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Improving the Performance of a Vibration Neutraliser by Actively Removing Damping

M.R.F. Kidner and M.J. Brennan

ISVR Technical Memorandum 829

July 1998

University of Southampton

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INSTITUTE OF SOUND AND VIBRATION RESEARCH

DYNAMICS GROUP

Improving the Performance of a Vibration Neutraliser by Actively Removing Damping

by

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July 1998

Authorized for issue by
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Abstract

This report describes the design of an active vibration neutraliser. The aim of the active element in the neutraliser is to reduce the internal damping of the device and thus make it more effective. Six different control configurations are considered and the input mechanical impedance of each configuration is calculated. This is used to assess the efficacy of each configuration. To study the behaviour of an active neutraliser a beam-like neutraliser is designed and built with piezoceramic patches providing the active element. An analytical model of this device is presented. Simulations and experimental results show that for two control configurations the amplitude of a mass-like structure with the neutraliser attached remains finite at a resonance of the composite system even when damping is entirely removed from the neutraliser, by the control system.

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1 Introduction

The vibration absorber was developed by Frahm and Hartog at the beginning of the century [1]. It works on the principle of impedance mismatching. If a single degree of freedom (SDoF), mass-spring-damper system is added to a vibrating system the impedance at the point of attachment becomes large at the natural frequency of the additional SDoF system. In this report the additional system is referred to as a neutraliser, rather than an absorber. The reasons for this are as follows: the neutraliser works by the reaction force of the neutraliser mass equalling the force applied to the host structure, bringing it to rest, hence the term neutraliser. Alternatively it could be viewed as the energy from the structure being transferred into the motion of the mass of the neutraliser, hence the term absorber. What is fundamental, though, is that no energy is actually dissipated unless there is damping in the neutraliser. It is this damping that limits the behaviour of the neutraliser. This report deals with the issue of using active control to reduce damping in the neutraliser and hence improve its performance.

When the neutraliser is tuned to suppress vibration at a frequency much higher than the structural resonance, only one peak in the response is produced, which is above the tuned frequency. Brennan [2] has shown that the separation of the tuned frequency and this peak is a function of mass ratio (the ratio of the neutraliser mass to the mass of the structure). This report shows that this peak can be limited to a finite value even when the damping is entirely removed from the neutraliser, by an active system.

Brennan [2] has shown that the effectiveness of the neutraliser is a function of the ratio of neutraliser and structure impedances. For this reason the emphasis in this report is placed on determining the input impedance of the active neutraliser. The improvement over the passive case can be determined by assessing the increase in impedance of the neutraliser and hence attenuation of the host structure, as a function of control gain. The control methods investigated in this report look specifically at velocity feedback.

The report is split into five sections. Following this introduction, section two develops the theory for a simple mass spring model of the neutraliser, and derives the input impedances of six possible neutraliser control arrangements. Three of which are selected for further study. The maxima and minima of the velocity of the host structure with these three neutraliser configurations are studied as the feedback gain is increased. The third section develops an equivalent two degree of freedom, mass-spring-damper system to model the behaviour of a beam vibrating in it's first mode. Experimental results obtained from a beam-like neutraliser are presented in section four and finally, section five contains the conclusions.

2 Comparison Of Active Neutraliser Configurations

As described above the neutraliser is a SDoF system attached to a host structure. This can be represented by the system shown in Fig (1). The structure is considered as an impedance, Z_s at the point of attachment and the neutraliser is a simple mass-spring-damper configuration. Brennan [2] has shown that the attenuation of the vibration level of a mass-like host structure of mass m_s, at the neutraliser's tuned frequency and point of attachment, is a function of mass ratio and the damping.

$$\text{Atten} \approx \frac{\mu}{2\zeta} \tag{1}$$

Where μ is the mass ratio, the ratio of the neutraliser mass to that of the structure, $\frac{m_n}{m_s}$ and ζ is the damping ratio of the neutraliser. It can be seen from Eqn(1) that reduction of the damping ζ will increase the attenuation of the host structures vibration.

The response of a mass-like host structure when a neutraliser is attached is shown in Fig(2). It shows that the neutraliser works best when there is little damping in the system. The dip in the structural response is at the neutralisers resonant frequency. In both cases the mass ratio $\mu = 0.1$.

The purpose of the control system is to actively remove damping from the neutraliser by opposing any forces created by the inherent damping. This requires an actuator to be fitted to the neutraliser and/or host structure. An active neutraliser can be implemented in several ways, and in this section the differences between the control configurations are examined.

Because damping is proportional to velocity, the actuator can be fitted in a feedback loop where velocity is fed back with the appropriate gain g. Such a system is shown in Fig(3)

Six variations of the control scheme are possible. Either relative velocity across the spring, $(V_n - V_s)$, or absolute velocity, V_n, of the neutraliser is fedback. The control force can be applied either to the neutraliser mass, the host structure or between the neutraliser mass and the host structure. The following section considers which configuration is most advantageous.

2.1 Input Impedance

The input impedance of a neutraliser can be derived by combining the impedance of each element according to rules given in [3] and considering the neutraliser in the way shown by Fig (4). Fig (4a) shows a free body diagram of the neutraliser, which can then be considered as two impedances in series as shown in Fig (4b). Z is the total impedance of the spring and damper given by $Z = Z_k + Z_c \equiv \frac{k}{i\omega} + c$. Z_m is the impedance of the neutraliser mass, given by $Z_m = i\omega m$. The secondary forces, F_s can then be made to act on either of the impedances to model the control configurations. For instance if it acts on both ends of the impedance Z it is equivalent to a force acting across the spring and damper of the neutraliser.

