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PUBLICATION P6.3
PULSE DISPERSION FOR SINGLE-MODE OPERATION OF MULTIMODE CLADDED OPTICAL FIBRES

Indexing terms: fibre optics, optical waveguides

Pulse dispersions as low as 0.4 ns/km have been measured in multimode cladded fibres at a normalised frequency \( V = 125 \) and for a constant bend radius of 5.7 cm. Particularly when the number of launched modes is small, the pulse dispersion as well as the polarisation and angular width of the output beam, are strong functions of the degree of mode conversion.

Introduction: An earlier report1 of pulse dispersion in multimode, liquid-core, optical fibres showed a strong dependence on curvature, and indicated a minimum value of 1.6 ns/km at a bend radius of just under 1 m. However, the cladding capillary from which the fibres were made was rewound from the drawing machine under a constant tension, and, although the supporting drums were of polystyrene, some small distortion nevertheless occurred. An appreciable amount of mode conversion was thereby caused, the presence and dominating effect of which has been convincingly demonstrated by the present experiments, as well as by the low pulse propagation delay obtained for a narrow input beam when the input angle of incidence is varied, compared with that in an ideal fibre. The former measurements have thus been repeated for a range of input conditions, which includes that for single-mode excitation\(^2\) and bending stresses. A considerably smaller dispersion has been obtained, which is comparable with that predicted\(^3\) and measured\(^4\) in graded-index cladding single-mode fibres. Both the polarisation and the angular width of the output beam are also strong functions of mode conversion, particularly for small input angular beamwidths, and may be used as a measure of the pulse dispersion.

Experiment: The measurements of dispersion, defined as the increase in the halfpower width of the pulse caused by propagation along the fibre, were carried out as described previously\(^1\) with a mode-locked helium–neon laser operating at 0.633 \( \mu \text{m} \). The input beam was plane polarised, owing to the Brewster windows on the laser tube, and the polarisation of the output beam was measured with an analyser.* and is defined as

\[
\frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}
\]

where \( I_{\text{max}} \) and \( I_{\text{min}} \) are the maximum and minimum intensities, respectively, observed as the analyser is rotated. The output angular-intensity pattern was plotted by an automatic scanning system using a p-i-n photodiode to give a display on an oscilloscope. The halfwidth of the beam is defined as the angle at which the beam intensity falls to \( e^{-1} \) times that on the axis.

The fibre consisted of hexachloroethane-1,3-diene in a cladding\(^5\) of internal diameter 57 \( \mu \text{m} \) and having a numerical aperture of 0.46. The normalised frequency is thus \( V = 125 \), and roughly 7800 modes are capable of propagating. The fibre was wound on a drum of 3.5-cm radius, initially at normal tension, but, to reduce the degree of mode conversion without changing the radius of curvature, it was gradually slackened until finally it was quite loose on the drum. Similar results have been obtained for lengths of 61, 150 and 400 m, but only those for the 61 m length are given here, as they are the most complete and the effect of mode filtering\(^6\) due to the lossy cladding is largely avoided.

Results: The four curves in each of Fig. 1a and b, which were obtained for different tensions on the drum, show, as before,\(^1\) that, when the angular width of the input beam, and therefore the number of launched modes, is made larger, there is an increase in the output beamwidth and also, because of the greater spread in group velocities, an increase in the dispersion. However, they now indicate, in addition, that, as the fibre is progressively slackened, at constant bend radius, the dispersion and the output beamwidth both fall, thus clearly showing that distortion of the fibre due to bending stress causes mode conversion. When the fibre is only loosely coiled on the drum, and the distortion is a minimum, it can be seen

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from curve (iv) of Fig. 1b that the output width is only marginally greater than that at the input, and the amount of mode conversion occurring is quite small. The output width and the dispersion are thus determined almost entirely by the number of modes launched, and curvature as such has little effect. It is clear, therefore, that the mode conversion evident in curves (i) to (iii) is due almost entirely to distortion, and not bending, of the fibre. This has been confirmed by inducing distortion in other ways; for example, through the application of a moderate transverse pressure.

To minimise the dispersion, the number of propagating

\[ a \]

Fig. 1 Pulse dispersion and output angular beamwidth

\[ a \] Pulse dispersion

\[ b \] Beamwidth

Angular width refers to core; values in air are approximately 50% greater.

Dispersion decreases from curve (i) to curve (iv)
CHAPTER 7

MATERIAL DISPERSION IN PHOSPHOSILICATE FIBRES

7.1 Measurement of Material Dispersion (P7.1)

The development of low-loss phosphosilicate fibres (Chapter 3) naturally raised several questions about the properties of the new material. Phosphosilicate glasses had only been used previously to a very limited extent and then only for their mechanical and chemical characteristics rather than for their optical properties. We were thus dealing with a completely unexplored material, at least in the optical sense, and work was therefore directed towards characterizing the glass. Just as its physical properties and their difference from those of silica are important to the fabrication process, a comparison of the optical properties of the two materials is equally significant in propagation studies. The parameters of particular interest are the compositional variation of refractive index relative to that of silica, and the dependence of the index on wavelength. The former is required to enable the fibre to be tailored to a given refractive-index profile and numerical aperture, while the latter has a profound effect on pulse propagation, as will become apparent.

While it proved possible to determine the compositional index variation by numerical aperture measurements (see P3.5), the wavelength variation of the index proves more difficult. In bulk samples of glass it is relatively simple to measure the refractive index \( n \) at various wavelengths \( \lambda \), using for example an Abbé refractometer. Thus both the first and second derivative of the index with wavelength may be obtained. The first derivative \( dn/d\lambda \) determines the 'profile dispersion' and affects the form of the index profile required for optical group delay equalisation between modes (see reference 1 for an explanation of profile dispersion). We are concerned here with the second derivative \( d^2n/d\lambda^2 \), which is important when the fibre is used with sources having a relatively broad spectral spread. It determines the transit time differences which exist between the various spectral components of the input source. A pulse having a finite spectral width propagating in a dispersive medium becomes broadened in time as a result of the wavelength dependence of the refractive index and thus of the group velocity.
The various spectral components of the pulse arrive at the output at different times, and this is known as material (or chromatic) dispersion.

Unfortunately it is extremely difficult to prepare bulk samples of phosphosilicate glass in order to measure its index. Phosphorus pentoxide has a high vapour pressure and evaporates from the glass during the melting stage unless high pressures are applied. Furthermore, it is not unlikely that the properties of the glass in the fibre differ considerably from those in bulk, owing to the severe thermal shock of the fibre-drawing process.

An experiment which allowed these difficulties to be overcome is described in publication P7.1. Measurements of the chromatic dispersion of phosphosilicate glass were achieved in the fibre itself by injecting pulses of various wavelengths and observing their relative propagation delay. After being generated by Raman shifting a ruby laser, the pulses were transmitted through lengths of up to 1km of phosphosilicate fibre having various $P_2O_5$ concentrations and radial index profiles. By this means the spectral dependence of group delay was measured, from which $d^2n/d\lambda^2$ could be calculated.

7.2 Discussion

The results provided confirmation of what had already been suspected, namely that the material dispersion of phosphosilicate glass is virtually identical to that of pure silica. The chromatic dispersion of a material is determined largely by the strong absorption caused by electronic transitions sited in the ultra-violet portion of the spectrum. It could reasonably be inferred, therefore, that since the spectral attenuation of phosphosilicate glass is similar to that of silica, its chromatic dispersion would also be unchanged. In contrast, a germania-silica glass has both a higher absorption in the blue region of the spectrum and a larger material dispersion.

The experiment indicated that the addition of phosphorus pentoxide to silica in no way compromises the excellent optical characteristics of the material. Since silica possesses the lowest chromatic dispersion of all common glasses, this conclusion is of some importance as it permits a high bandwidth to be obtained with broad-linewidth sources such as light-emitting diodes. Furthermore, no measurable dependence of the chromatic
dispersion on phosphorus pentoxide content could be found, thus providing additional confirmation that the binary glass has a value similar to that of silica. This may be compared with the only other measurement made to date on a fibre core material, namely the germania/silica binary glass. The value obtained at Bell Telephone Laboratories\(^2\) for this system was limited by the measurement technique to only one wavelength, 900nm, but indicated an increase of nearly 50% over the figure for pure silica.

The measurements reported in P7.1 constitute the only thorough investigation yet conducted of the material dispersion characteristics of the core of an optical waveguide. A particular strength of the technique developed for the measurement is that it enables results to be obtained over a broad wavelength range. This is critically important when a comparison is to be made between the characteristics of a binary glass and those of the host material. Furthermore, as discussed in the following section, the nature of material dispersion is such that it becomes vanishingly small at some wavelength in the region 1.2-1.4\(\mu\)m, and it is obviously desirable to know the exact wavelength at which this occurs. Although our results do not extend to as long a wavelength as this owing to the inadequacies of the ruby laser source, the measurement technique is ideally suited to near-infra-red determinations. It is hoped to extend the measurements towards the wavelength of zero material dispersion in the future.

7.3 The Material Dispersion Limitation to Fibre Bandwidth (P7.2)

In publication P7.2 the implications of the results obtained in P7.1, and outlined above, are pursued further. The material dispersion of the fibre core material becomes the dominant factor in limiting the bandwidth of the guide once the group velocity differences are effectively equalised by grading the core refractive index. The limitation can be serious for a source having a broad spectral spread, such as an L.E.D., although it is not negligible even for the relatively narrow linewidth of a laser. This is clearly shown by curves 2a and 2b of publication P7.2, in which the fibre bandwidth per kilometre is plotted against wavelength of operation for several prospective sources. To take an example, a 1km graded-index fibre excited by an L.E.D.
operating at 0.9μm is limited to a transmission rate of just over 100Mb/sec by material dispersion. This is despite the fact that the waveguide dispersion alone would permit 2.5Gb/s.

The central purpose of publication P7.2 is to point out that an accessible wavelength region exists in phosphosilicate fibres at which the material dispersion vanishes and therefore can no longer significantly curtail the bandwidth. All glasses possess a similar zero in chromatic dispersion; the significance of the phosphosilicate result is that the zero occurs at a wavelength which is accessible in terms of the fibre attenuation. Publication P3.4 of Section I showed for the first time that in a phosphosilicate fibre with its characteristically low water content the attenuation in the vicinity of 1.3μm could be below 5dB/km. Thus provided the material dispersion of the core material is similar to that of silica and has a zero at 1.27μm, operation at the zero is possible. Most other glass systems have their material dispersion zero within a region of unacceptably high attenuation, usually between 1.3-1.4μm, and this effectively precludes its adoption.

As fibre losses become lower and lower, the presence of the accessible zero material dispersion region described in P7.2 assumes considerable importance. The achievement of lower losses is accompanied by an increase in projected repeater spacing, and consequently a greater demand is made on bandwidth per kilometre. The recent Japanese result of 0.47dB/km at 1.25μm obtained with the phosphosilicate vapour-deposition technique suggests that repeater spacings of greater than 40km are feasible. In this case a modest system transmission rate of only 200Mb/sec demands an equivalent bit rate per kilometre of 8Gb/sec. The only means of achieving this at present is with a single-mode fibre operating at or near the material dispersion zero, which of course is also the spectral region in which the reported loss minimum occurs. The Japanese result is therefore significant both in emphasising the importance of the 1.3μm spectral region and in highlighting a growing reliance on single-mode fibres. Furthermore, the point made in P7.2 is all the more telling once it is appreciated that the loss minimum of 0.47dB/km in a phosphosilicate fibre is coincident with its zero in material dispersion. With such advantages available within this spectral region it would be surprising if means are not found to take
Publication P7.2 has played a not inconsiderable part in spurring activity towards the development of sources and detectors operating at longer wavelength (see for example reference 4). The revelation of the existence of a low-loss region near 1.3μm as a result of the ease with which the hydroxyl impurity may be eliminated from phosphosilicate glass has also apparently not gone unheeded. Although it is slightly irksome that the lowest loss yet reported\(^3\) was obtained, not at Southampton, but at a Japanese laboratory, it is gratifying that the full potential of the phosphosilicate glass and fibre fabrication process should have been so dramatically realised.

References


EVALUATION OF MATERIAL DISPERSION
IN LOW LOSS PHOSPHOSILICATE CORE OPTICAL FIBRES

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A method is described for the measurement of material dispersion in the core of an optical fibre, over a wide wavelength range. The method is relatively insensitive to the pulse dispersion caused by group delay differences between modes in the fibre. It is found that the dispersion of phosphosilicate core material is no greater than that of fused silica and is independent of composition over the range measured.

1. Introduction

The bandwidth of a multimode optical fibre waveguide is determined by material dispersion, and by transit time differences between modes. The limiting factor, particularly with broad linewidth sources, is material dispersion since the group delay differences can be made small by the correct choice of refractive index profile across the core [1] or alternatively by a mode scrambling technique [2]. Material dispersion arising from the non-linear wavelength dependence of refractive index, has been shown to produce a pulse broadening of 3.6 ns [3] in a 1 km length of fibre having a core of germania-doped silica when a light-emitting diode of 40 nm spectral width is used. Even for a GaAs laser of relatively narrow spectral width the broadening may not be negligible.

The material dispersion (as defined below) of glasses available in bulk form can be obtained by conventional techniques but, owing to the possibility of changes being caused by severe thermal processing during fibre drawing, a measurement directly on the fibre is to be preferred. In addition, for those materials difficult to form in bulk, such as the phosphosilicate glass recently used in this laboratory as the core of ultra-low loss optical fibres, the dispersion must be determined on the fibre itself.

One technique which has been reported [4] involves an analysis of the back-scattered radiation produced when a beam from a cw argon laser impinges transversely upon a fibre. This method has so far only been applied to unclad fibres and the results will be difficult to interpret in the presence of a cladding. It also gives the dispersion in a small localized region of the fibre whereas for communications purposes an effective value over a long length of fibre is of more interest. In another method [3] two self-pulsing GaAs lasers emitting at slightly different wavelengths are coupled to a length of fibre and the material dispersion is obtained from the time separation, \( \Delta \tau \), between the two pulses at the output end of the fibre. Because the two wavelengths are closely spaced the difference in pulse propagation times is small and this method can only be used with fibres of very low group delay dispersion, such as graded-index fibres, where the pulse broadening is less than \( \Delta \tau \). As a result the core material dispersion cannot be obtained as a function of composition since this varies with radius.

It is commonly observed that the addition of a dopant to fused silica in order to increase its refractive index also results in an increased absorption in the blue region of the spectrum, bringing with it a change in the material dispersion characteristics. This is perhaps not altogether surprising as the refractive index in the visible and near infra-red region is closely related (by the Kramers–Kronig relation) to the intensity and proximity of the strong electronic absorption in the ultra-violet. An increased refractive index implies an
intensified or shifted ultraviolet absorption band, and the tail of this band magnifies the loss in the blue spectral region. In addition the slope of the refractive index curve varies more rapidly with wavelength, giving rise to increased dispersion. Thus, for example, high refractive index optical glasses exhibit large dispersion [5] and a germania-doped silica fibre gives a higher dispersion than that of pure silica [3].

However, as has previously been reported [6,7], the addition of P₂O₅ to silica results in an increased refractive index without an accompanying increase in the low-loss characteristic of pure silica. The object of the present work therefore, was to verify the inference of this unchanged blue absorption, namely that the material dispersion characteristic would be similarly unaffected.

By employing a single laser to generate a series of monochromatic pulses of different wavelengths in the range 700–900 nm we have been able to determine the material dispersion of various phosphosilicate compositions over a wide wavelength range. The material dispersion results in differing fibre transit times for each wavelength pulse, and when this difference is measured relative to a time marker pulse the dispersion as a function of wavelength may be calculated.

2. Theory

Consider a short pulse in the form of a plane wave propagating in an infinite dispersive medium having a refractive index n which is a non-linear function of the angular frequency ω. The group delay τ per unit length is given by:

$$\tau = \frac{d\beta}{d\omega} = c^{-1} \left( n + \omega \frac{dn}{d\omega} \right),$$

where the phase constant β is related to the wavelength λ by:

$$\beta = 2\pi n/\lambda = \omega n/c = kn.$$  

Thus in terms of wavelength we can write:

$$\tau = \frac{d\beta}{d\omega} = c^{-1} \left( n - \frac{\lambda}{\lambda} \frac{dn}{d\lambda} \right).$$

If the spectral width of the pulse is Δλ then signal distortion will arise as a result of this wavelength dependence of group delay. The pulse width Δτ after travelling unit length is given by the differences in transit times corresponding to the wavelength spread Δλ namely:

$$\Delta \tau = \frac{d\tau}{d\lambda} \Delta \lambda.$$  

Alternatively if two monochromatic pulses of small wavelength difference are simultaneously launched then eq. (4) gives the difference Δτ in propagation times of the peaks after unit distance.

Thus the transmitted pulse width or pulse separation can be determined from a knowledge of the 'material dispersion' dτ/dλ* and the spectral width Δλ.

From eq. (3)

$$\frac{d\tau}{d\lambda} = -\frac{\lambda}{c} \frac{d^2 n}{d\lambda^2},$$

and Δτ can also be obtained from known refractive index data.

$$\Delta \tau = -\frac{\lambda}{c} \frac{d^2 n}{d\lambda^2} \Delta \lambda.$$  

However with optical fibres, for the reasons already stated, a much more useful and direct method is to measure dτ/dλ directly. This can be done by determining the group delay τ in the fibre as a function of wavelength and taking the slope of the resulting curve. In order to avoid errors arising from the fact that the delay differences are small compared with the total delay, a pulse of fixed wavelength and delay can be injected together with the pulses of varying wavelength, to act as a reference time marker. Then the variation of delay with wavelength can be measured relative to this pulse avoiding the need to accurately measure the total transit time.

In a multimode step-index fibre another dispersive mechanism must be taken into account. The pulse distortion caused by delay differences between the propagating modes is up to 5 ns/km in the fibres considered here, and the pulse separation due to material dispersion must therefore be at least 5 ns in a 1 km length. This can be achieved by judicious choice of the wave-

* The 'material dispersion' is taken to mean dτ/dλ = (-λ/c) d²n/dλ² throughout this paper. This is in contrast to the 'dispersion' of optical glass which normally refers to the first derivative of the refractive index, dn/dλ.
length difference between the reference pulse and the region in which it is desired to determine the dispersion. As may be seen in fig. 2, the pulses can be sufficiently well separated, even in a multimode fibre, to enable accurate delay difference measurements to be made. It may also be seen that the pulse width is equal for the two wavelengths, so that the waveguide has equal effect on both the time marker and the probe pulse. Thus the pulse separation, as measured from peak to peak, is due only to material dispersion.

