Modelling of Train Induced Vibration

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ABSTRACT

This paper reports on recent developments in techniques for modelling ground vibration from railways. The modelling considers both surface and underground railways, and accounts for the main dynamic systems involved, i.e. tracks (both ballasted and slab), tunnels and multi-layered ground. Results are presented to illustrate the modelling capabilities and the efficiency of computations for the models proposed. The work presented is part of the MOTIV project (Modelling of Train Induced Vibration), which is a collaboration between the Universities of Southampton and Cambridge. Future development of models and plans within the project are also addressed.

1. INTRODUCTION

The MOTIV project aims to develop a better understanding of key issues relating to the generation and propagation of railway-induced ground vibration. The project primarily addresses the efficient modelling of the main sub-systems involved in railway infrastructure and the dynamic interaction between them. These systems are the resilient elements of the track (ballasted or slab), the tunnel structure (for the case of underground railways), the surrounding soil and the nearby buildings, including those with pile foundations.

Predictions of ground-borne vibration from railways using numerical models are important as they help significantly in developing a better understanding of the physics of the generation and propagation and provide insight into results of field measurements and vibration problems encountered in practice. Numerical models are particularly useful in situations where a new railway line is to be constructed close to existing buildings or where a new building is to be built close to an existing track or tunnel. Moreover, modelling is also an essential pre-requisite to the development of prediction tools that can be used by engineers when designing vibration mitigation measures aimed at reducing vibration to an acceptable level. A large number of numerical models for predicting vibration from surface and underground railways have been presented in the literature. An overview of the state of the art on railway induced ground vibration models can be found in (1). These prediction models range from simple multi-degree-of-freedom models to two-dimensional and more comprehensive three-dimensional models. Most of these models are formed by coupling sub-models for the train, the track, the tunnel, the soil and the nearby buildings.

Within the MOTIV project, the modelling of both surface and underground railways is considered. From the available numerical models proposed for the prediction of ground-borne vibration from surface and underground railways, two were selected as the starting point for the project. The first model is the 'TGV: Train-induced Ground Vibration' model, developed within the ISVR at the University of Southampton for the prediction of ground vibration due to surface trains, based on the work of Sheng et al. (2). The second model, for the case of underground trains, is the 'PiP: Pipe-in-Pipe' model developed by Forrest and Hunt (3), and Hussein and Hunt (4) at Cambridge University. Both models are considered as semi-analytical models, where invariancy in the direction of the track is assumed. This allows the use of efficient solution procedures, based on a Fourier transform with respect to the coordinate along the track, for calculating vibration from railways in a three-dimensional field from a two-dimensional geometry. The results are transformed to the space domain using the inverse Fourier transform. Within the objectives of MOTIV, these models are being revisited and improved adding a number of developments needed to increase their efficiency, capabilities and applications.

This paper reports current and future development of the numerical models and plans within the MOTIV project. In the first part, the TGV and PiP models for simulating ground vibration generated by surface and underground trains are introduced and their capabilities in predicting the vibration in the free-field are addressed in two representative numerical examples. The last part of the paper addresses the forthcoming plans and objectives of the project, which involve the theoretical and experimental investigations on the effect of pre-load and the non-linear behaviour of the resilient elements of tracks, and the effect of coupling the railway and the surrounding piled buildings.

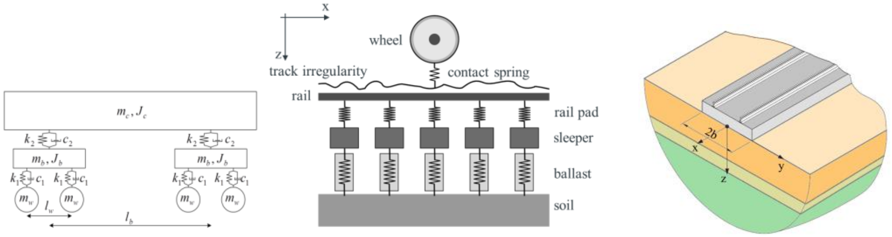
2. TGV – A NUMERICAL PREDICTION MODEL FOR SURFACE RAILWAYS

2.1 Model description

The TGV model is a semi-analytical model developed for the prediction of ground vibration generated by surface trains. The model can be used to analyze three components of vibration generated by the wheels of a train, namely (a) dynamic loads generated at fixed points on the track, (b) moving constant axle loads, and (c) moving dynamic loads applied through the wheels. It should be noted that the solution for the cases of moving axle loads is achieved by working in a frame moving with the velocity of the train.