2.1.1 Absolute Velocity Feedback

Fig(4b) shows the secondary force applied across the neutraliser spring. This is the most general case and is the arrangement for the actual neutraliser used in the experimental work in this paper. The other arrangements require independent locations for the secondary force to react from, which in practice would be an additional proof mass. The equations describing this system are

$$F_1 = ZV_1 - ZV_2 + F_s \tag{2-a}$$

$$F_2 = ZV_2 - ZV_1 - F_s \tag{2-b}$$

$$F_3 = Z_m V_3 \tag{2-c}$$

Because we are considering absolute velocity feedback the secondary force F_s is given by gV_2, where g is the gain in the feedback loop.
By applying the boundary conditions of $F_2 = -F_3$ and $V_2 = V_3$, the resulting expression for the input impedance of the neutraliser, the ratio of F_1/V_1 is

$$Z_{AS} = \frac{ZZ_m}{Z_m + Z - g} \tag{3}$$

The subscript AS refers to absolute velocity feedback, with the force applied across the spring of the neutraliser. When the neutraliser is tuned ($\omega = \omega_n$) the impedance reduces to

$$Z_{AS}|_{\omega=\omega_n} = \frac{Z_m(Z_c - Z_m)}{Z_c - g} = \frac{\omega_n m(1 + 2i\zeta)}{2(\zeta - \nu)} \tag{4}$$

where

$$\omega_n = \sqrt{\frac{k}{m}},\ \zeta = \frac{c}{2\omega_n m} \text{ and } \nu = \frac{g}{2\omega_n m}$$

are the natural frequency of the neutraliser, the passive damping ratio of the neutraliser and the damping ratio due to the secondary force respectively. It can be seen that the magnitude of the impedance at resonance is governed by the net damping ratio, when $\zeta = \nu$ the impedance becomes infinite.
From Fig(4) it can be seen that the two other cases can be derived by setting the secondary force on the mass or base to zero.
When the secondary force is applied only to the mass of the neutraliser, the three general equations describing the system are given by Eqns(2-a–c). However now the secondary force F_s in Eqn(2-a) is set to zero. By solving the above equations as before an expression for the input impedance can be obtained.

$$Z_{AM} = \frac{F_1}{V_1} = \frac{Z(Z_m - g)}{Z_m + Z - g} \tag{5}$$

The subscript AM refers to absolute velocity feedback when the force is applied to the neutraliser mass only. When $\omega = \omega_n$ this becomes

$$Z_{AM}|_{\omega=\omega_n} = \frac{(Z_m - g)(Z_c - Z_m)}{Z_c - g} = \frac{i\omega_n m(1+2i\zeta)(1+2i\omega_n^2\nu)}{2(\zeta-\nu)} \tag{6}$$

The magnitude of the impedance when the neutraliser is tuned is governed by $\zeta - \nu$, so when the gain in the feedback loop is equal to the passive damping coefficient, $g = c$, the impedance becomes infinite.

The third case is that of the secondary force applied to the base of the neutraliser/host structure and being proportional to the absolute velocity of the neutraliser mass. Once again using the general equations shown in Eqn(2-a–c), but setting the secondary force in Eqn(2-b) to zero we get:

$$Z_{AB} = \frac{F_1}{V_1} = \frac{Z(Z_m + g)}{Z_m + Z} \tag{7}$$

When $\omega = \omega_n$ this reduces to

$$Z_{AB}|_{\omega=\omega_n} = \frac{(Z_m - g)(Z_c - Z_m)}{Z_c - g} = \frac{i\omega_n m(1+2i\zeta)(1+2\nu)}{2i\zeta} \tag{8}$$

It can be seen that when the neutraliser is tuned the magnitude of the impedance is primarily governed by the passive damping coefficient. This implies that the use of the active element does not improve upon the attenuation of the host structure acheived by the passive neutraliser.

2.1.2 Relative Velocity Feedback

In the configurations discussed in this section the feedback force is controlled by the relative velocity between the neutraliser mass and the host structure. This means that it behaves more like a conventional damper element. The arrangement is the same as that shown in Fig(4), except that the secondary force is now given by $F_s = g(V_2 - V_1)$.

When the secondary force is placed across the spring it behaves like another damper whose sign and magnitude is controlled by the gain g, which is equivalent to the damping coefficient c. The three equations are the same as Eqn's (2-a–c) except that, as stated above the secondary force is proportional to the relative velocity. This gives the following input impedance.

$$Z_{RS} = \frac{Z_m(Z - g)}{Z_m + Z - g} \tag{9}$$

The subscript RS refers to relative velocity feedback applied across the neutraliser spring. When the neutraliser is tuned this reduces to an expression governed by the net damping in the neutraliser $\zeta - \nu$.

$$Z_{RS}|_{\omega=\omega_n} = \frac{Z_m(Z_c - g - Z_m)}{Z_c - g} = \frac{\omega_n(1 + 2i(\zeta - \nu))}{2(\zeta - \nu)} \tag{10}$$

If the secondary force is applied to the mass the three fundamental equations are the same as before except the secondary force in Eqn(2-a) is set to zero. These can be arranged to give an expression for the input impedance.

$$Z_{RM} = \frac{ZZ_m}{Z_m + Z - g} \tag{11}$$

The subscript RM refers to relative velocity feedback and the secondary force applied to the neutraliser mass only. Eqn (11) is the same as the expression for Z_{AS} shown in Eqn(3). Which, when $\omega = \omega_n$ reduces to Eqn(4).

In the final configuration that is considered in this paper, the secondary force is applied to the host structure/base of the neutraliser and is controlled by the relative velocity between the host structure and mass of the neutraliser. Hence the subscript RB. The three equations are the same as Eqn's(2-a–c) except that the secondary force in Eqn(2-b) is set to zero. The input impedance is given by

$$Z_{RB} = \frac{Z_m(Z - g)}{Z + Z_m} \tag{12}$$

Which at when $\omega = \omega_n$ reduces to

$$Z_{RB}|_{\omega=\omega_n} = \frac{Z_m(Z - c - g - Z_m)}{Z_c} = \frac{\omega_n(1 + 2i(\zeta - \nu))}{2\zeta} \tag{13}$$

It can be seen from Eqn(13) that the magnitude of this impedance depends primarily on the passive damping.

All of the above results and the asymptotic behaviour of the expressions at frequencies, above, below and at the tuned frequency of the neutraliser are summarised in Table (1). At low frequencies the impedances tend to mass-like characteristics for all apart from Z_{AS} and Z_{AB}, which tend to $Z_m - g$. At high frequencies the impedances tend to the value of the damping element, Z_c apart from Z_{RS} and Z_{RB} where this is modified by the feedback gain to $Z_c - g$. For effective control the magnitude of the impedance at the tuned frequency must be dependent on g, as is the case when the secondary force is applied across the spring or to the neutraliser mass only. It should also be noted that applying forces to the neutraliser mass only is difficult to realise in practice as a proof mass is required.

