

## Nonlinear Switching in Strongly Coupled Periodic Dual Waveguide Couplers

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### Introduction

Ultrafast photonic switching using third-order optical nonlinearities has been reported in dual-guide nonlinear directional couplers<sup>1,2</sup> (NDC's) and in a unique rocking-filter fibre switch<sup>3</sup>. In the second device, the two modes of a highly birefringent fibre are coupled together by a periodic distortion that is induced during the pulling process by rocking the preform at the birefringent beat period. Nonlinear effects dephase the resonant condition, and switch the device. In this paper a somewhat analogous device is proposed and analysed. This is a periodically perturbed NDC that may be hundreds of coupling lengths  $L_C$  long.

Uniform directional couplers (DC's) have been of interest for some time as linear vehicles for nonlinear switching<sup>4</sup>. Those NDC's so far reported are  $1 \times L_C$  long, so that for nonlinear switching their linear "coupling index" (the difference in guided index between the odd and even normal modes) has to be of the same order as the "nonlinear index" (the change in index due to the optical nonlinearity). Under these circumstances the transverse profiles of the normal modes are significantly distorted by the nonlinearity. In the case of dual-cored fibre couplers, the obtainable nonlinear index is very small, so that  $L_C$ 's of the order of a metre are necessary to observe optical switching; this renders the linear behaviour undesirably sensitive to environmental disturbance.

By contrast, in the device proposed here the coupling index is taken to be much greater than the nonlinear index (easily arranged in dual-cored fibres with  $L_C$ 's of a few mm). In much the same way that unwanted scattering of power between orthogonally polarized modes of an optical fibre can be minimized by making the fibre birefringent (and thus placing them far apart in a wavevector sense), this increase in coupling strength means that unwanted scattering of power between the odd and even modes of the directional coupler is suppressed.

In common with, e.g., Bragg diffraction, optical superlattice coupling<sup>4</sup>, and the rocking-filter switch<sup>3</sup>, power can be scattered from one normal mode of a DC into the other one by a suitable periodic perturbation whose period  $\Lambda$  equals  $2L_C$ ; the resulting grating vector  $K=2\pi/\Lambda$  causes a Bragg-like resonance between the odd and even modes, and can i) switch power between output ports in a velocity-matched coupler and ii) phase-match a highly mismatched DC. These effects may then be dephased by the nonlinear index, and cause the NDC to switch. We develop a theory of this phenomenon based on coupling between the odd and

even normal modes of the DC. The validity of this approach rests on the assumption that the transverse profiles of the two normal modes are only negligibly perturbed by the optical nonlinearity, i.e., the nonlinear index (at a power level that would switch the weak coupler) is substantially less than the coupling index of the strong coupler.

**Derivation of scalar normal-mode coupling equations**

The NDC is taken to support one even and one odd guided mode (propagation constants  $\beta_e$  and  $\beta_o$ ). Their fields can be expressed in the dimensionless form:

$$E_q(\xi, z, t) = e_q(\xi) \exp[-j(\beta_q z - \omega t)] + c.c. \quad (1)$$

where  $q=o$  (odd) or  $e$  (even).  $\omega$  is the angular optical frequency and  $\xi$  is a coordinate normal to the propagation coordinate  $z$  in the plane containing the two guide axes. Orthogonality of the normal modes is expressed as follows:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e_p(\xi) e_q(\xi) d\xi d\eta = A_{pq} \quad (2)$$

where  $\eta$  is the third cartesian coordinate in the problem (the scalar assumption gives  $\partial/\partial\eta = 0$ ). The dimensionless field profiles  $e_e(\xi)$  and  $e_o(\xi)$  are scaled to make  $A_{ee}=A_{oo}$  a unit of area, and mode orthogonality means that  $A_{oe}=A_{eo}=0$ . In general neither  $e_e(\xi)$ ,  $e_o(\xi)$  nor the guide properties will be symmetrical.

