Analysing liquidity crashes and liquidity risk contagion in short-term interbank rates

by

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Thesis for the degree of Doctor of Philosophy

September 2015
The financial crisis of 2007-08 is recognised to be the worst crisis since the Great Depression of the 1930s. As a result, liquidity risk and contagion perceived in the interbank market has gained increased attention. In this thesis, the LIBOR-OIS spread, the German-US bond spread, the Euro-dollar currency swap and the EONIA rate are studied to reveal causalities, interdependencies and regime changes in the short-term interbank market.

Interbank markets are channels of contagion due to the overlapping claims banks have on one another. If liquidity dries up in the overnight market, as happened during the latest financial crisis, the domino effect transmits liquidity shocks to other markets. Through three distinct investigations, the main objective of this thesis is to investigate liquidity crashes and contagion in the short-term interbank market. The first analysis demonstrates that there is causality among the series and that they are also cointegrated, while structural breaks are detected in the identified long-run equilibrium relationships. To better identify the breaks, in the second analysis, a novel univariate two-state regime switching model is presented. The variability in the LIBOR-OIS spread along with thresholds of different levels reveal regime changes consistent with liquidity crashes. Thus, the model acts as an early-warning indicator of an imminent liquidity shortage striking the interbank market. Depending which state the system is in, the series is modelled either as a first-order autoregressive process, or as a Gaussian white noise process. Finally, a multivariate endogenous regime switching model describes how liquidity shocks drive the transition between crisis and non-crisis regimes. The investigation uncovers the self-fulfilling nature of endogenous liquidity shocks and their propagation across markets before and during financial crises. Moreover, the results suggest that liquidity shocks originating from the LIBOR-OIS spread govern the dynamics of the system.
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Declaration of Authorship

I, Andrea Eross, declare that the thesis entitled Analysing liquidity crashes and liquidity risk contagion in short-term interbank rates and the work presented in the thesis are both my own, and have been generated by me as the result of my own original research. I confirm that:

- this work was done wholly or mainly while in candidature for a research degree at this University;
- where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated;
- where I have consulted the published work of others, this is always clearly attributed;
- where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;
- I have acknowledged all main sources of help;
- where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;
- none of this work has been published before submission

Signed:....................................................................................................................... 

Date:..........................................................................................................................
Acknowledgements

I would like to thank my previous supervisor Prof Gerhard Kling for enabling me to think outside the box and dare to explore uncharted waters. His vast knowledge gave me motivation during my PhD years.

Special thanks go to my supervisors Prof Simon Wolfe and Dr Andrew Urquhart who guided, supported and encouraged me during my final PhD year. Their advice was invaluable when most needed. I give thanks to a special friend who helped me in my struggle with Matlab coding and to the patient friends who proofread my thesis.

I would like to extend my gratitude to the Economic and Social Research Council, UK, which provided the scholarship for the PhD and funding for the three-month overseas institutional visit to the University of California, USA. I thank Prof David Draper for receiving me there and providing all the resources (office space, enrolling me into three postgraduate modules, weekly seminars) and for his suggestions which undoubtedly much improved this thesis.

Lastly, my heartfelt gratefulness goes to you Mum and Dad, for believing in me and supporting me endlessly in every possible way. Your unconditional love, moral support and sacrifice contributed to the completion of this thesis. I owe you all.
Nomenclature

ADF  Augmented Dickey-Fuller test  
AIC  Akaike Information Criteria  
AR(1)  Autoregressive process of order 1  
ARCH  Autoregressive Conditional Heteroskedasticity  
ARMA  Autoregressive Moving-Average process  
BIC  Bayesian Information Criteria  
CDS  Credit default swap  
DCC-MGARCH  Dynamic Conditional Correlation - Multivariate Generalised ARCH  
DF  Dickey-Fuller test  
DIC  Deviance Information Criteria  
DSGE  The Dynamic Stochastic General Equilibrium Theory  
ECB  European Central Bank  
EGARCH  Exponential Generalized ARCH Process  
EM  Expectation Maximization algorithm  
EONIA  Euro OverNight Index Average  
ESW  Early warning system  
EUSWEC  Euro-Dollar Overnight Interest Swap  
FX  Foreign Exchange  
GerUS3M  The spread between the daily German and US government bond rate  
GARCH  Generalized Autoregressive Conditional Heteroskedasticity process  
I(0)  Stationary process  
I(1)  Unit root (non-stationary) process  
KPSS  Kwiatkowski-Phillips-Schmidt-Shin test  
LIBOR-OIS  The London Interbank rate - Overnight Indexed Swap spread  
MCMC  Markov chain Monte Carlo method  
ML  Maximum Likelihood estimation  
NBER  National Bureau of Economic Research  
OLS  Ordinary Least Squares  
RBC  The Real Business Cycle Theory  
SD  Stable Deterministic System  
SS  Stable Stochastic System  
TVTP  Time-varying Transition Probability model
<table>
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<tr>
<td>VAR</td>
<td>Vector Autoregression</td>
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<td>VECM</td>
<td>Vector Error Correction model</td>
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Chapter 1

Introduction

1.1 Overview of the chapter

This chapter presents the research problem, motivation for investigation, research objectives and the relevant research questions. It identifies gaps in the interbank liquidity literature and justification for this research. Furthermore, it briefly outlines the data used in the analysis and the adopted methodology with contributions to existing literature. The chapter concludes with the outline of the thesis.

1.2 Statement of the problem and motivation for study

The financial crisis of 2007-08 is recognised to be the worst crisis since the Great Depression of the 1930s. As a result, liquidity risk and contagion perceived in the interbank market became the focus of interest. Market players anxiously observed how market circumstances changed dramatically in very short periods of time. In the period preceding the BNP Paribas bank announcing its losses on the 9th of August 2007, we witnessed how liquidity risk build up within the financial system.

During the last decade, a credit bubble increased inconspicuously in the housing market, which eventually burst and brought down some of the major financial players. Ultimately, this was credited to the liquidity shortage which developed and spread within the short-term interest rate market. There is a widespread agreement that the root causes of the recent credit crunch were the inadequate liquidity buffers and lack of regulation in the financial system. However, it is also argued that the root causes started

\[ \text{On that date, France’s biggest bank stopped withdrawing its investment funds as a result that they were not able to value them. This further led to liquidity dissapearing in the interbank market. The broadcast caused a drop in its share value as well as a plunge in the value of benchmark European assets and marked-to-market losses in the US asset-backed securities market (Boyd, 2007).} \]
a decade earlier, when the US Treasury lowered the interest rate to a previously un-
seen level and kept it there for a prolonged period of time (Economist, 2013). From a
monetary policy view, the focus was on low consumer index inflation while ignoring the
widespread development of asset price inflation. Consequently, excess liquidity built up
in the market.

From 2004 to 2006, US base interest rates rose from 1% to 5.35%, causing a rise in prop-
erty prices and slowdown in the US housing market. At the time, the financial system
was ineffectively regulated, and this led to an upsurge in newly created non-traditional
mortgage-backed financial products. During the years, these securities were re-packaged
and re-sold to investors around the world. The default on mortgages which had been
sold to home owners with poor credit histories increased to record levels, and this was
ultimately perceived by the whole financial system. Financial deterioration originating
from the housing downturn was concentrated in the heavily leveraged financial institu-
tions (Gorton and Metrick, 2012). By the beginning of 2007, there were early warning
signs of a credit turmoil approaching. In February 2007, the Federal Home Loan Mort-
gage Corporation declared that it will no longer purchase mortgage-backed securities.
At the beginning of April 2007, another US institution, the New Century Financial
Corporation filed for bankruptcy. By July 2007, credit rating agencies Standard and
Poor’s and Moody’s downgraded more than 1300 securities backed by mortgages orig-
inating from both the USA and Europe. Financial institutions, such as IndyMac and
Lehman Brothers were the first casualties, however some institutions were bailed out by
their governments, such as American Home Mortgage Investment Corporation and the
British bank Northern Rock, among others. By the end of 2008, the crisis affected all
major economies around the world, and consequently in the following years, these were
plagued by economic recession characterised by negative or very low output coupled
with a high unemployment rate.

The motivation for this study is twofold. First, the Basel Accords do not address the
issue of liquidity risk contagion within financial markets. Behind the crisis is the explo-
sion of the housing bubble built on complex financial products, such as collaterised debt
obligations (CDSs), coupled with the failure or lack of adequate financial regulation.
The Basel II Accord was set up in 2004 solely as a recommendation for international
regulatory standard aimed at maintaining capital adequacy, stress testing and financial
liquidity in the banking system (Allen et al., 2010). As the treaty had merely consultative
and no obligatory power, it was never implemented, primarily due to political
issues, and was subsequently replaced by the Basel III Accord, which addresses stricter
standards, such as market discipline and banking disclosure, as well as reform measures
on capital adequacy (for counterparty risk) and liquidity; it was set up in the wake of
the financial crisis of 2007-08 and was adopted by the major economies, including the
USA (BIS, 2013). Essentially, reckless banking procedures coupled with loose regulation
increases the occurrence of liquidity crashes, and ultimately bring devastating consequences on the affected economies (Ayadi, 2013). Consequently, there is an urgent need for regulatory renewal and consolidation in the financial system. Second, the uncertainty surrounding the progression of interest rate and spread variability is believed to be exogenous. Hence, it is assumed that interest rate variability is not conditional on shocks evolving from within the rates under investigation, nor on the actions of financial institutions or other market players. When the interbank market is not in turmoil, this view is rather benign. However, in crisis periods when information and beliefs are more likely to become homogenous, related interest rates behave in a similar fashion. In such cases, liquidity shocks are amplified from within the system as part of some self-fulfilling forecasts or endogenous responses (Danielsson, 2011). Previous theoretical and empirical analyses failed to detect, prevent and forecast liquidity crises. Thus, there is demand to develop an endogenous liquidity risk and contagion model which is able to detect and forecast liquidity shocks before they actually accumulate and subsequently transfer to neighbouring markets.

1.3 Overview of the existing literature and basic concepts

The literature investigating the financial crisis of 2007-08 focuses primarily on the analysis of liquidity and credit risk indicators, and decomposition of spreads into credit and liquidity components, revealing which component had a larger influence on widening of the short-term interest rate spreads.

Broadly, there are three major strands of empirical literature analysing liquidity crises in the last decade. One refers to the market microstructure of interbank markets, while looking at trading behaviour of high frequency data, such as Hartmann et al. (2001), Danielsson and Saltoglu (2003), Frank et al. (2008) and Baba et al. (2008), among others. Another focuses on measuring and assessing liquidity risk (such as Brunnermeier (2009), Schwarz (2009), Brunnermeier and Oehmke (2012), Gorton and Metrick (2012), Mistrulli (2011), Zhou and He (2012) and Min and Hwang (2012), among others), while a limited number of studies adopt regime change and Markov models (such as Dahlquist and Gray (2000), Ang and Timmermann (2011) and Guo et al. (2011), among others). Yet, for the period that followed the financial crisis of 2007-08, the majority of studies focussed on decomposing spreads into risk components prevalent in bringing down institutions and financial markets. Most of these empirical studies are linear in nature and they are not able to capture non-linearities which are particular to circumstances.
surrounding financial crises (Baba et al., 2008; Schwarz, 2009; Mistrulli, 2011; Upper, 2011; Gorton and Metrick, 2012). Moreover, these investigations do not address the core of the problem: what generates and drives these processes (liquidity crises), and how liquidity shocks propagate from one market to another?

The following is a description of the concepts that are relevant to this thesis. Contagion or spillover is used interchangeably in this study; it is defined as the transmission of financial shocks that originate nationally (or domestically), and which may ultimately reinforce and result in the failure of an institution. Kaminsky and Reinhart (2000) argue that in the various definitions of contagion, the notion of symmetric shock is generally not contained. Financial shocks propagate from one market to the other, eventually progressing from micro- to aggregate level. Likewise, shocks can simultaneously spread between different industries and markets, to later involve cross-border markets and regions. Another concept analysed in the present research is structural breaks, which are a fundamental notion in econometrics as they indicate an unanticipated shift, be that in the intercept, in the mean or in the variance of the time series. The presence of a structural break makes modelling difficult, and obliges the researcher to proceed with caution. In case of linearity, the Chow test\(^3\) is sufficient to detect structural breaks in the intercepts (Chow, 1960). When the data is non-linear with a known single structural break, tests of cointegration may be used. Where there are several structural shifts in the time series, the only efficient model known to identify the shifts (as time intervals) comes from the regime switching Markov chain family. Such models are successfully used in business cycle models, however they are non-existent in the liquidity risk and financial crisis literature.

Minsky (1992) defines financial instability as a process in which the prices of financial or capital assets dramatically change compared to the present prices of output. Financial instability is often associated with periods of recession. The author argues that financial instability can be the result of liability structures (for example asset-backed securitisation in the last financial crisis) which develop over long periods of time, while these structures cannot be endorsed by cash flows or asset prices set by the market. Liquidity shortage\(^4\) is associated with the failure of a financial institution and can consequently result in the collapse of other banks that are not necessarily directly affected by the initial shock. Liquidity risk\(^5\) can be the outcome of credit risk\(^6\) for example, which in turn is associated with interbank lending. Interbank contagion becomes a domino effect when more institutions are affected cross-regionally. According to economic theory, the

\(^3\)The Chow test uses the sum of squared errors to test whether the error terms are equal in a time series in the presence of a single structural break.

\(^4\)If a financial institution is not in possession of sufficient liquid assets and cannot find the cash it requires, and subsequently cannot meet its short-term obligations, that institution is said to be in a liquidity crisis (Tirole, 2008).

\(^5\)The probability of not being liquid implies there is liquidity risk. The proportion of illiquidity determines whether liquidity risk is high or not.

\(^6\)The Bank for International Settlements (BIS, 1999) defines credit risk as the possibility that a debtor or a counterparty is unable to honour its dues set out in the terms of agreement.
risk of contagion is determined by the way financial institutions are linked with each other, thus financial contagion can be modelled as an equilibrium phenomenon (Allen and Gale, 2000b; Freixas et al., 2000). Interbank links are essential, for they fundamentally help banks handle specific liquidity shocks. On the downside, interbank markets are channels of contagion primarily due to the overlapping financial claims banks have on one another. If a financial institution is hit by a liquidity shock, it will primarily turn to the overnight interbank market to fund the imminent liquidity shortage. However, if liquidity dries up in the overnight market, before liquidating its long-term assets, the bank will try to draw on its deposits at other banks. As Allen and Gale (2000b) argue, if there is no collective uncertainty concerning liquidity in the interbank market, the first-best distribution of risk sharing can be realised by the process of asset allocation. Fundamentally, the interbank market\(^7\) can only re-allocate liquidity and not create it on its own (Farhi et al., 2009). Domino effects can occur in very short time periods, and financial institutions are faced with little or no room for maneuvering in such events.

According to Diamond and Dybvig (1983) financial crises are pure random events, and therefore bank runs are merely self-fulfilling forecasts. Others, such as Mitchell (1959), Gorton (1988), Hamilton (1989) and Allen and Gale (2000b) have the alternative view that financial crises are intrinsically part of the business cycle. Allen and Gale (2000b) and Allen and Gale (2000a) argue that the interconnectedness of markets\(^8\) determined by cross-ownership of deposits, has a great impact on financial contagion. Feedback-loops have the characteristic of magnifying reactions to liquidity shocks, as well as externalities which are transferred across balance sheets (Danielsson, 2011). Moreover, if the market overall is incomplete\(^9\), each region is associated to several small regions. In such circumstances, early liquidity shocks can have a great impact on neighbouring markets, and eventually these surrender to a financial crisis. Subsequently, if several markets are involved in the process, a succession of liquidity shocks induces flash liquidation of assets, which in turn will result in dropping asset values. Therefore, previously unaffected markets and regions are drawn into the process due to the fact that their claims on the market - which is in financial distress - have dropped in value.

The last important concept to mention is that of an early warning system (EWS). Due to the above mentioned interconnectedness of markets and the complexity of such a system, an operational EWS has not been developed in any of the financial and economics disciplines. Therefore, the literature covering this topic is very limited and concentrates

\(^7\)Interbank markets are a fundamental part the financial system, primarily acting as intermediaries between parties. The main purpose is to reallocate liquidity between lenders and borrowers. Central banks implement their monetary policy via interbank markets whose effective functioning conserves financial stability.

\(^8\)In such system, “the relative values of liabilities and assets (and hence the net worth), the availability of credit, and asset prices are interrelated and fluctuate together” as Shin (2008, p.315) puts it.

\(^9\)In the field of economics, in an incomplete market the number of securities present in the market (which represents the world) would be less then the number of the states of the world (which is finite and exhaustive). In such situation, the optimal allocation of securities and risk-sharing is not achievable, and thus the market is not in equilibrium (Arrow, 1964).
on the period preceding the recent financial crisis. However, successful EWSs’ were
developed in the medical field for example, specifically in detecting contagious diseases
(Martínez-Beneito et al., 2008). Based on this development, I attempt to create a similar
system taking into consideration the fact that financial markets - in times of crises - are
not affected by seasonality as opposed to some contagious diseases which are believed
to be seasonal.

The present study fills the gap identified in both theoretical and empirical liquidity
 crisis and contagion literature. Essentially, the analysis integrates two strands of litera-
ture: the literature on shocks and their propagation within the financial system, and the
literature on regime switching Markov chain models successfully adopted in the macroe-
conomic (business cycle) literature. To the best of my knowledge, no recent analysis has
placed liquidity shocks inducing contagion between money markets in its focus.

1.4 Research objectives, research questions and data used
in the analysis

The focus of this thesis is liquidity risk detection and contagion in the short-term in-
terbank market. The main objective of the investigation is to assess the behaviour of
short-term interest rates and spreads in times of financial turmoil. Geographically, this
investigation concentrates on the European and US interbank market. Thus, the present
analysis provides a comprehensive inspection of the behaviour of leading interbank indi-
cators. The ultimate goal is the development of a multivariate regime switching model
which is able to detect/signal and trace liquidity shocks in the interbank market. In
order to achieve this, the investigation must provide an answer to the following research
questions:

1. Is there causality among short-term interbank rates and spreads? If so, what are
   the implications for the smooth functioning of the interbank market?

2. Are short-term interbank rates and spreads interconnected? If they are, what
   are the implications for the propagation of liquidity shocks within the interbank
   market?

3. Are there equilibrium relationships among short-term interbank rates and spreads?
   In what way do structural breaks affect the equilibrium relationships?

4. Was there an early warning signal before the financial crisis of 2007-08 erupted?
   Can the short-term interbank spread (LIBOR-OIS) predict financial crises? What
   are the implications for liquidity risk management?

5. How liquidity shocks accumulate over time and spread between the short-term
   interest rate, bond and currency markets?
6. Can structural (endogenous) shocks define the dynamics of the interbank financial system?

7. What drives the volatility of short-term interbank rates and spreads in turbulent times? Can liquidity shortage in the short-term interbank system be traced back to shocks originating from the short-term interest rate spread, the bond and currency markets and the currency swap rate?

8. In the light of findings, what are the overall implications for market players and the Basel Committee in terms of liquidity crisis prevention and forecasting?

The present research investigates 10 years of daily data, spanning from 1st January 2002 to 30th December 2011. The variables of interest are the spread between the US London Interbank Offered Rate and Overnight Interest Swap (LIBOR-OIS), the Euro-Dollar Overnight Interest Swap (EUSWEC), the three-month German-US bond (GerUS3M) spread and the Euro Overnight Index Average rate (EONIA).