3. Experiment

Since the wavelength region of principal interest for optical fibre communications is that of the various lasers and light-emitting diodes based on gallium arsenide and associated materials, namely 800 nm to 900 nm, a ruby laser was used to provide the marker pulse from which the probe pulses at various wavelengths were derived by Raman generation. The ruby laser was simultaneously Q-switched and mode locked DDI/DCI dye solution giving trains of pulses of 25 ps duration at 694 nm and separated by 9 ns [8]. The output from the laser was first passed through a beam-splitter, A in fig. 1, where a portion was extracted and fed to a vacuum photodiode in order to provide triggering pulses for a Tektronix 7904 oscilloscope with a plug-in unit having a 500 MHz response. Of the transmitted beam 8% was removed at a second beam-splitter

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Frequency shift (cm⁻¹)</th>
<th>Output wavelength (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benzene</td>
<td>991</td>
<td>746</td>
</tr>
<tr>
<td>Benzene</td>
<td>991 + 991 (2nd Stokes)</td>
<td>805</td>
</tr>
<tr>
<td>Benzene</td>
<td>3064 (1st Stokes)</td>
<td>882</td>
</tr>
<tr>
<td>Water</td>
<td>3651 (1st Stokes)</td>
<td>930</td>
</tr>
</tbody>
</table>

Fig. 2. Output pulses from 330 m of 50 µm core diameter phosphosilicate fibre (N.A.O. 162). The first pulse is an 805 nm probe pulse, while the second is the 649 nm time marker pulse. Time scale 2 ns/div.
Fig. 3. Group delay of pulses of various wavelengths relative to time marker pulse of 694 nm. The curve is for all fibres tested.

B to provide the reference pulse of fixed wavelength $\lambda_1 = 694$ nm. The remaining beam traversed a Raman cell of length 20 cm containing either benzene or water. In the latter case a lens of focal length 30 cm was placed in front of the cell in order to increase the Raman gain. The wavelengths generated are shown in table 1. The output from the Raman cell was filtered appropriately to remove the ruby laser radiation (filter RGN-9) and to select the desired Raman wavelength (narrow-band interference filter). The pulse trains at the two wavelengths were recombined using a tilted plate as shown and launched simultaneously into the fibre by means of a X10 objective lens.

The output from the fibre was detected with a vacuum photodiode having an $S1$ response and displayed on an oscilloscope. The intensities of the input beams to the fibre at the ruby laser and Raman wavelengths were adjusted to be approximately equal at a power level of 30 W over 20 ps, a level below the non-linear attenuation threshold for pulses of this duration. The propagation delay between the probe pulses, at the wavelengths shown in table 1, and the reference pulse was measured in stepped-index multimode fibres of ~50 $\mu$m core diameter for a range of $P_2O_5$ concentrations corresponding to numerical apertures from 0.10 to 0.18. Two fibres having a graded-index core were also measured. The lengths ranged from 0.73 to 1.15 km. The fibres had an attenuation of less than 10 dB/km.

Fig. 4. Wavelength dependence of the material dispersion of phosphosilicate glass. The dashed curve was obtained using fibres of several different $P_2O_5$ levels. The solid curve was calculated from the refractive index data of pure silica.
in the range measured, some samples having a loss as low as 2.7 dB/km at 830 nm. The pulse dispersion caused by group delay differences between modes was obtained from the broadening of individual pulses, and was \( \sim 5 \text{ ns/km} \) for the stepped-index fibres and \( \sim 1 \text{ ns/km} \) for the graded-index fibres.

Typical oscillograms showing the reference and probe pulses at the output from the fibre are shown in fig. 2. The group delay relative to that at 694 nm for all the fibres tested is given by fig. 3 which is, effectively, a curve of \( \tau \) versus \( \lambda \). The slope of this curve has been measured and the values of \( d\tau/d\lambda \) so obtained are indicated by the points marked in fig. 4. The solid line in fig. 4 is a curve of \( -(\lambda/c) d^2n/d\lambda^2 \) for pure silica which has been calculated from published [9] refractive index data. It can be seen that there is no significant difference between the results obtained with the various fibres and the material dispersion of phosphosilicate glass does not differ from that of silica.

4. Discussion

The fact that the measured material dispersion of phosphosilicate glass does not differ from that calculated for silica is in contrast with the result obtained for germania doping [3]. The small differences in the results obtained for the various fibres are probably due to experimental scatter since there was certainly no trend towards the value for silica as the phosphorus pentoxide concentration was reduced. The value of 69 ps nm\(^{-1}\) km\(^{-1}\) measured at 900 nm differs from that of silica by only 4\%, this difference being within the accuracy of the experiment. The present result is consistent with the observation [6,7] that the addition of phosphorus pentoxide to silica does not increase the transmission loss in the blue region of the spectrum.

Measurements have already been carried out over a wide wavelength range of nearly 200 nm but the technique can easily be used over an extended range. Thus second-harmonic generation of the output from a Nd: YAG laser would improve accuracy in the 700 nm region, and allow wavelengths to 530 nm to be covered. The flexibility of the method can also be increased by using a Raman generated pulse as the reference.

As an example of the use of fig. 4 let us take the case of a 1 km length of phosphosilicate core fibre with a light-emitting diode source of spectral width 40 nm and mean wavelength 900 nm. The material dispersion alone (69 ps nm\(^{-1}\) km\(^{-1}\)) would give rise to a pulse broadening of 2.8 ns/km, to which must be added any mode dispersion effects.

5. Conclusions

A simple method has been described for the measurement of material dispersion \( d\tau/d\lambda \) over a wide wavelength range. It can be used with both stepped-index cladded fibres, for a range of dopant concentrations, as well as graded-index fibres and therefore enables the variation of material dispersion with dopant concentration to be determined, since its application is not restricted to fibres of low pulse dispersion. Measurements with the phosphosilicate core fibre show that the addition of phosphorus pentoxide to silica does not increase the material dispersion.

Acknowledgements

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References

ZERO MATERIAL DISPERSION IN OPTICAL FIBRES

Indexing terms: Dispersion (wave), Fibre optics, Optical waveguide

The material dispersion of optical fibres having cores of silica or phosphosilicate glass falls to zero at a wavelength between 1.2 and 1.3 μm. A considerable increase in bandwidth can be obtained, especially with an I.E.D. source, by operation in this region.

Introduction: Recently, we reported a method for the determination of the material dispersion in the core of an optical waveguide. The technique has been applied to the low-loss phosphosilicate-core fibres developed in these laboratories, over the wavelength range 0.7-0.85 μm. When compared with computations of the dispersion of silica (Fig. 1), using the Sellmeier equation of Mallison, there is found to be a close correspondence over the measured range. A similar correspondence has also been observed for the predicted and measured losses of the two materials for wavelengths between 0.43 and 0.9 μm. It would appear therefore that the addition of P2O5 does not greatly affect the dispersion and loss of silica in the region where measurements have hitherto been made, and it is not unreasonable to hope that the same may be true for dispersion at longer wavelengths.

Material dispersion calculations for silica have therefore been extended to 2.3 μm and indicate that a zero value is obtained at 1.27 μm. Phosphosilicate glass is expected to behave similarly.

Effect of material dispersion on bandwidth: In a multimode optical fibre, the waveguide and material dispersions have a large influence on the bandwidth; the former is caused mainly by transit-time differences between modes, and the latter by the variation of the core refractive index with wavelength. The two mechanisms are separable provided that the core-cladding refractive-index difference is small.

The relative importance of the material-dispersion limitation depends on the spectral width of the source and on the magnitude of the waveguide dispersion. In a recent publication, it is predicted that, by a suitable choice of refractive-index profile, the waveguide pulse dispersion τp can be minimised to give a value of

\[ \tau_p = n_0 \Delta \frac{L}{2c} \]

where \( n_0 \) = maximum refractive-index difference

\( \Delta \) = fibre length

Thus, for the phosphosilicate-core borosilicate-cladded fibre reported earlier, with \( \Delta = 0.018 \) (n.a. = 0.2), the optimised waveguide dispersion could be as low as 0.1 ns/km, and, to obtain maximum bandwidth, the material-induced pulse dispersion must be kept to a similar value, namely ≈ 0.1 ns/km. The most likely sources for optical-fibre communication are injection lasers or light-emitting diodes having spectral widths of typically 4 and 40 nm, respectively. Operation of a phosphosilicate fibre with an I.E.D. source at 0.9 μm results in a pulse broadening due to material dispersion alone of 2-6 ns/km, and, even for the narrower-spectral-width laser, operating at 0.83 μm, a pulse width of 0.38 ns/km is obtained. Thus the material dispersion of this fibre, or, indeed, of a fibre having a pure silica core, is a serious drawback, and to realise the full potential bandwidth, some attempt must be made to reduce either the source linewidth or the glass material dispersion.

Operation at longer wavelength: As shown in Fig. 1, an appreciable decrease in material dispersion can be obtained by operation at a longer wavelength. However, there are several other factors to consider. The first is the associated fibre attenuation. We have already demonstrated that this can be less than 4 dB/km, provided that the OH content can be kept sufficiently small. The effect of the OH impurity arises partly because of the first overtone at 1.37 μm, but is mainly due to an absorption peak at 1.23 μm attributed to the combination vibration of the second overtone (ν2') of the fundamental OH resonance at 2.73 μm with the fundamental SiO4 tetrahedral vibration (νs'). The strength of the 1.23-μm line is about twice that at 0.95 μm, and, to limit the peak height above the baseline to an acceptable level of, say, 0.2 dB/km, the OH content must be kept to 0.1 parts in 106. Although this concentration is low, it should be attainable, since, in the borosilicate-cladded phosphosilicate-core fibre,

\[ \frac{dv}{dλ} = (-λr)c^2 n(d^8) \]

as a function of wavelength

The solid curve is calculated for silica from the data of Mallison and the points have been measured in a fibre having a phosphosilicate-glass core.

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which was prepared with relatively low-quality BCl₃, a figure of 0.5 parts in 10⁶ has been observed. Potentially, the loss at 1.27 μm is lower than that at 0.9 μm, since both the scattering and intrinsic absorption are reduced. A further advantage for military applications may be a reduced sensitivity of the attenuation to ionising radiation, compared with silicon at shorter wavelengths.

There seems to be no reason why light-emitting diodes should not be made with adequate radiance at \( \approx 1.27 \) μm and a suitable material might be GaAsInAsAsSbP. Detectors based on silicon are not suitable, and there will be some loss of sensitivity with germanium avalanche photodiodes.

Effect of source linewidth: To compare the pulse rates that may be attained in a fibre with a phosphosilicate or silica core, for a range of wavelengths and source linewidths, the curves of Fig. 2 have been calculated. The pulse shape will depend on the broadening mechanism, but the pulse rate is conservatively taken as \( B = (2\pi)^{-1/2} \), where \( B \) is the total width of the output pulse for unity mark/space ratio. To obtain \( r \) for a given wavelength, the material dispersion from Fig. 1 is multiplied by the source linewidth representing a semi-conductor laser (2 or 4 nm) or an i.e.d. (30, 50 or 500 nm). The pulse width \( r \), so obtained is added to the waveguide dispersion \( r_s \) to give \( r^2 = r_s^2 + r^2 \), assuming Gaussian pulse shapes.

It has also been assumed in Fig. 2a, that at each wavelength, the refractive-index profile and the fibre geometry are nearly optimum, allowing a waveguide dispersion of 0.2 ns/km. The material dispersion thus has an appreciable effect, even with a laser source, except at wavelengths close to 1.27 μm. Taking a typical linewidth for a high-quality gallium-arsenide laser of 4 nm, the maximum pulse rate possible at 0.83 μm is about 1200 MHz over 1 km, compared with the waveguide dispersion limit of 2500 MHz. On the other hand, for the i.e.d., the pulse rate rises spectacularly from 20 MHz over 0.9 μm to 2500 MHz over 1.27 μm. The corresponding figures over 7 km, assuming a linear dependence of waveguide dispersion on length, are 29 and 357 MHz.

Even if a shift in i.e.d. wavelength from 0.9 to only 0.98 μm, so that silicon detectors can still be used, would be well worth while as the pulse rate increases by 50% to 300 MHz over 1 km.

Fig. 2b is for a waveguide dispersion of 1 ns/km, which is currently possible and gives a lower maximum pulse rate of 500 MHz over 1 km. The improvement by shifting the laser wavelength from 0.83 μm, is small, but becomes substantial (from 180 to 500 MHz) with an i.e.d. of 40 nm wavelength.

Modulation capability of i.e.d.s: Another question is whether light-emitting diodes can be modulated sufficiently fast to make use of a reduced overall dispersion. Since, as indicated above, fibre attenuations of 2 dB/km, and below are within reach, it is reasonable to consider repeater spacings of, say, 7 km, although this will depend on the light source used and the launching and detector efficiencies. With a waveguide dispersion of 0.2 ns/km, the required pulse rate is 350 MHz and should be possible.

Recent work has shown that high modulation rates can be obtained by (a) constructing diodes to have low capacitance, (b) employing drivers with low output impedance and (c) appropriate shaping of the modulation pulse. With a large driving current, the optical pulse risetime can be \( \approx 2 \) ns with negligible (< 1 μs) delay. The minimum pulse-width is related to the carrier spontaneous recombination time which is \( \approx 1 \) ns. Further, in many applications, the i.e.d., with some sacrifice in driving efficiency, can be modulated at a higher speed than that imposed by the spontaneous recombination time. Thus a pulse rate of 280 MHz, and a risetime of 0.7 ns, has been demonstrated under conditions where the i.e.d. was not fully turned on. It is clear, therefore, that high pulse rates are possible and minimisation of the material dispersion is desirable, since, at 0.9 μm, the limit due to material dispersion is \( \approx 30 \) MHz in 7 km.

Conclusion: It is commonly, but not universally, assumed that, for widespread applications of optical-fibre systems, a suitable semiconductor laser capable of long life, continuous operation, at room temperature must be developed. Besides being small, efficient and capable of direct modulation, such devices have the advantage over light-emitting diodes of high brightness, narrow output beam and small wavelength spread. Unfortunately, the reliability and lifetimes of present-day injection lasers are far from adequate. In contrast, existing light-emitting diodes are reliable and easy to operate. However, their large linewidth results in a low bandwidth at wavelengths normally considered for operation in optical fibres, namely 0.8-0.9 μm. We wish to make the point that there is an accessible wavelength region where the transmission loss in phosphosilicate and silica-core fibres is low and the material dispersion is negligible. The bandwidth, even with an i.e.d. source, is therefore limited only by waveguide dispersion. There appears to be no inherent difficulty in fabricating an i.e.d. in this region of the spectrum, and, for moderate repeater separations, modulation rates compatible with the waveguide capability are possible.

The method of fibre manufacture is ideally suited to production of various core profiles, but it remains to be seen whether fibres can be made sufficiently accurately to keep the waveguide dispersion down to a low value. However, over several kilometres, mode coupling may well decrease the waveguide dispersion and make the pulsewidth increase as the square root of the fibre length. Since the broadening due to material dispersion remains a linear function of length, it imposes a more serious limitation as the fibre length increases.

With a refractive-index profile that is not optimum, we have already measured pulse dispersions of 1 ns/km, so that the target of 0.2 ns/km may not be too difficult to obtain.

In a single-mode fibre, the bandwidth could be increased by several orders of magnitude through a shift to 1.27 μm, which would remove material dispersion and leave mode dispersion as the main limitation.

Acknowledgments: We are indebted to C. Barwell for computing the material dispersion of silica from the Sellmeier equation. Grateful acknowledgment is made to the UK Science Research Council for supporting the work and to the Pirelli General Cable Company for the award of a research fellowship.

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References


CHAPTER 8

SINGLE-MODE FIBRE EVALUATION

8.1 The Single-mode Fibre (P8.1)

In the previous chapter it emerges that as the fibre loss diminishes the single-mode fibre becomes increasingly attractive for long-distance, high-capacity transmission. In view of this, it is surprising that very little work on the monomode fibre has been reported. Most work has been concentrated on multimode, graded-index fibres, largely because of their less critical jointing tolerances compared to those of the monomode fibre. Nevertheless, a recent paper\(^1\) suggests that the single-mode fibre is less susceptible to the excess attenuation produced by micro-bending and therefore should be considerably easier to cable. Together with the ease and speed of fabrication, this could outweigh the difficulty of splicing and make the monomode fibre economically attractive.

Single-mode fibres having a phosphosilicate core are made in much the same way as the multimode fibres described in Chapter 3. A brief description of the fabrication method is given in P3.5 and section 3.6 of Chapter 3. Since only a single layer of phosphosilicate glass is deposited, the process is much quicker than that for a multimode fibre. Furthermore, the dispersion properties of the fibre are not critically dependent on the index profile, as in the graded-index fibre, provided single-mode operation is maintained. This considerably eases the fabrication process. However, since the core diameter is typically only 3-6μm it is not easy to achieve a closely-controlled, predetermined core size and thus ensure that no higher mode propagation is permitted. In addition, the depletion by vaporisation of the phosphorus pentoxide from the centre of the tube during the collapse process leaves the core refractive index and profile in some doubt. Routine measurements of the fibre parameters are therefore required to provide information which will allow small corrections to be made to the fabrication process and permit a particular fibre specification to be attained. For example, until better control is achieved it is often necessary to pull a short test length of fibre to evaluate the waveguide parameters. The remainder of the preform may then be drawn to a diameter which yields the required propagation characteristics.
Clearly there is a demand for a rapid, convenient method of determining the parameters of a single-mode fibre. Existing methods were unsatisfactory since they were tedious and inaccurate. For example it was often necessary to measure the fibre core diameter in a scanning electron microscope. A new method which avoids the previous difficulties and satisfies all the requirements is described in publication P8.1.

8.2 Fibre Measurements (P8.1)

The procedure outlined in P8.1 is unique in that it allows the unambiguous and simultaneous measurement of both the core diameter and the refractive-index difference. It relies on the fact that the far-field radiation angle of the \text{HE}_{11} mode is related to the normalised frequency \( V \) of the fibre; therefore a measurement of the half-intensity width of the radiation pattern allows a determination of \( V \). Unfortunately this in itself is insufficient for most purposes, since \( V \) is a function of (i) the core diameter \( a \) and (ii) the index difference \( \Delta n \). Thus a further measurement of either (i) or (ii) is required to fully specify the fibre. Although this can be provided by tedious methods such as etching the core, it was discovered that a far more convenient means is to observe the side lobes of the radiation pattern. The angle of the first minimum in the radiation field provides the required additional measurement and determines both \( a \) and \( \Delta n \). The method has proved both accurate and easy to use. Results of determinations made on various fibres may be found in the paper, together with a more detailed description of its use.

A simple observation of the far-field pattern is therefore all that is required to specify a single-mode fibre completely. The technique is now regularly used in our laboratories and, because of its simplicity, will quite possibly be adopted eventually as the standard test method for all monomode fibres. It thus forms a valuable contribution to the growing field of optical fibre measurement techniques.

References
Determination of core diameter and refractive index difference of single-mode fibres by observation of the far-field pattern

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Summary

A method is described for determining unambiguously from the far-field pattern of single-mode fibres the core diameter and the refractive index difference between core and cladding. It involves measurements only of the half-power width of the main lobe and the width of the first minimum. Universal curves are presented which can be used with single-mode fibres or any fibre operating in the single-mode regime. Independent determinations of core diameter over the range 4 to 8μm by an etching technique are in excellent agreement with those obtained from the far-field pattern.

