	Reinfor	Clength	Cwidth	Length	Width	Depth	Mod
Reinfor	0.000							
Clength	0.000	0.000						
Cwidth	0.000	0.000	0.000					
Length	0.682	0.000	0.696	0.000				
Width	0.677	0.001	0.004	0.699	0.465			
Depth	0.993	-0.095	-0.009	0.006	0.671	0.677		
Mod	0.072	0.991	0.021	-0.007	0.063	0.044	-0.025	
Strain	-0.028	-0.985	-0.013	0.015	-0.024	-0.016	0.067	-0.985
Stress	0.131	0.965	0.055	0.045	0.129	0.111	0.033	0.971

	Strain
Stress	-0.934

Table E.I: Tensile Correlation Matrix

Correlations (Pearson) **for Flexural Tests:**

	Thick	Reinfor	Clength	Cwidth	Length	Width	Depth	Roller
Reinfor	0.000							
Clength	0.000	0.000						
Cwidth	0.000	0.000	0.000					
Length	0.918	0.000	0.366	0.000				
Width	0.751	0.002	-0.000	0.611	0.690			
Depth	0.980	-0.126	-0.040	-0.000	0.884	0.737		
Roller	0.000	0.000	0.000	0.000	0.000	-0.000	0.008	
Mod	0.134	0.949	-0.038	0.001	0.112	0.100	-0.022	0.041
Strain	-0.237	-0.952	-0.052	0.012	-0.236	-0.179	-0.096	0.065
Stress	-0.583	0.265	-0.406	0.118	-0.680	-0.405	-0.577	0.417

	Mod	Strain
Strain	-0.955	
Stress	0.207	-0.024

Table E.2: Flexural Correlation Matrix

THICK	REINFORC	CLENGTH	CWIDTH	LSMEAN	STDERR
3	.	.	.	700.608	
5	.	.	.	770.188	
7	.	.	.	756.563	
.	1	.	.	574.172	
.	2	.	.	910.734	
3	1	.	.	532.765	
3	2	.	.	868.451	
5	1	.	.	615.125	
5	2	.	.	925.250	
1	1	.	.	574.625	
7	2	.	.	938.500	
.	.	1	.	732.849	6.0297
.	.	2	.	752.057	6.0297
.	.	.	1	734.595	6.0297
.	.	.	2	750.310	6.0297
.	.	1	1	729.531	8.5272
.	.	1	2	736.167	8.5272
.	.	2	1	739.659	8.5272
.	.	2	2	764.454	8.5272
3	.	1	.	689.171	10.4437
3	.	2	.	712.045	10.4437
5	.	1	.	767.625	10.4437
5	.	2	.	772.750	10.4437
7	.	1	t	741.750	10.4437
7	.	2	.	771.375	10.4437
3	.	.	1	674.660	10.4437
3	.	.	2	726.556	10.4437
5	.	.	1	778.625	10.4437
5	.	.	2	761.750	10.4437
7	.	.	1	750.500	10.4437
7	.	.	2	762.625	10.4437
.	1	1	.	558.363	8.5272
.	1	2	.	589.980	8.5272
.	2	1	.	907.334	8.5272
.	2	2	.	914.133	8.5272
.	1	.	1	573.676	8.5272
.	1	.	2	574.668	8.5272
.	2	.	1	895.514	8.5272
.	2	.	2	925.953	8.5272

Table E.3: Tensile Stress Means and Standard Errors

Source	DF	AdjSS	AdjMS	SS / S-P Error	
Whole-Plot:					
Thick	2	43508	21754	24.93	
(Linear)	1	25047	25047	28.71	
(Quadratic)	1	18461	18461	21.16	
Reinforc	1	1359288	1359288	1557.81	
Thick* Reinfore	2	5783	2891	3.31	
(Linear)	1	1589	1589	1.82	
(Quadratic)	1	4194	4194	4.81	
Sub-Plot:	•			**F**	P
Nested Length	3	5708	1903	2.18	0.109
Nested Width	3	12500	4167	4.77	0.007
Clength*Cwidth	1	989	989	1.13	0.295
Reinforc*Clength	1	1848	1848	2.12	0.155
Reinforc*Cwidth	1	2601	2601	2.98	0.094
Error	33	28795	873		
Total	47	1461020			

R-Square: 0.980

Unusual Obsen'ations **for Stress:**

Obs.	Stress	**Fit**	**Stdev.Fit**	**Residual**	**St.Resid**
15	982.590	911.534	16.513	71.056	2.90R

R denotes an obs. with a large st. resid.

Table E.4: Tensile Stress ANOVA

THICK	REINFORC	CLENGTH	CWIDTH	LSMEAN	STDERR
3	.	.	.	1.54375	
5	.	.	.	1.51394	
7	.	.	.	1.50769	
.	1	.	.	2.03625	
.	2	.	.	1.00733	
3	1	.	.	2.05875	
3	2	.	.	1.02875	
5	1	.	.	2.03875	
5	2	.	.	0.98913	
7	1	.	.	2.01125	
7	2	.	.	1.00412	
.	.	1	.	1.52850	0.017706
.	.	2	.	1.51508	0.017706
.	.	.	1	1.51412	0.017706
.	.	.	2	1.52946	0.017706
.	.	1	1	1.54925	0.025041
.	.	1	2	1.50775	0.025041
.	.	2	1	1.47900	0.025041
.	.	2	2	1.55117	0.025041
3	.	1	.	1.55625	0.030669
3	.	2	.	1.53125	0.030669
5	.	1	.	1.53263	0.030669
5	.	2	.	1.49525	0.030669
7	.	1	.	1.49662	0.030669
7	.	2	.	1.51875	0.030669
3	.	.	1	1.49625	0.030669
3	.	.	2	1.59125	0.030669
5	.	.	1	1.55425	0.030669
5	.	.	2	1.47363	0.030669
7	.	.	1	1.49187	0.030669
7	.	.	2	1.52350	0.030669
.	1	1	.	2.04958	0.025041
.	1	2	.	2.02292	0.025041
.	2	1	.	1.00742	0.025041
.	2	2	.	1.00725	0.025041
.	1	.	1	2.04792	0.025041
.	1	.	2	2.02458	0.025041
.	2	.	1	0.98033	0.025041
.	2	.	2	1.03433	0.025041

Table E.5: Tensile Strain Means and Standard Errors

Source	DF	AdjSS	AdjMS	SS / S-P Error	
Whole-Plot:					
Thick	2	0.01188	0.00594	0.79	
(Linear)	1	0.01040	0.01040	1.38	
(Quadratic)	1	0.00148	0.00148	0.20	
Reinforc	1	12.70403	12.70403	1688.36	
Thick*Reinfore	2	0.00362	0.00181	0.24	
(Linear)	1	0.00104	0.00104	0.14	
(Quadratic)	1	0.00257	0.00257	0.34	
Sub-Plot:				F	P
Nested Length	1	0.01005	0.00335	0.45	0.722
Nested Width	1	0.0661	0.022	2.93	0.048
Clength*Cwidth	1	0.03876	0.03876	5.15	0.030
Reinforc*Clength	1	0.00211	0.00211	0.28	0.600
Reinforc*Cwidth	1	0.01794	0.01794	2.38	0.132
Error	33	0.24831	0.00752		
Total	47	13.10280			

R-Square: 0.981

Unusual Observations for Strain:

Obs.	Strain	Fit	Stdev.Fit	Residual	St.Resid
19	1.75000	1.97600	0.04849	-0.22600	-3.14R
20	2.16000	1.97600	0.04849	0.18400	2.56R
21	2.19000	2.04467	0.04849	0.14533	2.02R

R denotes an obs. with a large st. resid.

Table E.6: Tensile Strain ANOVA

THICK	REINFOR	CLENGTH	CWIDTH	ROLLER	LSMEAN	STDERR
5	.	.	.	.	856.125	
10	.	.	.	.	743.625	
15	.	.	.	.	704.000	
.	1	.	.	.	739.667	
.	2	.	.	.	796.167	
5	1	.	.	.	826.500	
5	2	.	.	.	885.750	
10	1	.	.	.	697.000	
10	2	.	.	.	790.250	
15	1	.	.	.	695.500	
15	2	.	.	.	712.500	
.	.	1	.	.	811.208	9.5193
.	.	2	.	.	724.625	9.5193
.	.	.	1	.	755.333	9.5193
.	.	.	2	.	780.500	9.5193
.	.	.	.	6	723.542	9.5193
.	.	.	.	12	812.292	9.5193
.	.	1	1	.	794.656	13.6710
.	.	1	2	.	827.760	13.6710
.	.	2	1	.	716.010	13.6710
.	.	2	2	.	733.240	13.6710
.	.	1	.	6	765.521	13.6710
.	.	1	.	12	856.896	13.6710
.	.	2	.	6	681.563	13.6710
.	.	2	.	12	767.688	13.6710
.	.	.	1	6	719.802	13.6710
.	.	.	1	12	790.865	13.6710
.	.	.	2	6	727.281	13.6710
.	.	.	2	12	833.719	13.6710
5	.	1	.	.	900.375	16.4879
5	.	2	.	.	811.875	16.4879
10	.	1	.	.	792.000	16.4879
10	.	2	.	.	695.250	16.4879
15	.	1	.	.	741.250	16.4879
15	.	2	.	.	666.750	16.4879
5	.	.	1	.	817.125	16.4879
5	.	.	2	.	895.125	16.4879
10	.	.	1	.	742.875	16.4879
10	.	.	2	.	744.375	16.4879
15	.	.	1	.	706.000	16.4879
15	.	.	2	.	702.000	16.4879
5	.	.	.	6	811.000	16.4879
5	.	.	.	12	901.250	16.4879
10	.	.	.	6	711.625	16.4879
10	.	.	.	12	775.625	16.4879
15	.	.	.	6	648.000	16.4879
15	.	.	.	12	760.000	16.4879

THICK	REINFOR	CLENGTH	CWIDTH	ROLLER	LSMEAN	STDERR
.	1	1	.	.	784.552	13.6710
.	1	2	.	.	694.781	13.6710
.	2	1	.	.	837.865	13.6710
.	2	2	.	.	754.469	13.6710
.	1	.	1	.	713.271	13.6710
.	1	.	2	.	766.063	13.6710
.	2	.	1	.	797.396	13.6710
.	2	.	2	.	794.938	13.6710
.	1	.	.	6	684.323	13.6710
.	1	.	.	12	795.010	13.6710
.	2	.	.	6	762.760	13.6710
.	2	.	.	12	829.573	13.6710

Table E.7: Flexural Stress Means and Standard Errors

Source	DF	AdjSS	AdjMS	SS / S-P Error	
Whole-Plot:					
Thick	2	199298	99649	45.82	
(Linear)	1	185136	185136	85.13	
(Quadratic)	1	14162	14162	6.51	
Reinforc	1	38307	38307	17.61	
Thick* Reinforc	2	11673	5837	2.68	
(Linear)	1	3570	3570	1.64	
(Quadratic)	1	8103	8103	3.73	
Sub-Plot:				F	**P**
Nested Length	1	90972	30324	13.94	0.000
Nested Width	1	24409	8136	3.74	0.023
Roller	1	94519	94519	43.46	0.000
Clength*Cwidth	1	672	672	0.31	0.583
Clength*Roller	1	74	74	0.03	0.856
Cwidth*Roller	1	3337	3337	1.53	0.226
Thick*Roller	2	4622	2311	1.06	0.360
Reinforc*Clength	1	108	108	0.05	0.825
Reinforc*Cwidth	1	8140	8140	3.74	0.064
Reinforc*Roller	1	5133	5133	2.36	0.136
Error	27	58720	2175		
Total	47	544592			

R-Squared: 0.892

Unusual Observations for Stress:

Obs.	Stress	Fit	Stdev.Fit	Residual	St.Resid
30	641.00	749.62	29.72	-108.62	-3.02R

R denotes an obs. with a large st. resid.

Table E.8: Flexural Stress ANOVA

THICK	REINFORC	CLENGTH	CWIDTH	ROLLER	LSMEAN	STDERR
5	.	.	.	.	2.01019	
10	.	.	.	.	1.52375	
15	.	.	.	.	1.51906	
.	1	.	.	.	1.48958	
.	2	.	.	.	0.87908	
5	1	.	.	.	1.86250	
5	2	.	.	.	1.15788	
10	1	.	.	.	1.32500	
10	2	.	.	.	0.72250	
15	1	.	.	.	2.28125	
15	2	.	.	.	0.75688	
.	.	1	.	.	1.72813	0.010391
.	.	2	.	.	1.64054	0.010391
.	.	.	1	.	1.67408	0.010391
.	.	.	2	.	1.69458	0.010391
.	.	.	.	6	1.62950	0.010391
.	.	.	.	12	1.73917	0.010391
.	.	1	1	.	1.69439	0.014923
.	.	1	2	.	1.76186	0.014923
.	.	2	1	.	1.65378	0.014923
.	.	2	2	.	1.62731	0.014923
.	.	1	.	6	1.66590	0.014923
.	.	1	.	12	1.79035	0.014923
.	.	2	.	6	1.59310	0.014923
.	.	2	.	12	1.68798	0.014923
.	.	.	1	6	1.65648	0.014923
.	.	.	1	12	1.69168	0.014923
.	.	.	2	6	1.60252	0.014923
.	.	.	2	12	1.78665	0.014923
5	.	1	.	.	2.04688	0.017998
5	.	2	.	.	1.97350	0.017998
10	.	1	.	.	1.58750	0.017998
10	.	2	.	.	1.46000	0.017998
15	.	1	.	.	1.55000	0.017998
15	.	2	.	.	1.48813	0.017998
5	.	.	1	.	1.97038	0.017998
5	.	.	2	.	2.05000	0.017998
10	.	.	1	.	1.51750	0.017998
10	.	.	2	.	1.53000	0.017998
15	.	.	1	.	1.53438	0.017998
15	.	.	2	.	1.50375	0.017998
5	.	.	.	6	1.92350	0.017998
5	.	.	.	12	2.09688	0.017998
10	.	.	.	6	1.49250	0.017998
10	.	.	.	12	1.55500	0.017998
15	.	.	.	6	1.47250	0.017998
15	.	.	.	12	1.56563	0.017998

THICK	REINFORC	CLENGTH	CWIDTH	ROLLER	LSMEAN	STDERR
.	1	1	.	.	2.54592	0.014923
.	1	2	.	.	2.43324	0.014923
.	2	1	.	.	0.91033	0.014923
.	2	2	.	.	0.84784	0.014923
.	1	.	1	.	2.48788	0.014923
.	1	.	2	.	2.49129	0.014923
.	2	.	1	.	0.86029	0.014923
.	2	.	2	.	0.89788	0.014923
.	1	.	.	6	2.40158	0.014923
.	1	.	.	12	2.57759	0.014923
.	2	.	.	6	0.85742	0.014923
.	2	.	.	12	0.90074	0.014923

Table E.9: Flexural Strain Means and Standard Errors

Analysis of Variance for Flexural Strain:

Source	DF	AdjSS	AdjMS	SS / S-P Error	
Whole-Plot:					
Thick	2	2.5485	1.2743	491.73	
(Linear)	1	1.9296	1.9296	744.64	
(Quadratic)	1	0.61889	0.61889	238.83	
Reinforc	1	31.1245	31.1245	1.2E+04	
Thick* Reinforc	2	0.0654	0.0327	12.61	
(Linear)	1	0.06498	0.06498	25.08	
(Quadratic)	1	0.00038	0.00038	0.15	
Sub-Plot:				**F**	**P**
Nested Length	3	0.1019	0.03397	13.06	0.000
Nested Width	3	0.0297	0.00990	3.81	0.0213
Roller	1	0.1443	0.1443	55.69	0.000
Clength*C width	1	0.0235	0.0235	9.08	0.006
Clength*Roller	1	0.0023	0.0023	0.90	0.351
Cwidth* Roller	1	0.0592	0.0592	22.83	0.000
Thick*Roller	2	0.0262	0.0131	5.06	0.014
Reinforc*Clength	1	0.0067	0.0067	2.59	0.119
Reinforc*Cwidth	1	0.0031	0.0031	1.20	0.282
Reinforc*Roller	1	0.0469	0.0469	18.12	0.000
Error	27	0.0700	0.0026		
Total	47	34.3079			

R-Squared: 0.998

Unusual Observations for Strain:

Obs.	Strain	Fit	Stdev.Fit	Residual	St.Resid
8	2.75000	2.67134	0.03245	0.07866	2.01R
21	2.20000	2.29428	0.03245	-0.09428	-2.40R
33	2.45000	2.37500	0.03600	0.07500	2.08R
34	2.30000	2.37500	0.03600	-0.07500	-2.08R

R denotes an obs. with a large st. resid.

Table E.10: Flexural Strain ANOVA

Appendix F Derivation of K^ for Four-Point Flexural Tests

Consider the example of four-point bending as shown in Figure F.I;

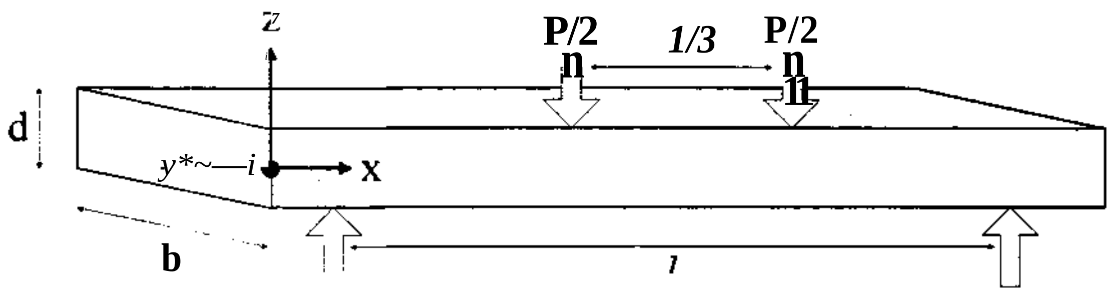


Figure F.I: Four-Point Bending

For the centre section of the beam between the two loads the stress is only a function of z;

$$\sigma(z) = \frac{2Plz}{bd^3} \tag{F.1}$$

Taking the reference stress to be the maximum stress in the material occurring on either face of the beam:

$$\sigma_r = \sigma_{max} = \frac{Pl}{bd^2} \tag{F.2}$$

i.e.

$$\sigma(z) = \frac{2\sigma_{max} z}{d} \tag{F.3}$$

Therefore;

$$\int \left(\frac{\sigma(z)}{\sigma_0}\right)^m dV = \int_{-d/2}^{d/2} \int_0^b \int_{l/3}^{2l/3} \left(\frac{\sigma(z)}{\sigma_0}\right)^m dx\,dy\,dz \tag{F.4}$$

$$= \frac{lbd}{3(m+1)}\left(\frac{\sigma_{max}}{\sigma_0}\right)^m$$

Or more generally:

$$\int \left(\frac{\sigma}{\sigma_0}\right)^m dV = K_1 V\left(\frac{\sigma_{max}}{\sigma_0}\right)^m \tag{F.5}$$

where;

$$K_1 = \frac{1}{3(m+1)} \tag{F.6}$$

For either outer section of the beam, between the loads and the supports, the stress is a function of x and z:

$$\sigma(x,z) = \frac{6Pxz}{bd^3} = \frac{6\sigma_{max}xz}{Id} \tag{F.7}$$

Therefore;

$$\int\left(\frac{\sigma(x,z)}{\sigma_0}\right)^m dV = \int\int\int_0^{1/3}\left(\frac{\sigma(x,z)}{\sigma_0}\right) dxdydz = \frac{lbd}{3(m+1)^2}\left(\frac{\sigma_{max}}{\sigma_0}\right)^m \tag{F.8}$$