1.5 Research methods

The above research questions are answered with the help of quantitative methods. The investigation was conducted using the softwares Stata (StataCorp., 2013), WinBUGS (Lunn et al., 2000) and MATLAB (MATLAB, 2014). Datasets were obtained from the Bloomberg Database (Bloomberg, 2012). The thesis consists of three distinct investigations. After presenting summary statistics (see Chapter 3), interdependencies and causality among the variables of interest are identified. Vector Autoregression (VAR) is run on the differenced time series with the aim of revealing whether changes in the independent variables cause movements in the dependent series. To see the adjustment course of the LIBOR-OIS, the EUSWEC and the GerUS3M spreads, and to assess how each type of shock feeds back to the forecast error variance, the impulse response function is implemented. As illustrated by Figure 3.9, shocks are not persistent. To see which market moves first and which one follows in propagating liquidity shocks, the Cholesky decomposition is used on the structural VAR residuals. Results reveal that the LIBOR-OIS spread moves first, followed by the currency swap and German-US bond spread. Next, the short-run and long-run equilibrium relationships among the three spreads are determined. Cointegration provides a framework for estimating long-run equilibrium parameters of non covariance stationary processes. One indication that time series are integrated is that error terms (nuisance or noise terms) cumulate over time. For the whole analysed period (from 1st January 2002 to 30th December 2011), all the three time series are found to be integrated of order 1. If the time series are cointegrated, a simple regression model is mis-specified, thus the Ordinary Least Squares (OLS) method cannot be used. Therefore, the long-run equilibrium relationships are estimated using
the Johansen (1988) and the Gregory-Hansen (1996) techniques, and show that the spreads are cointegrated and move together in a synchronised fashion. However, co-movements of the time series break down due to structural changes, as both tests reveal. The concept of structural break is one of the most significant in econometric time series analysis. If an unpredicted shift emerges in the series at a point in time, the estimated coefficients will not be constant over time, and consequently the model is mis-specified and forecast errors will be of high magnitude. Therefore, it is essential that structural breaks are identified in long-run relationships.

When time series switch from a I(0) to a I(1) process, and then back into stationarity, as seen in the time series which covers the pre-crisis, crisis and post-crisis period (see Figures 3.1 and 3.2), it can be argued that the series become consistent to a random walk\textsuperscript{10}. In such situations standard econometric models fail to explain the dynamics of the markets of interest, that is to show whether something persistent is driving the series, also whether there are any transitory elements in the processes. Furthermore, standard models are mainly descriptive in the sense that they only reveal trends, levels, components, etc. Yet, these models do not reveal anything important about the data generating process. Therefore, it is assumed that there is a hidden data generating process, which is to be revealed.

To overcome the aforementioned shortcomings, a novel approach to detect liquidity crashes in the short-term interbank market is presented in Chapter 4. The advantage of this model is that the data is not smoothed by means of taking the first difference of the series, nor significant outliers are eliminated. The aim is to find out how the outliers (as very significant observations in this case) contribute to the estimation of parameters and how they drive the dynamics of the system in a probabilistic setting. The LIBOR-OIS spread proves to be a good predictor of crisis and tranquil periods for the analysed time interval. Considering that the investigation deals with latent variables, states and non-linearities, the methodology calls for the implementation of a model grounded on MCMC simulations on a Bayesian platform. No past or recent papers discussing methods for detecting financial crises with various thresholds have been identified. The model designed in this analysis successfully defines when the series is in a pre-, mid-, or post-crisis period. This constitutes one of the significant contributions of this work. The model actually contains the permanent and transitory components (without using the old fashioned decomposition method to liquidity, failure and other elements) which can trace the liquidity risk built up prior to and in the height of the financial crisis of 2007-08 in the short-term interbank market. The model benefits from the autoregressive process identified in the first part of the analysis. Regime switching models are ideal when data is correlated. For simplicity, the state space consists of two independent states

\textsuperscript{10}Random walk is assumed to be the pillar of efficient market hypothesis which suggests that prices of financial products cannot be predicted over time. The term is associated with Brownian motion (used in Physics) where particles move in a random fashion from one place to another, and one is unable to foresee where the particle would move next.
which delimit crisis and non-crisis periods; essentially, this is achieved by exploiting the variability of the LIBOR-OIS spread. Depending on which phase the system is in, the series is modelled either as a first-order autoregressive process, or as a Gaussian white noise process. The transition between phases is described by a Markov process, and the probability of being in a crisis or non-crisis state is estimated using the Gibbs sampler. Part of the observations are used to forecast out-of-sample and then compare the estimated forecasts with the ex-post identified crises.

Finally, in Chapter 5, a multivariate endogenous regime-switching model is built with the aim of tracing liquidity risk contagion within the financial market. The estimates from the two-state Markov regime switching model with constant transition probabilities are compared to the estimates of the model using time-varying transition probabilities. The motivation of using an endogenous Markov regime switching model is that liquidity shocks are generated and intensified within the financial system and not only externally, as the majority of models assume. The time-varying probabilities allow the time series to depend on time as well as on some of the parameters of the model (such as the $\beta$s, for example). Maximum Likelihood (ML) infers the model estimates, while the modified Hamilton filter (1989) determines the time-varying transition probabilities. Thus, by building an endogenous model with time-varying transition probabilities, one can reveal what drives the dynamics of short-term interbank rates. Moreover, the model determines turning points and measures the persistence of crisis and non-crisis regimes. Fundamentally, the novel model is an EWS for it detects and signals disturbances (liquidity shocks) ex-post, which ultimately may lead to an emergency/crisis in the financial market. The results suggest that liquidity shocks originating from the LIBOR-OIS spread along with the variance of the system drive regime changes in the German-US bond spread. Based on the results of the three analyses several recommendations are put forward to the Basel Committee on Banking Supervision and to various financial institutions. The recommendations, along with those of the Basel Committee contribute to consolidating the international financial system overall. Ultimately, this helps avoid economic recessions, which fundamentally are the result of financial crises.

1.6 Contributions of this study

The main contributions of this comprehensive analysis are:

1. The development of a methodical analysis of leading short-term rates and spreads to determine causality, linear interdependencies and equilibrium relationships in the short-term interbank market.
2. An examination which allows the prediction of which market moves first and last in propagating shocks within the short-term interbank market, as well as the projection of the magnitude and effects of liquidity shocks on other spreads and interest rates.

3. A study which reveals structural changes and their permanent effect in the identified long-run and short-run equilibrium relationships.

4. To further the empirical literature by proposing a novel two-state univariate regime switching Markov chain model which successfully forecasts crises in the interbank market.

5. To further the empirical literature by proposing a novel multivariate endogenous regime switching model which outperforms previous linear early warning systems and regime switching models with constant transition probabilities, in order to detect liquidity crises and contagion in the short-term interbank market.

6. To further the theoretical literature by indicating the self-fulfilling nature of endogenous liquidity shocks which accumulate and magnify through a feedback loop and ultimately result in an aggregate financial distress.

7. To complement the Basel Accords on the issue of liquidity risk management and advise financial institutions on strengthening and maintaining financial stability within the financial system, with the ultimate aim of preventing financial crises.

1.7 Outline of the thesis

The outline of the thesis is as follows. Chapter 2 presents the literature review in terms of theories related to liquidity risk, financial crisis and contagion, as well as theories of market equilibrium and market expectations. The Chapter also discusses the Basel Accords and their implications for liquidity crisis and contagion management in the interbank market. Chapter 3 starts with statistical fundamentals and moves onto econometric modelling. Fundamentally, the Chapter reveals whether there is causality and interdependencies among the time series of interest; moreover, it determines short-run and long-run equilibrium relationships and structural breaks which disrupt the equilibrium relationships. Chapter 4 describes a novel univariate regime-switching model of liquidity risk detection, its application to the data and out-of-sample forecasting. The purpose is to show whether regime changes in the times series correspond to crisis periods ex-post, as well as to see whether the novel univariate regime switching model is able to accurately forecast both the financial crisis of 2007-08 and the Eurozone crisis that followed. In Chapter 5 an endogenous multivariate regime switching
model is presented. The purpose is to detect and predict liquidity crashes in the short-
term international interbank market in a multivariate setting. Theoretical implications,
contributions, limitations and further work are discussed in Chapter 6.
Chapter 2

Literature Review

2.1 Introduction

The importance of liquidity risk and contagion in the cross-border interbank market has clearly gained consideration during the recent global financial crisis, however it is not new. There are several approaches which measure the strength and propagation of shocks using data of past financial crises. However, none of the approaches consider all the characteristics of such a volatile time period, while at the same time assessing liquidity spillovers cross-regionally and evaluating the intensity and length of financial crisis episodes induced by interbank liquidity shocks.

The chapter is organised as follows. Section 2.2 provides a critical assessment of the financial crisis literature grounded on a microeconomic foundation in an equilibrium state. The theories that underpin the present study are discussed in the following two sections. Section 2.3 presents the General Equilibrium Theory with particular emphasis on the Business Cycle Theory and the Dynamic Stochastic General Equilibrium Theory. Section 2.4 reviews the Rational Expectations Theory, while Section 2.5 presents the Basel Accords and their implications to modelling and forecasting liquidity crises. Section 2.7 concludes the chapter.

2.2 Theories of market liquidity, financial crisis, contagion and bubbles

The section below discusses the literature using theories of market illiquidity in general, financial bubbles, financial crises and contagion. Before introducing the theoretical models, the two stages of financial crises are presented. The models introduced below are built on a theoretical microeconomic foundation (as opposed to existing macroeconomic analyses that are merely empirical) and assume either a finite- or continuous-horizon
setting. The agents of the economy are consumers and households, traders and specialist/financial institutions who trade risky and/or riskless assets of various maturities among themselves in normal and turbulent times. Agents possess endowments with the fundamental aim to maximise their utility function. The focus of this review is on the intermediation process in the presence of frictions (imperfect competition, for example), constraints (funding, for example), information asymmetry and shocks which affect the price formation, decision and behaviour of agents. Furthermore, it is assumed that the system overall is either dynamic or static, and the intermediation process is modelled as an equilibrium process.

According to Brunnermeier and Oehmke (2012), there are two well defined stages that play a significant role in the development of financial crises. The first one is the run-up stage in which bubbles and imbalances develop. The second stage is the crisis stage in which the risk accrued in the previous stage materialises and the bubble explodes. The first phase is characterised by relatively low volatility and the process is gradual. Financial imbalances are often unnoticed and asset increases are credited to technological or financial innovation, or liberalisation. Asset price increases and their valuation is an upward spiral driven essentially by incentives based on either rational behaviour or behavioural belief biases. As Brunnermeier and Oehmke (2012, p.4) note “the resulting leverage and maturity mismatch may be excessive because each individual speculator does not internalize the externalities he causes on the financial system”.

![Figure 2.1: Liquidity shock spiral during financial distress. The figure is adopted from Brunnermeier and Oehmke (2012, p.41).](image)

The crisis stage is activated by an event that causes the bubble to explode. The event involves a drastic asset price adjustment and subsequently the outcome is intensified, as can be seen in Figure 2.1. The authors emphasise the importance of an amplification mechanism (in the form of a self-fulfilling loop) in the development of the crisis, which defines the intensity of price adjustments. There are two channels which aid the feedback mechanism. The direct channel is initiated by direct business realtionships...
(interconnectedness of markets, for example), whereas the indirect channel is induced by contagion (in the like of bank runs or domino effects, for example), or externalities caused by endogenous reactions of market players (Brunnermeier and Oehmke, 2012).

Probably the most significant liquidity measures driven by theory are the price impact $\lambda$ and the price reversal $\gamma$, whereas bid-ask spreads, market depth, turnover and trade volume are fundamentally empirical (Vayanos and Wang, 2012). On the other hand, the theoretical framework which deals with liquidity risk in general (in terms of financial crises, contagion and bubbles) is primarily centred on asset pricing and value at risk models. Banks’ increased leverage and change in the structure of their assets and liabilities contribute to liquidity risk and its contagion to interconnected institutions and markets. The section below surveys the theoretical framework concerning liquidity risk.

Diamond and Dybvig (1983) argue that financial crises are chance events. Moreover, it is assumed that in both stable and crises periods, there is an equilibrium in the financial system. Allen et al. (2009) note that according to existing academic research, there are two major strings of theories which describe financial crises. The first one states that bank runs are self-fulfilling forecasts. If panic is perceived in the interbank market, agents react by withdrawing their assets and at the same time they cease lending to each other. On the other hand, in times of stability, agents withdraw their deposit merely to satisfy their consumption needs. The authors argue that it is hard to decide which theory to apply particularly for policy analysis, since these do not explain what prompts such turbulent financial periods. This is due to the fact that it is assumed that both crises and stable periods are essentially equilibriums. Yet, existing theories do not explain how agents decide on the particular equilibrium. According to the second set of theories, financial crises form an integral part of the business cycle (Mitchell, 1959; Gorton, 1988; Hamilton, 1989; Hamilton and Owyang, 2011), and therefore are predictable events. Financial or economic crises are followed by recessions, whereas stable periods are associated with economic growth. One explanation of financial crises being part of the business cycle is the role of information asymmetry played within the financial system (Allen et al., 1993). In the situation when market players lack crucial information to make an informed decision regarding their transactions, one talks about the lemons’ market\(^\text{1}\) in which case other asset prices are caught in the process, and subsequently their value is also depressed (Socio, 2011). In other cases, if investors sense and anticipate there is a recession looming, they will withdraw their deposits, thus stimulating a downturn in economic activity (Kodres and Pritsker, 2002). This is owed to the expectations of market players, and the Rational Expectations Theory applies.

The recent financial crisis highlights the importance of contagion and its association to the structure of connections among financial institutions and their funding maturity.

\(^1\)The notion of ‘lemons’ market’ is linked to fire sales in the insurance and investment sector. Months prior to the French bank BNP Paribas’ announcement of stopping the trade of asset-backed securities, there was a lemons’ market, in which case several liquidity stricken mutual funds and investment banks started unloading their unwanted asset holdings.
Market failures are the result of propagation of liquidity shocks across financial markets, whereas contagion is seen as the most important of market breakdowns. Allen and Gale (2000b), Allen et al. (2010) and Makarov and Plantin (2013) among others, argue that contagion can be modelled as an equilibrium phenomenon. When a crisis develops, only some institutions or a certain section of the economy is affected; however, shocks rapidly propagate to the rest of the financial sector and eventually damage the larger economy. A small liquidity shock in the money market for example, has the potency to generate the self-fulfilling anticipation of a crisis. Fundamentally, contagion is the process when the crisis moves from one region to another. To understand the process of contagion, Allen and Gale (2000b) focus on a theoretical single channel contagion model\(^2\) of liquidity preference, which provides different outcomes depending on whether the market is complete or not. The aim is to understand how cross-holding of deposits in different segments and regions of the banking system actually govern the dynamics of spillovers. The authors argue that the market is competitive and informationally efficient.

Yet, foreign exchange rate effects - as an indispensable part of interbank markets - are not considered in their investigation, and therefore this is a rather restricted view of assessing contagion in a cross-regional setting. In contrast, He and Krishnamurthy (2011) expand their setting to an infinite-horizon while modelling intermediation relationships within the financial system in the presence of shocks. Given some market constraints, the authors state that their model contributes to the asset pricing literature (by building asset pricing measures) in a dynamic setting. The handicap of the model is that it uses a single tradable asset (and therefore the market is incomplete) and does not provide efficient simulated estimates when it comes to the allocation of wealth. Allen et al. (2009) identify two approaches of dealing with contagion. According to the first approach, Eisenberg and Noe (2001) and Gai and Kapadia (2010) among others, evaluate systemic risk by analysing direct interbank linkages. These investigations use networks to simulate and assess the effects of shocks to the financial system. Upper and Worms (2004), Furfine (2003) and Mistrulli (2011) among others, apply balance sheet data to assess interbank contagion. For the second approach, Cifuentes et al. (2005) study indirect balance-sheet linkages within the banking system. Shin (2008) develops a theoretical lattice based market microstructure framework which integrates responses from asset price variations and contagion effects among market participants. Kodres and Pritsker (2002) note that global diversification due to internationally traded assets (which often share a high correlation) can be channels of cross-regional contagion. Similarly, Allen et al. (2010) argue that, although the arrival of new financial instruments during the last decade improved the prospects of portfolio diversification, it also amounted to the contagion of liquidity risk cross-regionally due to the fact that portfolios around the world share similar characteristics. Allen et al. (2010) build a two-period network model and assess

\(^2\)The authors assume that the state space consists of four regions, three dates \((t = 0, 1, 2)\) and banks behave in the same way. Consumers deposit one unit of asset in a bank from the region. Banks from different regions deal with each other. The main finding is that contagion develops in complete markets only.
how banks which interact with each other react to a shock (bad news, for example). The main finding is that there is a higher probability for debt (ensuing from short and long term financing) to be rolled over or being defaulted in an unclustered network than in a clustered one. An important implication is that systemic risk - as a result of contagion - is influenced by the structure of financial networks.

It is now widely accepted that the recent financial crisis was the result of the explosion of the real estate bubble that originated from the US market. Real estate loans have increased exponentially during the decade preceding the financial crisis of 2007-08. The Efficient Market Hypothesis\(^3\) does not consider the existence of bubbles, irrational behaviour of market players or externalities within the financial system. Hence, in the wake of the financial crisis of 2007-08, scholars and market players alike reassessed the existence of the black swan\(^4\). Though there were clear signs of an imminent crash, the crisis was unexpected and correspondingly no measures were taken to prevent the turmoil and its contagion to related markets. There are several theories of how financial bubbles develop and burst. Some, such as Allen and Gorton (1993) and Allen et al. (1993) among others, propose finite-horizon discrete and continuous time models to reveal issues arising from information asymmetries and agency problems. However, these are unsuccessful in capturing the dynamics of bubbles. Likewise, Bernanke and Gertler (1989) and Bernanke et al. (1999) argue that asymmetric information is a source which triggers agency problems. Allen and Gale (2000a) design a model which shows that intermediation instrumented by the interbank market leads to agency issues, and ultimately to the formation of asset bubbles. Agency problem is perceived as a source of ‘boom and bust’ as Allen and Gale (2000a) suggest, and bubble explosions originate primarily from variations in the real economic environment. The authors’ contribution to the theoretical literature is twofold. First, they apply the risk shifting phenomena in an asset-pricing framework. Second, they trace how credit expansion supports bubble formation. However, the analysis does not assess how a financial disturbance caused by the burst of a bubble spills over to other markets. Brunnermeier and Oehmke (2012) reveal how informational asymmetries aid the persistence of bubbles, and how some non-fundamental news can cause great price adjustments or collapses.

\(^3\)Eugene Fama published his influential paper on the Efficient Market Hypothesis in 1970. According to the theory, prices follow a random walk and consequently one cannot beat the market (and gain significant profits), since prices incorporate and reflect all available information. Fama defines three types of market efficiencies: weak-form, semi-strong and strong form. Recent economic and financial events dispute the validity of the theory, both theoretically and empirically.

\(^4\)The theory was introduced by Taleb (2001) in his best-seller book which discusses unpredicted and surprising financial events with dramatic consequences.
Chapter 2 Literature Review

2.3 The General Equilibrium Theory

The General Equilibrium Theory is derived from the Partial Equilibrium Theory and is described extensively in the Economic literature by the abstract supply-demand equilibrium equation set in an exchange economy. The basic theory which considers only two goods, was first formulated by Jevons (1871). However, the demand-supply relationship with an arbitrary number of consumers and goods was first developed by Walras (1874) with the aim of assessing the theoretical properties of the equation in their economic explanation. The modern general equilibrium theory based on ‘modern’ mathematics was framed much later by Debreu (1959). One of the main properties discussed in the literature is the efficiency (or Pareto optimality) of the theory. Yet, there are vast discussions on crucial ideas such as whether general equilibrium exists (first discussed by von Neumann (1945), Arrow and Debreu (1954) and McKenzie (1954)), whether the equilibrium is stable or not (first discussed by Wald (1951)), or if there are several general equilibria (first discovered by Auspitz and Lieben (1889) but modelled by Schumpeter (1954)), on how prices and rates adjust back to equilibrium, and how an exogenous shock affects the general equilibrium. Arrow (1974) argues that there isn’t a single general equilibrium, but a finite number of ‘locally unique’ general equilibria, yet none of them are stable.