1. Introduction

Single-mode fibres have several potential advantages when considered for use as transmission lines at optical frequencies. In particular the absence of dispersion due to multimode operation means that the attainable bandwidth can be very large, being ultimately limited by material and mode dispersion when operating with a monochromatic source and estimates\(^1\) give values in the region of 100GHz over a 1km length. In recent years attention has centred on multimode fibres because they can be used with light-emitting diodes and they present fewer handling problems, in launching and jointing for example. However the fabrication of single-mode fibres has been rendered comparatively simple\(^2\) by the new homogeneous chemical vapour deposition technique. Furthermore it may be deduced from recent theoretical work\(^3\) that the "microbending" loss can be made small by restricting the core diameter. Together with the longer lifetimes now being reported for semiconductor lasers, these factors indicate that single-mode fibres may become increasingly important in the future.
With any type of fibre, two of the fundamental parameters are the core diameter $2a$ and the difference in refractive index $\Delta n$ between that of the core $n_1$ and the cladding $n_2$ from which the normalised frequency $V$ at the (free-space) wavelength of operation $\lambda$ may be obtained.

Thus

$$\Delta n = n_1 - n_2$$

and

$$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{\frac{1}{2}} = \frac{2\pi a}{\lambda}(2n_1\Delta n)^{\frac{1}{2}}$$

In multimode fibres the core diameter, which commonly lies in the range 40 to 100 $\mu$m, can be measured by conventional optical techniques while $\Delta n$ can be determined from measurements of the numerical aperture or of the refractive index profile. However with single-mode fibres, because the core diameter is comparable with a wavelength, the effects of diffraction render such measurements more difficult, particularly at the smaller $V$ values. Sometimes the refractive indices of core and cladding may be accurately known but in general this is not the case, particularly with fibres produced by the C.V.D. technique. A method of determining $a$ and $\Delta n$ is therefore required which is simple to apply and which ideally can be used to assess fibres immediately after drawing.

No rapid and satisfactory method presently exists. One possible technique is to detect the onset of higher mode propagation in the output far-field radiation pattern from the fibre when the exciting wavelength is varied. In this way the wavelength of the second mode cut-off ($V=2.4$) is found, and assuming that the dispersive properties of the glasses in the core and cladding are similar, an extrapolation of the $V$ value to other wavelengths can be made. However, the method is an insensitive one because the wavelength at which a higher mode appears in the output pattern is somewhat indeterminate. A mode becomes extremely lossy as it nears its cut-off point, since the mode volume becomes large and guidance is weak. Thus no clear cut-off wavelength is observed and the measurements found to be very sensitive to slight bends and pressure applied to the fibre, as these cause premature radiation of the higher mode. For example we have found that some fibres operating at a wavelength such that $V = 2.8$, i.e. well into the overmoded region, have a radiation pattern indistinguishable from that of a single-mode fibre after a length of about 1 metre. Our conclusion from this and other experiments is, therefore, that the
mode cut-off technique is not sufficiently accurate and, moreover, requires a laser source which is tunable over a wide range of wavelengths. Furthermore it does not yield the core diameter or refractive index difference directly.

An alternative method is presented here of determining \( a \) and \( \Delta n \) from a simple measurement of the far-field pattern at a single wavelength.

2. **Far-field Radiation Pattern of HE\(_{11}\) Mode**

2.1 Angular width at half-maximum intensity

As with multimode fibres it is clear that the far-field radiation pattern of the HE\(_{11}\) mode is a function of both \( a \) and \( \Delta n \) and moreover can be easily observed experimentally.\(^4\) Our first step, therefore, is to calculate this field distribution. It is assumed that \( \Delta n < n_1 \) which is normally the case in practice and greatly simplifies the analysis. The far-field distribution \( \psi(r, \Theta, \Phi) \) may be obtained from the Fraunhofer diffraction equation\(^5\) expressed in terms of the spherical co-ordinates \((r, \Theta, \Phi)\) as defined in Fig.1, namely

\[
\psi(r, \Theta, \Phi) \approx \frac{jk}{\lambda r} \int_0^{2\pi} \int_0^\infty \psi_0(\rho, \Phi_0) \exp \left[ jk \rho \sin \Theta \cos (\Phi - \Phi_0) \right] \rho \, d\rho \, d\Phi_0
\]

... (3)

where \( k = 2\pi/\lambda \)

and \( \psi_0 \) expresses the near-field at the output end of the fibre where the corresponding co-ordinates are \((\rho, \Phi_0)\).

By using the approximate field equations derived by Snyder\(^6\) for the HE\(_{11}\) mode in structures having \( \Delta n < n_1 \) the normalized far-field distribution may be derived as:

\[
|\psi|^2 = \left[ \frac{U^2 W^2}{U^2 - \alpha^2} \left( \frac{J_0(\alpha)}{J_1(\alpha)} \right)^2 \right] \quad \text{for} \quad U \neq \alpha \quad \ldots (4)
\]

\[
= \left[ \frac{U^2 W^2}{2V^2} \frac{1}{UJ_1(U)} \left( J_0^2(\alpha) + J_1^2(\alpha) \right) \right]^2 \quad \text{for} \quad U = \alpha
\]

where \( V^2 = U^2 + W^2 \) \ldots (5)

and \( U, W \) are the arguments of the Bessel and modified Hankel functions. We have normalised the radiation angle in the form

\[
\alpha = ka \sin \Theta \quad \ldots (6)
\]
A convenient parameter to measure experimentally is the angle $\theta_h$ at which the far-field intensity has fallen to one-half that at the central maximum ($\theta = 0$). The normalized half-intensity angle is thus defined as

$$\alpha_h = ka \sin \theta_h$$ ... (7)

It may be shown from eqn.(4) that $\alpha_h$ is an unambiguous function of $V$ and the relationship is indicated by the solid line in Fig.2. Thus for values up to 10 or so $V$ may be very simply determined if $\alpha_h$ is known, assuming that for $V>2.4$ only the $HE_{11}$ mode is launched and propagated along the fibre. If the core radius $a$ can be found in some other way then measurement of the half-intensity width $\theta_h$ enables $\alpha_h$ to be obtained, so that eqn.(4), or in practice the universal curve of Fig.2, gives $V$ and hence $\Delta n$. As described in Section 3 an etching technique can sometimes be used to determine the core diameter but only with those fibres where the core and the cladding have markedly different etch rates.

2.2 Angular width of first minimum

The output end of an optical fibre forms a radiating aperture but it is not generally appreciated that as with any other form of radiating aperture, such as a microwave aerial for example, the output field pattern contains side lobes. Thus it can be shown from eqn.(4), and experimentally as in Section 3, that in addition to the main beam the far-field pattern exhibits a range of subsidiary peaks, as illustrated in Fig.3(a), at angles and relative intensities which depend on $a, a_1, a_2$ and $\lambda$. Some typical far-field radial intensity distributions are shown in Fig.3(b), in which the radiation angle has again been normalized in the form $\alpha = ka \sin \theta$, for various values of $V$. It may be further shown that the angular width $\theta_x$ to the first minimum can be used in conjunction with $\theta_h$ to obtain $V$ directly without any knowledge of $a$. Thus, like $\alpha_h$, the ratio $\sin \theta_x / \sin \theta_h$ is also an unambiguous function of $V$. The variation of this ratio, together with $\alpha_h$, is given in Fig. 4 for the range of $V$ values most likely to be encountered in practical single-mode fibres. Thus the interesting and invaluable result is obtained that the simple determination of $\theta_x$ and $\theta_h$ enables $V$ and $a$, and hence $\Delta n$ to be obtained without the need for any other measurements. In the example illustrated in Fig.4 it is assumed that the ratio $(\sin \theta_x / \sin \theta_h)$ is
found experimentally to be 5.25 indicating, using curve A, that $V = 2.14$. From curve B it can be seen that the corresponding value of $ka \sin \theta_h$ is 0.813 and from the measured value of $\theta_h$ it is possible to calculate $a$.

3. Experimental Techniques and Verification

Experiments have been carried out on a number of single-mode fibres made by the technique of homogeneous chemical vapour deposition. A piece of each fibre of about 1m length was taken and laser radiation was launched into the core by a x10 objective lens. In order to avoid the propagation of higher-order modes which, while beyond cut-off, may be weakly guided over this length, the fibre was slightly curved. Portions near the ends were immersed in a liquid having a refractive index higher than that of the cladding in order to remove any cladding modes.

The angular widths $\theta_h$ and $\theta_x$ were obtained by monitoring the far-field output pattern with an Integrated Photomatrix Ltd model 7000 scanning photodiode array. Fig. 5 shows the outputs from the array displayed on an oscilloscope under conditions of (a) low gain from which $\theta_h$ can be measured and (b) high gain, showing the positions of the minima ($\theta_x$). In Fig.5(a) the response of the photodiode array is linear and the Gaussian shape of the main beam can be seen. However in order to show up the first minima in Fig. 5(b) the gain is so high as to cause saturation and distortion at smaller angles. The values of $\theta_x$ were confirmed by taking photograph of the far-field pattern as shown in Fig.5(c).

As a check on the theory given in Section 2 the core diameter can be measured directly in two ways. Firstly the core at the end of a piece of fibre was etched away with hydrofluoric acid since phosphosilicate glass dissolves much more rapidly than pure silica. The core diameter was then measured by an optical microscope and could also be determined using a scanning electron microscope. A typical picture of an etched fibre end obtained with an SEM is shown in Fig.6 and shows the high degree of resolution which can be obtained at the edge of the core, indicating that the diameter can be measured reasonably accurately. Secondly the core diameter in the preform was measured optically as well as the outside diameter of the preform. The overall diameter of the resulting fibre was then measured after drawing so that the fibre core diameter is given by
the product of the preform core diameter and the preform/fibre outside diameter ratio. There are uncertainties in the second method due to possible diffusion between core and cladding during the drawing process so that only the values obtained by etching are used here. Nevertheless the agreement between the two methods was very good (within 2%).

Several single-mode fibres having core diameters ranging from 4 to 8.4\(\mu m\) have been tested. In the first set of measurements a Chromatix CMX4 tunable laser was used to vary the V values over a wide range. The angle \(\Theta_h\) was determined at wavelengths between 0.42 and 0.9\(\mu m\) for two fibres whose core diameters of 6.6\(\mu m\) and 8.1\(\mu m\) were obtained by etching. The results are given as the experimental points in Fig.2 and are in excellent agreement with the theory. The V values for these two fibres were found to be 1.98 and 2.78 at \(\lambda = 0.63\mu m\).

In the second set of measurements \(\Theta_h\) and \(\Theta_x\) were measured for a number of fibres. For each fibre the angular measurements were made in ten independent experiments and the repeatability was within \(\pm 2\%\). The core diameters of two of the samples were again obtained by etching and the comparison with the far-field measurements is shown in Table 1. The cores were slightly elliptical and the figures in the final column denote the lengths of the major and minor axes. The agreement between the two methods is excellent particularly since the orientation of the fibre ends for the far-field measurements was not known. For the other three samples independent diameter measurements were not made and the results obtained for \(a\) and \(\Delta n\) are given in Table 2. These fibres were all drawn from the same preform which explains the similarities in \(\Delta n\) but small changes could have occurred because of slight non-uniformities in the deposit layer or due to diffusion during fibre drawing.

It should be noted that the theory presented here assumes a uniform core refractive index, so that \(\Delta n\) is easily specified. However, it is clear from the etched end shown in Fig.6 that a degree of non-uniformity exists across the core. In particular the fibre exhibits a dip in refractive index at the core centre caused by depletion of the \(P_2O_5\) from this area during the preform collapse. The fact that the far-field measurement yields accurate values for the fibre core diameter suggests that the index non-uniformity has no great effect on the radiation pattern. Nevertheless care must be
exercised in interpreting the deduced value of $\Delta n$, as this presumably represents some mean value. It would appear that this mean value, or more accurately the $V$-number associated with it, has the usual significance in terms of the higher mode cut-off values.

4. Conclusions

It has been shown that the core diameter and refractive index difference can be obtained unambiguously from the far-field pattern of the $HE_{11}$ mode in fibres of low $V$ value, and the method is now being used in these laboratories for the routine characterisation of single-mode fibres. Independent determinations of core diameter by an etching technique are in good agreement with those found from the far-field, while further confirmation has been obtained by preform measurements and the similarity of the index difference for fibres of various diameters pulled from the same preform. The technique is particularly suited to characterisation of single mode fibres produced by the CVD method.

Acknowledgements

We are indebted to Dr. C.R. Hammond and Mr. S.R. Norman for fabricating the fibres used in the experiments, and to the Pirelli General Cable Company for the endowment of research fellowships.

References


### TABLE 1 Comparison of core diameters obtained from far-field pattern with those measured by etching

<table>
<thead>
<tr>
<th>Sample</th>
<th>sinθ_h (mean)</th>
<th>sinθ_h /sinθ_h</th>
<th>V (mean)</th>
<th>Δn</th>
<th>Core diameter (μm) obtained from:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Far-field measurements</td>
</tr>
<tr>
<td>1</td>
<td>0.0211</td>
<td>5.38</td>
<td>2.10</td>
<td>0.00107</td>
<td>7.6</td>
</tr>
<tr>
<td>2</td>
<td>0.0385</td>
<td>5.14</td>
<td>2.18</td>
<td>0.00349</td>
<td>4.3</td>
</tr>
</tbody>
</table>

### TABLE 2 Values of V, refractive index difference and core diameter determined from the far-field pattern

<table>
<thead>
<tr>
<th>Sample</th>
<th>sinθ_h (mean)</th>
<th>sinθ_h /sinθ_h</th>
<th>V (mean)</th>
<th>Δn</th>
<th>Core diameter (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.0205</td>
<td>6.05</td>
<td>1.91</td>
<td>0.00108</td>
<td>6.9</td>
</tr>
<tr>
<td>4</td>
<td>0.0220</td>
<td>4.79</td>
<td>2.32</td>
<td>0.00112</td>
<td>8.2</td>
</tr>
<tr>
<td>5</td>
<td>0.0231</td>
<td>4.50</td>
<td>2.48</td>
<td>0.00120</td>
<td>8.4</td>
</tr>
</tbody>
</table>
**Figure 1**
Spherical co-ordinate system used in calculations of far-field pattern.

**Figure 2**
Variation of normalised half-maximum angle with $V$. The solid line is calculated from Eqn. (4) while the points were measured with fibres of core diameter 6.6μm (o) and 8.1μm (●) over the wavelength range 0.42 to 0.9μm.
Figure 3
A) Calculated angular radiation pattern for fibre of core diameter 2 \( \mu \)m and \( V = 2.4 \). The figure shows the half intensity angle \( \theta_h \) and the first minimum angle \( \theta_m \).

B) Intensity distribution in the far-field as a function of the normalized radiation angle for various \( V \) numbers.
FIGURE 4
VARIATION OF HALF-INTENSITY ANGLE $\alpha_h$ AND THE RATIO $(\sin \theta_h / \sin \theta_x)$ WITH $V$. 
FIG. 5 OUTPUT FROM SCANNING PHOTODIODE ARRAY

a) UNDER LOW-GAIN CONDITIONS
b) UNDER HIGH-GAIN CONDITIONS, NOTE THAT THE CENTRAL PEAK IS SATURATED
c) PHOTOGRAPH OF THE FAR-FIELD INTENSITY PATTERN
Figure 6
Photograph obtained with a scanning electron microscope of an etched fibre end. The core diameter is approximately 7.5μm.
SECTION III

GRADED-INDEX FIBRES
CHAPTER 9

TUNNELLING LEAKY MODES IN GRADED-INDEX FIBRES

9.1 Introduction

The final Section of this thesis is concerned with some of the properties of graded-index fibres and particularly with the propagation of leaky modes in parabolic-index fibres. Chronologically the work stems from observations made during the development of a novel and convenient means of measuring the fibre refractive-index profile. However, it will not be presented in this order, since it may be more readily followed if the basic theory is given first. Thus the experiments which initiated the project will not be described until the following chapter. It may nevertheless be instructive to recall that the ultimate objective of the theory is a method for determining the index profile of a fibre by observation of the intensity distribution across its end face (the 'near field'). Currently this remains the central objective, although the development of the theory has served to highlight the importance of leaky modes in graded fibres. In particular, it was not previously recognised that modes of this type can have a considerable effect on measurements of fibre attenuation, pulse dispersion and splice loss.

The work to be described was performed in conjunction with M.J.Adams and F.M.E.Sladen.

9.2 The Acceptance Angle of Leaky Modes in Graded-Index Fibres

(P9.1)

The existence of tunnelling leaky modes in step-index fibres and the effect they have on propagation characteristics is well-known from the extensive work of A.W.Snyder (see for example, reference 1).

To use ray terminology, he has shown that in a fibre with a homogeneous core a tunnelling leaky ray is one of the class of skew rays which are allowed by a purely geometrical optics analysis to propagate at an angle greater than the meridionally-defined numerical aperture. Whereas these rays do indeed propagate, in reality they do so with a degree of radiation loss. In mode terminology, a tunnelling leaky mode is one which is below cut-off, but which still maintains a large proportion of
its energy trapped within the core. Although the mode must radiate since it is below cut-off, the loss may be relatively small and the mode may therefore persist for considerable distances. The term 'tunnelling leaky mode' was coined in analogy with quantum mechanical tunnelling. The electromagnetic wave 'tunnels' from the core/cladding interface (in a step-index fibre) to emerge at some radius in the cladding, from which it radiates. The transmitted wave appears to originate from this 'caustic', having 'tunnelled' from the core through a radially evanescent field region. Tunnelling arises because the wave is unable to follow the curved interface without its phase-velocity exceeding the velocity of light in the cladding. The point at which this occurs is the radiation caustic.

Although leaky modes in step-index fibres were well characterized, it was not generally appreciated that they also exist in graded-index fibres. In retrospect, however, there seems little reason to believe that they should not. Nevertheless, nothing was known about their number, attenuation, or propagation characteristics. Our discovery of leaky modes in a parabolic-index fibre resulted from a persistent error which occurred in the near-field scanning technique for index-profile determination (Chapter 10). This led us to develop a theory which allowed a description of the modes. The analysis which evolved is given in publication P9.1. It extends the tunnelling mode concepts, which previously had been applied only to step-index fibres, to include fibres having an arbitrary circularly-symmetric index profile.

The analysis has its foundations in the now classic work of Gloge and Marcatili in which the WKB method is employed to predict the performance of fully-excited graded-index fibres. Their theory is based on bound mode propagation only and assumes that once a mode is below cut-off it may be ignored. This is equivalent to the assumption that no ray may be accepted by the fibre once it has an angle to the axis greater than the local numerical aperture. Since leaky modes are disregarded, their results underestimate the power carried by the waveguide for all but very long fibre lengths.

