ROBUSTNESS AND EFFICIENCY OF AN ACOUSTICALLY COUPLED TWO-SOURCE SUPERDIRECTIVE ARRAY

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In a number of applications it is desirable to reproduce sound in a specific region whilst minimising it elsewhere. This can, in theory, be achieved using loudspeaker arrays and optimal, or superdirective beamforming techniques. However, these superdirective methods generally require a large electrical power at low frequencies, where the wavelength is large compared to the array, and are generally sensitive to practical uncertainties that may occur in the electroacoustic response of the loudspeaker array. In order to overcome these limitations, regularisation is often used to constrain the electrical power requirements of these arrays and improve their robustness to response uncertainties. However, in the context of a two-source line array an alternative method of reducing the required electrical power by coupling the two loudspeakers together via a common acoustic enclosure has been proposed. This paper investigates the performance of the coupled two-source loudspeaker array, and compares its performance to the standard uncoupled two-source array in terms of the acoustic contrast, electrical power requirement and robustness to uncertainties in the system's responses. It is firstly shown through a series of simulations that when there is no uncertainty in the responses, although the two arrays achieve the same acoustic contrast performance, the electrical power required by the coupled array is about 100 times lower than that required by the uncoupled array at low frequencies. It is then shown that the coupled array is significantly more robust to response uncertainties than the uncoupled array and, even when the electrical power required by the uncoupled array is limited to be equal to that required by the coupled array, it achieves a higher level of acoustic contrast performance.

1. Introduction

The generation of localised listening zones using arrays of loudspeakers has become particularly important in a number of applications including in mobile devices [1, 2], computer monitors [3], car cabin interiors [4], home entertainment systems [5], and aircraft seats [6]. In many of these applications, and particularly in the mobile device application, the electrical power required by the loudspeaker array is of significant importance. In practice, it is often necessary to employ constraints in the design of the filters in order to limit the maximum electrical power to be within the capabilities of the loudspeakers. This is generally achieved using some form of regularisation in the optimisation of the loudspeaker driving signals [4, 5] and it has also been shown that the application of such regularisation improves the robustness of the array to uncertainty in the acoustic responses and the positions and responses of the loudspeakers [7, 8]. This is particularly important in superdirective, or optimal beamforming systems due to their high sensitivity to uncertainties in the assumed response of the system [7, 8, 9], however, this regularisation also limits the directivity of the system. For the specific case of a two-source line array, by acoustically coupling the two loudspeakers via a shared enclosure it has been shown that the required electrical power can be significantly reduced at low frequencies [1]. This, therefore, appears to be a promising method of reducing the required electrical power without compromising the directivity of the array, however, the robustness of this system has not been considered.

In general the robustness of superdirective, or personal audio systems has been investigated for systems in which the electroacoustic interactions between the transducers is insignificant. However, in the case of the acoustically coupled two-source directional loudspeaker, this is no longer a valid assumption and, therefore, this paper presents an investigation of the robustness of these systems compared to the alternative uncoupled loudspeaker system. In section 2 the acoustic contrast control method of optimising a loudspeaker array for personal audio is reviewed and the performance metrics are described. In section 3 a two-port model of the acoustically coupled loudspeaker array is derived. In section 4 simulations are presented to compare the performance and robustness of a two-source loudspeaker array using either an acoustically coupled enclosure or two independent enclosures. Finally conclusions are drawn in section 5.

2. Superdirective Beamforming

There are a number of methods in the literature for designing superdirective, or optimal beamformers in the context of the generation of independent, or personal listening zones [5, 10, 11, 12]. If the primary performance criterion of the array is the level of separation between the bright, or listening zone and the dark, or quiet zone, then the acoustic contrast control strategy is guaranteed to give the highest level of performance. The alternative methods of sound zone generation generally provide some compromise between the difference in levels between the bright and dark zones and the sound field distribution within the bright zone [12]. However, the acoustic contrast control strategy will be employed here since it gives the highest performance in sound zone separation.