2.2 Maxima And Minima Of The Impedance Of A Neutraliser Attached To A Host Structure

The main objective of the active control of a neutraliser is to increase the acheived attenuation of the host structure at the neutraliser's tuned frequency. However, the presence of the neutraliser also produces a combined host structure and neutraliser resonance at a frequency above the tuned frequency of the neutraliser. The magnitude of this resonance is a function of the mass ratio and damping in the neutraliser. In this section the magnitudes of the maxima and minima of the combined impedance are considered. The maxima and minima in impedance correspond to dips and peaks in the response of the host structure. As the impedance for relative velocity feedback with the force applied to the mass, and absolute velocity with the force applied across the spring are the same, we shall consider only three cases, Z_{AS}, Z_{RS} and Z_{AM} . It is also assumed that the excitation frequency is such that the host structure is mass-like at this frequency. This allows the impedance of the host structure to be simplified to that of a mass, m_s. The combination of this simplified structure and the neutraliser is the complete system which is considered here.

When the control force governed by the relative velocity is applied across the spring of the neutraliser, and the gain is equal to the damping coefficient, the effect is to remove the damping from the system. This sets the response of the host structure to zero at the working frequency of the neutraliser but results in an infinite response at the adjacent resonance which can be seen by examining Fig(5). To compare the benefits of the other feedback strategies with this case, the response at the resonance of the structure is examined for a gain setting of $Z_c = g$.

If the Z_{AS} configuration, (absolute velocity feedback with the control force across the spring), is considered the input impedance eg F_p/V_s to the complete system is given by

$$Z_{t_{AS}} = Z_s + Z_{AS} \tag{14}$$

where Z_s is the impedance of the host structure. Substitution of the expression for Z_{AS} found in Table(1) gives

$$Z_{t_{AS}} = \frac{Z_m Z + Z_s(Z_m + Z - g)}{Z_m + Z - g} \tag{15}$$

Now, it has been shown by Brennan [2] that the resonance of the complete system ($\omega = \omega_s$) occurs when

$$Z_k = \frac{-Z_s Z_m}{Z_m + Z_s} \tag{16}$$

Substituting from Eqn(16) into Eqn(15) we can evaluate the magnitude of the minima in the impedance, by setting $g = Z_c$. This gives:

$$Z_{t_{AS}}|_{\substack{\omega=\omega_s \\ g=Z_c}} = Z_c\left(1 + \frac{1}{\mu}\right) \tag{17}$$

This is the minimum value of the input impedance of the system, which corresponds to the maximum response of the host structure, this is shown by the point $M_{AS_{max}}$ in Fig(5). For comparison, Z_{RS}, (relative velocity feedback with the resulting force applied across the neutraliser spring) is zero for this condition. This shows that $g = Z_c$ does not produce an infinite response at the resonance of the complete system. The solid line in Fig(5) illustrates the mobility of the host structure when $g = c$ and absolute velocity feedback controls the force applied across the spring of the neutraliser.

When the secondary force is proportional to the absolute velocity of the neutraliser mass and is applied only to the neutraliser mass the total impedance of the system shown in Fig(1) is given by

$$Z_{t_{AM}} = Z_{AM} + Z_s \tag{18}$$

On substitution of the expression for Z_{AM} found in Table (1) this becomes

$$Z_{t_{AM}} = \frac{Z(Z_m - g) + Z_s(Z_m + Z - g)}{Z_m + Z - g} \tag{19}$$

On substitution of the expression for Z_k given in Eqn(16)and setting $g = Z_c$ in the above expression we can evaluate the magnitude of the minima in the impedance. We find that

$$Z_{t_{AM}}|_{\substack{\omega=\omega_s \\ g=Z_c}} = \frac{Z_c(Z_m(Z_m + 2Z_s) - Z_c(Z_m + Z_s))}{Z_m^2} \tag{20}$$

If $Z_s \gg Z_m$, i.e. $\mu \ll 1$, and $\zeta \ll 1$ as is often the case in practice, then Eqn (20) becomes

$$Z_{t_{AM}}|_{\substack{\omega=\omega_s \\ g=Z_c}} = \frac{2Z_c}{\mu} \tag{21}$$

This is the minimum value of the input impedance of the system, which corresponds to the maximum response. This again shows that $g = Z_c$ does not produce an infinite response at the resonance of the structure, as shown by the point $M_{AM_{max}}$ in Fig(5).

By taking the ratio of $Z_{t_{AM}}|_{\substack{\omega=\omega_s \\ g=Z_c}}$ to $Z_{t_{AS}}|_{\substack{\omega=\omega_s \\ g=Z_c}}$ the reduction of vibration level at the complete systems resonance due to adopting AM control over AS control can be quantified.

$$\frac{Z_{t_{AM}}|_{\substack{\omega=\omega_s \\ g=Z_c}}}{Z_{t_{AS}}|_{\substack{\omega=\omega_s \\ g=Z_c}}} = \frac{1+\mu}{2} \tag{22}$$

By applying the condition $\mu \ll 1$, Eqn(22) tends to $\frac{1}{2}$. This means the vibration level at the combined resonance when adopting AM control is 6dB lower than when AS control is used. This advantage has to be balanced against the practical difficulties of applying a secondary force to the neutraliser mass only.