The widely used modern notation of the Arrow-Debreu equilibrium model is described as follows (Balasko, 2009). Let \( k = 1, \ldots, l \) be the number of commodities, \( m \) as the finite number of consumers and \( p \in S \) is the price vector. The economy is defined by the endowment vector \( \omega = (\omega_i) \in X^m \). The consumer exploits the utility \( u_i(x_i) \) with the budget restriction \( p x_i \leq \omega_i \) and \( i \)'s demand is given by the function \( f_i(p, p \omega_i) \). Therefore, the aggregate demand (that is, the sum of all individual demands) is given by \( \sum_i f_i(p, p \omega_i) \). Then, the aggregate excess demand equals:

\[
z(p, \omega) = \sum_i (f_i(p, p \omega_i)) - \sum \omega_i
\]

The aggregate excess demand, known as Walras’ law\(^5\), must fulfill:

\[
p z(p, \omega) = 0
\]

It then follows that supply and demand are equal, and the equilibrium equation is defined as:

\(^5\)The law states that surplus supply needs to be balanced to compensate surplus demand elsewhere in the economy. If the law is violated, the economy moves out from its equilibrium state.
\[ z(p, \omega) = \sum_i \left( f_i (p, \omega_i) - \sum \omega_i \right) = 0 \]  

While surveying the evolution of the General Equilibrium Theory from 1870 to 1970, Balasko (2009, p.xi) argues that according to the postmodern general equilibrium theory, a particular equilibrium can be persistent and this characteristic is possessed only by few economies, stating that “more generally, only some equilibria and economies are going to satisfy an economically meaningful property”.

The evolution of the theory took place in parallel with that of mathematics. This allowed the discovery of new attributes of the theory, and consequently it improved its economic significance considerably. The basic Arrow-Debreu equilibrium model can be expanded by adding uncertainty\(^6\) and time to it, thus allowing financial markets play a part in real economic setups (Levin, 2006). This means that while assessing the importance of economic fluctuations, one can address the problem of economic (or financial) dynamics. The volatility experienced in the economy (or a particular market) is the result of some market mechanisms that ultimately determine either stationary or non-stationary equilibria\(^7\). There are numerous generalisations of the Arrow-Debreu model. One example is the Temporary Equilibrium model accompanied by its several versions. Balasko (2009) for example, uses the assumptions of short-run and long-run utilities when incorporating the time concept. In his two-period model the author includes financial assets in the exchange economy and identifies several locally isolated equilibrium situations. There are a number of ways of incorporating time in the state-space, however those must be underpinned by appropriate assumptions. However, an important question is: how an economy moves from a stationary solution to a non-stationary one.

### 2.3.1 The Business Cycle Theory

Various definitions of the terms cycle and business cycle exist in the literature. According to one, a business cycle is characterised by specific trending economic variables which move together “with timing relationships among the variables that tend to remain the same from one expansion-recession cycle to another” (Sargent, 1979, p.215). Another contrasting definition of a cycle “in a single series is the occurrence of a peak in the spectral density of a series” (Sargent, 1979, p.254). In reality however, the spectral densities of aggregate economic variables are not characterised by well-defined peaks.

\(^6\)Uncertainty is best modelled in a probabilistic state-space representation.

\(^7\)Balasko (2009, p.180) argues that:

the proof of the existence of nonstationary equilibrium allocations in the intertemporal Arrow-Debreu model with restricted market participation is at best a first step in a general equilibrium approach to the theory of business cycles and fluctuations.
(as turning points) in the spectrum of frequencies related to the business cycle, nor is the amplitude from one period to the other constant. The scale of a business cycle is inversely related to its frequency, meaning that the spectrum falls radically with its intensification. However, this does not mean that fluctuations in a particular time series are not related to the business cycle, as empirical research extensively proved in the last decades (Bernanke et al., 1999; Billio and Casarin, 2010a; Kaufmann, 2011; Simpson, 2014). Thus, the two contrasting and long seen as controversial definitions demonstrate how deficient they are in reality.

Yet, a fitting and simplistic definition states that a business cycle is the recurring succession of economic growth interrupted by short-term decline followed by economic upturn (Lucas Jr, 1977). According to the classical view of economics, business cycles are seen as the economy’s reaction to market disturbances. Every business cycle is unique, however they do have common properties. Centered on the idea that key economic variables move in a synchronised fashion, Lucas Jr (1977) argues:

> Though there is absolute no theoretical reason to anticipate it, one is led by the facts to conclude that, with respect to the qualitative behaviour of comovements among series [that is economic variables], business cycles are all alike. To theoretically inclined economists, this conclusion should be attractive and challenging, for it suggests the possibility of unified explanation of business cycles, grounded in the general laws governing market economies, rather than in political or institutional characteristics specific to particular countries or periods.

Fundamentally, the Business Cycle Theory investigates how real shocks instigate fluctuations in the economy. The fluctuations characterised by persistence appear over a period of five or six years as a cycle (called periodicity), and can manifest themselves in prices, revenues, output, interest rates and unemployment, for example. Yet, business cycles are not periodic but recurring. They affect the whole economy, not just a particular section of it, nor are they limited to a geographical region (Simpson, 2014). As a matter of fact, the economy is constantly affected by fluctuations as one industry expands, while other shrinks.

The two key characteristics of the cyclical behaviour of economic and finance variables as Abel et al. (2008) define them are:

1. The **direction** of the key economic or finance variables.

   If the variables move in the same direction as the aggregate economic activity, they are called procyclical economic variables, such as nominal interest rates, Fed funds rate and stock prices, for example. If they move opposite to the aggregate
Chapter 2 Literature Review

economic activity, they are called countercyclical. The variables which do not fall in either of the two categories are called acyclical; one example is the real interest rate.

2. The **timing** of the peaks and troughs with respect to the turning points of the business cycle.

If the aggregate economic variables move ahead of the aggregate economic activity, they are called *leading variables* (stock prices, for example). These are efficient in forecasting the path of macroeconomic variables of the development of an economy or market. If they are behind it, they are called *lagging variables* (nominal interest rates, for example), and if they coincide with the aggregate economic activity, they are called *coincidence variables*.

Various business cycle theories exist and they are characterised by two principal elements. The first one gives a depiction of the kinds of causes (in form of shocks) which have a significant impact on the economy. The second element specifies a model showing how the economy or a particular market responds to different shocks/disturbances. The model must explain the shocks affecting the economy and how the market or the economy adjusts back into a long-run equilibrium (Abel et al., 2008). The two contrasting views are still upheld by the economics literature regarding the nature of shocks being of exogenous or endogenous nature. The two main business cycle theories are:

- The Classical business cycle theory
- The Keynesian business cycle theory

According to the Classical business cycle theory, the economy adjusts back into equilibrium quickly and productivity shocks are short-lived, whereas the Keynesian business cycle theory asserts that the economy needs adequate policy (monetary and fiscal) responses against supply and demand shocks. Moreover, the economy requires more time to adjust back to its equilibrium level, primarily due to sticky prices and wages (Abel et al., 2008).

The Real Business Cycle theory (RBC) - as the extension of the neoclassical growth model - was first documented and modelled by Kydland and Prescott (1982) using a state-space specification. Its central view is that real shocks\(^8\) affect the dynamics of the economy. The state variable, which is defined by the transitory and permanent innovations to technology, is latent and follows an autoregressive process with identically and independent normally distributed error terms. A system of decision rules helps determine the equilibrium process which maximises the wealth of an agent subject to

---

\(^8\)Real shocks are the ones that affect the real economy in terms of production, labour, savings of consumers, etc. In contrast, nominal shocks affect the demand and supply of money (Abel et al., 2008).
restrictions (technological and informational in this case) without the presence of externalities. The theory is then tested to see if it is consistent with the observed behaviour of some post-war US macroeconomic time series. Speaking about testing the equilibrium theory, Kydland and Prescott (1982, p.1360) argue that:

Quantitatively explaining the co-movements of the deviations is the test of the underlying theory. For want of better terminology, the deviations [from equilibrium] will be referred to as cyclical components even though, with our integrated approach, there is no separation between factors determining a secular path and factors determining deviations from that path.

The model estimates stem from the variance-covariance characteristics of the model. Measurement errors can highly influence the approximated correlations and standard deviations and fluctuations of shocks (mainly technological) are due to permanent (and not transitory) components, as expected (Kydland and Prescott, 1982).

2.3.2 The Dynamic Stochastic General Equilibrium Theory

One of the extensions of the General Equilibrium Theory is the Dynamic Stochastic General Equilibrium theory (DSGE). The DSGE, as one of the dominant theories applied in present day macroeconomic analyses, was derived from the new Keynesian view of sticky prices combined with classical business cycle analyses. Primarily, the theory addresses issues related to business cycles (such as economic growth and recessions) and assesses consequences of fiscal and monetary policy changes. The theory aims to describe the behavior of markets (or economies) by examining the interaction of micro- and macroeconomic decisions made by agents such as households, businesses, governments, banks, etc (Balasko, 2009). The context in which the decisions are made is dynamic, as opposed to static theories\(^9\), such as the Walrasian general equilibrium theory (Walras, 1874). Economic or financial issues are analysed over a longer period of time, such that a particular economy (or market) is traced over time from moving from one equilibrium to another. The time element and change are crucial\(^10\). As an example, a bond spread during a certain period may depend on the rate of a currency swap in the previous period. Apart from being dynamic, the setup is also stochastic, meaning that markets and economies are disturbed by random shocks. Shocks come in the like of technological advances, information asymmetry, changes in oil prices and macroeconomic policy (Abel et al., 2008). Alternatively, shocks are generated endogenously from within the system due to a self-fulfilling feedback mechanism.

\(^9\)These are theories that look at a particular issue at a given time, or over a short period of time.

\(^10\)One way to describe change in a particular variable over time is to use differential equation (that is to differentiate with respect to time).
2.4 The Theory of Rational Expectations

While working independently on optimal solutions for industrial planning and inventory management, Herbert A. Simon and John F. Muth lay down the foundations of what was later defined in Muth’s (1961) influential article as The Theory of Rational Expectations and price movements. The author’s aim was to estimate future economic conditions as well as to make sensible predictions based on available information in the light of dynamic changes within the system. Muth (1961) used price variations induced by past shocks to formulate the rational expectations theory. The theory assumes that outcomes of various economic circumstances somehow depend on economic agents’ expectations, whose ultimate aim is to maximise their utility function. The setting is relatively simplistic. The units produced in the economy equal the units consumed in the economy. Moreover, the system of equations is linear and the errors are assumed to be normally distributed. Furthermore, it is assumed that the variables of the economic system are departures from equilibrium values. Fundamentally, predictions of certain outcomes are efficient if they are superior to predictions made by the companies. Otherwise, due to insider information, one can make substantial profits (due to commodity speculation or selling information, for example). However, if the collective expectation of the firms is similar to the forecasts of the theory, profit prospects are zero. Muth (1961) also evaluates his model in the presence of market anomalies in terms of market imperfections and biases, such as incomplete markets, arbitrage or erroneous and incomplete information.

Expectations are actually informed forecasts of future events, and are fundamentally identical to forecasts of the applicable (underlying) economic theory. The rationale behind this is that, if the fundamental dynamic of the economic system changes, it is anticipated that economic agents will adjust their expectations fairly rapidly (Danielsson, 2011). Therefore, there is a recurrent feedback from past outcomes to existing expectations. Consequently, economic agents’ subjective prospects (in terms of probability distributions) are identical to the ‘true’ values of variables of interest. The hypothesis assumes that information is scarce and the underlying system defines how expectations form. Moreover, ‘public prediction’ does not influence the functioning of the economy. Later, Lucas’ empirical model (1978) proved that agents’ subjective expectations corresponds to the objective expectations or probability distributions of the economic system.

To appreciate the implications of Muth’s (1961) Theory of Rational Expectations, Sheffrin (1996) emphasises the importance of making a distinction between anticipating variables that are exogenous and those that are endogenous to the system. In the case of exogenous variables, expectations formed by market players are essential, however they will not influence the values of those variables. On the other hand, expectations of endogenous variables do influence the dynamics of the endogenous variables (Kim, 2004; Kim et al., 2008; Danielsson, 2011).
Sheffrin (1996, p.ix) notes that “from a rather controversial beginning, the rational expectations hypothesis is firmly embedded in the economist’s theoretical tool kit”. Yet, during the years, the theory has been widely criticised for being too simplistic, for not being coherent with the subjectivist idea of probability and for not having a clear explanation of ‘procedural rationality’. However, some argue that learning and adaptive behaviour cannot be added to the underlying economic system.

However, the learning behaviour issue in the context of rational expectations is worth discussing. Sheffrin (1996) notes that if there is continuous structural change, learning behaviour becomes most interesting. Besides, structural changes are seen as part of life and they are induced by continually adjusting activities of market players. This however challenges the view that structural changes cannot be predicted, that they are permanent and structural. To explain it, Sheffrin (1996) distinguishes between stable deterministic (SD) and stable stochastic (SS) economic systems. The SD system can be seldomly disturbed and always returns to a stationary equilibrium, whereas the SS system is characterised by continuous instability with shocks evolving according to some constant probability laws, such that the system is said to be in a stochastic equilibrium. Yet, in extreme market circumstances it is arguable whether the Theory of Rational Expectations is valid and applicable.

2.5 The Basel Accords: theoretical implications

Financial crises mainly result from financial liberalisation and exponential credit expansion. The credit bubble is eventually offset by a negative bubble triggered by a weighty drop in asset values, leading to a complete halt in the financial market and to the collapse of several market players (Kaminsky and Reinhart, 1999). In the majority of cases, policy responses are initiated in order to aid the functioning of the market and avoid further crashes. As it has been broadly agreed in the literature, the main factors contributing to the development of the financial crisis of 2007-08 are excess liquidity combined with the lack of regulation, as well as the inability of regulators and analysts to prevent such events (Allen et al., 2009; Ayadi, 2013).

The Committee on Banking Regulations and Supervisory Practices was set up in 1974 by the central bank governors of the G10 economies in the wake of market turbulence which followed the collapse of the Bretton Woods exchange rate system. The purpose was to achieve improved financial stability with the cooperative banking supervision of its members. Later the name of the committee was changed to Basel Committee on Banking Supervision. At the time of publication the membership has been extended to 28 jurisdictions (BIS, 2014a).

11The 10 countries that comprise the group are UK, France, Belgium, Germany, Italy, the Netherlands, Sweden, Japan, USA and Canada.
The Basel I accord was agreed in 1988, and it worked towards establishing a collective measure of capital adequacy. The capital measurement system was referred to as the Basel Capital Accord (BIS, 2014a). The capital to risk-weighted assets ratio was set to have the floor of 8%, and was implemented in 1992 not only by the member countries, but by all banks acting on the international market. The accord went through several amendments during the years in order to address the enhancement in the measurement of capital adequacy (inclusion of general loan-loss reserves, for example) and market risk (allowing banks to use internal measures such as the Value at Risk, for example). However, the accord was unsuccessful in consolidating the banking system’s capital base. Ayadi (2013, p.403-404) argues that the accord failed to acclimatize to progress made in the financial system while encouraging pernicious market behaviour. The author sums up his main arguments, noting that:

The use of broad-based risk buckets without taking account of relative risk, the focus on a single credit risk indicator, outdated treatments of securitisation and trading book risks, the zero-risk weight, short-term stand-by credits, and the cap on the counterparty-risk weight for swaps, and forward contracts spawned an army of financial engineers and encouraged many of the imprudent practices that are being ruthlessly exposed by an extreme reassessment of credit counterparty risk.

More than a decade later, in 1999 the Basel II Accord was drafted with the goal of addressing the shortcomings of its predecessor. The accord was believed to be “an evolutionary and flexible approach to banking regulation and supervision, which would reflect the rapid progress and sophistication of banking practices and risk management techniques, including securitisation”, as Ayadi (2013, p.404) notes. The Revised Capital Framework with its three pillars was released in June 2004. With the aim of continuously consolidating the financial system, in September 2008 the Committee issued the ‘Principles for sound liquidity risk management and supervision’ (BIS, 2014a), however the principles failed to address the fundamental issues of the market turmoil of 2007-08.

The third accord, namely Basel III, was proposed in the wake of the financial turmoil of 2007-08 and endorsed by its members in July 2010. The Bank for International Settlements’ decree aims to:

improve the banking sector’s ability to absorb shocks arising from financial and economic stress, whatever the source; improve risk management and governance; strengthen banks’ transparency and disclosures (BIS, 2014b).

Furthermore, the amendments target:
bank-level, or microprudential regulation, which will help raise the resilience of individual banking institutions to periods of stress; macroprudential, system-wide risks that can build up across the banking sector as well as the procyclical amplification of these risks over time (BIS, 2014b).

In 2013, further reforms were introduced, such as the revised Liquidity Coverage Ratio\textsuperscript{12}. The new accord prescribes the number of high quality liquid assets - which at the minimum - should equal the total net cash outflows. Ideally, financial institutions should maintain this ratio on an ongoing basis, at minimum of 100% level for at least a 30-day period, and specifically when there is an imminent threat of financial illiquidity in the market. In reality however, institutions have the prime goal to utilize their assets at the maximum, and consequently to keep the Liquidity Coverage Ratio at the minimum required level.

In essence, there are several shortcomings of the Basel Accords. They do not address market failures such as contagion, neither they encourage emerging economies to back and adhere to international standard principles and regulations. However, some of the emerging economies might not see the rationale for implementing such regulations for they might not be exposed to the mounting level of complexity that characterises the developed financial markets. Moreover, the accords do not clarify what the fundamental anomalies are, and why certain imposed guidelines are best when confronted with liquidity crises and subsequent contagion. Allen et al. (2009) question whether policies and guidelines centered on accounting capital and liquidity ratios are sufficient. The mechanisms to prevent crises are not complete unless measures and policies to prevent illiquidity contagion are in place. The dynamics of contagion are well documented and understood, yet there isn’t an operational model capable of signaling and predicting liquidity spillovers from one market to another.

2.6 Epistemology of modelling and simulation

For the reason that the present research involves a collection of models based on simulation, a note on the underlying philosophy of modelling and simulation is needed. Epistemology is concerned with the problem of gaining knowledge and according to Tolk et al. (2013) simulations are considered influential epistemological engines. Simulations are crucial sources of knowledge discovery and they are perceived as the third leg of the ‘science tool’ which helps strengthen, understand and unravel research and theory (Latane, 1996). Moreover, complex simulations are becoming theories in so far as they employ principles from the philosophy of science to corroborate that the simulation at

\textsuperscript{12}In case of financial distress, the Liquidity Coverage Ratio specifies the sufficient amount of tangential high quality and easily convertible (liquid) assets that provide a bank or institution with smooth operation for a 30-day period. The ratio is introduced progressively from January 2015, in order to ensure the systematic strengthening of the banking system in general (BIS, 2013).
hand is a true illustration of reality, and nevertheless they are practical until confirmed otherwise (Heath et al., 2009). On the other hand, Harrison et al. (2007) believe that simulations are inappropriate in describing complex systems and behaviours, and therefore inefficient to produce new theory. However, if the models based on simulation are validated, they prove to be powerful tools in investigating complex behaviours.

Since the phenomena of interest is complex from a behavioural and methodological viewpoint, the present research calls for a comprehensive model as opposed to conventional approaches based on deductive or inductive reasoning. Ultimately, the investigation will provide new theoretical and practical insights.

