BOUND MODES OF TWO-DIMENSIONAL PHOTONIC CRYSTAL WAVEGUIDES

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1. Introduction

It is now widely recognised that a volume of dielectric material with an appropriately designed periodic microstructure – a photonic crystal – will support a full three-dimensional photonic band gap (PBG) [1]. Over the frequency range spanned by the PBG, all electromagnetic modes are suppressed within the volume, allowing a single resonance (or photonic state) to be introduced by means of a structural point defect [2]. This unique ability to tamper strongly with the electromagnetic mode density enables the channelling of spontaneous emission into one or a few electromagnetic modes, and is attractive for enhancing the emission rate from light emitting diodes, and in achieving low threshold highly efficient operation in micro-cavity lasers [3].

Although photonic crystals with full PBG’s at optical frequencies seem set to have a revolutionary impact in optoelectronics, they are not yet available, largely because the technological demands on nanofabrication challenge the current limits of the state-of-the-art. As several groups have realised, however, it is less demanding to produce two-dimensional periodic patterns in thin film form (see Figure 1), and thus – perhaps – to achieve a full PBG in two dimensions [4,5,6,7]. One important potential application of such photonic crystal waveguides is in the suppression of lateral emission in arrays of closely
spaced vertical cavity emitting lasers [8].

The advantages of waveguiding are well known. Tight confinement of optical power over substantial distances, with precisely controlled parameters of propagation, allows the design and routine production of devices such as modulators, couplers, filters, resonators, mirrors, lasers and amplifiers. Using electron-beam lithography, complex waveguide patterns can be directly written on to a substrate, resulting in multi-functional optical chips. The ready accessibility of every point on the chip means that tapping light in and out, perhaps to interconnect with a neighbouring chip, is straightforward.

![Diagram of photonic crystal waveguides](image.png)

Figure 1 Examples of the photonic crystal waveguides discussed in this chapter. They consist of high index films on a low index substrate, etched vertically through with various periodic patterns.

There is, however, a fundamental conflict between the requirements for waveguiding and those for a full PBG in two dimensions. A waveguide operates by trapping light in a ‘potential well’ of high dielectric constant, depth $\Delta \varepsilon_{\text{film}}$. A photonic band gap appears through strongly modulating the dielectric constant so as to create a periodic array of deep wells and high barriers, with dielectric step $\Delta \varepsilon_{\text{pc}}$ (pc = photonic crystal). If $\Delta \varepsilon_{\text{pc}} \ll \Delta \varepsilon_{\text{film}}$, the waveguiding dominates, being perturbed only weakly by the photonic crystal. Given the high values of $\Delta \varepsilon_{\text{pc}}$ needed for a full PBG, this regime is not attainable in practice except in metal-clad waveguides, where owing to ohmic losses the optical absorption is unacceptably high. If $\Delta \varepsilon_{\text{pc}} \gg \Delta \varepsilon_{\text{film}}$, the waveguide is a weak perturbation to the photonic crystal, with the result that waveguiding will be lost; although a full two-dimensional PBG may be attained, it will be of very limited usefulness because of strong diffractive spreading out of the plane of the thin film. In the regime where $\Delta \varepsilon_{\text{pc}} \approx \Delta \varepsilon_{\text{film}}$, it is unclear whether the waveguide or the photonic crystal will dominate. There is at least a chance that some
beneficial trade-off can be found where useful mode suppression is achieved while retaining wave guidance. This is the regime we explore in this chapter.

The approach we use is as follows. Rather than building coupled mode equations from the guided modes of a uniform film of the same average index as the periodic film (the “conventional” approach as described in many textbooks [9]), we construct the guided modes of the fully etched layer from the Bloch waves of the periodic medium out of which the layer is constructed (Figure 2). This is done by adapting the well-known “zig-zag” ray model [10] for use with the Bloch wave rays of the periodic layer. The resulting guided Bloch modes encompass all the salient features of propagation in the periodic waveguide, including the photonic band structure, dispersion, phase and group velocity [11]. They have many unique and useful features, including momentum gaps (at fixed optical frequency) that cause substantial guided mode suppression in thick layers, and in-plane modal group velocities that can be zero.

