THE USE OF HANDWRITING RECOGNITION TECHNOLOGY IN MATHEMATICS EDUCATION: A PEDAGOGICAL PERSPECTIVE

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In mathematics education, the lack of an intuitive means to enter mathematics expressions online has been a major barrier to effective communication, causing mathematics to be lagging behind in the development of online collaborative learning environments. This study evaluates the use of handwriting recognition technology as a potential solution from a pedagogical standpoint. With pedagogical needs in mind, a new handwriting recognition user-interface (MathPen) was developed as a research tool to investigate the teaching and learning perspectives through a) an expert review with practising teachers, and b) a usability study with undergraduate students.

Keywords: Handwriting recognition user-interface design, instrumental genesis, technology-induced distractions

INTRODUCTION

The benefits and effectiveness of collaborative learning in mathematics is well-established (Edwards, 2009). Through proposing, exploring and evaluating different ideas with their peers, students are better able to develop a deeper understanding (Wegerif, 2013). As students justify and defend their mathematical reasoning, underlying misconceptions are uncovered and addressed (Mercer, 2000). Additionally, since collaborative group work is common in the work place, this learning method prepares young people for future employment (Hoyles, Noss, Kent and Bakker, 2007; ACME, 2011). The ability to employ collaborative learning through the Web, thus transcending the limits of time and space, has already benefited many text-based subjects (Harasim, 2002). Yet, developments for mathematics education in this regard is reported to be lagging behind (Allen and Seaman, 2010). Researchers such as Catalin, Deyan, Kohlhase and Corneli (2010), Costello, Fox and Walsh (2009), and Reba and Weaver (2007) have alluded to the lack of intuitive input methods for mathematics expressions as the main cause of the problem. Although joint-editing whiteboards are now available in pictorial formats, these do not lend itself to integration with digital computational tools, which could enrich the collaborative discussions. As Lo, Edwards, Bokhove and Davis (2013) argued, “serious considerations should be given to online handwriting recognition systems as a means of opening the way to online collaborative learning for mathematics education” (p.173).

THEORETICAL FRAMEWORK

According to the theory of instrumental genesis, an educational tool in the hands of students who do not yet know how to utilise the tool for educational purposes has little value (Rabardel and Bourmaud, 2003). The theory argues that for the tool to become an educationally useful instrument (or *instrumentalised*), specific conditions must be met. Teachers need to bring the tool’s affordance to the fore, while the students need to create personal concepts of the tool’s behaviour and develop their own ways of leveraging the tools’ capability for their own educational advantage (see theoretical overview in Drijvers, Godino, Font and Trouche, 2012). Typically, the onus of instrumentalisation rests with the teachers and the students, not with the tool design engineer. However, depending on how the tool is designed, the process of instrumentalisation may be made is easier or more difficult. Therefore, instrumental genesis, when viewed from an engineering standpoint, can also be used as a guide leading to a more readily ‘*instrumentalisable*’ tool. This study focuses on the investigation of the teaching perspective and the learning perspective in order to better understand the barriers to handwriting recognition technology being instrumentalised in mathematics education.

METHODOLOGY

This study is divided into three phases: 1) an engineering development phase, 2) an expert review phase with practising teachers, and 3) a usability study phase with university students. During phase 1, an online handwriting recognition system (MathPen) was designed and implemented. Although off-the-shelf software packages are available, these are prohibitively expensive for many and are not designed with education in mind (Lo et al, 2013). Since recognition algorithms are freely available through academic publications, an in-house development would a) provide greater engineering flexibility for research purposes, b) allow new-gained insight to be implemented in a reduced timescale, and c) eventually lead to a research-based tool which can be made publicly available free of charge. In order to take into account of users' needs, the design process, in line with a design-based research methodology (Reeves, 2006), began with findings from previous studies (Lo, 2012), which highlighted the need for multiline recognition (see Engineering Design section). Future designs will utilise findings from phases two and three, where the teachers' and learners' perspectives were explored, for future improvements.