2.7 Conclusions

The models presented in the subsequent chapters integrate the state-space interpretation of the DSGE and the Rational Expectations Theory in a novel empirical analysis which investigates the interconnectedness of short-term interest rates and spreads and contagion of liquidity shocks within the short-term interbank markets. In the models presented in the following chapters, fluctuations in interest rates are at the core of the business cycle. The stochastic elements of the models are innovations that can be either exogenous or endogenous, and are represented by either new information arriving into the market or central bank policy measures (or changes). The shocks may differ in their persistence and magnitude.

\[13\] In the present thesis I do not attempt to create a new business cycle model, but solely integrate the classical view in the short-term interbank market scenario and thus create a new financial liquidity contagion model. Yet, some of the assumptions of the models are very much specific.
Chapter 3

Modelling the Long-run Relationship of Short-term Interest Rate Spreads

3.1 Introduction

Finance theory suggests that a long-run equilibrium relationship should hold between some macro-economic variables, such as spot and future asset prices, the ratio of relative prices and exchange rate, or equity prices and dividends (Brooks, 2008). If times series are assumed stationary as opposed to being a random walk, a deterministic or stochastic trend (displaying an orderly pattern) or merely a cycle, the econometric model is spurious and yields inefficient parameter values. Moreover, in such a case, the identified relationships are misspecified (Maddala and Kim, 1998). Thus, it is imperative that non-stationarity is revealed in time series and an appropriate econometric model is applied. Therefore, cointegration, as a framework, allows the estimation of efficient parameters. Methodologically, the identified long-run relationships prevent regression residuals to increase with time, and consequently the econometrician avoids dealing with heteroskedasticity, which is known to weaken the efficiency of the estimates. Ultimately, this can have a significant impact in forecasting the interbank short-term interest rates.

The present chapter investigates how the US LIBOR-OIS spread, 3-month German-US bond spread and the Euro-US dollar currency swap evolve over time.

There are several motivations for conducting the investigation in this Chapter. First, the aim is to determine whether the time series under scrutiny are interdependent and whether some of the variables cause changes in others. If that is the case, one can
predict movements in benchmark short-term interbank spreads and rates, and anticipate unfavorable financial events unfolding (and prevent them if advantageous policy interventions are introduced). Second, if liquidity shocks affect the short-term interbank market, one would be able to trace the forecast error variance of the variables of interest to reveal what exactly drives movements (extreme volatility, for example) in benchmark spreads and rates. Third, if long-run equilibrium relationships exist among the time series - however these are affected by liquidity shocks in the like of structural breaks, for example - this would reveal that significant (liquidity) shocks may translate into extreme financial events as witnessed during the financial crisis of 2007-08.

The majority of the empirical literature disregards the important characteristics of short-term money market series, such as stationarity, normality and structural changes in the constant term, in the mean and the variance. After presenting the summary statistics of the spreads used in this analysis, the Dickey-Fuller test and KPSS method is used to test the series for stationarity. Prior to establishing the long-run equilibrium relationships, interdependencies and causality are revealed by following how the LIBOR-OIS, the GerUS3M and EUSWEC spreads evolve over time. Vector Autoregression allows the identification of whether lagged values of the dependent and independent variables have any effect on the dependent variables. The Impulse Response Function\(^1\) is used to trace shocks to the forecast error variance. Structural VAR and subsequently Cholesky decomposition\(^2\) is then implemented to obtain efficient parameter estimates, which ultimately determines which market leads and which one follows in transmitting liquidity shocks.

The long-run relationships among the time series are estimated using the cointegration methods developed by Johansen (1988) and Gregory and Hansen (1996). After deciding on appropriate models, as well as allowing for structural breaks, the cointegrating relationships and short- and long-run coefficients are estimated. Cointegration requires that departures from equilibrium are stationary (with zero mean and finite variance), and innovations have a short-term effect on the series; also, autocorrelations among the innovations decrease progressively in size, so that their sum is finite (Engle and Granger, 1987). Yet, individually the series in this analysis are integrated of order one, in addition to having an unbounded variance that drifts far apart\(^3\) (Maddala and Kim, 1998). Structural changes caused by the financial crisis of 2007-08 (among other events of the

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\(^1\)Sims (1980) was the first economist and statistician to trace the path of exogenous macroeconomic shocks in the context of a vector autoregression. He uses the method to reveal the timing and size of changes in the covariates which ultimately have an effect on the dependent variable.

\(^2\)Also called Cholesky factorisation, the method was developed by André-Louis Cholesky to decompose the correlation matrix into parameters or correlated values. Basically, the decomposition is achieved by taking the square root of the covariance matrix. The method is widely used in Monte Carlo simulations, for example.

\(^3\)In such case, the \(\text{var}(y_t) \to \infty \) as \( t \to \infty \), and consequently the OLS estimator does not follow the normal distribution, such as the \( t \) and \( F \) distribution but the non-standard asymptotic distribution based on a Wiener process (Maddala and Kim, 1998).
whole interval) led to the break-down in the co-movements of the time series, rendering the VAR useless without the implementation of a Vector Error Correction model (VECM). The VECM enables the estimation of first-differenced time series which are cointegrated. The approximated parameters and identified structural breaks provide an insight into the long and short-run behaviour of the LIBOR-OIS spread, German-US bond spread and the Euro-dollar currency swap series.

The remainder of the chapter is organised as follows. Section 3.2 summarises the literature review concerning liquidity risk and contagion. Section 3.3 presents the research questions. Section 3.4 describes the data sets used in the analysis. Section 3.5 presents the methodology. The empirical results are introduced in Section 3.6. First, the summary of the moments are provided along with the assessment of assumptions which support the use of regression equations. Second, the interconnectedness of the times series is discussed revealing whether any of the series can be used to predict the future values of the others. Third, the short-run and long-run relationships are established, and the presence of structural breaks is examined. Section 3.7 concludes the chapter and discusses the contributions and limitations of the study.

### 3.2 Literature review

The following section summarises the literature which focuses on liquidity risk, financial crises and domino effects in the short-term interbank market. It is acknowledged in academic literature and within the industry, that the dynamics of financial crises are fuelled by the propagation of liquidity shocks and subsequent contagion to neighbouring markets and regions. The surveyed studies discuss the presence of structural breaks while analysing the domino effects of liquidity shocks. Empirically, there are two strands of literature: one looks at financial crises and the propagation of shocks from a market microstructure view-point, while the other decomposes the dynamics of contagion into liquidity elements.

Assessing 80 currency crises occured in industrial and developing countries prior to 2000 and using monthly data for the period 1970-1998, Kaminsky and Reinhart (2000) analyse whether the dynamics of contagion are explained by market fundamentals or herding behaviour. The focus is on how trade and financial sector links affect the dynamics of contagion in the international bank trade, lending and hedging context. The authors note that financial crises can be simultaneous throughout economies and countries caused by a joint unfavourable shock, such as world wide (or region wide) rise in interest rates, for example. Their main findings are that contagion is regional rather than global and that one country experiencing financial distress cannot forecast a crisis elsewhere. This is in sharp contrast to the findings of Eichengreen et al. (1996), who believe that currency crisis spillovers are prevalent primarily between countries which
are associated via commercial trade links. Moreover, due to the complexity of regional contagion, it is hard to distinguish whether trade links or financial interconnectedness spreads crises from one region to the other. However, as the authors argue, in the case of Argentina (the currency crisis spread from Mexico due to high correlation of assets) and Indonesia (the crisis spread from Thailand due to high correlation of assets), the primary cause of contagion was financial links owing to the fact that trade relationships among these countries were absent.

**Danielsson and Saltoğlu (2003)** apply the market microstructure approach to analyse the short-lived liquidity crisis of December 2000 which stormed the Turkish overnight market. The authors strongly argue that empirical market microstructure provides a foundation for investigating price development and informational relationships in financial markets, which is paramount in describing and understanding financial crises. By using simple regression on log interest rates, **Danielsson and Saltoğlu (2003)** claim that an institutional level order flow model provides an insight and detailed explanation of decision making in an elaborate way. Their model describes how the crisis evolves, and at the same time allows the effect of individual trading strategies on yields to be investigated. Furthermore, structural breaks are allowed in the model, so one is able to assess the time period before the crisis hit, as well as the turbulent period that followed. However, the post crisis period is omitted from the investigation and the model’s sample statistics found that the variables of the model are not normally distributed and highly autocorrelated. The period surrounding the crisis yields a less accurate result and with diminished statistical significance considering that the number of observations available for the crisis period were very few (16 obs.).

**Hartmann et al. (2001)** investigate the microstructure of the overnight Euro money market, specifically looking at monetary policy changes and their effect on intra-day trading patterns. By using simple volatility tests, the authors prove that monetary policy changes are mirrored in the widening of spreads, and increased volatility is the result of noisy market behaviour. Essentially, the supply and demand of overnight liquidity can be the source of a financial crisis. When yields increase, and subsequently the value of collateral drops - while banks face margin calls, and some of the off-balance sheet transactions move against the bank - fundamentally, the overnight interbank market is the main source for funds. Furthermore, the liquidity feedback loop is reinforcing the same effects as a spiral does, stimulating further demand for overnight liquidity. Similarly to the financial crisis of 2007-08, banks started to unload assets to cover for

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4Kaminsky and Reinhart (2000) argue that due to the fact that trade is more intra-regional rather than inter-regional, financial contagion is contained within a (larger) region and rarely becomes global.

5Market microstructure studies the operational practices of specific markets by taking into consideration some factors - primarily trading behaviour, transaction costs, prices, quotes and volume - which describe and influence the functionality of such markets (O’Hara, 1995).

6Order flow (also named transaction flow) is fundamentally motivated by liquidity and takes place when a trader assumes or predicts the price of a security will move in a certain direction, and therefore she chooses to complete a transaction in the market (O’Hara, 1995).
liquidity, which resulted in a steep market decline. However, monetary policy changes are not the only causes of spread widening and volatility shocks, as the authors argue.

Implementing a principal component analysis, Hui et al. (2011) claim that the spillover of the crisis from the US to the European market was moderate between the period mid-2007 to mid-2008, and funding liquidity risk played a major part in the development of the global financial crisis. However, during the crisis of 2007-08, the authors find considerable deviations from covered interest rate parity. Fundamentally, in equilibrium conditions there are no arbitrage opportunities between the interest rate, the spot and forward currency values of two countries. In the model specification, multicollinearity among variables is corrected, however the effect of potential structural breaks on parameter estimates was not studied. The authors argue that the widening of the LIBOR-OIS spread mainly reflects funding liquidity risk in the interbank market, and subsequently it can be used to measure funding liquidity conditions. This view is supported by McAndrews et al. (2008) who claim that the spread undeniably contains a significant and time-varying funding liquidity element. Moreover, the LIBOR-OIS spread incorporates counterparty-risk premia, which originates from the counterparty’s inability to pay back an interbank loan.

Similarly, Baba et al. (2008) analyse the crisis spillover to the foreign exchange- (FX) and long-term cross currency basis swap markets for the period July 2007 to January 2008. Considering the short time period under investigation, the main finding is that the departure from interest rate parity conditions is due to non-US financial institutions’ use of the swap markets to survive US dollar deficiencies they encountered during the crisis. The Euro LIBOR-OIS spread was not as increased as the US LIBOR-OIS spread, while the Japanese LIBOR-OIS spread was the lowest. The Euro and the pound sterling were used as funding currencies in the FX spot market during the crisis, with the aim of raising US dollars. Liquidity risk and funding liquidity further deteriorated as a result of the fact that all borrowing was centred in the US currency. Consequently, a rise in the LIBOR-OIS spread corresponds to a price decrease, which represents a drop in the value of collateral used in repo dealings. Relating to the financial crisis of 2007-08 as a systemic event, while using an extensive data set of credit spreads to trace the spillover of the credit crunch and the causes of repo haircuts, Gorton and Metrick (2012) note that fluctuations in the interbank LIBOR-OIS spread were correlated with increases in the non-subprime securitised assets and associated derivatives spreads (primarily in the UK, Australia and the Netherlands), and no correlation was found with increases in subprime related securities and indices spreads. Moreover, an insignificant amount of

7 The method was invented in 1901 by Karl Pearson, however it was published more than 30 years later by Hotelling (1933); it is used specifically when variables of interest are serially correlated. Transformation of these into principal components can reveal the element that carries the most variability.

8 Its measure it is not yet generally agreed on. BIS (2011) notes that increased premiums and aggressive bidding at central bank auctions reveal funding liquidity risk.

9 It is imperative that independent variables in a regression are not correlated. If this occurs, it is more likely that there is also causality among the covariates (Hayashi, 2000).
subprime risk had increased effects on unrelated asset classes. Other analyses argue that the LIBOR-OIS spread encompasses credit and liquidity risk premia (Hui et al., 2011; McAndrews et al., 2008; Michaud and Upper, 2008; Sengupta and Tam, 2008). Schwarz (2009) finds that during the first half of the credit crisis of 2007-08, more than 60% of widening of the one-, the three-month Euro LIBOR-OIS spreads and the Eurozone sovereign debt spreads can be accredited to market liquidity effects. Widening risk spreads can be characterized by greater compensation claimed by risk-averse investors in case of default. Alternatively, it can be described as a compensation for investors holding less liquid securities. The results of Michaud and Upper (2008) are similar, however the authors caution for the reliability of results when assessing short time periods to assess market dynamics in face of financial crises, as well as using quotes and not real transction data when conducting analyses.

The analysis of contagion has important implications for bank regulations and the development of safety nets when confronting liquidity shocks. Employing a multivariate Vector Autoregressive Markov regime switching model\(^\text{10}\), Guo et al. (2011) assess the magnitude of economic shocks at aggregate level, and their ensuing contagion between the stock market, real estate-, CDS- and energy market during the global financial crisis. The authors argue that regime switches are more frequent at the start of the crisis, and that the stock market and oil price shocks induce the fluctuations observed in the CDS and stock markets in general. Upper and Worms (2004) and Mistrulli (2011) analyse balance sheet information of German banks to approximate a matrix of bilateral credit links and to assess whether the collapse of a bank can lead to contagion. The focus of these analyses is on the spillover effect caused by interbank linkages in the presence or absence of safety nets. Upper (2011) looks at contagion caused by bank failure and channels of contagion. The author evaluates methodologies based on simulation techniques, however as he notes, the analyses (with their methods) do not contribute to assessing bank policies, nor can they be applied to perform stress testing in the banking sector. Implementing simulation techniques, Mistrulli (2011) assesses the severity of financial contagion in the Italian banking system. The main finding is that Italian banks were more prone to financial contagion during the financial crisis of 2007-08 than in 2003. Interestingly however, both Upper and Worms (2004) and Mistrulli (2011) argue that the German and Italian banking system were not experiencing systemic risk potentially caused by financial spillover in the interbank market. The main limitation of such arguments is that these analyses solely look at domestic banks and therefore cross-border channels for spillover are not assessed.

Alternatively, Autoregressive Conditional Heteroskedasticity (ARCH) models are reasonably good in forecasting volatility in time series, however the model predictions are poor if volatility shocks are of extreme magnitude such as in the case of a financial crisis, for example. Hamilton and Susmel (1994) analyse weekly US stock returns and use

\(^{10}\) A Markov chain is memoryless and therefore the state of the latent variable depends solely on the previous time, namely on \(t - 1\) (Hamilton, 1989).
the Student $t$ distribution to construct a model with the ARCH parameters originating from different regimes. The dynamics of transitions are driven by a latent Markov chain. Using Dynamic Conditional Correlation Multivariate Generalized ARCH (DCC M-GARCH) to test the diffusion of liquidity shocks across US financial markets, Frank et al. (2008) find that there was a significant connection between markets and funding liquidity pressure which reached its peak right before the crisis hit. Additionally, the authors examine the relationships of financial institutions that were faced with solvency problems. Results corroborate the fact that in times of equilibrium, market illiquidity shocks are fundamentally short-lived, for these give prospects for traders to gain profits and subsequently enhance liquidity and the price discovery process. Contrarily however, in times of distress, systemic risk is being created by liquidity shocks which intensify and spread across financial markets. The analysis focuses on the period January 2003 to January 2008, thus the events that followed and were significant during the financial crisis of 2007-08 and the subsequent period are not included in the investigation. Zhou and He (2012) argue that the BEKK MGARCH model (named after Baba, Engle, Kraft and Kroner with) is unable to model the dynamics of risk contagion. As a result, Zhou and He (2012) propose a new contagion-MGARCH model which has the same mean and variance equation as the BEKK MGARCH model. The authors introduce a contagion equation which contains the latent spillover variable with the aim of capturing the time-varying contagion process, two lagged realised variances originating from different markets, and an independent stochastic innovation representing a Gaussian white noise process. The authors argue that the contagion element of the equation can capture the clustering feature of the risk contagion process. The parameters of the model are estimated by using Markov Chain Monte Carlo method (MCMC), however the model was not implemented on real data, therefore it remains a theoretical model of an extended Constant Conditional Correlation MGARCH technique.

### 3.2.1 Conclusions and contributions to the existing literature

The present investigation contributes to the empirical literature in several ways.

1. The spreads that are used to construct liquidity measures in earlier analyses omitted the fact that these time series are not stationary in their levels, and therefore are I(1). Once the series are I(1), generally they are also cointegrated. Consequently, several of previous analyses are rendered worthless due to the use of inadequate econometric tools in assessing liquidity risk and contagion in general;

2. The models presented in this chapter reveal causality, co-integrating relationships and determine which markets lead or lag in transmitting liquidity shocks in the short-term interbank market;
3. The interval covers a longer time period (from January 2002 to December 2011), and not only the period surrounding the latest financial crisis as the majority of studies considered. In this way, the models successfully assess the speed of adjustment of spreads to liquidity shocks and investigates whether a long-run relationship exists between the LIBOR-OIS spread, the 3-month German-US bond spread and the Euro-US currency swap spread.

### 3.3 Research questions

The study aims to answer the following research questions:

1. Is there a causality among the variables of interest? Can covariates predict the future values (movements) of the dependent variables?

2. Do endogenous (structural) shocks originating from covariates explain the forecast error variance of the dependent variable?

3. Is there an equilibrium relationship among the time series? How do structural breaks affect equilibrium relationships? What are the empirical and theoretical implications?

### 3.4 Data

The data set is constructed with historical closing daily spread between the US LIBOR and overnight indexed swap (OIS) rate, the daily three-month German-US bond spread and the daily euro-dollar swap spread. The data covers the period of 1st January 2002 to 30th December 2011, therefore including the pre- and post-crisis period to allow the documentation of possible structural breaks in the time series. This is an important feature, since previous analyses looked at shorter time periods, and mainly covered the first period of the financial crisis of 2007-08. Yet, at the time of conducting this study, one of the major economies affected by the credit crunch, namely the British economy, is emerging from the crisis. Therefore, it is imperative that a longer time period is included in the analysis, to capture the different magnitudes of a financial distress. The reason for using longitudinal and cross-sectional money market interest rates and spreads is that these carry a liquidity component (premia) on term unsecured interbank loans. Moreover, banks are interconnected cross-regionally, primarily due to cross-ownership of financial institutions.
3.4.1 The LIBOR-OIS spread

The London Interbank Offer Rate (LIBOR) is used as a reference rate in financial contracts all over the world and is the rate at which banks and institutions of similar size agree to lend each other. The rate is paid on unsecured interbank loans of various maturities (up to 12 months) and is calculated and published at 11.00am on every business day by the British Bankers’ Association in London, based on a survey submitted by banks operating in the short-term interbank market. The contributing reference panel consists of at least eight major reputable banks with expertise, and the selected banks are representative of the market. Fundamentally, a bank with surplus cash credits (deposits) its money to a bank in need for an agreed time period; however, only the rate is paid and the loans are never traded. Changes in the LIBOR rate can be attributed primarily to open market operations implemented by central banks in order to re-price short-term loans between banks. Increases in LIBOR rates can be credited to banks calling for greater compensation in case of default risk on their loans.