![Figure 2](image.png)

Figure 2 Conventional (left) and Bloch-wave (right) zig-zag ray pictures of a mode guided in a thin film. Unlike the rays in the isotropic film, the Bloch wave rays (which follow the group velocity) of the photonic crystal film can be normally incident upon the boundary while still undergoing total internal reflection.

In order for guided Bloch modes to form, the light must be able to bounce to and fro between the upper and lower interfaces of the periodic layer; this requires a non-zero component of wavevector $\beta$ normal to the film, i.e., along the translationally invariant axis of the two-dimensional photonic crystal. Since very few published numerical studies of two dimensional photonic band structure allow for $|\beta| > 0$, we shall devote some space to discussing the wavevector diagrams in that case.

The chapter is organised as follows. First we describe how to obtain the dispersion relations for unbounded two-dimensional photonic crystals (section 2). These are necessary for studying the conditions under which bound modes
exist in thin waveguiding layers formed from slices of these crystals (section 3). We develop a new kind of "band diagram" which shows clearly the regions where bound modes may be expected. In the discussion (section 4), we pay particular attention to localised resonances which occur when the in-plane group velocity is zero, and find that the total number of guided modes in a photonic crystal waveguide can be significantly less than in an equivalent layer of the same average index. Conclusions are drawn in section 5.

![Diagram of high index strips on a low index substrate.](attachment:diagram.jpg)

**Figure 3** Singly periodic layer consisting of high index strips on a low index substrate. The y-axis points normal to the high index planes, and the z-axis normal to the film.

2. Dispersion Relations in Unbounded Photonic Crystals

2.1 SINGLY PERIODIC

In this case, the dispersion relation is readily obtained for a multilayer stack using the standard translation matrix technique, as described (in our own notation) in [12]; see also [13]. It takes the functional form:

$$ k_y = \frac{\arccos \frac{A}{\Lambda}}{\Lambda} \quad (1) $$

where $k_y$ is the Bloch wavevector (pointing normal to the layers), $\Lambda$ the period and:
\[ A = \cos(p_1 h_1) \cos(p_2 h_2) - \frac{1}{2} \left( \frac{p_1 \xi_1}{p_2 \xi_2} + \frac{p_2 \xi_2}{p_1 \xi_1} \right) \sin(p_1 h_1) \sin(p_2 h_2), \]  \hspace{1cm} (2)

with \( h_1 \) and \( h_2 \) the thickness of the layers in each repeating unit \( A = h_1 + h_2 \), whose refractive indices are \( n_1 \) and \( n_2 \). The local wavevector components inside and normal to the layers are \( p_1 \) and \( p_2 \), given by:

\[ p_j = \frac{1}{\sqrt{n_j^2 - (\beta^2 + k_{2j}^2)}} \quad j = 1,2 \]  \hspace{1cm} (3)

where \( \beta \) is the component normal to the waveguiding layer of photonic crystal (the z-direction in Figure 3), \( k_{2j} \) the remaining in-plane wavevector component and \( k_0 = \omega/c \) the vacuum wave constant. The parameters \( \xi_1 \) and \( \xi_2 \) are defined by:

\[ \xi_j = \begin{cases} 1 & (\text{TE}), \\ 1/n_j^2 & (\text{TM}) \end{cases} \]  \hspace{1cm} (4)

where TE (TM) polarisation occurs when the electric (magnetic) field points in the plane of the layers. For convenience we define the following parameters:

\[ \nu = k_0 n_{sv} A, \quad n_{sv} = (\tau_1 n_1 + \tau_2 n_2), \quad \tau_j = h_j / A \]  \hspace{1cm} (5)

where \( \nu \) is the normalised optical frequency, \( n_{sv} \) the average index and \( \tau_j \) the normalised thickness of strip \( j = 1 \) or \( 2 \).

