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INSTITUTE OF SOUND AND VIBRATION RESEARCH

DYNAMICS GROUP

TGV – A Computer Program for Train-induced Ground Vibration

by

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ABSTRACT

A semi-analytical model has been developed for the prediction of ground vibrations generated by surface trains. The vehicles are described as multiple-rigid body systems and only the vertical dynamics is considered. The track is modelled as multiple beams supported by vertical springs with consistent mass. The ground is represented by horizontal layers on a homogeneous half-space or a rigid foundation. This model uses the moving axle loads and/or the vertical rail irregularities as its inputs. Outputs include the dynamic wheelset-rail forces, the displacements and displacement spectra (or displacement power spectra) of the track and the ground (on the surface or within the ground).

A computer program, called TGV, has been produced to implement this model. This report presents a description of the program and its input data.

CONTENTS

TGV–A COMPUTER PROGRAM FOR TRAIN-INDUCED GROUND VIBRATION

1. INTRODUCTION

The problem of ground vibration generated by surface trains has received more and more attention in the past few years [1, 2]. The sources of ground vibration are the quasi-static loads (i.e. the moving axle loads) and the dynamic wheel-rail forces. The dynamic wheel-rail forces are generated from the irregular profiles of the wheel and rail running surfaces. These introduce a relative displacement input to the vehicle/track system. A wavelength λ generates a frequency of excitation $f = c/\lambda$, where c denotes the train speed. For the frequency range of 5 Hz to 80 Hz of interest for ground vibration, and a train speed range of 36 to 250 km/h (10 to 70 m/s), the important wavelengths lie within the range of 0.125 to 14 m (or wavenumber from 0.07 to 8 cycle/m).

Comprehensive analysis requires a realistic model of ground vibration generation and propagation. This model should be able to account for the interactions between vehicles, track and ground, with consideration of the effect of train speeds. Such a model has been developed by the present authors [3-8] and the relevant computer program, written in FORTRAN 77, has been run for different cases with the results compared with measurements [4]. This report presents a detailed description of the program. In Section 2, the vehicles, track and ground models are presented in the form of diagrams and the model parameters as input to the program are defined. Sections 3 and 4 describe the program structure, the preparation of the input data, output data files and the functions of the subroutines. Example calculations are presented in Section 5.

2. MODELS FOR THE VEHICLES, TRACK AND GROUND

2.1 VEHICLE MODEL

Vehicles are modelled as multiple rigid body systems. Only the vertical dynamics is considered. As shown in Figures 1 to 5, the mass and pitch inertia of the car body are denoted by M_C and J_C. The vertical displacement and pitch angle of the car body are denoted by $z_C(t)$ and $\varphi_C(t)$. The mass and pitch inertia of each bogie are denoted by M_B and J_B. The vertical displacement and pitch angle of the jth bogie are denoted by $z_{Bj}(t)$ and $\varphi_{Bj}(t)$. The mass of each wheelset is denoted by M_W, and the vertical displacement of the lth wheelset is denoted by $z_{Wl}(t)$. For a vertical displacement, the positive direction is downwards, while for a pitch angle, the positive direction is clockwise. The dynamic stiffness of the primary suspension (between wheelset and bogie) per axle is denoted by k_1, and that of the secondary suspension (between bogie and car body) per bogie is denoted by k_2. For different types of suspension, k_1 and k_2 are different functions of frequency, stiffness and damping of the suspension. When a suspension, for example a primary suspension, consists of a spring and a viscous damper which are connected in parallel, as shown in Figure 6(a), then

$$k_1 = k_{S1} + i\Omega c_{S1} \quad (1)$$

where, k_{S1} denotes the stiffness of the spring, c_{S1} the viscous damping coefficient of the damper, Ω the angular frequency. If the suspension has a structure shown in Figure 6(b), in which an extra spring is connected in series with the damper, then

$$k_1 = \frac{k_{S1}k_{S1}^{'} + i\Omega c_{S1}(k_{S1} + k_{S1}^{'})}{k_{S1}^{'} + i\Omega c_{S1}} \quad (2)$$

where, $k_{S1}^{'}$ is the damper stiffness. A symbol, shown in Figure 6(c), is used to represent a suspension of any type. The hysteretic damping may also be incorporated into the suspension by introducing a complex spring stiffness.

Five vehicle types have been included in the program TGV. Vehicle type I, shown in Figure 1, represents a passenger coach which has two levels of suspension (primary and secondary). The displacement vector of the coach is defined as

$$\{z_V(t)\} = (z_C(t), \varphi_C(t), z_{B1}(t), \varphi_{B1}(t), z_{B2}(t), \varphi_{B2}(t), z_{W1}(t), z_{W2}(t), z_{W3}(t), z_{W4}(t))^T \quad (3)$$

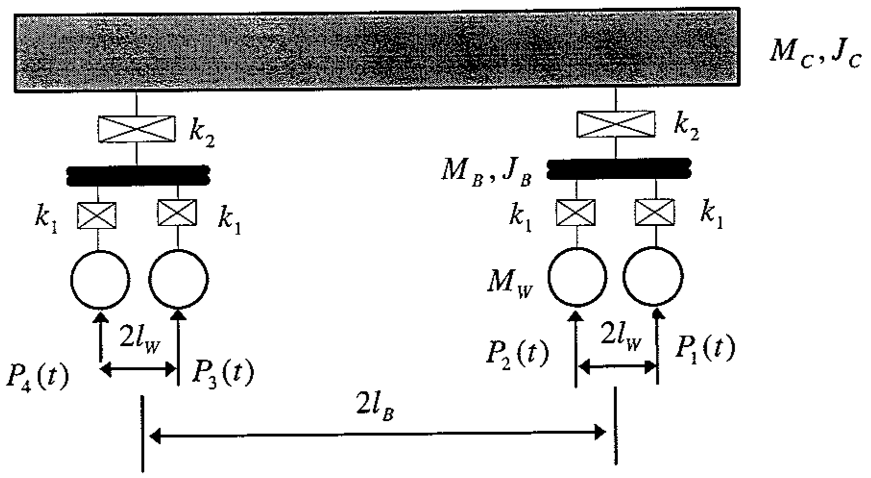

Figure 1. Type I: A vehicle system with primary and secondary suspensions.

Figure 2 (type II) shows a freight vehicle with only one level of suspension which is installed between the car body and the bogies. The displacement vector is defined as

$$\{z_V(t)\} = (z_C(t), \varphi_C(t), z_{B1}(t), \varphi_{B1}(t), z_{B2}(t), \varphi_{B2}(t))^T \tag{4}$$

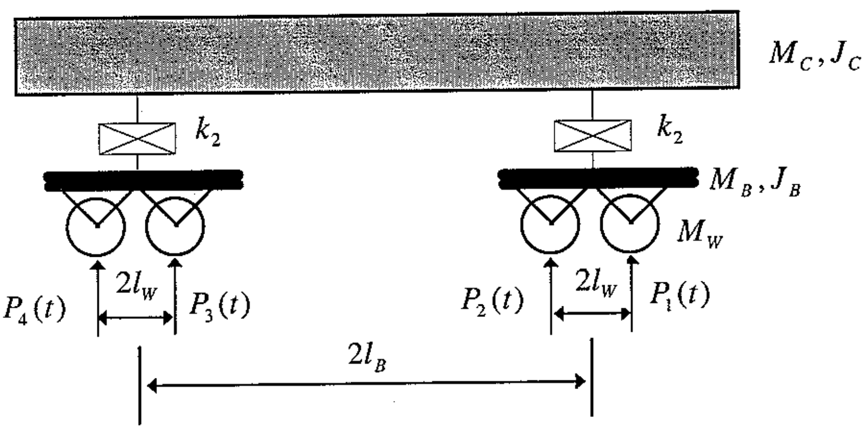

Figure 2. Type II: A vehicle system with only secondary suspensions.