2.1 Acoustic Contrast Control

The acoustic contrast is defined as the ratio of the acoustic potential energy density in the bright zone to that in the dark zone [10]. Alternatively, if we represent the sound field in the bright zone as a vector of pressures, $\mathbf{p}_B$, measured at N_B microphone positions within the bright zone, and similarly describe the dark zone using a vector of pressures measured at N_D locations in the dark zone, $\mathbf{p}_D$, as shown in Figure 1, then the acoustic contrast can be defined at a single frequency as [1]

(1) $$C = \frac{N_D \mathbf{p}_B^H \mathbf{p}_B}{N_B \mathbf{p}_D^H \mathbf{p}_D} = \frac{N_D \mathbf{i}^H \mathbf{G}_B^H \mathbf{G}_B \mathbf{i}}{N_B \mathbf{i}^H \mathbf{G}_D^H \mathbf{G}_D \mathbf{i}},$$

where superscript H is the Hermitian, complex conjugate, transpose; $\mathbf{i}$ is the vector of the two complex signals driving the loudspeakers in the two-source array; and $\mathbf{G}_B$ and $\mathbf{G}_D$ are the matrices of transfer responses between the inputs to the two sources in the loudspeaker array and the N_B and N_D pressure measurement positions in the bright and dark zones respectively. From this ratio it can be seen that the vector of driving signals, $\mathbf{i}$, must be optimised in order to maximise the acoustic contrast.

This optimisation problem can be cast as a constrained quadratic optimisation in which the sum of the squared pressures in the dark zone, $\mathbf{p}_D^H \mathbf{p}_D$, is minimised, subject to the constraint that the sum of the squared pressures in the bright zone, $\mathbf{p}_B^H \mathbf{p}_B$, is held constant with a value B. It is also useful in practice to include an additional constraint such that the array effort, or sum of the squared driving signals, $\mathbf{i}^H \mathbf{i}$, which is proportional to the electrical power, is held constant with a value W. The cost function in this case can be expressed as the Lagrangian [7]

(2) $$J = \mathbf{i}^H \mathbf{G}_D^H \mathbf{G}_D \mathbf{i} + \lambda_B \left(\mathbf{i}^H \mathbf{G}_B^H \mathbf{G}_B \mathbf{i} - B \right) + \lambda_W \left(\mathbf{i}^H \mathbf{i} - W \right),$$

where λ_C and λ_W are the Lagrange multipliers relating to the bright zone and electrical power constraints respectively. The optimal solution is then given by setting the differential of J with respect to the real and imaginary parts of $\mathbf{i}$ to zero [1, 7] and this gives

$$\lambda_B \mathbf{i} = -\left[\mathbf{G}_D^H \mathbf{G}_D + \lambda_W \mathbf{I}\right]^{-1} \mathbf{G}_B^H \mathbf{G}_B \mathbf{i}. \tag{3}$$

This is a classical eigenvalue problem and the optimal solution for the vector $\mathbf{i}$ is proportional to the eigenvector of $\left[\mathbf{G}_D^H \mathbf{G}_D + \lambda_W \mathbf{I}\right]^{-1} \mathbf{G}_B^H \mathbf{G}_B$ corresponding to its largest eigenvalue, where λ_W has to be set such that the constraint on $\mathbf{i}^H\mathbf{i}$ is satisfied [7, 10]. The absolute value of $\mathbf{i}$ is then determined by setting the Lagrange multiplier, λ_B, such that the constraint on the sum of the squared pressures in the bright zone is fulfilled. In practice, since the sum of the squared driving signals, $\mathbf{i}^H\mathbf{i}$, will also be dependent on λ_B, the selection of the two Lagrange multipliers must be achieved through an iterative process to ensure that both constraints are fulfilled.

