Numerical calculation of dielectrophoretic and electrostatic forces acting on micro-scale particles.

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Abstract. Much of the current literature on dielectrophoresis (DEP) relates to micro or nano scale particles; typically in micro-fluidic type experiment geometries. In contrast, this work focusses on the application of DEP forces to larger, micro-scale particles in air. Since DEP scales with particle volume, it can apply a significant force on surprisingly large objects. When using very small particles it is often sufficient to use Pohl's method [1] whereby the particle is considered to be spherical and where it does not interact with the externally applied electric field. For the larger particles used in this work, the spherical approximation does not necessarily hold. DEP forces are therefore calculated using the finite element method (FEM) which permits the use of arbitrary particle shapes. In this model the electric field is solved in the presence of a polarizable particle, the DEP force is then calculated using the Maxwell stress tensor method [2]. The development of this model allows the investigation of the DEP forces acting on non-spherical particles for a specific experimental electrode geometry.

1. Introduction

Since the work of Pohl [1], beginning in the early 1950's, the field of DEP has become increasingly sophisticated. Today it is possible, for example, to use a.c. DEP in a microfluidic chip to sort biological cells by their dielectric spectroscopy properties [3]. Much of the current literature [4] is devoted to the optimum shape of test electrodes [5], to the rotational torques exerted on microparticles [6] and to other subtleties. Despite this, the possibility of making use of DEP simply as an attractive force that may be used for practical purposes also merits consideration. As such, DEP has great potential for exploitation, especially in particlulate based applications. For example, in particle transport, grading by size or dielectric properties or enhanced filtering with the possibility of high yield recovery of trapped particulates. The work presented here has been carried out as part of the SPABRINK project (Self-Printing Advertising Board with Reusable Ink) which aims to apply DEP forces as a means of temporarily fixing micro-scale particles to a surface in order to display an image.

2. Initial verification of the FEM model

Before employing the FEM calculation in more complex geometries it is first necessary to verify that in a simple test model the forces calculated by FEM are in agreement with those obtained using Pohl's method. The hypothetical case of a spherical capacitor with vacuum as the insulating medium is therefore considered. This system consists of a solid perfectly-conducting sphere concentric within a thin, perfectly-conducting spherical shell (see Figure 1); the model parameters are given in table 1. The advantage of this simplified case is that (when no particle is present) it allows the use of an analytical expression for the electric field (directly from Gauss's Law) which is simple to differentiate.

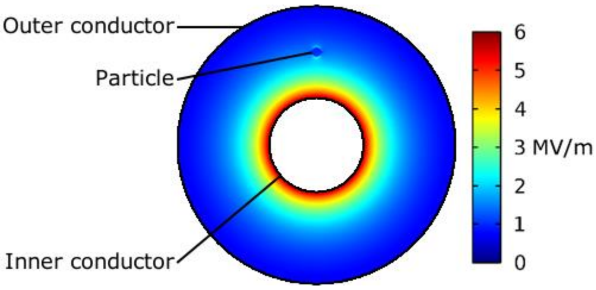

Figure 1. Geometry of the hypothetical spherical capacitor showing E-field strength from FEM.

Table 1. FEM model parameters:

Model Parameter	Value
Inner radius (r_i)	250 µm
Outer radius (r_o)	750 µm
Particle radius (a)	25 µm
Particle radial position (r)	500 µm
Voltage (r_i to r_o)	1000 V
Insulator permittivity	ε_0
Particle permittivity	2.5 x ε_0

Performing this differentiation, Pohl's method gives equation (1) for the DEP force (F) acting on a spherical particle (of radius a, and relative permittivity ε_p) inside the hypothetical spherical capacitor.

$$F = 2\pi a^3 \varepsilon_m \frac{(\varepsilon_p - \varepsilon_m)}{(\varepsilon_p + 2\varepsilon_m)} \frac{-Q^2}{4\pi^2 \varepsilon_0{}^2 r^5}, \qquad Q = \frac{4\pi\varepsilon_0 V}{\left[\frac{1}{r_i} - \frac{1}{r_o}\right]} \tag{1 \& 2}$$