Our analysis represents an analytical extension of their work to include modes below cut-off, since these will almost invariably be present. By using a transition from modes to rays, we have
defined an angular region in which leaky rays may be found in any index profile. This concept of an angular acceptance region is a particularly useful one in understanding the characteristics of leaky-mode propagation. The main conclusions of the analysis may be summarised as follows.

i) Tunnelling leaky modes may propagate in all circularly-symmetric optical fibres. Their number and the power they carry depends on the profile; in general both are greatest for the step-index fibre.

ii) A leaky ray in a graded-index fibre may be defined as one which propagates at an angle greater than the local numerical aperture and further, is one which would be predicted by geometrical optics to be trapped. In contrast to the step-index fibre, the local numerical aperture in a graded-index fibre depends upon the radial position on the fibre end-face and varies from a maximum at the centre to zero at the core/cladding interface. Note that the term 'graded-index' is used here to mean a profile having a maximum on axis and a smooth gradation to a constant lower value in the cladding.

iii) The angular region in which leaky rays may be launched lies outside the local acceptance cone of the bound rays. Whereas the acceptance cone for bound rays is circular, that of the leaky rays is elliptical, having a major axis at right angles to the radial direction. Reference to publication P11.1, Chapter 11, will clarify the geometry of the various acceptance regions for the special case of a parabolic-index fibre.

iv) In a parabolic-index fibre the acceptance region for leaky rays always lies within the angular limits of the meridionally-defined numerical aperture.

This last result is perhaps the most significant of all, since it has widespread implications for measurements on parabolic-index fibres, as follows. In the step-index fibre little attention need be paid to tunnelling modes, as they are all found outside the fibre numerical aperture. They are therefore rarely excited, since it is usual to arrange the source so that its angular aperture corresponds to that of the fibre. This is not the case in a parabolic-index fibre. Any multimode source arranged to fill the meridional numerical aperture will excite some, if not all, of the tunnelling leaky modes, and they must therefore be taken into account in fibre measurements.
9.3 Discussion

We may conclude that tunnelling leaky modes are of considerably greater importance in graded-index fibres than in step-index fibres, simply because it is difficult to avoid launching them. For example, the conventional excitation source for spectral attenuation measurement is a tungsten-halogen lamp, focussed through a lens of similar numerical aperture to that of the fibre. If the radiation completely fills the end-face, it will efficiently excite the complete set of leaky modes in a parabolic-index fibre, but none at all in a step-index fibre. Since the attenuation is determined by comparing the output from a long length of fibre to that from a short length, it is hardly surprising that researchers had previously observed with some puzzlement that a graded-index fibre always appears to possess a decibel or so higher loss than a step-index fibre. The measurement, of course, includes the loss of the leaky modes since they are present and are recorded in the output from the short length, but have largely radiated away, and are therefore not recorded, in the long length.

A further indication of the effect of leaky mode propagation is given in P9.1, Fig.3. The curves show the near-field intensity distribution, for various index profiles, found by summing the power contained in both bound and leaky modes. The near-field exhibits a substantial departure from that previously predicted\(^2\) using the assumption that only bound modes propagate in the fibre. This has a profound effect on the near-field scanning technique for index-profile determination (Chapter 10).

In addition to the errors in attenuation measurements, it is readily seen that other fibre measurements may be equally affected. This is particularly true of measurements which use short lengths of fibre, such as the determination of splice loss (Chapter II). Another measurement which will possibly be influenced is pulse dispersion; this is treated in the following section.

9.4 The Effect of Leaky-mode Propagation on Pulse Dispersion
(P9.2)

Further properties of tunnelling leaky modes in graded-index fibres are given in publication P9.2. The paper deals largely with the propagation delay of the leaky modes and the effect
they have on the pulse dispersion. It is also concerned, however, with the number of leaky modes which exist in a parabolic-index fibre and delineates their range of propagation constants.

Perhaps the most significant result given in P9.2 is that there are $V^2/12$ leaky modes in a parabolic-index fibre, where $V$ is the normalised frequency. Since the bound modes total $V^2/4$, 25% of the power is launched into the leaky modes if all modes are equally excited. This compares with 50% for a step-index fibre. The above result has subsequently been confirmed by several authors (see references 3 and 4 for example). Although less than for a step-index fibre, the power carried by leaky modes is by no means insignificant in a parabolic-index fibre, and can be expected to strongly influence measurements.

A further conclusion which may be drawn from P9.2 is that the transit times of leaky modes are always greater than that for bound modes in fibres having a profile parameter $\alpha$ greater than $2-2\Delta$, where $\Delta$ is the maximum relative index difference (see reference 2 for definitions of $\alpha$ and $\Delta$). Thus the presence of leaky modes at the output of these fibres will add a 'tail' to the pulse. It is however possible to eliminate the increase in pulse width by choosing a profile having $\alpha = 2-4\Delta$.

Profiles having $\alpha$ between $2-2\Delta$ and $2-4\Delta$ are unique in that they equalise the transit times of the leaky modes to lie partly or wholly within the range observed for the bound modes. Thus the bound mode transit times and those of the leaky modes overlap to a degree which depends on the choice of profile. When $\alpha = 2-4\Delta$ the overlap is complete and all leaky modes arrive within the same time period as the bound modes. The pulse width is therefore a minimum. A further decrease in $\alpha$ will result in some of the leaky modes arriving before the fastest bound mode, and a degradation of the leading edge of the pulse will result.

9.5 Leaky, Bound and Refracted Modes

The two publications presented in this chapter assume that leaky modes are a clearly identifiable class of mode and that they propagate unattenuated. The assumption is necessary at this stage in order to describe more clearly the properties of the modes. A worst-case estimate of the effects of leaky modes results since, in reality, the loss of the leaky modes is finite. So far we have assumed that all bound modes and leaky modes find
their way to the fibre output, while modes launched outside the angular region delineating bound and leaky modes (refracted modes) are immediately lost.

In practice the distinction between the three classes of modes is not so clear. Theoretically we define a bound mode as one with oscillating field behaviour within the core of the waveguide and an exponential decay (evanescent) behaviour everywhere else. It thus possesses no radially-directed propagation constant within the cladding and has no radiation loss. A refracted mode has exponentially-growing, oscillatory radial field behaviour throughout the cladding and is therefore not localised, i.e. trapped, within the core. It radiates from the core/cladding interface and is lost within a very short length. A leaky mode, on the other hand, lies between these two extremes; it possesses oscillatory fields within the core and is bounded by evanescent field conditions within the cladding, just as is a trapped mode. However, at some radial distance from the core/cladding interface the field becomes radially periodic once more. The mode radiates from this point, which is known as the outer radiation caustic.

To summarise, a bound mode does not radiate; it may be thought of as having an outer radiation caustic at infinity, where the field has decayed to zero. A refracted mode radiates from the core/cladding interface and therefore has a caustic equal to the core radius. Intermediate between these two lies the leaky mode which, depending on the mode under consideration, radiates from some radius between the core/cladding interface and infinity.

The distinctions between the modes, although clear in theory, are somewhat artificial in practice, particularly when the added complication of a finite cladding width is taken into account. The loss of the set of leaky modes varies between zero for the least leaky, to virtually infinite for the most leaky. They may therefore be regarded as having characteristics which grade smoothly between those of bound and those of refracted modes. Thus, whereas it is not accurate to assume that the number of modes propagating in the fibre is clearly delimited by the cut-off of the highest mode, as in reference 2, it is equally inaccurate to delimit the number of modes by the synthetic boundary between leaky and refracted modes, as we have done. Nevertheless, publications P9.1 and P9.2 serve to draw
attention to the existence of leaky modes in graded-index fibres and to define the circumstances under which they may be launched. The following chapter completes the picture by including the attenuation of leaky modes, thereby giving an indication of their practical significance.

References
Leaky Rays on Optical Fibres of Arbitrary (Circularly Symmetric) Index Profiles

**Indexing terms:** Fiber optics, Geometrical optics, Optical waveguides

The local plane-wave decomposition approach used to analyze optical fibres of arbitrary refractive-index profiles has been extended to include the case of so-called 'leaky' rays. The results thus obtained for acceptance angle represent a generalization of results derived previously for simple forms of the profile by geometrical-optics methods.

**Introduction:** Recent publications\(^{1,2}\) derive an expression for the acceptance angle of a graded-index fibre by a geometrical-optics technique. The results so obtained indicate that the acceptance angle at a point on the fibre input face varies not only with position, but also with the projected angle of incidence of the ray (\(\phi\) in Fig. 1). This is contrary to the approach developed by Gloga and Marcatili,\(^{3,4}\) which predicts that the local numerical aperture is a function of radius only. The purpose of this letter is to extend the plane-wave decomposition method to include weakly leaky or tunnelling rays\(^{5,6}\) and to show that these rays account for this discrepancy. In addition, we show that tunnelling rays exist in the general class of circularly symmetric guiding index profiles, and that their presence strongly influences the observed near-field power distribution in short fibres excited by incoherent sources.

![Fig. 1 Local wave-vector diagram for ray incident on fibre face at radius \(r_0\) and with angle \(\beta\)](image)

![Fig. 2 Squared magnitude of components of eqn. 2](image)

\(\text{Fig. 1} \quad \text{Local wave-vector diagram for ray incident on fibre face at radius } r_0 \text{ and with angle } \beta\)

\(\text{Fig. 2} \quad \text{Squared magnitude of components of eqn. 2}
\)

**Theory:** The local plane-wave decomposition\(^{3,4}\) is shown in Fig. 1, where the angles \(\alpha_0, \beta, \gamma_0\) are those appropriate for a ray entering the fibre at radius \(r_0\). The relationships between the launching conditions (given by \(r_0, \alpha_0, \gamma_0\)) and the wave-optical decomposition components are\(^{5}\)

\[
\cos \gamma_0 = \frac{\beta}{k n(r_0)} \quad \text{and} \quad \cos \alpha_0 = \frac{v}{r_0 k n(r_0)} \quad \text{(1)}
\]

where \(k\) is the wavenumber, \(v\) is the azimuthal wavenumber, \(n(r_0)\) is the refractive index and \(\beta\) is the propagation constant. The radial component of the wave vector at \(r_0\) is given by

\[
q(r_0) = \left[ k^2 n^2(r_0) - \beta^2 - \frac{v^2}{r_0^2} \right]^{1/2} \quad \text{(2)}
\]

In Fig. 2, the squared magnitude of the various components of eqn. 2 are shown as a function of radius \(r_0\). The Figure is drawn for a fibre core of arbitrary index profile and radius \(a\), surrounded by a cladding of constant index \(n_2\), which may be air. The case shown is that of a mode just below cutoff, i.e. \(\beta < \beta_{ns}\). It can be seen that this mode has

(a) a region of radial periodicity within the core, representing bound power

(b) a region of evanescent field within the cladding

(c) a further region having an oscillatory field solution within the cladding, representing radiated power.

The mode may therefore be identified as a leaky tunnelling mode. The limiting values for the propagation constant between which these modes may exist are given from Fig. 2 by

\[
k^2 n_2^2 - \frac{v^2}{a^2} < \beta^2 < k^2 n_0^2 \quad \text{(3)}
\]

Inserting eqn. 1 into eqn. 3 yields

\[
n_2^2 - (r_0/\alpha)^2 n_0^2 \cos^2 \gamma_0 < n_0^2 \cos^2 \gamma_0 < n_0^2 \quad \text{(4)}
\]

![Fig. 3 Near-field intensity plots](image)

**Fig. 3 Near-field intensity plots**

Calculated from eqn. 13 for class of profiles given by

\[
\frac{1}{n^2} - \frac{1}{(r_0/\alpha)^2} = \frac{1}{n^2} \text{ for } n = 2, 4, 6, \ldots
\]

Also shown (broken line) is the index-profile plot for \(n = 2\).

The physical significance of this result may be seen more readily by expressing it in terms of the angle of incidence \(\theta\) and the projected angle \(\phi\) of a ray incident on the fibre face (Fig. 1). We see that for an external medium of unity index,

\[
\sin \theta = n(r_0) \sin \gamma_0 \quad \text{(5)}
\]

\[
\cos \phi = \cos \gamma_0 \sin \phi \quad \text{(6)}
\]

Inserting eqns. 5 and 6 into eqn. 4 gives the required expression:

\[
n_2^2 - n_0^2 \cos^2 \phi \geq \sin^2 \theta > n_0^2 - n_2^2 \quad \text{(7)}
\]

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Eqn. 7 defines the angular region in which we find leaky tunneling rays at a given radius on the endface of a fibre with an arbitrary circularly symmetric index profile. It may be seen that one requirement is that the angle of incidence is greater than the local acceptance angle defined in Reference 3. In addition, I must be less than some value dependent on both the radius \( r_0 \) and the projected angle \( \phi \). As expected, no leaky-ray region exists for \( \phi = \pi/2 \), as this defines a meridional ray. 

**Solutions for simple profiles:** By using only the l.h.s. of eqn. 7, we define a local acceptance angle that includes both the bound rays, given by \( \sin^2 \theta_{\text{acc}} = r_0^{2n^2} - r_0^{2n^2} \), and the tunneling rays. It is then possible to verify the result by geometrical optics. Solutions to the following simple cases are already available:

(a) **Step-index fibre:** Eqn. 7 is equivalent to Snyder's conditions\(^4\) for leaky rays, if expressed in the angles \( \theta_a, \theta_b, \theta_t \) (see Reference 6 for definitions):

\[
\frac{\pi}{2} > \theta_a > \theta_b \quad \frac{\pi}{2} > \theta_t > \frac{\pi}{2} - \theta_b
\]

In addition, eqn. 7 gives the maximum acceptance angle at radius \( r_0 \) as

\[
\sin^2 \theta_{\text{acc}} = n_1^2 - n_2^2 \left[ 1 + \left( \frac{(r_0/a) \sin \delta}{1 + (r_0/a) \cos \delta} \right)^2 \right]
\]

where \( n_1 \) is the core refractive index and \( \delta \) is the angle defined by Matsumura\(^1\) as

\[
\delta = \cos^{-1} \left( -\frac{r_0}{a} \cos \phi \right) + \phi
\]

This is identical to the result obtained by \( u \) geometric optics analysis of skew-ray propagation,\(^1,4\) confirming that, in a step-index fibre, leaky tunneling rays are those rays travelling at an angle greater than the meridionally defined numerical aperture, although predicted by geometrical optics to be trapped.

(b) **Parabolic-index fibre:**

\[
n^2(r_0) = \begin{cases} \eta^2(1 - 2\Delta r_0/a^2) & r_0 \leq a \\ \eta^2(1 - 2\Delta) & r_0 > a \end{cases}
\]

where \( \eta(r_0) \) is the refractive index at the core centre and \( \Delta \) is the maximum refractive-index difference. Eqn. 7 yields

\[
\sin^2 \theta_{\text{acc}} = 2\Delta \eta^2(1 - r_0^2/a^2) \left[ 1 - (r_0/a) \cos \phi \right]
\]

This relationship was recently derived by Matsumura,\(^1\) using a geometrical technique.

(c) **Fourth-order index profile:** Ikeda's skew and meridional acceptance angle (eqns. 39 and 40 of Reference 2) may be similarly derived by setting \( \phi = 0 \) and \( \phi = \pi/2 \), respectively.

**Near-field intensity distribution:** To determine the effect of leaky rays on fibre propagation, the near-field intensity distribution \( P(r) \) may be calculated by the method of Reference 3, suitably corrected for the increased local acceptance angle. Assuming an incoherent source (all modes equally excited), it can be shown that

\[
P(r)/P(0) = \frac{n_0^2(r_0 - r_0)^2}{n_0^2(1 - r_0)^2} \left[ 1 \right] \sqrt{1 - (r_0)^2}
\]

where \( P(0) \) is the intensity at the fibre centre.

Fig. 3 shows the near-field intensity plots for a range of index profiles and clearly indicates a substantial departure from the plots of Reference 3. The most obvious difference occurs for the step-index fibre, plotted here for two different meridional numerical apertures. The dependence on numerical aperture is a result of truncation of eqn. 7 at some value of \( r_0 \) to ensure that the local acceptance angle does not exceed \( \pi/2 \).

**Conclusions:** By extending the plane-wave decomposition technique, a simple generalised expression has been derived defining in angular terms the region in which tunneling leaky rays are found. This region varies with both index profile and position on the fibre face, and accounts for the somewhat larger local acceptance angles found in References 1 and 2.

The inclusion of the additional angle defined by the leaky-ray region causes a large deviation from the expected near-field intensity distribution. The close resemblance between the field profile and the near-field intensity is no longer found. Although not shown here, a similar departure occurs in the far field. However, these results assume all rays propagate unattenuated, and whereas experiments show this to be substantially true for 1 m lengths of graded-index fibre, account must be taken of the slow radiation loss of tunneling rays in longer lengths.

The full significance of leaky tunneling rays on graded-index fibres has yet to be determined. In step-index fibres, it is normally possible to avoid the excitation of tunneling rays by simply arranging the launching conditions to just fill the numerical aperture of the fibre. This is not so for the parabolic-index fibre, as all leaky tunneling rays are contained within the angular limits of the meridionally defined numerical aperture. Thus an incoherent source excites all tunneling modes, even if arranged to only just fill this aperture.

At least for the step-index fibre, it has been shown\(^5\) that some of the tunneling modes may persist for several kilometres if excited by the source. It seems likely that this will also be so for the graded-index fibre. Since it is difficult to avoid their excitation, modes of this type may therefore have a significant effect on the measured loss and mode dispersion in graded-index fibres.

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**References**

7. **Kaplan, M. S.:** 'Fiber optics' (Academic Press, 1967)

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MODE TRANSIT TIMES IN NEAR-PARABOLIC-INDEX OPTICAL FIBRES

Calculations are presented of the transit times of guided and tunneling rays in graded-index optical fibres. It is shown that 25% of the power launched by an incoherent source into a parabolic-index fibre is carried by leaky rays, which may increase the r.m.s. width of the impulse response by up to 41%. The profile which most effectively equalizes the combined transit times of both guided and tunneling rays is given by a = 2-43.

Introduction: In a recent publication, it was shown that tunneling leaky rays are present on all multimode guiding structures having circular symmetry, and the plane-wave decomposition technique was used to delineate the angular region in which they exist. For a near-parabolic-index fibre, this angular region was found to be within the meridionally defined numerical aperture. Thus, in contrast to the step-index fibre, leaky rays are excited not only by an incoherent source, but also by other extended sources which may fill the numerical aperture, such as a broad-contact semiconductor laser. Consequently, these rays assume some importance in graded-index fibres, and may be expected to have a significant effect on both attenuation and pulse-dispersion measurements.

