**An experimentally validated contour method/eigenstrains hybrid model to incorporate residual stresses in glass structural designs**

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Abstract

Contour method-based finite element (FE) models together with knowledge of the surface deformation resulted from the stress relaxation along a newly cut plane were used to construct the residual stresses in commercially available float glass. The results show that the residual stress depth profile of float glass is parabolic. The constructed residual stress profiles, validated to some extent against results of scattered-light-polariscope (SCALP) experiments, were then used to establish the misfit strains (i.e. eigenstrains) existed in the original glass specimens. It is shown that, despite the modelling uncertainty of the contour method and the limitations associated with the SCALP measurements, the eigenstrain depth profile in a given float glass specimen can be determined to an acceptable accuracy. The paper shows that once the underlying eigenstrain distribution in a given thickness of glass has been determined, the complete residual stress distribution can simply be determined by incorporating the eigenstrain profile as a misfit strain in an appropriate FE model. It is also shown that the hybrid contour/eigenstrain model enables modelling the residual stress around stress concentration features such as holes and/or stress evolution during subsequent applied loadings, by simply using the knowledge of eigenstrains.

### Keywords: Contour method, Eigenstrains, Float glass, Residual stresses, Scattered-light-Polariscope

# Introduction

A comprehensive study of modelling residual stresses in commercially available glass sheets using the eigenstrain approach has been undertaken; the applications of the model for investigating the effect of thickness on residual stress distribution and predicting residual stress in thermally-tempered glass have been described elsewhere [1].To incorporate the effect of residual stress in different glass structural elements using the eigenstrain concept, it is necessary to accurately determine the eigenstrain depth profile in different glass types. This paper provides an essential tool that will enable the contour method [2] to be used to determinethe eigenstrain depth profiles in float glass. The paper explains the assumptions made in the analyses, and the model predictions were validated to some extent using the results of scattered-light polariscope experiments. The analyses presented herein will obviate the need for complex multi-physics analyses of the glass manufacturing process to model residual stress in glass sheets where reliable details of the thermal phenomena like convection and radiation, and the viscous response of glass at different temperatures are not usually obtainable for an accurate analysis.

The UK goal is to achieve 80% reduction in carbon emissions by 2050. The buildings presently account for ~45% of carbon emission [3] – the scale of the challenge in reducing fossil fuel dependency in the built environment is immense and it will require a significant overhaul in the construction industry. The use of artificial lighting and heating significantly contributes to a large carbon footprint [3]. Appropriate use of glass with its aesthetic characteristics, transparency, durability and relatively low cost to build roofs, floors, staircases and partitions in buildings can significantly reduce the reliance on artificial lightning. In addition, the solar heat which may be gained through glass has potential to be efficiently harvested for space and water heating [1].

The use of glass as a load bearing construction material in buildings became increasingly popular during the past 25 years [4]. Although glass offers great potential to be used in the built environment, design of glass structures is challenging for structural engineers due to its brittle material behaviour. Surface flaws which are inevitably present in glass can cause fracture – the lack of knowledge of the geometry and distribution of the surface flaws means the prediction of strength of glass is challenging [5]. The structural behaviour of glass is significantly different to that of conventional construction materials: steel and reinforced concrete where both materials are ductile and the design methodologies are well-established. The strength of glass exhibits a considerable time dependency due to the degradation in service environments. In addition, the duration and the spatial distribution of the applied loads also affect the strength of glass. A comprehensive analytical or numerical method to predict strength of glass is lacking. Consequently, the use of glass as a construction material is not being exploited as effectively as it might be, and areas of particular concern relate to the lack of tools to incorporate the effect of residual stresses, which arise during the manufacturing cooling processes. The evolution of stress and the failure load of glass depend on the residual stress. In addition, residual stress affects the fragmentation, which governs the post-breakage behaviour. The current glass structural design guidelines (e.g. [4]) do not explicitly take into account the effect of residual stress. For instance, *IStructE–Structural Use of Glass in Buildings* [4] recommends to limit the maximum design surface tensile stress determined from an analysis, without incorporating the residual stress, to the compressive residual stress at the surface.

Float glass, which is also known as soda-lime-silicate glass, is the most common and readily available glass type and is produced by floating of molten glass over a tin bath. During the glass manufacturing process, the ingredients (~72% silica sand, ~18% soda ash and ~10% limestone) are first heated in a furnace up to ~1500ºC to form molten glass, which will then be slowly cooled (i.e. annealed) to form glass sheets of uniform thickness [4]. Float glass has a low tensile strength (< 40 MPa) [4] and therefore it is mostly used in non-load bearing structural forms such as window panes. On the other hand, tempered glass, which is produced by heating up float glass up to a temperature ~620ºC and then rapidly cooled by jets of air (i.e. quenching), has a surface compressive prestress of magnitude ~80-150 MPa. Tempered glass is often used in structural engineering applications [4]. Typically, the depth of the surface compression zone is ~0.2 times the overall thickness of the sheet [6], and the compression layer will often retard the potential propagation of surface flaws. However, tempered glass is ~3-5 times more expensive than float glass (on unit area basis). The use of tempered glass also adds additional design/construction challenges. Penetrations beyond the surface compression layer will lead to fragmentation. Moreover, alterations (e.g. cutting, drilling or grinding) cannot be done into tempered glass. In addition to thermally-toughened glass, chemical-toughening is also used in practice to toughen glass. Although surface compressions in excess of 250 MPa may be achieved, due to the development of a very thin compression layer (~20-50 m thick) [6] and the relatively high price mean chemically-toughened glass is not widely used as a construction material.