Or more generally:

$$\int\left(\frac{\sigma}{\sigma_0}\right)^m dV = K_1 V\left(\frac{\sigma_{max}}{\sigma_0}\right)^m \tag{F.9}$$

where;

$$K_1 = \frac{1}{3(m+1)} \tag{F.10}$$

Hence for the whole beam;

$$\int\left(\frac{\sigma(x,y,z)}{\sigma_0}\right)^m dV = (K_1+2K_2)V\left(\frac{\sigma_{max}}{\sigma_0}\right)^m = K_s V\left(\frac{\sigma_{max}}{\sigma_0}\right)^m \tag{F.11}$$

where;

$$K_s = \frac{m+2}{(m+1)^2} \tag{F.12}$$

Appendix G Woven Roving Manufacturer Test Data

Specimen Code	Length (mm)	Width (mm)	Depth (mm)	Fibre Volume Fract.n	Initial Mod. (GPa)	Deflectn at (mm)	Failure Stress (MPa)	Failure Strain (%)
AUS8TA1/4	50	10.04	3.27	0.40	19.5	4.2	291	1.840
AUS8TC1/4	150	9.89	3.35	0.38	17.5	5.7	272	1.775
AUS8TA3/4	50	25.00	3.83	0.34	16.6	5.6	232	1.675
AUS8TC3/4	150	24.90	3.43	0.39	18.1	7.4	287	1.913
AUS12TA1/4	50	9.99	3.06	0.42	20.7	[illegible]	294	1.650
AUS12TC1/4	150	9.85	3.22	0.41	18.8	5.4	307	1.740
AUS12TA3/4	50	25.03	3.16	0.42	22.5	4.7	296	1.490
AUS12TC3/4	150	25.18	3.21	0.38	16.7	6.4	277	1.800
AUS11TB2/4	100	17.41	3.47	0.39	18.6	4.8	247	1.775
AUS11TA2/4	50	17.35	3.28	0.40	17.5	4.4	285	2.150
AUS11TC2/4	150	17.43	3.29	0.40	19.1	6.2	279	1.830
AUS11TB1/4	100	9.84	3.38	0.40	17.4	4.1	238	2.025
AUS11TB3/4	100	25.00	3.37	0.39	16.9	5.9	273	1.790
AUS8TB2/4	100	17.49	3.36	0.41	17.2	4.9	283	1.975
AUS12TB2/4	100	17.42	3.1	0.39	24.0	5.1	320	1.750
AUS8TA1/8	50	10.00	6.22	0.41	20.1	5.1	293	1.625
AUS8TC1/8	150	9.82	6.23	0.41	18.8	6.9	289	1.975
AUS8TA3/8	50	25.46	6.31	0.38	20.2	7.0	269	1.700
AUS8TC3/8	150	25.56	6.44	0.41	20.7	7.8	267	1.650
AUS12TA1/8	50	9.93	6.19	0.42	21.3	5.1	305	2.000
AUS12TC1/8	150	10.03	6.29	0.41	20.8	6.2	293	1.750
AUS12TA3/8	50	25.02	6.27	0.42	20.7	6.8	290	1.700
AUS12TC3/8	150	25.00	7.20	0.39	17.6	8.2	229	1.650
AUS11TB2/8	100	17.48	7.03	0.36	15.3	5.8	218	1.825
AUS11TA2/8	50	17.56	7.08	0.35	18.6	6.0	237	1.725
AUS11TC2/8	150	17.52	7.26	0.36	17.2	7.8	224	1.600
AUS11TB1/8	100	9.66	7.00	0.36	20.3	4.9	211	1.700
AUS11TB3/8	100	24.96	6.99	0.36	16.3	7.7	238	2.025
AUS8TB2/8	100	17.50	6.46	0.41	20.3	6.4	254	1.575
AUS12TB2/8	100	17.47	6.29	0.41	20.7	6.7	289	1.775

Table G.I : Woven Roving> Manufacturer Tensile Tests Results

Specimen Code	Length (mm)	Width (mm)	Depth (mm)	Fibre Volume Fract.n	Initial Mod. (GPa)	Deflect.n at Fail (mm)	Failur Stress (MPa)	Failur Strain (%)
AUS8FA1/4	50	10.07	3.47	0.40	16.7	3.0	346	2.200
AUS8FC1/4	100	10.12	3.28	0.38	18.0	16.2	369	2.275
AUS8FA3/4	50	29.76	3.39	0.40	16.1	4.3	476	2.650
AUS8FC3/4	100	29.91	3.54	0.38	14.9	14.3	323	2.400
AUS12FA1/4	50	10.03	3.13	0.40	21.8	3.7	495	2.225
AUS12FC1/4	100	10.05	3.19	0.38	21.1	14.5	365	2.425
AUS12FA3/4	50	29.96	3.31	0.38	18.3	3.7	408	2.225
AUS12FC3/4	100	30.05	3.09	0.42	18.7	15.6	422	2.525
AUS11FB2/4	75	19.95	3.35	0.40	17.2	8.5	378	2.550
AUS11FA2/4	50	20.01	3.50	0.40	15.5	3.2	367	2.650
AUS11FC2/4	100	20.04	3.38	0.40	17.5	16.7	358	2.500
AUS11FB1/4	75	10.00	3.36	0.41	15.4	6.1	267	1.825
AUS11FB3/4	75	29.92	3.37	0.40	14.9	6.5	323	2.275
AUS8FB2/4	75	20.09	3.46	0.37	15.3	8.6	390	2.875
AUS12FB2/4	75	20.07	3.17	0.40	16.8	9.7	437	2.725
AUS8FA1/8	100	20.02	6.27	0.41	20.1	8.0	433	2.550
AUS8FC1/8	200	20.11	6.16	0.41	21.0	21.2	300	1.563
AUS8FA3/8	100	59.75	6.58	0.40	18.8	6.6	331	2.000
AUS8FC3/8	200	59.99	6.24	0.40	19.0	23.0	320	1.850
AUS12FA1/8	100	20.06	6.29	0.43	19.5	7.4	422	2.525
AUS12FC1/8	200	19.41	6.39	0.41	21.0	22.0	328	1.750
AUS12FA3/8	100	60.03	6.25	0.41	21.1	6.0	351	1.800
AUS12FC3/8	200	59.91	6.50	0.41	19.9	21.0	309	1.650
AUS11FB2/8	150	39.96	6.89	0.37	17.5	9.6	247	1.575
AUS11FA2/8	100	39.92	7.04	0.37	17.7	4.0	218	1.275
AUS11FC2/8	200	39.96	7.07	0.36	16.9	21.8	294	2.050
AUS11FB1/8	150	19.89	6.90	0.36	15.1	11.4	284	2.000
AUS11FB3/8	150	60.02	6.87	0.37	17.5	11.2	287	1.725
AUS8FB2/8	150	40.03	6.33	0.40	19.0	13.8	376	2.150
AUS12FB2/8	150	39.88	6.44	0.42	22.7	10.8	305	1.600

Table G.2: Woven Roving Manufacturer Flexural Tests Results

Appendix H Woven Roving Manufacturer Tests Data Analysis

Correlations (Pearson) for Tensile Tests

	Thick	Man	Clength	CWidth	Length	Width	Depth	Mod
Man	0.000							
Clength	0.000	0.000						
CWidth	0.000	0.000	0.000					
Length	0.000	0.000	1.000	0.000				
Width	0.006	-0.005	-0.001	1.000	-0.001			
Depth	0.983	-0.022	0.030	0.049	0.030	0.053		
Mod	0.128	0.308	-0.261	-0.185	-0.261	-0.179	0.009	
Vf	-0.089	0.237	-0.037	-0.256	-0.037	-0.251	-0.237	0.550
Strain	-0.197	-0.114	0.033	-0.190	0.033	-0.196	-0.203	-0.391
Stress	-0.317	0.230	-0.096	-0.190	-0.096	-0.186	-0.464	0.577

	Vf	Strain
Strain	0.020	
Stress	0.743	0.101

Table H.I: Tensile Correlation Matrix

Correlations (Pearson) for Flexural Tests:

	Thick	Man	Clength	CWidth	Length	Width	Depth	Mod
Man	0.000							
Clength	0.000	0.000						
CWidth	0.000	0.000	0.000					
Length	0.758	0.000	0.619	0.000				
Width	0.611	-0.001	-0.000	0.750	0.463			
Depth	0.989	-0.024	-0.010	0.018	0.745	0.619		
Mod	0.436	0.412	0.047	-0.197	0.355	0.165	0.340	
Vf	0.018	0.247	-0.112	-0.045	-0.060	-0.045	-0.094	0.465
Strain	-0.678	-0.104	-0.112	-0.024	-0.614	-0.494	-0.695	-0.358
Stress	-0.475	0.113	-0.290	-0.037	-0.533	-0.362	-0.546	0.157

	Vf	Strain
Strain	0.086	
Stress	0.370	0.764

Table H.2: Flexural Correlation Matrix

Parameter	Estimate (MPa)	Std Error of Estimate (MPa)
INTERCEPT	-66.7299688	109.9992003
MAN	0.4412671	
QMAN	3.6006574	
THICK	-9.2078990	
MANTHICK	0.6117555	
QMAN*THICK	-1.1815535	
MAN*CLENGTH	2.0729426	4.4807767
QMAN*CLENGTH	2.8783692	2.7220726
CLENGTH	-0.7770771	3.5121374
QLENGTH	3.2546465	2.0874804
MAN*CWIDTH	-1.0096862	3.6049669
QMAN*CWIDTH	-9.5313045	2.7537502
CWIDTH	3.5951286	3.8785015
QWIDTH	-1.5789014	2.1212518
CLENGTH* CWIDTH	-1.8626633	3.6289426
THICK* CLENGTH	-6.6393792	3.1654296
THICK*CWIDTH	-3.6137010	3.2147825
MOD	5.0341191	2.2087125
VF	612.8438161	266.8583860

Table H.3: Tensile Stress Parameter Estimates and Standard Errors

Variable	Level	N	Mean (MPa)
CThick	**-1**	15	278.73
	1	15	260.40
Man	**-1**	10	273.70
	0	10	245.00
	1	10	290.00
CLength	**-1**	10	279.20
	0	10	257.10
	1	10	272.40
C Width	**-1**	10	279.30
	0	10	263.60
	1	10	265.80

Table H.4: Tensile Stress Means

Source	DF	Type III SS	MS		
Whole-Plot:				**SS / S-P error**	
MAN	1	2.55	2.55	0.01	
QMAN	1	224.04	224.04	1.13	
THICK	1	2320.38	2320.38	11.67	
MAN*THICK	1	5.90	5.90	0.03	
QMAN*THICK	1	32.55	32.55	0.16	
Sub-Plot:				F	**P**
MAN*CLENGTH	1	42.56	42.56	0.21	0.6526
QMAN*CLENGTH	1	222.35	222.35	1.12	0.3130
CLENGTH	1	9.73	9.73	0.05	0.8289
QLENGTH	1	483.39	483.39	2.43	0.1473
MAN*CWIDTH	1	15.60	15.60	0.08	0.7846
QMAN*CWIDTH	1	2382.28	2382.28	11.98	0.0053
CWIDTH	1	170.86	170.86	0.86	0.3738
QWIDTH	1	110.17	110.17	0.55	0.4723
CLENGTH*CWIDTH	1	52.39	52.39	0.26	0.6179
THICK*CLENGTH	1	874.84	874.84	4.40	0.0599
THICK*CWIDTH	1	251.27	251.27	1.26	0.2849
Covariates:					
MOD	1	1033.01	1033.01	5.19	0.0436
VF	1	1048.76	1048.76	5.27	0.0423
Error	11	2187.41	198.86		
Total	29	25145.37			

R-Square: 0.913

Table H.5: Tensile Stress ANOVA

Parameter	**Estimate**	**Std Error of Estimate**
INTERCEPT	0.0178366667	0.00030271
MAN	-.0002100000	
QMAN	-.0003018519	
THICK	-.0002966667	
MANTHICK	0.0005500000	
QMAN*THICK	0.0001966667	
MAN*CLENGTH	-.0002312500	0.00041451
QMAN*CLENGTH	0.0004937500	0.00030895
CLENGTH	-.0001375000	0.00039080
QLENGTH	-.0001185185	0.00024371
MAN*CWIDTH	-.0001312500	0.00041451
QMAN*CWIDTH	-.0002395833	0.00030895
CWIDTH	-.0002541667	0.00039080
QWIDTH	0.0000814815	0.00024371
CLENGTH*CWIDTH	0.0001937500	0.00041451
THICK*CLENGTH	-.0001900000	0.00037074
THICK* CWIDTH	0.0000200000	0.00037074

Table H.6: Tensile Strain Parameter Estimates and Standard Errors

Variable	Level	N	Mean
Thick	-1	15	0.01813
	1	15	0.01754
Man	-1	10	0.01773
	0	10	0.01847
	1	10	0.01731
CLength	-1	10	0.01757
	0	10	0.01825
	1	10	0.01769
CWidth	-1	10	0.01810
	0	10	0.01801
	1	10	0.01740

Table H.7: Tensile Strain Means

Source	DF	Type III SS	MS		
Whole-Plot:				**SS / S-P error**	
MAN	1	0.00000088	0.00000088	0.32	
QMAN	1	0.00000422	0.00000422	1.53	
THICK	1	0.00000264	0.00000264	0.96	
MAN*THICK	1	0.00000605	0.00000605	2.20	
QMAN*THICK	1	0.00000232	0.00000232	0.84	
Sub-Plot:				**F**	**P**
MAN*CLENGTH	1	0.00000086	0.00000086	0.31	0.5864
QMAN*CLENGTH	1	0.00000702	0.00000702	2.55	0.1340
CLENGTH	1	0.00000034	0.00000034	0.12	0.7306
QLENGTH	1	0.00000065	0.00000065	0.24	0.6348
MAN*CWIDTH	1	0.00000028	0.00000028	0.10	0.7565
QMAN*CWIDTH	1	0.00000165	0.00000165	0.60	0.4519
CWIDTH	1	0.00000116	0.00000116	0.42	0.5268
QWIDTH	1	0.00000031	0.00000031	0.11	0.7435
CLENGTH*CWIDTH	1	0.00000060	0.00000060	0.22	0.6479
THICK*CLENGTH	1	0.00000072	0.00000072	0.26	0.6169
THICK*CWIDTH	1	0.00000001	0.00000001	0.00	0.9578
Error	13	0.00003574	0.00000275		
Total		0.00006807			

R-Square: 0.475

Table H.8: Tensile Strain ANOVA

Parameter	Estimate	Std Error of Estimate
INTERCEPT	350.9666667	8.52373342
MAN	8.9000000	
QMAN	24.9629630	
THICK	-30.6333333	
MAN*THICK	-13.4000000	
QMAN*THICK	2.8333333	
MAN*CLENGTH	1.3750000	11.67160266
QMAN*CLENGTH	-16.5416667	8.69949899
CLENGTH	-16.3333333	11.00409252
QLENGTH	2.3796296	6.86225734
MAN*CWIDTH	-7.6250000	11.67160266
QMAN*CWIDTH	-7.3750000	8.69949899
CWIDTH	0.0000000	11.00409252
QWIDTH	-3.9537037	6.86225734
CLENGTH*CWIDTH	8.8750000	11.67160266
THICK*CLENGTH	2.5500000	10.43939879
THICK*CWIDTH	-13.9500000	10.43939879

Table H.9: Flexural Stress Parameter Estimates and Standard Errors

Variable	Level	N	Mean
CThick	-1	15	381.6
	1	15	320.3
Man	-1	10	366.4
	0	10	302.3
	1	10	384.2
CLength	1	10	384.7
	0	10	329.4
	1	10	338.8
	-1(4 ply)	5	418.4
	0(4 ply)	5	359.0
	1(4 ply)	5	367.4
	-1(8 ply)	5	351.0
	0(8 ply)	6	299.8
	1 (8 ply)	4	312.75
Width	-1	10	360.9
	0	10	337.0
	1	10	355.0
	-1(4 ply)	5	368.4
	0(4 ply)	5	386.0
	1(4 ply)	5	390.4
	-1(8 ply)	5	353.4
	0(8 ply)	5	288.0
	1(8 ply)	5	319.6

Table H.10: Flexural Stress Means

Source	DF	Type III SS	MS		
Whole-Plot				**SS / S-P error**	
MAN	1	1584.20	1584.20	0.73	
QMAN	1	28842.92	28842.92	13.23	
THICK	1	28152.03	28152.03	12.92	
MANTHICK	1	3591.20	3591.20	1.65	
QMAN* THICK	1	481.67	481.67	0.22	
Sub-Plot:				**F**	**P**
MAN*CLENGTH	1	30.25	30.25	0.01	0.9080
QMAN*CLENGTH	1	7880.45	7880.45	3.62	0.0796
NESTED CLENGTH	2	4932	2466	1.13	0.3524
QLENGTH	1	262.10	262.10	0.12	0.7343
MAN *CWIDTH	1	930.25	930.25	0.43	0.5250
QMAN*CWIDTH	1	1566.45	1566.45	0.72	0.4119
NESTED CWIDTH	2	3892	1946	0.89	0.4332
QWIDTH	1	723.53	723.53	0.33	0.5744
CLENGTH*CWIDTH	1	1260.25	1260.25	0.58	0.4606
Error	13	28335.07	2179.62		
Total	29	124870.97			

R-Square: 0.773

Table H.ll: Flexural Stress ANOVA

Parameter	Estimate	**Std Error of Estimate**
INTERCEPT	0.0214833333	0.00059000
MAN	-.0005200000	
QMAN	0.0007314815	
THICK	-.0027633333	
MAN*THICK	-.0002600000	
QMAN*THICK	0.0002033333	
MAN*CLENGTH	0.0005500000	0.00080790
QMAN*CLENGTH	-.0008791667	0.00060217
CLENGTH	-.0002083333	0.00076169
QLENGTH	-.0000018519	0.00047500
MAN*CWTDTH	-.0006500000	0.00080790
QMAN*CWIDTH	-.0002375000	0.00060217
CWIDTH	-.0000250000	0.00076169
QWTDTH	-.0005351852	0.00047500
CLENGTH*CWIDTH	0.0007750000	0.00080790
THICK*CLENGTH	-.0007400000	0.00072260
THICK*CWIDTH	-.0012400000	0.00072260

Table H.12: Flexural Strain Parameter Estimates and Standard Errors

Variable	Level	N	Mean
CThick	-1	15	0.02425
	1	15	0.01872
Man	-1	10	0.02252
	0	10	0.02045
	1	10	0.02148
CLength	-1	10	0.02212
	0	10	0.02133
	1	10	0.02100
	-1(4 ply)	5	0.02392
	0(4 ply)	5	0.02454
	1(4 ply)	5	0.02428
	-1 (8 ply)	5	0.02032
	0 (8 ply)	6	0.01770
	1 (8 ply)	4	0.01825
Width	-1	10	0.02136
	0	10	0.02197
	1	10	0.02112
	-1(4 ply)	5	0.02194
	0(4 ply)	5	0.02662
	1(4 ply)	5	0.02418
	-1(8 ply)	5	0.02078
	0(8 ply)	5	0.01732
	1(8 ply)	5	0.01806

Table H.13: Flexural Strain Means

Source	DF	TypeHISS	MS		
Whole-Plot:				**SS / S-P error**	
MAN	1	0.00000541	0.00000541	0.52	
QMAN	1	0.00002477	0.00002477	2.37	
THICK	1	0.00022908	0.00022908	21.94	
MANTHICK	1	0.00000135	0.00000135	0.13	
QMAN*THICK	1	0.00000248	0.00000248	0.24	
Sub-Plot:				**F**	**P**
MAN*CLENGTH	1	0.00000484	0.00000484	0.46	0.5080
QMAN*CLENGTH	1	0.00002226	0.00002226	2.13	0.1680
NESTED CLENGTH	2	0.00001173	0.00000587	0.56	0.5835
QLENGTH	1	0.00000000	0.00000000	0.00	0.9969
MAN*CWIDTH	1	0.00000676	0.00000676	0.65	0.4355
QMAN*CWIDTH	1	0.00000162	0.00000162	0.16	0.6997
NESTED CWIDTH	2	0.00003076	0.00001538	1.47	0.2651
QWIDTH	1	0.00001326	0.00001326	1.27	0.2802
CLENGTH*CWIDTH	1	0.00000961	0.00000961	0.92	0.3549
Error	13	0.00013576	0.00001044		
Total	29	0.00049792			

