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# How varies optimal welfare pricing with income distribution? The case of the untolled alternative

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**Abstract**

In some countries, such as Spain, it is very common that in the same corridor there are two roads with the same origin and destination but with some differences. The most important contrast is that one is a toll highway which offers a better quality than the parallel road in exchange of a price. The users decide if the price of the toll worth to pay for the advantages offered. This problem is known as the untolled alternative and it has been largely studied in the academic literature, particularly related to economic efficiency and the optimal welfare toll. However, there is a gap in the literature academic regarding how affects income distribution to the optimal toll. The main objective of the paper is to fill this gap.
In this paper a theoretical model in order to obtain the optimal welfare price in a toll highway that competes for capturing the traffic with a conventional road is developed. This model is done for non-usual users who decide over the expectation of free flow conditions. This model is finally applied to the variables we want to focus on: average value of travel time ($VTT$) which is strongly related with income, dispersion of this $VTT$ and traffic levels, from free flow to congestion. Derived from the results, we conclude that the higher the average VTT the higher the optimal price, the higher the dispersion of this VTT the lower the optimal price and finally, the more the traffic the higher the optimal toll.



*Keywords:* Untolled alternative; Optimal price; Welfare; Income distribution

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## 1. Introduction

The problem of road pricing has been largely studied. It is well acknowledged that in order to achieve the maximum social welfare, users must internalize the externalities they produce and do not perceive through a toll. However, that toll can harm users with low income. Depending on the objective function to be optimized (e.g. maximize welfare, maximize social equity, amortize the construction of the road, etc) the optimal toll might vary substantially. The academic literature about pricing, efficiency and equity is vast and diverse. However, as far as we have found, there is a gap in the literature regarding the optimal price where a road and a highway with different quality characteristics compete for capturing the traffic in a corridor. Particularly we did not find any research estimating the optimal welfare price when varying the conditions of average Value of Travel Time ($VTT$) and dispersion of $VTT$. The objective of the paper is to fill this gap. Apart from the introduction, the paper contains four sections more. Firstly, we conduct an extensive literature review on the subject and set the objective followed by the research. Secondly, we develop the methodology and explain the hypotheses adopted in the model. Thirdly, we implement it in a study case whose aim is to fill the objectives previously established. Finally, we offer a set of conclusions and lessons.

## 2. Literature Review

The problem of an untolled alternative has been largely studied. One of the first researches done is the mathematical demonstration carried out by Marchand (1968). He focuses on the case in which is impossible to get the *first best*. The theoretical solution to the problem is obtained through lagrangians. However, this model is not applied to any case study and it is static, so it does not take into account peak hours demand.

Verhoeff, Nijkamp and Rietveld (1996) update and improve the model. It is remarked that from the microeconomic theory, in order to get the optimal efficiency distribution of traffic, the toll would be the difference in marginal cost between the two options. They calculate two functions to optimize: the objective of the first is to obtain the maximum welfare, whereas the second function intends to get the maximum revenue from toll. They find for the welfare optimization that setting a price in one road is completely inefficient in the case that the cost for driving in the toll road is really low.

The point of view gave by Braid (1996) is complementary since he strictly focuses on congestion. The users always travel through the option less costly for them, and their cost depends on three parameters: schedule delay cost, the value of time waiting in the queue to access to the highway, and the toll. He assumes the hypothesis that there is no congestion and it is impossible to be delayed if the traffic level does not reach the capacity of the road. It is demonstrated that when the schedule delay cost is a V- shaped function and both roads have the same capacity, then the two out three users would drive through the toll highway and the gains of welfare would be the two thirds of the hypothetical gains with the *first best* solution.

Verhoeff and Small (1999) focus on the consequences of pricing in the second best option with heterogeneity between public and private managers of infrastructure. The transportation network selected by the authors has two stretches. The first stretch is divided into two paths which have the same origin and destination but with different alternatives. The second stretch is the same for both alternatives. The research concludes that considering heterogeneity in users helps to avoid some mistakes.

Liu and MacDonald (1998) study the problem by adding two periods of time: the peak hour and out of it. The study case consists in adding two toll lanes to an existing bridge of four lanes. They find ninety percent more in gains of welfare when the whole bridge is tolled than when only two lanes are tolled.