The third type (type III), shown in Figure 3, is also a freight vehicle with only one level of suspension which is between the axle boxes and the bogies. The displacement vector is defined as

$$\{z_V(t)\} = (z_C(t), \varphi_C(t), \varphi_{B1}(t), \varphi_{B2}(t), z_{W1}(t), z_{W2}(t), z_{W3}(t), z_{W4}(t))^T \tag{5}$$

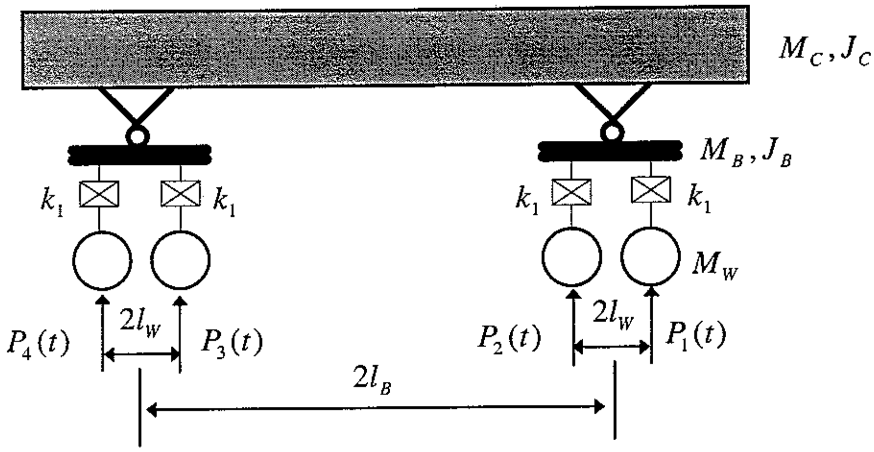

Figure 3. Type III: A vehicle system with only primary suspensions.

Figure 4 (type IV) shows a freight vehicle with two axles. The displacement vector is defined as

$$\{z_V(t)\} = (z_C(t), \varphi_C(t), z_{W1}(t), z_{W2}(t))^T \tag{6}$$

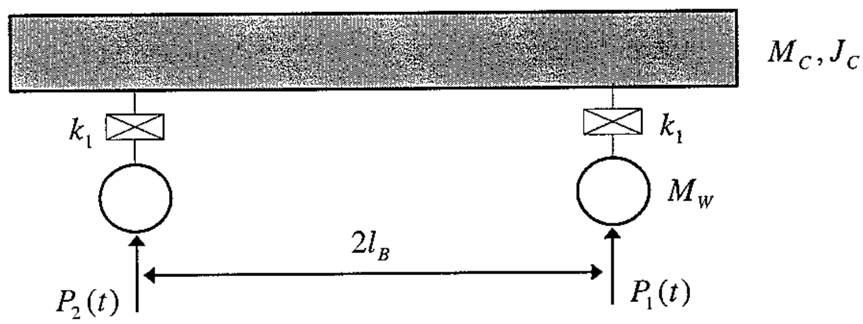

Figure 4. Type IV: A two-axle vehicle.

Finally, Figure 5 (Type V) shows a simplified vehicle model which only comprises a suspended mass M_C and an unsproung mass M_W. The displacement vector is defined as

$$\{z_V(t)\} = (z_C(t), z_{W1}(t))^T \tag{7}$$

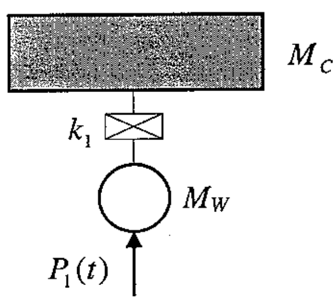

Figure 5. Type V: A single-axle vehicle model.

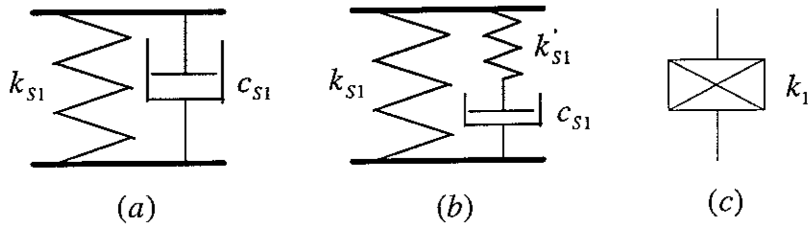

Figure 6. Structure of suspensions.

The number of the weelsets and the degrees of freedom of the five types of vehicle are summarised in Table 1.

TABLE 1

Number of wheelsets and degrees of freedom of the vehicles

Vehicle type number	Number of wheelstes	Degrees of freedom
1	4	10
2	4	6
3	4	8
4	2	4
5	1	2

2.2 TRACK MODEL

A diagram of the track model is shown in Figure 7. The railway track is aligned in the x direction. For the frequency range of interest, the rails and the sleepers (or the slab in the case of slab track) are modelled as Euler beams. The mass per unit length of track of the rail beam and its bending stiffness are denoted by m_R and EI, respectively. The mass of the sleeper beam (or slab beam) per unit length of track is denoted by m_S. If the beam represents sleepers, its bending stiffness is set to zero. If the beam models a slab, its bending stiffness is denoted by $(EI)_S$. The rail pads between the rails and the sleepers (or slab) are modelled as continuously distributed complex spring stiffness, expressed as $k_P = k'_P(1+i\eta_P \operatorname{sgn}(\omega))$, where $i=\sqrt{-1}$, k'_P is the stiffness of rail pad for a unit length of track, η_P is its loss factor and ω is the radian frequency. Since constraints in its cross-sections are weak, the ballast is modelled as continuously distributed vertical stiffness and mass along the track. The mass and vertical stiffness of the ballast per unit length of track are denoted by m_B and $k_B = k'_B(1+i\eta_B \operatorname{sgn}(\omega))$. An embankment, if it is present, is modelled in the same way as the ballast. In other words, the bending stiffness of the embankment is ignored. The cross-section of the embankment has a trapezium shape (Figure 7). The width of the upper side of the trapezium is $2b_1$, that of the lower side is $2b$ and its height is h. The Young's modulus, density and loss factor of the embankment material are denoted by E_E, ρ_E and η_E.

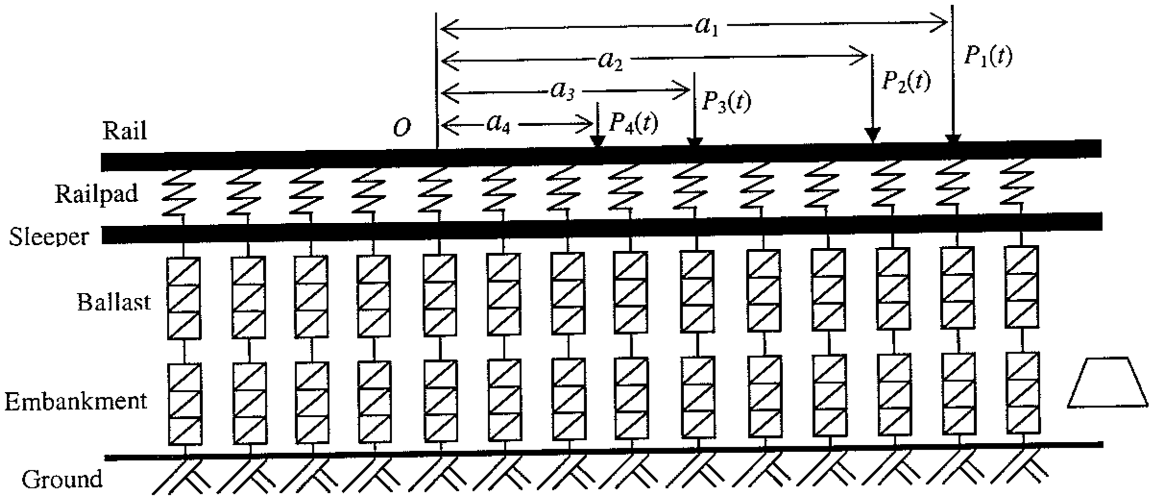

Figure 7. Diagram of the track model.

2.3 THE STIFFNESS OF THE CONTACT SPRING BETWEEN A WHEELSET AND THE RAILS

A Hertz contact spring is inserted between a wheelset and the rails (Figure 8). The stiffness of the Hertz contact spring is denoted by k_H, which is given by

$$k_H = \frac{3}{G}(P_0/2)^{1/3} \tag{8}$$

where P_0 is the static load applied on the rails by the wheelset, equal to the axle load. For a wheel with a coned tyre

$$G = 4.57R^{-0.149} \times 10^{-8} \quad (\mathrm{m/N^{2/3}}) \tag{9}$$

For a wheel with a worn tyre

$$G = 3.86R^{-0.115} \times 10^{-8} \quad (\mathrm{m/N^{2/3}}) \tag{10}$$

where R denotes the rolling radius of the wheel.