In the TGV model, the train vehicles are described as linear 2D multiple rigid-body systems and only the vertical dynamics are considered (Figure 1a). The track is modelled as multiple beams supported by vertical springs with consistent mass (Figure 1b). The rails are represented as a single Euler-Bernoulli beam and the rail pads are modelled as a distributed vertical stiffness. The sleepers are modelled as a continuous mass per unit length of the track and the ballast is modelled as a continuous distributed vertical spring stiffness with consistent mass. An embankment, if present, can be modelled in the same way as the ballast. The vertical profile of the rail may be decomposed into a spectrum of discrete harmonic components. Furthermore, a Hertzian contact spring can be introduced between each wheelset and the rail.

The ground is represented by horizontal layers on a homogenous half-space or rigid bedrock. The railway track is aligned in the x direction and has an invariant contact width $2b$ with the ground (Figure 1c). An efficient semi-analytical model developed by Sheng et al. in (5) is used for the prediction of the ground response excited by moving constant or harmonic loads acting directly on the ground, or for loads acting via a coupled track structure. The model uses a two-dimensional Fourier transform in the wavenumber domain with β, γ the wavenumbers corresponding to the coordinates x, y along and normal to the track. The coupling of the ground with the railway track is carried out by taking into account the continuity of the displacements and the equilibrium of the stresses in the plane of contact between them, rendering it possible to calculate the Fourier transformed response of the ground surface and the track elements. Results in the frequency-space domain are calculated using the inverse Fourier transform.



(a) (b) (c)

Figure 1. (a) 10 DOF multi-body model of vehicle, (b) coupling of the wheelset with the track (rail, pad, sleeper, ballast) and the ground and (c) the geometry of the coupled track-soil system.

The TGV model uses the moving axle loads and vertical rail irregularities as its inputs. Outputs include the dynamic forces at the wheel-rail interface and the response (displacement/velocity/acceleration amplitudes or spectra) of the track and the ground. Railway track unevenness is described as a stationary random process by means of the PSD of the irregularity in terms of the wavenumber along the railway track. In this concept, the axle loads of the vehicles represent a stationary random process as well; nevertheless, this is not the case for the vibrations observed in the free field. This is due to the fact that, for a receiver at a fixed position in the free field, the vibration level depends on the position x of the axle load along the rail, or on the time t, and is therefore a non-stationary random process. The calculation of a non-stationary response PSD when the correlation of axle loads is taken into account can prove highly computationally demanding (6). However, in the TGV model, the calculation of the vibration PSD is treated efficiently as a stationary process using two approaches. The first approach is based on modelling the train at fixed positions on the track which are excited by a "moving roughness"; the roughness is 'pulled through' between the wheels and track with the velocity of the train, assuming that each wheel is excited by the same roughness apart from a time lag. In the second approach, the vibration PSD is calculated at a fixed point for the moving train. First the response is calculated at a point that is moving with the speed of the train (moving frame) and a Fourier transform with respect of time is applied (5) leading to PSD formulation that does not vary in time (2).

2.2 Numerical example

In order to highlight some of the output predictions of the TGV model, a numerical example is presented for a range of practical train parameters and velocities. The results are compared with a coupled 2.5D finite element/boundary element (FE/BE) model developed in (7). The coupled FE/BE model uses the moving roughness excitation approach. For the TGV model, results for both approaches are presented and it should be noted that in the results below the 'moving roughness' approach is denoted as TGV-MR.