R-Square: 0.727

Table H.14: Flexural Strain ANOVA

Appendix I Final Woven Roving Tests Data

Specimen Code	Length (mm)	**Width (mm)**	**Depth (mm)**	**Fibre Volume Fract.n**	**Initial Mod. (GPa)**	**Deflect.n at Fail (mm)**	**Failure Stress (MPa)**	**Failure Strain (%)**
VT/41/1A	100	10.28	3.88	0.33	14.8	4.3	213	1.810
VT/41/1B	100	9.79	4.00	0.34	15.5	4.9	243	1.932
VT/41/2A	100	25.01	3.88	0.34	15.0	5.6	219	1.656
VT/41/2B	100	24.42	4.19	0.33	14.6	5.6	215	1.733
VT/41/3A	300	9.77	3.96	0.33	16.5	8.1	249	1.902
VT/41/3B	300	9.34	4.04	0.36	15.2	8.8	240	1.948
VT/41/4A	300	25.07	4.18	0.36	15.3	8.9	218	1.672
VT/41/4B	300	25.36	4.22	0.32	15.1	8.8	216	1.718
VT/42/1A	100	10.31	3.80	0.36	15.5	3.6	199	1.687
VT/42/1B	100	10.30	3.91	0.36	15.2	3.6	181	1.472
VT/42/2A	100	25.16	3.92	0.35	15.2	4.9	191	1.580
VT/42/2B	100	25.29	3.74	0.36	16.6	4.8	188	1.472
VT/42/3A	300	10.55	3.79	0.36	16.2	6.3	188	1.472
VT/42/3B	300	10.33	3.80	0.38	15.4	5.6	178	1.656
VT/42/4A	300	25.03	3.82	0.38	15.9	8.6	214	1.840
VT/42/4B	300	25.33	4.01	0.36	15.7	8.6	203	1.810
VT/43/1A	100	10.06	4.32	0.31	12.0	2.7	100	1.381
VT/43/1B	100	10.19	4.36	0.33	12.4	2.7	98	1.319
VT/43/2A	100	24.87	4.43	0.31	11.7	4.0	101	1.764
VT/43/2B	100	24.51	4.37	0.34	11.3	4.2	101	1.595
VT/43/3A	300	10.10	4.29	0.31	12.7	5.4	103	1.350
VT/43/3B	300	10.31	4.24	0.32	12.3	4.8	102	1.411
VT/43/4A	300	24.77	4.41	0.33	12.6	6.6	102	1.411
VT/43/4B	300	24.77	4.44	0.32	12.8	6.3	102	1.3.50
VT/44/1A	100	9.88	4.16	0.35	12.7	2.9	108	1.411
VT/44/1B	100	10.02	4.38	0.36	12.2	3.1	106	1.626
VT/44/2A	100	24.90	4.17	0.34	12.5	4.2	115	1.549
VT/44/2B	100	25.03	4.35	0.32	9.7	4.5	110	1.672
VT/44/3A	300	10.09	4.15	0.35	11.9	3.8	91	0.890
VT/44/3B	300	10.32	4.04	0.36	13.6	4.8	104	1.166
VT/44/4A	300	25.02	4.33	0.35	12.5	6.0	99	1.074
VT/44/4B	300	4.19	24.91	0.36	11.8	5.8	101	1.135

Specimen Code	Length (mm)	Width (mm)	Depth (mm)	Fibre Volume Fract.n	Initial Mod. (GPa)	Deflect.n at Fail (mm)	Failure Stress (MPa)	Failure Strain (%)
VT/81/1A	100	10.00	7.27	0.36	16.5	5.8	249	1.779
VT/81/1B	H)0	10.27	7.25	0.34	16.7	6.0	254	1.810
VT/81/2A	100	24.77	7.36	0.36	16.7	8.1	256	1.794
VT/81/2B	100	25.50	7.32	0.35	16.1	8.1	255	1.856
VT/81/3A	300	10.82	6.97	0.33	17.2	10.6	267	1.902
VT/81/3B	300	10.46	6.85	0.35	17.9	10.3	281	1.886
VT/81/4A	300	25.44	7.02	0.35	17.8	10.1	244	1.610
VT/81/4B	300	25.76	6.97	0.32	17.9	11.5	257	1.702
VT/82/1A	100	10.33	7.72	0.37	14.2	3.3	98	1.195
VT/82/1B	100	10.54	7.87	0.38	13.3	3.8	93	1.166
VT/82/2A	100	24.77	7.93	0.38	10.4	4.8	104	1.595
VT/82/2B	100	24.94	7.76	0.38	13.9	4.6	103	1.595
VT/82/3A	300	10.18	7.44	0.38	14.7	5.4	103	1.472
VT/82/3B	300	10.05	7.40	0.37	14.6	5.8	109	1.564
VT/82/4A	300	24.96	7.64	0.37	13.8	7.0	101	1.442
VT/82/4B	300	25.19	7.50	0.38	15.1	7.2	106	1.626
VT/83/1A	100	1.0.28	7.60	0.34	15.1	5.2	211	1.748
VT/83/1B	100	10.37	7.40	0.35	16.6	5.5	238	1.779
VT/83/2A	100	25.25	7.53	0.35	15.5	5.6	214	1.718
VT/83/2B	100	24.99	8.03	0.35	14.9	6.7	209	1.687
VT/83/3A	300	10.52	6.61	0.35	17.8	9.4	252	2.086
VT/83/3B	300	10.72	6.65	0.34	16.2	9.4	241	1.932
VT/83/4A	300	25.44	7.09	0.35	16.9	9.6	216	1.595
VT/83/4B	300	25.60	7.25	0.35	16.6	9.8	207	1.564
VT/84/1A	100	10.21	7.71	0.36	12.9	3.8	107	1.534
VT/84/1B	100	10.18	7.47	0.37	13.7	4.0	122	1.595
VT/84/2A	100	24.66	7.86	0.36	12.7	6.0	124	1.902
VT/84/2B	100	24.76	7.88	0.37	13.9	5.6	124	1.994
VT/84/3A	300	10.27	7.38	0.36	12.7	6.2	117	1.595
VT/84/3B	300	10.24	7.35	0.38	15.3	5.4	125	1.488
VT/84/4A	300	24.66	7.55	0.38	14.6	8.5	134	1.748
VT/84/4B	300	24.80	7.45	0.35	14.7	8.5	129	1.503

Specimen Code	Length (mm)	**Width (mm)**	**Depth (mm)**	Fibre **Volume Fract.n**	Initial **Mod. (GPa)**	**Deflect.n at Fail (mm)**	**Failure Stress (MPa)**	**Failure Strain** (%)
VT/121/1A	100	10.60	10.10	0.37	16.7	9.5	283	1.948
VT/121/1B	100	10.36	10.16	0.37	16.1	6.9	260	1.871
VT/121/2A	100	25.45	10.13	0.36	17.0	7.2	222	1.932
VT/121/2B	100	25.59	10.07	0.37	17.0	9.6	236	2.024
VT/121/3A	300	10.47	11.21	0.36	14.3	12.6	256	2.147
VT/121/3B	300	10.46	11.04	0.37	16.0	11.1	224	1.748
VT/121/4A	300	25.42	11.28	0.37	15.8	11.2	176	1.656
VT/121/4B	300	25.49	11.17	0.37	15.4	13.4	188	1.748
VT/122/1A	100	10.34	11.70	0.34	13.7	9.6	258	1.871
VT/122/1B	100	10.76	11.79	0.35	13.4	9.6	197	1.902
VT/122/2A	100	25.59	11.62	0.33	14.0	11.0	145	1.472
VT/122/2B	100	25.33	11.90	0.34	13.9	12.7	157	1.748
VT/122/3A	300	10.55	11.21	0.34	14.5	11.1	228	2.086
VT/122/3B	300	10.33	10.93	0.33	14.7	14.8	262	2.116
VT/122/4A	300	25.55	11.36	0.34	14.9	14.0	174	1.871
VT/122/4B	300	25.46	11.54	0.34	14.5	15.1	187	1.963
VT/123/1A	100	10.37	10.17	0.34	13.3	4.3	118	1.840
VT/123/1B	100	10.35	10.15	0.32	14.4	4.4	117	1.840
VT/123/2A	100	25.47	10.29	0.34	14.1	6.6	124	1.902
VT/123/2B	100	25.57	10.28	0.33	13.8	6.6	126	1.840
VT/123/3A	300	10.58	11.07	0.34	12.1	6.7	101	1.534
VT/123/3B	300	10.51	11.10	0.35	12.0	7.2	103	1.810
VT/123/4A	300	25.58	11.36	0.33	13.0	9.0	111	1.656
VT/123/4B	300	25.37	11.12	0.33	13.0	9.2	113	1.595
VT/124/1A	100	10.62	11.69	0.36	12.9	4.5	101	1.687
VT/124/1B	100	10.50	11.72	0.37	12.1	4.3	97	1.656
VT/124/2A	100	25.36	11.84	0.35	12.1	6.8	114	2.055
VT/124/2B	100	25.28	11.75	0.34	12.5	6.2	118	1.779
VT/124/3A	300	10.52	11.20	0.58	11.7	7.1	105	1.687
VT/124/3B	300	10.33	10.94	0.36	11.8	6.8	107	1.626
VT/124/4A	300	25.58	11.39	0.34	12.3	9.9	119	1.810
VT/124/4B	300	25.55	11.61	0.35	11.6	10.0	118	1.840

Table I.I: Final Woven Roving Tensile Tests Results

Specimen Code	Length (mm)	**Width (mm)**	**Depth (mm)**	**Fibre Volume Fract.n**	**Initial Mod. (GPa)**	**Deflect.n at Fail (mm)**	**Failure Stress (MPa)**	**Failure Strain** (%)
VF/41/1A	55	10.31	4.01	0.35	14.5	3.5	303	2.250
VF/41/1B	55	9.66	3.70	0.34	14.0	4.5	429	2.750
VF/41/2A	55	29.89	4.01	0.32	15.3	3.5	295	1.963
VF/41/2B	55	30.39	4.01	0.36	13.7	5.1	398	3.275
VF/41/3A	TOO	9.87	3.87	0.36	14.7	15.0	389	2.850
VF/41/3B	100	9.74	3.98	0.34	~	11.8	331	~
VF/41/4A	100	30.06	3.78	0.31	14.2	15.0	387	3.050
VF/41/4B	100	30.35	4.20	0.35	13.5	15.2	336	2.200
VF/42/1A	55	10.17	3.75	0.34	15.3	3.3	296	2.250
VF/42/1B	55	10.18	3.70	0.34	12.6	3.7	297	2.475
VF/42/2A	55	29.78	3.79	0.38	15.8	3.7	302	2.275
VF/42/2B	55	29.80	3.63	0.37	15.8	4.6	334	2.863
VF/42/3A	100	10.21	3.67	0.36	10.8	9.2	214	2.550
VF/42/3B	100	10.31	3.58	0.34	14.5	9.8	290	2.450
VF/42/4A	100	29.86	4.09	0.36	13.8	12.8	281	2.450
VF/42/4B	100	29.82	4.04	0.35	13.3	12.0	280	2.525
VF/43/1A	55	9.41	4.24	0.33	11.0	3.1	214	2.300
VF/43/1B	55	10.33	4.25	0.32	12.7	3.7	241	3.400
VF/43/2A	55	29.22	4.41	0.33	10.7	4.9	299	3.325
VF/43/2B	55	29.98	4.39	0.33	~	4.5	286	~
VF/43/3A	100	10.09	4.32	0.32	12.0	10.5	227	2.225
VF/43/3B	100	10.20	4.32	0.33	11.2	12.4	223	2.600
VF/43/4A	100	29.99	4.41	0.33	11.1	12.6	281	2.713
VF/43/4B	100	29.09	4.30	0.33	11.9	12.8	295	2.750
VF/44/1A	55	9.85	3.70	0.32	15.7	3.4	395	2.650
VF/44/1B	55	10.18	4.26	0.33	11.1	4.4	282	2.575
VF/44/2A	55	29.67	4.37	0.34	14.7	4.3	287	2.875
VF/44/2B	55	29.81	4.36	0.35	13.9	4.1	300	2.225'
VF/44/3A	100	10.09	4.25	0.32	10.5	11.6	257	2.800
VF/44/3B	100	9.97	4.28	0.34	9.6	12.2	246	3.600
VF/44/4A	100	29.64	4.09	0.35	14.6	14.0	312	2.675
VF/44/4B	100	29.78	4.09	0.36	14.8	10.2	291	2.250

Specimen Code	Length (mm)	Width (mm)	Depth (mm)	Fibre Volume Fract.n	Initial Mod. (GPa)	Deflect.n at Fail (mm)	Failure Stress (MPa)	Failure Strain (%)
VF/81/1A	105	20.0	7.4	0.34	17.4	7.4	380	2.400
VF/81/1B	105	20.1	7.7	0.34	16.3	7.2	353	2.475
VF/81/2A	105	59.9	8.0	0.32	15.7	13.5	375	2.700
VF/81/2B	105	59.5	7.8	0.33	16.6	13.9	414	2.575
VF/81/3A	200	19.5	7.5	0.35	15.4	26.8	333	2.325
VF/81/3B	200	19.8	7.4	0.35	15.8	30.2	329	2.200
VF/81/4A	200	59.8	7.5	0.33	16.0	26.4	336	2.350
VF/81/4B	200	59.7	7.5	0.34	15.2	29.2	343	~
VF/82/1A	105	19.8	7.5	0.34	15.4	6.2	228	1.750
VF/82/1B	105	20.0	6.7	0.34	19.0	6.0	392	2.125
VF/82/2A	105	60.5	7.6	0.37	17.2	6.2	297	1.950
VF/82/2B	105	60.0	7.4	0.35	15.8	5.6	266	1.850
VF/82/3A	200	19.7	6.8	0.33	19.1	25.2	440	2.450
VF/82/3B	200	19.8	6.7	0.35	19.0	25.8	459	2.525
VF/82/4A	200	60.4	7.3	0.36	17.9	26.0	368	2.425
VF/82/4B	200	60.5	7.3	0.35	17.0	23.6	342	2.050
VF/83/1A	105	19.7	7.1	0.37	15.2	7.8	326	2.500
VF/83/1B	105	19.7	7.8	0.40	13.1	7.6	232	2.825
VF/83/2A	105	60.5	7.6	0.36	15.9	6.2	256	1.813
VF/83/2B	105	60.5	7.6	0.35	15.4	6.8	293	2.125
VF/83/3A	200	19.6	7.9	0.39	11.3	24.2	190	2.175
VF/83/3B	200	19.8	7.6	0.41	12.0	25.4	203	~
VF/83/4A	200	60.5	7.5	0.38	14.1	24.4	264	2.050
VF/83/4B	200	60.3	7.5	0.35	13.9	23.4	253	2.050
VF/84/1A	105	19.5	7.4	0.37	15.0	7.2	292	2.500
VF/84/1B	105	19.6	7.4	0.36	13.3	7.2	292	2.538
VF/84/2A	105	60.7	7.7	0.34	13.1	7.2	303	2.625
VF/84/2B	105	60.6	7.4	0.36	15.7	8.0	325	2.275
VF/84/3A	200	19.8	7.7	0.37	12.9	18.0	222	2.325
VF/84/3B	200	19.4	7.5	0.38	13.2	18.8	235	2.063
VF/84/4A	200	61.0	7.7	0.37	15.0	20.2	245	2.075
VF/84/4B	200	61.2	7.7	0.36	14.6	19.0	242	1.988

Specimen Code	Length (mm)	Width (mm)	Depth (mm)	Fibre Volume Fract.n	Initial Mod. (GPa)	Deflect.n at Fail (mm)	Failure Stress (MPa)	Failure Strain (%)
VF/121/1A	162	31.10	10.61	0.36	19.1	10.8	353	2.175
VF/121/1B	162	30.65	10.40	0.36	19.2	11.8	383	2.475
VF/121/2A	162	90.50	10.59	0.36	20.3	12.0	352	~
VF/121/2B	162	89.56	10.45	0.36	18.8	9.6	324	1.900
VF/121/3A	300	30.46	10.51	0.36	19.1	34.8	332	2.075
VF/121/3B	300	30.31	10.39	0.36	19.8	32.4	314	1.700
VF/121/4A	300	90.03	10.38	0.37	19.3	34.8	314	1.875
VF/121/4B	300	90.08	10.48	0.36	18.7	33.6	328	2.025
VF/122/1A	162	30.75	11.21	0.34	17.5	9.8	302	1.825
VF/122/1B	162	30.64	11.24	0.34	16.9	10.4	333	2.200
VF/122/2A	162	89.93	10.58	0.35	19.1	11.0	351	~
VF/122/2B	162	89.93	11.46	0.33	17.5	10.4	295	1.925
VF/122/3A	300	30.67	11.29	0.32	17.4	28.8	260	1.700
VF/122/3B	300	30.63	11.42	0.33	17.4	32.4	281	1.825
VF/122/4A	300	90.20	11.37	0.34	15.3	30.4	273	2.050
VF/122/4B	300	90.08	11.54	0.34	17.6	30.8	279	1.900
VF/123/1A	162	30.74	12.08	0.33	14.7	10.2	253	2.388
VF/123/1B	162	30.90	11.83	0.34	13.8	9.8	234	2.250
VF/123/2A	162	90.51	11.36	0.34	16.3	11.0	308	2.150
VF/123/2B	162	89.90	11.29	0.35	15.4	10.8	293	2.275
VF/123/3A	300	30.71	11.45	0.34	13.4	34.4	240	2.225
VF/123/3B	300	30.53	11.43	0.35	14.5	31.6	227	2.225
VF/123/4A	300	90.32	11.87	0.34	14.3	32.0	259	2.025
VF/123/4B	300	89.67	11.94	0.34	14.6	32.8	270	2.225
VF/124/1A	162	30.65	11.60	0.35	~	9.6	260	—
VF/124/1B	162	30.64	11.53	**0.34**	14.8	10.6	275	2.375
VF/124/2A	162	90.20	11.39	**0.34**	18.1	10.6	297	1.925
VF/124/2B	162	89.93	11.34	**0.34**	17.3	9.8	284	1.900
VF/124/3A	300	30.87	11.36	0.35	15.2	31.2	250	1.913
VF/124/3B	300	30.34	11.43	**0.34**	15.4	33.2	255	2.250
VF/124/4A	300	90.41	11.44	**0.34**	15.8	33.2	284	2.075
VF/124/4B	300	90.15	11.57	**0.34**	15.7	26.8	239	~

Table 1.2: Final Woven Roving Flexural Tests Results

Specimen Code	Length (mm)	Width (mm)	Depth (mm)	Initial Mod. (GPa)	Deflect.n at Fail (mm)	Failure Stress (MPa)	Failure Strain (%)
VTGR/1	180	25.07	4.25	15.67	0.69	226.5	1.779
VTGR/2	180	25.05	4.06	15.06	0.78	259.4	2.208
VTGR/3	180	25.08	4.12	15.17	0.69	235.9	1.871
VTGR/4	180	25.05	4.22	15.18	0.70	238.9	1.932
VTGR/5	180	25.06	4.39	14.25	0.66	209.1	1.81
VTGR/6	180	24.96	4.23	15.44	0.71	239.2	1.932
VTGT/1	180	25.07	4.43	13.70	0.78	235.3	2.147
VTGT/2	180	25.06	4.28	13.86	0.74	244.7	2.178
VTGT/3	180	24.03	4.15	15.42	0.74	255.7	2.086
VTGT/4	180	24.95	4.19	15.41	0.69	250.0	2.162
VTGT/5	180	25.01	4.28	15.11	0.69	238.2	2.178
VTGT/6	180	25.00	4.27	14.51	0.74	243.6	2.116
VTGW/1	180	20.01	4.17	15.26	0.65	242.7	2.055
VTGW/2	180	19.77	4.12	14.22	0.67	254.8	2.178
VTGW/3	180	19.88	4.22	14.29	0.65	238.4	2.055
VTGW/4	180	19.84	4.31	14.89	0.72	260.2	2.178
VTGW/5	180	19.86	4.21	14.88	0.65	243.7	2.024
VTGW/6	180	20.01	4.14	15.55	0.66	244.4	2.024

Table 1.3: Tensile Geometry Tests Results

Specimen Code	Length (mm)	Width (mm)	Depth (mm)	Fibre VolFn	Max Load (kN)	ILSS (MPa)
VS/122/1A	50	10.28	11.14	0.34	5.3	34.4
VS/122/1B	50	10.87	10.07	0.33	6.0	41.1
VS/122/1C	50	10.24	11.01	0.34	**4.7**	31.3
VS/122/1D	50	11.02	11.03	0.33	5.6	34.4
VS/122/2A	50	25.43	11.26	0.28	13.5	35.4
VS/122/2B	50	25.91	11.10	0.34	13.2	34.4
VS/122/2C	50	25.44	11.13	0.34	12.8	33.9
VS/122/2D	50	25.83	11.42	0.33	11.7	29.7
VS/122/3A	50	30.56	11.34	0.33	16.2	35.1
VS/122/3B	50	30.92	11.21	0.34	16.0	34.6
VS/122/3C	50	31.18	11.23	0.33	16.6	35.6
VS/122/3D	50	30.57	11.40	0.33	16.5	35.5