The previous analysis shows that for the AM and AS control configurations the amplitude at the complete structure resonance does not become infinite when $g = Z_c$. To calculate the gain at which the response at this resonance does become infinite we can set the numerator of the impedance expression to zero and solve for g. The resulting value of g is termed the critical gain and for the AS control configuration is given by

$$g_{c_{AS}} = \frac{Z_m Z + Z_s(Z_m + Z)}{Z_s} \tag{23}$$

which can be evaluated at the resonance of the combined structure and neutraliser by substituting for Z_k from Eqn(16). This results in

$$g_{c_{AS}}|_{\omega=\omega_s} = Z_c(\mu + 1) \tag{24}$$

When the secondary force is applied to the neutraliser mass only, e.g the AM control configuration the critical gain is given by

$$g_{c_{AM}} = \frac{Z_m Z + Z_s(Z_m + Z)}{Z_s + Z} \tag{25}$$

At resonance of the complete system Eqn(25) becomes

$$g_{c_{AM}}|_{\omega=\omega_n} = \frac{Z_c(1+\mu)^2}{1 + \frac{Z_c}{Z_s}(1+\mu)} \tag{26}$$

If $\zeta \ll 1$ and $\mu \ll 1$, then $Z_s \gg Z_c$ and this can be reduced to

$$g_{c_{AM}}|_{\omega=\omega_s} = Z_c(1+\mu)^2 \tag{27}$$

In both Z_{AM} and Z_{AS} cases the critical gain is a function of the mass ratio μ. This is illustrated in Fig(6) which shows the velocity of the host structure at the complete structure resonance under control normalised to the passive case. It shows that the for the absolute velocity feedback control configurations the response does not become very large until the gain is greater than the passive damping coefficient of the neutraliser. This also means that the condition $g = c$, which reduces the vibration of the host structure at $\omega = \omega_n$ to zero, can be reached under broadband excitation without causing instability at ω_s.

3 Analytical Model of a Beam-like Neutraliser

The active neutraliser used for the experimental work in this paper takes the form of a double cantilever beam, a schematic of which is shown in Fig(7). Note that this is also equivalent to a free-free beam with a force acting at it's center point. The secondary force is provided by the two moments applied at a distance d_p from the centre of the beam, (these moments would be generated by a piezo-ceramic patch glued to the beam). The centre point is the point of attachment to the host structure. This section shows that the beam-like neutraliser can be approximated by the system shown in Fig(8). This is similar to the neutraliser discussed in the previous section, but with the addition of a mass m_1. Conceptually m_1 can be attached to the host structure, leaving a system which is identical to the neutraliser discussed previously. The aim of this section is to relate the secondary force, F_s, the stiffness of the neutraliser spring, k and the neutraliser mass, m_2 to the applied moments, M and material properties of the beam. To develop this equivalent two degree of freedom (2DoF) system the point impedances (F_p/V_1, $F_p/V(0)$) and transfer impedances, (F_s/V_1, $M/V(0)$) of the SDoF model and the beam respectively must be considered.

3.1 Response To A Point Force

If an undamped free free beam is considered as in Fig(7) the point force mobility can be written as the summation over all the modes of vibration [4].

$$\frac{V(0)}{F_p(0)} = \frac{1}{i\omega m_T} + \sum_{r=1}^{\infty} \frac{i\omega\phi_r^2(0)}{m_T(\omega_r^2 - \omega^2)} \tag{28}$$

Where $\phi_r(0)$ is the r^{th} mode shape of a free-free beam at $d = 0$, m_T is the total mass of the beam and ω_r is the resonant frequency of the r^{th} mode. The main assumption in the development of this equivalent model is that higher order modes do not significantly contribute to the dynamic response in the frequency range of interest.

Because of this, only the rigid body mode and first mode of vibration are considered, so mobility can be written as

$$\frac{V(0)}{F_p(0)} = \frac{1}{i\omega m_T} + \frac{i\omega\phi_1^2(0)}{m_T(\omega_1^2 - \omega^2)} \tag{29}$$

This can be rearranged to give the impedance

$$\frac{F_p(0)}{V(0)} = \frac{i\omega m_T(\omega_1^2 - \omega^2)}{\omega_1^2 - \omega^2(1 + \phi_1^2(0))} \tag{30}$$

The above expression shows that the beam impedance is zero at $\omega = \omega_1$, the first resonance of the beam and is infinite when

$$\omega^2 = \omega_a^2 = \frac{\omega_1^2}{1 + \phi_1^2(0)} \tag{31}$$

the first anti-resonance of the beam.

Now consider the two degree of freedom system shown in Fig(8). The impedance of the system is given by

$$\frac{F_p}{V_1} = i\omega \frac{(\frac{m_1+m_2}{m_1})\omega_n^2 - \omega^2}{\frac{1}{m_1}(\omega_n^2 - \omega^2)} \tag{32}$$

This is infinite when $\omega = \omega_n$ and zero when

$$\omega = \omega_m = \sqrt{\omega_n^2 \frac{m_1 + m_2}{m_1}} \tag{33}$$

where $\omega_n = \sqrt{\frac{k}{m_2}}$. If we set $\omega_a = \omega_n$, ie equate the frequencies at which the point impedance of the beam and the impedance of the equivalent 2DoF system are both zero, then we get

$$\omega_1^2 \frac{1}{1 + \phi_1^2} = \omega_m^2 \frac{m_1}{m_1 + m_2} \tag{34}$$

If we set $\omega_1 = \omega_m$, ie equate the frequencies at which the point impedance of the beam and the impedance of the equivalent 2DoF system are both infinite, then we get a relationship between the ratio of the masses $\frac{m_2}{m_1}$ and the square of the mode shape evaluated at the centre of the beam i,e,

$$\frac{m_2}{m_1} = \phi_1^2(0) = 1.478 \tag{35}$$

Hence the equivalent mass spring system has masses in the following ratio $m_2 = 0.596 m_T$ where m_T is the total mass of the beam. This means that only 0.596 of the beam mass is effective in the beam-like neutraliser, the remaining $0.404 m_T$ is effectively added to the mass of the host structure.

3.2 Response Due To The Piezo-Ceramic Elements

The development of an equivalent system for moment excitation is fundamental in successfully modeling the active control of the neutraliser in a simple 2DoF form. Consider the moments applied in Fig(7), the general form of the response to a single moment is given by [4] as

$$\frac{V(0)}{M} = \sum_{r=1}^{\infty} \left(\frac{i\omega \phi_r'(d_p)\phi_r(0)}{m_T(\omega_1^2 - \omega^2)} \right) \tag{36}$$

where $\phi_r'(d_p)$ is the spatial derivative of the mode shape at $d = d_p$, and $\phi_r(0)$ is the mode shape at $d = 0$. The suffix d_p refers to the location of the moments. In the case illustrated in Fig(7) the excitation is symmetrical about d_p. By considering only the first bending mode and noting that the beam is being excited by two moments $M(d_p)$ Eqn(36) reduces to

$$\frac{M(d_p)}{V(0)} = \frac{m_T \omega_1^2}{2i\omega \phi_1'(d_p)\phi_1(0)} \left(\frac{\omega^2}{\omega_1^2} - 1 \right) \tag{37}$$