The same authors (Liu and MacDonald, 1999) improve their model by including a relationship between the periods previous to the peak hour and the peak hour. They compare a transportation network without any toll, completely tolled on only tolled in some stretches. They find important differences between the scenario without toll and the scenario in the *second best*, as a decreasing of total traffic or a switch in the peak hour from the toll roads to the free roads. They also find welfare loses when comparing this solution with the *first best*.

Verhoeff (2002) establishes a new model by introducing linear functions of cost and demand. This methodology can be applied to any network when analyzing the problem of finding both, the *first best* and the *second best* solution.

Light (2009) establishes the minimum social cost to explain how the optimal price would be obtained. However the hypothesis of the author is to study not only the optimal price, but also the investment in roads. Yang and Zhang (2003) develop a numerical model and include two examples of transportation networks. They focus on looking for the *second best*, due to the impossibility of having the *first best*. They obtain where the toll should be imposed, to whom and the price of it.

Salas, Robusté and Saurí (2009) find the optimal price in the metropolitan area of Barcelona by calculating the social cost and the simulation of several scenarios. They test what would happen in the area after the introduction of the toll with three different scenarios: decreases of traffic of 5, 10 and 15 percent. The best scenario it would be the 10 percent decrease.

So, as far as we have found, there is a gap in the literature regarding the optimal price in a tolled highway that competes with a conventional road. Particularly we could not found any research setting the optimal welfare price when varying the conditions of VTT and dispersion of VTT. The objective of the paper is to fill this gap.

## 3. Methodology, Assumptions and Case Study

The solution to the problem is to set the price that minimizes the total cost for society defined as follows:

$$SC = UC + EC + HOB + GB \qquad (1)$$

Where:

$SC$ – Means the total social cost. This is the objective function that has to be minimized.

$UC$ – Is the total cost that the users bear per trip. It is divided into four terms which are travel time, toll, fuel cost and maintenance of the vehicle.

$EC$ – These are the externalities produced by the vehicles. They are the summation of environmental cost, i.e. gas emissions, noise and so on, and accidents.

$HOB$ – This is the net operating profit for the road operator either public or private. They are tolls paid in the highway minus maintenance cost of the toll highway.

$GB$ – This is the result for the Government or Public Administration with the responsibility of maintaining the conventional road. It is calculated as the summation of tolls paid in the road plus taxes recovered from fuel minus maintenance cost of the road.

The main characteristics of the model are the following:

- The interurban network is very simple: a toll highway and a parallel conventional road that might be tolled as well. There are no alternative transport modes. This network is similar to the conditions in many interurban corridors where there are two options for travelling from a particular pair origin – destination. The figure 1 can help to explain the model network.
- Users are supposed not to be familiar with the traffic conditions in the corridor, therefore we assume that they travel under the expectation of free flow conditions. So, the travel time that exceeds their free flow expectations will not be taken into account by them at the time of adopting the travel decision. In other words, each cost that exceeds the cost of free flow conditions is considered as an externality in SC. However, once they decide to drive, they will automatically internalize this extra cost because they will be directly affected by it.
- The users will decide on the basis of their value of travel time (VTT) under free flow conditions, the gasoline cost under free flow conditions and toll in both alternatives.
- The potential users will be divided into 100 groups. Each group will have different VTTs and a daily expenditure limit for transport. If the expected cost for the group of users exceeds this limit, that group will end up not traveling. Otherwise, they will travel and must decide between road and highway.
- Finally, it is important to note that the assumption related to the way users decide where they travel is right, because they are non-usual users of an interurban corridor and therefore they cannot know any conditions in both roads, neither real traffic conditions. However, if the model were adapted for commuters of an urban transport network, it would be necessary to change this assumption. At the end of this section we briefly describe some future research lines for the present model.