2.2 MULTIPLE PERIODIC

The multiply periodic geometry considered here comprises a hexagonal arrangement of parallel circular-cylindrical "rods" of low index surrounded by a medium of high index. For generality, a normalised propagation constant \( \beta A \) and normalised frequency \( k A \) are defined, where \( A \) is the centre-to-centre spacing of adjacent rods. Suitable normalised parameters in this case are:

\[ \nu = k_0 n_{sv} A, \quad n_{sv} = \sqrt{\sigma_1 n_1^2 + \sigma_2 n_2^2}, \quad \sigma_j = a_j / A \]  \hspace{1cm} (6)

where \( A = a_1 + a_2 \), \( a_j \) being the unit cell sub area for which the index is \( n_j \).

The numerical method employed is the real-space method [14], in which the field and index distributions within the unit cell are discretised on a grid of points. These are grouped into sub-cells, within which the fields are related by transfer matrices. The sub-cells are small enough to preclude numerical instabilities caused by exponentially growing modes. The fields of adjacent sub-cells are then related using a numerically-stable scattering matrix. For hexagonal symmetry, the field and index distributions are discretised along non-orthogonal
axes, corresponding to the primitive lattice vectors of the underlying hexagonal Bravais lattice.

All possible transverse wavevectors \( \mathbf{k} \) in the \((x, y)\) plane at given \( \nu \) and \( \beta \) are sought. The calculation thus implicitly considers at once all possible polarisations and transverse directions of propagation, and the results are equally applicable for any orientation of the structure.

3. Conditions for Bound Modes in Photonic Crystal Waveguides

We now develop a geometrical tool suitable for establishing the conditions under which a photonic crystal waveguide will support guided modes. It involves adapting the usual band diagram by replacing optical frequency (which we keep constant) with \( \beta \), the component of wavevector along the axis of invariance of the structure (the \( z \)-axis in our notation).

In reciprocal space at fixed optical frequency, the allowed wavevectors (of the photonic crystal layer) map out a series of one or more curved dispersion or constant photon energy surfaces \( \beta(k_x, k_y) \), where the maximum possible value of \( \beta \) is given by the product of the vacuum wavevector \( k_0 \) and the highest refractive index in the photonic crystal. At each value of \( \beta \), the curved intersections of these dispersion surfaces with the constant \( \beta \) plane yield a unique wavevector diagram. The boundaries of the first Brillouin zone, being set by the crystal lattice, are (of course) invariant with \( \beta \). Each point \( \mathbf{k} \) on the curves in the first Brillouin zone is associated with a single Bloch wave, and is accompanied by equivalent points in all the higher order Brillouin zones, which tile all of reciprocal space. Each of these points represents the wavevector of one of the partial plane waves in the Fourier expansion of the Bloch wave field. Bound modes can appear only if all of these wavevectors have a magnitude greater than \( k_0 n_{\text{in}} \), the maximum value in the substrate. It is therefore sufficient to consider only the first Brillouin zone, since the smallest Bloch wavevector will always lie within it. The group velocity component (and hence ray direction) of a Bloch wave in the transverse \((x, y)\) plane is given by:

\[
\mathbf{v}_g = \nabla_\mathbf{k} \omega(\mathbf{k})
\]

and points normal to the dispersion surfaces.

Throughout what follows we adopt the normal practice of labelling the points of high symmetry with the letters \( J \) and \( X \), the origin of the \( \beta = \text{constant} \) wavevector plane being labelled \( \Gamma \). The group velocity of the Bloch waves in the \((x, y)\) plane can be vanishingly small at these high symmetry points, owing to the action of one or more sets of primary Bragg planes. Since low group
velocities mean enhanced interactions between matter and light, the high symmetry points are of particular interest in, for example, microcavity lasers.