We show here that for a parabolic-index fibre 25% of the power launched by an incoherent source is contained within the leaky modes, and that if all these modes propagate unattenuated the r.m.s. width of the impulse response may be increased by up to 41%. It is further shown that the index profile may be adjusted to equalize the transit times of both leaky and guided modes, leading to a somewhat different result from that obtained by equalizing the transit times of guided modes alone.1

Number of leaky modes: As a first step towards determining the pulse dispersion, we may calculate the number \( m(f) \) of all modes having a propagation constant \( \beta \) greater than a certain specified value, including those below cutoff. Modes below cutoff have \( k^2 \leq n_e(f) k^2(1 - 2\Delta) \), where \( k \) is the wavevector, \( \Delta \) the normalised maximum refractive-index difference and \( n(0) \) the index at the core centre. In this case, the modes are of two types: tunneling and refracted.2 The total number summed over all modes may be determined by the plane-wave decomposition technique,1,2 and is given by the analytic continuation of Gloge and Marcadili's expression below cutoff. The result is shown by the upper curve in Fig. 1. The lower curve is obtained by summing only the guided and tunneling modes, and represents the subset of modes assumed here to be propagating unattenuated. The figure is drawn for a parabolic-index fibre for which \( a = 2 \), where \( a \) is defined in Reference 3.

Whereas refracted modes may have \( \beta > 0 \), there is a critical value of \( \beta \) below which no further tunneling modes are found. The limit occurs when the \( \beta^2(1 - 2\Delta) \) curve is tangential to the \( k^2(1 - 2\Delta) \) curve (Fig. 2, Reference 1), so that there is no longer an oscillatory region within the core. For this condition to apply it can be shown that

\[
\beta^2 \geq n_e(f) k^2(1 - 2\Delta) \quad (1)
\]

It is clear from Fig. 1 that there are \( \nu \) guided modes in a parabolic-index fibre, since this number has a propagating constant greater than the bound-mode cutoff value of \( \beta^2 \geq n_e(f) k^2(1 - 2\Delta) \) (\( \nu \) is the normalised frequency). Similarly, the number of guided plus tunneling modes is \( \nu + 1 \), these modes having \( \beta^2 \geq n_e(f) k^2(1 - 2\Delta) \). Thus 25% of the power launched by an incoherent source will be contained within the tunneling modes, since the ratio of the number of guided to tunneling modes is \( 3 : 1 \).

An alternative method of obtaining the above result is to integrate the near-field intensity profile shown in Fig. 3 of Reference 1 over the radius \( r \), from which the ratio of the number of bound modes to leaky modes may be calculated. We note in passing that Snyder's result2 for the total number of tunneling modes in a step-index fibre may be derived by either method.

Differential mode delays: If, for the present, we ignore the attenuation of leaky tunneling modes, we may obtain an upper limit for the dispersion effects on near-parabolic fibres. Following the method of Reference 3, we differentiate the \( m(f) \) curve shown in Fig. 1 (using the curve for all modes) to obtain the dispersion \( d\beta/df \).

---

Fig. 1 Number of modes \( m(f) \) in parabolic-index fibre having propagation constant \( \beta \) greater than value specified on abscissa. Below the bound-mode cutoff the number of modes is equal to the number of modes having some bound energy (lower curve). The tunneling modes exist below \( \beta^2(n_e(f) k^2) = 1 - 43 \).

Fig. 2 Pulse dispersion as function of index-profile parameter \( s \) for fibres having numerical apertures of 0·1, 0·2, and 0·3. For a given fibre, the dispersion is determined either by the broken line or by the limits of the shaded regions.

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Fig. 2 shows the calculated pulse dispersion, as a function of the index-profile parameter $n$. For a given numerical aperture, the shaded area represents a region of allowed pulse dispersion, the actual pulsewidth depending on the power remaining in the leaky modes at the fibre output. Each shaded region is bounded by the calculated curves for guided rays (lower bound) and for leaky-plus-guided rays (upper bound). Outside the shaded regions the pulsewidth is unaffected by the presence of leaky rays, and is given unambiguously by the broken curve. It should be noted that the pulse dispersion referred to here is the total spread in transit times between the fastest and slowest modes, and represents an upper limit on the pulsewidth.

For a near-parabolic-index variation in the form of a power law, we assume

$$\pi = 2 - K\Delta$$

It has been shown that the optimum choice of $K$ to minimize the group-delay differences between guided modes is given by $K = 2$, resulting in a dispersion per unit length of $n(0)\Delta^2/8c$. However, Fig. 2 shows that this choice of profile is no longer optimal if all tunneling modes are present. In this case, the pulse width may be increased by $9n(0)\Delta^2/8c$. A new optimum value of $K = 4$ is now appropriate and the maximum pulse dispersion is reduced to $n(0)\Delta^2/2c$. We note that this result has been obtained by Geckeler for helical rays only.

![Diagram](image)

**Fig. 3** Impulse response per unit length for index profiles described by $\pi = 2, 2-2\Delta$ and $2-4\Delta$

The solid lines show the pulse dispersion with both bound and tunneling modes present and the broken lines show the effect of bound modes only.

**Impulse response:** Having obtained the number of tunneling modes and the transit times for all modes below cutoff, it remains to determine the impulse response, assuming that the guided and tunneling modes propagate unattenuated. The results are shown in Fig. 3 for $K$ of 0, 2 and 4 in eqn. 2, corresponding, respectively, to the perfect parabolic index, the optimised distribution for guided modes and the optimised distribution for guided-plus-tunneling modes. It is seen that for $K = 0$ and 2 leaky rays have the effect of adding a tail at the end of the pulse, since their transit times are all greater than that of the slowest bound mode. For the perfect parabola the r.m.s. width of the response is increased by 41% from $0.5n(0)\Delta^2/8c$ to $0.76n(0)\Delta^2/8c$ when all these rays are present. For the new optimum condition $K = 4$, the effect of leaky rays is to increase the amplitude of the response, rather than the overall width, since this choice of profile has the effect of adjusting the transit times of the leaky rays to lie within the normal range of those of the bound rays. Note that it is this effect which gives the unambiguous pulse dispersion shown by the broken line in Fig. 2. The complete overlap of the transit times of both leaky and bound rays is only possible in fibres with close-to-parabolic index profiles.

In conclusion, it must be reiterated that the results herein ignore the attenuation of leaky tunneling modes, so that they represent a worst-case estimate. However, there are indications that these modes may persist for long distances and carry a significant proportion of the power. In addition, Simard has shown that many of the modes treated here as leaky no longer radiate if the fibre has a cladding of finite thickness. In this case they may be expected to have low loss and, consequently, to have a significant effect on pulse broadening in long fibres.

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**References**

6. STEWART, W. A.: 'A new technique for determining the $V$ values and refractive index profiles of optical fibres', Conference on optical fiber transmission, Williamsburg, USA, 1973
CHAPTER 10
THE NEAR-FIELD SCANNING TECHNIQUE FOR INDEX-PROFILE DETERMINATION

10.1 Measurement of Index Profiles (P10.1)

The great strength of the homogeneous CVD technique is its ability to produce fibres having virtually any form of refractive-index profile. Thus, as one of the originators of the production process we were faced with the problem of determining the index profiles of the wide variety of fibres that could thereby be produced. Rapid and accurate measurements are required to provide information on the conformity, or otherwise, of the index profile to the ideal near-parabolic form. Existing methods were few and suffered from being either inaccurate or tedious. Of the available methods, the most commonly used is the interference technique, which requires a thin cross-sectional slice to be cut from the fibre. The slice is polished and placed in an interference microscope. Concentric-ring interference fringes are generated by the difference in optical path lengths seen by the light as it traverses regions of varying refractive index. The spacing of the fringes gives a measure of the index profile.

Apart from the inconvenience and difficulty of polishing the slice, the method suffers from several disadvantages. The most serious of these is caused by the curvature of the image field which results from the focussing action of the sample. The slice is effectively a tiny lens having a focal length comparable to its thickness. This produces a field distortion such that the image of the fringes is not localised in a plane, but forms a spherical image surface. Since the high-power microscope objectives which are required to observe the fibre core have a limited depth of focus, it is not possible to view the whole of the image simultaneously and the complete fringe field cannot be recorded. The only way to reduce the effect is to decrease the focal length of the slice by reducing its thickness. Unfortunately a thinner slice displays fewer interference fringes across the core; a typical CVD fibre slice with a sufficiently long focal length may only exhibit 3 fringes, from which the entire index profile must be inferred. Obviously the spatial resolution available is very poor and makes the method ill-suited to observations of small-scale imperfections caused, for example, by the layer structure of CVD fibres. The technique will effectively average all small-scale imperfections in the profile.
Although the smoothing yields a more aesthetically pleasing result, the apparent perfection of the profile is often a delusion, particularly with regard to one of the most interesting features of fibres produced by CVD, the central dip. The interference method is unable to resolve the full extent of the dip in refractive index at the core centre caused by the evaporation of the $P_2O_5$ additive from the inside layer in the tube during the high-temperature collapse process. The latter has only become apparent as a result of the development of the near-field scanning technique described here.

Despite its inconvenience and poor resolution, the interference method has been extensively used in other laboratories and many examples may be found in the literature of the reassuring index profiles obtained. Nevertheless, it was largely the inconvenience of the method and not its poor resolution which led to our development of a new method, the near-field scanning (NFS) technique. The idea stems from the routine observation of the fibre end-face in a microscope, when it was noticed that if the fibre was illuminated from the far end the intensity variation across the core appeared to correlate with the index profile. In addition, Gloge and Marcatali had previously demonstrated theoretically that the intensity distribution and the index profile are identical, provided all modes are equally excited. Early experiments in our laboratories showed this to be substantially true and led eventually to a far simpler and more satisfactory means of measuring the refractive-index profile than the interference method. The experimental technique was first reported in publication P10.1, where details of the apparatus and the results obtained may be found. Briefly, a short length of fibre is illuminated from the far end by an incoherent source such as a tungsten lamp and the index profile determined by observation of the light intensity variation across the fibre output face (the near field). The following points serve to emphasise some of the features of the method.

1) We have verified that the near-field intensity distribution closely resembles the refractive-index profile and can therefore be used as a method of measurement, provided all modes are equally excited. Although it does not appear to have been fully appreciated, the latter condition is critical to the measurement. It may be achieved by using a source having
a Lambertian (cosine) angular intensity distribution, for example a light emitting diode (LED) or a tungsten lamp. If the condition is not met, the near-field distribution will modulate with fibre length. For instance, in a parabolic-index fibre the source distribution will be periodically reproduced at intervals along the fibre, since the fibre has imaging properties.

2) In common with our earlier work on step-index fibres (Chapter 4), the theory of Gloge and Marcatili does not include the effects of mode coupling or differential modal attenuation. These must therefore be avoided by measuring the near-field intensity distribution on a short length of fibre, i.e. a few metres. The theoretical assumptions impose several other conditions which must be met for the experiment to yield accurate results. They are concerned with the validity of ray optics and are similar to those outlined in Section II of this thesis.

3) The NFS method very rapidly yields the refractive index profile with a minimum of sample preparation. The fine detail which may be observed is excellent and far superior to that obtainable by other means.

4) As will be appreciated from the theory developed in the previous chapter, the presence of leaky modes in graded-index fibres is unavoidable, particularly in the short lengths of fibre that we are constrained to use for the measurement. In addition it was shown that leaky-mode propagation influences the near-field intensity distribution; consequently it no longer accurately reflects the refractive-index profile. A length-dependent error results, the magnitude of which depends on the power remaining in leaky modes at the fibre output. The inaccuracy will depend on the fibre parameters; for example, it will be relatively trivial for small-core fibres having a low V-number, in which leaky mode attenuation is high. The form of the error is such that a smoothly increasing departure from the correct value occurs with increasing radius.

5) The existence of an error which smoothly affects the general form of the profile suggests that the interference and NFS methods of profile determination are complementary. The former yields the general form of the profile, but cannot show the detail, and vice versa. Thus is is possible to exper-
mentally generate a specific correction factor for subsequent NFS measurements by comparing a single interference observation with a near-field plot. The correction obtained in this way may be used to allow for the effects of leaky modes on subsequent samples of fibre having similar, but not necessarily identical, profiles. This is perfectly satisfactory for a production test facility, where the fibre parameters would not depart greatly from the original standard fibre. In general, however, the required correction will depend on the characteristics of the fibre under test.

Ideally a calculated correction factor, which may be adjusted to the fibre under test, is required. The correction must take into account the attenuation of leaky modes in fibres having a range of possible index profiles, core diameters, numerical apertures and lengths. Only in this way is it possible to predict the residual leaky-mode contribution to the near-field. Publication P10.1 shows such a correction in use and compares the index profile obtained by the NFS technique with that obtained by the interference method. The agreement is excellent and confirms the validity of the correction employed. The following section gives details of the derivation of the correction.

10.2 Computation of the Universal Correction Factors (P10.2)

The necessity for a correction factor detracts a little from the versatility of the NFS technique. It is clear that the convenience of the method makes it particularly suited to production quality control and indeed, at present, it is the only contender for this application. As such it has been extensively adopted (see for example reference 4) regardless of the error which is known to exist. The development of a convenient form of correction would therefore render the method even more attractive. In principle, it is possible to calculate the precise index profile from a knowledge of the numerical aperture, core radius, length and near-field distribution. However, an individual computation for each fibre is highly inconvenient and makes the method unattractive. On the other hand, if the correction factors can be normalised in such a way as to require only one set of curves to describe a variety of fibres, the computation effort becomes minimal and the accuracy of the NFS technique is considerably improved.
Publication P10.2 is concerned with the calculation of the losses of leaky modes in fibres having arbitrary refractive index profiles. It outlines the evolution of a universal set of correction curves which may be used to account for the existence of leaky modes in the near-field and to convert intensity distribution measurements into accurate refractive-index profiles. The procedure to determine such a correction factor is as follows. We first calculate the attenuation coefficient of each leaky mode from a 'tunnelling coefficient', found by application of the WKB method. The accurate near-field distribution may then be obtained by summation of the power remaining in all modes after a given length of fibre. The result will lie between that of Gloge and Marcatili\textsuperscript{2}, which did not recognise the existence of leaky mode propagation, and that of publication P9.1, which assumed that all leaky modes were present unattenuated. A comparison of the true length-dependent near-field with the refractive-index profile will then indicate the magnitude of the correction which it is necessary to apply. Details of the calculation may be found in publication P10.2, together with the implications of the presence of leaky modes for attenuation measurements.

P10.2 is particularly concerned with the development of analytic approximations which will allow the normalisation of the correction factors into a convenient universal form. Although, in general, the attenuation of the leaky modes depends on the index distribution, it is clear that a correction factor which requires prior knowledge of the form of the profile being measured is highly unsatisfactory. Consequently we seek analytic approximations which remove the dependence on profile and therefore allow the correction to be applied to any fibre. The approximations used are as follows.

a) The outer turning point (caustic) of a ray propagating in the core of a graded-index fibre is assumed to be at the core/cladding interface. In practice, of course, rays may have outer caustics at any radial distance from the core centre. However, we are only interested in the least leaky modes, as these will contribute most to the near-field. In this case the assumption is a reasonable one, since these rays do indeed have caustics near the interface.

b) The distance between successive approaches of a ray to its outer caustic (the ray period) is required for the calculation...
of leaky-mode attenuation and this unfortunately depends strongly on the form of the profile. A coarse approximation must be made here, as for all but the parabolic-index fibre the ray period depends on the ray under consideration. An average value is therefore taken to cover the rather wide range of periods found in fibres described by the same numerical aperture and core diameter, but which differ in index profile. Full details of these approximations may be found in both P10.2 and in the following publication, P10.3.

10.3 Results

1) With the aid of the above analytic approximations, it is possible to calculate the required correction factors and to normalise them by a single parameter \( X \) which depends only on the fibre length, numerical aperture and core radius. Thus a single set of curves may be generated to cover a wide range of possible fibre characteristics and profiles, considerably simplifying the application of the correction.

2) The use of the curves results in a considerable improvement in the accuracy of the NFS technique, as shown by Figs. 3 and 4 of publication P10.1. Note that the fibres used in the experiments require a particularly large correction. A typical fibre produced by CVD requires a smaller correction, as it has a lower \( X \)-value. Consequently, the departure of the near-field from the index profile may be relatively small. Nevertheless, for accurate measurements a correction is required.

3) The necessity for a correction factor cannot be avoided by the use of longer lengths of fibre, as has been suggested. The length required would be several hundred metres and other propagation effects would then dominate the results.

4) The analysis in publication P10.2 also includes the length dependence of the total power remaining in leaky modes. It may be seen that the power is not negligible after a length of one kilometre for a typical parabolic-index fibre. This has particular significance for the measurement of the fibre attenuation, as outlined earlier. For example, if an attenuation measurement is carried out with a Lambertian source on a 100m fibre, using a 50cm reference length, the presence of leaky modes leads to an apparent excess loss of 1.3dB/km.
5) The ordering of the correction factors into a convenient and usable form requires the application of some rather coarse approximations. Although the validity of the approximations has been demonstrated experimentally, both in the case of a step-index and a parabolic-index fibre, it remains to verify the performance of the correction over a wider range of fibres. In particular, verification for high X-values is needed, as these are commonly encountered in CVD fibres. The following section and publication P10.3 are concerned with further computations to this end.

10.4 Numerical Computation of the Correction Factors (P10.3)

The investigation of the range of validity of the approximations (a) and (b) above takes the form of numerical computations of the correction factor for a range of profiles, described by a profile exponent g of 1.5, 2, 3, 4 and \( \infty \). The analytic approximations made previously are avoided by using the more accurate form of the tunnelling coefficient \( T \) (for definitions see P10.2). The integration is performed numerically for each mode. The ray period is obtained more precisely for each ray in a given profile by utilising an approximation due to Gloge\(^5\). In this way an accurate correction is obtained which is applicable to a particular profile only. Fortunately it is found that the results may still be normalised by the X-value given previously and thus may be conveniently compared.

The results of the above exercise are given in P10.3, Fig.1, and may be summarised as follows.

1) Our previous assumption that the correction factor would be approximately independent of profile is verified. The correction required for fibres having a given X-value and a range of profiles is remarkably similar up to a normalised core radius of 0.8. Fortunately, the departures which occur at greater radius are acceptable, at least in graded-index fibres which have a small index difference at the radius where the correction-factor errors are largest. This may be seen as follows:-

We define \(^5\) the profile error \( E \) as the ratio of the maximum deviation \( dn \) from the true profile \( n(r) \), to the index difference at the core centre:
\[ E = \frac{dn}{n_o \Delta} \]  
where \[ \Delta = \frac{n_o - n_2}{n_2} \]

and \( n_o, n_2 \) are the refractive indices at the core centre and the core/cladding interface.