The periodicity and the optical nonlinearity will cause either i) exchange of power, or ii) cross-and self-phase modulation between the even and odd modes. A suitable Ansatz for these interactions is:

$$E(\xi, z, t) = \sum_{q=o,e} \sqrt{(2\omega\mu P_t / A_{qq} \beta_q)} V_q E_q(\xi, z, t) \quad (3)$$

where  $\mu$  the magnetic permeability in vacuo,  $P_t$  the total power and  $V_e$  and  $V_o$  are dimensionless variables proportional to the even and odd field amplitudes; the constant under the square-root ensures that  $|V_e|^2 + |V_o|^2 = 1$  if power is conserved.

Coupled normal-mode equations may be derived for  $V_o$  and  $V_e$  thus: assume that  $\chi^{(1)}(\xi, z) = \chi_o^{(1)}(\xi) + \chi_m^{(1)}(\xi) \cos(Kz)$  describes the dual-guide and the linear periodic perturbation; put  $E(\xi, z, t)$  into the nonlinear wave equation; neglect second order derivatives of  $V_o$  and  $V_e$ ; collect the coefficients of  $\exp[-j(\beta_o z - \omega t)]$  and  $\exp[-j(\beta_e z - \omega t)]$ , multiply them by  $e_o(\xi)$  and  $e_e(\xi)$  respectively, integrate them over the whole transverse cross-section and equate the resulting expressions to zero. The following pair of equations results:

$$j \frac{dV_p}{dz} = \kappa V_q \exp(\pm j\theta z) + \left[ \kappa_{pp}^{(3)} |V_p|^2 + 2\kappa_{qp}^{(3)} |V_q|^2 \right] V_p \quad (5)$$

where  $[p,q] = [e,o]$  or  $[o,e]$ , the -ve sign is adopted when  $p=e$  and the parameter  $\theta = K - (\beta_o - \beta_e)$  describes dephasing from resonant coupling. The linear coupling constant  $\kappa$  is given by:

$$\kappa = \left\{ k_v^2 / 2 / \{ \beta_o \beta_e A_o A_e \} \right\} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi_m^{(1)}(\xi) e_o(\xi) e_e(\xi) d\xi d\eta \quad (6)$$

and the nonlinear coupling constants  $\kappa_{pq}^{(3)}$  by:

$$\kappa_{pq}^{(3)} = \left\{ 3k_v^2 \omega \mu P_t / \beta_p \beta_q A_p A_q \right\} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi^{(3)}(\xi) e_p^2(\xi) e_q^2(\xi) d\xi d\eta \quad (7)$$

where  $k_v$  is the vacuum wavevector. By adding (5)  $\times V_p^*$  to its complex conjugate, one arrives at the result  $d/dz \{ |V_o|^2 + |V_e|^2 \} = 0$ , necessary for power conservation.

Notice that, due to mode orthogonality,  $\kappa$  is zero unless the periodic perturbation  $\chi_m^{(1)}$  is non-symmetrical in the plane containing the two guide axes; coupling of power between the odd and even modes can be optimised by arranging  $\chi_m^{(1)}$  to be anti-symmetrical in this plane. No such restriction applies to the  $\kappa_{pq}^{(3)}$ 's, however, for the integrands are positive-definite. In a non-velocity-matched coupler,  $e_e(\xi)$  and  $e_o(\xi)$  are asymmetric, and  $\kappa_{oe}^{(3)}$  will be significantly smaller than  $\kappa_{oo}^{(3)}$  or  $\kappa_{ee}^{(3)}$ . Notice the interesting possibility of choosing materials so as to make  $\chi^{(3)}$  itself a function of  $\xi$ ; this could potentially lead to reductions in the coupler switching threshold.