Table **1.4:** Interlaminar Shear Strength Tests Results

Appendix J Final Woven Roving Tests Data Analysis

Correlations (Pearson) for Final Woven Roving Tensile Tests:

	Thick	Butts	Skew	Clength	CWidth	Length	Width	Depth
Butts	0.000							
Skew	0.000	0.000						
Clength	0.000	0.000	0.000					
CWidth	0.000	0.000	0.000	0.000				
Length	0.000	0.000	0.000	1.000	0.000			
Width	0.059	-0.031	-0.033	-0.022	0.959	-0.022		
Depth	0.769	0.119	0.084	0.056	0.087	0.056	-0.044	
Vf	0.164	0.357	-0.091	0.105	-0.120	0.105	-0.125	0.155
Mod	0.018	-0.378	-0.560	0.111	-0.026	0.111	0.024	-0.153
Strain	0.471	-0.254	-0.251	-0.103	0.023	-0.103	0.097	0.223
Stress	0.051	-0.436	-0.591	0.008	-0.082	0.008	-0.043	-0.083

	Vf	Mod	Strain
Mod	-0.010		
Strain	-0.033	0.365	
Stress	-0.092	0.821	0.591

Table J.I: Tensile Correlation Matrix

Correlations (Pearson) **for Final Woven Roving Flexural Tests:**

	Thick	Butts	Skew	Clength	CWidth	Length	Width	Depth
Butts	0.000							
Skew	0.000	0.000						
Clength	0.000	0.000	0.000					
CWidth	0.000	0.000	0.000	0.000				
Length	0.781	0.000	0.000	0.578	0.000			
Width	0.608	0.002	0.000	-0.000	0.736	0.475		
Depth	0.992	-0.005	0.072	0.002	0.015	0.776	0.611	
Vf	0.103	-0.011	0.040	0.060	0.027	0.106	0.058	0.063
Mod	0.608	0.085	-0.539	-0.131	0.129	0.402	0.462	0.531
Strain	-0.629	-0.149	0.162	-0.093	-0.101	-0.556	-0.477	-0.605
Stress	-0.069	-0.089	-0.578	-0.195	0.103	-0.176	0.023	-0.139

	Vf	Mod	Strain
Mod	0.094		
Strain	-0.203	-0.563	
Stress	-0.160	0.544	0.191

Table J.2: Flexural Correlation Matrix

Whole-Plot Mean Values and Standard Errors for Vospers Tensile Stress:

THICK	BUTTS	SKEW	LSMEAN	STDERR
4	.	.	158.947	3.92419
8	.	.	171.453	4.64526
12	.	.	166.196	3.82364
.	1	.	188.316	3.71203
.	2	.	142.748	3.71203
.	.	1	195.924	4.47305
.	.	2	135.140	4.47305
4	1	.	167.145	5.40182
4	2	.	150.750	5.23166
8	1	.	227.380	9.21203
8	2	.	115.527	5.36039
12	1	.	170.424	5.17045
12	2	.	161.968	6.30798
4	.	1	203.067	6.35177
4	.	2	114.827	8.69110
8	.	1	173.611	6.22806
X	.	2	169.296	5.55497
12	.	1	211.094	5.72983
12	.	2	121.298	7.39272
.	1	1	227.894	7.27270
.	1	2	148.739	4.36285
.	2	1	163.955	4.20553
.	2	2	121.542	6.77597

Table J.3: Tensile Stress Whole-Plot Means and Standard Errors

THICK	BUTTS	SKEW	**CLENGTH**	**CWIDTH**	**LSMEAN**	**STDERR**
.	.	.	1	.	166.200	2.36657
.	.	.	2	.	164.864	2.36657
.	.	.	.	1	170.290	2.30919
.	.	.	.	2	160.774	2.30919
.	.	.	1	1	169.091	3.26449
.	.	.	1	2	163.309	3.38712
.	.	.	2	1	171.489	3.28486
.	.	.	2	2	158.240	3.32922
4	.	.	1	.	159.740	4.44562
4	.	. .	2	.	158.154	4.03803
8	.	.	1	.	171.207	4.04822
8	.	.	2	.	171.700	5.66576
12	.	.	1	.	167.653	4.00562
12	.	.	2	.	164.739	4.39201
4	.	.	.	1	158.173	4.07425
4	.	.	.	2	159.721	4.35748
8	.	.	.	1	173.336	4.79743
8	.	.	.	2	169.570	4.47631
1 2	.	.	.	1	179.361	4.26117
1 2	.	.	.	2	153.032	4.04603
.	1	.	.	1	196.076	3.74010
.	1	.	.	2	180.556	3.71799
.	2	.	.	1	144.504	3.62632
.	2	.	.	2	140.993	3.84182
.	1	.	1	.	190.780	3.61970
.	1	.	2	.	185.853	3.84987
.	2	.	1	.	141.621	4.19288
.	2	.	2	.	143.876	3.40561
.	.	1	.	1	206.944	4.26683
.	.	1	.	2	184.905	4.16911
.	.	2	.	1	133.636	4.11294
.	.	2	.	2	136.644	4.32617
.	.	1	1	.	196.418	3.82527
.	.	1	2	.	195.430	4.67483
.	.	2	1	.	135.982	4.47476
.	.	2	2	.	134.298	3.98190

Table .1.4: Tensile Stress Sub-Plot Means and Standard Errors

Source	DF	AdjSS	AdjMS	F	P
Whole-Plot:					
Thick	2	1608	804	1.96	0.337
(Linear)	1	838	838	2.05	0.289
(Quadratic)	1	829	829	2.03	0.291
Butts	1	22345	22345	54.57	0.018
Skew	1	24027	24027	58.67	0.017
Thick*Butts	2	35976	17988	43.93	0.022
(Linear)	1	196	196	0.48	0.560
(Quadratic)	1	31008	31008	75.72	0.0130
Thick* Skew	2	21388	10694	26.11	0.037
(Linear)	1	9	9	0.02	0.896
(Quadratic)	1	20502	20502	50.07	0.0194
Butts* Skew	1	7924	7924	19.35	0.048
Error (Thick*Butts* Skew)	2	819	410		
Sub-Plot:					
CLength	1	39	39	0.15	0.698
Thick*CLength	2	35	17	0.07	0.934
(Linear)	1	6	6	0.02	0.876
(Quadratic)	1	32	32	0.13	0.724
CWidth	1	2161	2161	8.47	0.005
Thick*CWidth	2	3317	1659	6.5	0.003
(Linear)	1	2962	2962	11.61	0.001
(Quadratic)	1	394	394	1.54	0.218
CLength*CWidth	1	326	326	1.28	0.262
Butts*CLength	1	300	300	1.17	.282
Butts*CWidth	1	862	862	3.38	0.070
Skew* CLength	1	3	3	0.01	0.916
Skew*CWidth	1	3761	3761	14.74	0.000
Covariates:					
Initial Modulus	1	1240	1240	4.86	0.0307
Error	72	18377	255		
Total	95	361914			

R-Square: 0.949

Unusual Observations for Tensile Stress:

Obs.	Stress	Fit	Stdev.Fit	Residual	StResid
43	104.400	77.337	9.863	27.063	2.15R
73	258.300	212.123	7.972	46.177	3.34R
75	145.300	184.616	7.887	-39.316	-2.83R
76	156.600	184.386	7.902	-27.786	-2.00R
78	262.400	222.506	8.115	39.894	2.90R

R denotes an obs. with a large st. resid.

Table J.5: Tensile Stress ANOVA

THICK	BUTTS	SKEW	LSMEAN	STDERR
4	.	.	1.54575	0.030553
8	.	.	1.67069	0.030553
12	.	.	1.82063	0.030553
.	1	.	1.73940	0.024946
.	2	.	1.61865	0.024946
.	.	1	1.73869	0.024946
.	.	2	1.61935	0.024946
4	1	.	1.62200	0.043208
4	2	.	1.46950	0.043208
8	1	.	1.77800	0.043208
8	2	.	1.56338	0.043208
12	1	.	1.81819	0.043208
12	2	.	1.82306	0.043208
4	.	1	1.71000	0.043208
4	.	2	1.38150	0.043208
8	.	1	1.62463	0.043208
8	.	2	1.71675	0.043208
12	.	1	1.88144	0.043208
12	.	2	1.75981	0.043208
.	1	1	1.82433	0.035279
.	1	2	1.65446	0.035279
.	2	1	1.65304	0.035279
.	2	2	1.58425	0.035279

Table .1.6: Tensile Strain Whole-Plot Means and Standard Errors

THICK	BUTTS	SKEW	CLENGTH	CWIDTH	LSMEAN	STDERR
.	.	.	1	.	1.70360	0.021341
.	.	.	2	.	1.65444	0.021341
.	.	.	.	1	1.67360	0.021341
.	.	.	.	2	1.68444	0.021341
.	.	.	1	1	1.66079	0.030181
.	.	.	1	2	1.74642	0.030181
.	.	.	2	1	1.68642	0.030181
.	.	.	2	2	1.62246	0.030181
4	.	.	1	.	1.60369	0.036964
4	.	.	2	.	1.48781	0.036964
8	.	.	1	.	1.67169	0.036964
8	.	.	2	.	1.66969	0.036964
12	.	.	1	.	1.83544	0.036964
12	.	.	2	.	1.80581	0.036964
4	.	.	.	1	1.52706	0.036964
4	.	.	.	2	1.56444	0.036964
8	.	.	.	1	1.65819	0.036964
8	.	.	.	2	1.68319	0.036964
12	.	.	.	1	1.83556	0.036964
12	.	.	.	2	1.80569	0.036964
.	1	.	.	1	1.77971	0.030181
.	1	.	.	2	1.69908	0.030181
.	2	.	.	1	1.56750	0.030181
.	2	.	.	2	1.66979	0.030181
.	1	.	1	.	1.77325	0.030181
.	1	.	2	.	1.70554	0.030181
.	2	.	1	.	1.63396	0.030181
.	2	.	2	.	1.60333	0.030181
.	.	1	.	1	1.76425	0.030181
.	.	1	.	2	1.71313	0.030181
.	.	2	.	1	1.58296	0.030181
.	.	2	.	2	1.65575	0.030181
.	.	1	1	.	1.70417	0.030181
.	.	1	2	.	1.77321	0.030181
.	.	2	1	.	1.70304	0.030181
.	.	2	2	.	1.53567	0.030181

Table J.7: Tensile Strain Sub-Plot Means and Standard Errors

Source	DF	AdjSS	AdjMS	F	P
Whole-Plot					
Thick	2	1.21223	0.60612	20.29	0.047
(linear)	1	1.20890	1.20890	40.47	0.024
(Quadratic)	1	0.00333	0.00333	0.11	0.770
Butts	1	0.34993	0.34993	11.71	0.076
Skew	1	0.34177	0.34177	11.44	0.077
Thick*Butts	2	0.20482	0.10241	3.43	0.226
(linear)	1	0.09907	0.09907	3.32	0.210
(Quadratic)	1	0.10576	0.10576	3.54	0.201
Thick* Skew	2.	0.70776	0.35388	11.85	0.078
(linear)	1	0.17119	0.17119	5.73	0.139
(Quadratic)	1	0.53658	0.53658	17.96	0.051
Butts* Skew	1	0.06131	0.06131	2.05	0.288
Error (Thick*Butts*Skew)	2	0.05974	0.02987		
Sub-Plot:					
CLength	1	0.05802	0.05802	2.65	0.108
Thick*CLength	2	0.05645	0.02823	1.29	0.281
(linear)	1	0.02976	0.02976	1.36	0.247
(Quadratic)	1	0.02670	0.02670	1.22	0.273
CWidth	1	0.00282	0.00282	0.13	0.721
Thick*CWidth	2	0.02050	0.01025	0.47	0.628
(linear)	1	0.01809	0.01809	0.83	0.366
(Quadratic)	1	0.00241	0.00241	0.11	0.741
CLength*CWidth	1	0.13425	0.13425	6.14	0.016
Butts* CLength	1	0.00825	0.00825	0.38	0.541
Butts* CWidth	1	0.20075	0.20075	9.18	0.003
Skew* CLength	1	0.33536	0.33536	15.34	0.000
Skew*CWidth	1	0.09213	0.09213	4.21	0.044
Error	73	1.59587	0.02186		
Total	95	5.44197			

R-Square: 0.707

Unusual Observations for Tensile Strain:

Obs.	Strain	Fit	Stdev.Fit	Residual	St.Resid
26	i .62600	1.29035	0.07237	0.33565	2.60R
29	0.80000	1.14960	0.07237	-0.25960	-2.01R
53	2.08600	1.73390	0.07237	0.35210	2.73R
70	1.74800	2.04831	0.07237	-0.30031	-2.33R
75	1.47200	1.86227	0.07237	-0.39027	-3.03R
93	1.68700	1.69950	0.14038	-0.01250	-0.25X

R denotes an obs. with a large st. resid.
X denotes an obs. whose x value gives it large influence

Table J.8: Tensile Strain ANOVA

THICK	BUTTS	SKEW	LSMEAN	STDERR
4	.	.	333.918	15.4273
8	.	.	302.406	10.5761
12	.	.	261.427	14.3093
.	1	.	307.722	8.6231
.	2	.	290.778	8.6231
.	.	1	308.169	11.4588
.	.	2	290.331	11.4588
4	1	.	348.380	19.9257
4	' 2	.	319.455	17.4873
8	1	.	307.950	14.8228
8	2	.	296.862	15.3572
12	1	.	266.837	18.2753
12	2	t	256.016	17.2089
4	.	1	339.755	15.8567
4	.	2	328.080	22.5073
8	.	1	323.835	17.7868
8	.	2	280.977	16.2494
12	.	1	260.917	23.8894
12	.	2	261.936	14.8102
.	1	1	326.059	14.7228
.	1	2	289.386	16.2166
.	2	1	290.279	13.8887
.	2	2	291.276	12.8730

Table ,1.9: Flexural Stress Whole-Plot Means and Standard Errors

THICK	BUTTS	SKEW	CLENGTH	CWIDTH	LSMEAN	STDERR
.	.	.	1	.	304.429	4.9657
.	.	.	2	.	294.071	4.9657
.	.	.	.	1	299.060	4.9628
.	.	.	.	2	299.440	4.9628
.	.	.	1	1	306.424	6.8633
.	.	.	1	2	302.433	7.2003
.	.	.	2	1	291.696	7.1498
.	.	.	2	2	296.446	6.8636
4	.	.	1	g	337.024	9.8574
4	.	.	2	.	330.811	11.4269
8	.	.	1	.	305.032	8.5757
8	.	.	2	.	299.779	8.4058
12	.	.	1	.	271.229	10.6914
12	.	.	2	.	251.624	9.5309
4	.	.	.	1	330.723	11.4638
4	.	.	.	2	337.113	9.8291
8	.	.	.	1	305.051	8.4106
8	.	.	.	2	299.761	8.5366
12	.	.	.	1	261.408	9.4781
12	.	.	.	2	261.446	10.7616
.	1	.	.	1	299.368	6.9590
.	1	.	.	2	316.077	6.8662
.	2	.	.	1	298.752	6.9250
.	2	.	.	2	282.803	7.2348
.	1	.	1	.	314.489	6.8726
.	1	.	2	.	300.955	7.0777
.	2	.	1	.	294.368	7.0801
.	2	.	2	t	287.187	6.8731
.	.	1	.	1	310.822	8.1718
.	.	1	.	2	305.516	8.0718
.	.	2	.	1	287.299	9.4620
.	.	2	.	2	293.364	7.1982
.	.	1	1	.	309.171	8.4877
.	.	1	2	.	307.167	7.7924
.	.	2	1	t	299.686	7.4460
.	.	2	2	.	280.976	8.9833

Table .1.10: Flexural Stress Sub-Plot Means and Standard Errors

Source	DF	AdjSS	AdjMS	F	P
Whole-Plot					
Thick	2	29871	14936	4.27	0.190
(Linear)	1	27687	27687	7.91	0.107
(Quadratic)	1	462	462	0.13	0.751
Butts	1	6629	6629	1.89	0.303
Skew	1	2935	2935	0.84	0.456
Thick* Butts	2	1707	854	0.24	0.804
(Linear)	1	1260	1260	0.36	0.609
(Quadratic)	1	401	401	0.11	0.767
Thick* Skew	2	8156	4078	1.17	0.462
(Linear)	1	613	613	0.18	0.716
(Quadratic)	1	7467	7467	2.13	0.282
Butts* Skew	1	7686	7686	2.20	0.277
Error (Thick*Butts*Skew)	2	7000	3500		
Sub-Plot:					
Nested Length	3	12237	4079	2.64	0.056
Nested Width	3	4636	1545	1.00	0.398
Clength*CWidth	1	458	458	0.41	0.526
Butts*Clength	1	242	242	0.21	0.645
Butts*CWidth	1	6228	6228	5.51	0.022
Skew*Clength	1	1652	1652	1.46	0.231
Skew*CWidth	1	700	700	0.62	0.434
Covariates:					
Initial Modulus	1	31417	31417	27.79	0.000
Error	72	81387	1130		
Total	95	290594			

R-Square: 0.720

Unusual Observations for Flexural Stress:

Obs.	Stress	Fit	Stdev.Fit	Residual	St.Resid
2	429.000	346.507	16.620	82.493	2.82R
3	295.000	382.653	16.670	-87.653	-3.00R
41	228.000	323.739	18.081	-95.739	-3.38R
46	459.000	389.206	17.262	69.794	2.42R

R denotes an obs. with a large st. resid.

Table J.11: Flexural Stress ANOVA

THICK	BUTTS	SKEW	LSMEAN	STDERR
4	.	.	2.66606	0.051013
8	.	.	2.27094	0.051013
12	.	.	2.06644	0.051013
.	1	.	2.39238	0.041652
.	2	.	2.27658	0.041652
.	.	1	2.27148	0.041652
.	.	2	2.39748	0.041652
4	1	.	2.73913	0.072144
4	2	.	2.59300	0.072144
8	1	.	2.32000	0.072144
8	2	.	2.22188	0.072144
12	1	.	2.11800	0.072144
12	2	.	2.01488	0.072144
4	.	1	2.56413	0.072144
4	.	2	2.76800	0.072144
8	.	1	2.28313	0.072144
8	.	2	2.25875	0.072144
12	.	1	1.96719	0.072144
12	.	2	2.16569	0.072144
.	1	1	2.36263	0.058905
.	1	2	2.42213	0.058905
.	2	1	2.18033	0.058905
.	2	2	2.37283	0.058905

Table .1.12: Flexural Strain Whole-Plot Means and Standard Errors

THICK	BUTTS	SKEW	CLENGTH	CWIDTH	LSMEAN	STDERR
.	.	.	1	.	2.37081	0.042457
.	.	.	2	.	2.29815	0.042457
.	.	.	.	1	2.37377	0.042457
.	.	.	.	2	2.29519	0.042457
.	.	.	1	1	2.41013	0.060044
.	.	.	1	2	2.33150	0.060044
.	.	.	2	1	2.33742	0.060044
.	.	.	2	2	2.25887	0.060044
4	.	.	1	.	2.67350	0.073538
4	.	.	2	.	2.65863	0.073538
8	.	.	1	.	2.31625	0.073538
8	.	.	2	.	2.22562	0.073538
12	.	.	1	.	2.12269	0.073538
12	.	.	2	.	2.01019	0.073538
4	.	.	.	1	2.66094	0.073538
4	.	.	.	2	2.67119	0.073538
8	.	.	.	1	2.33688	0.073538
8	.	.	.	2	2.20500	0.073538
12	.	.	.	1	2.12350	0.073538
12	.	.	.	2	2.00938	0.073538
.	1	.	.	1	2.40992	0.060044
.	1	.	.	2	2.37483	0.060044
.	2	.	.	1	2.33763	0.060044
.	2	.	.	2	2.21554	0.060044
.	1	.	1	.	2.48046	0.060044
.	1	.	2	.	2.30429	0.060044
.	2	.	1	.	2.26117	0.060044
.	2	.	2	.	2.29200	0.060044
.	.	1	.	1	2.27792	0.060044
.	.	1	.	2	2.26504	0.060044
.	.	2	.	1	2.46963	0.060044
.	.	2	.	2	2.32533	0.060044
.	.	1	1	.	2.26108	0.060044
.	.	1	2	.	2.28188	0.060044
.	.	2	1	.	2.48054	0.060044
.	.	2	2	.	2.31442	0.060044

Table J.13: Flexural Strain Sub-Plot Means and Standard Errors

Source	DF	AdjSS	AdjMS	F	P
Whole-Plot:					
Thick	2	5.94660	2.97330	35.70	0.027
(Linear)	1	5.75280	5.75280	69.08	0.014
(Quadratic)	1	0.19380	0.19380	2.33	0.267
Butts	1	0.32179	0.32179	3.86	0.188
Skew	1	0.38102	0.38102	4.58	0.166
Thick*Butts	2	0.01114	0.00557	0.07	0.937
(Linear)	1	0.00740	0.00740	0.09	0.794
(Quadratic)	1	0.00375	0.00375	0.04	0.892
Thick* Skew	2	0.27147	0.13573	1.63	0.380
(Linear)	1	0.00012	0.00012	0.00	0.974
(Quadratic)	1	0.27135	0.27135	3.26	0.213
Butts* Skew	1	0.10613	0.10613	1.27	0.376
Error (Thick*Butts*Skew)	2	0.16655	0.08328		
Sub-Plot:					
Nested Length	3	0.16872	0.05624	0.65	0.586
Nested Width	3	0.24417	0.08139	0.94	0.426
Clength*C Width	1	0.00000	0.00000	0.00	0.999
Butts* Clength	1	0.25709	0.25709	2.97	0.089
Butts*CWidth	1	0.04541	0.04541	0.52	0.471
Skew* Clength	1	0.20963	0.20963	2.42	0.124
Skew*CWidth	1	0.10362	0.10362	1.20	0.277
Error	73	6.3164	0.08653		
Total	95	14.54974			

R-Square: 0.584

Unusual Observations for Flexural Strain:

Obs.	Strain	Fit	Stdev.Fit	Residual	St.Resid
3	1.96300	2.72067	0.14398	-0.75767	-2.95R
4	3.27500	2.72067	0.14398	0.55433	2.16R
17	2.30000	2.94167	0.14398	-0.64167	-2.50R
30	3.60000	2.75329	0.14398	0.84671	3.30R

R denotes an obs. with a large st. resid.