The prediction of residual stresses in thermally-tempered glass has received a notable research interest in the literature due to the load bearing structural engineering applications of the material. The generation of residual stress in tempered glass involves a complex multi-physics phenomenon, thus, modelling the residual stress is challenging. A knowledge of thermal-tempering parameters and the viscous response of glass at different temperatures will be required in an analysis. These parameters are difficult to determine as they depend on complex thermal phenomena like convection and radiation. The specific volume of glass which is required for modelling the residual stress depends on both thermal vibration and microstructural rearrangement. One of the simplest theories used in the literature was the instant freeze model of Bartenev (as quoted in [7]) where an instance change from being a liquid that has no ability to carry stresses to a fully-elastic rigid solid at the glass transition temperature. The works of Narayanaswamy [8, 9] considered the structural relaxation which accounts for the effects of the rate of cooling on the properties of glass. In the recent years, with the availability of FE computer packages, advanced constitutive models and material parameter characterisation techniques were introduced to take into account temperature dependent viscoelasticity, structural volume relaxation and temperature history of the tempering process [7]. The models described above and other models reported in the literature provide useful insight into the residual stresses in tempered glass. However, there is no validated model amongst the glass research/engineering community to incorporate the effect of residual stress. For instance, the test method recommended in the current Eurocode [10] to quantify the residual stress is a destructive test, in which the residual stress is quantified based on the fragments in an area of 1100 x 360 mm after the fracture initiated by an impact of a pointed steel tool perpendicular to the plane of the glass specimen. There is a need for a model which is capable of incorporating the effect of residual stress in strength analysis of glass structures.

Despite its low tensile strength, float glass offers potential to be used as a structural material due to its favourable post-breakage behaviour, ability to subject to sectioning and holes drilled into it without shattering, low cost and easy availability. In particular, the recent research investigations of hybrid float glass structural elements (e.g. with PVB (Polyvinyl butyral), steel, glass fibre reinforced polymer (GFRP), etc.) demonstrate the potential structural engineering applications of float glass [11]. PVB–float glass hybrid systems have relatively safe post-breakage behaviours because the PVB layer can retain broken glass pieces during the failures. In other types of hybrid float glass systems, the presence of a strong material (e.g. steel, timber, GFRP) in between the float glass sheets will limit the failure to one side of the hybrid laminate. These developments of float glass hybrid systems imply the need for the knowledge of residual stresses in float glass.

Although glass manufacturers anticipate that the gradual cooling of float glass during the manufacturing process would eliminate the generation of residual stress, the experiments suggest that some stresses are inevitably present in the final products [1]. In a marked contrast to the significant research focus on tempered glass, the residual stresses in float glass were not studied in detail in the literature. As a result, the current design guidelines of float glass ignore to take into account the effect of residual stress. This paper presents a validated hybrid contour method/eigenstrain model to characterise the full distribution of the residual stress present in commercially available float glass. It is also shown that the hybrid model enables modelling residual stress in new geometries (e.g. stress concentrations features) and /or stress evolution under applied loading by simply using the knowledge of eigenstrains.

**Experimental investigations of residual stress in glass**

Although X-ray/neutron diffraction methods are successfully used to measure residual stresses in metal alloys, the methods cannot be used to measure stresses in glass due to its amorphous (non-crystalline) microstructure. Alternatively, methods which are based on the photoelastic principle are generally used to measure residual stress in glass [12]. It should be noted, however, that the available measurement techniques have mainly been used for measuring stresses in tempered glass and the methods were proven to be less reliable to measure residual stresses in float glass [12] due to the relatively low magnitudes of the stresses present.

The principle of photoelasticity is based on the optical property of birefringence, in which a ray of light that passes through a birefringent material (e.g. glass) experiences a refractive index that depends on the polarisation and propagation direction of light. The magnitude of the refractive index at each point may be related to the magnitude of the stress at that point. Some of the methods which have been used in the literature to measure residual stress in glass are grazing-angle-surface-polarimeter (GASP), tomographic photoelasticity, magnetophotoelasticity and scattered-light-photoelasticity.

GASP, which uses polarised light reflected from the tin (Sn) particles which remain on one side of float glass specimens, may be used to measure the surface compression at that side of the glass specimen [13]. Although the technique was successfully used to measure surface compression in tempered glass, the method is not capable of determining the depth of the surface compression layer [12]. Measuring instruments which are based on magneto-photoelasticity [14] and photoelastic tomography [15] were used to measure the full stress depth profile including the depth of the compression layer in tempered glass. Magneto-photoelasticity uses the interaction between light and a magnetic field (i.e. Faraday Effect) to determine the stress distribution along the thickness of tempered glass specimens [14]. The experimental results suggests that although the method may be used to measure residual stress in toughened glass the method is not reliable for measuring stresses of low magnitude present in float glass [14].