3. Two-Port Model of Two-Source Loudspeaker Arrays

The two-source endfire loudspeaker array can be implemented using either two independent, closed-back enclosures as shown in Figure 2a or using a single coupled enclosure, as shown in Figure 2b, in which case the two loudspeaker drivers interact through the internal acoustic coupling. These two loudspeaker arrays can be modelled using a two-port network model to determine the electrical signal requirements [1]. The two-port network model assumes that the loudspeaker diaphragms act as pistons, and the radiation from the individual sources is modelled as a free field monopole. These assumptions have been shown to provide sufficiently accurate results at low frequencies where the wavelength of the radiated sound is large compared with the loudspeaker diaphragm.

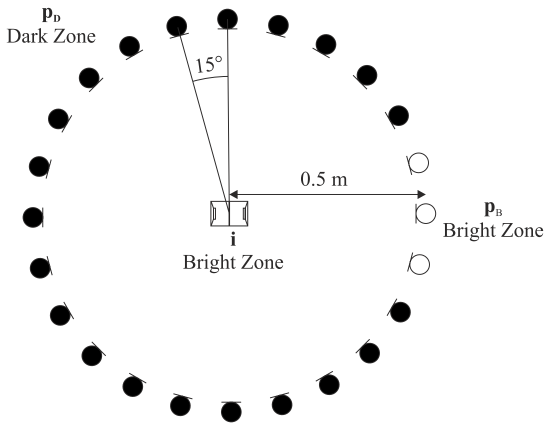


Figure 1: Geometry of the bright and dark zones and the two-source loudspeaker array.

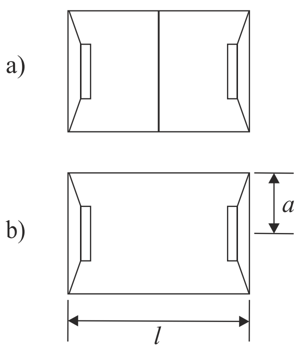


Figure 2: The uncoupled (a), and coupled (b) two-source loudspeaker arrangements.

Using two-port network theory, the vector of volume velocities of the two loudspeakers, $\mathbf{q}$, can be written in terms of the two driving currents, $\mathbf{i}$, and the two effective pressures acting on the diaphragms, $\mathbf{p}$, as

$$\mathbf{q} = \mathbf{S}\mathbf{i} + \mathbf{Y}_{a0}\mathbf{p}, \tag{4}$$

where $\mathbf{S}$ and $\mathbf{Y}_{a0}$ are the diagonal matrices of loudspeaker sensitivities and acoustic admittances for the two loudspeakers respectively. The acoustic admittance matrix is given by

$$\mathbf{Y}_{a0} = \frac{1}{Z_{a0}}\mathbf{I}, \tag{5}$$

where $\mathbf{I}$ is the identity matrix and Z_{a0} is the open-circuit acoustical impedance given by

$$Z_{a0} = \frac{Z_{m0}}{(\pi a^2)^2} = \frac{R + j\left(\omega M - K/\omega\right)}{(\pi a^2)^2} \tag{6}$$

where Z_{m0} is the open-circuit mechanical impedance of the loudspeaker, a is the radius of the loudspeaker diaphragm, R is the damping, M is the moving mass and K is the stiffness of the loudspeaker suspension. The loudspeaker sensitivity matrix is given by

$$\mathbf{S} = \frac{T}{Z_{a0}}\mathbf{I} = \frac{Bl_{coil}/\pi a^2}{Z_{a0}}\mathbf{I} \tag{7}$$

where T is the loudspeaker transduction coefficient, B is the magnetic flux density and l_{coil} is the length of the voice coil in the magnet gap.

The vector of effective pressures acting on the two diaphragms is given by the difference between the radiated pressures and the pressures acting on the diaphragms inside of the enclosure, which is

$$\mathbf{p} = \mathbf{Z}_R\mathbf{q} - \mathbf{Z}_L\mathbf{q} \tag{8}$$

where $\mathbf{Z}_R$ is the matrix of self and mutual radiation impedances and $\mathbf{Z}_L$ is the matrix of input and transfer impedances within the enclosure. In practice, for both enclosure designs, the radiation impedances will be small compared to the load impedances and, therefore, may be neglected, as in [1]. Thus, neglecting the radiation impedances and substituting eq. (8) into eq. (4) and rearranging gives the vector of volume velocities as

$$\mathbf{q} = \left[\mathbf{I} + \mathbf{Y}_{a0}\mathbf{Z}_L\right]^{-1}\mathbf{S}\mathbf{i}. \tag{9}$$