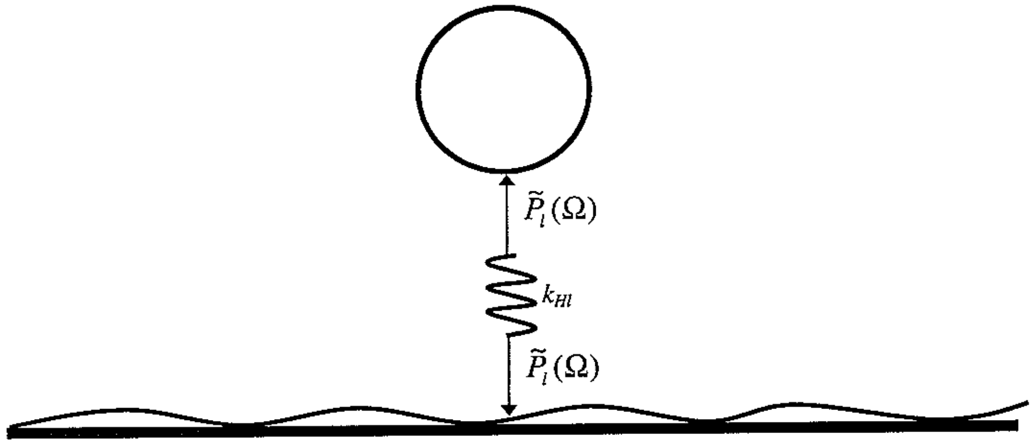

Figure 8. The Hertz contact spring between the lth wheelset and the rails.

2.4 GROUND MODEL

It is assumed that the ground consists of a number, n, of parallel layers of different materials. The nth layer overlies a homogeneous half-space or a rigid foundation, which is identified as 'layer' number (n+1). For the jth layer the material constants are: elastic modulus, E_j, Poisson ratio, ν_j, density, ρ_j, loss factor, η_j and layer thickness, h_j. The displacements of the ground are denoted by u for the longitudinal component, υ for the lateral component and w for the vertical component.

2.5 SOURCES OF EXCITATION

This model uses the quasi-static loads (i.e. the moving axle loads) and the vertical irregular profile of the rails as the sources of excitation. At time $t = 0$, the longitudinal coordinates of the vertical forces (Figure 7) between the wheelsets and the rails are denoted by $a_1, a_2, \cdots, a_M$, where M is the number of the wheelsets of the train. The vertical irregular profile of the rail top is described by the power spectral density, $P_z(\beta)$, where β is the wavenumber in radians/m in the track direction.

2.6 RELATIONSHIP BETWEEN THE DISPLACEMENT POWER SPECTRA OF THE GROUND AND THE POWER SPECTRAL DENSITY OF THE RAIL

The vertical displacement power spectrum of point (x, y) on the ground surface is denoted by $P_w(x, y, f)$, where, f is the frequency at which the spectrum is evaluated. It has been shown that [3]

$$P_w(x,y,f) = |S_w(x,y,f;0)|^2 + \frac{1}{2\pi}\sum_{k=1}^{\infty}\left\{|S_w(x,y,f;\Omega_k)|^2 + |S_w(x,y,f;-\Omega_k)|^2\right\}P_z(\beta_k)\Delta\beta \quad (11)$$

where, $\beta_k = k\Delta\beta$, $\Omega_k = \beta_k c$ and c is the train speed. In equation (11), the first right-hand term represents the vertical displacement power spectrum generated by the quasi-static loads and the second is due to the track irregularity. $S_w(x,y,f;\Omega_k)$ denotes the vertical displacement spectrum of the ground surface due to a rail profile of single wavenumber $\beta_k = \Omega_k / c$ and of unit amplitude. When divided by the time taken for the whole train to pass a fixed point, equation (11) gives an estimate of the power spectral density (PSD) of vibration at point (x, y) on the ground surface.

The velocity power spectrum, $P_{\dot{w}}(x,y,f)$, and the acceleration power spectrum, $P_{\ddot{w}}(x,y,f)$, of the ground surface are given by

$$P_{\dot{w}}(x,y,f) = (2\pi f)^2 P_w(x,y,f) \quad (12)$$

$$P_{\ddot{w}}(x,y,f) = (2\pi f)^2 P_{\dot{w}}(x,y,f) = (2\pi f)^4 P_w(x,y,f) \quad (13)$$

The power spectra of the longitudinal and lateral responses, $P_u(x,y,f)$ and $P_v(x,y,f)$, may also be calculated similarly.

3. PROGRAM STRUCTURE AND FUNCTIONS

The structure of the computer program is shown in Figure 9. The subroutines for each part of Figure 9 will be described in Section 4. To run this program, a RAM of about 200 Mb is needed. The hard disk required for output data depends on what quantities are calculated and will be stated in Section 3.4.

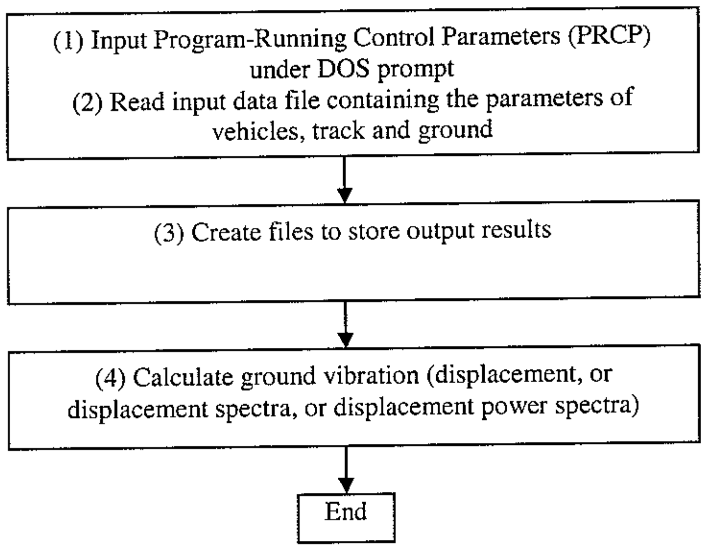

Figure 9. Structure of the program

3.1 PROGRAM CONTROL PARAMETERS

The program is initiated by typing TGV at the DOS prompt. A number of Program-Running Control Parameters (PRCP) are entered in response to prompts from the software. These are listed in Table 2.

TABLE 2

Program-Running Control Parameters

On the screen	PRCP	Explanations
Please input railway type number: 0 = no track 2 = lateral-vib: rail + pad + sleeper + ballast 3 = vertical-vib: rail + pad + sleeper + ballast 4 = vertical-vib: rail + pad + sleeper + ballast + embankment 5 = vertical-vib: rail + pad + slab 10 = torsional-vib: rail + pad + sleeper + ballast	*irail* (Integer)	*irail* = 0: There is no track. A rectangular moving or stationary harmonic load is applied directly on the ground surface or within the ground. *irail* = 2: For the lateral dynamics of a track comprising rails, railpads, sleepers and ballast. *irail* = 3: For the vertical dynamics of a track comprising rails, railpads, sleepers and ballast. *irail* = 4: For the vertical dynamics of a track comprising rails, railpads, sleepers, ballast and embankment. *irail* = 5: For the vertical dynamics of a track comprising rails, railpads and a slab. *irail* = 10: For the torsional dynamics of a track comprising rails, railpads and sleepers.
The ground foundation elastic or rigid? (1 = elastic, 0 = rigid)	*iyesno* (Integer)	*iyesno* = 1: soil layers rest on a homogeneous half-space. *iyesno* = 0: soil layers rest on a rigid foundation.
Calculate what? 0 = Displacements as a function of *x* and *y*. 1 = Disp. for frequencies 2, 4, 8...,100 Hz 2 = Disp. for speeds 2, 4, 6..., maximum speed (m/s) 3 = Doppler displacement spectra. 4 = One-side displacement power spectra due to different excitation frequencies	*ispect* (Integer)	**See Section 3.4**
Couple the railway with ground? 1 = yes, 0 = no	*icouple* (Integer)	*icouple* = 1: The track is coupled with the ground. *icouple* = 0: The track rests on a rigid foundation.
Calculate stresses? 1 = yes, 0 = no	*istress* (Integer)	*istress* = 1: Calculate stresses on a horizontal plane in the ground. *istress* = 0: No stresses are calculated.