Now Eqn (37) can be arranged to give:

$$\frac{2i\omega M(d_p)\phi_1'(d_p)\phi_1(0)}{m_T\omega_1^2 V(0)} = \frac{\omega^2}{\omega_1^2} - 1 \tag{38}$$

If a mass spring system as shown in Fig(8) is considered, the transfer impedance between the secondary force and the velocity V_1 is

$$\frac{F_s}{V_1} = \frac{-\omega^2 m_1 + \omega_n^2(m_1 + m_2)}{i\omega} \tag{39}$$

Noting that $\omega_1 = \omega_m$ this can be rearranged to give

$$\frac{F_s}{V_1} = -\frac{m_T\omega_n^2}{i\omega}\left(\frac{\omega^2}{\omega_1^2} - 1\right) \tag{40}$$

Setting

$$\omega_n^2 = \frac{\omega_1^2}{1 + \phi^2(0)}$$

as before and rearranging yields,

$$-\frac{i\omega F_s(1 + \phi_1^2(0))}{V_1 m_T \omega_1^2} = \frac{\omega^2}{\omega_1^2} - 1 \tag{41}$$

It can be seen that Eqn(41) and Eqn(38) are equivalent. Thus this yields the relationship between a moment applied to a beam and the secondary force in the simple mass-spring system.

$$F_s = -\left(\frac{2\phi'(d_p)\phi_1(0)}{1 + \phi_1(0)^2}\right)M \tag{42}$$

The derivative of the mode shape $\phi'(d_p)$ is proportional to the flexural wave number k, which is in turn proportional to the square root of frequency. This means that the equivalent secondary force in the 2DoF model has a frequency dependance, ie

$$\frac{F_s}{M} \propto \omega_r^{\frac{1}{2}}$$

This means that if the beam-like neutraliser is tuned to a high frequency, the secondary force will be very effectively produced by applying a moment to the beam. However at low frequency the secondary force will be much smaller.

The above analysis shows that an equivalent 2DoF system can be developed for a beam-like neutraliser, by dividing the total mass of the beam into two masses as a function of the mode shape. It has also shown that moments applied to a beam and the secondary forces acting on the 2DoF system are related by a function of mode shape and it's spatial derivative.

3.3 The Effect of Residual Modes

The above analysis assumess that the error in calciulation of response of the first mode caused by ignoring higher order modes is small. Fig 9 shows that this error is of the order of 2×10^{-4}%, at the frequency of the first mode, although it increases with frequency.

4 Experimantal Work

The neutraliser discussed in section 3 was physically realised as a perspex beam ($6 \times 40 \times 200$mm) with two piezo ceramic (PZT) actuators ($0.5 \times 31 \times 40$mm) symmetrically attached to the top surface as shown in Fig(10) to generate the moments M [5]. Perspex was used because it has high internal damping allowing any improvements due to active control to be easily measured and reducing the dynamic range of the measurements. The purpose of the experimentation was to verify the predicted active and passive impedances.

Fig(10) shows the experimental set up used to measure the point mobility of the beam. A Hewlett Packard 3567/A analyser produced a narrow band ($200 \rightarrow 400$Hz) random excitation signal used to drive an electro-dynamic shaker attached to the center point of the beam. The applied force was measured directly using a B&K force gauge (type 8200), and acceleration at the center and end points of the beam was also measured. These accelration signals were intergrated using the *B&K* charge amplifiers (type 2635) to give the velocities. The relative velocity between the end and center of the beam was passed through a low pass filter, with a -3dB cut-off point at 1kHz. This was then fed into a B&K power amp type 2713 and was used to control the PZT actuators. The point mobility was then measured over the frequency range of $200 \rightarrow 400$Hz, and the feedback gain was set so that $g \approx c$. Setting the gain exactly equal to the damping coefficient c was not possible as the system would become unstable. The mobility with absolute velocity feedback employed was then measured. In both cases the secondary force was effectively acting across the neutraliser spring. The results are shown in Fig(11).

Fig(11) also shows the theoretical mobility, using the equivalent 2DoF system developed in section 3. For the theoretical results the gain was set such that $g = 0.9c$. The smaller amplitude of the structure-neutraliser resonance suggests that absolute velocity feedback with the secondary force applied across the spring of the neutraliser is the preferred method of control. It also has practical advantages as it only requires one velocity to be measured. The agreement between the experiment and the equivalent 2DoF system is considered good.

5 Conclusions

This report has presented an active control method for reducing the damping in a vibration neutraliser, hence increasing the attenuation of the host structure at the point of attachment that it can provide. It has been shown that the use of absolute velocity feedback has distinct advantages over the use of relative velocity feedback. The most important of which is achieving a finite amplitude at the complete system resonance, whilst still obtaining maximium attenuation at the working frequency of the neutraliser.

An equivalent 2DoF model of a beam has been developed and shown to accurately model the dynamics of a beam-like neutraliser. This greatly simplifies the analysis of beam-like neutralisers.

Experimental results agree with the predictions from the 2DoF model and show that using a secondary force to remove damping is a valid method of improving the performance of vibration neutralisers. This, combined with active tuning technology, produces a very versatile vibration control device.

References

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6 Tables

	Impedance	Impedance at the tuned frequency	Impedance as $\omega \Rightarrow 0$	Impedance as $\omega \Rightarrow \infty$
Absolute Feedback				
Force across spring	$Z_{AS} = \frac{Z_m Z}{Z_m + Z - g}$	$\frac{Z_m(Z_c - Z_m)}{Z_c - g}$	Z_m	Z_c
Force on mass	$Z_{AM} = \frac{(Z_m - g)Z}{Z_m + Z - g}$	$\frac{(Z_m - g)(Z_c - Z_m)}{Z_c - g}$	$Z_m - g$	Z_c
Force on base	$Z_{AB} = \frac{(Z_m - g)Z}{Z_m + Z + g}$	$\frac{(Z_m - g)(Z_c - Z_m)}{Z_c + g}$	$Z_m - g$	Z_c
Relative feedback				
Force across spring	$Z_{RS} = \frac{(Z - g)Z_m}{Z_m + Z - g}$	$\frac{Z_m(Z_c - g - Z_m)}{Z_c - g}$	Z_m	$Z_c - g$
Force on mass	$Z_{RM} = \frac{Z_m Z}{Z_m + Z - g}$	$\frac{Z_m(Z_c - Z_m)}{Z_c - g}$	Z_m	Z_c
Force on base	$Z_{RB} = \frac{Z_m(Z - g)}{Z_m + Z}$	$\frac{Z_m(Z_c - g - Z_m)}{Z_c}$	Z_m	$Z_c - g$