Phase 2 was a three-part expert review with three practising teachers. First, since hardware capabilities and recognition accuracy are commonly thought to be a barrier to the instrumentalisation of the technology in the classroom (Lo, 2012), the teachers were given a range of devices (Android-based Samsung Galaxy Note 10.1, iOS-based 3rd generation iPad and Windows 7-based Tablet PC) to explore the capability of commercial handwriting recognition products as well as MathPen's modest implementation. Figure 1 shows an example of mathematics expressions supplied for testing.

$$\frac{\sin\theta+\cos\theta+\tan\theta}{x+y+z} \qquad \alpha_{n+1}-3\beta=\frac{2}{3}\alpha_n+\beta_1 \qquad \sqrt{1+\sqrt{2+\sqrt{3+\sqrt{4}}}} \qquad 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}$$

$$x^{\frac{1}{2}}\left(1+\frac{1}{2^2}\right)\left(1+\frac{1}{2^4}\right)\left(1+\frac{1}{2^8}\right) \qquad \left[b^x\left\{\left(\frac{a}{b}\right)^x+1\right\}\right]^{\frac{1}{x}} \qquad A=\sqrt{a+\frac{1}{\sqrt{a+\frac{1}{\sqrt{a}}}}+\sqrt{b}} \qquad \lim_{x\to\infty}\int_0^x e^{-y^2}dy=\frac{\sqrt{\pi}}{2}$$

Fig 1: Sample expressions for exploring recognition accuracy

Next, three mathematics questions were posted online, to which the teachers were to respond with the supplied model answers. Question 1 involves very few steps with notations that are well within MathPen's recognition power. Question 2 has the same number of steps but with notations which are at the threshold of MathPen's recognition capability. Question 3 is also at the threshold of MathPen's recognition capability but with an increased number of steps (Fig 2). The session concluded with a focus group to reflect on their instrumentalisation process/ experience from a teaching viewpoint.

Q1. Given $y = 4x - 7$, find x.

$$y = 4x - 7$$
$$y + 7 = 4x$$
$$\frac{y+7}{4} = x$$
$$x = \frac{y+7}{4}$$

Q2. Given $y = \frac{\sqrt{2}x^2+1}{4}$, find x.

$$4y = \sqrt{2}x^2 + 1$$
$$4y - 1 = \sqrt{2}x^2$$
$$\frac{4y-1}{\sqrt{2}} = x^2$$
$$x = \pm\sqrt{\frac{4y-1}{\sqrt{2}}}$$

Q3. Evaluate $y = \int_0^2 (\frac{2x+1}{4})\ dx$

$$y = \int_0^2 (\frac{2x}{4}+\frac{1}{4})\ dx$$
$$y = [\frac{x^2}{4}+\frac{x}{4}]_0^2$$
$$y = \left[\frac{2^2}{4}+\frac{2}{4}\right]-\left[\frac{0^2}{4}+\frac{0}{4}\right]$$
$$y = \left[\frac{4}{4}+\frac{2}{4}\right]$$
$$y = \left[1+\frac{1}{2}\right]$$
$$y = 1\frac{1}{2}$$

Fig 2: Online questions posted to the experts

Having established in phase 2 that the barrier to the instrumentalisation process is not the level of recognition accuracy as is commonly perceived, but the level of technology-induced distractions, phase 3 further investigates the distraction elements by inviting seven undergraduate students (studying engineering or mathematics) to complete two pieces of collaborative group work (ten minute each), using MathPen for one task and keyboard entry for the other. Since commercial products have already been shown to feature more technology-induced distractions, keyboard entry has been chosen as a comparator for phase 3. The students were split into two groups so that one group would complete one task with MathPen first and keyboard entry the second, while the other group performs the same tasks in the same order but with keyboard entry first and MathPen second. Throughout the exercise, all the keyboard and mouse interactions were recorded through screencast recordings. Additionally, students were also asked to think aloud throughout the process to externalise their thoughts. The session concluded with a 30-minute focus group discussion to reflect on their instrumentalisation process and experience from a learning standpoint.

ENGINEERING DESIGN

In terms of the engineering design, it is known that without handwriting recognition, the standard quadratic equation would have to be entered as "*[TEX] x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}[/TEX]*" in order to communicate online. While current technology is capable of recognising mathematics one line at a time, recognition of multiple lines of mathematics is not supported in any present systems (Lo, 2012). When users submit multiple lines of mathematics for recognition, present systems will merge these into a single line for recognition (Fig 3, 4). This is a pedagogical concern, because new lines of mathematics cannot be written until the recognition result is transferred to the communication medium. By then, without a visual point of reference to the ongoing mathematical argument, the mathematical argument for the next step is likely to have been forgotten.