If \( P(0) \) and \( P(r) \) are the intensities measured at the core centre and at radius \( r \) respectively, we infer the index profile from (see PI0.1)

\[ \frac{n(r) - n_2}{n(0) - n_2} = \frac{P(r)}{P(0)} \cdot \frac{1}{C(r,z)} \]  

where \( C(r,z) \) is the correction factor. It may easily be shown that if the correction factor \( C(r,z) \) contains a percentage inaccuracy \( \delta \) then

\[ E = \frac{P(r)}{P(0)} \cdot \frac{\delta}{C(r,z)} \% \]  

For a parabolic index fibre of core radius \( a \), the error is given by

\[ E = (1 - \rho^2)\delta \]

where \( \rho = r/a \) is the normalised radius. Thus an error in correction factor of 8% at a normalised radius of 0.85 produces a profile error of only \( \approx 2\% \), which is an acceptable value. An inaccuracy of this magnitude would hardly be visible in the corrected plot.

Thus the errors inherent in the use of a generalised correction factor are tolerable in that they conveniently occur at a large radius. This of course is not the case for step-index fibres or for fibres having high index exponents. In practice, the NFS technique is normally used for determinations of index exponents in the range 1.5 to 4.

2) The more-accurate numerical computations, whilst confirming the similarity which exists for different profiles, indicate that the approximations made previously become increasingly inaccurate as the fibre X-value becomes larger. The effect may be seen by a comparison between Fig.3 of PI0.2 and Fig.3 of P10.3. The analytically-derived corrections are somewhat too small as a result of the two approximations producing an error of similar sign. A numerically-derived curve is
3) It is clear that a single set of correction factors may be applied to a range of different fibres, as originally surmised. However, it is perhaps too ambitious to suggest that the curves will serve every possible fibre. For example, considerable errors would result if a curve computed for a parabolic-index fibre were used to correct a step-index fibre. Nevertheless, the curves can be used for a surprisingly large range of profiles without incurring a significant error. Since the range of accuracy of the curves encompasses the fibres normally of interest, the restrictions on their use are of minor importance.

Recent improvements in the CVD manufacturing process have changed the emphasis from the determination of arbitrary profiles to one of measurement of closely-controlled near-parabolic profiles. In this case it is preferable to use a correction curve calculated specifically for a parabolic-index fibre. Publication P10.3 gives such a set of curves.

10.5 Discussion

At present the NFS technique provides the only viable alternative to the interference method for index profile determination. In addition, it is considerably more convenient and provides a higher resolution of fine detail in the profile. It should, however, be emphasised that the limitations of the method are yet to be fully determined. Ideally the accuracy of the method should be tested by comparing a series of measurements of widely different index profiles with those found by a method of reliable accuracy. Unfortunately such a method does not exist; there appears to be little to choose between the accuracy of the NFS technique and the interference method. The limited comparisons made to date between the two suggest that the agreement is good. However, for experimental convenience the fibres were always chosen to have large core diameters and numerical apertures, and thus a large V-value. Since the
assumption of a mode continuum is explicit in the theory presented earlier, it is not inconceivable that the theory would be more generally applicable to fibres having large $V$. Certainly preliminary experiments on fibres having small $V$-values, i.e. less than 20, show considerable departures both from the bound mode theory of Gloge and Marcatili and from the leaky mode extension presented here. Thus it is dangerous to judge the general applicability of the method on the excellent agreement obtained with high $V$-value fibres. Work is at present continuing on verification of the technique for fibres having $V$-values in the range 20-40.

Experimentally some care must be taken to ensure that no additional loss of the leaky modes occurs. It is found that both tight bending of the fibre, and excessive pressure, produced for example by end clamps, can effectively strip some of the leaky modes from the fibre and reduce the accuracy of the experiment. It would be ideal if all leaky modes could be stripped in this way and the need for a correction factor obviated. Unfortunately this is not possible, as some of the bound modes are also invariably stripped by the deliberate introduction of kinks in the fibre. We are left therefore with no alternative but to ensure that the leaky modes do not suffer excess loss and to correct the profile for their presence. No doubt reports of accurate results obtained without the use of a correction are a result of a fortuitous balance between the excess loss of bound modes and the power remaining in leaky modes.

It has been found experimentally that both the thickness and loss of the fibre cladding can cause excess attenuation of leaky modes. The effect of cladding loss is not unexpected, as the proportion of leaky-mode power which propagates in the cladding is high. The thickness of the cladding, on the other hand, influences the result only if it is such that the mode fields extend significantly through the cladding into the medium surrounding the fibre. The theory assumes that the core is imbedded in an infinite, uniform cladding medium and this assumption is invalidated once the cladding thickness is less than about 50% of the core radius. Fortunately neither the requirement for a low-loss cladding nor for one of adequate
thickness proves particularly restrictive. Most CVD fibres possess the required characteristics.

A further and less clearly understood excess loss has been found in fibres which exhibit departures from circularity in the core. Although slight ellipticity of the core appears to have negligible effect on the measurement, a small geometrical imperfection can very effectively discriminate against leaky modes. The imperfections which prove most damaging are those which have a tight local radius of curvature, such as an angular protrusion of the core into the cladding. Since this is uncommon in well-made fibres it is not regarded as a major disadvantage.

In summary, a systematic evaluation of the limitations of the NFS technique is under way. Preliminary results suggest that the restrictions on the general applicability of the method are not serious, but should be recognised by the user. It is probable that all optical methods of profile determination will be similarly limited, since ultimately the inaccuracies can be traced to the size of the specimen relative to the wavelength of light. It is also worth recalling that an accurate knowledge of the profile is required only to predict the pulse performance of the waveguide. Since the pulse dispersion is very sensitive to the form of the profile, direct measurements of pulse performance provide the conclusive test of profile accuracy.

References


Optical fibers having a smooth gradation of refractive index from a maximum on axis to a constant lower value in the cladding can exhibit very low pulse dispersion. Theoretical considerations\(^1\)\(^,\)\(^2\) show that an optimum near-parabolic index profile exists for which the transit time of all modes is very nearly equalized, resulting in a considerable increase in bandwidth. However, the index grading must be accurately controlled, and whereas this can be achieved by using the chemical vapor deposition technique,\(^3\) a need exists for a simple and rapid method of index profile determination. Existing methods\(^4\),\(^5\) either require lengthy sample preparation or have yet to demonstrate sufficient precision to be of value in correcting profile inaccuracies by adjustment of the manufacturing process.

The near-field scanning technique to be described here provides a simple and rapid method for obtaining
reproducible and detailed refractive index profile measurements. A short length of fiber is illuminated with an incoherent source, and the index profile is determined by observation of the light-intensity variation across the fiber output face. The profile obtained directly in this way may be suitable for many applications; however, for accurate determinations it is necessary to take into account the presence of tunnelling leaky modes. These modes contribute additional power to the observed near-field intensity distribution, resulting in an error in the inferred refractive index profile. The magnitude of this error decreases with fiber length as the leaky modes attenuate, but may still be significant after 100 m. We show here that a length-dependent correction factor can be calculated and employed to effectively eliminate this inaccuracy.

Gloge and Marcantil have shown that a close resemblance exists between the near-field intensity distribution and the refractive index profile of a fiber in which all bound modes are equally excited, and this fact has already been used to obtain a qualitative indication of the index profile of several fibers. Before quantitative measurements can be made, however, it is worth considering the factors which may influence the near-field intensity distribution in practical fibers. Principally these are (i) differential mode attenuation by absorption and scattering, (ii) mode conversion effects, and (iii) the presence of leaky modes. The first two may be largely eliminated in all but the worst fibers by using a relatively short length. However, for many index profiles, particularly near-parabolic, it is impossible to avoid launching leaky modes with even an apertured Lambertian source, since these modes are all contained within the angular limits of the numerical aperture.

Since 25% of the power launched from an incoherent source into a parabolic index fiber is contained within tunnelling modes, a considerable departure from the predicted intensity profile occurs, and a length dependence is observed as these modes decay. The loss of the tunnelling modes varies from near zero for the least leaky to a large value for the most leaky, and hence the intensity profile will be influenced largely by those modes just below cutoff. By summation of the power remaining in all modes, it is possible to calculate the near-field intensity as a function of length. A correction factor may then be developed which can be applied to a given length of fiber to convert the observed intensity distribution into the refractive index profile. Thus the refractive index $n(r)$ at radius $r$ can be related to the near-field intensity $P(r)$ by

$$\frac{n(r) - n_0}{n(0) - n_0} = \frac{\pi^2}{\int_0^{\pi/2} \sin^2 \theta \cos \theta \exp\left(-\frac{\alpha(\theta, \phi)}{a} r \theta \right) \, d\theta}$$

where $\alpha(\theta, \phi)$ is the attenuation of the mode associated with rays launched at angles $(\theta, \phi)$, $a$ is the fiber length, and $a$ is the core radius. The $\theta$ integral in Eq. (2) may be split into three angular acceptance regions delineating bound, tunnelling, and refracted rays. The attenuation coefficient $\alpha(\theta, \phi)$ of a ray launched into the bound ray region is taken as zero, that of a refracted ray infinite, and that of the leaky rays remains to be calculated, but will vary between 0 and $\infty$. We see that after an infinite length of fiber, Eq. (2) reduces to $C(r, z) = 1$ as anticipated, since only bound modes remain. In addition, $C(r, z) = \left(1 - \left(r/a\right)^4/2\right)$ in the limit $z = 0$, as derived previously for the case when all leaky modes are present unattenuated.

The detailed computation of the losses suffered by leaky rays in graded index fibers will not be presented here. For a step-index fiber, the attenuation coefficients may be evaluated exactly from the known electromagnetic fields. However, we note that for the purpose of evaluating Eq. (2) for a graded-index fiber, the attenuation may be calculated explicitly to a reasonable degree of accuracy by use of the zeroth-order WKB approximation. As a result of these calculations a simplified approximate expression has been derived for the correction factor, and it is hoped to present this at a later date. Fortunately it is found that $C(r, z)$ does not vary greatly with the form of the index profile, and furthermore can be normalized to the fiber length, core radius, and numerical aperture.

FIG. 1. Calculated near-field intensity distribution for a parabolic-index fiber having a numerical aperture of 0.2 and a core radius of 40 $\mu$m. The curves are plotted for lengths of 1 cm, 1 m, 100 m, and $\infty$. The curve for infinite length is equivalent to the index profile. Also shown is the curve calculated assuming all leaky modes are present unattenuated.
As an indication of the length dependence of the intensity profile to be expected in practice is shown in Fig. 1, where the calculated distribution is plotted as a function of length for a typical parabolic-index fiber having a numerical aperture of 0.2 and a core radius of 40 μm. As shown by the difference between the curve for all leaky modes and that for 1 cm, many of the tunnelling modes are lost within a short distance from the source. However, the less-leaky modes are extremely persistent and cause significant error even after a length of 100 m. For this fiber a length of well over 1 km would be required before the direct measurement would give the index profile with negligible error, although after this length other propagation effects would in practice influence the result. Curves calculated for fiber having a larger numerical aperture or core diameter converge more slowly to resemble the index profile since the tunnelling modes have lower loss.

The above theory applies to fibers in which all fiber modes are equally excited, and in practice this requires a Lambertian source such as a tungsten filament lamp or an LED, although it should be noted that many commercially available LEDs are far from perfect Lambertian emitters. Figure 2 shows the experimental arrangement. The incoherent source is either focussed onto the end of the fiber in the case of the lamp or is butted directly up to the fiber when using an LED. A magnified image of the fiber output face is displayed in the plane of a small-active-area (250 μm diameter) PIN photodiode which is arranged to scan the field transversely. Amplification is by a phase-sensitive detection system and the intensity profile is plotted directly onto an xy recorder. The photodiode and x axis of the recorder are controlled by means of stepper-motor drives and the relationship between step length and photodiode area gives the system a 1% spatial resolution. The fiber is kept as straight as possible to avoid radiation effects and is typically less than 1 m long. Optically flat end faces are obtained by transversely scratching and breaking the fiber under tension.

In order to test the accuracy of the technique and the validity of the calculated near-field correction factors, the intensity distribution was measured for two fibers having known refractive index profiles, one a parabolic-index fiber and the other a step-index fiber. Figure 3 shows the uncorrected intensity profile obtained from a 35 cm length of Selfoc. For comparison, the figure also shows the index profile obtained by observation of the interference fringe spacing in a 110-μm-thick cross-sectional slice of the fiber. Although the effect is somewhat emphasised by the high numerical aperture of this fiber (0.43), it can be seen that the tunnelling rays cause a marked difference between the two curves. This error, however, is reduced to a negligible value by application of the correction factor, calculated as outlined earlier.

Figure 4 shows the results obtained when the measured intensity profile in a short length of step-index fiber is corrected. In this case good agreement with the known index profile is obtained up to a normalized radius of 0.8, as evidenced by the flatness of the corrected curve. However, some rounding of the curve occurs at a greater radius, and this is believed to be a result of the very high cladding loss in this fiber sample rather than an excessively large correction factor.

![FIG. 2. Experimental apparatus.](image)

![FIG. 3. Measured intensity distribution (upper dashed curve) and corrected intensity distribution (lower solid curve) for a parabolic-index fiber compared with the index profile determined by an interference method. Length, 35 cm; numerical aperture, 0.43; core radius, 51 μm; wavelength, 0.8 μm.](image)

![FIG. 4. Measured intensity distribution (upper dashed curve) and corrected intensity distribution (lower solid curve) for a step-index fiber. Numerical aperture, 0.26; core radius, 45 μm; length, 30 cm; wavelength, 0.93 μm.](image)
The near-field scanning technique provides a rapid and convenient means of index profile determination. The method is attractive as it requires little specimen preparation and relatively simple equipment. In general, the plots contain considerably more fine detail than do those obtained by the interference fringe counting technique, and this is particularly useful when analyzing fibers made by the CVD process.

A correction factor which incorporates the length dependence of the intensity profile is required to allow for the existence of tunnelling leaky modes. The magnitude of this factor is greatest for a large normalized frequency or for short fiber lengths, and is significant in most practical cases. It is hoped to present detailed calculations of the normalized correction factors in a future publication.

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LENGTH-DEPENDENT EFFECTS DUE TO LEAKY MODES
ON MULTIMODE GRADED-INDEX OPTICAL FIBRES

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The radiation losses of tunnelling leaky modes in graded-index optical fibres are calculated theoretically, and it is shown that the near-field intensity profile has a length dependence. Consequently measurements of the near-field intensity distribution do not give the refractive index profile directly, and a correction factor must be applied. We have investigated this factor and find that it depends only on a single normalisation parameter involving fibre length, core radius and normalised frequency. A further use of the correction factor is to determine the total power attenuation due to the loss of leaky modes.

I. Introduction

The theory of bound mode propagation in graded-index multimode optical fibres is now well-known, and may be adequately described for many applications by the localised plane wave method [1]. However it has recently been recognised [2−4] that tunnelling leaky modes are usually present in graded-index structures, although the significance of these modes in a practical situation has not been fully clarified. In earlier work in this area [2,5], we have assumed that all leaky modes propagate unattenuated and showed that i) in contrast to the step-index fibre, tunnelling leaky rays in parabolic index fibres are all contained within the angular limits of the numerical aperture. It is therefore impossible to avoid their excitation when using an apertured lambertian source. ii) 25% of the power launched by a lambertian source into a parabolic index is carried by the leaky modes. iii) the near-field intensity distribution is related to the refractive index profile by the factor $1/\sqrt{1-(r/a)^2}$, where $r/a$ is the normalised radius. It is apparent, therefore, that leaky modes will have a significant effect on at least two fibre measurements which commonly use apertured lambertian sources, namely a) evaluation of total attenuation and b) determination of the refractive index profile by observation of the near-field intensity distribution [6]. In the former the total output power is measured after transmission through a long length of fibre and then compared with that from a short length. Since the attenuation of the leaky modes is included in the measurement, the result is inclined to be pessimistic. In the latter the refractive index profile is inferred from the intensity distribution across the output face of a metre or so of fibre. In this case the presence of leaky modes can considerably influence the intensity distribution, and therefore introduces a profile error. The object of the present contribution is to calculate the losses inherent in modes of this type, and hence to ascertain the magnitude of the errors introduced in the above two cases.

As a first step, the attenuation coefficient for each leaky mode is found by application of the WKB method, and it is shown that a simple approximation may be applied to give an expression which is not particularly sensitive to the exact form of the index profile. This expression may be used to sum the power remaining in all modes after a given length of fibre, and hence we obtain the near-field distribution at any point along the fibre. A generalised length-dependent correction factor involving only fibre length, core radius and normalised frequency may then be introduced to relate the refractive index profile to the intensity distribution. Finally the theory is applied to calculations of the leaky mode contribution to fibre attenuation measurements, and this gives a useful physical insight into the persistence of tunnelling modes.
2. Leaky mode attenuation coefficient

For a step index fibre the attenuation of leaky modes may be calculated exactly from the known electromagnetic fields, and has been dealt with previously in some detail [7]. For graded-index fibres, on the other hand, some approximate procedure must be adopted, the simplest being the zeroth order WKB approximation. The results so obtained are of comparable accuracy to the more formal first-order approach given by Petermann [3], and in addition more physical understanding is gained in the present treatment. The approximation is valid under the usual WKB restriction, viz. small variation of refractive index distribution over distances of the order of one wavelength. In the case of a graded-index multimode fibre this condition is usually easily satisfied. An assumption which is made throughout is that the fibre core is surrounded by a cladding of infinite extent. Although this is not met in practice, for the purposes of the present calculation the error introduced by the assumption is small, provided that the cladding thickness is at least 50% of the core radius. Most low-loss CVD fibres satisfy this condition.

2.1. General form of the attenuation coefficient

Fig. 1 shows the squared magnitudes of the local plane-wave vector components as functions of radius for a leaky mode in a general graded-index fibre [2]. Here \( a \) is the core radius and \( r_1, r_2, r_3 \) correspond to the caustics separating regions of oscillatory and evanescent fields. In analogy with the concept of quantum mechanical tunnelling, the probability of a photon from \( r_2 \) emerging at the outer caustic \( r_3 \) is given by the tunnelling probability \( T \). This is calculated as the inverse ratio of the squares of the field amplitudes \( E(r_2), E(r_3) \), at these radii [4]:

\[
T = \frac{|E(r_3)|^2}{|E(r_2)|^2} = \exp \left\{ -2 \int_{r_2}^{r_3} \left( \frac{\beta^2}{r^2} + \beta^2 - k^2 n^2(r) \right)^{1/2} dr \right\},
\]

where \( \beta \) is the propagation constant, \( \nu = \) azimuthal mode number, \( k = 2\pi/\lambda \), \( \lambda = \) wavelength and \( n(r) \) is the refractive index at radius \( r \). The dimensionless attenuation coefficient \( \alpha \) (normalised to the core radius) of a given leaky mode \((\mu, \nu)\) can then be calculated from the mode tunnelling coefficient \( T \) by the simple relation

\[
\alpha(\mu, \nu) = \frac{2\nu}{a\beta} \frac{T}{1 - T}.
\]

In order to obtain the net power flow out of the fibre, the component of the Poynting vector normal to the fibre axis must be calculated, and this gives rise to the term \( 2\nu/a\beta \) in eq. (2) above. The same expression may be deduced directly from Poynting's vector theorem (following Snyder [8]), or alternatively from geometrical optics when the first term arises from the mean distance between points at which a ray meets its caustics (the ray period). Note that implicit in eq. (2) is the assumption that only the least leaky modes are of importance.
2.2. Attenuation coefficient for parabolic index variation

The integral in eq. (1) may be calculated numerically for any refractive index profile \( n(r) \). However, an analytic expression may be obtained for the special case of a parabolic index variation:

\[
T = \left( \mu^4 - 4v^2 \mu^2 \right) \frac{4v}{\lambda_n} \left( \frac{(\nu - u^2)(2\nu + 2\nu^2 - u^2)}{(\nu + x)^2} \right)^\nu \left( 2\nu + 2\nu^2 - u^2 \right)^2 \exp(\alpha),
\]

where \( x = [\nu^2 - (u^2 - v^2)]^{1/2} \), and the conventional notation \( u^2 = a^2 [k^2 n^2(0) - \beta^2] \), \( v^2 = a^2 k^2 [n^2(0) - n_2^2] \) has been used; \( n(0) \) is the refractive index at core centre and \( n_2 \) that of the cladding.