Here ε_m is the relative permittivity of the insulator, Q is the total charge on the inner sphere and r is the radial distance of the particle from the mutual centre of the two spheres. For comparison with experiment it is often more convenient to consider the applied voltage (V) rather than Q. This can be achieved using equation (2), where r_i and r_o are the radii of the inner and outer conductors respectively. Equations (1 & 2) allow calculation (via Pohl's method) of the DEP force acting on a spherical particle within the hypothetical, spherical capacitor (see figure 2).

A 3d FEM model of the hypothetical, spherical capacitor was constructed in COMSOL (figure 1 shows a 2D slice of the E-field). In COMSOL, the DEP force is calculated by integrating the Maxwell stress tensor $\boldsymbol{T}$ over the surface of the particle [2]; equation (3) defines $\boldsymbol{T}$ in matrix notation.

$$\boldsymbol{n}_1 \boldsymbol{T}_2 = \frac{1}{2}\boldsymbol{n}_1(\boldsymbol{E}\cdot\boldsymbol{D}) + (\boldsymbol{n}_1\cdot\boldsymbol{E})\boldsymbol{D}^T \tag{3}$$

Where $\boldsymbol{n}_1$ is the outward normal on the surface of the particle, $\boldsymbol{E}$ is the electric field and $\boldsymbol{D}$ is the electric displacement. The subscripts denotes the domain of evaluation, i.e. $\boldsymbol{T}_2$ is evaluated in the media surrounding the particle. The superscript ^{T} denotes the matrix transpose, which makes the final term a tensor product. The markers in figure 2 show the results obtained using the 3d FEM simulation; clearly there is good agreement with Pohl's analytical expression.

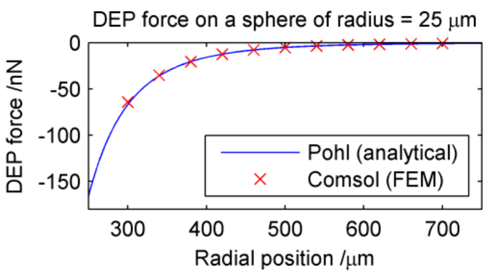

Figure 2. DEP force acting on a 25 µm (radius) particle with relative permittivity of 2.5; plotted as a function of its radial position within a spherical capacitor. The solid line is calculated using Pohl's method via equation (1). The markers show the DEP force as calculated using the FEM model. The DEP force is negative as it acts inward towards the centre of the spherical capacitor.

3. Beyond Pohl's method

The advantage of using the finite element approach is that it allows calculation of the forces acting on arbitrarily shaped particles. To test this, the spherical particle was replaced with an ellipsoid with semi-axes r_x, r_y and r_z. In all calculations presented here $r_x = r_y$, the aspect ratio of the ellipsoid is therefore defined as R = r_z/r_x. Whilst the aspect ratio R is varied, r_x, r_y and r_z will be chosen in order to keep the volume of the ellipsoid equal to that of the spherical particle used previously. Varying R between 1/3 and 3 changes the ellipsoid from oblate to prolate respectively as shown in figure 3a. It is possible to vary the rotation of the ellipsoid as illustrated in figure 3b. Alternatively the position of the ellipsoid can be defined relative to its lowest point (which will be nearest to the charged electrode).