In the example, two different types of soil are considered. The first soil has properties similar to a measurement site at Horstwalde in Germany, which is a sandy soil that can be represented as a homogeneous half-space of moderately soft soil (8). The second is a layered half-space with a soft ground corresponding to a measurement site at Greby in Sweden (9). For both soil cases the same track is used, which is modelled as a continuously supported track with the properties reported in (8). The rail roughness profile (Figure 2) for all simulations was chosen according to FRA class 3 (10). The train parameters used for the simulations are chosen to correspond to those used in (9). These are based on a modified Bombardier Regina EMU (‘Gröna Tåget’). Unlike (9) a four-car train is used with a total length of 106.4 m. The train speed is 150 km/h.

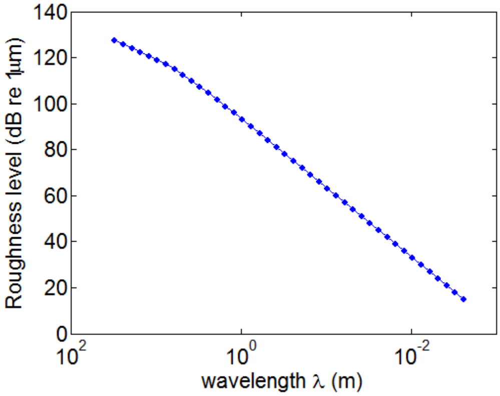


Figure 2. Assumed roughness spectrum in one-third octave.

The predicted dynamic response of the rail for both soil types is shown in Figure 3 in one-third octave. The results from the FE/BE model are also presented for the Horstwalde site. The motion of the train is included only in the TGV case. Figure 4 shows the one-third octave dynamic response in the free field for the Horstwalde site at distances of 0 m (under the track), 8 m, 32 m and 100 m from the track. For the distances 0, 8 and 32 m the velocity predicted using the FE/BE model is also shown. For all cases the TGV-MR and the FE/BE models show very good agreement. They show the same trends as the TGV model, although lacking some of the spectral detail which is caused by the motion of the dynamic loads past the receiver point. Note that this detail is only significant at the rail, it being essentially lost at distance within the free-field.

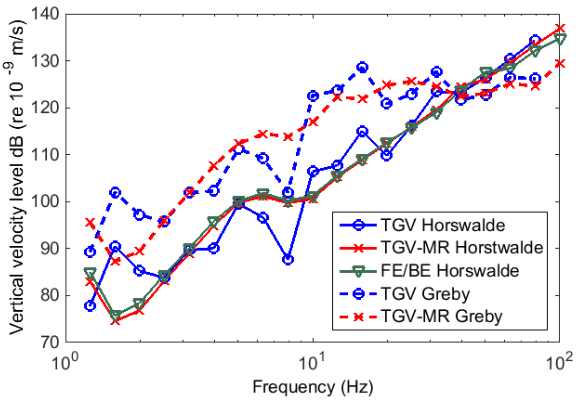


Figure 3. One-third octave comparison for the dynamic velocity rail response level for both sites.