Input source layer No. and observer layer No.	*lptem, lrtem* (Integer)	*lptem*: Number of the layer within which forces are applied. *lrtem*: Number of the layer of which responses are calculated.
Input depths of source and observer planes (in metres)	*h1tem, h2tem* (Double precision)	*h1tem*: Vertical distance from the source to the upper surface of the source layer. *h2tem*: Vertical distance from the observer to the upper surface of the observer layer.
Contact force type? lateral = 2, vertical = 3, torsion = 5	*imotion* (Integer)	On the ground surface or on a horizontal plane within the ground, the longitudinal, lateral and vertical forces per unit area are denoted by p_x, p_y and p_z. Then *imotion* = 2: $p_x = 0$; $p_y = p_y(x)$, $\|y\| \leq 2b$; $p_z = 0$; *imotion* = 3: $p_x = 0$; $p_y = 0$; $p_z = p_z(x)$, $\|y\| \leq 2b$; *imotion* = 5: $p_x = 0$; $p_y = 0$; $p_z = p_z(x)y/b$, $\|y\| \leq 2b$;
Select damping model for the track: 1 = hysteretic, 0 = viscous	*idamper1* (Integer)	
Select damping model for the ground: 1 = hysteretic, 0 = viscous	*idamper2* (Integer)	
Input file name (including the path) where the input data are stored	*file_name* (Character, ≤ 50)	
Input a, b of rectangular load (in metres)	*xlenth* *ylenth* (Double precision)	Appears when *irail* = 0. The half-lengths of the sides of the rectangular loading area in the *x*- and *y*-directions.
Input speed of moving loads (m/s), dbeta/2pi (cycle/m), dgama/2pi (cycle/m) If the load is not moving, speed = 0	*speed* *dbeta* *dgama* (Double precision)	Input train or load speed, wavenumber spacing (in cycle/m) in the *x*-direction and wavenumber spacing (in cycle/m) in the *y*-direction.
Position of the line normal to the track (in metres)	*xdist* (Double precision)	Appears when *ispect* = 1, 2, 3 or 4. The *x*-coordinate of a horizontal line normal to the *x*-axis.
Please input the maximum speed (m/s)	*maximumspeed*	Appears when *ispect* = 2 for displacements at different load speeds.

3.2 DATA FILE CONTAINING THE PARAMETERS OF VEHICLES, TRACK AND GROUND

The parameters of the vehicles, track and ground are pre-edited as a text file. The layout of the file is shown in Table 3. An example of the file is given in Section 4.

TABLE 3

Layout of the input data file

Variables	Type and Meaning
nvehicle	Integer, number of the vehicles considered. If responses to known forces on the rails are calculated, then put *nvehicle* = 0.
ivehicle(1), *nwheel*(1), *ndegree*(1) *ivehicle*(2), *nwheel*(2), *ndegree*(2) ... *ivehicle*(*nvehicle*), *nwheel*(*nvehicle*), *ndegree*(*nvehicle*) **(When *nvehicle* = 0, miss this part)**	*ivehicle*: integer, vehicle type number; *nwheel*: integer, number of wheelsets of the vehicle; *ndegree*: integer, degrees of freedom of the vehicle. (See Table 1)
mcar(1), *jcar*(1), *mbogie*(1), *jbogie*(1), *mwheel*(1), *lbogie*(1), *lwheel*(1), *kfirst1*(1), *kfirst2*(1), *cfirst1*(1), *cfirst2*(1), *ksecond1*(1), *ksecond2*(1), *csecond1*(1), *csecond2*(1) ... *mcar*(*nvehicle*), *jcar*(*nvehicle*), *mbogie*(*nvehicle*), *jbogie*(*nvehicle*), *mwheel*(*nvehicle*), *lbogie*(*nvehicle*), *lwheel*(*nvehicle*), *kfirst1*(*nvehicle*), *kfirst2*(*nvehicle*), *cfirst1*(*nvehicle*), *cfirst2*(*nvehicle*), *ksecond1*(*nvehicle*), *ksecond2*(*nvehicle*), *csecond1*(*nvehicle*), *csecond2*(*nvehicle*) **(When *nvehicle* = 0, miss this part)**	All are double precision. *mcar*: mass of car body; *jcar*: pitch inertia of car body; *mbogie*: mass of bogie; *jbogie*: pitch inertia of bogie; *mwheel*: mass of wheelset; *lbogie*: half-distance between bogie centres; *lwheel*: half-distance between wheelsets in a bogie; *kfirst1*: stiffness of the primary suspension per axle; *kfirst2*: series stiffness of the viscous damper of the primary suspension per axle; *cfirst1*: loss factor of the primary suspension spring; *cfirst2*: damping coefficient of the viscous damper of the primary suspension per axle; *ksecond1*: stiffness of the secondary suspension per bogie; *ksecond2*: series stiffness of the viscous damper of the secondary suspension per bogie; *csecond1*: loss factor of the secondary suspension spring; *csecond2*: damping coefficient of the viscous damper of the secondary suspension per bogie; **If a vehicle does not contain some of the elements, the values for these missing elements may be set to zero.**
nload	Integer, total number of wheelset-rail forces
ncmpnt	Integer, total number of harmonic components in each wheelset-rail force. **See Section 3.4**
axled(1), *kh*(1) *axled*(2), *kh*(2) ... *axled*(*nload*), *kh*(*nload*)	All are double precision. *axled*(1): at time $t = 0$, the longitudinal coordinate of the first wheelset-rail force; *kh*(1): the stiffness of the Hertz contact spring between the first wheelset and the rails. **When vehicles are not included, *kh* can be any values, e.g. zero.**
freload(1) *freload*(2) ... *freload*(*ncmpnt*)	All are double precision. Frequency of each harmonic component. **See Section 3.4.**
zr(1, 1), *zi*(1,1) *zr*(1, 2), *zi*(1,2) ... *zr*(1, *ncmpnt*), *zi*(1, *ncmpnt*) *zr*(2, 1), *zi*(2,1) ... *zr*(*nload*, *ncmpnt*), *zi*(*nload*, *ncmpnt*)	**See Section 3.4.**
nlayer	Integer. Number of layers of the ground. For a homogeneous half-space, set *nlayer* = 0

e(1), *zita1*(1), *nu*(1), *ro*(1), *h*(1) ... *e*(*nlayer*), *zita1*(*nlayer*), *nu*(*nlayer*), *ro*(*nlayer*),*h*(*nlayer*) **(If *nlayer* = 0, miss this part)**	All are double precision. Young's modulus, loss factor, Possion ratio, density and thickness of the layers.
e(*nlayer*+1), *zita1*(*nlayer*+1), *nu*(*nlayer*+1), *ro*(*nlayer*+1) **(If soil layers rest on a rigid foundation, miss this part)**	All are double precision. Young's modulus, loss factor, Possion ratio, density of the underlying half-space.
mr, ei1, zitar1, kp1, zitap1, ms, mb, kb1, zitab1, width	All are double precision. **For track type 3:** *mr*: mass of the rail beam per unit length of track; *ei1*: bending stiffness of the rail beam; *zitar1*: loss factor of the rail beam; *kp1*: stiffness of the rail pad; *zitap1*: loss factor of the rail pad; *ms*: mass of the sleeper beam; *kb1*: vertical stiffness of the ballast; *zitab1*: loss factor of the ballast; *width*: half width of the track-ground contact plane.
mr, ei1, zitar1, kp1, zitap1, ms, mb, kb1, zitab1 *roe, ee1, zitae1, width1, width, height*	**For track type 4:** *roe*: density of the embankment; *ee1*: Young's modulus of the embankment; *zitae1*: loss factor of the embankment; *width1*: half width of the top side of the embankment cross-section; *width*: half width of the bottom side; *height*: height of the embankment.
mr, ei1, zitar1, kp1, zitap1, mb, kb1, zitab1, width	**For track type 5:** *mr*: mass of the rail beam per unit length of track; *ei1*: bending stiffness of the rail beam; *zitar1*: loss factor of the rail beam; *kp1*: stiffness of the rail pad; *zitap1*: loss factor of the rail pad; *mb*: mass of the slab per unit length of track; *kb1*: bending stiffness of the slab; *zitab1*: loss factor of the slab; *width*: half width of the track-ground contact plane (i.e. half width of the slab).

3.3 OUTPUT DATA FILES

The output data files created by Subroutine *file_create* are listed in Table 4. All of them are formatted files. Data are written into these files consecutively using WRITE (*id*,'(E15.6)') *u*, where, *id* is the identifier of the file into which the data in variable *u* is written.