![Diagram](image)

**Figure 4** Plot of $\beta A$ along a specific path in the $(k_x, k_y)$ plane for a singly periodic layer at fixed optical frequency $\nu = 4$; at a vacuum wavelength of 1550 nm this corresponds to a structure with $A = 493$ nm — for the other parameters see the text. The trajectory follows the $k_x$ axis, turning through a right angle at the origin $\Gamma$, and progressing along the $k_y$ axis through $X$ (the edge of the Brillouin zone). The upper (lower) TE and TM branches at the $X$-point give the $\beta A$ value for slow (fast) Bloch waves, the slow waves having a larger $\beta A$ value. Waveguiding is only possible within the unshaded regions where $\beta A$ in the substrate is not real-valued. The horizontal lines approximately represent successive transverse resonances in a layer of thickness $L = 7A$, for which the intermodal spacing $\Delta \beta A = \pi/7$. Note that the total number of modes supported is reduced by the presence of the momentum gap in $\beta$.

3.1 SINGLY PERIODIC CRYSTAL WAVEGUIDE

As a first example, we consider a singly periodic waveguiding film, consisting of parallel strips of high index material on a low index substrate (see Figure 3).
Figure 5: Plot of $\beta \Lambda$ along a specific path in the $(k_x, k_y)$ plane for a singly periodic layer at a fixed optical frequency $\nu = 2.5$; this corresponds to $\Lambda = 308$ nm at a vacuum wavelength of 1550 nm. The full black curve is for TE polarization, the full gray curve for TM and the black dotted curve for the substrate. See the caption of Figure 4 for more details.

In this case the Brillouin zone is an infinitely long rectangle of half-width (the distance from $\Gamma$ to $X$) $k_y \Lambda = \pi$. In reciprocal space, imagine now an arbitrarily oriented plane containing the $\beta$ (i.e., $k_y$) axis. The wavevector diagram in this plane has two forms, depending on the state of polarization. The $s$ or TE state occurs when the electric field is normal to the plane of the diagram, and the $p$ or TM state occurs when it is in the plane. The full three-dimensional TE and TM dispersion surfaces at fixed optical frequency are the surfaces of revolution formed by rotating these in-plane wavevector diagrams about the $k_y$-axis. The main features of the full three-dimensional dispersion surfaces can be summarised on a two-dimensional plot by showing their intersections with the $(k_x, \beta)$ and $(k_y, \beta)$ planes (left and right hand sides of Figures 4 and 5). This is carried out by plotting the position of the intersections of the dispersion surfaces with a trajectory including the $k_x$ axis, and the line joining the $\Gamma$ to the $X$ point, i.e., $\beta$ versus $k_y$ for $k_x = 0$. As already pointed out, bound modes are possible only in regions where the wavevector in the $(k_x, k_y)$ plane has a magnitude greater than $k_y n_m$. These regions are easily identified if $\beta_s$, the value of $\beta$ in the substrate, is plotted on the same diagram:

$$\beta_s = \sqrt{k_y^2 n_m^2 - k_x^2 - k_x^2}.$$  \hspace{1cm} (8)

Note that this describes the surface of a hemisphere as expected. When $\beta_s$ is real, bound modes are impossible since the light will always leak into the substrate (which is taken as usual to have a higher refractive index than the cover). Provided a transverse resonance condition can be found, bound modes
will occur in regions where $\beta$ in the substrate is imaginary.

The structure treated in Figures 4 and 5 consists of strips of silicon separated by air, with parameters:

$$\tau_2 = 0.4, \quad n_1 = 1, \quad n_2 = 3.5, \quad n_{sv} = 2,$$

the substrate and cover being chosen for illustrative purposes to be silica (index 1.46) and air. At a vacuum wavelength of 1550 nm, these parameters correspond to $\Lambda = 308$ nm, $h_1 = 185$ nm for $v = 2.5$, and $\Lambda = 493$ nm, $h_1 = 296$ nm for $v = 4$. In these units, the locus of $\beta_1$ is a sphere of radius $1.46v/n_{sv}$.