2.3. Approximate form of the attenuation coefficient

Although eq. (1) can be evaluated for any index profile \( n(r) \), a more useful general result can be achieved by a simple approximation. From fig. 1 it is intuitively seen that for most forms of the index profile \( n(r) \), the central caustic \( r_2 \) is not too far from the core-cladding boundary at radius \( a \). Using this fact as a basis for approximation and replacing \( E(r_2) \) by \( E(a) \), the tunnelling coefficient \( T \) becomes

\[
T \simeq |E(r_3)/E(a)|^2 = [(u^2 - v^2)/(\nu + x)^2]^\nu \exp(2x).
\]

It may be seen that \( T \), and hence the attenuation coefficient \( \alpha \), is now a function of the specific form of the index profile \( n(r) \). This is an important result as it implies that, at least within the limits of the approximation used here, the loss of a leaky mode having a given designation \((u, v)\) is independent of the profile of the structure within which it is propagating. Thus we might expect that the influence of leaky modes on, for example, the near-field intensity, would be relatively insensitive to the core index profile, and this is indeed the case, as will be shown later.

In the following sections eq. (4) will be used in place of eq. (1), as this allows a considerable reduction in numerical computation whilst yielding results of comparable accuracy. Furthermore, we note that provided only the least leaky modes are considered \((x \simeq \nu)\), eqs. (4) and (2) reduce to give the leaky mode attenuation coefficient derived by Golec [9] for the step index fibre.

3. The length-dependent near-field intensity distribution

Reverting temporarily to geometrical optics, the near-field intensity \( I(\theta) \) at radius \( r \) on the output face of a fibre excited by a lambertian source may be found by summing a cosine source function over angles of incidence \( \theta \) and projected angles \( \phi \) [2]. The attenuation of the leaky modes is included by way of the ray equivalent of the attenuation coefficient \( \alpha \) derived above:

\[
\frac{I(\theta)}{I(0)} = \frac{4}{\pi [n^2(0) - n_2^2]} \int_0^{\pi/2} d\phi \int_0^{\pi/2} \sin \theta \cos \theta \exp[-\alpha(\theta, \phi)z/a] d\theta,
\]

where \( \alpha(\theta, \phi) \) is the attenuation of mode \((u, v)\) associated with rays launched at angles \((\theta, \phi)\), and \( z \) is the fibre length. The integral over angles of incidence \( \theta \) in eq. (5) may be split into three regions corresponding to bound, leaky and refracted rays [2]. The attenuation coefficient \( \alpha(\theta, \phi) \) of a bound ray is taken to be zero, that of a refracted ray infinite, and that of a leaky ray is given by the mode attenuation \( \alpha(u, v) \) defined in eq. (2). If we consider only bound rays to be propagating, the upper limit on \( \theta \) in eq. (5) becomes \( \sin^{-1} [\sqrt{n^2(\theta) - n_2^2}] \) and the expression reduces to that given by Golec and Marcatali [1].

Converting eq. (5) back into the mode notation [2] \((u, v)\), and assuming an external medium of \( n = 1 \), we deduce that the intensity distribution \( I(\theta) \) is

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\[
\frac{I(r)}{I(0)} \approx \frac{n(r) - n_2}{n(0) - n_2} C(r, z),
\]

where
\[
C(r, z) = 1 + \frac{4}{\pi [n^2(r) - n_2^2]} \int_0^{\nu_{\text{max}}} \frac{\nu}{a^2 r k^3} \int \frac{u du \exp \left[-\alpha(u, \nu) z/a\right]}{[n^2(r) - n_2^2 + (u^2/a^2 k^2) - (u^2/r^2 k^2)]^{1/2}}.
\]

The upper limit \(\nu_{\text{max}}\) is given by the appropriate zero of the square-root term in the integral.*

Thus \(C(r, z)\) may be considered as a "correction factor" relating the near-field distribution \(I(r)\) to the refractive-index profile \(n(r)\). As expected this factor reduces to unity after an infinite length of fibre when no leaky modes remain; a close resemblance will then exist between the intensity and index profiles, a result first reported in ref. [1].

The intensity distribution as a function of fibre length for any given index profile may now be evaluated numerically from eqs. (6) and (7) using the attenuation coefficients given by (2) and (1), or the approximate form (4). A specific example of this is given in Fig. 2 where the near field is calculated using the more accurate eq. (3) for a typical parabolic index fibre of 1 metre in length. The index profile of this fibre is also shown, and it can be seen that the presence of leaky modes causes a marked difference between the two curves. In order to illustrate the process of converting near-field intensity to refractive index, and also to test the accuracy of the approximate expression (4), as compared to the more-precise eq. (3), an index profile calculated from the near-field using eq. (4) is plotted. It is clear that this result is very close to the true profile, thereby illustrating the level of accuracy obtainable by use of the approximate tunnelling coefficient.

4. Generalised near-field correction factors

Although eq. (7) may be used as it stands for computing near-field correction factors, it has proved possible to further simplify the results by use of a general normalisation parameter. It can be shown that an analytic approximation for eq. (7) may be realised by considering only the least leaky modes, since these contribute most to the integrals. The result of this approximation is dependent principally on the normalisation parameter \(X = (1/a) \ln (e/a)\), and this suggests that this parameter may be a general normalisation even when eq. (7) is evaluated so as to include all modes. Fortunately this hypothesis is well founded provided \(z/a > 10^3\); the validity of the normalisation has been verified to within 2% for a wide range of index profiles by numerical integration of eq. (7) using the approximation (4) introduced earlier.

As a consequence both of the approximation (4) and the above normalisation, a single set of curves \(C(r, z)\) is all that is now required to completely specify the near-field intensity distribution for a length of a fully-excited fibre having an arbitrary circularly sym-
Fig. 3. Near-field correction factors \( C(r, z) \) given as a function of normalised fibre radius for \( X \) values from 0.05 to 0.5 in increments of 0.05. The normalisation parameter \( X \) describes the fibre core radius, length and numerical aperture, and is given by \( X = (1/a) \ln (c/a) \). The curves may be used for fibres having any graded-index profile. Also shown is the result for \( z = 0 \), when all leaky modes are present.

Fig. 4. Plot of output power \( P(z) \) relative to input power \( P(0) \) as a function of length for parabolic index fibres having the \( \nu \)-values shown. Provided \( z/a > 10^3 \) the curves are applicable to fibres having any combination of numerical aperture, core radius and wavelength. For shorter lengths the curve is drawn for the specific example of a numerical aperture of 0.2 and \( \lambda = 0.93 \mu \text{m} \). The decrease of power with length is caused by the radiation losses experienced by tunnelling modes. The curves are asymptotic to \( P(z)/P(0) = 0.75 \).

The exciting wavelength is 0.9 \( \mu \text{m} \). We calculate the \( X \) value as 0.2, and the figure indicates that the near field is 8% greater than the index profile at a normalised radius of 0.6, rising to 20% at 0.85.

5. Power attenuation

The near-field distributions given by eqs. (6) and (7) may be integrated over radius \( r \) to give the total power remaining in all modes as a function of fibre length \( z \). In this case the normalisation parameter \( X \) may be used for values of \( z/a > 10^3 \), although the result is now sensitive to details of the refractive index profile as a consequence of the differing number of leaky modes supported by various structures. Results for a parabolic index fibre are shown in fig. 4, where the normalised power \( P(z)/P(0) \) after length \( z \) is plotted as a function of normalised length \( z/a \) for a number of \( \nu \)-values. The curves are computed for a fixed numerical aperture (0.2) and wavelength (0.93 \( \mu \text{m} \)) and the \( \nu \)-values are determined by different core radii. For other numerical aperture/wavelength/radius combinations the curves will be similar to those of fig. 4 for \( z/a > 10^3 \), but
somewhat different for lower $z/a$ values as a result of the invalidity of the normalisation parameter $X$.

As a numerical example, consider a fibre of numerical aperture 0.3 and core radius 25 μm at the wavelength of GaAs emission (0.93 μm). This yields a $\mu$-value of 50 so that, from the graph, after 50 cm ($z/a = 2 \times 10^4$) the power in the leaky modes has decayed from an initial 25% of the total to 9.7%; after 1 km ($z/a = 4 \times 10^7$) the proportion is further reduced to 6.1%. This illustrates the persistence of some proportion of the leaky modes for considerable distances.

As noted earlier the radiation losses suffered by leaky modes will contribute an error to the total attenuation measurement of a fully excited fibre. Taking the initial fibre length to be 1 km and the shortened length 50 cm, as in the example given above, the fibre loss would be pessimistic by 0.17 dB/km. If, however, the length available was only 100 metres ($z/a = 4 \times 10^6$) the extrapolated loss would be in error by the more significant figure of 1.3 dB/km.

6. Conclusion

It has been shown that the near-field intensity distribution in graded-index fibres excited by lambertian sources has a length-dependence caused by the radiation losses of leaky modes. Although many of the leaky modes are lost within distances of less than 1 cm, other can persist for a kilometre or more, giving a near-field that departs considerably from that predicted by a bound mode analysis. Length-dependent correction factors have been computed which enable near-field intensities to be calculated, given the refractive index profile. The inverse of this process yields a technique for the experimental measurement of refractive index profiles [6].

The correction factors may also be used to calculate total power attenuation as a function of length. This would indicate that a small error is incurred under normal attenuation measurement conditions, but that care should be taken in extrapolating results obtained on short fibre lengths. The normalisation parameter deduced here, $X = (1/\mu) \ln (z/a)$, provides a convenient characterisation of a given length of fibre.

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References

CORRECTION FACTORS FOR THE DETERMINATION OF OPTICAL-FIBRE REFRACTIVE-INDEX PROFILES BY THE NEAR-FIELD SCANNING TECHNIQUE

Introduction: In recent publications,1,2 we have described the determination of optical-fibre refractive-index profiles by a near-field scanning technique. The method provides a convenient means of obtaining detailed index profiles by observation of the light intensity distribution across the output face of a short length of fibre illuminated by a Lamberntian source. However, it is necessary in most practical cases to correct the measured intensity profile so as to allow for the inevitable presence of leaky modes. This has led to the development3 of a set of numerical calculations enabling near-field measurements to be converted to refractive-index profiles.

To calculate a set of curves which would be applicable to a range of index profiles, and which would therefore require prior knowledge of the profile being measured, it was necessary to make a number of approximations. The object of the present contribution is to clarify these approximations, and to investigate their validity by presenting numerical calculations of the correction factors for several possible fibre profiles.

Leaky-mode attenuation coefficient: As shown in Reference 2, the near-field intensity I(r) at a radius r may be calculated by summation of the power remaining in all propagating modes after a length z of fibre. When all bound modes are equally excited, this leads to a simple dependence4 of near-field on refractive-index profile n(r). However, in practice, leaky modes are also excited5 and will contribute additional power to the near field. To determine the power remaining in leaky modes, and hence the magnitude of their contribution, the attenuation a(α, ρ) of each leaky mode (α, ρ) must be found. A correction factor C(α, ρ) may then be formulated to account for the discrepancy between near field and refractive-index profile.

The correction factor C(α, ρ) is given by the product of T with the number of reflections (1/Δz) per unit length at the central caustic r2:

\[ a(α, ρ) = aT/Δz \]  

Δz may be interpreted as the ray half period for the parabolic fibre only. In general, Δz is given in the WKB approximation by

\[ Δz = 2 \int_{r_1}^{r_2} \left( k^2 n^2(r) - k^2 n^2(r') + \frac{1}{2} \frac{d}{dr'} n^2(r') \right) dr' \]  

where r1 is the radius of the inner caustic, r2 is that of the central caustic, k is the longitudinal propagation constant, k is the free-space wavenumber and v is the azimuthal wave number. Similarly, the WKB approximation yields, for the tunnelling coefficient,7-8

\[ T = \exp \left[ -2 \int_{r_1}^{r_2} \left( (v^2 r^2) + k^2 n^2(r) \right) dr \right] \]  

Eqsns. 2 and 3 may be numerically integrated to generate a correction curve for a specific index profile. However, apart from the time of calculation, this is unsatisfactory when dealing with fibres having unknown index profiles, as in the near-field scanning technique. We therefore seek approximations which will allow a single average correction curve to cover a range of possible profiles.

Correction factors: In Reference 2 approximate forms of

\[ \Delta z = \alpha/βx \]  

which is similarly independent of profile. The use of this approximation explains the apparent discrepancy between the generalised coefficient of Reference 2 and that developed for the specific case of the parabolic-index profile by Petermann,6 and more recently by Ryder and Dove.7

To investigate the validity of the above approximations when applied to various fibres, we have numerically computed the correction factors for a range of power-law profiles. The tunnelling coefficient T for a given profile is obtained by using numerical quadrature for the integral of eqn. 3, while Δz is derived from Gloge's expression8 for the number of propagating modes:

\[ Δz = \frac{e^2 β}{2m} \frac{g^2 + 2}{2} \left( \frac{g + 2}{g} \right)^3 \left( \frac{m}{g} \right) \frac{1}{1 + \frac{m}{g}} \]  

where g is the index exponent, and ω and v have their conventional meanings. This approximation is exact for g = 2 and very good for other values, provided g is not too large.

The correction factors for index profiles represented by g = 1.5, 2, 3 and 4 are given in Fig. 1, together with the step-index correction factors.

![Fig. 1](image1)

**Fig. 1** Correction factors C(r, z) plotted as function of normalised fibre radius r/a for fibres having a range of index profiles and X = 1/ln(n(a)) values x = 0.05, 0.25 and 0.5.

Near-field intensity profiles must be divided by C(r, z) to give the refractive-index distribution.

Index curves for comparison. The X values shown refer to the normalisation parameter9 X = 1/ln(n(a)), which renders the curves applicable to fibres of any radius a, length z > 10^4 a and normalised frequency v. It may be seen that the correction factors are remarkably similar up to a normalised radius

![Fig. 2](image2)

**Fig. 2** Near-field correction factors for fibres having an index exponent of 3 and X values from 0.05 to 0.5, in increments of 0.05. The curves represent a useful median value for application when the approximate profile is unknown.

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of 0.8 for a wide range of index profiles, after which a spread of ±6% may occur. Thus our initial assumption that the correction factor would be approximately independent of profile is verified. Moreover, the errors incurred by applying an inappropriate correction factor are only significant at a normalised radius of greater than 0.8. Fortunately, since the magnitude of most graded-index profiles at this radius is small, the error will be hardly noticeable.

\[ D(g) = \frac{1}{D} \ln \left( \frac{e}{\text{NA}(0)} \right) \]
\[ + \ln \left( \frac{1}{2} \left( \frac{g+2}{g} \right)^{1/n} \left( \frac{g+2}{g} \right)^{1/2} + \ln \left( \frac{e}{a} \right) \right) \]

which reduces exactly to the expression derived by Love and Pask for the parabolic case (g = 2). The normalisation parameter introduced in Reference 2 retains only the third term of eqn. 6, on the basis that, in practice, the variation in this term dominates. For very short lengths of fibre, less than 3 cm for a fibre having a 60 μm core diameter, it is possible that a more accurate normalisation based on eqn. 6 may be useful. However, in a practical case the small improvements in accuracy thus obtained would hardly merit the added complexity.

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References
5 STEWART, W. J.: 'A new technique for determining e-values and refractive index profiles of optical fibres'. Presented at the OSA/IEEE meeting on optical fibre transmission, Williamsburg, Va., 1975
7 SNYDER, A. W., and LOVE, J. D.: 'Attenuation coefficient for tunnelling leaky rays in graded fibres', Electron. Lett. (to be published for publication)
CHAPTER 11
SPLICING TOLERANCES IN GRADED-INDEX FIBRES

11.1 Calculation of Splice Loss (P11.1)

It was shown in Chapter 9 that tunnelling leaky modes in parabolic-index fibres are all contained within the limits of the meridionally-defined numerical aperture. Consequently modes of this type assume a greater importance in graded-index fibres than in step-index fibres, where they are rarely excited. The implications for the measurement of fibre attenuation have already been investigated. The present chapter is concerned with a further implication, namely the excitation of leaky modes at splice imperfections and the effect this has on the apparent joint loss.

An imperfect fibre splice results in a redistribution of power amongst the modes of the waveguide. Frequently the new mode spectrum has a higher overall loss than that before the splice, caused, for example, by excitation of higher-order modes which are subject to higher modal attenuation. Thus a spatial transient follows the splice as the power within the modes decays and redistributes to a form which minimises the overall loss. Measurements of splice loss made within the spatial transient region will yield an optimistic result.

A principal cause of such a spatial transient in graded-index fibres is the excitation of leaky modes. This is not the case for a splice between two step-index fibres, as we have already seen that all leaky rays are found in an angular region outside the numerical aperture. They therefore cannot be excited by a simple lateral displacement at a butt joint. On the other hand, the smallest misalignment between graded-index fibres produces a mismatch between the emission angle of one fibre and the acceptance angle of the other, since the local numerical aperture is a function of radius. Clearly this will excite tunnelling leaky modes in the receiving fibre.

The imperfections which are possible at a junction between two fibres are numerous. They include effects which are avoidable in principle, such as the inclusion of dirt between the end-faces and inadequate flatness of the fibre ends. We are concerned here with imperfections which are always present to a
degree, namely the small misalignments which exist between two fibres. Of the three possibilities, angular misalignment, end-face separation and transverse misalignment, the latter shortcoming is the most difficult to avoid. The assumption will therefore be made that the only misalignment present is transverse. Further assumptions are that the two fibres are identical in both core radius and numerical aperture and that the emitting fibre is fully excited.

Details of the calculation of the splicing loss between both step-index fibres and parabolic-index fibres is given in publication P11.1. As usual, the solution to the problem involves the assumption of a mode continuum. This permits the use of geometrical optics and allows us to use the concept of local acceptance regions. The jointing efficiency may then be found by calculating the overlap of the emitting fibre radiation cone with the local acceptance cone of the receiving fibre, followed by a summation of the overlap over the common core area. For the step-index fibre the solution simply involves a calculation of the overlap area, since the local acceptance angle for bound modes is constant over the end-face.