The Overnight Interest Swap (OIS) rate is the rate of the derivative contract on the federal funds rate, and in usual market circumstances is generally below the LIBOR rate. In turbulent market conditions, the LIBOR-OIS spread is a good indicator of risk premiums as a result of credit risk and funding liquidity risk. In the OIS contract, the interest rate swap’s floating leg, which is the federal funds rate, is exchanged for a fixed interest rate. The cash-flow consists of the difference between the two rates exchanged at maturity, and therefore the contract is riskless, for there is no principal involved in the exchange. The present value of the floating rate is calculated generally by taking the geometric average over the length of the contract. The federal funds rate is seen as a good indicator of the health of the short term interbank markets, since it bears no risk compared to traditional interest rate spreads.

The term LIBOR-OIS spread has the role of evaluating the health of banks, for it mirrors the risk linked with lending to other banks. As Alan Greenspan argued, the LIBOR-OIS spread remains a barometer of alarm for bank insolvency, and increased spread levels point to difficulties in the banking industry (Thornton, 2009). He goes further to claim that variations in the LIBOR-OIS spread reveal changes in risk premiums rather than in liquidity premiums, which in turn mirrors banks’ need for liquidity. Contrarily, Hui et al. (2011) argue that the LIBOR-OIS spread mainly reflects funding liquidity risk in the interbank market and is used for measuring funding liquidity conditions. Besides, the spread includes counterparty-risk premia which originate from the counterparty’s ability to pay back an interbank loan. Moreover, the spread contains a significant and time-varying funding liquidity element, as McAndrews et al. (2008) point out. In times of increased uncertainty, such as in financial distress caused by a credit crisis, increase in the risk premium is caused by a decrease in the rate of default-risk-free asset compared
to the rate of the risky asset. Such scenario would induce investors to claim their assets from banks, resulting in deposits’ flight to safety.

During the financial crisis of 2007-08, the spread varied persistently around unseen extremely high levels until the announcement of Lehman Brothers’ collapse. Since then, for the period of investigation, the spread level continued to vary and decreased somehow, however it did not stabilise relative to its pre-2007 level. Results of the present study reveal that the US LIBOR-OIS spread is a good indicator of financial instability. For the interval January 2010 to January 2012, the European Central Bank bailed out Ireland once and Greece twice. Interestingly, all these three events were reflected in the US LIBOR-OIS spread movements over that time period. One would want to find out why this spread had been affected by the downturn of the Eurozone market and its economies; however, this is beyond the scope of this investigation.

### 3.4.2 The German-US bond spread

The second variable used in this analysis is the daily spread between the three-month German and US government bond rate. Any deviation between the German and US government bond rate is mirrored in varying future economic development and interest-rate outlook for the two economies. Fundamentally, different economic and monetary policies between the two biggest economies drive the widening or narrowing of the spread (Cappiello et al., 2006). Correspondingly, US and Euro-zone country specific debt and job market prospects influence German-US bond spread fluctuations (Goldberg and Leonard, 2003). The US and German Government bonds carry no risk (theoretically), for they are considered the two safest assets in the world. After the collapse of Lehman Brothers, the US Federal Reserve and the European Central Bank lowered the base rate to previously unseen levels. Financial stimulus of central banks, in terms of government bond purchases in both in the US and Euro-zone, kept the spread level low for a prolonged period of time compared to historical standards.

### 3.4.3 The euro-dollar currency swap

The euro-dollar currency swap (EUSWEC) is defined as a customised financial derivative which eliminates foreign exchange risk. Essentially, the swap enables two parties who have similar funds, but in different currencies, to exchange cash-flows, which is any interest that occurred during the time of the contract; this is termed the cost of carry, which in fact eliminates foreign exchange risk. Similarly to the OIS, the swap consists of two legs, namely a spot and a forward transaction. Although bearing comparative advantage to its owner, the currency swap carries default risk, since both principal and interest are exchanged between the contracted parties (Goldberg et al., 2010).
Chapter 3 Modelling the Long-run Relationship of Short-term Interest Rate Spreads

In turbulent market conditions, all the spreads used in the present study are good indicators of risk premiums as a result of credit-, funding liquidity-, default-, forex-, and ultimately systemic risk. Moreover, the spreads reflect movements in interest rates on the two significant geographical markets affected by the financial crisis of 2007-08, that is the Eurozone and US market. Finally, using the three significant and reference spreads helps identify where the liquidity crisis started in the short-term interbank market and trace the propagation of liquidity shocks cross-regionally.

3.5 Methodology

The following sections present the methods applied in determining whether the variables of interest are interrelated, if there is causality among them and whether they are cointegrated in the long run. The ultimate aim is to reveal whether structural breaks disturb the established long-run equilibrium relationships.

3.5.1 Interdependence and causality in the short-term money market rates

Before investigating interdependence and causality among the series, the assumption of stationarity must be assessed. The Dickey-Fuller (1979) and KPSS\textsuperscript{11} (1992) tests are used to test for the presence of a unit root. If the series are not stationary, there might be more than one trend in the series. If the series are stationary in their levels, then they are cointegrated and share a common stochastic trend.

Both residual based tests assess whether there is a unit root in the autoregressive model, which may or not contain a constant and/or a time trend. Following the notation of Maddala and Kim (1998), the three cases of the Dickey-Fuller test are:

\begin{align*}
y_t &= \rho y_{t-1} + e_t \quad (3.1) \\
y_t &= \beta_0 + \rho y_{t-1} + e_t \quad (3.2) \\
y_t &= \beta_0 + \beta_1 t + \rho y_{t-1} + e_t \quad (3.3)
\end{align*}

The first test is a simple unit root test, whereas the second and third are tests of a unit root with a drift, and a unit root with a drift and deterministic time trend, correspondingly. Each test has its own \textit{t}-statistic based on the sample size computed by Dickey and Fuller (1981) and based on critical values using likelihood approximation.

\textsuperscript{11}The four statisticians whose names gave the acronym of the test itself developed the Kwiatkowski-Phillips-Schmidt-Shin test in 1992, with the aim of complementing the Dickey-Fuller unit root test, which is believed to have less power in revealing near unit-root and long-run trend processes.
and Monte Carlo methods. The $\beta$'s represent the coefficients of the constant and time trend. According to the Dickey-Fuller test, the null hypothesis of a unit root $\rho = 1$ is tested against the alternative of $|\rho| \leq 1$. The tests assume that errors are normal, and identically and independently distributed (in other words, the errors must not be correlated). However, in some cases this assumption is relaxed or the estimation procedure is amended\footnote{The Philips-Perron test (1988) is an alternative non-linear test to modify the test statistic; however, it does not work well in finite samples or in processes with high order negative correlation (Maddala and Kim, 1998).}.

The KPSS test (1992) is a Lagrange multiplier test with the asymptotic test statistics calculated by Nabeya and Tanaka (1988); it is valid only if the errors are identically and independently distributed. The model is represented by:

$$y_t = \delta t + \zeta_t + \epsilon_t \quad (3.4)$$

where $\zeta_t$ is a random walk, such that $\zeta_t = \zeta_{t-1} + u_t$ with $u_t \sim iid(0, \sigma^2_t)$.

The null hypothesis is that of trend stationarity, more precisely $H_0: \sigma^2_u = 0$ which is equivalent to $\zeta_t$ being assumed constant. Here the null hypothesis is opposite of that seen in the Dickey-Fuller (1979) test.

After determining whether the series have a unit root, interdependencies and causality can be determined. Let $Y_t = (y_{1t}, y_{2t}, y_{3t})$ denote a $(3 \times 1)$ vector of variables representing the LIBOR-OIS, the EUSWEC and the GerUS3M series, with $t = 1, 2, \ldots, T$. The chapter follows Enders’ (2003) notation when writing the VAR system of equations, and later the impulse response functions. Considering that there are three time series in the system, there will be three equations to be solved. The covariates in the three equations are the lagged values of all three time series. Therefore, in the VAR(4) system, the time path of the LIBOR-OIS time series is determined by current and past realisations of the EUSWEC and GerUS3M series. Similarly, the time path of the EUSWEC time series is influenced by current and past realisations of the LIBOR-OIS and GerUS3M series, and finally the time path of the GerUS3M time series is affected by current and past realisations of the EUSWEC and LIBOR-OIS series. Every equation in the system is actually solved by the OLS method with joint hypotheses which imply restrictions throughout the VAR system. At 5% significance level, the parameters of the structural equations are calculated as the estimated coefficients $\pm 1.96$ standard errors (Stock and Watson, 2003). The lagged values of the series are weakly exogeneous, thus the model does not violate the endogeneity assumptions\footnote{OLS assumption states that there should be no correlation among the coefficients, variables and error terms (Hayashi, 2000).}. The four-lag vector autoregressive VAR(4) model is written in the following form. For every single equation there will be 13 parameters.
to be estimated. The null hypothesis that all parameters are zero is tested against the alternative that at least one of those parameters is non-zero:

\[ H_0 : \gamma_{10}, \gamma_{20}, \gamma_{30}, \gamma_{1p}, \gamma_{2p}, \gamma_{3p} = 0, \]

where \( p \) is the number of lags. In this case \( p = 4 \).

\[
\begin{pmatrix}
y_{LIBOROIS_t} \\
y_{EUSWEC_t} \\
y_{GerUS3M_t}
\end{pmatrix}
= \begin{pmatrix} \gamma_{10} \\ \gamma_{20} \\ \gamma_{30} \end{pmatrix} + \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix}
\begin{pmatrix}
y_{LIBOROIS_{t-1}} \\
y_{EUSWEC_{t-1}} \\
y_{GerUS3M_{t-1}}
\end{pmatrix}
+ \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix}
\begin{pmatrix}
y_{LIBOROIS_{t-2}} \\
y_{EUSWEC_{t-2}} \\
y_{GerUS3M_{t-2}}
\end{pmatrix}
+ \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix}
\begin{pmatrix}
y_{LIBOROIS_{t-3}} \\
y_{EUSWEC_{t-3}} \\
y_{GerUS3M_{t-3}}
\end{pmatrix}
+ \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix}
\begin{pmatrix}
y_{LIBOROIS_{t-4}} \\
y_{EUSWEC_{t-4}} \\
y_{GerUS3M_{t-4}}
\end{pmatrix}
+ \begin{pmatrix} \epsilon_{LIB_t} \\ \epsilon_{EUSW_t} \\ \epsilon_{GerUS_t} \end{pmatrix}
\]

(3.5)

where \( \Gamma_i \) is a \((3 \times 3)\) coefficient matrix to be estimated, with \( i = 1, 2, \ldots, I \). For example, the value of the coefficient \( \gamma_{11} \) represents a contemporaneous effect of a unit change in the \( y_{LIBOROIS_{t-1}} \) series (in this case, it will be a unit change in the first lag of the LIBOR-OIS series). Correspondingly, the impact of \( y_{EUSWEC_{t-1}} \) on \( y_{LIBOROIS_t} \) is given by the magnitude of the parameter \( \gamma_{12} \). \( \epsilon_t \) is a \((3 \times 1)\) unobservable zero mean and constant variance white noise process. Fundamentally, the innovations are shocks to the dependent variables.

To understand the VAR coefficients, to see the adjustment course of the LIBOR-OIS, the GerUS3M and the EUSWEC spreads and to assess how each type of shock to the forecast error variance feeds back, the impulse response function is implemented. Due to the fact that the VAR system is underidentified (primitive), at this stage the impulse response functions are only plotted, however one is unable to identify and determine the course and effects of the shocks. In fact, this method is a vector moving average in the sense that past and present innovations (shocks) determine the path of the three spreads over time, and thus one can see the synergy between the spreads. The innovations, or the vector of errors, can be re-written in terms of shocks, such as \( \epsilon_{LIB_t} \), \( \epsilon_{EUSW_t} \) and \( \epsilon_{GerUS_t} \) sequences, as in the equation below:
\[
\begin{pmatrix}
\epsilon_{LIB_i} \\
\epsilon_{EUSW_i} \\
\epsilon_{GeUS_i}
\end{pmatrix}
= \frac{1}{1 - \beta_12\beta_13\beta_21\beta_23\beta_31\beta_32} \begin{pmatrix}
1 & -\beta_12 & -\beta_13 \\
-\beta_21 & 1 & -\beta_23 \\
-\beta_31 & -\beta_32 & 1
\end{pmatrix}
\begin{pmatrix}
\epsilon_{LIB_i} \\
\epsilon_{EUSW_i} \\
\epsilon_{GeUS_i}
\end{pmatrix}
\] (3.6)

where \(\beta_s\) are shock coefficients and \(\epsilon_s\) are the shocks as a result of decomposition of the error terms. Combining the VAR(4) system with the vector of errors, we get:

\[
\begin{pmatrix}
y_{LIB_t} \\
y_{EUSW_t} \\
y_{GeUS_t}
\end{pmatrix}
= \begin{pmatrix}
y_{LIB_t} \\
y_{EUSW_t} \\
y_{GeUS_t}
\end{pmatrix}
+ \frac{1}{1 - \beta_12\beta_13\beta_21\beta_23\beta_31\beta_32} \sum_{i=0}^t \begin{pmatrix}
\gamma_{11} & \gamma_{12} & \gamma_{13} \\
\gamma_{21} & \gamma_{22} & \gamma_{23} \\
\gamma_{31} & \gamma_{32} & \gamma_{33}
\end{pmatrix}
\begin{pmatrix}
1 & -\beta_12 & -\beta_13 \\
-\beta_21 & 1 & -\beta_23 \\
-\beta_31 & -\beta_32 & 1
\end{pmatrix}
\begin{pmatrix}
\epsilon_{LIB_t} \\
\epsilon_{EUSW_t} \\
\epsilon_{GeUS_t}
\end{pmatrix}
\] (3.7)

For simplicity, the \(\gamma\) and \(\beta\) coefficients can be wrapped up into a matrix of impact multipliers noted \(\phi_i\), and consequently, the moving average illustration is as follows:

\[
\begin{pmatrix}
y_{LIB_t} \\
y_{EUSW_t} \\
y_{GeUS_t}
\end{pmatrix}
= \begin{pmatrix}
y_{LIB_t} \\
y_{EUSW_t} \\
y_{GeUS_t}
\end{pmatrix}
+ \sum_{i=0}^t \begin{pmatrix}
\phi_{11}(i) & \phi_{12}(i) & \phi_{13}(i) \\
\phi_{21}(i) & \phi_{22}(i) & \phi_{23}(i) \\
\phi_{31}(i) & \phi_{32}(i) & \phi_{33}(i)
\end{pmatrix}
\begin{pmatrix}
\epsilon_{LIB_{t-i}} \\
\epsilon_{EUSW_{t-i}} \\
\epsilon_{GeUS_{t-i}}
\end{pmatrix}
\] (3.8)

Aggregately, that is their sum over the total length of the time, the \(\phi_i\) coefficients represent impulse response functions, and since it is assumed that the time series are stationary (more precisely their first differences are), as time approaches to infinity, their sum is finite (Enders, 2003). These functions are used later in the Cholesky decomposition along with further restrictions to produce the effects of the \(\epsilon_{LIB_t}\), \(\epsilon_{EUSW_t}\) and \(\epsilon_{GeUS_t}\) shocks have throughout the length of the three time series.

To better explain the impulse responses and precisely identify which market moves first - that is to find leaders and followers in the short-term interbank market - restrictions are imposed on the three-variable VAR system, and subsequently the Cholesky decomposition is implemented. Restrictions mean setting some of the parameters of the system to zero. Next, Sims’ (1986) method of variance decomposition is applied. At this moment the system introduces an ordering\(^{14}\) of the variables, and in every single equation one

\(^{14}\)Fundamentally, ordering of the variables in the structural equations depends on the correlation between the regression residuals (Enders, 2003).
of the time series is exogenous. For example, if $\epsilon_{\text{LIBOR-OIS}}$ shocks do not explain the forecast error variance of the $y_{\text{GerUS3M}}$ spread for all predicted time sequences, then the $y_{\text{GerUS3M}}$ series is said to be exogeneous and therefore progresses freely from $\epsilon_{\text{LIBOR-OIS}}$ shocks and from the $y_{\text{LIBOR-OIS}}$ spread. However, if the $\epsilon_{\text{LIBOR-OIS}}$ shocks account for all the forecast error variance in the $y_{\text{GerUS3M}}$ spread for all predicted time sequences, the $y_{\text{GerUS3M}}$ spread would be absolutely endogenous. Knowing the estimated VAR parameters and the elements of the variance/covariance matrix, one can decompose the vector of innovations/shocks into impulse responses, as follows. The restrictions apply to both matrix $\mathbf{A}$ and $\mathbf{B}$.

$$
\mathbf{A} \epsilon_t = \mathbf{B} e_t
$$

where $\mathbf{A}$ is a lower triangular restricted matrix and $\mathbf{B}$ is a diagonal restricted covariance matrix of structural shocks $\Sigma$, which implies that shocks are uncorrelated. This means that shocks to the LIBOR-OIS spread are not correlated with shocks to the EUSWEC or to the GerUS3M spreads. $\epsilon_t$ are the VAR residuals, and $e_t$ are the structural shocks.

To find out whether there is causality among the series, the pairwise Granger causality method (Granger, 1969) is implemented on the VAR estimates. Fundamentally, the test uses one time series to predict future values of another. This can be used, for example, to test whether the past values of the LIBOR-OIS spread produce statistically significant information about the EUSWEC spread. The Wald-test (1951), as the basis of the Granger causality method, is a joint hypothesis test that the coefficients on all lags are jointly equal to zero. The VAR residuals are tested for serial correlation using the Durbin-Watson test (1950) and for an unit root.

### 3.5.2 Testing for structural breaks with cointegrated variables

There are different avenues one can pursue in testing the time series for cointegration. The Engle and Granger (1987) method is a two-step standard cointegration test which assumes that the cointegrating vector does not change over time, meaning that the cointegrating relationship has a stationary distribution in the long run. The method is based on the Granger representation theorem\textsuperscript{15} and fundamentally is a residual based method in comparison to other tests where the raw data is examined. The test identifies whether the cointegrating vector had shifted at an unknown point in time. Basically, the method assesses whether the residuals have a unit root, such that the lagged residuals are

\textsuperscript{15}The conjecture asserts that if the data is I(1) and a dynamic linear model with stationary errors is found, subsequently the variables must be I(1) (Brooks, 2008). Besides, the lagged value of the error term acts as a vector error correction term. The Vector Error Correction Model gives support for short-term relationships, whereas the cointegration represents the long-run relationships.
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regressed on the differenced residuals. If the residuals are proved to be stationary, then the two series are cointegrated. The criticism of this method is that the cointegrating relationships cannot be identified. For this reason, Johansen’s (1995) cointegration test is implemented, and restrictions are placed on the equations. The techniques can be implemented solely if the time series have the same level of integration. The VAR(4) is re-written as a Vector Error Correction Model following the notation used in StataCorp (2011):

\[
\Delta y_t = v + \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \epsilon_t
\]  

(3.10)

where \( \Pi = \sum_{j=1}^{p} A_j - I_k \) and \( \Gamma_i = -\sum_{j=i+1}^{p} A_j \). The \( v \) is a \((3 \times 1)\) vector of coefficients, and \( \epsilon_t \) is a \((3 \times 1)\) vector of normally distributed error terms, with zero mean and constant variance. Similarly, for the VAR(4) model, \( t = 1, 2, \ldots, T \) and \( i = 1, 2, \ldots, I \). Earlier, visual inspection of the series revealed that there might be deterministic trends present in the relationships.

