THE EFFECT OF INTERNAL RESISTANCE ON AN ENERGY HARVESTER WITH CUBIC RESISTANCE LOAD

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The objective of this paper is to study the effect of the internal transducer resistance on the dynamic behaviour and the output power of an energy harvester when it is attached to a cubic load resistance. It is seen that by considering the internal resistance, the dynamic equation of a conventional nonlinear energy harvester changes so that it is not easy to separate the linear damping from the cubic damping. An analytical model is presented for an energy harvester with an internal resistance in series with a cubic load resistance. Then, a comparative study is conducted between three different cases, representing three different levels of the internal resistance for energy harvesters, respectively, with the values of 0.2Ω, 2.8Ω and 6.8Ω. For each case the performance of the system with cubic resistance is compared with its equivalent in three different conditions. The three conditions are as follow: purely linear system, referring to the condition that the load resistance is linear and there is no internal resistance in the system; linear system, referring to the energy harvester with internal resistance in series with linear load resistance; and purely cubic system, referring to the system with cubic load resistance and negligible internal resistance. The comparisons between systems are carried out for both variable amplitude and variable frequency input excitation. It is seen that, similar to the linear systems, the internal resistance of the energy harvester provides an upper limit for the electrical damping of a device with cubic load resistance. By increasing the amount of the internal resistance, employing the cubic damping to increase the output power of system becomes less effective since the system behaves in a more linear manner.

1. Introduction

Harvesting energy from ambient vibration, as a ubiquitous source of energy, has been the subject of significant research in the last decade [1, 2]. However, many of the suggested vibration energy harvesters in the literature have two fundamental limitations. Firstly, they are designed to operate at certain frequencies and any difference between the excitation frequency and the natural frequency of the harvester will dramatically reduce the efficiency of system [3]. Secondly, due to the physical constraints, the oscillating mass only moves within a specified range [4, 5]. Therefore, in order to achieve a high performance, harvesters are designed for their maximum excitation level. However, when the transducer is excited below its maximum excitation level, it would operate in a sub-optimum condition. In contrast with the first issue, the second has received less attention in the literature, which is discussed in this paper. Recently, a nonlinear cubic electrical damping has been introduced to extend the dynamic range of energy harvesters [6]. It was shown that a shunted cubic resistance causes a cubic damping which is proportional to fourth power of the system transduction coefficient. However, the internal resistance of the energy harvester was not included in this analysis. This paper investigates the effect of the internal resistance of the energy harvester on the dynamic behaviour and the average power. In this paper, the dynamic model of an energy harvester when an internal resistance is in series with a cubic resistance is first presented. Then, the effect of the internal resistance is studied on the performance of system with linear and cubic load subjected to the different excitation levels and frequencies.

2. Energy harvester modeling

A schematic diagram of a base excited energy harvester using an electromagnetic generator is shown in Fig. 1. In this diagram, m is the seismic mass, k is the spring stiffness, c_m represents the mechanical viscous damping, K_t is called the transducer coefficient of generator, f_e is the electromotive force and v_e is the produced voltage by the generator. Also, in this schematic, the generalized load R can be either linear or cubic which henceforth are, respectively, referred to as R_l, and R_c. The governing differential equation of motion for this system with respect to the relative displacement of the seismic mass, i.e. $z(t)=x(t)-y(t)$, is

(1) $$m\ddot{z}(t)+c_m\dot{z}(t)+f_e(t)+kz(t)=-m\ddot{y}(t).$$

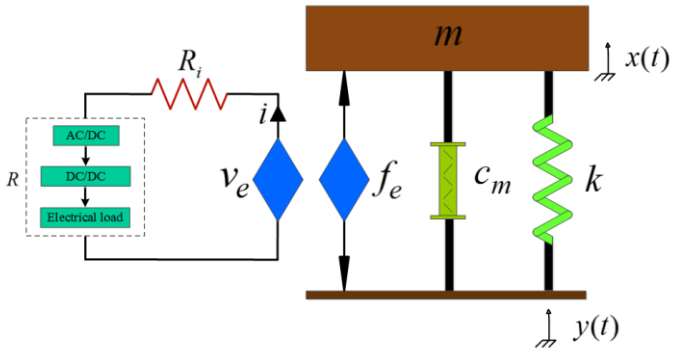

Figure 1. Schematic of an energy harvester.