The excitation of leaky modes in the receiving fibre is included by defining an angular acceptance region in which they may be launched. In a practical case we are not interested in the limits of acceptance of all leaky rays, since many of these will radiate very rapidly. We therefore delineate an angular acceptance region in which only low-loss leaky modes are launched. In this way an effective local numerical aperture can be defined to include the acceptance angle of the bound modes and that of the persistent leaky modes. If the effective local numerical aperture is now used to calculate the angular overlap between emitting-fibre radiation cone and receiving-fibre acceptance cone at a point on the fibre end-face, the results will include the effect of leaky-mode excitation. The length dependence of the residual power in tunnelling modes is implicit in the calculation, as the magnitude of the effective receiving local numerical aperture will depend on the length specified after the splice.

The details of the calculations given in P11.1 provide considerable insight into the properties of leaky modes in parabolic-index fibres. Fig.1 is particularly illuminating as
it clearly delineates the various angular acceptance regions. It also serves to emphasise the distinction between leaky modes which have low loss, and those which radiate within a short distance of the source and therefore are of little importance. We see that the total acceptance angle of leaky modes is largely of academic interest, since the effective acceptance angle for the example given is not very much greater than the bound mode acceptance angle. Nevertheless, the difference is sufficient to cause appreciable errors in the NFS technique.

11.2 Results

The calculation of splice loss as a function of lateral misalignment covers two cases of importance:

a) The receiving fibre is long so that the leaky-mode spatial transient has subsided. The attenuation measured under these conditions may be regarded as the 'true' splice loss, since it is the figure which has greatest practical significance.

b) The receiving fibre is short so that measurements are made within the spatial transient regime. Reported\textsuperscript{1,2} measurements have frequently been made under these conditions.

The results are summarised as follows.

1) When the receiving fibre is long, a junction between step-index fibres is more tolerant to misalignment than an equivalent parabolic-index fibre splice. Typically a 100\textmu m core-diameter parabolic-index fibre requires alignment to better than 6.4\textmu m to achieve a splice loss of 0.5dB. The loss for the same misalignment between step-index fibres is about 25\% lower. Fortunately the tolerances needed to splice parabolic-index fibres are not unacceptably greater than for the step-index fibre, and do not therefore represent a serious disadvantage.

2) Measurements made on graded-index fibres within the spatial transient regime will be optimistic. Under certain conditions, for example a measurement made within a metre or so of a splice, the underestimation may be such that the parabolic-index fibre appears more tolerant to misalignment than a step-index fibre. Care must therefore be exercised to ensure that the spatial transient has subsided before measurements are made. It may be necessary to allow a hundred or so metres after the splice before accurate measurements are obtained.
This requirement has not been appreciated in previously published work on jointing loss.

3) Clearly, a succession of closely-spaced splices will not produce an additive loss. In this case the assumption of only bound mode excitation in the emitting fibre, used in the present work, is no longer valid. The individual splice loss will depend on the emitting fibre mode spectrum, which in turn will depend on the misalignments found in the previous slice.

11.3 Conclusions
The results given here represent the first attempt to put the splicing losses experienced between graded-index fibres on a quantitative basis. The analysis provides several useful conclusions, not least of which is the unexpectedly small difference in splicing tolerance between step and graded fibres. Before the publication of this work it had even been suggested that a graded-index fibre would be as difficult to joint as a single-mode fibre.

It should be emphasised that the calculated results will probably overestimate the loss in a practical field splice. In this case the fibres will be partially rather than fully excited, as a result of mode filtering. The spatial transient is then caused both by leaky-mode and by higher bound-mode excitation. Both will decay after the splice with their own characteristic attenuation. It is probable, however, that the results presented here will still provide a reasonable first-order description.

References
Splicing tolerances in graded-index fibers

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Calculations are presented showing that, in general, a parabolic-index fiber is more sensitive to lateral misalignments within a splice than a step-index fiber. However, misalignments result in the excitation of leaky modes in graded-index fibers, and this can lead to optimistic joint loss measurements. Effective losses are given for various lengths of fiber following the splice, and it is shown that a parabolic-index fiber may appear more tolerant to misalignment than a step-index fiber when short lengths are used.

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The achievement of a low-loss splice between two optical fibers requires a high degree of both lateral and angular alignment. An imperfectly made joint fails to match the emitted radiation from one fiber to the acceptance cone of the other, and this results in a radiation loss. The magnitude of the loss depends on the degree of misalignment, the most likely imperfection in practice being a lateral displacement of the core centers. Such a misalignment also causes a redistribution of power amongst the fiber modes, and will excite higher-order modes which usually have a higher loss. Thus a spatial transient follows a fiber splice as the steady-state modal distribution establishes itself, and the full effect of the splice on fiber loss will not be seen unless measurements are made a sufficient distance after the joint for the transient to have subsided.

A principal cause of such a spatial transient in graded-index fibers is the excitation of tunneling leaky rays. In step-index fibers, all leaky rays are found in an angular region outside the numerical aperture, and therefore cannot be excited by a simple lateral displacement between the emitting and receiving fibers (assuming that only bound rays are present in the emitting fiber). This is not the case for a misalignment between two graded-index fibers, as it has been shown that all leaky rays are contained within the meridionally defined numerical aperture, and therefore will be excited by even a small misalignment.

The purpose of the present letter is twofold. First, it is to calculate the losses produced by a laterally misaligned splice between two parabolic-index fibers, and to compare this with a splice between step-index fibers. It will be assumed that both emitting and receiving graded-index fibers are long, and therefore the spatial transients associated with the initial excitation and with the splice have subsided. A second objective is to calculate the leaky mode power loss within the transient region following the splice, and hence to determine the influence of the transient on the measurement of splice performance.

To simplify the calculation of splicing efficiency, we assume that (a) the emitting fiber is long and leaky modes are no longer present; (b) the remaining modes, i.e., the bound modes, are equally excited; (c) the fiber end faces are in contact; and (d) the only misalignment is transverse. Provided the fiber supports sufficient modes to allow a geometrical optics approximation, the problem then becomes one of calculating the overlap of the emitting fiber radiation cone with the local acceptance cone of the receiving fiber at a common point on their end faces, followed by a summation of this overlap over the common core area. For a step-index fiber the solution simply involves a calculation of the common area, since the local acceptance angle for bound modes is constant over the end face, and therefore leaky modes are not excited by a misalignment, as noted above.

For a graded-index fiber, however, the local acceptance angle varies with position on the fiber end face. This is illustrated schematically in Fig. 1(a), where we see that the local acceptance cone for leaky rays is always larger than that for bound rays, except at the core center. The equations defining the angular acceptance regions shown in the figure have been given in an earlier publication in terms of the local angle of incidence. A plan view of a parabolic-index fiber is shown in Fig. 1(b), together with a superimposed section through the acceptance cones at a specific radius. From

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FIG. 1. (a) Schematic acceptance cones for rays incident on a parabolic-index fiber end face at radius r. The inner (circular) acceptance cone defines the acceptance region for bound rays, while the outer (elliptical) cone defines the region containing leaky plus bound rays. The shaded area, therefore, represents the angular region within which leaky rays are accepted at this radius. (b) Plan view of fiber end face shown in (a) with section through acceptance cones drawn at r/a = 0.8. As in (a) the shaded region between the circular and elliptical acceptance cones contains the leaky rays. The intermediate elliptical cone within the shaded region represents the effective acceptance angle, drawn in this case for x = 0.1 (see text). Also shown is the locus of the extreme of the bound ray acceptance cone (dashed lines), that of the bound plus leaky rays (vertical solid lines), and that of the effective acceptance cone (chain dotted).
FIG. 2. Splice loss as a function of lateral fiber misalignment for a parabolic-index fiber. The shaded region represents an area of apparent splice loss; the observed loss depends on the length of fiber after the joint, and the fiber parameters. Specific examples are given for fibers having $X$ values as shown (see text). The upper bound gives the splice loss seen after a length sufficient for the transient to have subsided. Shown for comparison is the loss calculated for a step-index fiber (dashed line). Note that this line and the upper and lower bounds to the shaded region are independent of fiber parameters.

the loci presented in the figure, it can be seen that whereas the circle delineating the guided ray acceptance angle decreases from a maximum at the core center to zero at the core/cladding interface, the major axis of the ellipse defining the acceptance of the leaky rays remains unchanged, and is equal to the maximum acceptance angle at the core center.

In a practical case, we are not interested in the limits of acceptance of all leaky rays, but only of those having low radiation losses and which therefore persist for considerable lengths. We note that the losses of leaky modes will vary from zero for the least leaky (at cutoff) to infinity for the most leaky (at the transition to radiation modes). Furthermore, we have previously shown that this loss exhibits a rapid increase at some critical longitudinal propagation constant $\beta$ (or, equivalently, angle of incidence $\Theta$), thus dividing the low-loss from high-loss modes. This transition enables us to define an effective angular acceptance region on the end face; the exact dimensions of the region will depend strongly on the core radius $a$, wavelength $\lambda$, numerical aperture $A_0$, and length of fiber $L$ from the point where leaky modes were first excited. A conic section defined in this way for a particular combination of fiber parameters is shown in Fig. 1(b), together with the locus of extrema of its major axis (the chain-dotted lines). The calculation of this new effective angular acceptance region is performed as follows.

The effective local numerical aperture $A(r, \phi, z)$ (sine of the effective local acceptance angle) is defined by summing the power injected at various angles of incidence $\Theta$ by a source having a Lambertian (cosine) distribution, suitably weighted by the attenuation coefficient $\alpha(\Theta, \phi)$:

$$A^2(r, \phi, z) = \int_{0}^{\pi} \int_{0}^{2\pi} \sin \Theta \exp[-\alpha(\Theta, \phi)z/a] d\Theta d\phi.$$  

The angle $\phi$ referred to here is the projected angle of a ray incident at radius $r$ on the end face. The values of the attenuation coefficient $\alpha(\Theta, \phi)$ vary from zero for the guided region to $\infty$ for the refracted region. In the intermediate (leaky) region the WKB approximation may be used to obtain expressions for $\alpha(\Theta, \phi)$.

For a parabolic-index fiber, treating only the least leaky rays, a useful approximation is

$$\alpha(\Theta, \phi) = \frac{\gamma}{\pi} \left[ \frac{\sin \Theta}{A_0} \right]^2 \left[ 1 + \left( \frac{r}{a} \right)^{4/3} \right]^{1/2} \times \left[ \frac{\sin \Theta}{A_0} \right] A_0 \cos \phi,$$

where $\gamma = 2r/a \nu \cos \phi \sin(\Theta/A_0)$. Here $A_0$ is the meridional numerical aperture (defined at $r = 0$), $\nu = aA_02\pi/\lambda$ is the normalized frequency, and $n_2$ is the cladding refractive index.

A simple expression for $A(r, \phi, z)$ may be obtained from Eq. (1) by introducing a linear approximation to the exponential term. This yields an effective upper limit to $\sin \Theta/A_0$ using Eq. (2), and renders the integral in (1) trivial. Additionally, when this linear approximation is performed, it is clear that the fiber parameters and length enter principally in the form $(1/r) \ln(z/a)$, suggesting the normalization parameter $X = (1/r) \ln(z/a)$ used previously in calculations of near-field intensity distributions. The intermediate acceptance cone of Fig. 1(b), as delineated by the effective local numerical aperture $A(r, \phi, z)$, was calculated for $X = 0.1$.

We now apply the concept of effective local acceptance angle to the calculation of jointing efficiency $\eta(d, X)$ between two identical fibers having a lateral displacement $d$:

$$\eta(d, X) = \left\{ \int_{S_E} \int_{S_R} r dr d\phi \left[ A_E(r, \phi, z)A_R(r, \phi, \infty) \right]^2 \right\} \times \left[ \int_{S_E} \int_{S_R} r dr d\phi \left[ A_E^2(r, \phi, \infty) \right] \right]^{-1}.$$  

Here $S_E$ and $S_R$ are the core areas of the emitting and receiving fibers, respectively, and $S_E \cap S_R$ (a function of $d$) is the area of overlap; $z$ is the length of fiber after the splice, and $\phi$ is the angle associated with the radius $r$ to the launch point [see Fig. 1(a)]. $A_E$ and $A_R$ are the effective local numerical apertures of the receiving and emitting fibers, defined by Eq. (1), and $A_R/|A_E|$ is the angular overlap. The use of $z = \infty$ in the denominator indicates that the emitting fiber is long and therefore only guided modes are present. The first integral in both the numerator and denominator of (3) refers to the geometry of the fiber end faces, while the second refers to the geometry of the angular acceptance and emittance cones.

The splicing efficiency between two identical parabolic-index fibers is shown in Fig. 2 as a function of lateral displacement, and may be compared with that between two step-index fibers. The shaded region represents results for a fiber length within the spatial transient regime, where the loss measurement will depend on the particular fiber parameters and the distance after the splice. The upper bound to the shaded region gives the loss when observations are made some distance after the splice, and no leaky modes are present. From this curve we see that the loss varies almost linearly with displacement over the range shown, and that under these conditions a parabolic-index fiber is
more sensitive to misalignment than a step-index fiber, the loss for a given displacement being about 35% higher. Typically, a 100-μm core diameter parabolic-index fiber requires alignment to better than 6.4 μm to achieve a loss of 0.5 dB, while an equivalent step-index fiber requires 8.6 μm.

The effect that the presence of leaky modes can have on a measurement may be illustrated by considering a splice made 2.5 m from the end of a fiber having a numerical aperture of 0.18 and a core diameter of 60 μm (X = 0.3 for λ = 0.9 μm). Referring to the figure, a parabolic-index fiber now appears slightly less sensitive to misalignment than a step-index fiber, at least up to a relative displacement of 0.06. Fibers having larger core diameters and numerical apertures exhibit the effect more strongly, and this clearly illustrates the pitfalls in measuring joint losses in short fiber lengths.

When both emitting and receiving fibers are long, so that leaky rays are no longer important, parabolic-index fibers are somewhat more sensitive to lateral misalignment than step-index fibers. A displacement tolerance of about 10% of the core radius must be achieved for acceptable splice performance (0.5 dB).

A spatial transient caused by the excitation of leaky modes follows an imperfect splice, and measurements made within this region will be optimistic. For example, a parabolic-index fiber may appear more tolerant to misalignment than a step-index fiber. Measurements made within the spatial transient may be extrapolated to longer lengths by use of the X parameter introduced here.

A further consequence of the transient following a splice is that a succession of closely spaced joints will not produce an additive loss. Finally, it should be noted that the curves of Fig. 2 are not directly applicable to partially excited fibers, where a greater tolerance to misalignment is anticipated.

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8J. Guittman, O. Krumpholz, and E. Pfeiffer, Electron. Lett. 11, 583 (1975). In this reference a parabolic-index fiber is shown to be less sensitive to misalignment than a step-index fiber; a correction will be published shortly [O. Krumpholz (private communication)].
CONCLUSIONS
It is clear from the work presented here that dramatic progress has been made in the technology of optical waveguides. The initial expectations for the attainable fibre attenuation and bandwidth have both been greatly exceeded. The fibre loss has been reduced from several thousand dB/km to less than one dB/km. Few physicists would have dared to predict that a solid material could be so transparent. It is interesting to observe that the purity of the phosphosilicate glass made by the homogeneous CVD process is such that it has a loss similar to that of the atmosphere on a clear day in a mountain region, when a typical visibility of 30km prevails. In both cases the optical transmission is limited largely by scattering.

Similarly, the low mode-mixing which exists in the fibres reported here is remarkable. We have shown that the perfection of the liquid-core waveguide is such that a single mode may be transmitted over several hundred metres, even though the fibre is capable of supporting some 8000 modes. This is equivalent to obtaining single-mode propagation in a 100GHz H_{01} millimetre waveguide, some 30cm in diameter and 3000km long! By virtue of the highly-stable pulling process, it clearly proves easier to make a near-perfect waveguide at optical frequencies than at millimetre wave frequencies.

The work presented in this thesis has concentrated on the optical transmission medium; it has shown the technical feasibility of high-capacity fibre optical communication systems. The desired transmission objectives have largely been met by the development of the phosphosilicate fibre, while a much clearer understanding of fibre propagation has resulted from the work described in Section II.

However, there are a number of other requirements which have to be met before fibres may be regarded as a practical communications alternative to the present copper-based systems. T-junctions, splices, demountable connectors and terminations are just a few examples of the associated components which have yet to be developed. Moreover, considerable research is required into fibre strength and cable design. Investigations in these areas are continuing in our own laboratories and elsewhere.
P1.1 Relaxation processes in glasses as shown by optical attenuation experiments

P1.2 The preparation of multimode glass- and liquid-core optical fibres

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P2.1 New low-loss liquid-core fibre waveguide

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P3.1 A resistance-heated high temperature furnace for drawing silica-based fibers for optical communications

P3.2 A new silica-based low-loss optical fibre waveguide

P3.3 The preparation of water-free silica-based optical fibre waveguide

P3.4 A borosilicate-cladded phosphosilicate-core optical fibre

P3.5 Optical fibres based on phosphosilicate glass

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P4.1 Launching into glass-fibre optical waveguide

P4.2 Pulse dispersion in glass fibres

P4.3 Propagation model for multimode optical-fibre waveguide

P4.4 Theory of dispersion in lossless multimode optical fibres

P4.5 Novel mode filter for use with cladded-glass and liquid-core optical waveguide

* * *
P5.1 Dispersion in low-loss liquid-core optical fibres

P5.2 Gigahertz bandwidths in multimode, liquid-core, optical fibre waveguide

P5.3 Effect of loss on propagation in multimode fibres

P5.4 Determination of mode conversion coefficients in optical fibres

P5.5 Optical fibres and the Goos-Hanchen shift

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P6.1 Mode excitation in a multimode optical-fibre waveguide

P6.2 Propagation in curved multimode cladded fibres

P6.3 Pulse dispersion for single-mode operation of multimode cladded optical fibres

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P7.1 Evaluation of material dispersion in low-loss phosphosilicate-core optical fibres
B. Luther-Davies, D.N. Payne and W.A. Gambling, Optics Communications 13, pp 84-88, January 1975.

P7.2 Zero material dispersion in optical fibres

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P8.1 Determination of core diameter and refractive index difference of single mode fibres by observation of the far-field pattern

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P9.1  Leaky rays on optical fibres of arbitrary (circularly symmetric) index profiles

P9.2  Mode transit times in near-parabolic index optical fibres

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P10.1 Determination of optical fiber refractive index profiles by a near-field scanning technique

P10.2 Length dependent effects due to leaky modes on multimode graded-index optical fibres

P10.3 Correction factors for the determination of optical fibre refractive-index profiles by the near-field scanning technique

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P11.1 Splicing tolerances in graded-index fibres

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