The following is the generic representation of the VECM, which allows for a linear trend and a constant in the model:

\[
\Delta y_t = \alpha \beta' y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + v + \delta t + \epsilon_t
\]  

(3.11)

where matrix \( \beta \) consists of the parameters in the cointegrating relationships and \( \alpha \) comprises the error correction terms also called adjustment parameters. \( \alpha \) is a \((k \times r)\) rank matrix with the deterministic components:

\[
v = \alpha \mu + \gamma
\]  

(3.12)

and

\[
\delta t = \alpha \rho t + \tau t
\]  

(3.13)

where \( \mu \) and \( \rho \) are \((r \times 1)\) parameter vectors, and \( \gamma \) and \( \tau \) are \((k \times 1)\) parameter vectors. \( \gamma \) is independent of \( \alpha \mu \) and \( \tau \) is independent of \( \alpha \rho \). Thus, the VECM can be re-written as:
\[
\Delta y_t = \alpha (\beta' y_{t-1} + \mu + \rho t) + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-1} + \gamma + \tau t + \epsilon_t
\] (3.14)

Johansen’s (1988) maximum likelihood framework based on the trace test is used to find the cointegrating vectors and relationships (Maddala and Kim, 1998), and is given by:

\[
-2 \log L_{max} \propto T \sum_{i=1}^{n} \ln (1 - \lambda_i)
\] (3.15)

where \(\lambda_i\) are the roots of the base equation and the likelihood ratio test statistic is given by:

\[
\lambda_{\text{trace}} = T \sum_{i=r+1}^{n} \ln (1 - \hat{\lambda}_i)
\] (3.16)

where \(\hat{\lambda}_{r+1}, \ldots, \hat{\lambda}_n\) are the eigenvalues of the base equation with the smallest values (Maddala and Kim, 1998).

As shown below, an unrestricted cointegration test along with four restricted cointegration tests are implemented. The number of lags to be included in the equations is determined in advance. The decision on the optimal lag order is based on the AIC, the Schwarz’s Bayesian information criterion (BIC), and the Hanna-Quinn information criterion (HQIC) - which are calculated using the likelihood ratio test\(^\text{16}\). For all five cointegration tests, the null hypothesis states that there are at most \(r\) cointegrating vectors in the system of equations.

1. Unrestricted trend
   
   This implies that there are quadratic terms in the levels of the LIBOR-OIS, EUSWEC and GerUS3M spreads. Moreover, the cointegrating relationships are I(0).

2. Restricted trend, \(\tau = 0\)
   
   The trends in the spreads are linear and not quadratic, thus the cointegrating relationships are trend stationary.

3. Unrestricted constant, with \(\tau = 0\) and \(\rho = 0\).

\(^{16}\text{The Stata software uses the ‘varsoc’ command to select the optimal number of lags. The null hypothesis states that the coefficients on the } p\text{-th lag of all variables are zero (StataCorp., 2013).}\)
In this case there are no quadratic trends but linear trends in the differenced series. Also, the relationships are limited to being stationary with a constant mean.

4. Restricted constant, \( \tau = 0, \rho = 0 \) and \( \gamma = 0 \).

There are no linear trends in the levels of the differenced LIBOR-OIS, GerUS3M and EUSWEC series. Cointegrating relationships will have a constant mean, however there won’t be trends or constants in the equations.

5. No trend, \( \tau = 0, \rho = 0, \gamma = 0 \) and \( \mu = 0 \).

There are no trends in the relationships, which are stationary with zero mean. Similarly, differences and levels of the series have zero means.

After the short-run and long-run relationships are established, the Gregory-Hansen (1996) cointegration method is implemented to test for the presence of structural breaks or regime switches. If the cointegrating vector changes at a single unknown time, the series may return to equilibrium and a linear combination of the series becomes stationary. This test allows for serial correlation among the innovations. A dummy is included in the Engle-Granger system of regressions, which helps identify a one-time regime shift in the intercept and slope coefficients. The conventional augmented DF test would not suffice in view of the cointegrating vector shifting at an unknown point in time. The test statistic is similar to a typical Chow test, and is centred on comparing the sum of squares of residuals, a method which measures the amount of variance in the data sets. In normal circumstances, this is not accounted for by the regression model. The null hypothesis of no cointegration is tested against the alternative hypothesis of cointegration with level shift/structural break/regime shift at a single unknown time. The three models of structural breaks follow the Gregory and Hansen (1996) notation, and are as follows:

**A** Structural break in level

\[
y_{t1} = \mu_1 + \mu_2 \phi t + \alpha^T y_{t2} + \gamma^T y_{t3} + \epsilon_t, \tag{3.17}
\]

where \( \mu_1 \) represents the intercept previous to a shift, \( \mu_2 \) represents the change in the intercept at the moment of the shift. \( \alpha \) refers to the cointegrating slope coefficient for \( y_{t2} \) (which is the LIBOR-OIS spread) and \( \gamma \) represents the cointegrating slope coefficient for \( y_{t3} \) (which represents the EUSWEC rate): \( t = 1, \ldots, n \) and \( \tau \in (0, 1) \) is the unidentified parameter and represents the relative timing of the break point; it can only take integer values. The error term satisfies \( \epsilon_t \sim N(0, \sigma^2) \). The dummy or indicator variable possesses the following features:
\[ \phi_{t\tau} = \begin{cases} 
0, & \text{if } t \leq [n\tau], \\
1, & \text{if } t > [n\tau]. 
\end{cases} \]  
(3.18)

The dummy variable has the role of accounting for fluctuations in the constant term and slope coefficients.

B Level shift with trend

\[ y_{t1} = \mu_1 + \mu_2 \phi_{t\tau} + \beta t + \alpha_1^T y_{t2} + \gamma_1^T y_{t3} + \epsilon_t \]  
(3.19)

Beside the change in the intercept, a shift in the slope vector \( \beta \) is allowed. \( \alpha \) refers to the cointegrating slope coefficient for the LIBOR-OIS spread and \( \gamma \) represents the cointegrating slope coefficient for the EUSWEC spread; \( t = 1, \ldots, n \) and \( \epsilon_t \sim N(0, \sigma^2) \).

C Regime shift

\[ y_{t1} = \mu_1 + \mu_2 \phi_{t\tau} + \alpha_1^T y_{t2} + \alpha_2^T y_{t2} \phi_{t\tau} + \gamma_1^T y_{t3} + \gamma_2^T y_{t3} \phi_{t\tau} + \epsilon_t, \]  
(3.20)

where \( \alpha_1 \) refers to the cointegrating slope coefficients for the LIBOR-OIS spread before the regime shift and \( \alpha_2 \) represents the change in slope coefficients. \( \gamma_1 \) refers to the cointegrating slope coefficients for the EUSWEC spread before the regime shift and \( \gamma_2 \) represents the change in the EUSWEC rate after a regime change has occurred. \( t = 1, \ldots, n \) and \( \epsilon_t \sim N(0, \sigma^2) \).

3.6 Empirical results

The section below first details the summary statistics, then follows on to present the results of the VAR, the structural VAR, the Granger causality test and those of the cointegration tests.

3.6.1 Summary statistics

Owing to the fact that the data changes persistently, it is hard to find a measure of central tendency for the entire length of the analysed time interval. Figure 3.1 is particularly interesting, for it depicts the financial crisis of 2007-08. Closer inspection of
Figure 3.2 reveals a similar behaviour. This kind of extreme behaviour - with extreme and persistent bursts - cannot be modelled and explained by conventional analyses. It is conventional that stationarity is a necessity for most empirical investigations. Fundamentally, a time series is said to be stationary if it has constant mean, variance and covariance structure. Analysis is rendered ineffectual where time series do not display these characteristics, or where the relationship between the error terms changes over time. The section summarises the three time series of interest for the period 1st of January 2002 to 30th December 2011. There are 2609 observations for the LIBOR-OIS and GerUS3M spread, and 2607 observations for the EUSWEC (currency swap) spread.

The augmented Dickey-Fuller test (see Table 3.2) confirms that the variables contain a unit root, and therefore are stationary in their levels. The test statistic for the three series were higher than all critical values (at 1%, 5% and 10%, respectively). To corroborate the findings, the KPSS test for stationarity (Kwiatkowski et al., 1992) is implemented and the $H_0$ of trend stationarity is rejected. For all three time series the test statistic was higher than the critical values (at 1%, 5% and 10%, respectively). Consequently, the series in this study are not I(0), in addition to having an unbounded variance that changes persistently.

As seen in the long-run behaviour of the LIBOR-OIS spread for example, the measure of variability - or the standard deviation - captures how wide the distribution around the mean is. Variance and standard deviation are the most important and crucial concepts in econometrics and statistics, however they are often overlooked. The plots (see Figures 3.1 and 3.2) of the daily time series show stochastic trends, that is they increase and decrease over time. Table 3.1 presents the summary statistics of the daily LIBOR-OIS,
Table 3.1: Summary statistics of the LIBOR-OIS, GerUS3M and EUSWEC spreads.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>USLIBOIS</td>
<td>28.50</td>
<td>40.65</td>
<td>1.91</td>
<td>364.43</td>
<td>3.80</td>
<td>22.32</td>
</tr>
<tr>
<td>GerUS3M</td>
<td>-0.27</td>
<td>1.23</td>
<td>-4.01</td>
<td>2.29</td>
<td>0.35</td>
<td>2.44</td>
</tr>
<tr>
<td>EUSWEC</td>
<td>2.25</td>
<td>1.23</td>
<td>0.35</td>
<td>4.35</td>
<td>-0.006</td>
<td>1.83</td>
</tr>
</tbody>
</table>

EUSWEC and GerUS3M spreads. The mean value - or point estimate - which econometricians believe is representative of the whole distribution is 28.50 for the LIBOR-OIS spread, with minimum and maximum values ranging from 1.91 to 364.43.

Table 3.2: Results for the Dickey-Fuller test of no stationarity.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Test statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>USLIBOIS</td>
<td>-2.092</td>
<td>0.248</td>
</tr>
<tr>
<td>EUSWEC</td>
<td>0.566</td>
<td>0.987</td>
</tr>
<tr>
<td>GerUS3M</td>
<td>-1.595</td>
<td>0.486</td>
</tr>
<tr>
<td>diff. USLIBOIS</td>
<td>-45.768</td>
<td>0.000</td>
</tr>
<tr>
<td>diff. EUSWEC</td>
<td>-48.176</td>
<td>0.000</td>
</tr>
<tr>
<td>diff. GerUS3M</td>
<td>-51.752</td>
<td>0.000</td>
</tr>
</tbody>
</table>

1% crit. val. -3.430
5% crit. val. -2.860
10% crit. val. -2.570

As details of Table 3.1 reveal, the data is widely dispersed, specifically for the LIBOR-OIS spread with a standard deviation of 40.65. Thus, the data is not symmetrically
distributed, and is clustered around the mean to form an acute peak, as explained by
the high level of kurtosis. For the GerUS3M and EUSWEC series, the dispersion around
the mean is not as pronounced as that of the LIBOR-OIS spread, with value of 1.23 for
both time series.

Table 3.3: Summary statistics of the differenced LIBOR-OIS, GerUS3M and
EUSWEC spreads.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>USLIBOIS</td>
<td>0.014</td>
<td>3.36</td>
<td>-39.64</td>
<td>37.55</td>
<td>0.313</td>
<td>52.17</td>
</tr>
<tr>
<td>GerUS3M</td>
<td>0.000</td>
<td>0.069</td>
<td>-0.69</td>
<td>0.872</td>
<td>0.45</td>
<td>35.92</td>
</tr>
<tr>
<td>EUSWEC</td>
<td>-0.000</td>
<td>0.020</td>
<td>-0.293</td>
<td>0.1975</td>
<td>-2.62</td>
<td>42.39</td>
</tr>
</tbody>
</table>

At this time, it is essential to modify the time series to permit a reliable evaluation of
the relationships among the spreads. A close examination of Figure 3.3 reveals that
all three spreads have more than one structural break, however at this time one cannot
tell where these breaks occur. As data in Table 3.3 shows, the mean values of all three
series get close to zero. The LIBOR-OIS spread has the highest standard deviation of
3.36. However, the time series remain peaked, with still extremely high values of kurtosis
(52.17, 35.92 and 42.39, respectively). Figure 3.4 depicts the histograms of the series
before and after first differencing; the spreads get as close as possible to normality. The
mean is used best when the data is normally distributed. Yet, the median describes
the series best when the data is not normally distributed. If the data were “perfectly”
normally distributed, the values of the mean and median would be identical; however,
that is not the case with the three time series. In such a case, the central value cannot
be representative of the whole dataset. Nevertheless, these findings are consistent with
financial and economic time series in general.

For all three variables, the test statistic of the Dickey-Fuller test have the values
-45.77, -48.18 and -51.75 correspondingly, and are smaller than the critical values at
1%, 5% and 10%, respectively. Therefore, the series are stationary in their levels. The
KPSS test corroborates the Dickey-Fuller test results. Thus, first differencing produces
a mean-reverting process\(^\text{17}\), however the variance changes persistently. As proven by the
Dickey-Fuller and KPSS tests and by revealing the adjustments in levels and by first
differencing the time series, all the variables of interest become constant. Thus, the time
series become stationary, reflecting a white noise process. Once the series are I(1), they
are almost certainly cointegrated; consequently the series share a common stochastic
trend and move jointly in the long-run (Maddala and Kim, 1998).

Earlier, the augmented Dickey-Fuller test confirmed that the time series are not station-
ary, however, their first difference is covariance stationary, which means that the mean

\(^\text{17}\)Mean reversion denotes that deviation from mean levels are temporary. In finance, specifically in
the field of investments, diversion from the average level - highs or lows - provides gain opportunities to
traders (Hayashi, 2000).
Figure 3.3: Behaviour of the LIBOR-OIS, GerUS3M and EUSWEC spreads after first differencing.

Figure 3.4: Histogram of the daily LIBOR-OIS, EUSWEC and GerUS3M spreads vs differenced spreads.
Chapter 3 Modelling the Long-run Relationship of Short-term Interest Rate Spreads

Figure 3.5: Auto-correlation function of the diff. LIBOR-OIS spread.

Figure 3.6: Auto-correlation function of the diff. EUSWEC spread.
and covariances are not time dependent. Figures 3.1 and 3.2 clearly show that the structure of the data changes over time; thus, the Dickey-Fuller test can only provide global summary statistics.

When interpreting the correlation function and regression results, it is assumed that the three series are normally distributed, also there is a linear relationship between the series. Moreover, the homoskedasticity assumption holds and there are no relationships between the independent variables and regression residuals. The autocorrelation function of the differenced LIBOR-OIS spread shows that the series is autoregressive, as seen in Figure 3.5. However, up to lag three, the autocorrelation does not decay with time, rather it increases. For the currency swap spread, the autocorrelation is significant at lag two, after which it decays. The GerUS3M spread does not show any significant autocorrelation between its lags, however its fourth lag seems significant.

### 3.6.2 VAR, Structural VAR and Granger causality

At the moment, there is no information on which of the three time series is exogenous. Vector Autoregression (VAR) enables regression of the LIBOR-OIS spread on its lagged values, and on the GerUS3M and EUSWEC spreads and their lags. All three variables are treated symmetrically in the VAR system, that is there are no specified dependent variables and covariates. Table 3.4 presents the lag-order selection statistics and shows how many autoregressive components one needs to include in the regression equations. The AIC, HQIC and Schwarz’s Bayesian information criterion suggest that the optimal lag is four. The configuration of the VAR(4) system contains impulse responses (or feedback), as a result that the LIBOR-OIS, the EUSWEC and the GerUS3M spreads
and their lags are permitted to interact with and influence each other. The objective is to find out whether changes in the independent variables cause movement in the LIBOR-OIS spread, for example. To put it another way, causal relationships are assessed among the series. Thus, the VAR model helps detect feedback effects within the three spreads.

Table 3.4: Lag-selection order criteria for the LIBOR-OIS, GerUS3M and EUSWEC spreads.

<table>
<thead>
<tr>
<th>Lag</th>
<th>LL</th>
<th>LR</th>
<th>df</th>
<th>p</th>
<th>FPE</th>
<th>AIC</th>
<th>FPE</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2883.44</td>
<td>.0000022</td>
<td>-2.21402</td>
<td>-2.21157</td>
<td>-2.20726</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2956.77</td>
<td>146.66</td>
<td>9</td>
<td>0.000</td>
<td>.0000021</td>
<td>-2.26346</td>
<td>-2.25366</td>
<td>-2.23642</td>
</tr>
<tr>
<td>2</td>
<td>3057.93</td>
<td>202.32</td>
<td>9</td>
<td>0.000</td>
<td>.0000019</td>
<td>-2.3343</td>
<td>-2.31715</td>
<td>-2.28697</td>
</tr>
<tr>
<td>3</td>
<td>3113.12</td>
<td>110.39</td>
<td>9</td>
<td>0.000</td>
<td>.0000019</td>
<td>-2.36981</td>
<td>-2.34531</td>
<td>-2.3022</td>
</tr>
<tr>
<td>4</td>
<td>3160.16</td>
<td>94.077*</td>
<td>9</td>
<td>0.000</td>
<td>.000018*</td>
<td>-2.39905*</td>
<td>-2.3672*</td>
<td>-2.31115*</td>
</tr>
</tbody>
</table>

The VAR results are presented in Tables 3.5 and Table 3.6. The null hypothesis that jointly the coefficients are zero is rejected; thus, it can be concluded that the LIBOR-OIS spread is a good predictor of changes in GerUS3M spread. At 5% significance level, for every unit increase in the first lagged LIBOR-OIS spread a 0.001 unit increase in the GerUS3M spread is predicted. Similarly, the GerUS3M spread is a good predictor of changes in the currency swap. For every unit increase in the fourth lag of the GerUS3M spread a 0.012 unit decrease in the EUSWEC spread is predicted. Last, for every unit increase in the first lagged EUSWEC spread a 0.05 unit increase in the GerUS3M is predicted. However, it seems that various lags of some of the variables have an ex-post effect on the present observations of the other two. This might be accredited to the fact that a single shock may last several periods as a result of autoregressive attributes of the time series. Or another explanation is that a shock’s effect is not experienced in its full force instantaneously, but after some time has lapsed. For example, for every unit increase in the forth lagged EUSWEC spread a 0.148 unit increase in the GerUS3M is predicted. Fundamentally, the VAR equations can be used to perform a Granger causality test among the variables.

The errors of the models are unobservable and essentially are departures from the linear relationships. Basically, the residuals are estimates for the errors. The patterns observed in Figure 3.8 for the VAR residuals might explain whether there are structural breaks in the model, or whether there are non-linear relationships. The residuals of the regressions are stationary, as supported by the Dickey-Fuller and KPSS tests, however there are time intervals which suggest the presence of structural breaks; for example, the period surrounding the last quarter of 2005 and the period last quarter of 2007 to first quarter of 2010. Moreover, the residuals are not correlated as confirmed by the Durbin-Watson test (Durbin and Watson, 1950). The stability condition of the VAR parameters are checked and all eigenvalues lie within the unit circle; that is, they are all less than 1,
Table 3.5: Vector Autoregression with four lags for the LIBOR-OIS, EUSWEC and GerUS3M spreads.