2.1 Energy harvester with linear load resistance

If the load resistance of system shown in Fig. 1 is linear, then the current through the load is given by

(2) $$i(t)=\frac{v_e(t)}{R_l+R_i}=\frac{K_t\dot{z}(t)}{R_l+R_i},$$

and the electromotive force is

(3) $$f_e(t)=K_t i(t)=\frac{K_t^2}{R_l+R_i}\dot{z}(t).$$

Hence, the dynamic equation of system can be re-written as

(4) $$m\ddot{z}(t)+c_m\dot{z}(t)+c_e\dot{z}(t)+kz(t)=-m\ddot{y}(t)$$

where $c_e = K_t^2 / (R_l + R_i)$ can be called the electrical damping of system. The minimum amount of electrical damping, $c_e = 0$, is obtained when the generator is open circuit and hence $R_l \to \infty$. However, the maximum electrical damping can be achieved by short circuiting the generator, i.e. $R_l = 0$. In this case, the maximum electrical damping is $c_e = K_t^2 / R_i$ and as it is seen the internal resistance of generator provides an upper limit for the maximum achievable electrical damping. Energy harvester with cubic load resistance

If the energy harvester is attached to a cubic load resistance, based on the Kirchhoff's circuit laws, the voltage is the sum of a linear and nonlinear function of current. By solving Eq. (5), the current is obtained as

(5)
$$v_e = R_i i + (R_c i)^{\frac{1}{3}}$$

which resulted in

(6)
$$i = \frac{1}{6}\frac{12^{\frac{1}{3}}\left(R_c R_i^2\left(-9v_e + \sqrt{3}\sqrt{\frac{4R_c + 27R_i v_e^2}{R_i}}\right)\right)^{\frac{1}{3}}}{R_i^2} - \frac{1}{6}\frac{12^{\frac{2}{3}}R_c}{R_i\left(R_c R_i^2\left(-9v_e + \sqrt{3}\sqrt{\frac{4R_c + 27R_i v_e^2}{R_i}}\right)\right)^{\frac{1}{3}}} + \frac{v_e}{R_i}.$$

Equation (6) can be simplified by denoting $G = R_c / R_i$,

(7)
$$i = \frac{12^{\frac{1}{3}}}{6}\frac{\left(G\left(-9v_e + \sqrt{12G + 81v_e^2}\right)\right)^{\frac{1}{3}}}{R_i} - \frac{12^{\frac{2}{3}}}{6}\frac{G}{R_i\left(G\left(-9v_e + \sqrt{12G + 81v_e^2}\right)\right)^{\frac{1}{3}}} + \frac{v_e}{R_i}$$

Replacing i from Eq.(7) in Eq.(2) and Eq.(3), the electrical force f_e, can be written as a function of $\dot{z}$ as

(8)
$$f_e = \underbrace{\frac{12^{\frac{1}{3}}\left(G\left(-9(K_t\dot{z}) + \sqrt{12G + 81(K_t\dot{z})^2}\right)\right)^{\frac{1}{3}}K_t}{6R_i} - \frac{12^{\frac{2}{3}}GK_t}{6R_i\left(G\left(-9(K_t\dot{z}) + \sqrt{12G + 81(K_t\dot{z})^2}\right)\right)^{\frac{1}{3}}}}_{A} + \frac{K_t^2\dot{z}}{R_i}$$

It can be shown that the nonlinear term presented in Eq. (8), marked as *A*, is always negative. Therefore, the electromotive force in a system with cubic resistance has an upper limit equal to $(K_t^2\dot{z})/R_i$ and in the other words, an energy harvester with cubic damping cannot contribute a damping force greater than the amount of force determined by the internal resistance of system. The parameters of a simulated energy harvester with cubic resistance is shown in Table 1. Fig. 2.(i) shows the nonlinear part of the electrical damping force as a function of the relative velocity and $G = R_c / R_i$. It is seen that the nonlinear part of the electrical damping force is increased by reducing the ratio of

R_c / R_i or relative velocity, however, it is always negative. Also, it is seen that for a given relative velocity, by increasing the ratio of G, the total electrical damping is decreased. Fig. 2. (ii) shows the total electrical damping force over the relative velocity. This ratio can be interpreted as the equivalent linear damping. It is seen that the equivalent linear damping cannot exceed a maximum limit. In theory, this maximum limit value is $c_e = K_t^2 / R_i$, which is equal to 3.65N.s. m^{-1} for the simulated energy harvester shown in Table 1. The coupling coefficient of the selected system is $\Lambda_{em} = K_t^2 / R_i c_m$ which is equal to 73.14. This value is based on the discussion presented in [4], and indicates that the selected system is a very well coupled system for energy harvesting purposes.