The recently developed scattered-light-polariscope (SCALP) instruments provide a reliable tool for the measurement of residual stress in all glass types including float glass. For instance, the compact SCALP manufactured by Aben [16] (SCALP-05) is capable of measuring full stress depth profile to an accuracy greater than that may be achieved using other methods. Depending on the settings of SCALP-05, the instruments may be used to measure stresses in the depth ranges up to 5 mm [17]. According to the manufacturer of SCALP-05 [17], the expected error in measured stresses of magnitude less than 20 MPa is ± 2MPa. As will be shown in the latter half of this paper, the experimental results obtained using two SCALP-05 devices, with two different laser angle settings, were used in the current study to validate the predictions from the model proposed in this paper. During an experiment, SCALP-05 instrument first emits a laser beam which passes through the glass panel at an angle (β) of ~45º. Glass birefringence changes the polarisation of the laser beam and the consequent variation in the intensity (optical retardation) of the scattered-light is measured by the in-built charge-coupled device camera (Fig. 1 shows a schematic diagram of the concept used in SCALP-05). Each transverse component of the stress () along the thickness of the glass piece will then be computed by using the knowledge of photo-elastic constant (*C*) of glass and the incidence angle of the laser beam (β) as shown in equation (1) [18].

(1)

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**Fig. 1**: Schematic diagram of SCALP-05

SCALP-05 were calibrated [18] using the experimental results of 4-point bending experiments conducted in accordance with EN 1288-3 [19] and the stresses measured using grazing-angle-surface-polarimeters. For instance, using SCALP-05 the surface compression in float, heat treated and tempered glass specimen were determined to be ~7 MPa, 120 MPa and 160 MPa [18]. The potential of SCALP-05 to measure stresses along the thickness of glass was also exploited in the literature: for instance, stresses up to a depth of ~2.2 mm was determined in float, tempered and chemically toughen glass specimens [20]. The research presented in the literature suggest the potential of SCALP-05 to measure residual stresses present in float glass, and as it will be shown in the latter half of this paper, the results of SCALP-05 were used to validate the predictions of the contour method/eigenstrain hybrid model proposed in this paper.

**Hybrid contour method/eigenstrain model to predict residual stress**

In the hybrid model presented in this paper, the contour method [2] was used to determine the residual stress distribution in a given float glass specimen. This was done by incorporating the surface deformation occurred in a newly cut plane, as a result of the relaxation of stress in the direction perpendicular to the cut plane, in an appropriate FE model. The computed stress profile was then validated to some extent against the results of SCALP-05 experiments. Although the contour method is able to determine the residual stress in a given glass specimen, it does not provide a mean of incorporating residual stress distribution in subsequent strength analysis. In the model, the concept of eigenstrains (i.e. misfit strains) was used as a technique of incorporating the effect of residual stress in the designs of float glass structural elements. The method, however, requires the knowledge of the eigenstrain distribution in the given glass specimen, and its determination using the knowledge of residual stress distribution determined from the contour method analysis is described below.

Eigenstrains act as a source of incompatibility and provide a method to represent the residual stress distribution in a component. The method has been successfully used in the literature to model residual stress distribution in different applications: welded super-alloy plates [21] and the residual stress generated due to laser shock peening [22]. As shown in the latter sections of this paper, it is possible to determine an accurate estimate of the eigenstrain presented in a given float glass specimen by combining the knowledge of the residual stress data obtained from the contour method analysis with an initially assumed, but sensibly chosen, eigenstrain distribution. Once the knowledge of the eigenstrains is available it can be incorporated in FE models to analyse the effect of residual stress. The method has the advantage that it provides an accurate method of representing the residual stress without the need of modelling the thermo-mechanical process of the glass manufacturing process. In addition, the method enables the determination of all six components of stress distribution that satisfy overall equilibrium, compatibility and boundary conditions of the given glass specimen. The step-by-step procedure of the methodology used in the current hybrid model is presented in Fig. 2.

**graph1**

**Fig. 2.** Step-by-step procedure of contour method /eigenstrains hybrid method.

# Theory of the contour method

The contour method [2, 23-25] has been used in the literature as a tool of modelling residual stress in metal alloys in various applications (e.g. residual stress due to plastic bending [24], residual stresses around welds [25]). The method is based on Bueckner’s superposition principle [2] and the technique requires cutting a specimen into two parts (Fig. 3a) and measuring the contour (i.e. displacement profile) of the resulting new surface (Fig. 3b). The distribution of the original residual stress perpendicular to the cut-plane may be determined by incorporating the measured surface contour normal to the cut plane as a displacement boundary condition in a FE model, which represents the initially stress-free half sample of the original specimen (Fig. 3c).

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**Fig. 3.** Contour method (a) Original sample (b) Deformed shape due to the relaxation of stress along the cut-plane (c) The deformed shape is forced back to construct the residual stress

**Contour method analysis of float glass : An example**

In order to demonstrate the use of the method to devise the residual stress distribution, the analysis of a commercially available float glass specimen of dimensions 150 mm × 100 mm × 10 mm is descrbed below.

***Contour cut***

To achieve accurate results from a contour method analysis it is essential that no additional surface deformations except for that due to the stress relaxation, occur during the cutting process. Thus, a straight cut-plane without induced stresses due to the cutting is required. Cutting methods based on electric discharge machining (EDM) in which the material removal is done by spark erosion were used in the experiments of metallic components [2]. However, EDM cannot be used to cut glass due to its low dielectric capacity. In addition, due to the brittle nature of glass, making a cut which is free from defects such as micro-cracks is not trivial. The suitability of diamond disk cutting, ground carbide tip and water jet cutting were investigated in the study. The results showed [1] that water-jet cutting (done by a commercial contractor with a jet diameter of ~1 mm and an 80 mesh garnet grade) produced cuts which were accurate enough for the contour method analysis. An optimal cutting speed was established after several trials [1]. In addition, to eliminate edge effects such as angular cuts, two sacrificial glass specimens on each side of the sample were used (Fig. 4a)

***Measurements of the surface contour***

After the sample was cut into halves, the displacement contour of the cut surface was measured using a 3-D micro-coordinate system “Alicona InfiniteFocus” [26] – An optical measurement system was used in the study to achieve a high accuracy whilst eliminating the surface defects which would have been introduced by the touch probes in contact-based measuring devices. After a study of trial measurements with different magnifications (×) available in Alicona InfiniteFocus, it was determined that 10× magnification provided an appropriate magnification and a manageable measurement speed. The noise of the measured contour was removed by smoothing the original data using an in-built software available in Alicona InfiniteFocus [26]. This was done by introducing a cut-off frequency: the cut-off limit, which represents the frequency bound below or above which the measured data are extracted or eliminated was chosen to achieve a smooth displacement profile [1].