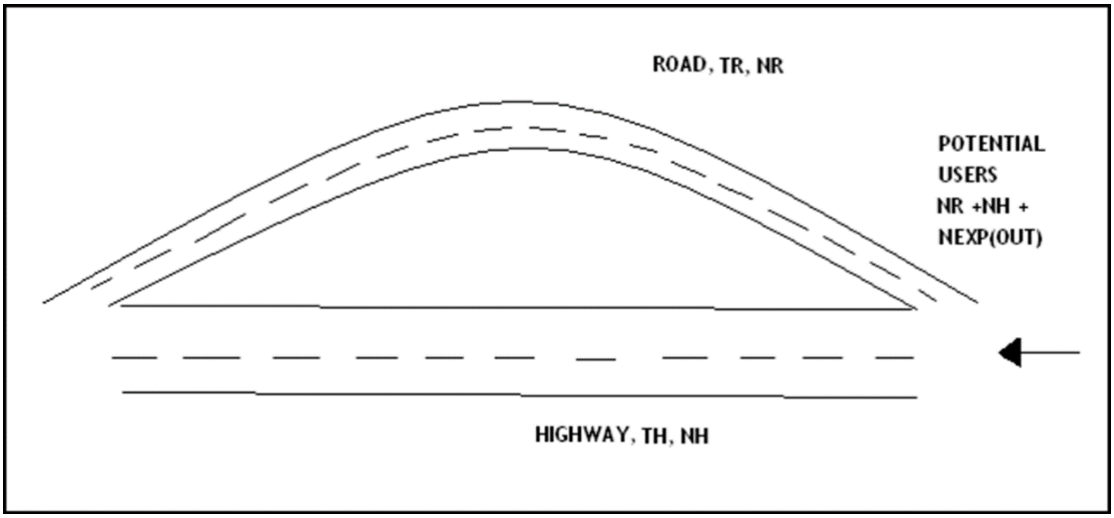


Figure 1. Transport Network

The results of the research are obtained for 25 Lognormal income distributions of potential users. The aim of this comparison is to study the optimal tolls for different scenarios of potential traffic, from free flow to congestion. VTT is strongly related to income so the results can be associated to different societies in terms of income per capita and income distribution.

The potential users of the corridor are divided according to their income and this is depicted by the letter $i = 1, \dots, n$ $(n = 100)$. The letter $j$ represents the different options for the potential users and it can be $H$ (Toll Highway) or $R$ (Conventional Road). As we have explained previously, the potential users have a daily expenditure limit for transport, which is defined as follows:

$$\text{If } T^j + GC^j > \Theta * I_i \; \forall j, \text{j} \rightarrow N_i = N_{i,exp} \text{ do not travel} \tag{2}$$

Where $T^j$ is the toll to be paid in the option $j$, $GC^j$ is the expected gasoline cost in the option $j$ under free flow conditions, $\Theta$ is the limit of expenditure in transport, $I_i$ is their daily income of the group $i$ and $N_i$ is the number of users of social group $i$. Therefore, the users pushed out the transportation system ($N_{exp}$) can be expressed as the total potential users minus the real users in the corridor:

$$\mathrm{N_{exp}} = \mathrm{N_{Pot}} - (\mathrm{N^H} + \mathrm{N^R}) \tag{3}$$

Where $N_{Pot}$ is the potential demand in the corridor, $N^H$means the number of users travelling through the highway and $N^R$ represents the users travelling through the conventional road. Once the group of users has decided if they travel or not, they decide where travelling according to equation of the perceived cost for them (4):

$$DC_i^j = T^j + \varphi\, x\, GC^j + VTT_i\, x\, ETT^j \tag{4}$$

Where $\mathrm{DC_i^j}$ is the decision cost for the social group $i$ in the option $j$. $VTT_i$ is the value of travel time for the group of users $i$ and it is strongly related with the income, $I_i$. $\varphi$ is a coefficient that expresses the user's perception with respect to the cost of gasoline. Finally, $ETT^j$ is the expected travel time under free flow traffic conditions in the road $j$. For each group of potential users who have decided to travel, it is calculated their $DC_i^j$ and they will travel through the road with less expected cost for them.

The developed model does have its own input variables (VTT distribution, tolls and Potential users), some parameters which do not vary such as the slope or taxes paid by fuel consumption, and the output variables (S.C., Users Pushed out and traffic distribution). In the table below (1), the selected range for the variables and the value of the parameters can be seen:

Table 1. Parameters and variables selected for the case study

| ***Variable*** | ***Value*** | ***Source*** |
|---|---|---|
| $VTT_i$ | 25 lognormal distributions with 5 different average (μ) and 5 different variances ($\sigma^2$) | Compilation after studying Small, Winston and Yan (2002); Calvo, Cortiñas and Sánchez, (2012);De Rus et al (2010); and Fosgerau (2006) |
| $T^H$ | Range from 0 to 8€ | Own calculations |
| N | From 3.000 to 7.000 veh. | Compilation after studying Kraemer et al. (2004) |
| ***Parameter*** | ***Value*** | ***Source*** |
| $DIST^H$ | 90 Km | Compilation after studying Vassallo, Ortega y Baeza (2012) |
| $DIST^R$ | 100 Km | Compilation after studying Vassallo, Ortega y Baeza (2012) |
| Free flow speed Highway | 120 Km/h | Compilation after studying Kraemer et al. (2004) |
| Free flow speed road | 100 Km/h | Compilation after studying Kraemer et al. (2004) |
| $CAP^H$ | 2,400 users/lane/hour. 2 Lanes | Kraemer et al. (2004) |
| $CAP^R$ | 1,700 users/lane/hour. 1 Lane | Kraemer et al. (2004) |
| φ | 0.9 | Compilation after studying Matas, Raymond and Ruiz (2012), and Huang and Burris (2013) |
| Θ | 20% | Litman (2007) |

Apart from these parameters, is important to know that a penalty for travel time that exceeds the free flow has been considered (Wardmand and Ibañez, 2012), that the gasoline taxes are 45% and the price of it is 1.5 € (Ortega, Gomez and Vassallo, 2012), and finally for gasoline consumption and the calculation of externalities the two stretches are completely flat (Vassallo, Lopez and Martinez, 2012; Monzón et al, 2012).

In order to know the optimal toll, a simulation several scenarios has been done for each distribution of VTT. The level of potential users tested has varied from 3,000 to 7,000 in increases of 500, and we also tested tolls from 0€ to 8€ in increases of 0.1€. After this calculation, we know the SC for each potential traffic and toll, and it is only necessary to find the minimum.

This research can be improved with futures lines of research. The most important improvements detected are the inclusion of a logit model in the equation of the decision cost of users, a different and more complex expulsion criterion, and add a mathematical demonstration of the general study case such in the case of the theoretical solution through lagrangians. However, these improvements can be included in a further model, where the users are usual and they perfectly know the traffic in the corridor, and consequently they would decide over a basis of both real travel time and monetary cost.

## 4. Results

The results of the research are divided into five sections. Firstly, in order to know how the model works and how the optimal toll is obtained, the evolution of Social Cost for different tolls and different levels of traffic is represented. Secondly, we analyze the effect of average VTT in the optimal tolls. In the third section we find out the effect of variance over the optimal price. In section four, we study the combination of the three variables analyzed in the model: average VTT (μ), variance VTT ($\sigma^2$) and potential traffic. Finally, in section five the model is tested with a VTT distribution with the income characteristics of Spain.

### *4.1. Evolution of Social Cost and traffic*

In order to know how is the evolution of the Social Cost for each toll highway, figure (2) is depicted. This figure (2) is made up of four graphs, and each one does have different levels of potential traffic. The four levels of potential traffic chosen are 3,000, 4,500, 6,000 and 7,000, and therefore the results range from free flow to congestion. The left bar indicates the total Social Cost in Euros and the horizontal bars represents the toll in the highway. The lognormal distribution of VTT chosen for this section does have an average (μ=19€/hour) and variance ($\sigma^2$=249.28) intermediates within the range previously selected. This distribution does not have any users pushed out, and the results depend on the distribution of traffic between the two roads and the externalities in both.