Table 1. Parameters of the simulated energy harvester.

Parameter	Value
Mass (m)	1 kg
Generator resistance (R_i)	2.8 Ω
Mechanical damping (c_m)	0.05 N.s. m^{-1}
Spring stiffness (k)	$4\pi^2$ N.m^{-1}
Coupling coefficient (K_t)	3.2 V.s.m^{-1}

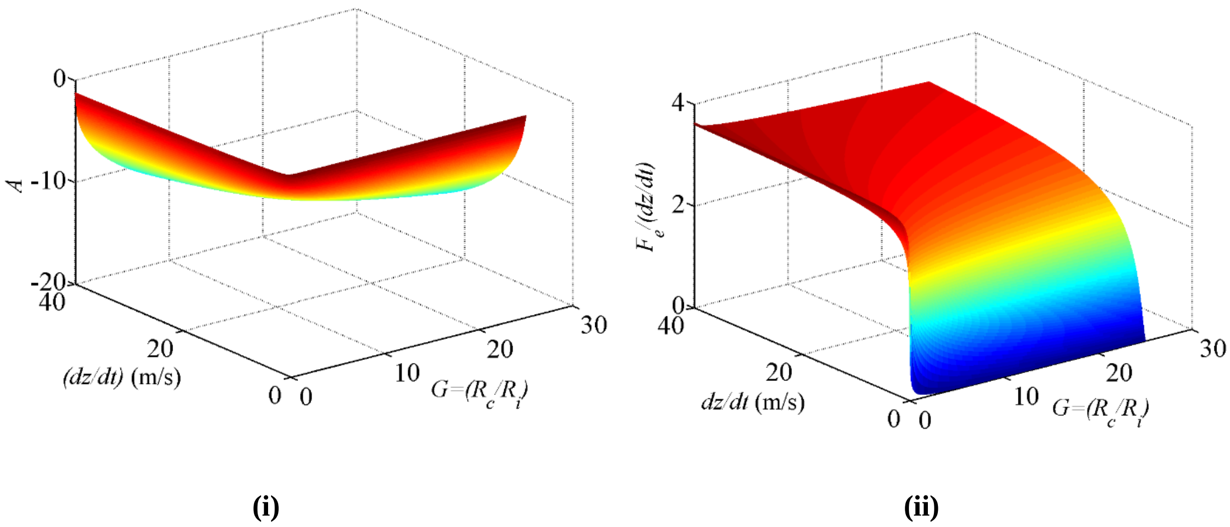

Figure 2. (i) Nonlinear electrical damping force, A, versus the variation of G and relative velocity, **(ii)** Electrical damping force over velocity for different values of G and relative displacement.

3. Comparing the performance of the energy harvester with different internal resistance

In this section a numerical study is conducted to investigate the effect of the internal resistance of system on the dynamic range of both linear and nonlinear energy harvesters. For this purpose, three energy harvesters are selected with different internal resistances, henceforth referred as Case (1), Case (2) and Case (3). For each case, the performance of the energy harvesters are compared in four different conditions. These four conditions are as follow: an energy harvester with no internal resistance and only linear load resistance, henceforth referred as condition (a), an energy harvester with internal resistance in series with a linear load resistance, henceforth referred as condition (b), an energy harvester with internal resistance in series with cubic load resistance, henceforth referred as condition (c), and an energy harvester with purely cubic load resistance and negligible internal resistance, henceforth referred as system (d). In this study, it is assumed that the maximum permissible throw for the systems with physical parameters shown in Table 1 is $Z_{max} = 1\text{m}$ and the maximum level of excitation is $Y_{\max} = 0.246\,\text{m}$. Also, the natural frequency of system is $f_n = \sqrt{(k/m)}/2\pi = 1\text{Hz}$. The internal and load resistance for all selected systems are shown in Table 2. For each case the load values, either linear or cubic, are selected so that in all conditions, i.e. (a) to (d), the energy harvesters have the same throw when they are subjected to the maximum excitation level with excitation frequency matches the natural frequency of system. Fig. 3 shows the relative displacement and harvested power of different cases in all four conditions as a function of input displacement, when the systems are excited at their natural frequency. It is seen that for all four systems of each case, the parameters of the electrical circuit have been chosen to have the same throw for the maximum input displacement amplitude, i.e. when $Y = Y_{\max}$. From Fig. 3.(i) to Fig. 3.(iii), it is seen that in all cases the relative displacement for systems with linear resistance load are similar, i.e. condition (a) and (b). It is due to the fact that in linear system, the damping coefficient is not a function of input excitation. However, for the systems with nonlinear resistance, i.e. conditions (c) and (d), for the level of excitations below $Y_{\max}$, the relative displacement is dramatically attenuated in cases with higher internal resistance. This is due to the fact that in these cases, the electrical damping is a function of input excitation amplitude. Also, from Fig. 3.(iii) it can be seen that in condition (c), i.e. the system with cubic load and internal linear resistance, for the level of excitations close to $Y_{\max}$, the relative displacement of the system is close to the linear conditions, i.e. conditions (a) and (b); however, by reducing the level of excitations, the dynamic behaviour of system tends to the system with purely cubic damping, i.e. (d). Fig. 3.(iv) to Fig. 3.(vi) illustrate the output power of energy harvester for the simulated cases. It is seen that in all conditions the presence of the internal resistance in the energy harvester reduces the output power of systems, which is due to the power dissipation in the internal resistance. Since the response of the linear harvesters linearly depends on the amplitude of the excitation, the reduction ratio of the output power as a result of the internal resistance would be the same for all input levels. However, for the nonlinear conditions, the output power of the system with purely cubic resistance, when $Y = Y_{\max}$ is much greater than the condition with internal resistance and cubic load resistance. By reducing the level of excitation, the output power of the condition (d), is dramatically decreased.