Table J.14: Flexural Strain ANOVA

Tensile Geometry Tests:

Variable	Geom	N	**Mean**	**Median**	**TrMean**	**StDev**	**SEMean**
Modulus	1	6	15.128	15.175	15.128	0.484	0.198
	2	6	14.668	14.810	14.668	0.765	0.312
	3	6	14.848	14.885	14.848	0.524	0.214
Stress	1	6	234.83	237.40	234.83	16.56	6.76
	2	6	244.58	244.15	244.58	7.49	3.06
	3	6	247.37	244.05	247.37	8.30	3.39
Strain	1	6	0.01922	0.01901	0.01922	0.00153	0.00063
	2	6	0.02144	0.02155	0.02144	0.00037	0.00015
	3	6	0.02086	0.02055	0.02086	0.00073	0.00030

Variable	Geom	**Min**	**Max**	Q1	Q3
Modulus	1	1.4.250	15.670	14.858	15.497
	2	13.700	15.420	13.820	15.413
	3	14.220	15.550	14.273	15.333
Stress	1	209.10	259.40	222.15	244.25
	2	235.30	255.70	237.48	251.43
	3	238.40	260.20	241.63	256.15
Strain	1	0.01779	0.02208	0.01802	0.02001
	9	0.02086	0.02178	0.02108	0.02178
	3	0.02024	0.02178	0.02024	0.02178

Table J.15: Tensile Geometry Tests Descriptive Statistics

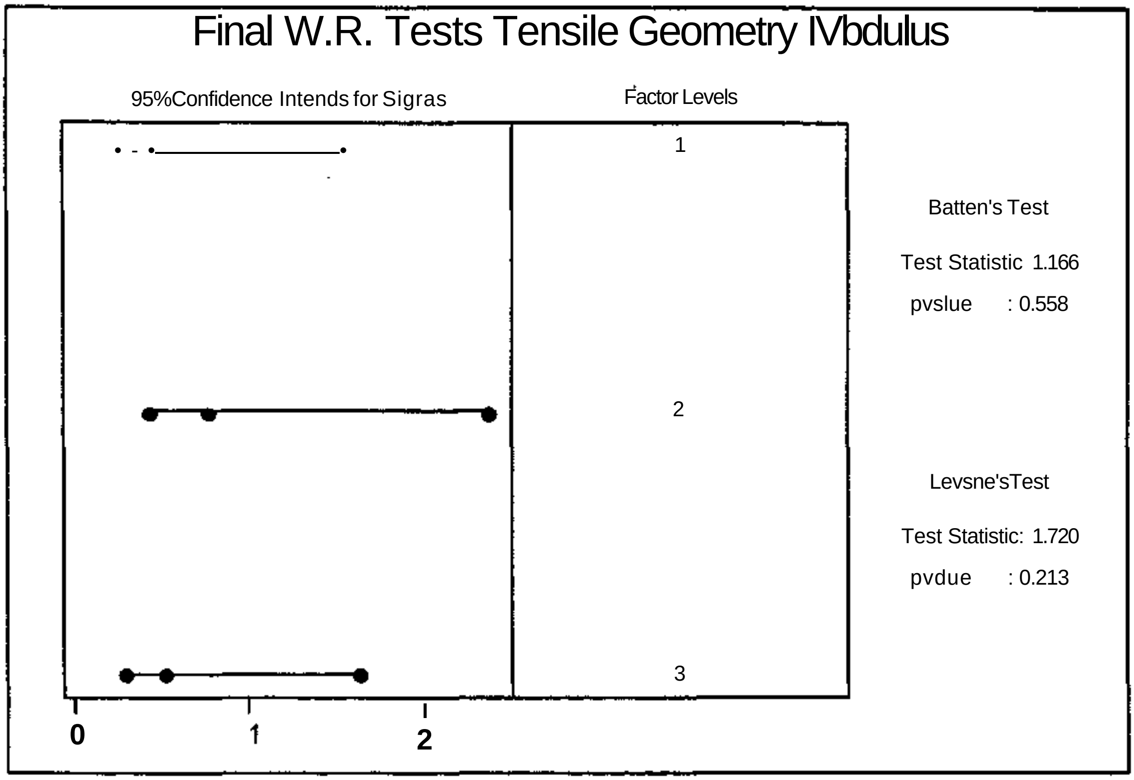


Figure .T.I: Tensile Geometry Tests Homogeneity of Modulus Variance

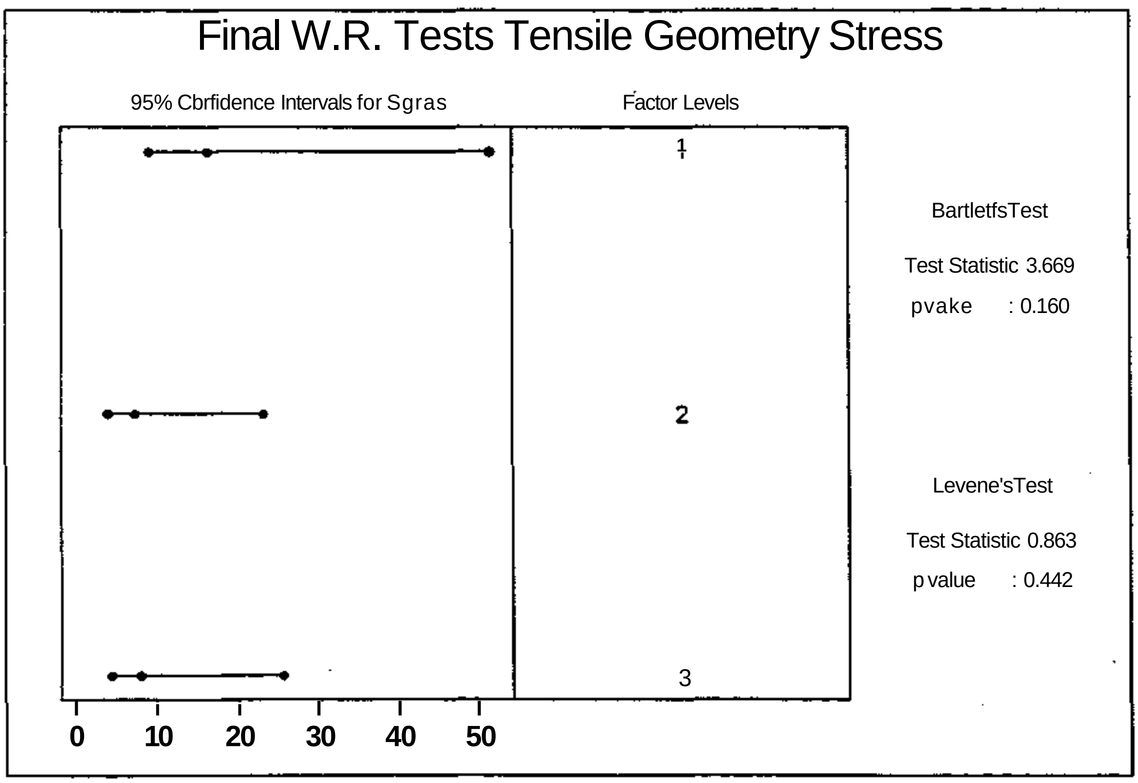


Figure .1.2: Tensile Geometry Tests Homogeneity of Failure Stress Variance

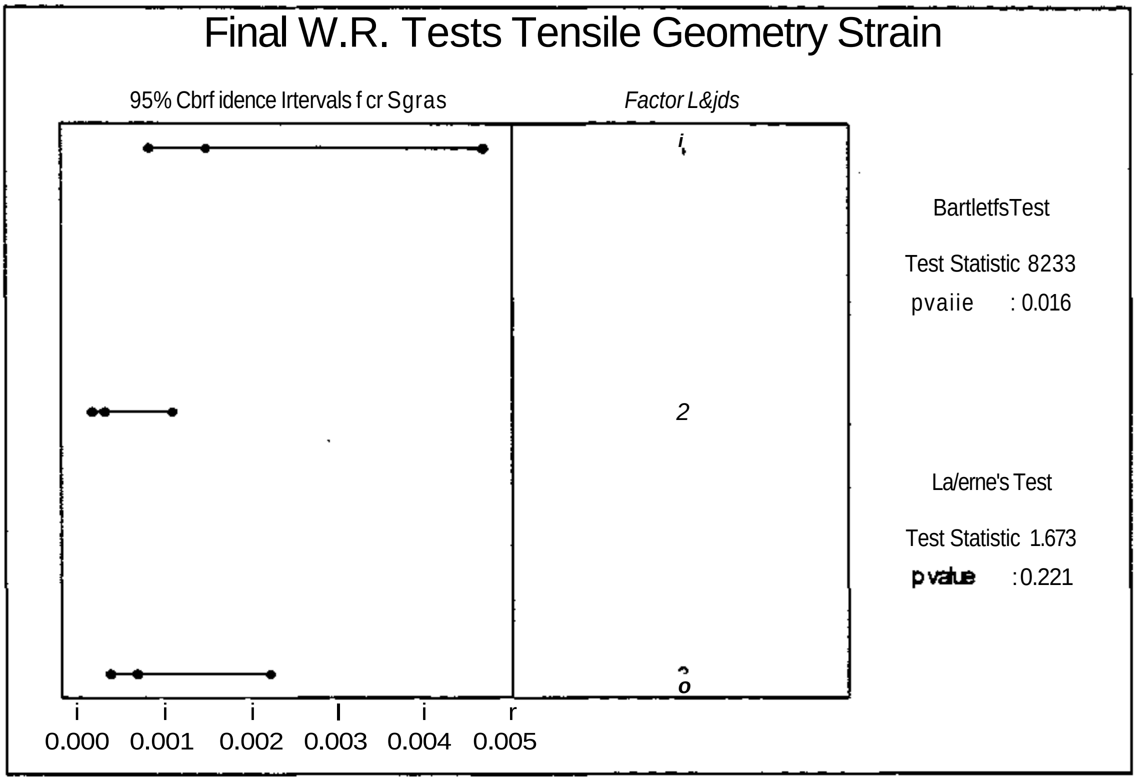


Figure .1.3: Tensile Geometry Tests Homogeneity of Failure Strain Variance

	Modulus		Stress		**Strain**	
	Equal Var.	Diff. Var.	**Equal** Var.	**Diff. Var.**	**Equal Var.**	**Diff. Var.**
Rectangular vs Tabs	**0.24**	**0.25**	0.22	0.24	0.006	0.018
Rectangular vs Waisted	**0.36**	**0.36**	0.13	0.14	0.04	0.05
Tabs vs Waisted	**0.64**	**0.65**	0.56	0.56	0.11	0.12

Table .1.16: Tensile Geometry Tests P-values for Identical Means

Appendix K Reinforcement and Matrix Materials Data

Unidirectional Tests, Reinforcement and Matrix Materials

Reinforcement Details:

	Glass	**Carbon**
Type	E-Glass	High Strength
Supplier	Vetrotex Ltd.	S.P. Systems Ltd.
Product Reference	LR/450/PB/100	UC300 / 350A
Weight (g/m^2)	450	300
Binder	Transverse	Transverse
Fibre Style	~	TenaxHTA5131 F12000

Table K.1: Unidirectional Reinforcement Details

Resin Details:

Resin:	Ampreg 20
Type:	Epoxy
Supplier:	S.P. Systems Ltd.
Hardener:	Ampreg 20 Standard
Ratio:	4 parts resin to 1 part hardener, by weight
Gel Time:	40 minutes, at room temperature
Cure:	(i) 16 hours, at room temperature
	(ii) 5 hours, at 80°C

Further information concerning the resin system, as supplied by S.P. Systems Ltd., follows.

SP Ampreg 20™

SP SYSTEMS
MONTECATINI ADVANCED MATERIALS

High performance epoxy laminating system

• FEATURES

- **Good mechanical properties**
- **DDM (MDA) and Phenol-free**
- **Low viscosity**
- **Good water and weathering resistance**
- **Generous working times**
- **Good HDT**
- **Useable with Ampreg UltraSlow Hardener**

INTRODUCTION

Ampreg 20 is one of the new generation of wet lay-up laminating resins designed by SP Systems. It characterises the latest developments in epoxy chemistry and as a primary feature of this advance in technology, Ampreg 20 is formulated without DDM (MDA) or phenol. These significant improvements in the health and safety aspects have been achieved without detrimental effects to the mechanical properties.

The Ampreg 20 laminating system has been developed specifically for the manufacture of high strength, lightweight composites with glass, aramid or carbon fibres by wet lay-up techniques.

The low initial viscosity also allows laminates to be produced by contact pressure, vacuum or pressure bag techniques, filament winding, or vacuum assisted resin injection. Thorough wetting of reinforcement fibres is ensured by the low viscosity and excellent air release properties of the resin/hardener mixture. This, in particular, assists vith the impregnation of aramid and carbon fibres.

Ampreg 20 exhibits good mechanical properties with an ambient temperature cure. However a post cure can be utilised to produce optimum properties from trip resin system.

• TECHNICAL DATA

SP Ampreg 20 resin is combined with SP Ampreg 20 fast, standard or slow hardeners in the foDowing ratio:

Resin	:	Hardener
4	:	1 (by weight)
100	:	29 (volume)

PHYSICAL PROPERTIES

	Viscosity	Mini Viscosity	Gal Tims 150B®25°C	Lam. Working Hm8@20°C
Resin	1240 cps	•	-	
Slow Hardener	70cps	380 cps	200 min	5-7 hours
Standard Hardener	400 cps	800 cps	40min	1.5 hours
Fast Hardener	2000 cps	1500 cps	**15** min	0.5 hours

AMPREG PREGEL

Ampreg Pregel is a thixotropic resin modifier that can be used with Ampreg 20 hardeners. It must be mixed with the chosen Ampreg 20 hardener at a ratio of 100:25 by weight. It can be used in the following situations:

- As a resin modifier to reduce drainage in laminates.
- As an adhesive mix for bonding core materials to Ampreg 20 laminate skins.
- For the secondary bonding of preformed Ampreg 20 laminate components.

For more information, please see the separate data sheet

£

VACUUM BAG INSULATION AND TECHNIQUES

Consolidation of the laminate can be obtained either by hand using paddle rollers or by vacuum or pressure bags. A typical vacuum bag arrangement is shown in figure 1. Due to the reduced volatile content, high vacuums may be used. However, as with all wet lay-up systems, extreme vacuums may remove system components before gelation. Heating can be economically and effectively achieved with either space heaters under an insulation tent or heated blankets with insulation over. Details of the various types of system are available from SP Systems Technical Services.

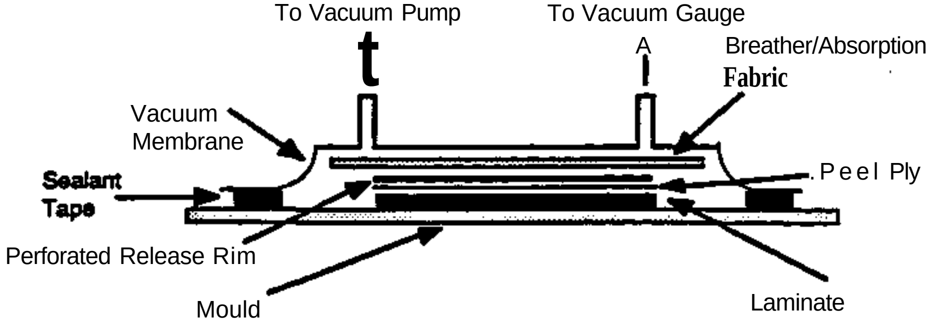


Figure 1

BONDING TECHNIQUES AND CORE MATERIALS

Where it is necessary for a bonding operation to be carried out following the cure of the Ampreg 20 laminate. Peel Ply (DPI00) can be applied to the surface to be bonded during the lay-up process. After curing and just prior to bonding, the Peel Ply is stripped off leaving a dean, dust and grease-free, textured surface which does not need preparation before bonding.

Various core materials can be used with the Ampreg 20 system, including PVC and other foams, honeycombs and end-grain balsa as well as more specialised cores. Contact SP Systems Technical Services for further information.

CURING SCHEDULE

Ambient Temperature Cure - Ampreg 20 has been developed to return good mechanical properties after cure at ambient temperatures, the minimum recommended temperature being 18°C, and excellent properties after a slightly elevated temperature post-cure.

An initial cure of at least 48 hours (with slow hardener) or 16 hours (with fast hardener) at 18°C is recommended before de-moulding. Laminates subjected to an ambient temperature cure should be allowed 14 days before the system can be considered to be adequately cured and should be kept in a warm dry environment during this period. When using the slow hardener exclusively, an elevated temperature cure is strongly recommended.

Elevated Temperature Cure • Post curing the laminate will greatly increase mechanical properties as shown in Table 1 (see Technical Data). The Ampreg 20 system will achieve similar properties with a cure of either 5 hours at 80°C or 16 hours at 45-50°C but as the table shows, the cure has been optimised for 45-50°C. The latter temperatures are easily achievable with low cost heating and insulation techniques. The table below shows that these cure cycles improve the properties by up to 60%. There is a considerable increase in glass transition temperature and toughness of the system, with the elongation at break increasing to approximately 6.5%. The post cure need not be carried out immediately after laminating and it is possible to assemble several composite components and post cure the entire assembly together. It is recommended, however, that elevated temperature curing should be completed before any painting/Finishing operations. Furthermore care should be taken to adequately support the laminate if it is to be post cured after demouldng and the laminate must be allowed to cool before the support is removed.

System	Cure Scheme	Flexurai Strength MPa	Flexurai Modulus GPa	Tensile Strength MPa	Tensile Modulus GPa	Elongation to Break	HDT (°C)
Resin/Slow Hardener	5hrs©80°C	147	3.0	77	3.1	>5%	80
	16hrs©50°C	156	3.4	82	3.3	>5%	71
	4wks®RT	112	3.6	58	3.5	2.6%	51
Resin/Std Hardener	5hrs@80°C	159	3.5	80	3.3	>5%	78
	16hrs©50°C	167	3.5	85	3.3	5.0%	71
	4wks@RT	126	3.7	71	*25*	2.9%	51

AMPREG ULTRA-SLOW HARDENER

A separate data sheet for Ampreg UltraSlow Hardener is available.