***Uniform surface contour in the lateral directions***

The procedure adopted in the study to determine the displacement contour of the cut plane was described in detail in a previous publication of the present authors [1]. In this paper one of the main advantages of the contour method is exploited below to demonstrate that the distribution of residual stresses along the length direction in the cut plane (*y* direction in Fig. 4a) is uniform.

It is common in the literature of the contour method to measure the displacement depth profile over a finite width at a given location, and subsequently, to use the knowledge of the average profile in the analyses. Fig. 4b shows the average displacement profile measured at the mid location of the cut-plane (indicated in Fig. 4a) over zones of width 2, 3, 5 and 10 mm respectively. From the figure it can be seen that the average contour over a finite zone of width greater than 2 mm was unaffected by the actual width of the measurement zone. The average contour over a zone of 3 mm wide was used in all subsequent analyses presented in this paper.

In order to investigate the distribution of the displacement depth profile along the *y* direction, the surface contours were measured at three different locations along the cut plane (at the mid location, one fourth location from the left edge, and one fourth location from the right edge along the *y* direction) (Fig. 4a). The results shown in Fig. 4c illustrate that the three depth profiles are similar. The small discrepancies between the depth profiles may be due to the inevitable local effects of the cutting process. The results suggest that the surface deformation was mostly uniform in the lateral direction (*y* direction) and varied only along the thickness (*z* direction). Thus, as a first approximation, it is appropriate to model the surface contour as the average depth profile, but distributed over the entire area of the cut-plane. It should be noted, however, that this assumption may not be accurate in the vicinity of free edges in which the material would have undergone different physical conditions than the middle area of the glass sheet during the manufacturing process. An analysis of the edge effects is beyond the scope of this paper and it is not discussed.

***Polynomial representation of the contour depth profile***

In the current problem, the measured displacement data at the middle location of the cut-plane (Fig. 4a) was used to determine the displacement depth profile (i.e. distribution of the surface displacement in the *z* direction) of the cut-plane. Fig. 4d shows the measured displacement depth profile at the middle of each half sample. The results show that both depth profiles were similar (difference is less than 5%), with negative displacement at mid depth and positive displacement near the edges (surface). The average of the two (Fig. 4d) was used in the analyses eliminating potential shear effects and other local effects of the cutting process. In order to incorporate the surface depth profile as a displacement boundary condition in an initially stress-free FE model, it is useful to represent the average displacement depth profile as a polynomial function of *z* coordinate, measured from the bottom of the sample. The order of the polynomial function was determined based on a sensitivity study; this analysis will be described in detail in the next section.

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| **Fig. 4.** (a) Half of the glass sample after cutting along y axis using the water-jet cutter (b) Average displacement profile over different widths (c) Measured surface displacement profile at different locations in the cut-plane (d) Displacement depth profiles at the middle | |

***Determination of the residual stress profile***

A 3-D FE model in ABAQUS/Standard [27] was used in the study to compute the residual stress distribution by incorporating the polynomial form of the displacement boundary condition over the entire area of the cut-plane representing the initially stress-free half sample. In this analysis, the average displacement profile approximately represented by a 2nd order polynomial of *z* coordinate with coefficients: 0.025, -0.243 and 0.255 was used. A linear-elastic material behaviour with parameters broadly representative of glass (Young modulus =70 GPa and Poisson’s ratio=0.22) was used in the FE analysis. The half sample was modelled using 20-noded quadratic solid elements (3D20R), with a refined mesh near the edges and the cut surface (element size ~0.05 mm).

It should be noted that only the stress component normal to the cut surface (i.e. σ*xx*) can be determined (Fig. 5a) from this analysis. An analysis based on a new cut perpendicular to the *y* direction will be required to determine σ*yy* stress component. For simplicity of the presentation of results only 2D figures (i.e. *xz* plane) are used in this paper. Fig. 5b shows the σ*xx* depth profile obtained from the analysis. The results show a parabolic stress depth profile, with compression at the surface (~6 MPa) and tension (~4 MPa) at the mid-thickness. The depth of the compression zones at each side is ~2 mm (~20% of the specimen thickness) and is balanced by a middle tension zone of 6 mm (~60% of the thickness). The results also confirm that the total force balances itself over the thickness of the sample satisfying the overall equilibrium requirement. As it will be shown below, the predicted parabolic residual stress distribution is consistent with the stresses measured using SCALP experiments. The results also agree with those reported in the literature: for instance, using an analysis of fracture toughness of float glass, Geandier et al. [28] suggested a parabolic stress depth profile with a surface compression of 4.5 MPa and a mid-thickness tension of 2.3 MPa.