The impedance matrix, $\mathbf{Z}_L$, in eq. (9) differentiates the behaviour of the two enclosure designs and can be expanded as

$$\mathbf{Z}_L = \begin{bmatrix} Z_I & Z_C \\ Z_C & Z_I \end{bmatrix} \tag{10}$$

where Z_I are the input impedances experienced by the two loudspeakers and Z_C are the coupling, or transfer impedances between the two loudspeakers. These impedances can be derived for the two array configurations by describing the pressure and particle velocity in the enclosures using two plane waves propagating in the positive and negative directions perpendicularly to the loudspeakers. For the coupled array the input and coupling impedances are given by

$$Z_I = -j\frac{\rho_0 c_0}{\pi a^2}\cot\left(kl\right), \qquad Z_C = \frac{-j\rho_0 c_0}{\pi a^2 \sin\left(kl\right)}, \tag{11}$$

where ρ_0 is the density of air, c_0 is the speed of sound in air, k is the acoustic wavenumber and l is the length of the two-source array, as shown in Fig. 2. For the uncoupled two-source array there is no internal coupling between the two loudspeakers and so $Z_C = 0$. The input impedance for the uncoupled array is given by

$$Z_I = -j\frac{\rho_0 c_0}{\pi a^2}\cot\left(\frac{kl}{2}\right), \tag{12}$$

where the factor of a half is due to the length of the uncoupled enclosure cavity being half of the coupled enclosure cavity, as shown in Figure 2. For the uncoupled array the impedance matrix $\mathbf{Z}_L$ is therefore diagonal, while in the case of the coupled array the impedance matrix is fully populated and symmetric.

The pressures radiated from the two-source loudspeaker array to the bright and dark zones can be calculated as

$$\mathbf{p}_B = \mathbf{Z}_B\mathbf{q} \qquad \mathbf{p}_D = \mathbf{Z}_D\mathbf{q} \tag{13}$$

where $\mathbf{Z}_B$ and $\mathbf{Z}_D$ are the acoustic transfer impedances between the acoustic volume velocities of the two loudspeakers and the pressures measured at the N_B and N_D microphone locations in the bright and dark zones respectively. Substituting Eq. (9) into Eq. (13) then gives the vectors of bright and dark zone pressures in terms of the loudspeaker driving signals as

$$\mathbf{p}_B = \mathbf{Z}_B\left[\mathbf{I} + \mathbf{Y}_{a0}\mathbf{Z}_L\right]^{-1}\mathbf{Si} \qquad \mathbf{p}_D = \mathbf{Z}_D\left[\mathbf{I} + \mathbf{Y}_{a0}\mathbf{Z}_L\right]^{-1}\mathbf{Si}. \tag{14}$$

Since the full electroacoustic transfer response matrices, $\mathbf{G}_B$ and $\mathbf{G}_D$ are dependent on the impedance matrix $\mathbf{Z}_L$, which differs for the two enclosure configurations, the optimal driving signals will differ for the coupled and uncoupled two-source loudspeaker arrays. The effect of this on both the efficiency and robustness of the arrays will be investigated in the following sections.