Fig. 4 shows the relative displacement and output power of the energy harvesters for case (1) and case (3) as a function of excitation frequency for two levels of excitation amplitude, $Y = 0.1Y_{\max}$ and $Y = Y_{\max}$. It is seen that for the level of excitation $Y = Y_{\max}$, the presence of internal resistance in the system with cubic resistance causes a slightly shift in the resonance frequency of system. For instance in Fig. 4.(ii) and Fig. 4.(iv), the resonance frequency of the linear systems are 0.98 Hz, whereas the resonance frequency system with purely cubic resistance, i.e. (d), is 0.92 Hz. However, as it is evident from Fig. 4.(i) and Fig. 4.(iii), this effect is less obvious when $Y = 0.1Y_{\max}$.

Table 2. Parameters of the simulated energy harvester.

	Case (1)			Case (2)			Case (3)		
	$R_i(\Omega)$	$R_l(\Omega)$	$R_c(V^3/A)$	$R_i(\Omega)$	$R_l(\Omega)$	$R_c(V^3/A)$	$R_i(\Omega)$	$R_l(\Omega)$	$R_c(V^3/A)$
(a)	-	6.8	-	-	6.8	-	-	6.8	-
(b)	0.2	6.6	-	2.8	4.0	-	6.6	0.2	-
(c)	0.2	-	1900	2.8	-	410	6.6		0.1
(d)	-	-	1980	-	-	1980	-		1980