At fixed $\beta$, any Bloch wave whose transverse wavevector lies within the range spanned by the substrate sphere will be leaky. Since every wavevector in the first Brillouin zone is accompanied by partners in every other Brillouin zone, this means that guided modes can occur only within the unshaded regions in Figures 4 and 5. As indicated in (7), the ray directions of the Bloch waves are given by the normals to the dispersion surfaces. One particular mode of the periodic layer thus consists of upward and downward propagating Bloch waves, confined by total internal reflection at the interfaces. One of the most striking things about the loci is the large momentum gap that appears in $\beta\Lambda$. Within this gap there are no real values of $\beta$ and hence no guided modes can exist.

The transverse resonance condition for bound modes may be formally written:

$$\beta L - \Phi = mn$$

where $\Phi$ is the sum of the phase changes upon total internal reflection at the

![Figure 6](image)

**Figure 6** Brillouin zone of hexagonal crystal with trajectory marked in. The intersections of the dispersion surfaces with the $\Gamma$-I-X-$\Gamma$ path, which change as the wavevector component $\beta$ normal to the layer increases, are used to plot the “band” diagrams in Figure 8.
two interfaces and \( L \) is the layer thickness. The spacing \( \Delta \beta \) between successive modes is thus (ignoring changes in \( \Phi \)) roughly \( \pi/L \). This allows us to count up the number of possible bound modes in each case (see section 4).

![Diagram of hexagonal crystal with \( \Gamma \), \( J \), \( X \) points]

Figure 7 Numerically calculated transverse wavevector diagram for a hexagonal crystal with \( v/n_w = 2.2 \) at \( \beta A = 1.76, \sigma_1 = 0.8, n_1 = 3.64 \) and \( n_2 = 1 \).

3.2 HEXAGONAL CRYSTAL WAVEGUIDE

We now extend the results of the last section to layers formed from a hexagonal photonic crystal. On the Brillouin diagram we trace out the usual \( \Gamma-J-X-\Gamma \) wavevector path (Figure 6). As before, waveguiding will exist where the value of \( \beta_s \in \) the substrate is imaginary. To identify these regions, we use (8) to convert the in-plane wavevector \( k \) (joining points along the \( \Gamma-J-X \) path to the \( \Gamma \) point) into \( \beta_s \), which is then plotted on the Brillouin diagram along with \( \beta \) in the photonic crystal. Defining the normalised parameters:

\[
b = \beta A, \quad p = k_p A
\]

where \( k_p \) is the scalar distance along the path \( \Gamma-J-X-\Gamma \) and \( A \) is the distance between adjacent cylinder centres, one obtains:
\[ b^2 + p^2 = (\nu n_{w}/n_{g})^2, \quad \Gamma-I, \quad 0 \leq p \leq \frac{2\pi}{\sqrt{3}} \]

\[ b^2 + \left(\frac{p}{2}\right)^2 + \left(\frac{2\pi - p\sqrt{3}}{2}\right)^2 = (\nu n_{w}/n_{g})^2, \quad J-X, \quad \frac{2\pi}{\sqrt{3}} \leq p \leq \pi\sqrt{3} \quad (12) \]

\[ b^2 + (\pi(1+\sqrt{3})-p)^2 = (\nu n_{w}/n_{g})^2, \quad X-\Gamma, \quad \pi\sqrt{3} \leq p \leq \pi(1+\sqrt{3}). \]