TABLE 4

A list of the output data files

File name	Data in the file
load	Magnitudes of the dynamic forces between the wheelsets and rails.
load.pha	Phase angles of the dynamic forces between the wheelsets and rails.
wvehicle	Magnitudes of the displacements of the first vehicle.
wvehicle.pha	Phase angles of the displacements of the first vehicle.
u10bar	Magnitudes of the Fourier-transformed longitudinal displacements of the ground.
u10bar.pha	Phase angles of the Fourier-transformed longitudinal displacements of the ground.
v10bar	Magnitudes of the Fourier-transformed lateral displacements of the ground.

v10bar.pha	Phase angles of the Fourier-transformed lateral displacements of the ground.
w10bar	Magnitudes of the Fourier-transformed vertical displacements of the ground.
w10bar.pha	Phase angles of the Fourier-transformed vertical displacements of the ground.
w1.dsp	Magnitudes of the displacements of the rail beam.
w1.pha	Phase angles of the displacements of the rail beam.
w2.dsp	Magnitudes of the displacements of the centre-line on the track-ground contact plane.
w2.pha	Phase angles of the displacements of the centre-line on the track-ground contact plane.
u10.dsp	Magnitudes of the longitudinal displacements of the ground.
u10.pha	Phase angles of the longitudinal displacements of the ground.
v10.dsp	Magnitudes of the lateral displacements of the ground.
v10.pha	Phase angles of the lateral displacements of the ground.
w10.dsp	Magnitudes of the vertical displacements of the ground.
w10.pha	Phase angles of the vertical displacements of the ground.
w1spe	Magnitude of the displacement spectrum (or power spectrum) of the rail beam.
u10spe	Magnitude of the longitudinal displacement spectrum (or power spectrum) of the ground.
v10spe	Magnitude of the lateral displacement spectrum (or power spectrum) of the ground.
w10spe	Magnitude of the vertical displacement spectrum (or power spectrum) of the ground.

3.4 RESULTS PRODUCED BY TGV

3.4.1 *DISPLACEMENTS OF THE TRACK AND GROUND GENERATED BY STATIONARY OR MOVING LOADS APPLIED ON THE RAILS* (*ispect* = 0, *nvehicle* = 0)

In this case, the number of vehicles is set to zero (*nvehicle* = 0) in the input data file. The loads may be harmonic of a single frequency or contain several harmonic components. In the input data file, *freload*(1), *freload*(2),…, *freload*(*ncmpnt*) are excitation frequencies. *zr*(1, 1), *zi*(1,1), *zr*(1, 2), *zi*(1,2), …, *zr*(1, *ncmpnt*), *zi*(1, *ncmpnt*), *zr*(2, 1), *zi*(2,1), …, *zr*(*nload, ncmpnt*), *zi*(*nload, ncmpnt*) are real and imaginary parts of the loads. *zr*(*i, j*) is the real part of the *j*th harmonic component of the *i*th load while *zi*(*i, j*) is the imaginary part. The PRCP parameter, *ispect*, is set to zero from the keyboard. Outputs are

(1) *Magnitudes and phase angles of the Fourier-transformed displacements of the ground.* For each of the values of $\beta = -255\times\Delta\beta,\cdots,0,\Delta\beta,\cdots,256\times\Delta\beta$, the magnitudes and phases are calculated for $\gamma = -255\times\Delta\gamma,\cdots,0,\Delta\gamma,\cdots,256\times\Delta\gamma$, and stored consecutively. The Fourier-transformed longitudinal displacements are stored in files *u10bar* (magnitude) and *u10bar.pha* (phase). The Fourier-transformed lateral displacements are stored in files *v10bar* and *v10bar.pha*. Files *w10bar* and *w10bar.pha* contain the vertical displacements. Each of these files contains 512×512 data, having a size of 4.25 Mb. Note that $\Delta\beta$, $\Delta\gamma$ are the wavenumber spacings in rad/m.

(2) *Magnitudes and phase angles of the actual displacements of the track and ground.* For each of the values of $x = -255\times\Delta x,\cdots,0,\Delta x,\cdots,256\times\Delta x$, displacements of the ground are calculated for $y = -255\times\Delta y,\cdots,0,\Delta y,\cdots,256\times\Delta y$, and stored consecutively, where, $\Delta x = 2\pi/(2048\times\Delta\beta)$ and $\Delta y = 2\pi/(2048\times\Delta\gamma)$. The magnitudes of the longitudinal, lateral and vertical displacements of the ground are stored in files *u10.dsp, v10.dsp* and *w10.dsp*, and the phase angles are in *u10.pha, v10.pha* and *w10.pha.* Each file contains 512×512 data and has a size of 4.25 Mb. For the track, displacements are calculated for $x = -255\times\Delta x,\cdots,0,\Delta x,\cdots,256\times\Delta x$ and stored in *w1.dsp, w2.dsp, w1.pha* and *w2.pha.* For the track, each file contains 512 data.

3.4.2 *DISPLACEMENTS OF THE TRACK AND GROUND GENERATED BY VEHICLES RUNNING ON IRREGULAR RAILS* (*ispect* = 0, *nvehicle* > 0)

In this case, the PRCP parameter, *ispect*, is set to zero from the keyboard. In the input data file, *freload*(1), *freload*(2),…, *freload*(*ncmpnt*) are excitation frequencies determined by the wavelengths of the spatial harmonic components in the rail profile and the train speed (frequency = train speed divided by wavelength). *zr*(*i*, *j*) is the real part of the *j*th harmonic component of the displacement input (roughness amplitude) at the *i*th wheelset while *zi*(*i*, *j*) is the imaginary part. **If the quasi-static loads are considered, and are identified as the first component, then *freload*(1) = 0, *zr*(*i*, 1) = the *i*th axle load, and *zi*(*i*, 1) = 0.** Outputs are as the same as for the situation in Section 3.4.1.

3.4.3 *DISPLACEMENTS OF THE TRACK AND GROUND FOR DIFFERENT EXCITATION FREQUENCIES* (*ispect* = 1, *nvehicle* = 0 or > 0)

In this case the load speed is fixed. The frequencies of excitation are 2, 4, 8, …, 100 Hz (with a frequency spacing of 2 Hz). The PRCP parameter, *ispect*, is set to 1 from the keyboard. **In the input data file, *ncmpnt* = 1** and *freload*(1) can be set any value (e.g. 2 Hz). If *nvehicle* = 0, *zr*(*i*, 1) and *zi*(*i*, 1) are the real part and imaginary part of the *i*th load, the values of which are specified by the user in the input data file (e.g. set *zr*(*i*, 1) = 1 N and *zi*(*i*, 1) = 0). **If *nvehicle* > 0, *zr*(*i*, 1) and *zi*(*i*, 1) can be any values. The displacement inputs at the wheelsets are given by the program. The magnitudes of the displacement inputs are 0.1 mm and the phase angles are determined by the program in terms of the excitation frequency, train speed and initial positions of the wheelsets.**

For each excitation frequency, the magnitudes and phase angles of the displacement amplitudes of the ground are calculated for 256 points on a horizontal line normal to the track. The x-coordinate of the points is *xdist*, and the y-coordinates are $y = 0, \Delta y, \cdots, 255\times\Delta y$, where, $\Delta y = 2\pi/(2048\times\Delta\gamma)$. The magnitudes of the longitudinal, lateral and vertical displacements are stored in files *u10.dsp*, *v10.dsp*, *w10.dsp*, and the phase angles are in *u10.pha*, *v10.pha* and *w10.pha*. Each of these files contains 256×50 data. For the track, for each of the excitation frequencies, displacements are calculated for $x = -255\times\Delta x, \cdots, 0, \Delta x, \cdots, 256\times\Delta x$ and stored in *w1.dsp, w2.dsp, w1.pha and w2.pha*, each of which contains 512×50 data.

For each excitation frequency, the vertical dynamic wheelset-rail forces as well as the displacements of the first vehicle (if *nvehicle* > 0) are also produced. The magnitudes of the wheelset-rail forces are stored in file *load*, and the phase angles of the forces are stored in file *load.pha*. Both these files contain $M \times 50$ data, where, M is the number of the wheelsets considered. The displacements of the first vehicle are stored in file *wvehicle* for the magnitudes and in *wvehicle.pha* for the phase angles. These two files contain $N \times 50$ data, where N is the degrees of freedom of the vehicle.