This hardener is intended for the manufacture of extremely large structures, and gives an extended working time (10-12 hours). It is used at a different mix ratio (100:30 by weight) than the other Ampreg 20 hardeners and details of the post-cure schedules required are given in the separate data sheet available for this product

W.R. Manufacturer Tests, Reinforcement and Matrix Materials

Woven Roving Details: These are as supplied by Vosper Thornycroft.

	AUS 8	AUS 11	AUS12
Manufacturer	Colan WR / Chinese Roving	Asahi Fiber Glass Co.	Key Trading Corp.
Roving Tex	2170	2300	2170
Roving Type	Assembled	Direct	Direct
Filament Diameter, (µm)	10.7	16	16.7-23.5
Roving Source	China	Japan	USA
Cloth Weight (gm"^{2}). Nominal	788	*111*	782
Cloth Weight (gm"$^{:}$). Sample	770	756	801
Ends / 100 mm. Nominal	19.7	18	19.7
Ends / 100 mm. Sample	20	17.8	20
Picks / 100 mm. Nominal	15.1	15.2	15.8
Picks / 100 mm. Sample	16	15	16
Wet-out Time (s).	374	267	247

* Bullseye test

Table K.2: W.R. Manufacturer Tests Woven Roving Details

Resin Details : These are as supplied by DSM Resins in the technical data sheet.

Synolite 73-2785

INTRODUCTION: Is a medium reactivity isophthalic resin suitable for the fabrication of large marine structures. Approved to M.O.D. Specification DG ships 180B. B.S. 3532 Type C, Lloyds Register of Shipping.

APPLICATION: Hand Lay Up, Spray Up.

TYPE: Unsaturated Polyester, Isophthalic, Corrosion resistant, Thixotropic, Medium Reactivity.

CHARACTERISTICS: Good Corrosion Resistance, High Heat Distortion Point.

Test	Result	Test Method
Liquid Resin:		
Appearance	Hazy, amber liquid	
Specific gravity	1.09	BS 3532 App. F
Viscosity	Thixotropic	
Storage life at 20°C (months)	j	
Volatile content (%)	43	BS 3532 App. J
Gel time at 25°C (minutes) (using 2% SA11 & 2% SC5)	19	BS 3532 App. K
Acid value (mg KOH / g)	15	BS 3532 App. H
Flash point (°C)	31	Abel closed cup
Cast Resin:		
Temp, of defln. under Load (°C)	118	BS 2782 Method 102C
Specific gravity	1.19	BS 2782 Method 509A
Hardness	50	Barcol 934-1
Water absorption (mg)	20	BS 2782 Method 502G
Flexural strength (MPa)	110	BS 2782 Method 304C
Flexural modulus (GPa)	4.0	BS 2782 Method 302D
Tensile strength (MPa)	50	BS 2782 Method 301A
Tensile modulus (GPa)	4	BS 2782 Method 302A
Impact strength (.1)	0.8	BS 2782 Method 306A

Table K.3: W.R. Manufacturer Tests Resin Properties

Laminate Details: Over the page the instructions to the Vosper Thornycroft laminators are reproduced. These are followed by production details as noted by the laminators.

RESIN : DSM 73.2785 CV

JkCCfite.qA.Tog • •-V. 49 P ‹D--75*/!6

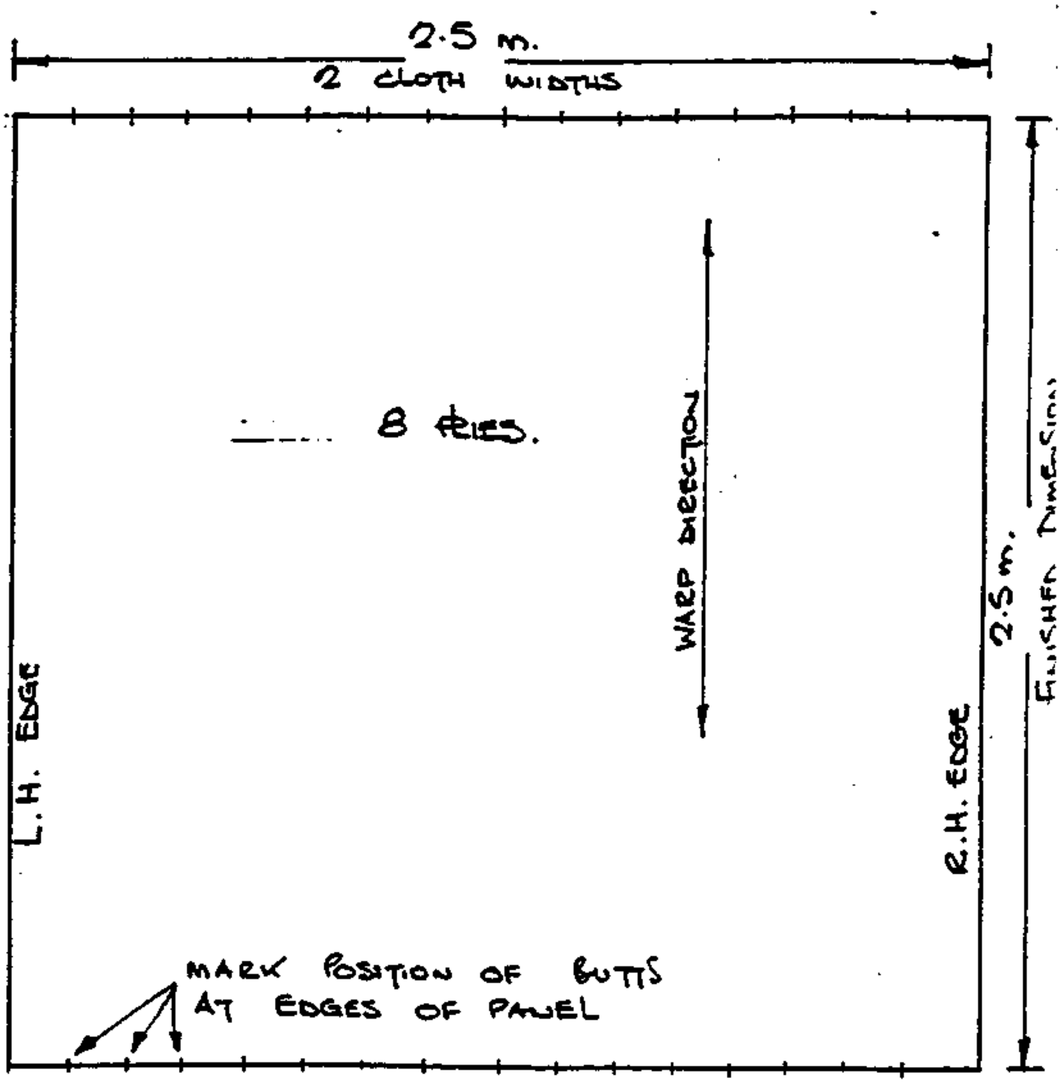
CATALYST : £*TA-wo>: »*\ So Vh "^

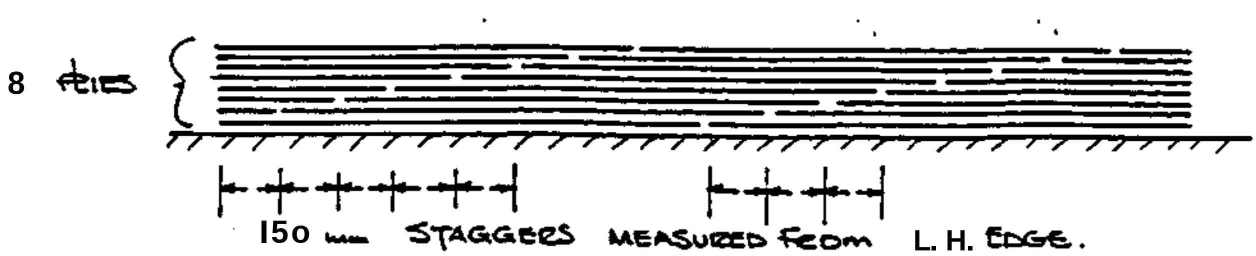
ees^ eAr^.o : So % S-, WEIGHT

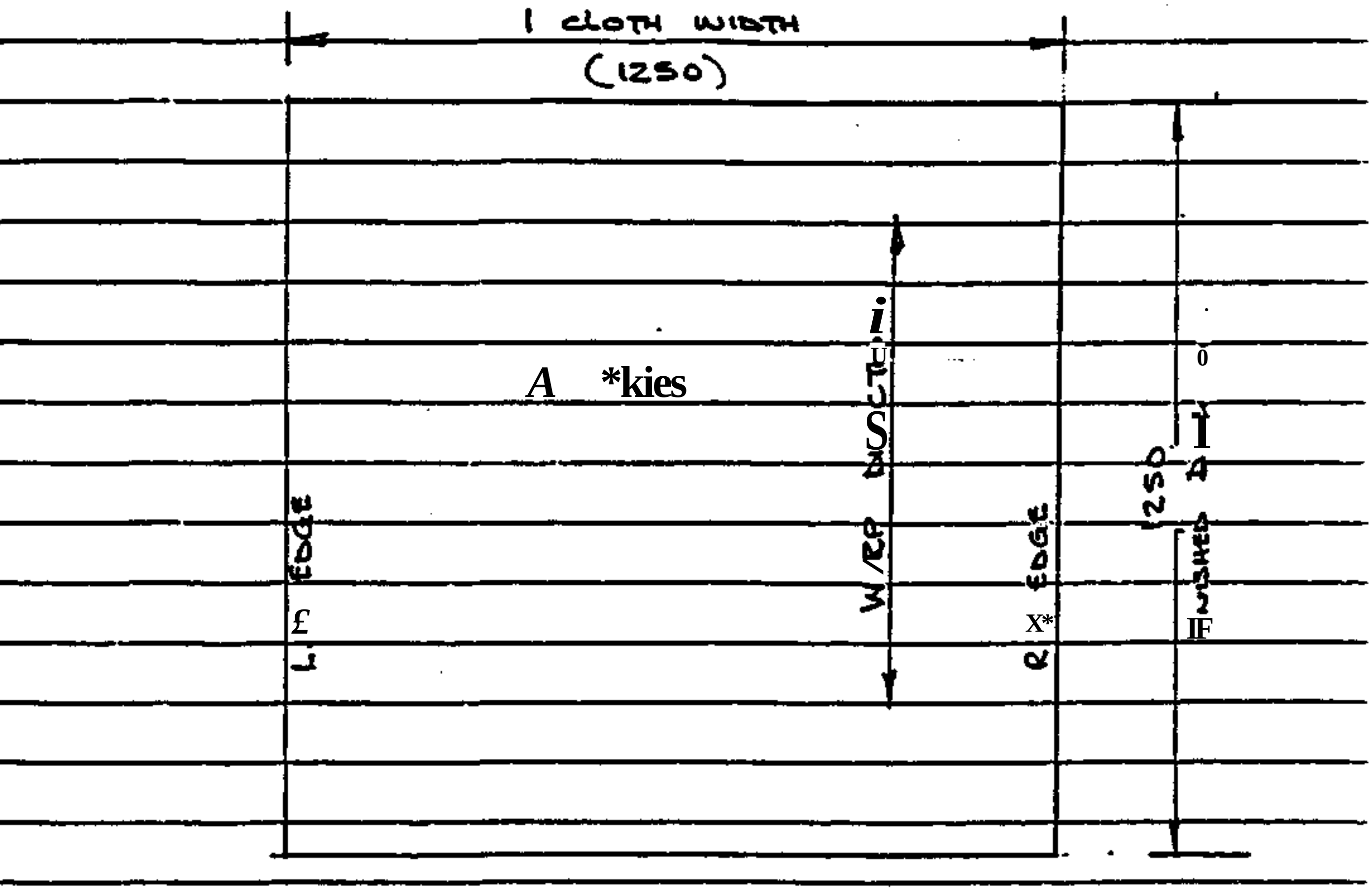
BUT THE LAMINATE IS To BE WETTED OUT

-WoCoo&uUv Sv»T <^ «4O"T TO HAVE EXCESS

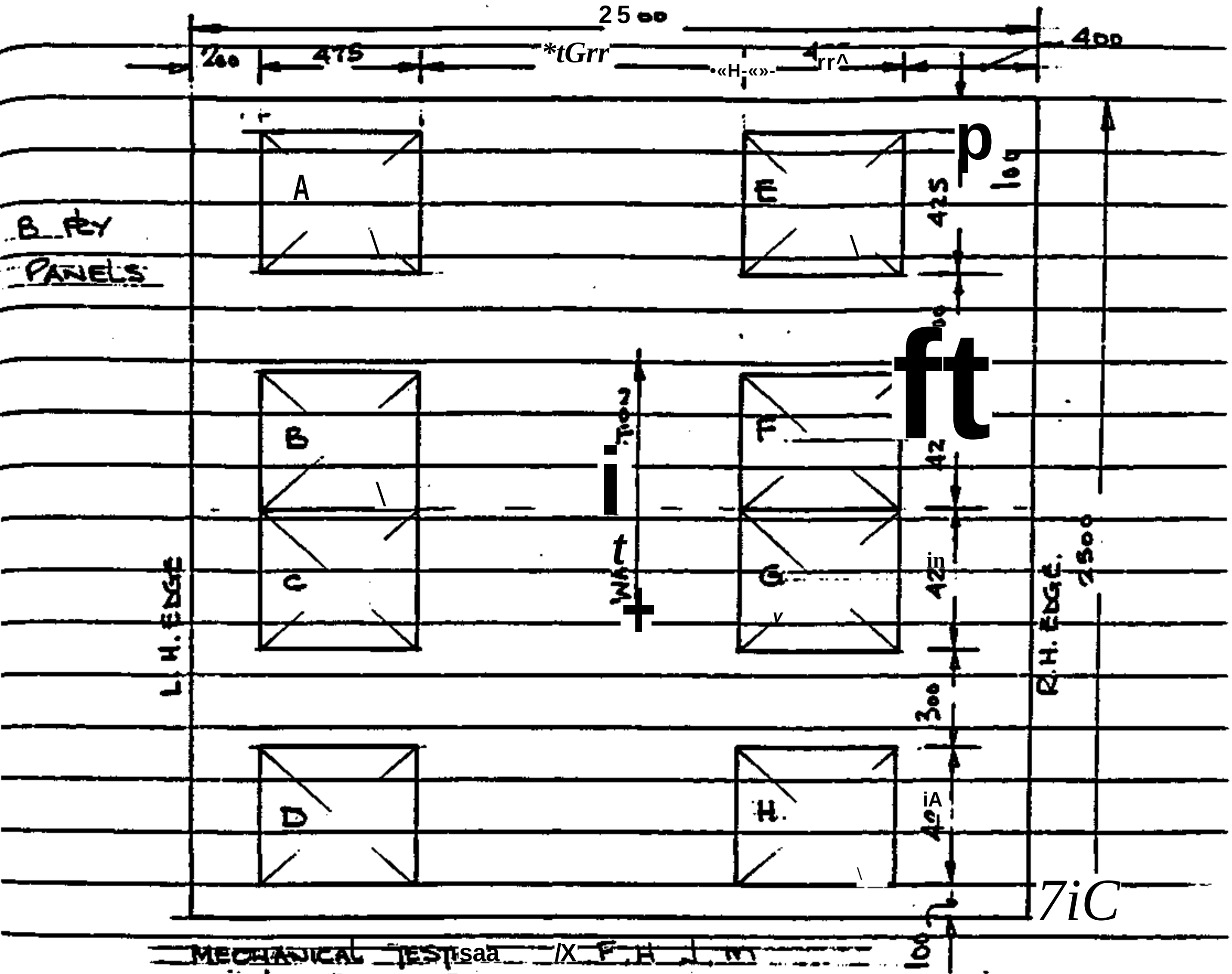
RESIN ON / . — FACE.



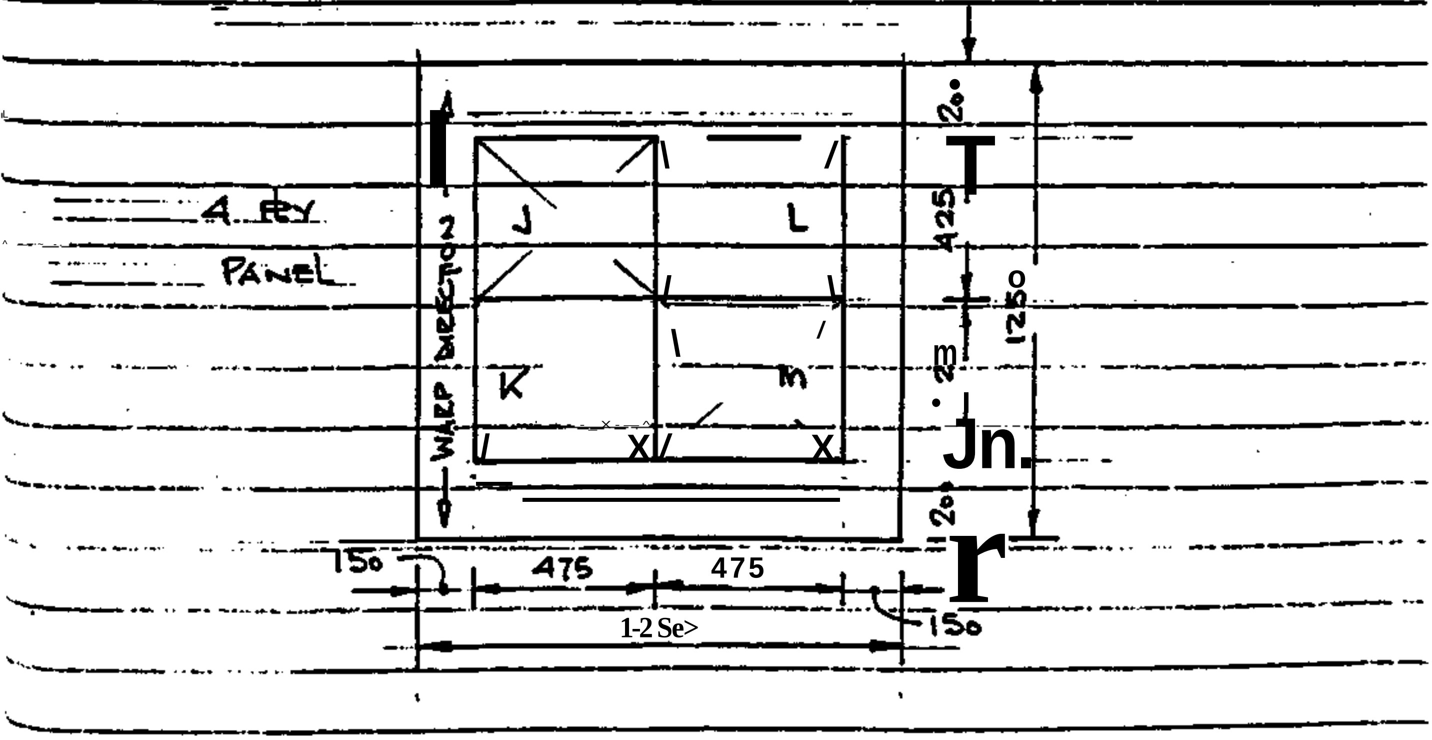



1 CLOTH WIDTH
(1250)
L. EDGE
WARP DIRECTION
R. EDGE
1250
FINISHED

CUTTING DETAILS FOR SAMPLE PIECES.



MECHANICAL TESTS [illegible], F, H, J, M
DEA/GATS B, D, K
SPARE C, E, G, L



	AUS8	AUS11	AUS12
Reinforcement ID	XR 2007		
Resin Batch No.	40141/6	40141/6	40141/6
Accelerator Batch No.	0029300212003	0029300212003	0029300212003
Catalyst Batch No.	BN041	BN041	BN041
Dates of lay up	26/10/93	2/11/93	8/11/93
	27/10/93	3/11/93	15/11/93
	1/11/93		16/11/93
Shop temperature	23 -30°C	27 -30°C	22 -24°C
Samples cut	28/10/93	3/11/93	10/11/93
	.2/11/93	4/11/93	14/11/93
Post cure	Within 3 days of Laminating	Within 3 days of Laminating	Within 3 days of Laminating

Table K.4: W.R. Manufacturer Tests Production Details

Final W.R. Manufacturer Tests, Reinforcement and Matrix Materials

Woven Roving Details : These are as supplied by Vosper Thornycroft.