In the contour method analysis, the order of the polynomial which used to represent the displacement depth profile was determined such that the predicted stress results are largely independent of the chosen polynomial fit. For instance, the displacement profile was first approximately represented as 2nd, 3rd, 4th, 5th and 6th order polynomials of the depth coordinate (*z*). Fig. 5c shows *xx* stress depth profile predicted by the analyses based on each assumed polynomial form respectively. From the figure it can be seen that the predicted *xx* stress depth profiles are almost identical. It should be appreciated that in order to minimise the edge effects, the measured displacement data close to the surface (0.5 mm from each side) was ignored in the polynomial fits. The results suggest that the effect due to the ignorance of data close to the surface is negligible and, also the use of the simplified 2nd order polynomial quoted above is justifiable.

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| **Fig. 5.** (a) Residual stress (σ*xx*) distribution (b) σ*xx* depth profile (c) σ*xx* depth profile for different polynomial input of the surface contour | |

# Experimental validation of the contour method results

The residual stress depth profile determined using the contour method (Fig. 5b) was validated using the results of scattered-light-polariscope (SCALP-05) [17] experiments. Two SCALP-05 devices [17] with two different laser angle settings were used to measure the residual stress. The measurable depth range of the two polariscopes were ~2.2 mm and ~5.3 mm respectively. At a given location measurements taken from both sides of the specimen were used to construct the full residual stress depth profile. As specified by the manufacturer of SCALP-05, the photoelastic constant was assumed to be 2.7 TPa-1 (*C* in Eq. (1)). As stated previously, the expected error in a measured stress of magnitude less than 20 MPa is ± 2MPa.

Prior to taking readings, the surface of the specimen was first cleaned to remove dirt and fingerprints. In order to ensure a good optical contact between the glass and the polariscope a manufacture-recommended immersion liquid (refraction index = 1.52) [17] was used. Figs. 6a and 6b show the *xx* depth profiles measured using the two polariscopes respectively at the central region of a float glass specimen of the same dimensions (150 mm x 100 mm x 10 mm) as the one used in the contour method analysis. This specimen and the one used in the contour method analysis were both cut from a single large specimen; hence, the experimental results may be used to validate the contour method analysis (Fig 5b). In the experiments, the measurements were repeated four times at the same location (denoted as S1, S2, S3 and S4 in Fig. 6a and 6b) to minimise experimental errors. From the figures, it can be seen that, in each experiment the four stress depth profiles are approximately the same. Hence, the average of the four profile was taken as the actual stress distribution. The average xx depth profiles determined from the two experiments together with that determined from the contour method analysis are shown in Fig. 6c (denoted as Exp1, Exp2 and CM in the figure respectively). As can be seen from the figure the stresses measured using the polariscopes match each other in the surface region (within ~2.2 mm from the surface) and also with the results of the contour method. For instance, the measured surface compression is ~5 MPa and this matches with ~6 MPa predicted by the contour method (within the error ± 2MPa of SCALP-05). It should be appreciated that although a certain error may be associated with the contour method results an analysis of it is beyond the scope of this paper.

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| **Fig. 6.** σ*xx* measured using SCALP-05 of (a) 2.2 mm (b) 5.3 mm measurable depth range  (c) Comparison between the measured σ*xx* depth profiles and the predictions of the contour method | |

The *xx* depth profile measured using the SCALP-05 with ~5.3 mm measurable depth range may be compared with the full stress depth profile predicted by the contour method (Fig. 6c). From the figure it can be seen that, over a large part of the depth profile the two set of stress values are fitted within the error range ± 2MPa except for in the mid-thickness region in which the contour method predicted relatively higher tensile stress (~4 MPa) than the measured value (~2 MPa). However, an analysis of the measured stress profile shows that the force does not balance itself: a total compression force higher than the total tensile force can be noted. As described in the earlier section “Experimental investigation of residual stress in glass”, although SCALP experiments are generally capable to provide accurate results for the stresses in the surface region, the errors associated with the measurements of mid-thickness stresses in float glass can be significant. The results suggest that SCALP-05 underestimated the stress in the mid-thickness region.

Rather than undertaking a detailed study of the measured stresses in the mid-thickness region, one of the main advantages of the reliable measurements taken in the surface region was exploited in order to predict an accurate estimate of the full residual stress depth profile. This was done by extrapolating the measured stress values in the surface region whilst taking into account the through-thickness force balance. The measured stress values in the surface region was fitted using a second order polynomial of the thickness coordinate *z* whilst ensuring that the constructed full stress depth profile satisfies the overall force equilibrium. A second order polynomial distribution was chosen by considering the knowledge of the residual stress depth profile determined from the contour method. Fig 7a and 7b show the full *xx* depth profile constructed this way after extrapolating the measured stress values within 2.2 mm from the surface determined by using the two SCALP-05 devices respectively. The coefficients of the 2nd order polynomial functions (of *z*) of the two constructed *xx* depth profiles are: -0.338, 3.40 and -5.71 for Exp1; and -0.329, 3.30 and -5.52 for Exp2. The figures illustrate that the agreement between the constructed full *xx* depth profiles and the predictions of the contour method is better than that between the contour method and the measured raw data (Fig. 6c). As can be seen from the figures, the surface compression of each constructed profile remains ~5 MPa and this was not affected by the polynomial fits. The mid-depth tensile stress in each constructed full stress depth profile is ~3 MPa and this agrees with the prediction of the contour method (~4 MPa). Fig. 7 suggests that the agreement between the three stress profiles is good and the contour method results fit within the error range ± 2MPa of the stresses measured using SCALP-05.