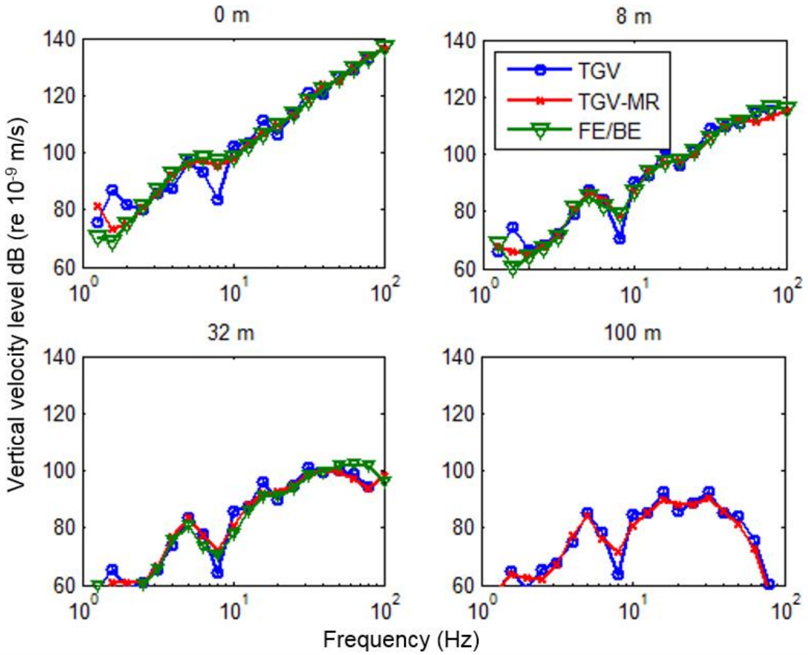


Figure 4. One-third octave dynamic velocity response level in the Horstwalde free field at 0, 8, 32 and 100 m from the track.

3. PIPE-IN-PIPE MODEL OF UNDERGROUND RAILWAYS

3.1 Model description

The PiP model is a semi-analytical model that predicts the vibration from a tunnel embedded in a multi-layered half-space (Figure 5). The tunnel and the surrounding soil are modelled as concentric thick cylindrical shells. A floating slab track is modelled as an Euler–Bernoulli beam and is coupled to the tunnel wall. The main assumption used in the model is that the near-field displacement of the tunnel is not influenced by the existence of a free surface or ground layers. The accuracy of this assumption increases with the increasing depth of tunnel. Similar to the TGV model, the tunnel wall and the surrounding soil are assumed to be invariant in the longitudinal direction. Hence, all calculations are performed in the frequency-wavenumber domain and results in the frequency-space domain are calculated using the inverse Fourier transform. It has been shown that the efficiency of computations of PiP is significantly higher than discretization models such as the coupled FE-BE model (11, 12, 13).

In PiP a train of infinite length is represented by an infinite number of wheelset masses, with a constant spacing, moving on a track as shown in Figure 6. Due to the low stiffness of primary suspensions of modern trains, and as the interest for trains in tunnels is usually limited to frequencies above 10 Hz, it is reasonable to ignore sprung masses in such a model. As shown in Figure 6, a model of double-beams supported on an elastic foundation is used to calculate forces at the wheel-rail interface. The source of excitation in the model is the track irregularity which causes relative displacements between the axles and the rail. The relative displacements are defined as uncorrelated random inputs and the outputs are calculated at points in the soil in the same cross-section as one of the wheelsets. These calculations are based on the assumption that vibration does not vary along a line parallel to the tunnel. The assumption is reasonable at distances from the tunnel that are large compared with the axle spacing (3).

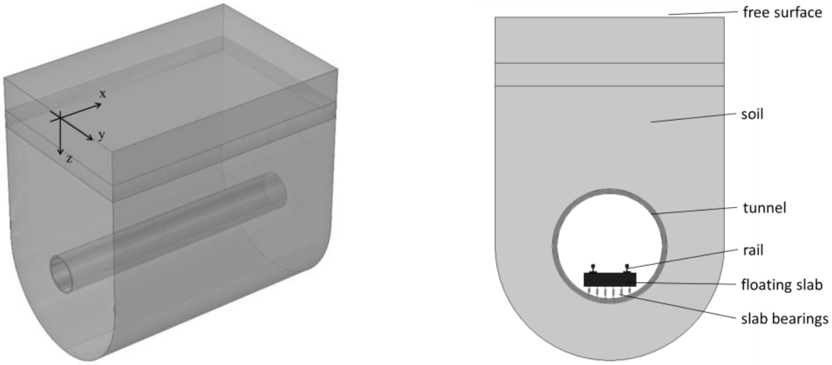


Figure 5. Layout of the model; a tunnel embedded in a multi-layered half-space. The figure shows two soil layers on a half-space.