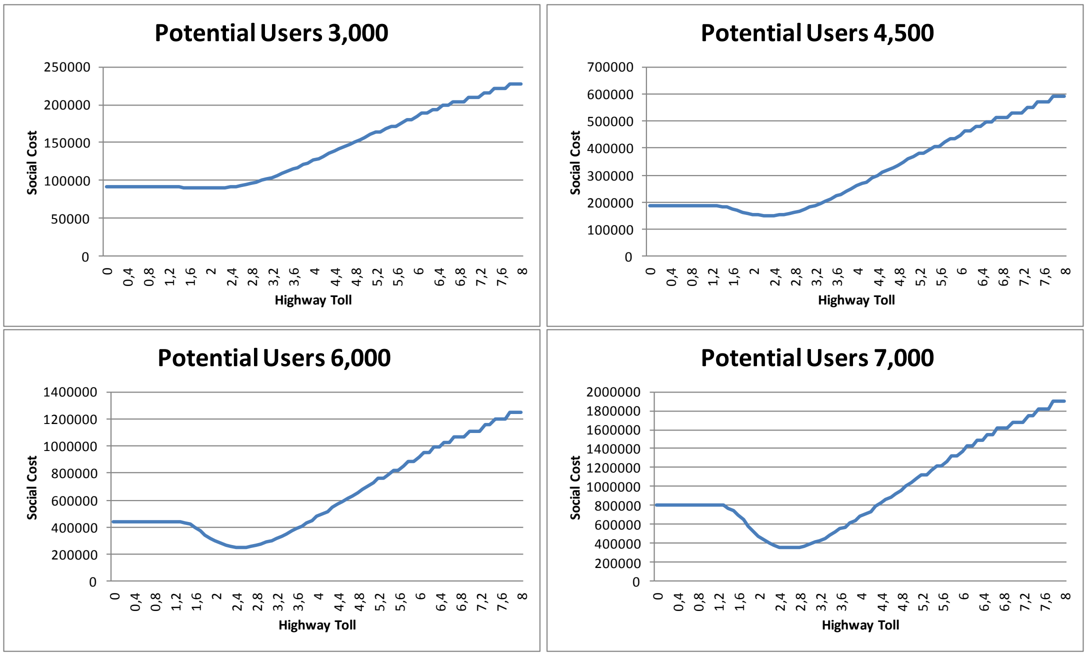


Figure 2. Evolution of Social Cost and traffic

Several questions can be answered from figure 2. Firstly, it is clear that the more the potential traffic, the greater the Social Cost and the optimal toll. This leads to an increase of the use of the road with the increase of potential traffic. In other words, the more the congestion, the higher the share of the traffic in the road. This result is entirely logical, given that more users in the corridor mean greater congestion and therefore a higher cost of fuel, travel time, road maintenance and externalities. Moreover, for higher levels of potential traffic, in order to avoid important losses of welfare it is crucial to set correctly the toll.

## *4.2. Effect of average VTT*

In this section we analyze what is the effect of average VTT in the optimal toll. In order to study this effect we have selected the medium variance of the range ($\sigma^2$=249.28) and the five averages tested in the model (μ=13, 16, 19, 22 and 25€/hour). In this way, we can perfectly isolate the effect of μ in the optimal toll. Figure (3) represents the evolution of the optimal toll with different levels of potential traffic for the five distributions chosen.

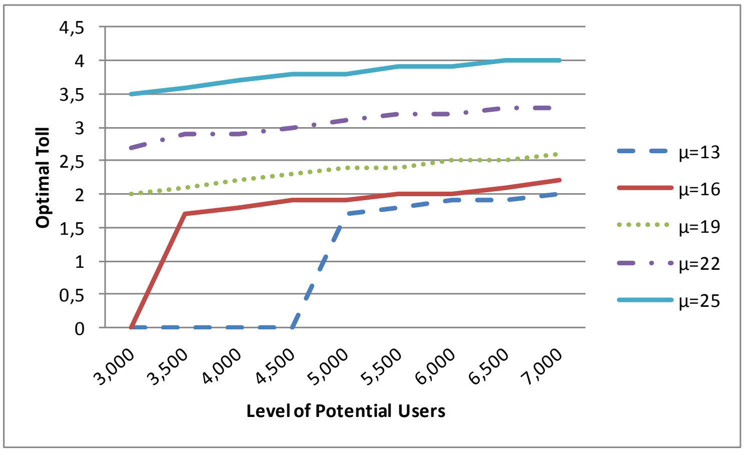


Figure 3. Effect of average VTT

Before analyzing the figure, it's worth knowing that the distributions with μ=13€/hour and μ=16€/hour have 38% and 15% of the potential users pushed out the transportation system. The other three distributions do not have any group of potential users pushed out. From the preceding figure two conclusions can be drawn. Firstly, the more the potential traffic, the higher the optimal toll. This conclusion is in line with the previous findings and with the literature review on the topic. Secondly, it is noteworthy that ceteris paribus, the higher the μ the higher the optimal toll. However, this increase of the toll is not proportional to μ, since the optimum toll rises less than μ. Finally, there is a discontinuity in the model with low potential traffics and low μ, where the optimal toll is zero. The reason for that lies in the total number of vehicles in the corridor. When in the corridor there are fewer users than certain limit, then there are no congestion and externalities, and therefore it is not necessary to set any toll. In the following section we will confirm this trend.