Fothergill Engineering Fabrics YO 530: The cloth is a warp biased plain weave woven roving and is to be approved by the MOD for use in ships. It is to have roving flexibility and wet out characteristics to suit VT production requirements. Generally it is to conform to BS3749.

Specification

Type of Reinforcement	:	Plain weave woven roving
Type of Glass	:	E
Type of Roving	:	WtoBS3691
Roving Tex	:	2300 ± 10% (or 2400 ± 10%)
Filament Diameter	:	11 (i (may be greater provided that required properties are achieved)
Ends/ 100mm	:	19 ± 5% (may be 18.5 ± 5% for 2400 tex provided that properties are met)
Picks / 100mm	:	14.5 ± 5% (may be 13.5 ± 5% for 2400 tex provided that properties are met)
Mass	:	780 gm^{-2} ± 5%
Selvedge Tassels	:	20mm ± 10%
Weave Distortion	:	10mm in lm
Weave Defects	:	Generally as given in BS3749

Resin Details : These are as supplied by Scott Bader in the technical data sheet which follows.

SCOTT
BADER

Crystid®
489 PA

IsophtLalic boatbuilding polyester resin for *watched performance* system with superior blister resistance.

Introduction

CRYSTIC 489 FA is one of a new generation of pre-accelerated, thixotropic, contact moulding, unsaturated polyester resins based onisophthalic acid for boatbuilding. It is supplied as a solution dissolved in monomeric styrene. This resin was specially developed for use with CRYSTIC GELCOAT 65 PA to meet the need for a matched performance isophthalic galcoat/laminating resin system for moulding boat hulls with outstanding durability and performance.

The use of matched performance isophthalic resin/gelcoat systems nas been shown by long term immersion in water, both in the laboratory and in service, to significantly improve the resistance of GRP to blistering, caused by the osmotic process.

The handling characteristics of CRYSTIC 489 PA have been designed to give rapid impregnation, freedom from drainage and adequate pot life for normal boatbuilding techniques.

CRYSTIC 489 PA is tinted blue to help see air bubble entrapment in the laminate and thereby facilitate their removal during the laminating process.

CRYSTIC 489 PA is approved by Lloyd's Register of Shipping for use in the construction of craft under their survey and also Det Norske Veritas. It also meets the requirements of BS 3S32: 1990: TypeB.

Fully-cured boat hulls and mouldings made with CRYSTIC 489 PA have high mechanical performance with excellent strength retention in wet environments at temperatures up to 40*C, *nd exhibit high impact resistance as a result of the high elongation to break of the resin.

Good interlaminar adhesion is a vital requirement for the structural integrity of a GRP Laminate. Creat care has therefore been taken in the design of CRYSTIC 489 PA to ensure that no loss in strength occurs at the interface between layers when a delay has occurred in lay-up.

CRYSTIC 489 PA has been designed for use with the hand lay-up technique. Although it can be used without modification in certain types of spray equipment, higher levels of styrene emission must be expected during spraying.

At a mould temperature of 18-20'C, CRYSTIC GELCOAT 65 PA will be sufficiently cured for lamination to commence approximately 1 hour after application. Commencement of lamination can be delayed, provided that the surface remains uncontaminated for up to 8 hours at 18-20°C. If users wish to delay the commencement of lamination further, they are advised to carry out their own tests to ensure adequate adhesion will be obtained.

Formulation

The following cold-curing formulation is recommended:

	Paris by weight
Ctytlc 489 PA	100
Catalyai M or Bwtanox MOO	1-2

CRYSTIC 489 PA needs only the addition of catalyst to start the curing reaction. The correct amount of catalyst *is* therefore added and thoroughly stirred into the resin shortly before use.

Gel time

The ambient temperature, and the amount and type of catalyst, control the gel time of the resin formulation. This can be approximately determined from Table 1, which shows the gel time of 100 pbw CRYSTIC 489 PA. containing 1 or 2 pbw Catalyst M, (or Butanox M50).

T*bl« 1

Pun of Catalyst M or Butanox M6Q to 100 ports of CRYSTIC 4(9 PA	1.0	2.0
Cal time in minutes at 16 °C	eo	34
Gel lime in minutes at 20°C	40	18
Gel time in minutes at 23"C	23	12

Curing should not be *carried out Mi temperatures below lS'C and the resin must b» allowed toaiiain workshop tempermlur* before being formulated tor use. 20*C is recommended.*

When a longer gel time is required, Catalyst O or Butanox LA should be used instead of Catalyst M or Butanox M50.

Additives

Since addition of certain pigments, fillers or extra styrene may interfere with the properties of CRYSTIC 489 PA, users axe urged to seek the advice of our Technical Service Department before making any such additions. Other suitably formulated resins are available if special properties e.g. reduced fire hazard, are required.

Post-curing

Satisfactory laminates for many applications can be made from CRYSTIC 489 PA by curing at workshop temperature (20°C). When optimum properties and long term performance are required however, the laminate should be post-cured. After release from the mould, laminates should be allowed to mature for 24 hours at workshop temperature (20°C). They should then be post-cured for three hours at80°C. although a longer period at a lower temperature (e.g., 40°C) will also develop satisfactory properties for marine applications. The post-cure is most effective if it is carried out immediately after the 24 hour maturing period.

Caution

Safe handling information for this material is contained in the current edition of the Scott Bade* Material Safety Data Sheet • A Guide to the Safe Handling of Unsaturated Polyester Resin and Resin Systems.

CRYSTIC 489 PA is subject to the Highly Flammable Liquids and Liquefied petroleum Gases Regulations 1972 since it has a flash point below 32°C (90°F) when tested in accordance with Schedule No. 1 of these regulations.

The Flash Point of CRYSTIC 489 PA is 31.8°C.

Liquid CRYSTIC 489 PA must be kept away from naked flames.

CRYSTIC 489 PA should be stored in the dark in suitable containers. It is recommended that the storage temperature should not exceed 20°C <68°F).

Packaging

CRYSTIC 489 PA *is* supplied in 25kg and 200kg steel containers.

Bulk supplies can be delivered by road tanker.

Table 2

Typical properties of liquid CRYSTIC 489 PA

Viscosity at 25°C		
(Femnti shear ma 37.36 «ec')	poise	4.3
(Fexranti shear tale 4600 s e Q	poise	3.6
Specific crravity at 26"C		1.11
Acid Valua	maKOH/o	19
Volatile content	Vt	44
Appearance		Blue
Stability in the dark at 20*C	months	3
Cal lima at 25°C Reain 100 pbw, Catalyst M or Butanox M80 1 pbw	minutes	23
T«si methods as in BS 3782		

Table 3

Typical properties of post-cured * CRYSTIC 489 PA (unfilled casting)

Barcol hardness (Model GY2T 934-N		40
Water absorption 24h at 23*C	ma	18
Deflection temperature under load (1.81 MPa1	*C	7B
Specific gravity at 2S°C		1.2
Elongation at break*	%	3.6
Tensile strength	MPa+	76
Tensile modulus	MPa	3200
Volumetric shrinkacre	H	7.7
Test methods as in BS 2782		

*> The curingschedale MM24h*t20'C followed by3hai80'C, except in determining deflection tempereture, where the schedule was 24h at 20*C. Sh at 80*C. 3h at 120*C.*

**Filtered resin, void-free casting.*

** 1 W>a - / MN/m² - IN/nvn* and is approximately 1<S Ibf/in² or 10.2 kgt/cm².*

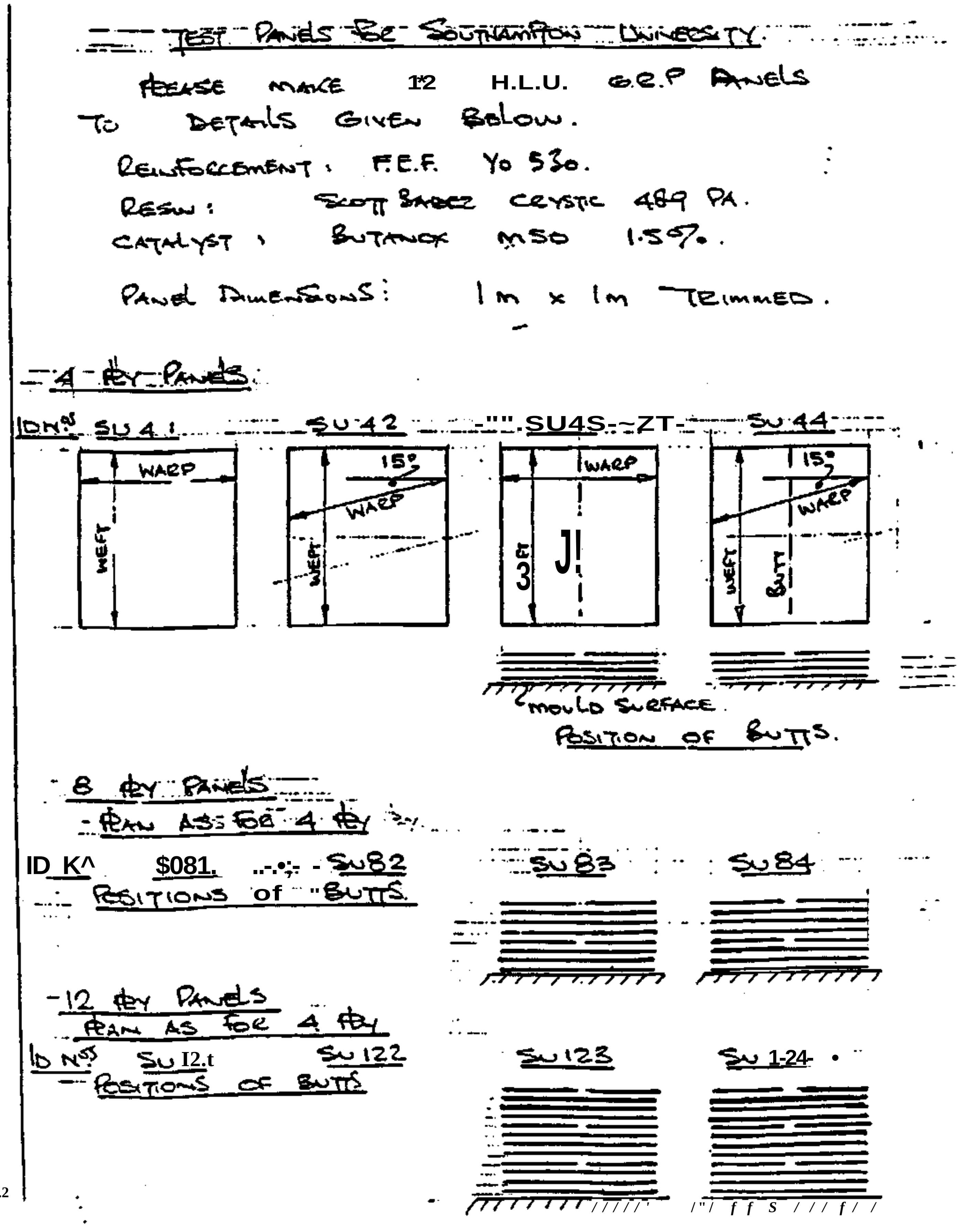
Laminate properties

Measured properties of laminates made in our laboratories using CRYSTIC 489 PA and various types of glass reinforcement are shown in Tables 4 to 11 and Figure 1.

It is pointed out that *both* sides of the laminates were immersed in water and that the elevated temperature (40°C) was chosen as the maximum considered advisable for accelerated testing.

The excellent property retention (shown in the tables) after *total* immersion in water at 40°C means that mouldings made with CRYSTIC 489 PA will exhibit excellent long-term performance under single-sided contact conditions at the lower temperatures normally experienced in service.

Laminate Details: The instructions to the Vosper Thornycroft laminators are reproduced below;


TEST PANELS FOR SOUTHAMPTON UNIVERSITY.

PLEASE MAKE 12 H.L.U. G.R.P PANELS TO DETAILS GIVEN BELOW.

REINFORCEMENT: F.E.F. Y0 530.

RESIN: SCOTT BADER CRYSTIC 489 PA.

CATALYST: BUTANOX M50 1.5%.

PANEL DIMENSIONS: 1m x 1m TRIMMED.

4 PLY PANELS

ID No SU 41 SU 42 SU 43 SU 44

POSITION OF BUTTS.

8 PLY PANELS
- PLAN AS FOR 4 PLY

ID No SU 81 SU 82 SU 83 SU 84

POSITIONS OF BUTTS

12 PLY PANELS
PLAN AS FOR 4 PLY

ID Nos SU 121 SU 122 SU 123 SU 124

POSITIONS OF BUTTS

Appendix L Failure Modes and Load Strain Plots

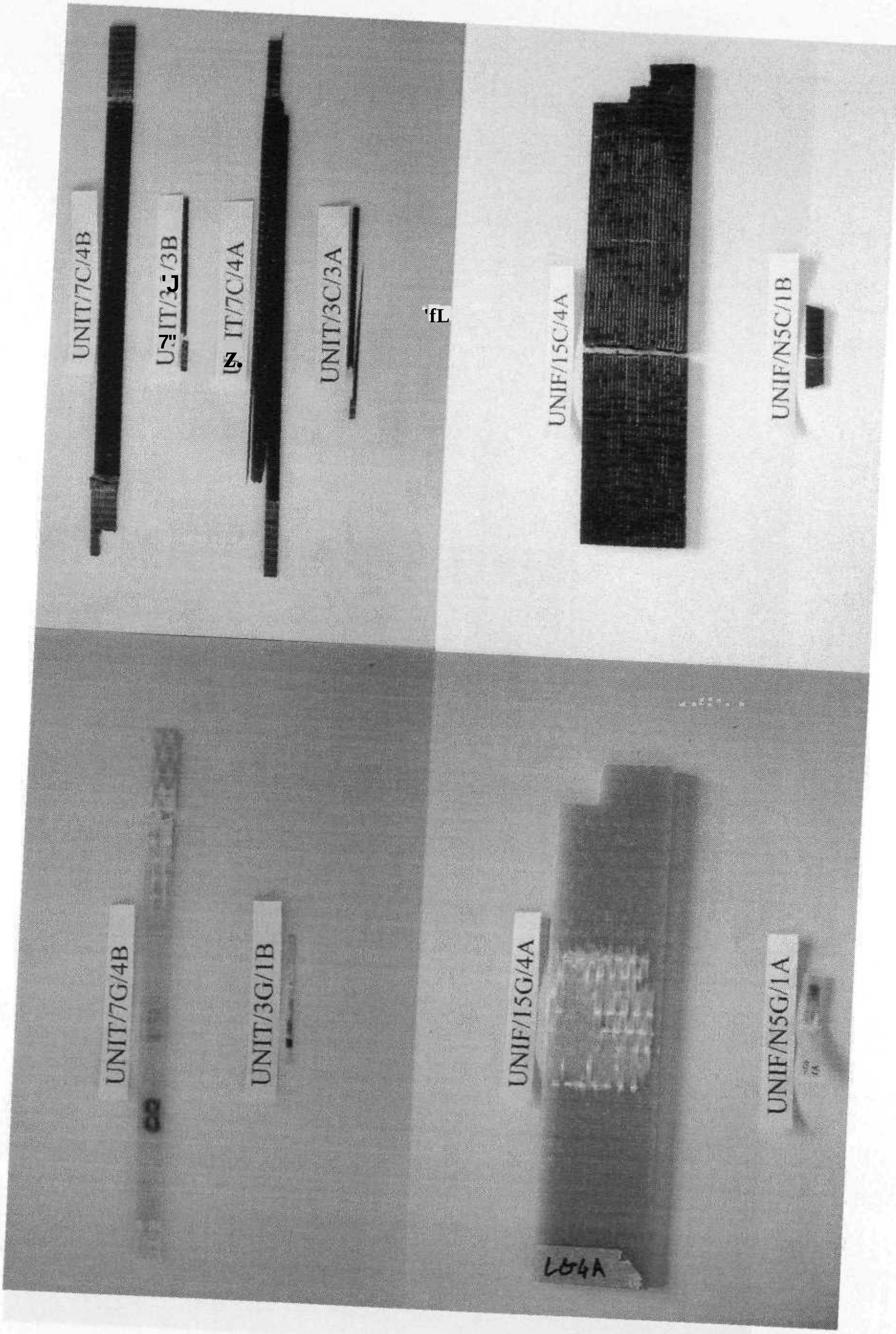


Figure L.1: Unidirectional Specimen Failures

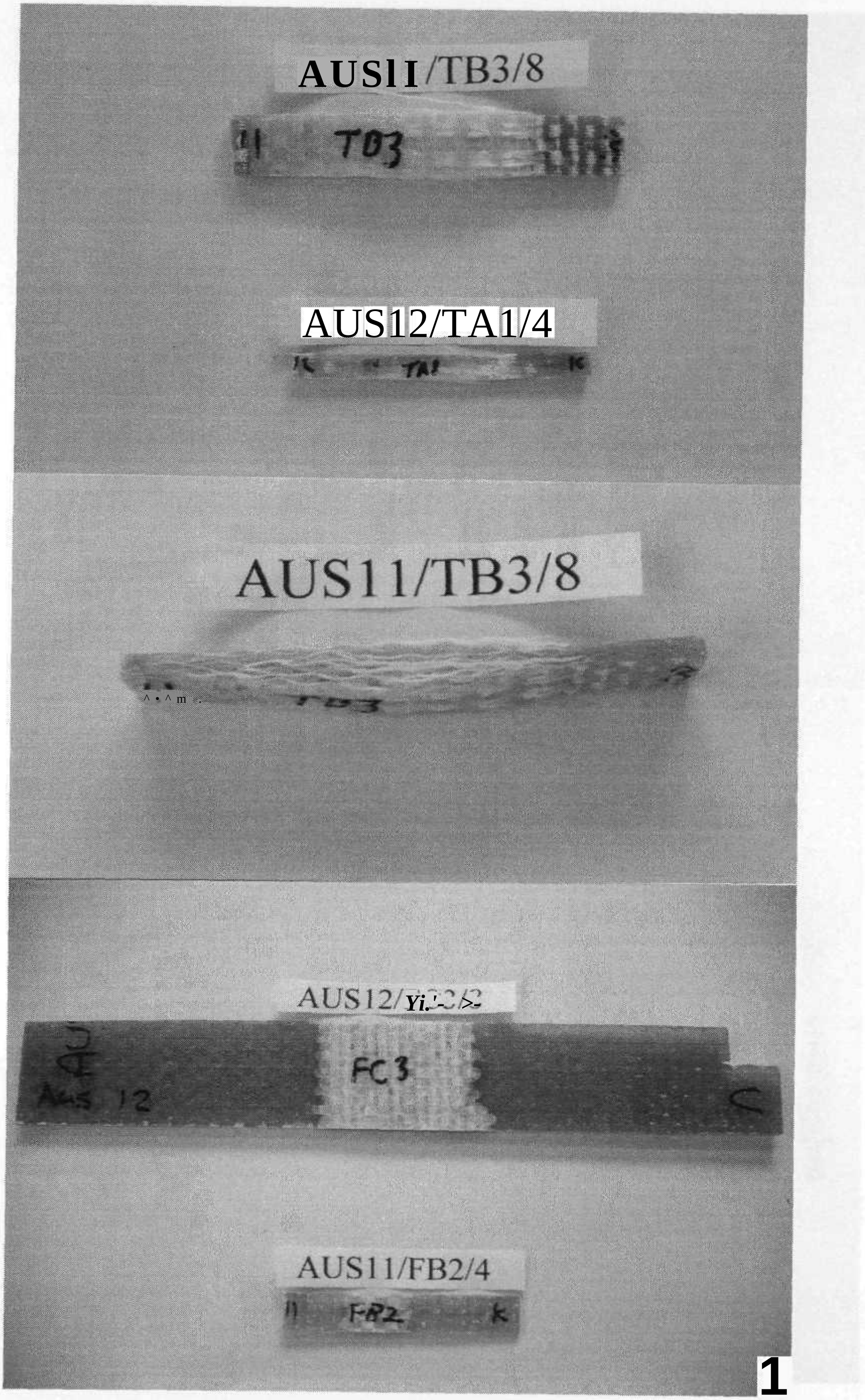
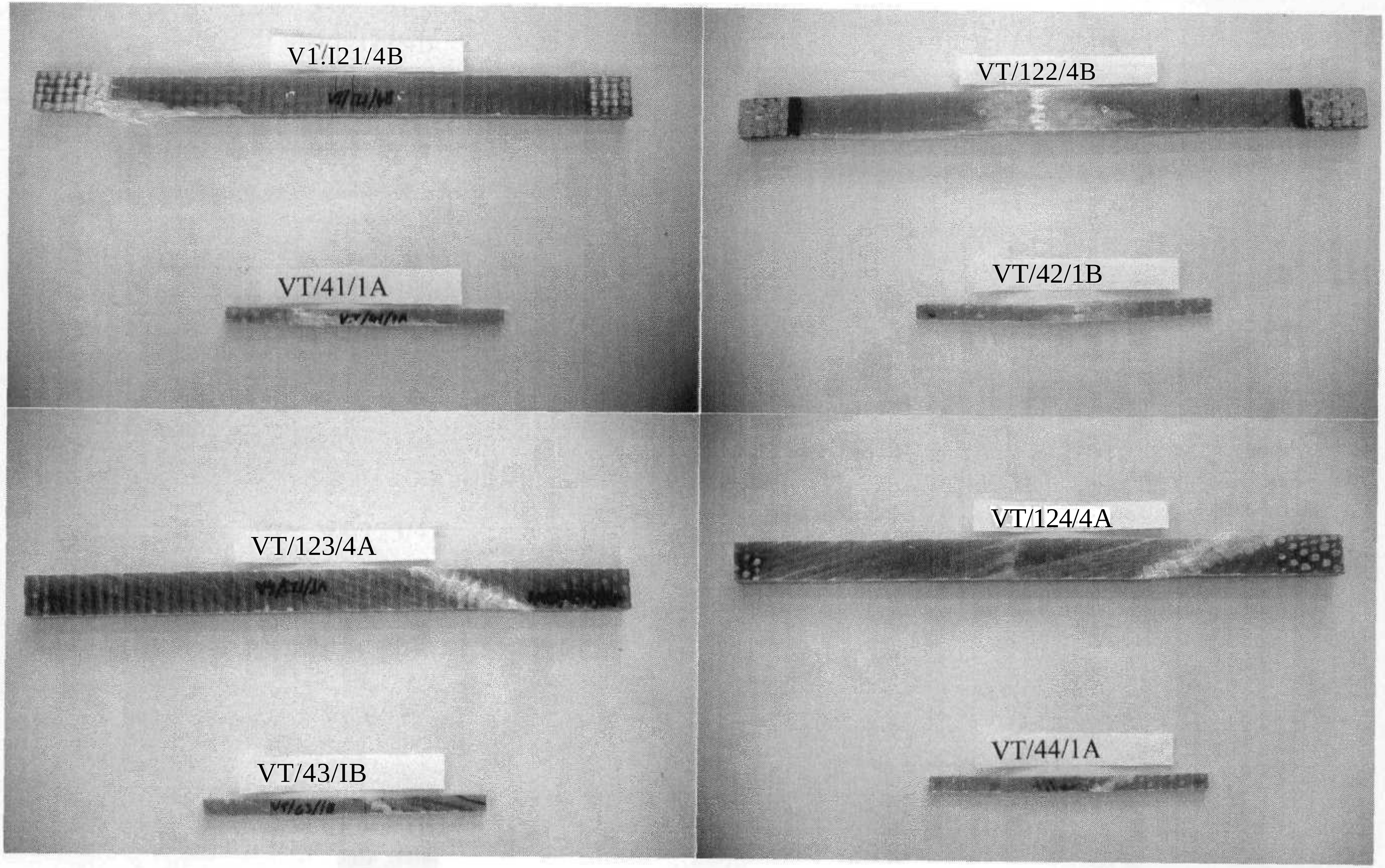