3.4.4 *DISPLACEMENTS OF THE TRACK AND GROUND FOR DIFFERENT LOAD (OR TRAIN) SPEEDS* (*ispect* = 2, *nvehicle* = 0 or > 0)

In this case the frequency of excitation is fixed. The speeds of the load (or train) are 2, 4, 8, …, c_{max} m/s, where c_{max} is the highest speed considered. The PRCP parameter, *ispect*, is set to 2 from the keyboard. **In the input data file, *ncmpnt* = 1.** If *nvehicle* = 0, *zr*(*i*, 1) and *zi*(*i*, 1) are the real part and imaginary part of the *i*th load, the values of which are specified by the user in the input data file (e.g. set *zr*(*i*, 1) = 1 N and *zi*(*i*, 1) = 0). If *nvehicle* > 0, *zr*(*i*, 1) and *zi*(*i*, 1) can be any values. The displacement inputs at the wheelsets are given by the program. **The magnitudes of the displacement inputs are 0.1 mm and the phase angles are determined by the program in terms of the excitation frequency, train speed and initial positions of the wheelsets.**

For each of the load speeds, the magnitudes and phase angles of the displacement amplitudes of the ground are calculated for $x = xdist$ and $y = 0, \Delta y, \cdots, 255\times\Delta y$, where, $\Delta y = 2\pi/(2048\times\Delta\gamma)$. The magnitudes of the longitudinal, lateral and vertical displacements are stored in files *u10.dsp*, *v10.dsp*, *w10.dsp*, and the phase angles are in *u10.pha*, *v10.pha* and *w10.pha*. For the track, for each of the load speeds, displacements are calculated for $x = -255\times\Delta x, \ldots, 0, \Delta x, \ldots, 256\times\Delta x$ and stored in *w1.dsp, w2.dsp, w1.pha and w2.pha.*

For each load speed, the vertical dynamic wheelset-rail forces as well as the displacements of the first vehicle (if *nvehicle* > 0) are also produced. The magnitudes of the wheelset-rail forces are stored in file *load*, and the phase angles of the forces are stored in file *load.pha*. The displacements of the first vehicle are stored in file *wvehicle* for the magnitudes and in *wvehicle.pha* for the phase angles.

3.4.5 *DISPLACEMENT SPECTRA OF THE TRACK AND GROUND* (*ispect* = 3, *nvehicle* = 0 or > 0)

The excitations may be harmonic of a single frequency or contain several harmonic components. The PRCP parameter, *ispect*, is set to 3 from the keyboard.

If *nvehicle* = 0, then in the input data file, *freload*(1), *freload*(2),..., *freload*(*ncmpnt*) are excitation frequencies; *zr*(1, 1), *zi*(1,1), *zr*(1, 2), *zi*(1,2), ..., *zr*(1, *ncmpnt*), *zi*(1, *ncmpnt*), *zr*(2, 1), *zi*(2,1), ..., *zr*(*nload*, *ncmpnt*), *zi*(*nload*, *ncmpnt*) are the real and imaginary parts of the loads; *zr*(*i*, *j*) is the real part of the *j*th harmonic component of the *i*th load while *zi*(*i*, *j*) is the imaginary part.

If *nvehicle* > 0, then in the input data file, *freload*(1), *freload*(2),..., *freload*(*ncmpnt*) are excitation frequencies determined by the wavelengths of the spatial harmonic components in the rail profile and the train speed (frequency = train speed divided by wavelength); *zr*(*i*, *j*) is the real part of the *j*th harmonic component of the displacement input (roughness amplitude) at the *i*th wheelset while *zi*(*i*, *j*) is the imaginary part. **If the quasi-static loads are considered, and are identified as the first component, then *freload*(1) = 0, *zr*(*i*, 1) = the *i*th axle load, and *zi*(*i*, 1) = 0.**

The displacement spectra are calculated at frequencies (called *spectral frequency*) 1 Hz, 2 Hz, ..., 100 Hz. For each of these frequencies, the magnitudes of the displacement spectra of the rail and the *x*-axis at point $x = 0$ are stored in *w1spe* and *w2spe*, which contain 100 data. For each of the spectral frequencies, the longitudinal, lateral and vertical displacement spectra for $y = 0, \Delta y, \cdots, 255\times\Delta y$, where, $\Delta y = 2\pi/(2048\times\Delta\gamma)$, are stored in files *ul0spe*, *vl0spe*, *wl0spe*, each of which contains 256×100 data.

3.4.6 *ONE-SIDE DISPLACEMENT POWER SPECTRA OF THE TRACK AND GROUND GENERATED BY VERTICAL RAIL IRREGULARITIES* (*ispect* = 4, *nvehicle* > 0)

The PRCP parameter, *ispect*, is set to 4 from the keyboard. **In the input data file, *ncmpnt* = 1, *freload*(1) = 0. *zr*(*i*, 1) = the *i*th axle load, and *zi*(*i*, 1) = 0**, to account for the quasi-static loads. First the displacement power spectra due to the quasi-static loads, e.g. the vertical displacement power spectrum of the ground

$$|S_w(x,y,f;0)|^2 \tag{14}$$

in equation (11), are calculated. Then for excitation frequencies $\frac{\Omega_1}{2\pi}$ = 0.25 Hz, $\frac{\Omega_2}{2\pi}$ = 0.50 Hz, …, $\frac{\Omega_{40}}{2\pi}$ = 10 Hz (a spacing of 0.25 Hz) and $\frac{\Omega_{41}}{2\pi}$ = 12 Hz, $\frac{\Omega_{42}}{2\pi}$ = 14 Hz, …, $\frac{\Omega_{95}}{2\pi}$ = 120 Hz (a spacing of 2 Hz), the one-side displacement power spectra are calculated, i.e.

$$|S_w(x,y,f;\Omega_k)|^2 + |S_w(x,y,f;-\Omega_k)|^2 \tag{15}$$

due to a vertical rail irregular profile with **an amplitude of 0.1 mm**. For each excitation frequency Ω_k, the corresponding wavenumber of the profile is $\beta_k = \Omega_k / c$, where, c is the train speed. The displacement power spectra are evaluated at spectral frequencies f = 1, 2, …, 100 Hz. For the ground, for each of the excitation frequencies and each of the spectral frequencies, the displacement power spectra are evaluated for 256 points on a horizontal line normal to the track. The x-coordinate of these points is $x = 0$, while the y-coordinates are $y = 0, \Delta y, \cdots, 255 \times \Delta y$, where, $\Delta y = 2\pi/(2048 \times \Delta\gamma)$. For the track, the displacement power spectra are evaluated for point at $x = 0$ (in fact, all the displacement power spectra are independent of the x-coordinate [8]). The one-side displacement power spectra of the rail and the x-axis are stored in *w1spe* and *w2spe*, and those of the ground are stored in files *u10spe* for the longitudinal displacement power spectrum, *v10spe* for the lateral displacement power spectrum and *w10spe* for the vertical displacement power spectrum. Therefore, there are 100 × 96 = 9600 data in files *w1spe* and *w2spe*, while in files *u10spe*, *v10spe* and *w10spe*, there are 256 × 100 × 96 = 2457600 data.

Having calculated the one-side displacement power spectra due to a rail profile with an amplitude of 0.1 mm, the total one-side displacement power spectra due to an actual rail profile spectrum can be evaluated using equation (11).

In the calculation of one-side displacement power spectra, the vertical dynamic wheelset-rail forces as well as the displacements of the first vehicle are also produced. The magnitudes of the wheelset-rail forces are stored in file *load*, and the phase angles of the forces are stored in file *load.pha*. Both these files contain $M \times 95$ data, where, M is the number of the wheelsets considered. The displacements of the first vehicle are stored in file *wvehicle* for the magnitudes and in *wvehicle.pha* for the phase angles. These two files contain $N \times 95$ data, where N is the degrees of freedom of the vehicle.

4. SUBROUTINES

TGV contains a number of subroutines. The diagram of the calling structure is shown in Figure 10. The functions of the subroutines are described as follows.