### *4.3. Effect of variance VTT*

In this section we examine the effect in optimal tolls of variance of VTT. As we did previously, in order to analyze this effect we have selected the medium average of the range (μ=19€/hour) and the five variances tested in the model ($\sigma^2$=84.44, 171.28, 249.28, 324.80, 391.95). There are no users pushed out the transportation system for the five distributions chosen. Figure (4) contains the evolution of the optimal toll in each distribution of VTT with different levels of potential traffic.

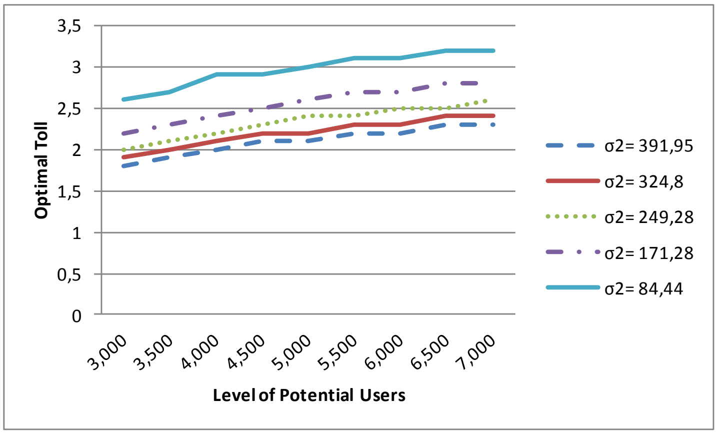


Figure 4. Effect of σ2 VTT

As can be seen from figure (4), the higher the variance (or dispersion) of the distribution of VTT, the lower the optimal toll. In other words, with the same average VTT (μ) and traffic, the more the GINI index, the lower the optimal toll. This also means that in regions with a not homogenous income distribution, the tolls should be lower than in regions with a homogenous income distribution. As in the previous figures, the more potential traffic, the higher the toll.

### 4.4. Combined effect of μ and σ2

In the last two subsections, on the one hand we have analyzed the effect of average VTT (μ) and on the other hand the effect of variance (σ2). However the combined effect of both variables has not been considered yet. In order to study this effect, figure 5 has been depicted. This figure contains four graphs. In each graph, the left bar indicates the optimal toll in Euros and the horizontal bars represents different averages (μ). The difference between the four graphs is the level of potential traffic.