### Switching in a uniform NDC

In this case,  $\chi_m^{(1)} = 0$ , and Eqs(5) simplify to show that there is no exchange of power between the normal modes of the coupler; only their phase velocities are affected. The distribution of power at the output ports depends on the number of coupling lengths  $L_c$  over the device length  $L$ . If  $L_c$  is changed sufficiently by the nonlinearity, power will switch from one output port to the other. This condition is expressed mathematically:

$$\left[ \kappa_{oo}^{(3)} - 2\kappa_{oe}^{(3)} \right] |V_o|^2 - \left[ \kappa_{ee}^{(3)} - 2\kappa_{eo}^{(3)} \right] |V_e|^2 = \pm \pi / L \quad (8)$$

$|V_o|^2$  and  $|V_e|^2$  are set by the condition that just one guide of the DC is excited at  $z=0$ ; thus only in a symmetrical DC will they be equal. A detailed analysis of Eq(8) reveals that as the geometry becomes more symmetrical and the coupling weaker,  $e_e^2(\xi) \rightarrow e_o^2(\xi)$ ,  $\kappa_{oo}^{(3)} \rightarrow \kappa_{ee}^{(3)}$  and  $|V_o|^2 \rightarrow |V_e|^2$ . In the limit, the LH side of Eq(8) is zero, showing that perfectly symmetrical, uniform weakly-coupled NDC's will not switch. Nonlinear switching is only possible in strongly-coupled symmetrical guides (at high threshold powers), or in asymmetrical guides, when however only a small proportion of the power can be switched.

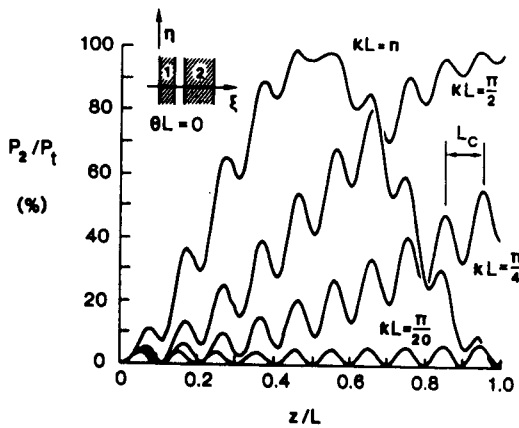
### Switching in a periodic NDC

The situation is much more attractive in periodic NDC's,

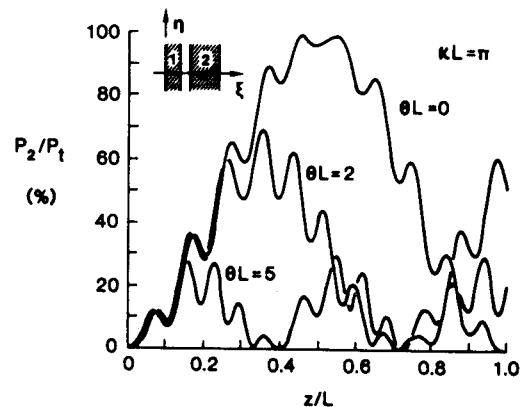
and although the full solutions of Eqs(5) are beyond the scope of this paper (we plan to present them elsewhere), it is of interest to explore briefly their linear behaviour. At least two different switching arrangements can be envisaged, according as whether the nonlinearity dephases or rephases the grating resonance. Taking the first case, and assuming the linear DC to be highly velocity-mismatched, the introduction of a periodic perturbation  $\Lambda=2L_c$  rephases the mismatch, and at  $\kappa L=\pi/2$  yields ~100% conversion (see Fig.1). If now dephasing  $\theta L>0$  is introduced (by changing the wavelength or stretching the fibre), the grating resonance is violated and the conversion efficiency drops to a small value (Fig.2). The effect of optical nonlinearity will be similar; a coupler at resonance will be switched by the nonlinear changes in the refractive index.

**References**

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**Figure 1** The effect of introducing a periodic perturbation at twice the coupling length  $L_c$  on the behaviour of a highly dephased DC. The power in guide 2 is plotted for excitation of guide 1 at  $z=0$ . The number of  $L_c$ 's over the device length  $L$  is reduced for schematic clarity; in a dual-cored fibre it would be of the order of a few hundreds.



**Figure 2** The effect of increasing levels of grating dephasing  $\theta L>0$  on the case  $\kappa L=\pi$  from Fig(1). An optical nonlinearity would have a somewhat similar effect.