To calculate displacements in the soil due to the forces applied at the rails, a model of a double-beam coupled to a tunnel embedded in a layered half-space is used. The model is shown in Figure 5b and it is used to calculate transfer functions between the rails and soil. The track is coupled to the tunnel-soil system in the wavenumber-frequency domain by using frequency response functions of the double-beam system and the tunnel-soil system. To calculate the transfer functions for the tunnel-soil system, it is assumed that displacements at the tunnel-soil interface due to a source inside the tunnel are the same whether or not there is a free surface. More specifically, first, the displacements at the tunnel-soil interface are calculated using a model of a tunnel embedded in a full space. Next, the internal source that produces the same displacements is calculated using a full-space model based on two concentric pipes. Finally, the vibration in the far field is calculated using these internal sources together with the Green's functions for an elastic layered half-space. The Green's functions calculations are performed using the elastodynamic toolbox developed at KU Leuven (14).

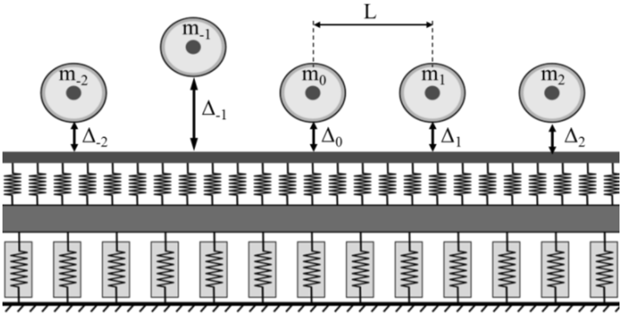


Figure 6. An infinite number of wheelset masses used to model a moving train.

3.2 Numerical example

In this section, the near-field and far-field response of a tunnel embedded in a layered half-space is calculated using the PiP model for a harmonic load applied at the invert of the tunnel, at (x=y=0 m, z=16.75 m). The response is calculated by the PiP model and then compared with that calculated by a coupled FE/BE model (12). The cross-section of the tunnel and the ground at x=0 is shown in Figure 7. The tunnel has an external radius of r_o=3 m and thickness t=0.25 m. The tunnel is made of concrete with a density ρ_t=2500 kg/m^3, longitudinal wave velocity c_p=5189 m/s, shear wave velocity c_s=2774 m/s and hysteretic damping loss factor η=0.03. The soil comprises a 6 m surface layer on top of a half-space. The surface layer (type 1, Figure 7) has longitudinal wave velocity C_{p1}=1964 m/s and shear wave velocity C_{s1}=275 m/s. For the half-space (type 2, Figure 7), the longitudinal wave velocity is C_{p2}=1571 m/s and the shear wave velocity is C_{s2}=220 m/s. The density for both layers was set to ρ=1980 kg/m^3 and the hysteretic material damping ratio ζ_s= ζ_p=0.03 in shear and volumetric deformation. The response is calculated at 4 points: A (x=0, y=0, z=17); B (x=0, y=0, z=11); C (x=10, y=10, z=6); and D (x=10, y=10, z=0).

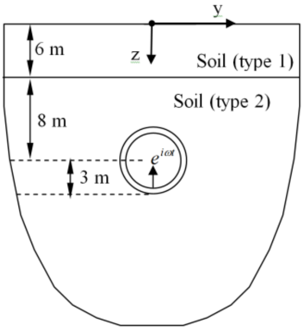
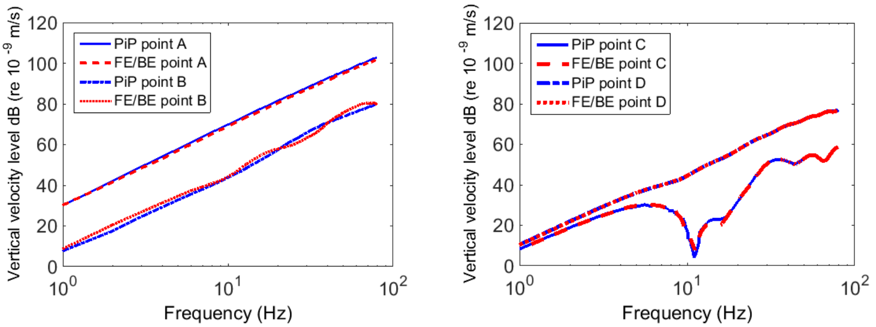


Figure 7: A cross-section of the tunnel and the ground at x=0. A harmonic load is applied at the tunnel invert at (x,y=0 m, z=16.75 m).