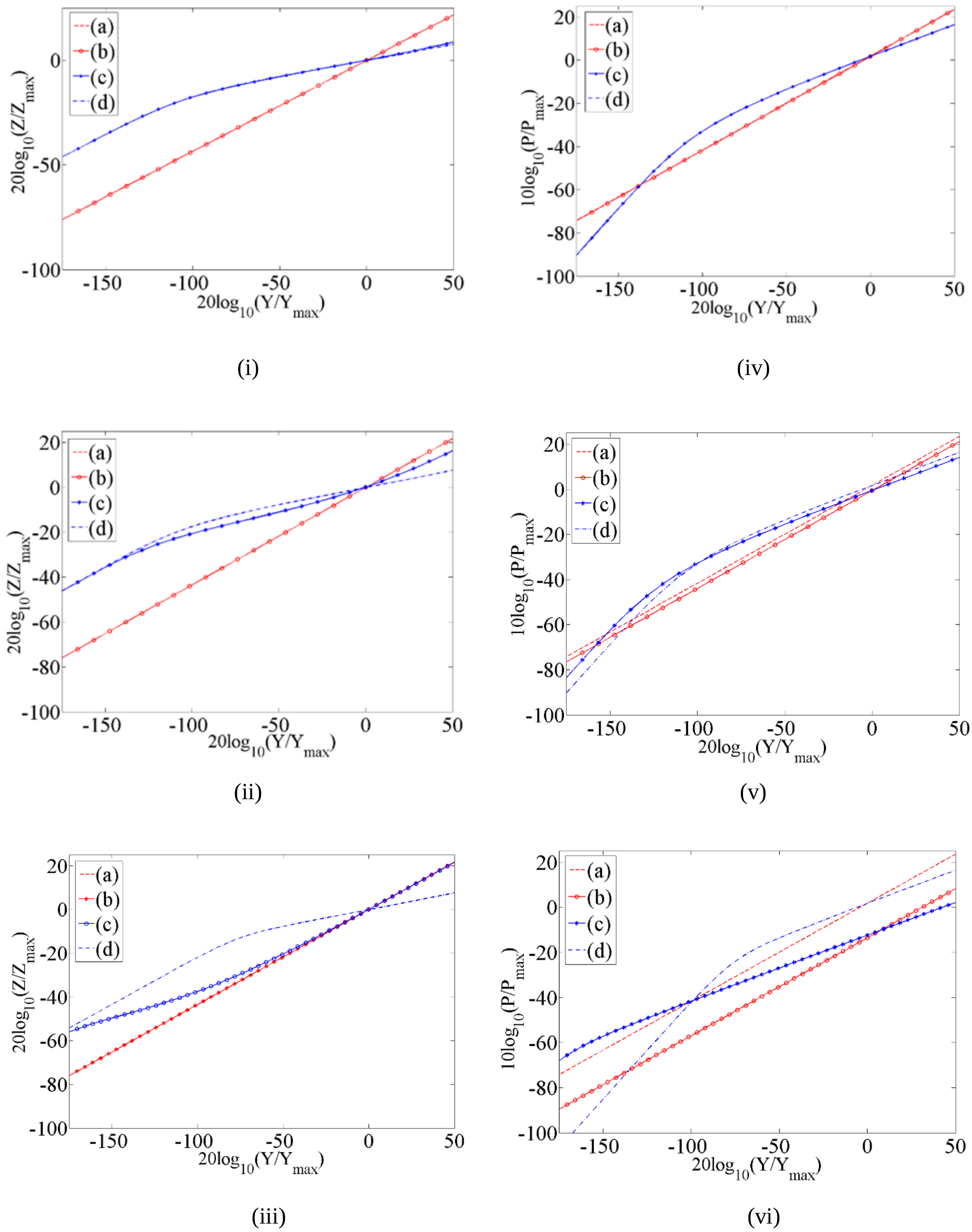

Figure 3. (i) Relative displacement for four systems shown as Case (1), **(ii)** Relative displacement for four systems shown as Case (2), **(iii)** Relative displacement for four systems shown in as Case (3), **(iv)** output power for four systems shown as Case (1), **(v)** output power for four systems shown as Case (2), **(vi)** output power for four systems shown as Case (3).