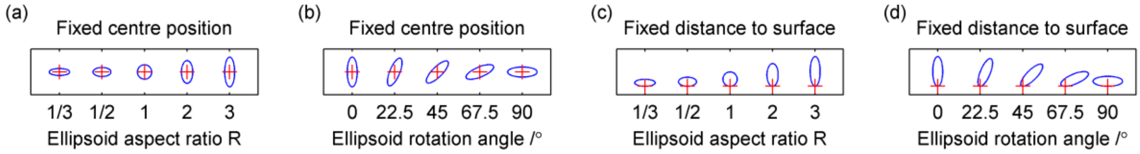

Figure 3. Diagrams showing the effect of ellipsoid parameters. The ellipsoid centre position is fixed whilst in (a) the aspect ratio R is varied and in (b) the ellipsoid is rotated. Figure parts (c) and (d) are similar except that the ellipsoid positions are defined relative to their lowest point.

The FEM model outputs x, y and z components of the DEP force and can also be configured to give rotational torques. In contrast Pohl's method only gives the translational force acting on the centre of mass of the particle. The strongest force (and the one of most interest for the display screen application) is the z-component of the DEP force (towards the centre of the capacitor). This is plotted as a function of ellipsoid aspect ratio and rotation angle in figure 4.

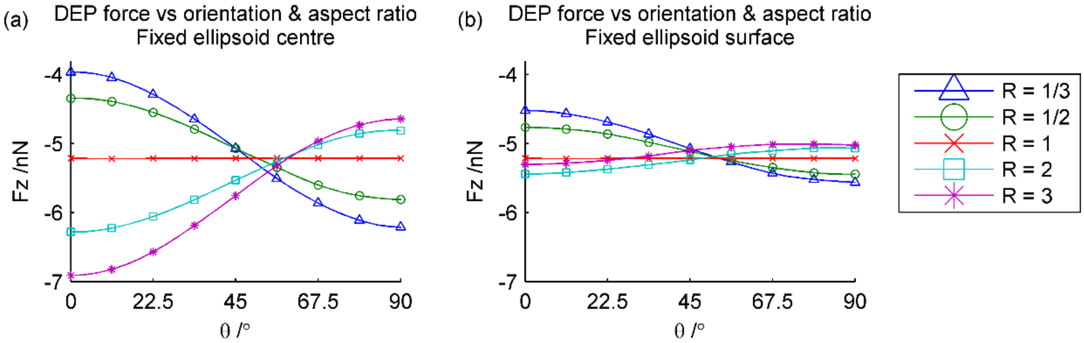

Figure 4. The DEP force acting on an ellipsoid particle within a spherical capacitor, as a function of ellipsoid aspect ratio and rotation angle. (a) When the centre of the ellipse is fixed 500 µm from the centre of the capacitor. (b) When the nearest point of the ellipse is always 475 µm from the centre.

When the centre of the ellipsoid is fixed (figure 4a) it is the prolate spheroid that experiences the strongest DEP force. This occurs when the tip of the ellipsoid comes into closest proximity to the charged inner electrode surface of the spherical capacitor. It is interesting to note that the ellipsoid has to rotate by more than 45° before the oblate shape produces the strongest DEP force.

Intuitively one might expect the prolate ellipsoid to experience the strongest E-field enhancement at its tip and therefore to generate the largest DEP force; however, this is not the case. When the distance between the ellipsoid surface and the charged inner surface of the capacitor is kept constant (figure 4b), it is the oblate spheroid that experiences the strongest DEP force. This is presumably because the rim of the oblate spheroid is narrower than the tip of the prolate spheroid when their volumes are equal.

4. Application to experimental geometry

The FEM approach allows calculation of the DEP forces for non-spherical particles in complex experimental geometries. Figure 5 shows the 3d electric field calculation for an ellipsoid particle in the vicinity of two strip electrodes. This model simulates the experiment geometry used by the display foil application; the interdigitated electrodes produce a divergent electric field near to the surface of the foil.

Figure 6 shows the DEP force acting on the ellipsoid particle as a function of its geometric parameters. The angle Θ is between the unique axis of the ellipsoid and the vertical (z) direction. When $\Theta = 90°$ and $\phi = 0°$ the unique axis of the ellipsoid is aligned across the electrode gap; the strongest DEP force occurs for the prolate spheroid when it is in this orientation.