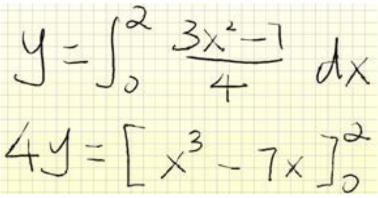

Fig 3: Multiple lines of mathematics for recognition

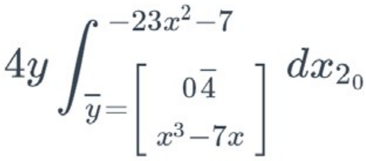

Fig 4: Erroneous recognition without multi-line recognition

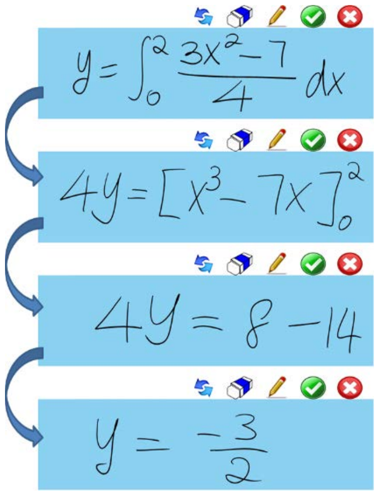

Fig 5: Multiple lines of mathematics in MathPen

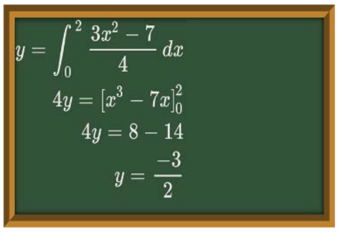

Fig 6: Floating blackboard

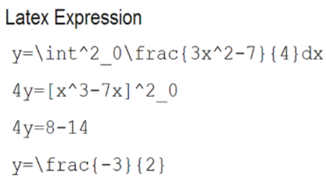

Fig 7: Complete set of Latex expressions

MathPen is designed to address this pedagogical issue (Fig 5-7). First, the users can submit unlimited lines of mathematics to the recognition engine so that they can concentrate on the mathematical reasoning from beginning to end. Figure 4 shows a 4-lined submission for an integral evaluation. Then, the formatted recognition result is displayed on a floating blackboard which 'floats' together with the users as the they scroll down the page to add more lines (Fig 6). Finally, the users receive a complete set of Latex code for copying and pasting into their choice of Web-based communication platforms (Fig 7).

PHASE 2 RESULTS: EXPERT REVIEW

During the technology exploration stage, all three experts were impressed with the standard of recognition currently achievable. As well as testing the technologies with the suggested mathematics expressions (Fig 1), they also created several of their own. Despite observing occasional recognition errors, the experts unanimously agreed that handwriting recognition for mathematics is quickly becoming a reality and should be given serious consideration. In terms of choice of equipment, the experts felt that only the Windows-based Tablet PC delivered the necessary processing power to keep up with the recognition needs. The experts also found the Tablet PCs' palm rejection feature, which allowed them to rest their palm on the screen while writing, very helpful.

During the online interaction stage, all three experts adopted handwriting recognition without hesitation. Although initially inclining towards commercial packages for their superior recognition power, all three experts eventually adopted MathPen as their preferred recognition engine for question one. As they continued to the second question, two of the experts continued with MathPen while the remaining expert switched from MathPen to commercial products and then back to MathPen. Although all three experts attempted question three, they eventually abandoned the task and none progressed to the end.

At the focus group that followed, experts commented on their experience in handling question one and expressed their frustration at the commercial products' lack of multi-line support; examples include:

> "This going back and forth between the forum page and the recognition page one line at the time is driving me crazy."

> "I'm a teacher and this maths is easy. Still, I can't remember the next step. One line at a time is too distracting."

> "The kids using this would be trying to work things out. If we struggle with the answers in front of us, it would be impossible for the kids."

In the case of question two, when the mathematics involved complex notations and the number of erroneous recognition results increases, there was a battle between MathPen's multi-line recognition support and commercial products' superior recognition power. Finally, in question three, where there are many lines of complex notations, the experts gave up. Relevant comments include:

> "Either way, you are distracted. Switching between pages at every line is distracting, but correcting recognition error has the same effect."

> "However you look at it, I think we are talking about interruption to thinking. It doesn't matter what is distracting you. It almost seems the moment you stop thinking about the maths at hand, you lose your train of thought."

> "I think this perfectly well explains why people don't discuss maths online."

During the focus group, the experts were asked to comment on the use of handwriting recognition from a teaching perspective, to which they answered:

> "I think the most important thing for me is having sufficient recognition power for the task and the multi-line thing. I mean it was quite ok for the first question. Everything ran smoothly and MathPen was great."