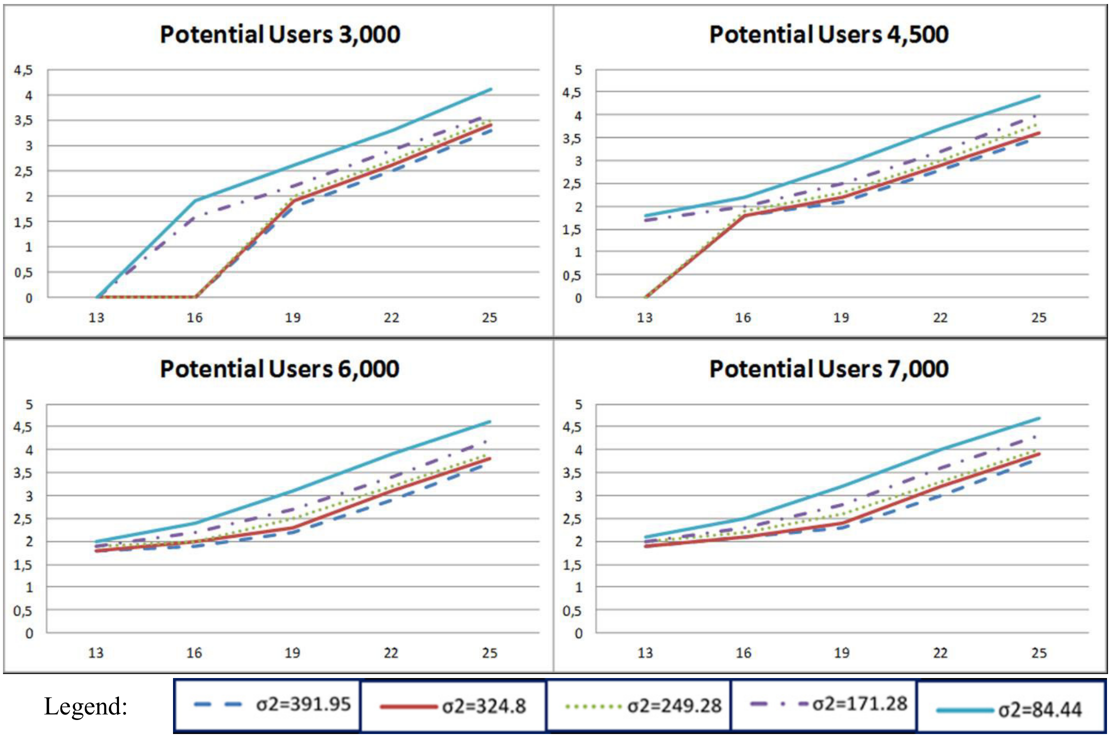


Figure 5. Effect of both μ and σ2 VTT

The previously observed trends are confirmed in figure 5. Firstly, the more the potential traffic, the higher the toll. Secondly, the higher the average of VTT, the higher the optimal toll. Finally, the higher the dispersion of VTT, the lower the optimal toll.

The discontinuity observed in figure (3) is clearly detected in this figure. With low average VTT and low potential traffic, the optimal toll is zero. We can set a limit between 2,700 and 2,900 of real users where the toll must be null. If the real traffic is higher than this limit, the toll cannot be zero. This limit depends on both, potential users and users pushed out the transportation system. In other words, when the level of service in the tolled highway is B, it is necessary to set some toll and as a consequence the traffic is shared between both roads. In societies with a high percentage of users pushed out (low μ and high $\sigma^2$), this limit is close to the level of service C. In the rest of societies, the limit is in the level of service B.

### *4.5. The Spanish Case*

Finally, we want to get the optimal price if the socioeconomic conditions of Spain are applied to the model. With these results, it could be known what would be an optimal price depending on the traffic level. The distribution has been extracted from the Spanish statistics of tax return, and its shape is very similar to a lognormal distribution. In the table below (2), the results for different levels of traffic can be seen.

Table 2. The Spanish Case

| Kind of distribution | Potential Traffic (Number of potential users) | Optimal price in the toll highway (€)/ (€/km) | Share of traffic in the conventional road (%) |
|---|---|---|---|
| Proxylognormal ($\mu$=13.09€/h; $\sigma^2$=469.92; GI=51.68; Exp=31%) | 3,000 | 0 | 0 |
| | 4,000 | 0 | 0 |
| | 5,000 | 1.7/(0.018) | 18.84 |
| | 6,000 | 1.9/(0.021) | 26.08 |
| | 7,000 | 1.9/(0.021) | 26.08 |

Just as it can be anticipated by the characteristics of the distribution of VTT, the conclusions to be drawn from the above table are similar to the conclusions for distributions with low average ($\mu$) and high variance ($\sigma^2$). With free flow traffic conditions, tolls are zero. By contrast, with the increase of traffic, the toll increases and the traffic is distributed efficiently among both roads. If we compare the optimal tolls with the existing tariffs in Spain for corridors with similar characteristics, it is important to stress that there should be a drop in the prices (€/km) of at least 73.5%.

## 5. Conclusions and Policy Lessons

This paper develops a model to evaluate the optimal welfare price in a corridor where a toll highway competes for capturing the traffic with a parallel conventional road. This model aims to obtain the optimal price for non-usual users of an interurban corridor who decide with the expectations of free flow traffic conditions. The most important addition to the academic literature in the field is to know how income distribution affects to the optimal price. This socioeconomic characteristics are given by average and dispersion of VTT. The model has been tested with appropriate values for the Spanish case, however it could be applied to any region by changing the value of the parameters.

The influence of dispersion of VTT, average VTT and traffic has been clearly detected. It can be said that the higher the average VTT the higher the optimal price, the higher the dispersion of this VTT the lower the optimal price and finally, the more the traffic the higher the optimal toll. Moreover, the prices of tolls in Spain should be dramatically decreased. So, from a pricing policy point of view, in regions with lower GINI index, the toll should be clearly higher than in regions with higher GINI index, where the toll must be lower; and the toll in the peak hours must be higher than in off peak hours.

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