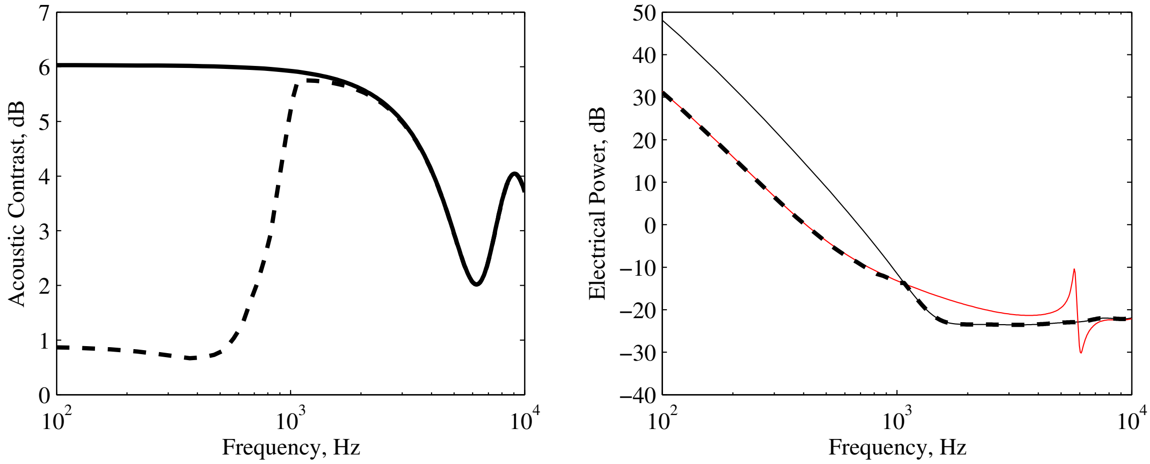
4. Simulations

To investigate the difference in performance between the coupled and uncoupled two-source loudspeaker arrays, the two systems will be simulated using the models derived in the previous section. The length of the two arrays has been defined as 3 cm and the loudspeaker drivers have been modelled based on a micro-loudspeaker with a radius of 1 cm.

4.1 Optimal Performance of the Two-Source Arrays

The optimal loudspeaker driving signals for the two arrays have been calculated according to eq. 3, using the electroacoustic transfer responses derived in the previous section for the uncoupled and coupled arrays. The Lagrange multiplier, λ_B, in both cases has been set to achieve a sound pressure level of 60 dB at the centre of the bright zone. Figure 3 shows the acoustic contrast and electrical power required by the two arrays when there is no uncertainty in the electroacoustic responses. The thick black line in Figure 3a shows the acoustic contrast for the uncoupled and coupled arrays when there is no uncertainty in the electroacoustic responses and the corresponding electrical power required by the uncoupled and coupled arrays is shown in Figure 3b by the thin black and red lines respectively. These results show that although the acoustic contrast performance of the two arrays is equal, the electrical power required by the coupled array has been reduced by a factor of about 100 compared to the uncoupled array at frequencies below around 1kHz. As discussed by Elliott *et al* [1] this electrical power reduction is a result of the coupling between the two loudspeakers causing the rear loudspeaker cone to naturally move in the opposite direction to the front loudspeaker cone. This motion is close to the optimal motion of the rear cone and, therefore, the response only needs electrical fine tuning through the driving signal. At frequencies above around 1 kHz the coupled array requires a higher level of electrical power than the uncoupled array, as it does not benefit from the resonance of the enclosed cavity. However, the electrical power required in this frequency range does not typically limit the performance of the superdirective array.

The bold dashed lines in Figure 3 show the performance of the uncoupled array when the electrical power has been constrained at low frequencies to be equal to that required by the coupled array. From Figure 3a it can be seen from the bold dashed line that the acoustic contrast of the power constrained uncoupled array has been significantly reduced compared to the unconstrained case, as expected. However, this will provide a useful comparison to the coupled array in the following section on array robustness.



(a) Acoustic contrast of the uncoupled and coupled two-source loudspeaker arrays (bold solid line) and the uncoupled array with a constraint on the array effort (bold dashed line).

(b) The electrical power plotted in decibels relative to 1 W required by the uncoupled (thin solid black line) and coupled (thin solid red line) two-source loudspeaker arrays. The thick dashed black line shows the electrical power required by the uncoupled array with a constraint on the array effort.

Figure 3: The acoustic contrast and electrical power of the two-source loudspeaker arrays when there is no uncertainty in the electroacoustic responses.