<table>
<thead>
<tr>
<th>Main</th>
<th>D.USLIBOIS</th>
<th>D.GerUS3M</th>
<th>D.EUSWEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>LD.USLIBOIS</td>
<td>0.0767***</td>
<td>0.00108*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.63)</td>
<td>(2.57)</td>
<td></td>
</tr>
<tr>
<td>L2D.USLIBOIS</td>
<td>0.111***</td>
<td>0.00173***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.68)</td>
<td>(4.14)</td>
<td></td>
</tr>
<tr>
<td>L3D.USLIBOIS</td>
<td>0.114***</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.91)</td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>L4D.USLIBOIS</td>
<td>0.0638***</td>
<td>-0.000792</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.30)</td>
<td>(-1.90)</td>
<td></td>
</tr>
<tr>
<td>LD.EUSWEC</td>
<td>-15.14***</td>
<td>0.0518**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-4.81)</td>
<td>(2.67)</td>
<td></td>
</tr>
<tr>
<td>L2D.EUSWEC</td>
<td>-24.08***</td>
<td>0.151***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-7.63)</td>
<td>(7.77)</td>
<td></td>
</tr>
<tr>
<td>L3D.EUSWEC</td>
<td>9.485***</td>
<td>-0.0260</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.98)</td>
<td>(-1.34)</td>
<td></td>
</tr>
<tr>
<td>L4D.EUSWEC</td>
<td>9.509***</td>
<td>0.123***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.00)</td>
<td>(6.36)</td>
<td></td>
</tr>
<tr>
<td>LD.GerUS3M</td>
<td>-0.00427</td>
<td>-0.0237***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.21)</td>
<td>(-4.14)</td>
<td></td>
</tr>
<tr>
<td>L2D.GerUS3M</td>
<td>0.00195</td>
<td>0.00507</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.88)</td>
<td></td>
</tr>
<tr>
<td>L3D.GerUS3M</td>
<td>0.0338</td>
<td>-0.0182**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.96)</td>
<td>(-3.17)</td>
<td></td>
</tr>
<tr>
<td>L4D.GerUS3M</td>
<td>-0.00693***</td>
<td>-0.0118*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-3.44)</td>
<td>(-2.05)</td>
<td></td>
</tr>
<tr>
<td>_cons</td>
<td>-0.0145</td>
<td>0.000085</td>
<td>-0.000751</td>
</tr>
<tr>
<td></td>
<td>(-1.23)</td>
<td>(0.51)</td>
<td>(-1.91)</td>
</tr>
</tbody>
</table>

| D.EUSWEC      | 0.000377** |          |          |
|               | (3.11)     |          |          |
| L2D.USLIBOIS  | 0.000291*  |          |          |
|               | (2.42)     |          |          |
| L3D.USLIBOIS  | -0.000377**|          |          |
|               | (-3.16)    |          |          |
| L4D.USLIBOIS  | -0.000115  |          |          |
|               | (-0.96)    |          |          |
| LD.EUSWEC     | 0.0462*    |          |          |
|               | (2.37)     |          |          |
| L2D.EUSWEC    | 0.166***   |          |          |
|               | (8.50)     |          |          |
| L3D.EUSWEC    | -0.00644   |          |          |
|               | (-0.33)    |          |          |
| L4D.EUSWEC    | 0.126***   |          |          |
|               | (6.40)     |          |          |
| _cons         | -0.000748  |          |          |
|               | (-1.90)    |          |          |

t statistics in parantheses

* p < 0.05, ** p < 0.01, *** p < 0.001
and thus the system is stationary. Consequently, the impulse response functions are interpretable.

As it can be seen from Figure 3.9, shocks are persistent, however in the long-run they die away. Both the GerUS3M and EUSWEC spreads are affected by shocks induced by the LIBOR-OIS spread. If after some shocks the equilibrium errors return to zero, a long-run equilibrium among the LIBOR-OIS, GerUS3M and EUSWEC spreads exists. As noted earlier, this is due to the fact that the system is underidentified and the path of the shocks cannot be traced. More accurately, one cannot detect the timing and length of the identified shocks.

The Granger causality test is applied on the coefficients of the VAR. The null hypothesis that the EUSWEC does not Granger-cause LIBOR-OIS can be rejected; similarly, the null hypothesis that the coefficients on the four lags of the GerUS3M in the equation for LIBOR-OIS are zero can be rejected. Last, the null hypothesis that the GerUS3M does not Granger-cause the LIBOR-OIS can be rejected. Similarly, for the third equation, the coefficients of both endogenous variables are not jointly zero. Therefore, the null hypothesis that the EUSWEC and GerUS3M series do not Granger-cause the LIBOR-OIS can be rejected.

The Cholesky decomposition is represented by the lower triangular $A$ matrix and $B$, a diagonal matrix, with the estimated values displayed in matrices 3.21 and 3.22.

Figure 3.8: Plot of the VAR(4) residuals versus time.
Table 3.6: Vector Autoregression with four lags for the LIBOR-OIS, EUSWEC and GerUS3M spreads.

<table>
<thead>
<tr>
<th>Main</th>
<th>D.USLIBOIS</th>
<th>D.GerUS3M</th>
<th>D.EUSWEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>LD.D.USLIBOIS</td>
<td>4.702***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LD.GerUS3M</td>
<td>(4.91)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L2D.GerUS3M</td>
<td>0.927</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L2D.GerUS3M</td>
<td>(0.97)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L3D.GerUS3M</td>
<td>4.682***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L3D.GerUS3M</td>
<td>(5.08)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L4D.GerUS3M</td>
<td>2.636**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L4D.GerUS3M</td>
<td>(2.74)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LD.USLIBOIS</td>
<td>0.0890***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LD.USLIBOIS</td>
<td>(4.45)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L2D.USLIBOIS</td>
<td>0.0772***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L2D.USLIBOIS</td>
<td>(3.87)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L3D.USLIBOIS</td>
<td>0.106***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L3D.USLIBOIS</td>
<td>(5.28)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L4D.USLIBOIS</td>
<td>0.0740***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L4D.USLIBOIS</td>
<td>(3.71)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>_cons</td>
<td>-0.0000835</td>
<td></td>
<td></td>
</tr>
<tr>
<td>_cons</td>
<td>(-0.00)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D.GerUS3M</td>
<td></td>
<td>-0.121</td>
<td></td>
</tr>
<tr>
<td>LD.EUSWEC</td>
<td></td>
<td>(-1.82)</td>
<td></td>
</tr>
<tr>
<td>L2D.EUSWEC</td>
<td></td>
<td>-0.0979</td>
<td></td>
</tr>
<tr>
<td>L2D.EUSWEC</td>
<td></td>
<td>(-1.48)</td>
<td></td>
</tr>
<tr>
<td>L3D.EUSWEC</td>
<td>-0.184***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L3D.EUSWEC</td>
<td>(-2.77)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L4D.EUSWEC</td>
<td>0.148*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L4D.EUSWEC</td>
<td>(2.25)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LD.GerUS3M</td>
<td>-0.0117</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LD.GerUS3M</td>
<td>(-0.60)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L2D.GerUS3M</td>
<td>-0.0175</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L2D.GerUS3M</td>
<td>(-0.89)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L3D.GerUS3M</td>
<td>0.0399*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L3D.GerUS3M</td>
<td>(2.04)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L4D.GerUS3M</td>
<td>-0.0625*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L4D.GerUS3M</td>
<td>(-3.18)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>_cons</td>
<td>0.000419</td>
<td></td>
<td></td>
</tr>
<tr>
<td>_cons</td>
<td>(0.31)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*t statistics in parantheses

* p < 0.05, ** p < 0.01, *** p < 0.001

Economic theory suggests the number of restrictions should be as follows: \((n^2 - n)/2\), with \(n\) representing the number of variables of the system (Hayashi, 2000). Since there are three variables, the number of restrictions is \((3^2 - 3)/2 = 3\). In the first equation, the restrictions imply that the LIBOR-OIS equation does not contain contemporaneous EUSWEC and GerUS3M series; more precisely, it means that the LIBOR-OIS spread is not contemporaneously affected by shocks originating from the EUSWEC and GerUS3M series correspondingly. In the second equation of matrix \(A\), the EUSWEC spread responds to changes in the LIBOR-OIS spread, but not to contemporaneous changes in
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Figure 3.9: Impulse response function of the coefficients after fitting the VAR.

Table 3.7: Results for the Granger causality Wald tests.

<table>
<thead>
<tr>
<th>Equation Excluded from equation</th>
<th>chi2</th>
<th>df</th>
<th>Prob. chi2</th>
</tr>
</thead>
<tbody>
<tr>
<td>D_USLIBOIS D_EUSWEC</td>
<td>85.957</td>
<td>4</td>
<td>0.000</td>
</tr>
<tr>
<td>D_USLIBOIS D_GerUS3M</td>
<td>55.511</td>
<td>4</td>
<td>0.000</td>
</tr>
<tr>
<td>D_USLIBOIS All</td>
<td>148.86</td>
<td>8</td>
<td>0.000</td>
</tr>
<tr>
<td>D_EUSWEC D_USLIBOIS</td>
<td>27.733</td>
<td>4</td>
<td>0.000</td>
</tr>
<tr>
<td>D_EUSWEC D_GerUS3M</td>
<td>36.267</td>
<td>4</td>
<td>0.000</td>
</tr>
<tr>
<td>D_EUSWEC All</td>
<td>60.639</td>
<td>8</td>
<td>0.000</td>
</tr>
<tr>
<td>D_GerUS3M D_USLIBOIS</td>
<td>28.723</td>
<td>4</td>
<td>0.000</td>
</tr>
<tr>
<td>D_GerUS3M D_EUSWEC</td>
<td>20.31</td>
<td>4</td>
<td>0.000</td>
</tr>
<tr>
<td>D_GerUS3M All</td>
<td>48.557</td>
<td>8</td>
<td>0.000</td>
</tr>
</tbody>
</table>

the GerUS3M spread. In the third equation, the GerUS3M spread responds to contemporaneous changes in the LIBOR-OIS and EUSWEC spread correspondingly:

\[
A = \begin{pmatrix}
1 & 0 & 0 \\
-0.003 & 1 & 0 \\
0.005 & -0.003 & 1 \\
\end{pmatrix}
\] (3.21)

As discussed earlier, a zero restriction on the coefficients of the established variables is imposed. The matrix below displays the values representing the speed of adjustment to liquidity shocks. If the short-term interbank market is affected by a liquidity shock, the LIBOR-OIS spread is a leader in moving back into equilibrium, whereas the currency swap and the German-US bond spreads are followers.
The results with the values of the Cholesky decomposition are presented in Table 3.8. Thus, by imposing three restrictions, the following two relationships are identified:

\[ \text{LIBOROIS}_t = 0.0009 \text{EUSWEC}_t + \epsilon_{\text{LIBOROIS}_t} \]

\[ \text{LIBOROIS}_t = 0.0007 \text{EUSWEC}_t - 0.016 \text{GerUS3M}_t + \epsilon_{\text{LIBOROIS}_t} \]

A 1% increase in the current LIBOR-OIS spread affects the current EUSWEC spread by 0.0009%, and 1% increase in the current LIBOR-OIS spread affects the currency swap by 0.0007% and the GerUS3M spread by -0.016%.

Table 3.8: Results with the Cholesky decomposition of the covariance matrix of residuals.

<table>
<thead>
<tr>
<th>Variable</th>
<th>USLIBOIS</th>
<th>EUSWEC</th>
<th>GerUS3M</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIBOR-OIS</td>
<td>3.26</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>EUSWEC</td>
<td>0.0009</td>
<td>0.020</td>
<td>0</td>
</tr>
<tr>
<td>GerUS3M</td>
<td>-0.016</td>
<td>0.0007</td>
<td>0.07</td>
</tr>
</tbody>
</table>

The forecast error variance decomposition calculated the fraction of the movement in a sequence, owing to its individual shocks reacting against shocks of the other two time series. The results suggest that the currency swap rate rises as the LIBOR-OIS spread increases. The German-US bond spread is positively related to the currency swap rate and negatively related to the LIBOR-OIS rate. The impulse response functions presented in Figure 3.9 are consistent with these findings.

3.6.3 Structural breaks with cointegrated variables

Cointegration received prominence in econometrics after Engle and Granger introduced it in 1987, and has since become a standard tool in modelling and investigating unit root processes. Previously, the time series of interest were found to be I(1), and rather possibly cointegrated. In this section is shown whether there is a stationary linear combination of the LIBOR-OIS, GerUS3M and EUSWEC time series. If the short-term spreads are cointegrated, and the market as a system is in a long-term equilibrium, then one has an insight into fundamental short-term money market rates behaviour. Ultimately, this has substantial implication for policy makers and market players alike.
Chapter 3 Modelling the Long-run Relationship of Short-term Interest Rate Spreads

However, as it is proven in this section, the long-term equilibrium breaks down temporarily as a result of short-term liquidity shocks affecting the short-term interbank money market.

Essentially, a standard test of cointegration fails to produce reliable results in case there is a structural break in the time series. Visually analysing the time series (see Figures 3.1 and 3.2), one can see that the spreads have shifted at least at one point in time; thus, the cointegrating vector might not be time-invariant, as expected for cointegration tests.

The results reveal that there are cointegrating relationships for the restricted trend case, for the restricted constant case and for the case when there is no trend nor a constant in the equation (see Tables 3.9, 3.10, 3.11, 3.12 and 3.13). For all three cases, at rank zero \( r = 0 \) the null hypothesis of no cointegrating relationship is rejected. At \( r = 1 \), the trace statistic\(^\text{18}\) is below the 5% critical values for all the three cases. Therefore, the null hypothesis that there is one cointegrating relationship in the system of equations cannot be rejected. The next step is to fit a VECM for the identified cases (equation with restricted trend, equation with restricted constant and no trend nor a constant in the equation) and estimate the long-run and short-run coefficients. Vector error correction implies that departure from equilibrium in one period is corrected in the subsequent period; or more precisely, deviations from the long-run equilibrium condition are introduced into the short-time dynamics (Johansen, 1995). It is imperative that all error terms in the VEC model are stationary. Basically, the VECM is formulated as the response of the LIBOR-OIS and EUSWEC shocks to the GerUS3M spreads, and implies that the cointegrating vector reduces the series to stationarity.

Table 3.9: Rank test for the equation with constant, maxlag(4).

<table>
<thead>
<tr>
<th>Rank</th>
<th>Param</th>
<th>LL</th>
<th>eigenvalue</th>
<th>trace stat.</th>
<th>5% crit. val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>3115.54</td>
<td>.</td>
<td>214.97</td>
<td>29.68</td>
</tr>
<tr>
<td>1</td>
<td>35</td>
<td>3215.03</td>
<td>0.074</td>
<td>16.00</td>
<td>15.41</td>
</tr>
<tr>
<td>2</td>
<td>38</td>
<td>3221.06</td>
<td>0.005</td>
<td>3.93</td>
<td>3.76</td>
</tr>
<tr>
<td>3</td>
<td>39</td>
<td>3223.03</td>
<td>0.002</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.10: Rank test for the equation with trend, maxlag(4).

<table>
<thead>
<tr>
<th>Rank</th>
<th>Param</th>
<th>LL</th>
<th>eigenvalue</th>
<th>trace stat.</th>
<th>5% crit. val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>3116.45</td>
<td>.</td>
<td>261.10</td>
<td>34.55</td>
</tr>
<tr>
<td>1</td>
<td>38</td>
<td>3234.95</td>
<td>0.087</td>
<td>24.10</td>
<td>18.17</td>
</tr>
<tr>
<td>2</td>
<td>41</td>
<td>3244.86</td>
<td>0.0075</td>
<td>4.28</td>
<td>3.74</td>
</tr>
<tr>
<td>3</td>
<td>42</td>
<td>3247.00</td>
<td>0.002</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{18}\)The test statistic is based on Johansen’s (1995) multiple-trace test technique and uses the formula \( -T \sum_{i=r+1}^{K} \ln(1 - \hat{\lambda}_i) \) to calculate it; the \( T \) represents the number of observations and \( \hat{\lambda}_i \) are the approximated eigenvalues.
Table 3.11: Rank test for the equation with restricted constant, maxlag(4).

<table>
<thead>
<tr>
<th>Rank</th>
<th>Param</th>
<th>LL</th>
<th>eigenvalue</th>
<th>trace stat.</th>
<th>5% crit. val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>27</td>
<td>3113.27</td>
<td>.</td>
<td>219.52</td>
<td>34.91</td>
</tr>
<tr>
<td>1</td>
<td>33</td>
<td>3214.76</td>
<td>0.075</td>
<td>16.53*</td>
<td>19.96</td>
</tr>
<tr>
<td>2</td>
<td>37</td>
<td>3221.04</td>
<td>0.005</td>
<td>3.98</td>
<td>9.42</td>
</tr>
<tr>
<td>3</td>
<td>39</td>
<td>3223.03</td>
<td>0.002</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.12: Rank test for the equation with restricted trend, maxlag(4).

<table>
<thead>
<tr>
<th>Rank</th>
<th>Param</th>
<th>LL</th>
<th>eigenvalue</th>
<th>trace stat.</th>
<th>5% crit. val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>3115.54</td>
<td>.</td>
<td>262.92</td>
<td>42.44</td>
</tr>
<tr>
<td>1</td>
<td>36</td>
<td>3234.56</td>
<td>0.087</td>
<td>24.88*</td>
<td>25.32</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>3244.75</td>
<td>0.008</td>
<td>4.50</td>
<td>12.25</td>
</tr>
<tr>
<td>3</td>
<td>42</td>
<td>3247.00</td>
<td>0.002</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.13: Rank test for the equation with no constant and no trend, maxlag(4).

<table>
<thead>
<tr>
<th>Rank</th>
<th>Param</th>
<th>LL</th>
<th>eigenvalue</th>
<th>trace stat.</th>
<th>5% crit. val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>27</td>
<td>3113.27</td>
<td>.</td>
<td>214.23</td>
<td>24.31</td>
</tr>
<tr>
<td>1</td>
<td>32</td>
<td>3214.75</td>
<td>0.075</td>
<td>11.76*</td>
<td>12.53</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
<td>3219.76</td>
<td>0.004</td>
<td>1.24</td>
<td>3.84</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
<td>3220.38</td>
<td>0.0005</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Gonzalo (1994) suggests that a higher number of lags should be included in the VECM; he argues that underspecifying the number of lags will more likely lead to bias in the finite-sample estimates, and subsequently the researcher will be faced with autocorrelation in the residuals. Therefore, the number of lags included in the equations is increased from four to six. The dependent variable is the three-month German-US bond spread, and this is based on the Cholesky decomposition result, where the LIBOR-OIS spread became the leader, while the EUSWEC and GerUS3M spreads were followers in transmitting liquidity shocks. The covariates are represented by the EUSWEC and LIBOR-OIS spreads. The three cases which are considered, are as follows:

A Restricted Trend equation

B Restricted Constant equation

C No Trend and no Constant equation

Table 3.14 presents the estimated error-correction coefficients, whereas the significant short-run coefficients are shown in Table 3.15. The error correction coefficients indicate long-run relationships, also showing the speed of the variables adjusting back into
equilibrium. The lagged terms’ significant coefficients signal short-run causality in the equilibrium model. Therefore, the test reveals both the short-term adjustment and long-term relationships between the LIBOR-OIS, GerUS3M and EUSWEC spreads, as well as their direction of causality.

Table 3.14: VECM with the estimated long-run coefficients.

<table>
<thead>
<tr>
<th>Equation</th>
<th>α</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>Restricted Trend</td>
<td>(0.0006, -0.056, -0.0004)</td>
<td>(1, 0.36, -5.62)</td>
</tr>
<tr>
<td>Restricted Constant</td>
<td>(-0.0002, 0.0255, 0.0002)</td>
<td>(1, -0.50, 4.19)</td>
</tr>
<tr>
<td>No Trend and Constant</td>
<td>(-0.0002, 0.257, 0.0002)</td>
<td>(1, -0.491, 4.719)</td>
</tr>
</tbody>
</table>

All error correction terms (or adjustment coefficients) are significant at 5% critical value. However, the estimated α’s are meaningful when the first error correction coefficient in the model takes a negative value. The restricted trend equation does not fit, however in the restricted constant and no trend no constant cases the model fits well. For these two cases one can argue that there is a long-term causality running in both directions between the GerUS3M, LIBOR-OIS and EUSWEC spreads. For example, when the average three-month German-US bond spread is too high (with coeff. value of -0.0002), this will decrease back to the LIBOR-OIS spread and currency swap level. In the no trend no constant case for example, the coefficient on the LIBOR-OIS spread is 0.257. This means that when the average GerUS3M spread is too high, the LIBOR-OIS spread level rapidly adjusts (increases in this case) towards the GerUS3M level, while at the same time the GerUS3M spread adjusts as well.