Figure L.2: W.R. Manufacturer Specimen Failures



L.3: Final W.R. Tensile Specimens

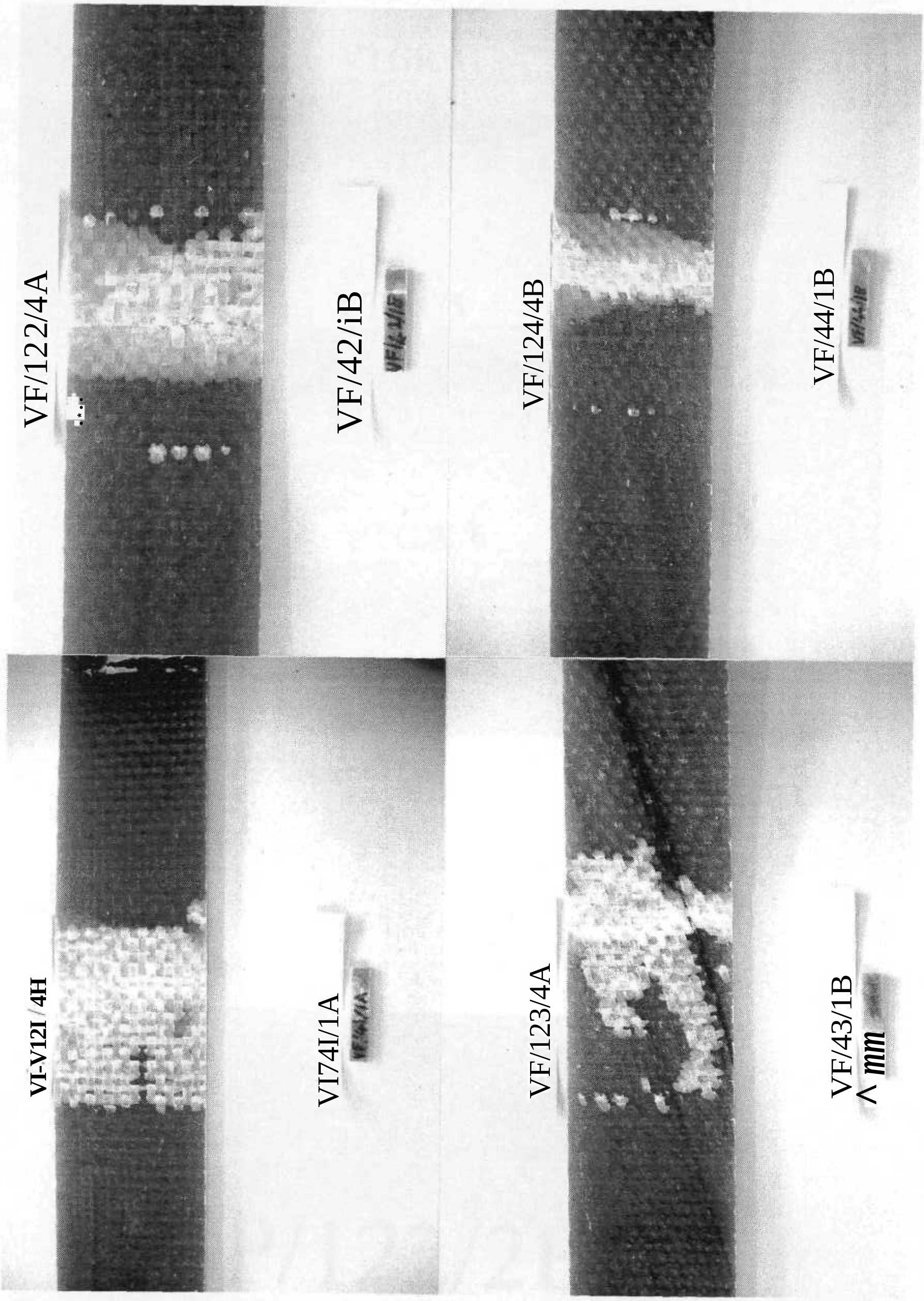
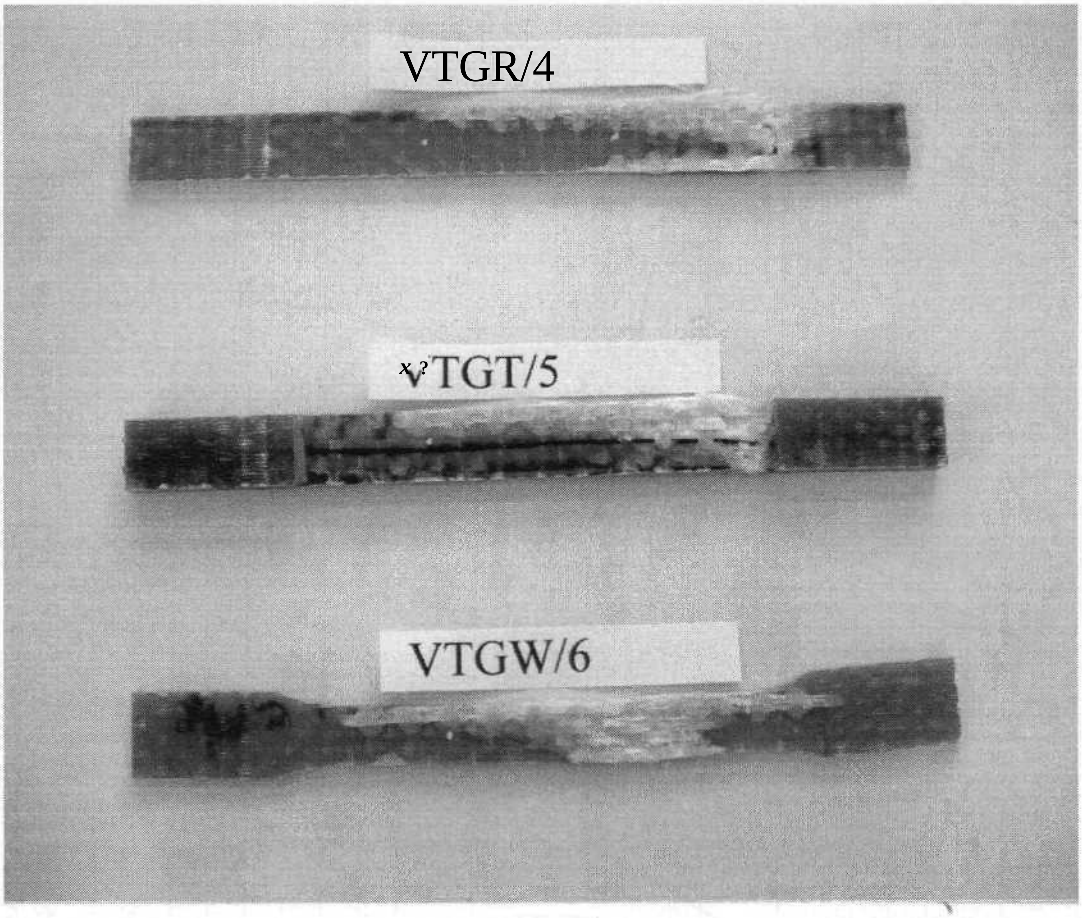


Figure L.4: Final W.R. Flexural Specimen Failures



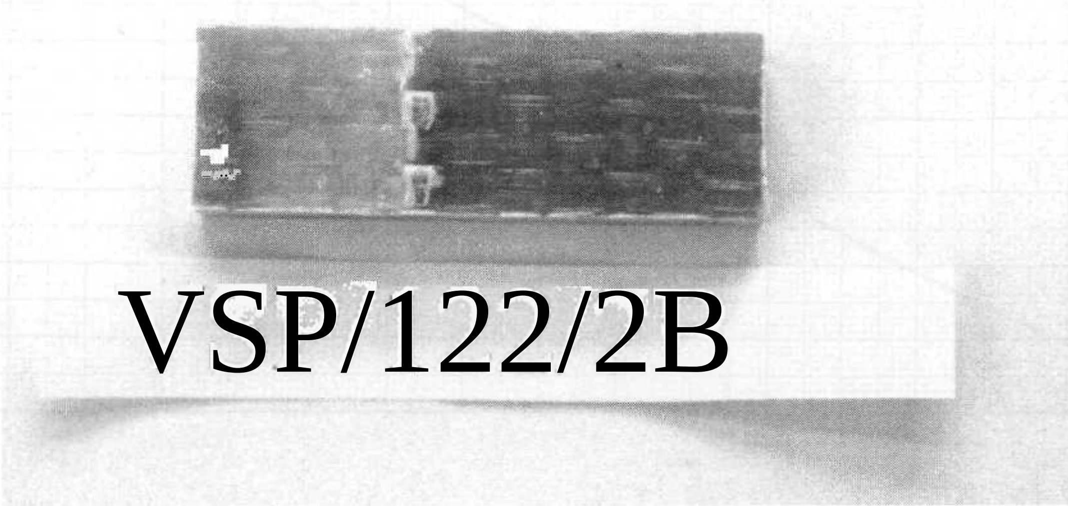


Figure L.5: Final W.R. Tensile Geometry (Top) and ILSS (Bottom) Specimen Failures

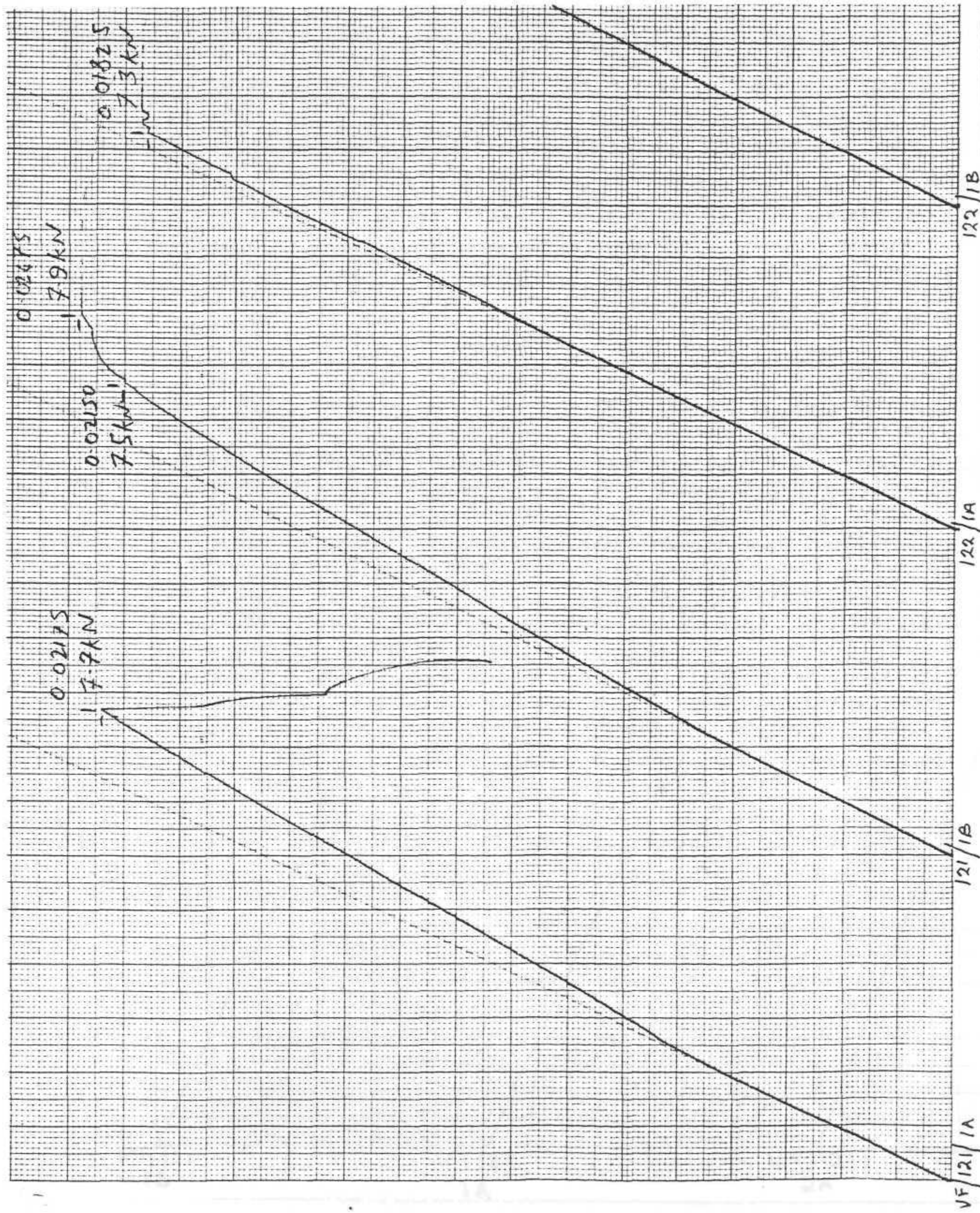


Figure L.6: Typical W.R. Flexural Load Strain Plots

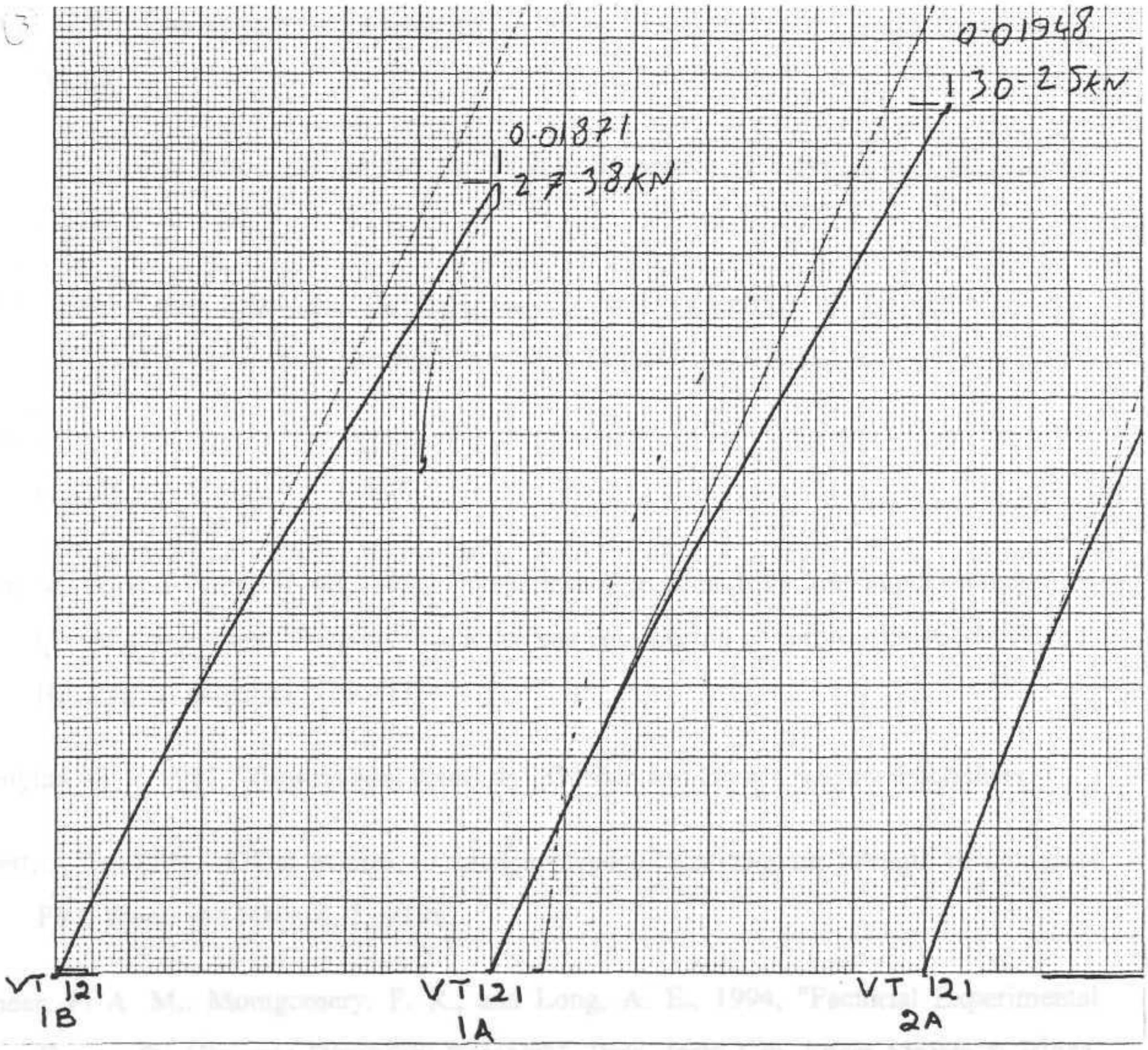


Figure L.7: Typical W.R. Tensile Load Strain Plots

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