4.2 Robustness to Uncertainty in the System Responses

It has been shown that the coupled two-source array is able to significantly reduce the high levels of electrical power required by the superdirective two-source loudspeaker array at low frequencies. However, in a practical system it is also important to understand the robustness of the array to uncertainties in the system's response and, although this has been considered for personal audio systems [7, 8, 9], it has not been studied for the coupled loudspeaker array.

If we consider the case when the electroacoustic transfer responses, $\mathbf{G}_B$ and $\mathbf{G}_D$, are perturbed by some uncertainty, then the perturbed responses can be expressed as

$$\hat{\mathbf{G}}_B = \mathbf{G}_B + \boldsymbol{\Delta}\mathbf{G}_B \qquad \hat{\mathbf{G}}_D = \mathbf{G}_D + \boldsymbol{\Delta}\mathbf{G}_D, \tag{15}$$

where $\boldsymbol{\Delta}\mathbf{G}_B$ and $\boldsymbol{\Delta}\mathbf{G}_D$ are matrices of the uncertain components. The average acoustic contrast in the presence of random uncertainties can then be expressed as

$$C_{error} = \frac{N_D \mathbf{i}^H \overline{\hat{\mathbf{G}}_B^H \hat{\mathbf{G}}_B} \mathbf{i}}{N_B \mathbf{i}^H \overline{\hat{\mathbf{G}}_D^H \hat{\mathbf{G}}_D} \mathbf{i}}, \tag{16}$$

where the overscore indicates the average over a number of random uncertainty matrices. If it is assumed that the uncertainties are uncorrelated with the unperturbed responses, as in [7, 8], then

$$\overline{\mathbf{G}_B^H \boldsymbol{\Delta}\mathbf{G}_B} = \mathbf{0} \qquad \overline{\mathbf{G}_D^H \boldsymbol{\Delta}\mathbf{G}_D} = \mathbf{0}. \tag{17}$$

If we also define the mean square response uncertainties as

$$\overline{\boldsymbol{\Delta}\mathbf{G}_B^H \boldsymbol{\Delta}\mathbf{G}_B} = \boldsymbol{\Delta}_B \qquad \overline{\boldsymbol{\Delta}\mathbf{G}_D^H \boldsymbol{\Delta}\mathbf{G}_D} = \boldsymbol{\Delta}_D, \tag{18}$$

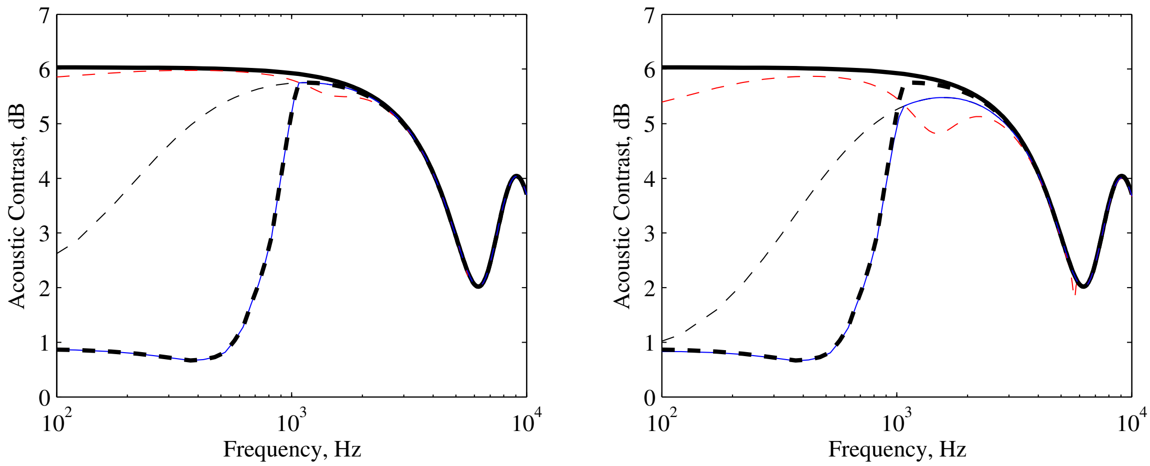
then substituting for the perturbed responses using Eq. (15) and using the assumptions given by Eq. (17) the average acoustic contrast in the presence of random uncertainties is given by

$$C_{error} = \frac{N_D \mathbf{i}^H (\mathbf{G}_B^H \mathbf{G}_B + \boldsymbol{\Delta}_B) \mathbf{i}}{N_B \mathbf{i}^H \mathbf{G}_D^H \mathbf{G}_D + \boldsymbol{\Delta}_D \mathbf{i}}. \tag{19}$$