Figure 8 Sequence of “band” diagrams for a hexagonal crystal layer. The horizontal axis is \( k_x A \). The unshaded regions represent the parameter ranges where guided modes are possible. The dashed circles are the substrate \( \beta A \) values. The horizontal lines are spaced by an amount appropriate to the mode spacing of a layer \( 7A \) thick. Note the anti-crossing points where the slope and hence the in-plane group velocity is zero; photons in the vicinity of these regions will be trapped at resonances. The dashed line AA corresponds to \( \beta A = 1.76 \), the value used in Figure 7. The total number of guided modes is reduced compared to a layer of the same average index, the lower order modes (small \( \beta \)) being the first to disappear.
Plotting $b$ versus $p$ yields the "band diagram" for the substrate, which indicates regions where there are no real-valued wavevectors in the substrate, and hence where guided modes might be expected assuming that $\beta A$ is real-valued in the photonic crystal layer. An example of a typical transverse wavevector diagram is given in Figure 7. The complete "band" diagram is plotted in Figure 8 for three different optical frequencies ($\nu/n_{\nu} = 1.4, 1.8$ and $2.2$) at $n_1 = 1$, $n_2 = 3.64$ and $\sigma_1 = 0.8$.

4. Discussion

To illustrate the quantizing effects of the thin layer, we adopt a simplified version of the transverse resonance condition (10) that ignores phase changes upon reflection (a more rigorous treatment of the singly periodic guide is available in [11]). As already pointed out, under these circumstances the step in transverse wavevector between successive guided modes is $\Delta \beta = \pi/L$, where $L$ is the layer thickness. We draw in a sequence of horizontal lines spaced by $\pi A/L$ for a layer thickness $L = 3.7$ $\mu$m, a pitch $A = 525$ $nm$ and $\nu/n_{\nu} = 2.2$ (Figure 8). Within the rectangular unshaded regions, the intersections of these lines with the loci yield the approximate $k_p A$ values of the guided modes.

4.1. MOMENTUM GAPS AND GUIDED MODE SUPPRESSION

Significant suppression of guided modes can occur in a photonic crystal waveguide compared to a layer of the same average index and thickness. By way of illustration, refer to Figures 4 and 5 for a singly periodic layer. The precise number of modes that would be supported by a non-periodic layer of the same average index $2$ and thickness $2.1$ $\mu$m, with air cover and silica substrate, is eight (4 TE and 4 TM) at a wavelength of 1550 $nm$. The singly periodic waveguide supports one (fast) TM mode at $\nu = 4$, and both a slow TM and a slow TE at $\nu = 2.5$. A thick guide supporting only one mode might be useful for increasing the output of an LED by allowing multiple gain regions while preserving single mode operation.

For the hexagonal crystal waveguide, at $\nu/n_{\nu} = 2.2$ all the guided modes which would be present in a layer of the same thickness and average index are suppressed below about $\beta A = 1.7$ (Figure 8).
4.2 LOCALISED RESONANCES

The group velocity of the guided Bloch modes along the layer can be very small, equalling zero if the frequency of the light is chosen so that one of the horizontal lines intersects with the momentum gap edges. In the vicinity of these points the light will travel very slowly along the guide, and enhanced interactions with matter (e.g., optical gain, nonlinearity and electrooptic modulation) will result. In the singly periodic case, in addition to stationary modes consisting of one upward and one downward Bloch wave, other modes appear – unexpectedly – consisting of two upward and two downward Bloch waves [11]. They occur because the guided Bloch modes associated with the fast and slow branches on either side of the momentum gaps (Figure 4) can, under the correct circumstances, be simultaneously resonant (the fast mode having a smaller number of lobes across the layer than the slow branch) at a certain optical frequency. Away from the Brillouin zone edge (the X point in Figure 4),

![Figure 9](image)

Figure 9 Example of the field intensity distribution of a fast (left) and a slow (right) guided Bloch mode in a singly periodic layer at the momentum gap edges. For the fast mode, the light is anomalously concentrated in the air gaps [11].
two such simultaneously resonant modes are coupled together by reflection at the boundaries, and have group velocities that point in opposite directions along the layer. This means that the net group velocity can be zero, allowing a stationary guided mode to form.

In the hexagonal case for $\sqrt{h} = 2.2$, there is a whole series of band edges, which indicate the positions of stationary resonances and the directions in which momentum gaps appear.