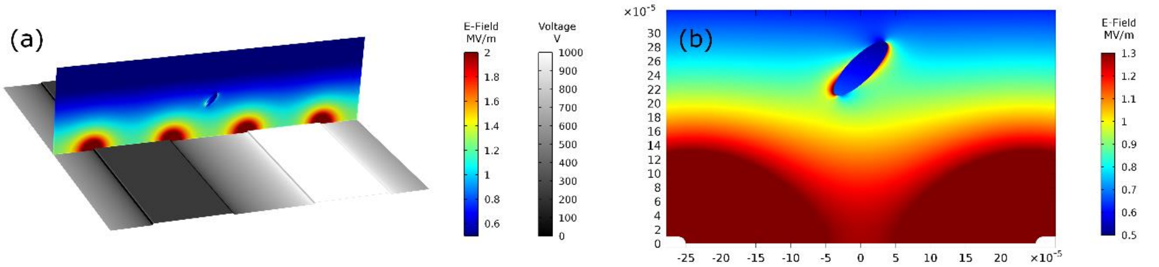

Figure 5. (a) Example of the 3d FEM model used to calculate DEP forces. In the horizontal plane the colour map of the electrodes indicates the voltage. In the vertical plane the colour map indicates the electric field strength. (b) Enlarged view showing the electric field near to the ellipsoidal particle. Particle aspect ratio R=3, and rotation angle is 45°.

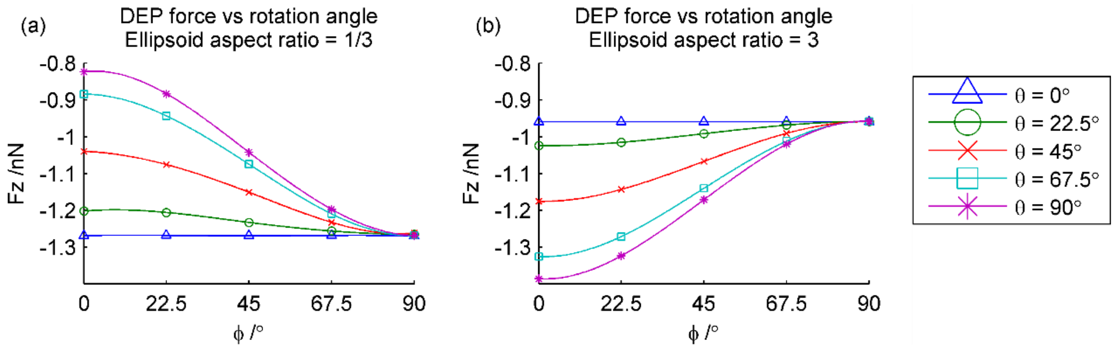

Figure 6. The z component of the DEP force (-ve is downward) acting on an ellipsoid particle in the vicinity of two strip electrodes. (a) Forces on an oblate spheroid. (b) Forces on a prolate spheroid.

5. Conclusions

In the SPABRINK display screen application the particles are made from polymeric materials such as PET ($\varepsilon_r \approx 2.5$). If a density of 1380 $kg.m^{-3}$ is assumed for PET then the weight of the particle produces a force of 0.886 nN. In the 3d model the particle is positioned 250 μm from electrode plane. If the electrodes were inverted so that gravity pulls the particle away from the display surface then the oblate spheroid would fall away from the electrodes when in certain orientations. In all orientations the prolate spheroid would be attracted towards the electrode and will be held in situ by the DEP forces.

This paper has demonstrated the use of FEM to calculate DEP forces in complex physical geometries. In the future this method will be extended to study the forces acting on (and motion of) groups of multiple, irregularly shaped particles; electrostatic charging of these particles could also be incorporated. In this manner FEM could facilitate the engineering of DEP based solutions to a range of practical problems; for example, in filtering and in particulate based industrial processes.

Acknowledgments

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