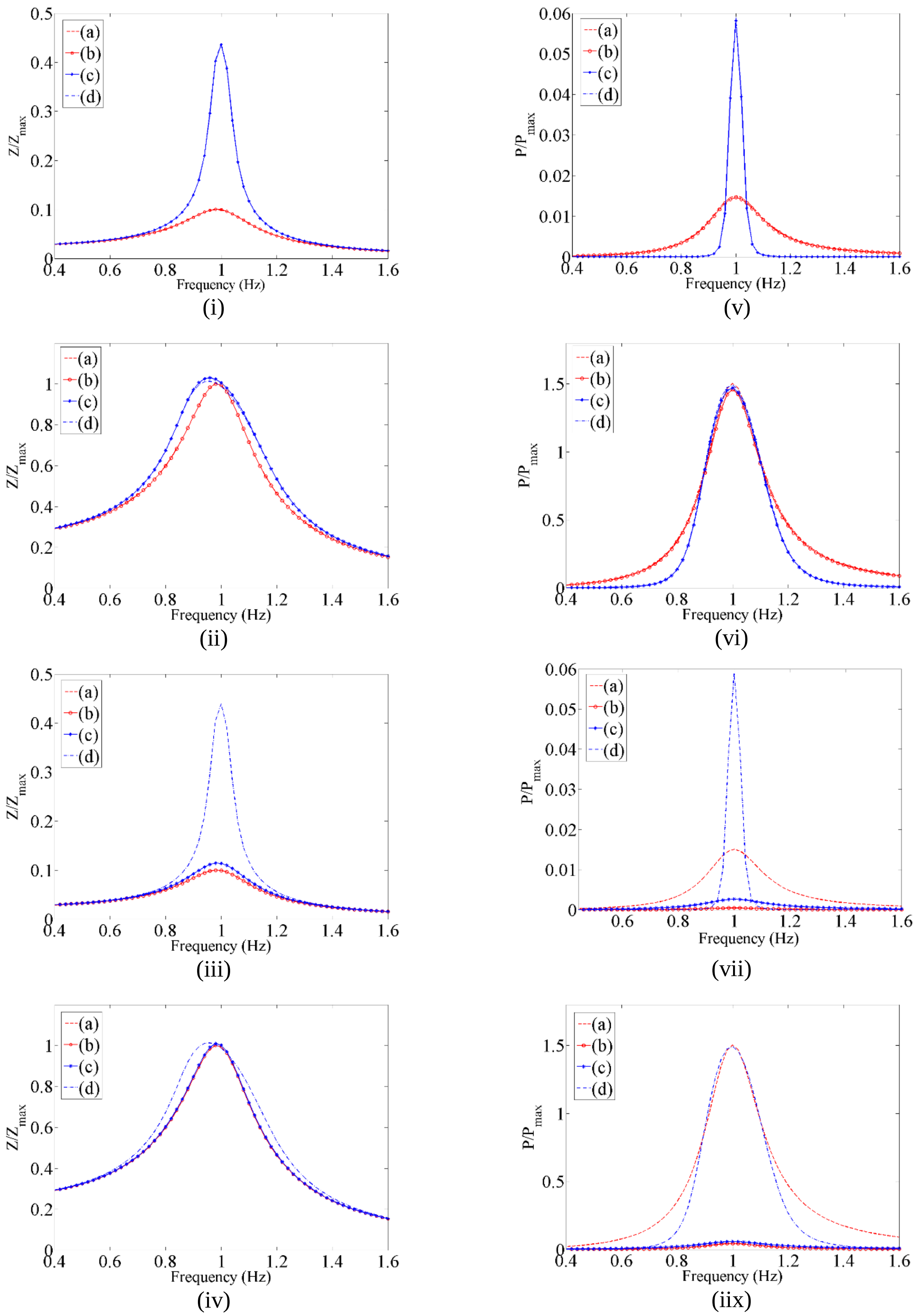

Figure 4. Relative displacement versus excitation frequency in **(i)** case (1) when $Y = 0.1Y_{max}$, **(ii)** case (1) when $Y = Y_{max}$, **(iii)** case (3) when $Y = 0.1Y_{max}$, **(iv)** case (3) when $Y = Y_{max}$, and output power versus frequency of excitation in **(v)** case (1) when $Y = 0.1Y_{max}$, **(vi)** case (1) when $Y = Y_{max}$, **(vii)** case (3) when $Y = 0.1Y_{max}$ and **(iix)** case (3) when $Y = Y_{max}$.

Also, form the output power profiles shown in Fig. 4.(v) to Fig. 4.(iix), it is seen that for all conditions regardless the value of the internal resistance, the maximum output power is obtained when the excitation frequency matches the natural frequency of the system. Also, for both level of excitations, the systems without internal resistance can produce more power when they are excited at their natural frequency. However, when $Y = 0.1Y_{\max}$, it is seen that the output power of system with purely cubic resistance, i.e. (d), is dramatically decreased for the excitation frequencies away from the natural frequency. This is due to the high quality factor (Q factor) of the system with nonlinear resistance at low level of excitations. It is seen that presence of internal resistance reduces the Q factor of energy harvester at low level of excitations and makes the performance of system closer to the linear system.

4. Conclusion

This paper studies the dynamic equation of an energy harvester with a linear internal resistance in series with a cubic load resistance. It is seen that the internal resistance in systems with nonlinear load provides an upper limit for the electrical damping. The simulation results of three energy harvesters with different level of internal resistance indicates that in a system with linear internal resistance in series with a load cubic resistance, the system behaves in a more linear manner at higher level of excitation. However, by reducing the excitation amplitude, the behaviour of system will be more similar to the systems with purely nonlinear damping. Also, it is illustrated that at the lower level of excitations, the system with cubic resistance is more sensitive to the frequency variation of the vibration source, compared with the linear system. This sensitivity is reduced by increasing the internal resistance of system. The result of this study reveals that employing an intelligent control system to switch between linear and cubic load resistance to maximize the output power of the system in variable amplitude and frequency excitation condition, which is the case in the real environment, is desirable. It is seen that the internal resistance can affect the type and quantity of the optimal load resistance. Therefore, the internal resistance of energy harvester plays an important role in selecting the optimal load resistance for energy harvesters and it should be considered in the design of the control strategy.

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