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| **Fig. 7.** The constructed σ*xx* depth profile using the results of SCALP-05 of (a) 2.2 mm (b) 5.3 mm measurable depth range | |

**Experimental validation: An 8 mm thick float glass specimen**

The thermal misfit strains generated due to the differential cooling that takes place during the manufacturing of glass result in residual stresses in float glass. It is therefore anticipated that the residual stress depth profile may depend on the thickness of the glass sheet [1]. In order to demonstrate the application of the contour method analysis for a glass specimen of a different thickness, a new specimen of the same surface dimensions (150 mm x 100 mm) as the one considered in the previous example, but of thickness 8 mm was analysed.

The contour method analysis and SCALP-05 experiments were conducted using the same respective procedures used in the analysis of 10 mm thick specimen. Fig. 8a shows the comparison between the *xx* depth profile determined from the contour method and that constructed using the data obtained from SCALP-05 of 2.2 mm measurable depth range. Similarly, Fig. 8b shows the comparison between *xx* depth profile determined from the contour method and that constructed by extrapolating the stress data measured within 2.2 mm from the surface using SCALP-05 of 5.3 mm measurable depth range. As can be seen from Fig. 8, the agreement between the stress profiles constructed using the experimental data and that obtained from the contour method is good. A surface compression of ~5 MPa and a mid-depth tensile stress of ~3 MPa can be noted in each prediction. The experimental results provide a satisfactory validation of the contour method. The results also suggest that both SCALP-05 polariscopes provided similar results. Since SCALP-05 of depth range 5.3 mm is able to provide stress data in a greater region, only this polariscope was used in the experimental investigations described in the subsequent sections of this paper.

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| \\soton.ac.uk\ude\PersonalFiles\Users\bab1v12\mydocuments\Articles\Paper 1 - Prediction of residual stress in structural glass\Review 1\Figures-B&W-Paper1\Updated 01 June\Fig8a.eps  (a) | \\soton.ac.uk\ude\PersonalFiles\Users\bab1v12\mydocuments\Articles\Paper 1 - Prediction of residual stress in structural glass\Review 1\Figures-B&W-Paper1\Updated 01 June\Fig8b.eps  (b) |
| **Fig. 8.** Comparison between the σ*xx* depeth profile determined from the contour method and that determined using the results of SCALP-05 of (a) 2.2 mm (b) 5.3 mm measurable depth range | |

The results described above may also be used to investigate the effect of the thickness of float glass on the residual stress distribution. A detailed study of the dependence of the residual stress on thickness of glass, including a method to incorporate the effect of thickness on residual stress, was presented in a previous work [1] of the present authors.

# Modelling residual stress using eigenstrains

Although the contour method can be used to model the residual stress depth profile in a given glass specimen, new experiments will be required to predict the full 3D stress distribution in the same specimen, in a new glass specimen, or to predict the stress evolution during subsequent applied loadings. To incorporate the effect of residual stresses in glass specimens a validated method, exploiting the results of the contour method analysis of one specimen, is described below.

***Determination of the eigenstrain depth profile***

Eigenstrains act as a source of incompatibility and the knowledge of eigenstrains can be used to represent the full 3D distribution of the residual stress states present in structural components [29]. However, determination of the eigenstrain distribution in a given float glass specimen, which is a prerequisite to model the residual stress, is not trivial. Achintha et al. [29] presented an inverse eigenstrain analysis to determine a representative eigenstrain distribution that generated in laser shock peened metal alloy specimens by using the knowledge of a limited number of measured residual stress data in the specimen. This method was adopted in the current study: it is proposed to determine the eigenstrain depth profile in a given float glass specimen by first assuming a sensible choice of a parametric form and then by matching the predicted residual stress when this assumed eigenstrain distribution was incorporated in a FE model, with the corresponding known xx residual stress data, determined from the contour method analysis, in a least squares sense.

All six components of the eigenstrain distribution are generally required to model the residual stress distribution using eigenstrains. However, as a first approximation, it is appropriate to assume that the eigenstrain in each transverse direction is the same (i.e. ε*ixx*= ε*iyy*). In addition, the eigenstrain in the thickness direction (εi*zz*) was neglected in the analysis, since no significant misfit strains can be developed in the thickness direction: i.e. ε*izz*= 0. All shear components of the eigenstrain are zero, since *x*, *y* and *z* are principal axes. Therefore, only the knowledge of the variation of ε*ixx* with *z* coordinate is required in the analysis (note: ε*ixx*= ε*iyy*). Initially the assumed ε*ixx* eigenstrain depth profile was represented as a series of Chebyshev polynomials [30]. The number of polynomials in the series is to some extent arbitrary, but it should be large enough to capture the expected form of the eigenstrain distribution accurately. In order to achieve a unique solution, the number of polynomials used in the analysis must be less than the total number of known stress data points. In the first step of the inverse eigenstrain analysis, *xx* stress depth profile generated in the specimen for each Chebysheve polynomial of the assumed eigenstrain distribution was determined by incorporating each polynomial in an appropriate individual FE model. The resultant residual stress distribution caused by the originally-assumed eigenstrain distribution is the sum of each residual stress values resulted in from each polynomial of the Chebyshev series. Minimising the mismatch between the predicted *xx* and the corresponding known *xx* residual stress values (i.e. results of the contour method analysis), an accurate estimate of the actual eigenstrain distribution was established. The flowchart of the step-by-step procedure used in the study is shown in Fig. 9.