Many of the curious characteristics associated with guided Bloch modes are discussed in [11]. One of their most intriguing features is that the modes associated with the fast Bloch waves (i.e., those on the small ellipsoidal dispersion surface around the X-point in Figure 4) appear to be guided by the air gaps in the waveguiding layer. This anomalous behaviour turns out to occur because the period of the field is below the resolution limit of light both in the cover and the substrate, permitting strongly guided modes to be supported. Examples of the field intensity patterns of one of these rather bizarre modes, together with one of the more usual modes (i.e., those concentrated in the high index regions) are available in Figure 9.

In addition to these intrinsically resonant (i.e., zero group velocity in the waveguide plane) guided modes, it is possible to create a resonance by introducing a structural defect in an otherwise perfectly uniform photonic crystal waveguide. This causes a state to form within the momentum gap, in a manner closely analogous with published reports of intra-band defect states that form at a particular frequency within a conventional photonic band gap [2]. The structural defect that gives rise to these intra-momentum-gap resonances can be either a sharply localised point defect, or a smoothly distributed defect. Distributed defects, where a slowly changing aperiodicity with a "bell" distribution is envisaged, may be analysed using a Hamiltonian optics approach [17]. Stationary resonances will allow strong coupling of electromagnetic fields to a dipole of the correct frequency if it is incorporated into the waveguide.

4.3 LEAKAGE MECHANISMS

In order to produce any sort of trapped state (length $\Delta y$) in the waveguide plane, a Fourier spectrum of $k_y$ wavevectors is needed. And, because of the need for a guided mode, the only way a spread of $k_y$ can be produced is by allowing a finite frequency bandwidth. Of course, a finite bandwidth implies a finite lifetime. Thus, the leakage rate for an excitation of length $\Delta y$ depends on the curvature at the energy band edges [11]. This illustrates the need for a general clarification of the questions being asked about spontaneous emission control in waveguides. The presence of Fourier plane wave components that radiate
either laterally along the waveguide (as just described), or into free space (as would be the case for an intra-momentum-gap resonance), precludes the existence of a perfectly confined state within a waveguide supporting a two-dimensional photonic band-gap. As a result, the key issue to address may be that of minimizing the effect of the radiating field components so as to maximize the Q-factor of the guided resonances.

5. Conclusions

The overall conclusion of this chapter is that, despite initial indications to the contrary, strong photonic crystal effects can co-exist together with waveguiding in appropriately designed photonic crystal layers. Furthermore, calculations of the pitch and index contrast needed for a full two-dimensional photonic bandgap, based purely on in-plane propagation ($\beta = 0$), are misleading if a thin layer of photonic crystal is sandwiched between two low index media. The actual pitch required in a waveguide is actually larger and the index contrast smaller than for the in-plane case, owing to there being a substantial component of photon momentum normal to the guide plane. This reduces the photon momentum in the guide plane, making it easier to attain a full two-dimensional band gap. It also lessens the technological difficulty of making practically useful photonic crystal waveguides. It is even possible to attain a full two-dimensional band gap in the silica/air system, where the index contrast is 1.46:1 [15]. Indeed, in the case of the silicon/air system, two-dimensional hexagonal photonic crystals turn out to exhibit many of the properties normally associated only with a full three-dimensional photonic crystal [16].

The analysis presented in this chapter is also relevant, for example, to the use of photonic band gaps in the suppression of lateral spontaneous emission in arrays of closely spaced vertical cavity emitting lasers, which resemble the structures depicted in Figure 1. The ability to maintain waveguiding in the presence of strong photonic crystal effects may be useful in many applications where miniaturisation of standard optoelectronic components, such as couplers, filters and mirrors, is sought.

Finally – and this may be the most significant point of all given the difficulties associated with producing three-dimensional photonic crystals with full band gaps – a technique already exists (anisotropic photo-electro-chemical etching [18]) which allows extraordinarily precise micron-sized patterns to be etched mm deep into silicon. Some of the effects described in this chapter may thus be within reach technologically in the near future.
6. References