The results are presented for the frequency range of 1 to 80 Hz. Higher frequencies are not attempted due to the high computational cost of the FE/BE model as a result of the finer mesh required at high frequencies. The current frequency range is the predominant perceptible range for ground-borne vibration in buildings and it is sufficient for the purposes of validation.



(a) (b)

Figure 8: The velocity response level at (a) near-field (point A and point B) and (b) at far-field (point C and point D)

Figure 8a shows the vibration level at the tunnel-soil interface, at the tunnel invert (A) and the tunnel crown (B). A good agreement is observed between the PiP and

FE/BE models, which confirms that the near-field vibration is not influenced by layers away from the tunnel. Figure 8b show the far-field displacements at point C, at the interface of the two types of soil, and point D at the free surface. These results confirm that the PiP model calculates the far-field vibration for a tunnel embedded in a layered ground with the same accuracy as significantly more computationally expensive FE/BE models. This example confirms that the near-field vibration is controlled by the dynamics of the tunnel and the soil layer surrounding the tunnel.

4. OBJECTIVES AND IMPACT OF MOTIV PROJECT

As well as the work improving the TGV and PiP models, MOTIV aims to address the non-linear behaviour of track elements and the dynamic tunnel-soil-pile interaction for single as well as twin tunnels. The main hypothesis of the proposed work is that the level of accuracy of predictions will be significantly improved by accounting for these mechanisms when modelling ground-borne vibration from railways. The hypothesis will be tested by a series of comparisons of the models against measurements. The models will be incorporated in end-user software that will be released at the end of the project.

The effect of pre-load dependence and non-linear behaviour of resilient elements of tracks is investigated using excitation models that account for non-linearity not only of railpads but also of all other elements within conventional tracks, i.e. ballast, slab and trough tracks. The modelling effort uses FE methods and the Dynamic Stiffness approach that accounts for both discretely and continuously supported tracks. The track behaviour and the reaction force at the trackbed are investigated for tracks due to the movement of trains with roughness excitation; the models are developed in the time domain to allow the non-linear track stiffness and damping values to be taken into account. The effect of non-linear track stiffness is quantified by comparing the results from this model with that resulting from the same model when ignoring the non-linear behaviour (15).

Regarding the interaction of the railway with the surrounding soil, a novel BE model is under development for a multi-layered ground with one and two cylindrical cavities in the horizontal direction to allow for the coupling of single and twin tunnels respectively, and multiple cavities in the vertical direction to allow for coupling of piles. The model uses periodic structure theory to decrease the computational cost as only two slices of the problem need to be discretized: i) a generic slice with one or two cylindrical cavities; and ii) a central slice containing cavities for the piles beside the tunnel cavities. The piles are modelled using a bar formulation for axial deformation and a beam formulation for bending. The tunnels are modelled using thin-shell theory. The slices are joined, before coupling the tunnels and piles, to generate a solution for a full problem with horizontal and vertical cylindrical cavity(ies). Coupling is achieved with tunnels with infinite lengths by using the transfer functions for the two systems, following the method developed by Hussein and Hunt (16) in modelling floating slab tracks with discontinuous slabs in underground railway tunnels. The model is used to understand and validate the effect of weak coupling between tunnels and between tunnels and pile foundations. Moreover, the tunnel model is compared with the PiP model while the piled-foundation model is validated against previous FE/BE models (17).

5. CONCLUSIONS

The current paper reports recent developments in techniques for modelling vibration from surface and underground railways within the MOTIV project. The results presented illustrate the modelling capabilities and the efficiency of computations for the proposed numerical models. The models presented show good accuracy with reduced computational requirements compared with coupled FE/BE models. Further advancements within the work of the MOTIV project aim to address the non-linear behaviour of the track elements and the dynamic tunnel-soil-pile interaction for single and twin tunnels.

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