Table 1: Summary of Input Impedances To Neutraliser Configurations

7 Figures

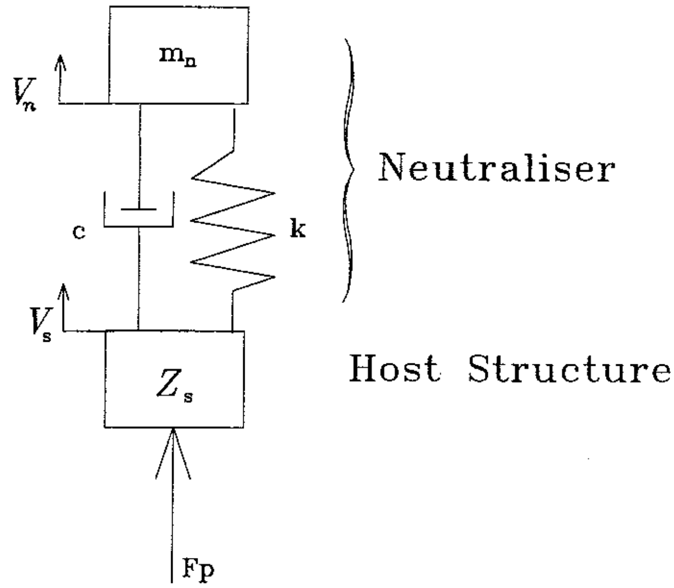

Figure 1: Neutraliser attached to a host structure of impedance Z_s.

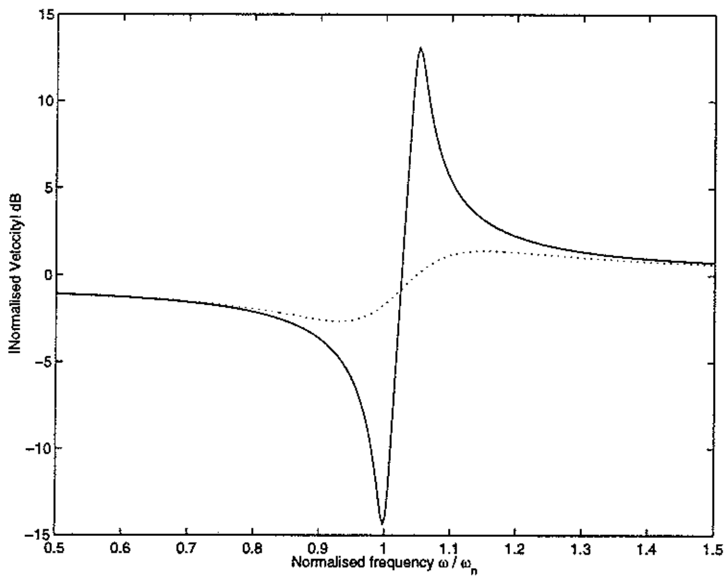

Figure 2: Host Structure velocity, normalised to the velocity without a neutraliser attached solid line:- neutraliser has low damping, $\zeta = 0.01$. Dotted line:- neutraliser has high damping, $\zeta = 0.1$. ω_n is the tuned frequency of the neutraliser. $\mu = 0.1$

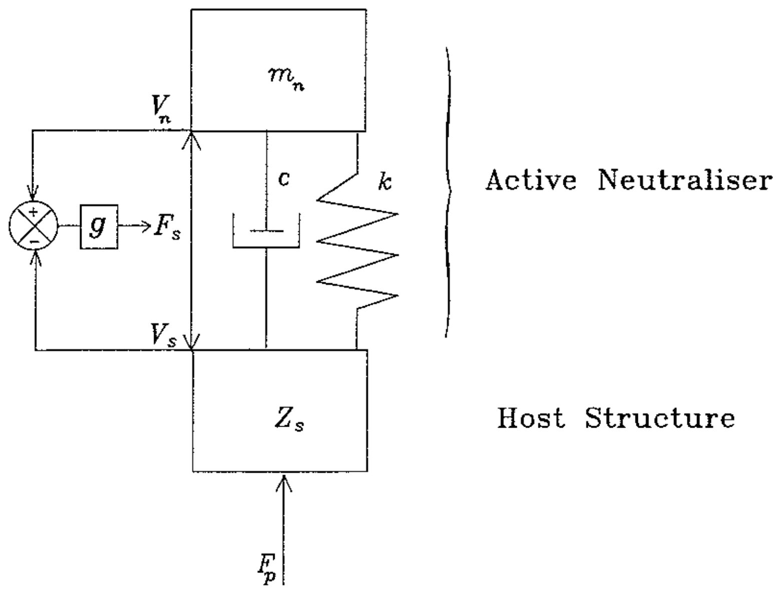

Figure 3: An active neutraliser attached to a host structure of impedance Z_s

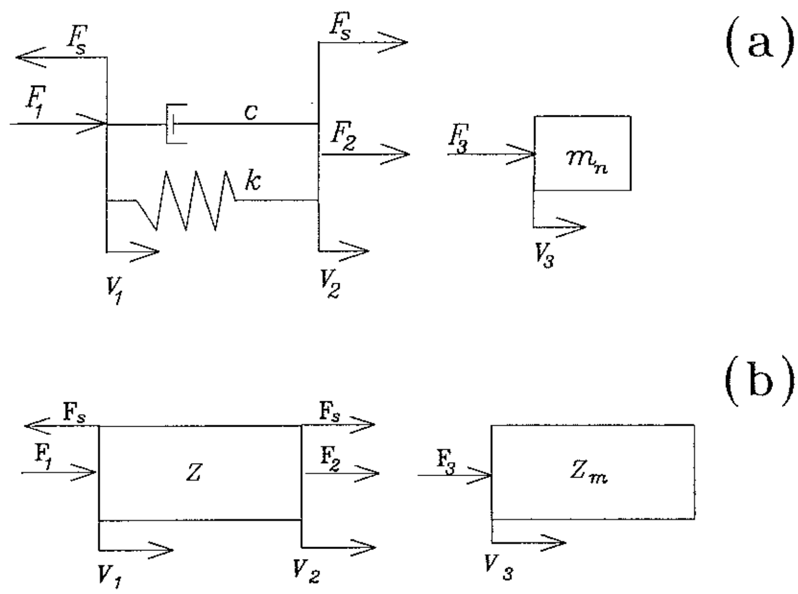

Figure 4: Analytical model of an active neutraliser
(a) Free body diagram (b) Impedance representation. $Z = Z_k + Z_c$