graph2

**Fig. 9.** Step-by-step procedure of the inverse eiegnstrain analysis used in the hybrid method

In order to determine a suitable Chebyshev series to estimate the actual eigenstrain distribution in the 10 mm thick float glass specimen (Fig. 3a), the analysis was carried out by assuming series of 6, 8, 10 and 12 Chebyshev polynomials respectively. The least squares analysis was carried out for each case by considering 21 data points (i.e. xx values determined from the contour method analysis) along the thickness of the specimen. Results suggest that the choice of a series of 10 Chebyshev polynomials is capable of accurately reconstructing stress field present in the specimen. The subsequent eigenstrain analyses presented below were based on Chebyshev series of 10 polynomials. Fig. 10 shows the best estimate of the ε*ixx* depth profile determined from the inverse eigenstrain analysis described above. From the figure it can be seen that the eigenstrain depth profile has a parabolic shape similar to the residual stress depth profile.

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| \\soton.ac.uk\ude\PersonalFiles\Users\bab1v12\mydocuments\Articles\Paper 1 - Prediction of residual stress in structural glass\Review 1\Figures-B&W-Paper1\Updated 01 June\Fig10.eps |
| **Fig. 10.** Eigenstrain depth profile (ε*ixx*) consutrcted using 10 Chebyshev polynomials |

***Prediction of 3D residual stress distribution using the knowledge of eigenstrain depth profile***

The knowledge of the eigenstrain depth profile derived above can be used to predict the full residual stress state in the whole specimen by incorporating it as an initial misfit strain in a FE model. It should be noted that the solution formulated this way satisfies equilibrium, compatibility, boundary conditions, and the resultant residual stress distribution is entirely self-consistent. Although the computed eigenstrain distribution as the sum of the 10 Chebyshev polynomials (Fig. 10) can be used to determine the residual stress field it is time consuming and computationally inefficient. Therefore, in the present study the resultant eigenstrain depth profile (Fig. 10) was approximated by a 2nd order polynomial function of *z* (coefficients: -0.000078, 0.00077 and -0.0011). Fig. 11a shows the residual stress distribution (*xx*) (only the *xz* plane is shown in the figure) determined by incorporating this 2nd order polynomial distribution of the eigenstrain. Fig. 11b shows the comparison between the *xx* depth profile predicted from the eigenstrain analysis and that constructed using the experimental results of SCALP-05 of measurable depth range of 5.3 mm. As can be seen from Fig. 11b, the stress determined using the eigenstrain method has the same parabolic shape as the experimental stress profile. The surface compression was determined to be ~6 MPa and this is exactly the same value that determined by SCALP-05. The mid-depth tensile stress determined by the eigenstrain method is ~4 MPa and this agrees with that of the experiments.

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| \\soton.ac.uk\ude\PersonalFiles\Users\bab1v12\mydocuments\Articles\Paper 1 - Prediction of residual stress in structural glass\Review 1\Figures-B&W-Paper1\Updated 01 June\Fig11a.eps  (a) | \\soton.ac.uk\ude\PersonalFiles\Users\bab1v12\mydocuments\Articles\Paper 1 - Prediction of residual stress in structural glass\Review 1\Figures-B&W-Paper1\Updated 01 June\Fig11b.eps  (b) |
| **Fig. 11.** (a) σ*xx* distribution predicted by the eigenstrain analysis (b) Comparison between *xx* depth profiles from the eigenstrain analysis and the SCALP-05 expxriments | |

# Prediction of stress evolution around a hole using the knowledge of eigenstrains

In the previous section, it was shown that the eigenstrain depth profile can be determined using the results of contour method analysis of one glass specimen. The use of this knowledge of the eigenstrain depth profile to predict the residual stress distribution in the vicinity of a hole in a glass plate is demonstrated below.

***Residual stresses around a hole***

It is appropriate to assume that the distribution of eigenstrains in float glass of a given thickness is unaffected by the actual geometry of the specimen. Hence, if the eigenstrain distribution in a flat specimen is known, the residual stress distribution in new geometries can be predicted using the knowledge of eigenstrains. The application of this analysis to predict the residual stress distribution in the vicinity of a hole is described using the analysis of a glass specimen (150 mm x 100 mm x 10 mm) with a central hole of diameter 20 mm. Because of symmetry only a quarter of the specimen was modelled (Fig. 12a). Fig. 12a shows the distribution of the σ*xx*componentof the residualstress determined by incorporating the eigenstrain depth profile determined above (Fig. 10) in a FE model. From the figure it can be seen that the σ*xx* distribution is not uniform in the *xy* plane, and in particular, σ*xx* in the inner surface of the hole at the 00 position with respect to *x* direction shows stress of low magnitude in comparison to stresses at locations away from the hole. Surface flaws are inevitably present in the inner surface of drilled-holes, and therefore, the knowledge of exact stress distribution will be required in the strength analysis of the glass specimen.

The predicted stress distribution may be validated to some extent against the experimental results of SCALP-05. One straightforward method of validation is to compare the experimental and the predicted σ*xx* stress depth profiles at selected locations in the vicinity of the edge of the hole. Fig. 12b and 12c show the σ*xx* depth profile predicted from the model and that constructed using the measured stress data at two locations A and D (Fig 12a) at the edge of the hole. The selected two locations, at the 900 and 00 position with respect to *x* direction respectively, represent the maximum and the minimum σ*xx* stress at the edge of the hole. It should be noted that, however, since SCALP-05 cannot measure stresses at the exact edge of the hole, the model predictions and the experimental results shown in Fig. 12b and 12c were obtained at 5 mm away from the edge. From Fig. 12b, it can be seen that the surface compression measured at location A using SCALP-05 is ~6 MPa and this value compares well with the stress predicted by the model (~8 MPa). The mid-thickness tensile stress of 3 MPa of the experimental profile agrees with the model prediction of ~2 MPa. Similarly, Fig. 12c show that the measured and the model predicted σ*xx* stress depth profiles at location D match well with each other. For instance, from each method, the surface compression and the mid-depth tensile stress were determined to be ~2 MPa and ~1 MPa respectively.