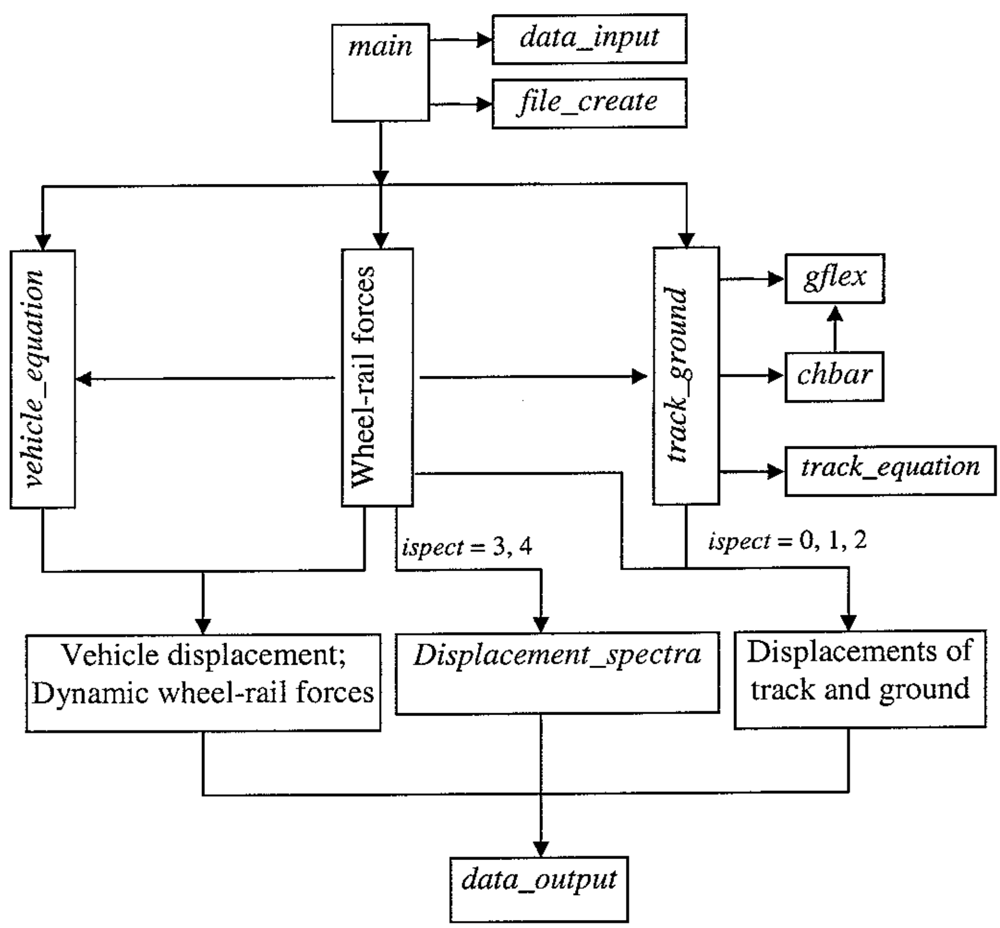

Figure 10. Calling structure of the subroutines of TGV

main: this is the main program.

data_input: read in Program-Running Control Parameters from the keyboard and read in vehicle, track and ground parameters from the input data file. See Tables 2 and 3.

file_create: create files for the output data. See Table 4.

track_ground: calculate the steady-state displacement of the rail and the Fourier transformed displacement of the ground due to a SINGLE UNIT moving or stationary harmonic load applied on the rail.

displacement_spectra: calculate the displacement spectra of the track (one point) and ground (256 points on a horizontal line normal to the track) due to moving harmonic loads of SINGLE FREQUENCY. The spectra are evaluated at frequencies 1 Hz, 2 Hz, …, 100 Hz.

vehicle_equation: calculate the receptance matrix of a vehicle at the wheelsets.

track_equation: calculate the displacements of a track.

gflex: calculate the Fourier-transformed dynamic flexibility matrix of a ground, i.e. the Fourier transformed longitudinal, lateral and vertical displacements, of a horizontal plane of h_2 depth (called *the observer plane*) in the observer layer to unit harmonic loads (moving or stationary) acting in a horizontal plane of h_1 depth (called *the source plane*) in the source layer.

chbar: calculate function $\tilde{H}(\beta)$ [3].

data_output: write the output data into files created by *file_create*.

5. EXAMPLE CALCULATIONS

Calculations of the one-side displacement spectra (see Section 3.4.6) are performed using the program for a Mk 3 passenger coach running at two speeds, 25 m/s and 83 m/s. The parameters for the vehicle, track and ground are listed in Tables 5 to 7. As can be seen from Table 7, the second vehicle speed (83 m/s) is higher than the Rayleigh wave speed in the upper layer of the ground. The data in the input data file are listed in Table 8. As has been stated in Section 3.4.6, the amplitude of the irregular profile of the rail is 0.1 mm.

TABLE 5

Parameters for the Mk 3 passenger coach

Mass of the car body (kg)	21400
Pitch inertia moment of the car body (kg-m^2)	8.3×10^5
Mass of each bogie (kg)	2707
Pitch inertia moment of each bogie (kg-m^2)	1970
Secondary vertical stiffness per bogie (N/m)	0.81×10^6
Secondary vertical viscous damping (Ns/m)	74000
Primary vertical stiffness per axle (N/m)	0.359×10^6
Primary vertical viscous damping per axle (Ns/m)	8400
Primary damper stiffness per axle	14×10^6
Distance between bogie centres (m)	2×8
Bogie wheelbase (m)	2×1.3
Mass of each wheelset (kg)	1375
Wheel diameter (m)	0.914

TABLE 6

Parameters for a ballasted railway track

Mass of rail beam per unit length of track	120 kg/m
Bending stiffness of rail beam	1.26×10^7 N-m^2
Loss factor of the rail	0.01
Rail pad stiffness per unit length of track	3.5×10^8 N/m^2
Rail pad loss factor	0.15
Mass of sleepers per unit length of track	490 kg/m
Mass of ballast per unit length of track	1200 kg/m
Ballast stiffness per unit length of track	3.15×10^8 N/m^2
Loss factor of ballast	1.0
Contact width of railway and ground	2.7m

TABLE 7

Parameters for a layered ground

Layer	Depth (m)	Young's modulus (10^6 Nm^{-2})	Poisson's ratio	Density (kg/m^3)	Loss factor	P-wave speed (m/s)	S-wave speed (m/s)	Rayleigh wave speed (m/s)
1	2.0	30	0.47	1550	0.1	340	81.1	77
Half space		360	0.49	2000	0.1	1755	245	233

Figure 11 shows the dynamic forces between the wheelsets and rails for the coach running at 25 m/s. Figure 12 shows the vertical displacement power spectrum at 10 m on the ground surface, due to the quasi-static loads and due to loads at three excitation frequencies. These excitation frequencies are 10 Hz, 50 Hz and 90 Hz. When the coach runs at 83 m/s, results are shown in Figures 13 and 14. For vehicle speed equal to 25 m/s, the wavelengths of the rail profile corresponding to 10 Hz, 50 Hz and 90 Hz are 2.5 m, 0.5 m and 0.28 m. For a vehicle speed equal to 83 m/s, the corresponding wavelengths are 8.3 m, 1.66 m and 0.92 m.

TABLE 8

Data in the input data file

1 1, 4, 10 21400.d0, 8.3d5, 2707.d0, 1970.d0, 1375.d0, 8.d0, 1.3d0, 0.359d6, 14.d6, 0.d0, 8400.d0, 0.81d6, 0.d0 , 0.d0, 74000.d0	Vehicle parameters
4 1 9.3d0, 2.42d9 6.7d0, 2.42d9 -6.7d0, 2.42d9 -9.3d0, 2.42d9 0.d0 79169.3d0, 0.d0 79169.3d0, 0.d0 79169.3d0, 0.d0 79169.3d0, 0.d0	Wheelset-rail force parameters
1 30.d6, 0.1d0, 0.47d0, 1550.d0, 2.d0 360.d6, 0.1d0, 0.49d0, 2000.d0	Ground parameters
120.d0, 1.26d7, 0.01d0, 3.5d8, 0.15d0, 490.d0, 1200.d0, 3.15d8, 1.0d0, 1.35d0	Track parameters

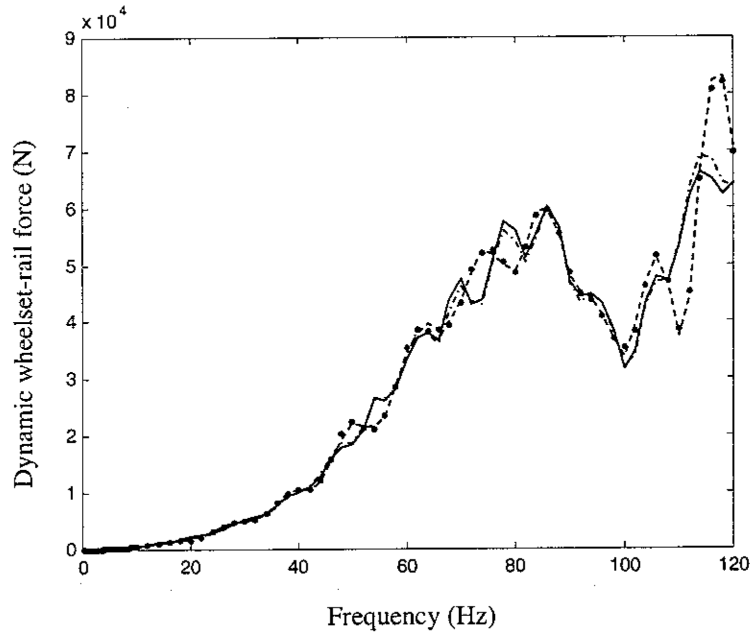

Figure 11. Dynamic forces between the wheelsets and rails for the coach running at 25 m/s. ——, for the first wheelset; – – –, for the second wheelset; – · –, for the third wheelset; …, for the last wheelset.