> "If I set a question for discussion, what I would like to see is the quality of discussion. So I guess if the kids are frustrated, they may be tempted to skip steps or even give up like us. You've probably hit the nail on the head there, there maybe nothing to do with accuracy, but more to do

with letting the kids focus on what they are doing. As MathPen's recognition power is up to the job for that level of mathematics, it doesn't really have to be the latest technology, does it?"

> "There is a whole host of Web 2.0 stuff that my colleagues are using. They talk about how the kids work together online and correct their own mistakes, but it's just impossible for maths. It's just too much work involved. Software like MathPen really would be the answer."

> "I teach in a boarding school with many international students, and there are always a few who have to return home during term time. Supporting these students through the Internet has been a real struggle, particularly when students get things wrong, there's always a reason for it. At the moment, it's a case of writing on a piece of paper and scanning. But that's dependent on them having a scanner on the other side of the world. We can kind of forget just how fundamental it is to be able to communicate. I mean, how is anyone supposed to teach without being able to communicate?"

> "It boils down to being able to communicate at ease and without distraction. The whole point about paying attention in class is that you don't get distracted and you listen to what others have to say. I suppose, from a teaching point of view, what this does is to make this possible in an online environment."

Therefore, contrary to common opinions, the reason for handwriting recognition not having a major role in mathematics education is *not* because the latest recognition algorithms is not accurate enough for school use and it is *not* because the hardware is expensive. In fact, with the Tablet PCs costing around £150, it is the cheapest option available. The teachers' favourable comments towards MathPen's multi-line shows that, pedagogically speaking, it is the technology-induced distractions inherent in current user interface design that is preventing the instrumentalisation of handwriting recognition from happening in real life setting.

PHASE 3 RESULTS: USABILITY STUDY

During the first five minutes of both online discussion exercises (ten minutes each), contrary to expectation, both student groups spent more time discussing mathematics and were able to progress further in their discussion when using keyboard entry than when using MathPen. Reviewing the screencast recordings of the onscreen interactions and audio recordings of the think aloud commentaries reveals that students equipped with MathPen had spent a substantial amount of time with the software, switching between writing mathematics and responding to comments posted on the Web. However, during the last five minutes, where student groups were not equipped with MathPen, both groups shown less overall pedagogical progress, digressed and did not continue with the mathematical discussion. By contrast, where students were equipped with MathPen, the mathematical discussion continued to the end.

At the focus group, the students' comments regarding keyboard entry were as follows:

> "I think it's bearable, but then I have been typing maths at uni for some time now. It took a lot to get used to it, and [the maths] just doesn't look right. So you have to reinterpret the thing all the time. It gets tiring and makes you want to give up. It would be much better if handwriting can just get rid of that awkward stuff altogether."

> "For this exercise, it's kind of fine. You know, hat 2 instead of squared is kind of well-known to us. Not sure if I would have known that at school-age though. Also things would be very different if the maths have more difficult stuff. You know, things like square root and fractions. I think when you have loads of those stuff, typing becomes rather stupid. I mean you can't interpret those stuff without the proper formatting."

> "I hate the stuff, just can't get on with it. I know how to type using that funny hat thing, but I always have to copy that thing out by hand and read it that way. I mean, you know, that's how maths is meant to look like. Not a chance for me. Sorry I gave up in the end. At least the handwriting thing, I know it's slow and it's not great yet, but at least it gives you something that you can just read and understand."

With regards to keyboard-based communication, despite the students' engineering/ mathematics background, there is a consensus that not only is typing mathematics difficult, interpreting the unformatted mathematics is equally troublesome. This is even to the extent that none of these students managed to engage with the tasks for the entire ten-minute duration.

By contrast, when equipped with MathPen, all the students were able to engage with the mathematics throughout the ten-minute exercise, and their comments were as follows:

> "I really like MathPen's multi-line idea, but it would be even better if I don't need to switch pages at all. It's like I can hear the messages are coming through, and I want to just scan read the message. But you can't do that. It's better than typing on a laptop, but I think it needs to be one step further."

> "It is quite frustrating to see postings, and I can't just compare what they've said with what I am writing. They are on different pages. If the communication system and MathPen are integrated, then you can see things side by side."

> "I think MathPen is fine when it gets things right, but when you make a mistake, you can't just rub out one stroke. You have to start the whole lot again."

> "It's like you are busy thinking about the maths and what to do next, then you noticed MathPen doesn't always get it right. You'd want to just correct that bit that it gets wrong. You don't want to start that that line of maths again."