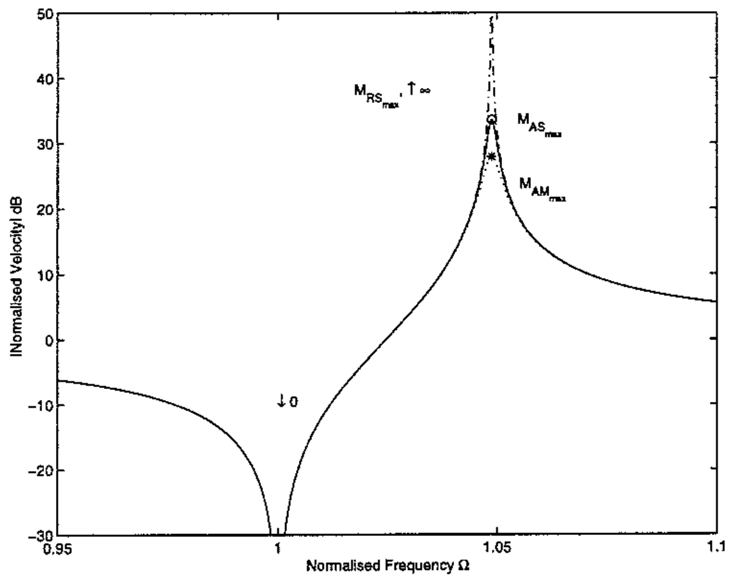

Figure 5: Host structure velocity normalised to the velocity without a neutraliser attached for the three control configurations.
M_{RS}, (dash-dot line):- relative velocity feedback with the force applied across the spring.
M_{AS}, (solid line):- absolute velocity feedback with the force applied across the spring.
M_{AM}, (dotted line):- absolute velocity feedback with the force applied to the neutraliser mass.
The max values o, $*$ correspond to the minimum impedance expressions shown in Eqns(17 and 21) respectively. $g = c$, $\zeta = 0.01, \mu = 0.1$

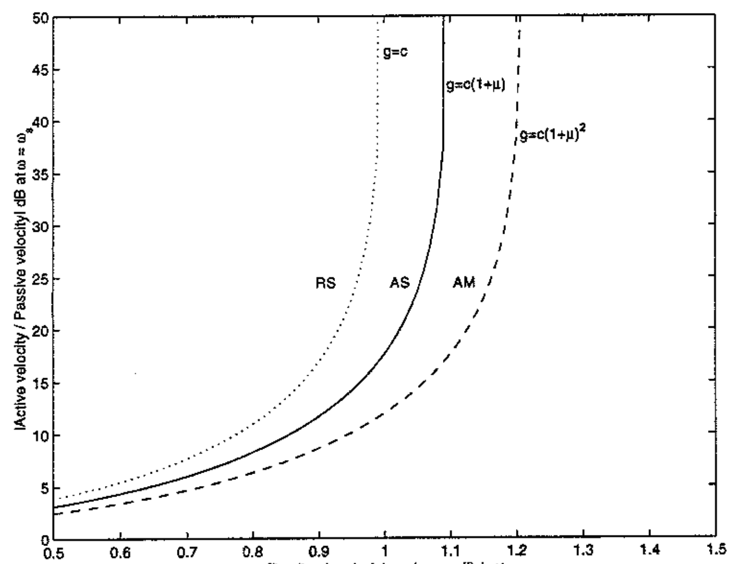

Figure 6: Velocity of the host structure for three control configurations, at $\omega = \omega_s$, normalised to the passive velocity vs the feedback gain. $\mu = 0.1$
M_{RS}, (dotted line):- relative velocity feedback with the force applied across the spring.
M_{AS}, (solid line):- absolute velocity feedback with the force applied across the spring.
M_{AM}, (dashed line):- absolute velocity feedback with the force applied to the neutraliser mass.

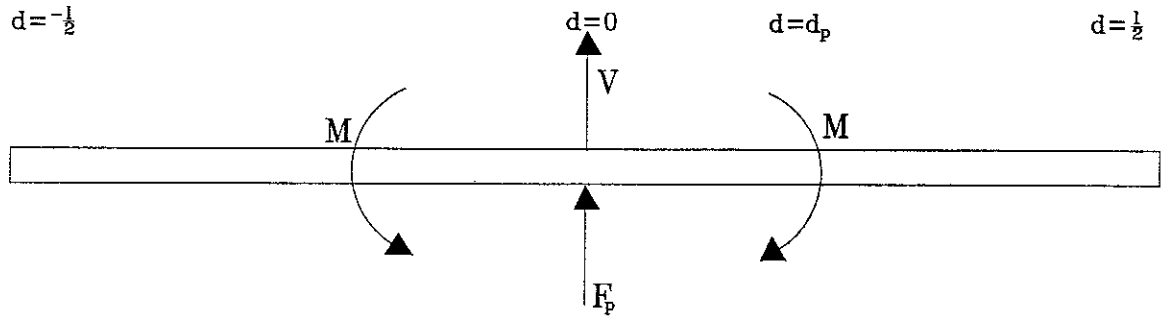

Figure 7: Idealization of free-free beam used in experimental work. F_p is the excitation force and M are moments provided by the piezo ceramic element