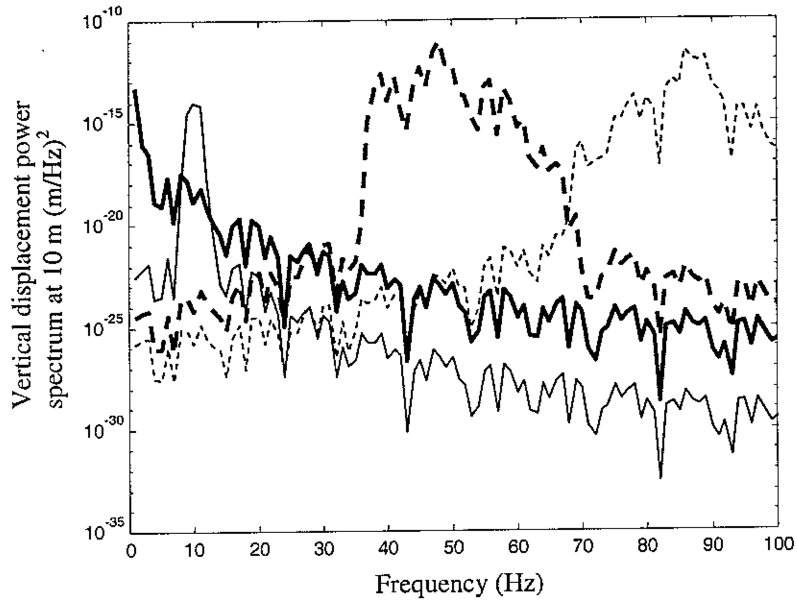

Figure 12. Vertical displacement power spectrum at 10 m on the ground surface for the coach running at 25 m/s. ——, due to the quasi-static loads; ——, for excitation frequency 10 Hz (wavelength 2.5 m); — — —, for excitation frequency 50 Hz (wavelength 0.5 m); – – –, for excitation frequency 90 Hz (wavelength 0.28 m).

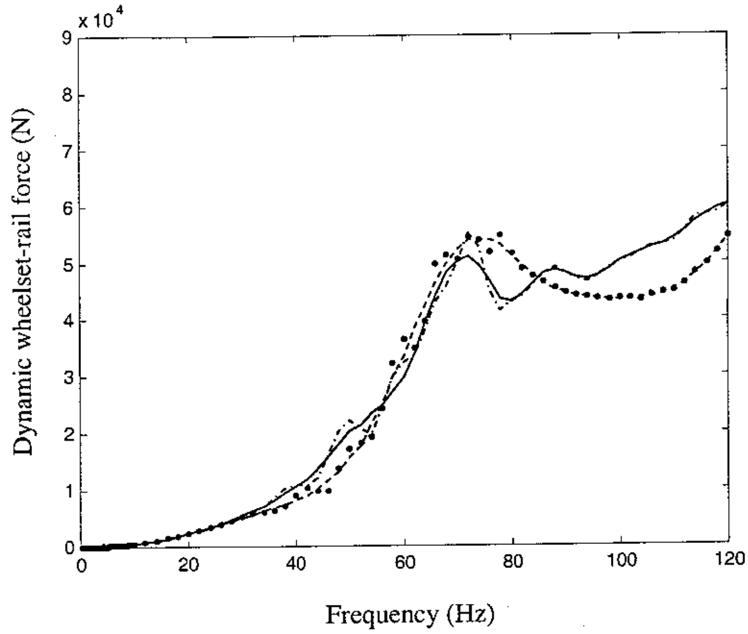

Figure 13. Dynamic forces between the wheelsets and rails for the coach running at 83 m/s. ——, for the first wheelset; – – –, for the second wheelset; – · –, for the third wheelset; …, for the last wheelset.

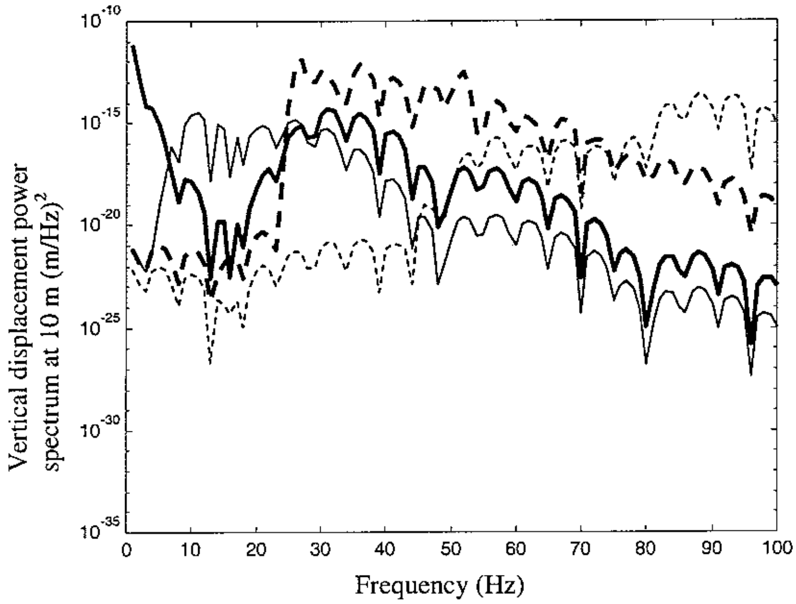

Figure 14. Vertical displacement power spectrum at 10 m on the ground surface for the coach running at 83 m/s. ▬▬, due to the quasi-static loads; ——, for excitation frequency 10 Hz (wavelength 8.3 m); ▬ ▬ ▬, for excitation frequency 50 Hz (wavelength 1.66 m); – – –, for excitation frequency 90 Hz (wavelength 0.92 m).

From Figures 11 to 14 it can be seen that:

(1) The vehicle speed does not have a significant effect on the dynamic wheelset-rail forces. In fact, for most of the excitation frequencies, the wheelset-rail forces in the high speed case are a little lower than those in the low speed case. See Figure 11 and 13.

(2) The vehicle speed has a great effect on the displacement power spectrum due to the quasi-static loads. In general, the displacement power spectrum in the high speed case is much greater than that in the low speed case, especially at frequencies around 20–40 Hz. This is because a propagating mode of the track/ground system is excited by the moving axle loads [5].

(3) Due to the Doppler effect, the displacement power spectrum due to dynamic wheelset-rail forces of a particular frequency has a broadband of frequency components centred at the excitation frequency. The bandwidth is greatly increased by the vehicle speed. However, the maximum magnitude of the spectrum does not have a significant dependence on the vehicle speed.

6. CONCLUSIONS

A semi-analytical model has been developed for the prediction of ground vibrations generated by surface trains. This model incorporates the necessary components of the mechanism and propagation of vibration in railway traffic. The vehicles are described as multiple-rigid body systems and only the vertical dynamics is considered. The track is modelled as multiple beams supported by vertical springs with consistent mass. The ground is represented by horizontal layers on a homogeneous half-space or a rigid foundation. The governing equations of the vehicles are solved in the frequency domain and those of the track-ground system in the frequency-wavenumber domain. The FFT technique is used to convert the responses of the track and ground from the frequency-wavenumber domain into the frequency-space domain. This model uses the moving axle loads and/or the vertical rail irregularities as its inputs. Outputs include the dynamic wheelset-rail forces, the displacements and displacement spectra (or displacement power spectra) of the track and the ground (on the surface or within the ground).

A computer program, called TGV, has been produced from this model. This report presents a functional description of the program.

7. REFERENCE

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