In order to assess the robustness of the coupled and uncoupled two-source arrays the matrices of mean square uncertainties given by Eq. (18) have been defined as in [7] as

$$\mathbf{\Delta}_B = e^2 \frac{||\mathbf{G}_B^H \mathbf{G}_B||_F}{4} \mathbf{I} \qquad \mathbf{\Delta}_D = e^2 \frac{||\mathbf{G}_D^H \mathbf{G}_D||_F}{4} \mathbf{I} \tag{20}$$

where e is the normalised rms error and $||\cdots||_F$ is the Frobenius norm [13]. The average acoustic contrast, given by Eq. (19), has then been calculated for the uncoupled and coupled arrays with no electrical power constraint, and the uncoupled array with a constraint on the electrical power, when the normalised random error is either 0.1 or 0.2. The results of these calculations are shown in Figure 4 along with the acoustic contrast calculated for two-sources arrays when there is no uncertainty in the system responses. From Figure 4a it can be seen that the response uncertainties reduce the performance of the unconstrained uncoupled array by almost 3.5 dB at frequencies below around 1 kHz, whereas the performance of the coupled array is reduced by less than 0.3 dB at around 1.4 kHz. The performance of the uncoupled array with a constraint on the electrical power is also shown in Figure 4a and in this case the performance of the array is not affected by the response uncertainties. However, its performance is significantly below the unconstrained arrays. To emphasise the effects of the response uncertainties Figure 4b shows the acoustic contrast when the normalised rms error has been set to $e = 0.2$. In this case the reduction in performance for both the unconstrained coupled and uncoupled arrays is increased. However, they still both outperform the constrained uncoupled array on average.



(a) $e = 0.1$ (b) $e = 0.2$

Figure 4: The average acoustic contrast for the uncoupled and coupled two-source loudspeaker arrays (bold solid black line) and the uncoupled array with a constraint on the array effort (bold dashed black line) without uncertainty, and for the uncoupled (thin black dashed line), coupled (thin red dashed line) and uncoupled with an array effort constraint (thin blue line) arrays with a normalised rms random variation in the transfer responses of (a) $e = 0.1$ and (b) $e = 0.2$

5. Conclusions

Two-source line arrays have been used in a variety of applications to generate localised listening zones and have also been used as a constituent element of larger arrays to achieve sound field control more generally [11]. These compact arrays require a high level of electrical power at low frequencies, which can be impractical, and are also sensitive to response uncertainties. In practice, regularisation is often used to limit the electrical power and improve the robustness of the array. An alternative method of reducing the required electrical power of a two-source line array has previously been proposed in [1], in which the two loudspeakers are coupled via a common acoustic enclosure. However, its robustness to plant uncertainties has not been considered.

This paper has investigated the performance of the acoustically coupled two-source line array compared to the standard uncoupled two-source array in terms of the acoustic contrast, required electrical power and robustness to response uncertainties. A two-port model of the two loudspeaker arrays has been derived and this has been used in a series of simulations. The significant reductions in the electrical power required by the coupled array have first been demonstrated and then the robustness of the arrays to different levels of random perturbation has been simulated. These results have shown that the performance of the uncoupled array is more significantly affected by response uncertainties than the coupled array. Critically, it has also been shown that the performance of the coupled array significantly outperforms the uncoupled array even when its electrical power requirement at low frequencies has been constrained to be equal to the coupled array. This means that in the presence of response uncertainties, the coupled array is able achieve a higher level of acoustic contrast than the uncoupled array with a lower electrical power requirement.

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