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| \\soton.ac.uk\ude\PersonalFiles\Users\bab1v12\mydocuments\Articles\Paper 1 - Prediction of residual stress in structural glass\Review 1\Figures-B&W-Paper1\Updated 01 June\Fig12c.eps  (a) | \\soton.ac.uk\ude\PersonalFiles\Users\bab1v12\mydocuments\Articles\Paper 1 - Prediction of residual stress in structural glass\Review 1\Figures-B&W-Paper1\Updated 01 June\Fig12a.eps  (b) |
| \\soton.ac.uk\ude\PersonalFiles\Users\bab1v12\mydocuments\Articles\Paper 1 - Prediction of residual stress in structural glass\Review 1\Figures-B&W-Paper1\Updated 01 June\Fig12b.eps  (c) | |
| **Fig. 12.** (a) *xx* stress distribution around the hole (b) σ*xx* depth profile at the edge of the hole along path A (c) σ*xx* depth profile at the edge of the hole along path D | |

**Modelling stress evolution around a hole during applied loading**

The results in the previous section confirm that the hybrid contour method/eigenstrain model provides a method to incorporate the effect of residual stress distribution around a hole. This knowledge is extended below to study the stress evolution in the vicinity of the hole during subsequent applied loadings. In this analysis, two FE models of the same glass specimen described above (Fig. 12a) were constructed: the first model did not include the residual stress whilst in the second FE model the effect of residual stress was incorporated. Both FE models were subjected to an arbitrary 10 MPa remote tensile stress in *x* direction.

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| **\\soton.ac.uk\ude\PersonalFiles\Users\bab1v12\mydocuments\Articles\Paper 1 - Prediction of residual stress in structural glass\Review 1\Figures-B&W-Paper1\Updated 01 June\Fig13a.eps**  (a) | \\soton.ac.uk\ude\PersonalFiles\Users\bab1v12\mydocuments\Articles\Paper 1 - Prediction of residual stress in structural glass\Review 1\Figures-B&W-Paper1\Updated 01 June\Fig13b.eps  (b) |
| \\soton.ac.uk\ude\PersonalFiles\Users\bab1v12\mydocuments\Articles\Paper 1 - Prediction of residual stress in structural glass\Review 1\Figures-B&W-Paper1\Updated 01 June\Fig13c.eps  (c) | |
| **Fig. 13.** σ*xx*stress distribution around the hole: (a) Without residual stress (b) With residual stress. (c) σ*xx* depth profile at locations A and B at 10 MPa applied tensile stress | |

Fig. 13a shows the σ*xx* stress distribution in the specimen, which did not include the residual stress, under the applied stress. Fig. 13b shows σ*xx* stress distribution in the specimen in which the effect of residual stress was included. From Figs. 13a and 13b it can be seen that, although in each cases a high stress concentration developed at location A, the *xx* values are different in the two cases. The corresponding *xx* stress depth profiles are shown in Fig. 13c. The model without residual stress show a largely uniform σ*xx* depth profile of tensile stress of ~33 MPa with a maximum mid-thickness tensile stress of ~34 MPa. On the other hand, in the model which included the residual stress the surface tension is ~20 MPa whilst the peak tension at the mid-thickness is ~39 MPa (Fig. 13c). Thus, the design which ignored the residual stress overestimated the surface tensile stresses whilst it underestimated the subsurface peak tension. Fig. 13c also shows σ*xx* depth profile at location B (Fig. 13b), a location away from the edge of the hole. The results show that in the model which included the residual stress, the surface stress at B is ~2 MPa (tension) whereas that of the model without residual stress is ~11 MPa (tension). The mid-depth stress at the same location of the model with the residual stress is ~14 MPa, whilst that of the model without the residual stress is ~11MPa. The results suggest that the residual stress affects evolution of stress under applied loadings, and the hybrid model provides a useful tool to incorporate the effect of residual stress.

# Conclusions

This paper presents a hybrid experimental/numerical-modelling tool to characterise the residual stresses present in commercially available float glass. It has been shown that, by employing contour-method experiments, the residual stress depth profile of a given glass specimen can be determined. The constructed residual stress profiles were validated to some extent against results of scattered-light-polariscope (SCALP-05) experiments. The paper shows that an accurate estimate of the eigenstrain depth profile in the glass specimen can be devised from an inverse eigenstrain analysis using the knowledge of residual stress data obtained from the contour method analysis. It is shown that using the knowledge of eigenstrain depth profile in a given glass type the full residual stress distribution that satisfies overall equilibrium, compatibility and boundary conditions of the given glass specimen can be determined from a finite element analysis. A further advantage of the proposed hybrid model is that the eigenstrain distribution is not sensitive to the changes made into the geometry of the specimen. This means it is possible to undertake a rapid assessment of the residual stress field around complex geometries such as that around a hole and stress evolution during subsequent loading in a computationally efficient manner, simply by installing the appropriate eigenstrain depth profile in the correct geometry.

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