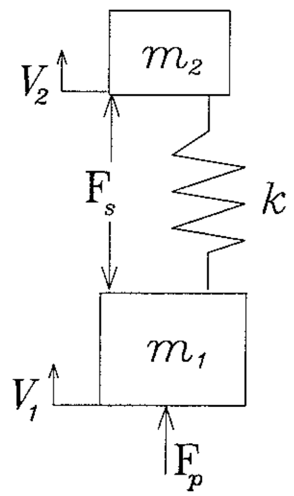

Figure 8: Basic two degree of freedom model of a beam-like neutraliser

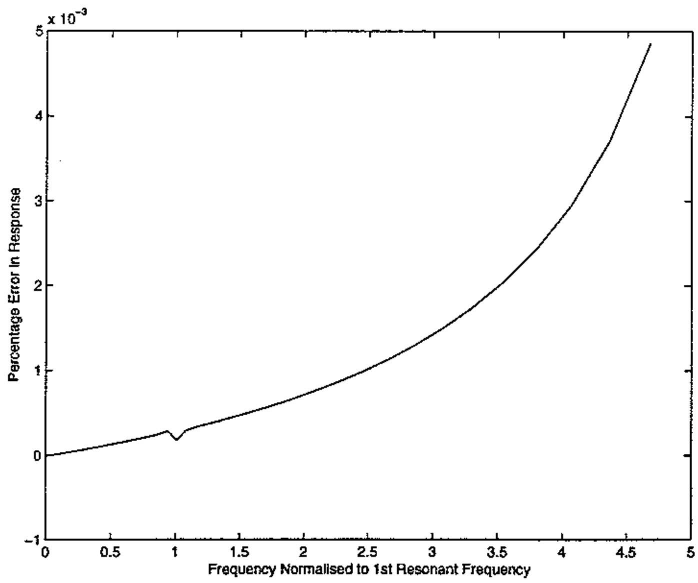

Figure 9: Percentage error as a function of frequency due to ignoring modes of order > 1

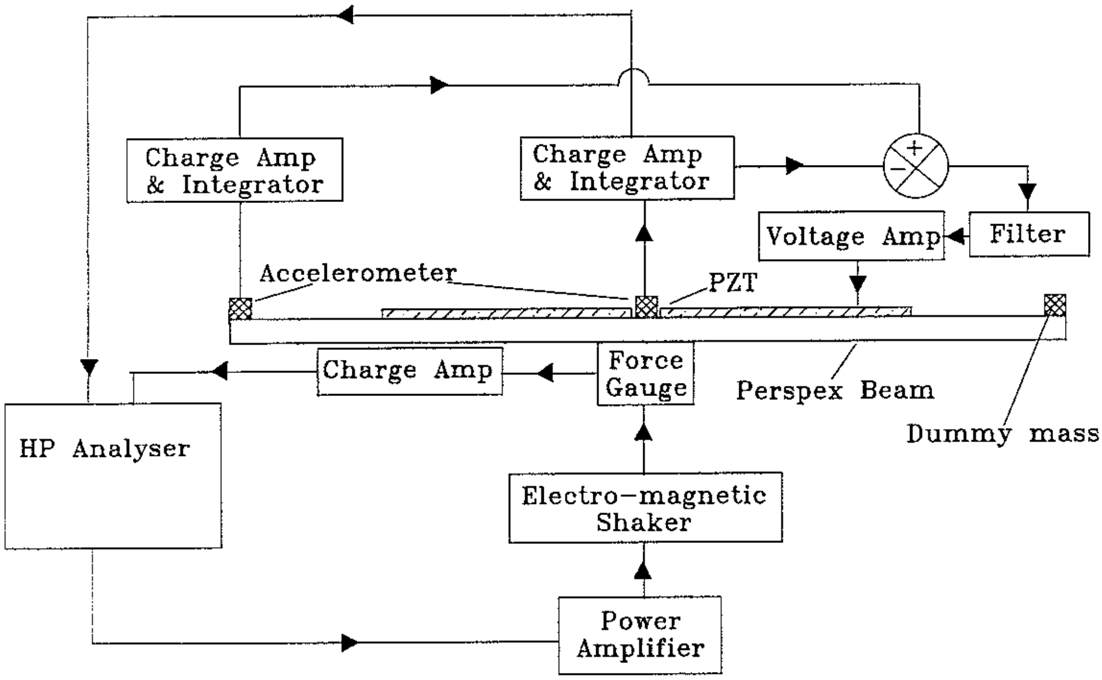

Figure 10: Diagram of the Experimental set up to measure the mobilities of AS, (absolute velocity feedback with the force applied across the spring), and RS, (relative velocity feedback with the force applied across the spring), control configurations

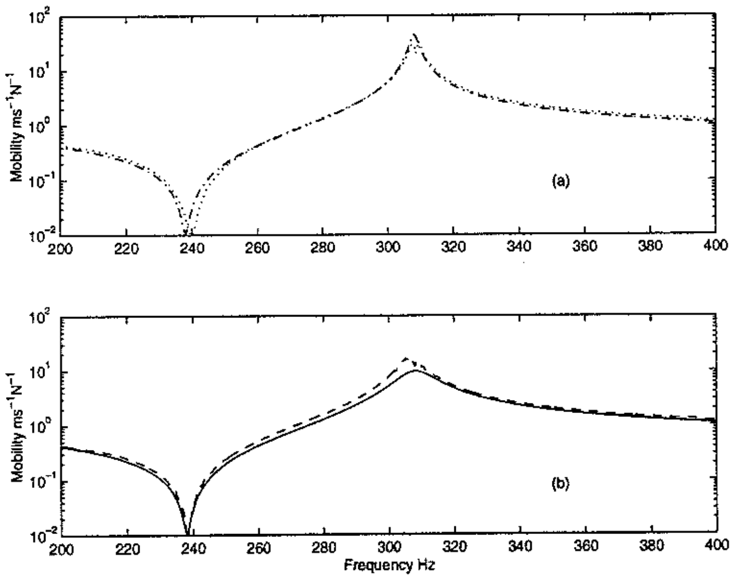

Figure 11: Comparison of experimental and theorectical plots for mobility.
(a),RS, relative velocity feedback with the force applied across the spring. Dash dot :- Model, Dotted :- Experimental data.
(b), AS, absolute velocity feedback with the force applied across the spring, Solid line:- Model, Dashed :- Experimental data. For models, $g = 0.9c$, $\zeta = 0.0311$ and $\mu = 0.67$