> "I like MathPen, at least you can say things without having to think 'Oh, how am I suppose to type this'. But when I am concentrating, ideas flow, and you want to capture that moment. When technology is so slow, it gets frustrating. I'd like it to be a bit quicker."

> "There's actually a lot going on at the same time. You are thinking about the maths, the questions, the solution, the way to express it and so on. And then on top of that, you are sort of bombarded with messages, you know, we are all talking at the same time. By the time you manage all that, you really haven't got that much patience to battle with technology. It's to do with the speed thing that [participant x] said. You want to be able to just write and not think about anything else."

Generally speaking, the consensus is that MathPen, when compared with traditional keyboard entry, is definitely a superior choice. There was also clear indications that the multi-line feature is much appreciated. However, it is also evident from the comments above that more needs to be done in order for the tool to be instrumentalisable.

SYNTHESIS

Overall, both teachers and students are supportive of the idea of using handwriting recognition in mathematics education. Concerning the online learning process, the students described it as a hectic time, trying to keep hold of their mathematical train of thought in their heads while simultaneously maintaining a channel of communication with their peers. In describing their experience, they use expressions such as, "*there's a lot going on*", "*it's information bombarding you all the time*", and "*by the time you've done all that*". However, despite the demands, the students' are generally favourable to this mode of learning. Their comments indicated a high level of interests in what their peers have to say and are keen to modify their thoughts accordingly. This is also reflected in the screencast recording where students are often seen to have paused from their ongoing activity to take note of every new message. There were also many occasions where students either deleted or modified their responses in view of what they had just read. As one student succinctly put it, "*Of course it feels intensive, cos you're thinking independently while listening and thinking about what they are thinking at the same time. But that's the whole point of teamwork: to bounce off each other, generate ideas and get things done quickly*". This corresponds well with one of the expert's comments regarding the quality of discussion and mathematical arguments. The learning process described here is precisely the mechanisms that make collaborative learning so effective (Harasim, 2002; Edwards, 2009), and the significance of this student's statement is that online collaborative learning can be just as effective for mathematics students too.

However, it is also clear that in order for the tool to be instrumentalisable in real-life settings, certain conditions have to be met. A recurring theme highlighted by both the teachers and the university students is the level of technology-induced distraction and its impact on the users' ability to maintain their mathematical train of thought. With regards to the impact of technology-induced distractions on the learning process, the students used expressions such as, "*you forget what you were doing*", "*lost track*", "*you'd want to keep the flow going, but can't*", "*you end up starring at what you've written and wondered what for*". As one of the experts pointed out: "*It boils down to being able to communicate at ease and without distraction*".

Sources of distraction can be many. For example, during phase 2, all participants identified the lack of multi-line support as a significant source of technology-induced distraction. However, results from phase 3 show that the provision multi-line support alone is not sufficient, hence all participants agreed that a more tightly integrated system where the online communication medium and MathPen can be viewed side-by-side simultaneously would be beneficial. Additionally, recognition error correction proved to be another source of distraction. From the students' comments, it appears that MathPen's accuracy level was sufficient for the task, but an intuitive means of error correction is lacking. Perhaps alluding to the pencil and paper experience, some of the students speak of "*rubbing out*" the mistakes instead of "*crossing out*" the entire line and starting again. Whatever the cause of the distraction, or whatever the engineering solution may be, it is clear from this study that, from a pedagogical viewpoint, the key to an instrumentalisable handwriting recognition tool is to facilitate online mathematics communication while keeping technology-induced distractions to the minimum.

CONCLUSIONS

This study provides additional evidence of limitations to entering mathematics online as a major barrier to online collaborative learning in mathematics. Evidence from this study shows that current handwriting recognition is already delivering sufficiently accurate results for school mathematics. Tablet PCs costing around £150 are providing sufficient computing power. Therefore, recognition accuracy, hardware demands and portability issues are not a cause for concern in usability.

Where previous studies led us to believe that the main issue is in "the lack of a natural and effective means of entering mathematical expressions online" (Lo et al, 2013, p.173), an even greater concern, from a pedagogical standpoint, is the impact that this is having on the students' ability to focus on their mathematics. In terms of students' progress in their mathematical understanding, the change of artefact from typewriting to handwriting means that students are progressing through collaborative discussions, instead of abandoning the exercise. Therefore, instead of focusing on vague terms such as "natural and effective", which are difficult to define, the results from this study suggest that identifying and reducing technology-induced distractions maybe a more fruitful line of research. Since this study has identified a number of technology-induced distractions, all of which are addressable through interface engineering, these will be the focus of our future studies.

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