The short-run coefficients are obtained from the lagged spread values (see Table 3.15). For all three equations, the estimated β’s, which represent the parameters of the cointegrating relationships, are significant at 5% critical level. For the restricted constant case and for the no trend no constant case, the long-run cointegrating equations are as follows:

\[
y_{D,GerUS3M} = 4.72 \ y_{D,EUSWEC} - 0.49 \ y_{D,LIBOROIS} + \epsilon_{D,GerUS3M}
\]

\[
y_{D,GerUS3M} = 4.20 \ y_{D,EUSWEC} - 0.50 \ y_{D,LIBOROIS} + \epsilon_{D,GerUS3M}
\]

Next, the specification of the two models that proved to fit well is tested. A graph of the levels gives a guidance as to whether the estimated cointegrating equations are stationary. As it can be seen in Figure 3.12 (which represents both the restricted constant and no trend and no constant cases), there is a major and long-lasting break in the long-run relationship which starts at the end of 2007. The timing corresponds to the start of the financial crisis of 2007-08. Moreover, the graph tells us that the break lasted until approximately the end of 2009, which corresponds to the end of the financial
Table 3.15: VECM estimates with the short-run $\Gamma$ coefficients.

<table>
<thead>
<tr>
<th>Variable</th>
<th>rtrend</th>
<th>rconstant</th>
<th>none</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.GerUS3M</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ce1</td>
<td>0.0006***</td>
<td>-0.0002**</td>
<td>-0.0002**</td>
</tr>
<tr>
<td>LD.GerUS3M</td>
<td>-0.017</td>
<td>-0.014</td>
<td>-0.014</td>
</tr>
<tr>
<td>L2D.</td>
<td>0.001</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>L3D.</td>
<td>0.033</td>
<td>0.035</td>
<td>0.035</td>
</tr>
<tr>
<td>L4D.</td>
<td>-0.081***</td>
<td>-0.077***</td>
<td>-0.078***</td>
</tr>
<tr>
<td>L5D.</td>
<td>-0.048*</td>
<td>-0.045*</td>
<td>-0.045*</td>
</tr>
<tr>
<td>LD.USLIBOIS</td>
<td>0.001*</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>L2D.</td>
<td>0.002***</td>
<td>0.002***</td>
<td>0.002***</td>
</tr>
<tr>
<td>L3D.</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>L4D.</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td>L5D.</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>LD.EUSWEC</td>
<td>-0.115</td>
<td>-0.139*</td>
<td>-0.138*</td>
</tr>
<tr>
<td>L2D.</td>
<td>-0.021</td>
<td>-0.050</td>
<td>-0.049</td>
</tr>
<tr>
<td>L3D.</td>
<td>-0.091</td>
<td>-0.121</td>
<td>-0.120</td>
</tr>
<tr>
<td>L4D.</td>
<td>0.249***</td>
<td>0.215***</td>
<td>0.217</td>
</tr>
<tr>
<td>L5D.</td>
<td>0.277***</td>
<td>0.246***</td>
<td>0.247***</td>
</tr>
<tr>
<td>const</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D.USLIBOIS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ce1</td>
<td>-0.056***</td>
<td>0.026***</td>
<td>0.026***</td>
</tr>
<tr>
<td>LD.GerUS3M</td>
<td>4.988***</td>
<td>4.730***</td>
<td>4.748***</td>
</tr>
<tr>
<td>L2D.</td>
<td>0.913</td>
<td>0.718</td>
<td>0.727</td>
</tr>
<tr>
<td>L3D.</td>
<td>4.381***</td>
<td>4.131***</td>
<td>4.141***</td>
</tr>
<tr>
<td>L4D.</td>
<td>2.699**</td>
<td>2.389*</td>
<td>2.399*</td>
</tr>
<tr>
<td>L5D.</td>
<td>3.754***</td>
<td>3.386***</td>
<td>3.397***</td>
</tr>
<tr>
<td>LD.USLIBOIS</td>
<td>0.065***</td>
<td>0.070***</td>
<td>0.070***</td>
</tr>
<tr>
<td>L2D.</td>
<td>0.073***</td>
<td>0.77***</td>
<td>0.077***</td>
</tr>
<tr>
<td>L3D.</td>
<td>0.108*</td>
<td>0.109***</td>
<td>0.109***</td>
</tr>
<tr>
<td>L4D.</td>
<td>0.063**</td>
<td>0.062**</td>
<td>0.062**</td>
</tr>
<tr>
<td>L5D.</td>
<td>0.134***</td>
<td>0.129***</td>
<td>0.129***</td>
</tr>
<tr>
<td>L2D.</td>
<td>-28.022***</td>
<td>-25.782***</td>
<td>-25.813***</td>
</tr>
<tr>
<td>L3D.</td>
<td>0.992</td>
<td>3.335</td>
<td>3.301</td>
</tr>
<tr>
<td>L4D.</td>
<td>0.866</td>
<td>3.435</td>
<td>3.398</td>
</tr>
<tr>
<td>L5D.</td>
<td>-3.860</td>
<td>-1.454</td>
<td>-1.496</td>
</tr>
<tr>
<td>const</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D.EUSWEC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ce1</td>
<td>-0.0004***</td>
<td>0.0002***</td>
<td>0.0002***</td>
</tr>
<tr>
<td>LD.GerUS3M</td>
<td>-0.022***</td>
<td>-0.023***</td>
<td>-0.023***</td>
</tr>
<tr>
<td>L2D.</td>
<td>0.012*</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>L3D.</td>
<td>-0.019**</td>
<td>-0.021***</td>
<td>-0.021***</td>
</tr>
<tr>
<td>L4D.</td>
<td>-0.008</td>
<td>-0.010</td>
<td>-0.010</td>
</tr>
<tr>
<td>L5D.</td>
<td>-0.018**</td>
<td>-0.020***</td>
<td>-0.020***</td>
</tr>
<tr>
<td>LD.USLIBOIS</td>
<td>0.000</td>
<td>0.000*</td>
<td>0.000*</td>
</tr>
<tr>
<td>L2D.</td>
<td>0.000***</td>
<td>0.000***</td>
<td>0.000***</td>
</tr>
<tr>
<td>L3D.</td>
<td>-0.000*</td>
<td>0.000*</td>
<td>0.000*</td>
</tr>
<tr>
<td>L4D.</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>L5D.</td>
<td>-0.000*</td>
<td>-0.000*</td>
<td>-0.000*</td>
</tr>
<tr>
<td>LD.EUSWEC</td>
<td>-0.017</td>
<td>-0.014</td>
<td>-0.014</td>
</tr>
<tr>
<td>L2D.</td>
<td>0.095***</td>
<td>0.090***</td>
<td>0.099***</td>
</tr>
<tr>
<td>L3D.</td>
<td>-0.078***</td>
<td>-0.073***</td>
<td>-0.073***</td>
</tr>
<tr>
<td>L4D.</td>
<td>0.060**</td>
<td>0.066**</td>
<td>0.066**</td>
</tr>
<tr>
<td>L5D.</td>
<td>0.067***</td>
<td>0.073***</td>
<td>0.073***</td>
</tr>
<tr>
<td>const</td>
<td>-0.001</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* p < 0.05, ** p < 0.01, *** p < 0.001
crisis (see Figure 4.6). The developed economies continued in recession, but that was not necessarily due to the financial crisis but due to the Eurozone crisis which followed.

The stability of the estimates are checked and it is revealed that all eigenvalues lie within the unit circle (see Figures 3.10 and 3.11). This proves that the number of cointegrating equations were successfully and correctly selected.

The obtained cointegrated relationships are used to generate forecast, and subsequently compare the variances of the impulse responses obtained from the VAR(4) with the dynamic error forecasts obtained from the VECM. Significant distinctions are revealed. Figure 3.9 shows that the error forecasts from the first differenced stationary VAR(4) model converge to zero with time, whereas the dynamic error forecasts in Figures 3.13 and 3.14 deviate towards infinity. These findings strongly indicate the presence of at least one structural break in the time series. It seems, that the noise term(s) in some of the equations have non-zero value(s). Even if structural changes occur at a single time $t$, these will have an everlasting affect on the behaviour of the time series. This phenomena is characteristic to unit root processes (Johansen, 1988).

The Gregory-Hansen test (1996) assesses whether the equilibrium relationships are disturbed by a one-time structural break or regime change. Considering that the aim of this study is to find the timing of a potential structural break, the coefficients of the three distinct cases are irrelevant and therefore not reported\(^\text{19}\). The number of lags used

\(^{19}\)The Johansen cointegration tests (1988) implemented earlier already identified the short-run and long-run equilibrium coefficients.
Figure 3.11: Stability check with eigen values for the case with no trend and no const.

Figure 3.12: Predicted cointegrating equation versus time for the restricted constant, no trend and no constant cases.
Figure 3.13: Dynamic 500 days forecasts of the levels for the restricted constant case.

Figure 3.14: Dynamic 500 days forecasts of the levels for the no trend no constant case.
are the same as in the Johansen cointegration tests, which are six lags. Three equations are considered:

A Structural break in the level;

\[ y_{GerUS3M} = \mu_1 + \mu_2 \phi_{t\tau} + \alpha^T y_{USLIBOIS} + \gamma^T y_{EUSWEC} + \epsilon_t, \quad (3.23) \]

where \( \alpha \) refers to the cointegrating slope coefficient for the LIBOR-OIS spread, and \( \gamma \) represents the cointegrating slope coefficient for the EUSWEC spread; \( \tau \) reveals the relative timing of a single structural break point.

B Level shift with trend;

\[ y_{GerUS3M} = \mu_1 + \mu_2 \phi_{t\tau} + \beta t + \alpha^T y_{USLIBOIS} + \gamma^T y_{EUSWEC} + \epsilon_t \quad (3.24) \]

where \( \alpha \) refers to the cointegrating slope coefficients for the LIBOR-OIS spread and \( \gamma \) represents the cointegrating slope coefficient for the EUSWEC spread. \( \beta \) denotes the shift in the slope vector.

C Regime shift, which actually is a shift in both the level and the mean of the covariates.

\[ y_{GerUS3M} = \mu_1 + \mu_2 \phi_{t\tau} + \alpha_1^T y_{USLIBOIS} + \alpha_2^T y_{USLIBOIS} \phi_{t\tau} + \gamma_1^T y_{EUSWEC} + \gamma_2^T y_{EUSWEC} \phi_{t\tau} + \epsilon_t, \quad (3.25) \]

The coefficients of interest are \( \alpha_1 \), which refer to the cointegrating slope coefficient for the LIBOR-OIS spread before the regime shift. \( \alpha_2 \) denotes the change in slope coefficients. \( \gamma_1 \) denotes the cointegrating slope coefficient for the EUSWEC spread before the regime shift, and \( \gamma_2 \) represents the change in the EUSWEC rate after the regime change.

The critical values are calculated to evaluate the outcomes. Table 3.16 reports the results. \( t \) spans from 1st January 2002 to 30th December 2011, and \( t = 1, 2, \ldots, 2609 \).

At 5% significance, the test for structural break in the level (A) confirms for all three test statistics (the ADF, the Phillips \( Z_\alpha \) and \( Z_t \)) test statistics) that there is a shift at time \( t = 719 \), \( t = 722 \) and respectively \( t = 722 \). Therefore, the null hypothesis of no cointegration is rejected in favour of cointegration with shift in the level. The dates

\[ Z_\alpha(\tau) = n(\hat{\rho}_\tau - 1) \quad \text{and} \quad Z_t(\tau) = (\hat{\rho}_\tau - 1)/\hat{s}_\tau, \]

where \( \hat{s}_\tau = \hat{\sigma}_\tau^2/\sum_{t=1}^{n-1} \hat{e}_t^2 \).

Following Gregory and Hansen (1996), the test statistic can be written as \( Z_\alpha(\tau) = n(\hat{\rho}_\tau - 1) \quad \text{and} \quad Z_t(\tau) = (\hat{\rho}_\tau - 1)/\hat{s}_\tau \), where \( \hat{s}_\tau = \hat{\sigma}_\tau^2/\sum_{t=1}^{n-1} \hat{e}_t^2 \).
correspond to 1st October 2004 and 6th October 2004. For the level shift with trend test (B), at 5% significance, only the ADF test identifies a regime change at time \( t = 1453 \), and this corresponds to 26th of July 2007. For the regime shift test (C), none of the three test statistics are significant at 5% significance level, therefore the null hypothesis of no cointegration cannot be rejected.

<table>
<thead>
<tr>
<th>Test Stat.</th>
<th>Break.</th>
<th>Date</th>
<th>1% crit. val.</th>
<th>5% crit. val.</th>
<th>10% crit. val.</th>
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<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
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<tr>
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<td>722</td>
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</tr>
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<td>722</td>
<td>-57.01</td>
<td>-46.98</td>
</tr>
<tr>
<td>(B)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>1453</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>1463</td>
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</tr>
<tr>
<td>(C)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ADF</td>
<td>-7.12</td>
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<td>1765</td>
<td>-6.45</td>
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<tr>
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</tr>
<tr>
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<td>1761</td>
<td>1761</td>
<td>-79.65</td>
<td>-68.43</td>
</tr>
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</table>

3.7 Conclusions, contributions and limitations

This chapter illustrated that benchmark spreads used in several previous analyses, which investigated the liquidity issue in the development of the financial crisis of 2007-08, are actually integrated of order one. This causes problems for statistical inference, particularly when standard errors are estimated with bias. The method of first-differencing the time series has introduced a unit root, therefore there is fundamentally a non-stationary process where innovations cumulate over time. If there is a unit root in the time series, there is also a stochastic trend. The results reveal that there is a long-term causality running in both directions between the GerUS3M, LIBOR-OIS and EUSWEC spreads. The Cholesky decomposition proved that, if a liquidity shock affects the short-term interbank market, the LIBOR-OIS spread is the leader, whereas the EUSWEC and GerUS3M spreads are followers in aligning back into equilibrium. Engle and Granger (1987) argued that in the long-run, some linear combination of non-stationary variables become stationary. Thus, cointegration provided a framework for estimating parameters of non covariance stationary processes. The evidence found in this chapter suggests that there are long-run cointegrating relationships among the GerUS3M, the LIBOR-OIS, and EUSWEC spreads. Independently, the non-stationary spreads have no predilection to return to a deterministic path, however the spreads together form a stationary relationship and follow an equilibrium path. However, these relationships break down due
to structural breaks. Thus, the implications of structural breaks for stationarity are significant. Nelson and Plosser (1982) argues that non-stationary spreads are affected by permanent effects originating from random shocks (structural breaks), and therefore these follow a random walk. In this particular case, the magnitude of shocks translating into structural breaks is large and infrequent; for example, such was the shock perceived in the short-term interbank market on the 26th of July 2007 as identified by the Gregory-Hansen level shift with trend test (1996). Consequently, in Chapter 4 an autoregressive regime switching model is presented with the aim of exploring and detecting multiple structural changes or regime switches in the US LIBOR-OIS spread. To further assess the presence of structural changes, Chapter 5 presents various multivariate regime switching models. The Johansen (1988) test’s coefficients revealed the speed of adjustment back to equilibrium levels, whereas the Gregory-Hansen (1996) test precisely identified when structural breaks occurred. Nonetheless, in the long-run, the cointegrating relationships return to equilibrium. This is a significant finding in the interbank liquidity literature and, in terms of theoretical implications, the results support the Dynamic Stochastic General Equilibrium Theory which states that financial markets are disturbed by random shocks, however in the long-run the system is in equilibrium.

Previous analyses looked at spreads in terms of their components and never assessed short-term interbank spreads jointly over a longer period of time. Important relationships have been revealed among the three nonstationary variables. The LIBOR-OIS, the currency swap and the German-US bond spread move in a synchronised fashion and this ultimately has implications for policymakers and market players alike.

Furthermore, the above analysis got us closer to advanced understanding of the characteristics of the forecast errors variance and also to recognising interrelationships and dependencies among the short-term interbank spreads. However, the forces influencing each of the variables were not identified. These are going to be identified in Chapter 5 where an endogenous variable drives regime changes in the independent variable. In classical cointegration models, the integration order is rigid, either I(1) or I(0), yet real economic and finance events described by time series, which exhibit persistent exogenous shocks, could be also investigated with the use of a fractional cointegration method to model long-run equilibriums. In most cases, the presence of fractionally integrated errors can be the reason for rejecting cointegration in conventional methods, such as the Dickey-Fuller or KPSS methods. Therefore, it would be interesting to address these in future research. Short-term interest rate models assume mean-reverting processes and the long-run mean and speed of adjustment is constant throughout the considered sample period. These are the so-called single regime models (Gray, 1996). Thus, the identified structural relationship between the LIBOR-OIS, German-US bond and ESUWEC spread can only be preserved by implementing a non-linear Markov chain model.
Chapter 4

A Univariate Two-regime Switching Model to Detect Crises in the Short-term Interbank Market

4.1 Introduction

Interest rate times series are known to vary persistently in times of uncertainty. It is critical to detect liquidity shocks before they crash the financial system. Moreover, it is imperative that early warning systems (EWS) with forecasting attributes are developed, specifically in the light of the recent financial crisis (and the Eurozone crisis that followed) which affected the majority of developed economies. Such EWSs detect liquidity crashes well before they develop into crises and ultimately spread to neighbouring markets via interbank channels. These two prolonged and financially devastating events are linked, and it is assumed that the Eurozone crisis was the direct result of the credit crunch. If appropriate financial measures and tools were in place to detect crises and forecast subsequent ones, the developed economies would not struggle to recover from the present recession which is still crippling Eurozone countries and the US economy alike.

In this chapter, a new regime switching model is proposed which provides the probability of being in a liquidity crisis state at any given instance. The model is assessed on the daily LIBOR-OIS spread during a period of 10 years. By using thresholds, crisis and tranquil periods are established, which subsequently model the baseline distribution.
Bayesian inference\textsuperscript{1} differs from statistics in the sense that all unknown parameters are treated as random variables, whose prior distribution which describes the data is outlined from outset. More accurately, the priors provide all available information about the data, such that it summarises the researcher’s knowledge of the uncertain parameters ($\theta$ for example) before any data is taken into consideration. Consequently, estimated parameters incorporate uncertainty defined in terms of probability distributions. Prior distributions are grounded on either expert knowledge about the data and/or subjective perceptions (in case of uninformative probability distributions, for example). As new data becomes available, the predicting mechanism updates the system with the information that becomes available. Matching prior distributions with the time series produces not a fixed value, such as an expected mean for example in standard statistical or econometric tests, but a matrix of posterior distributions for all the parameters. This provides an accurate and superior estimation of crisis and non-crisis periods.

This chapter is organised as follows. Section 4.2 surveys the literature, while Section 4.3 presents the research questions. The data used in this analysis is discussed in Section 4.4. In Section 4.5 the specification for the two-state regime-switching model is presented. Section 4.6 contains the empirical analysis with the results and out-of-sample forecasted estimates. Section 4.8 presents the conclusions, limitations of this study and future research.

4.2 Literature Review

The section below discusses regime change and Markov chain models and their application in the financial crisis and contagion literature. Regime switching regression models were first pioneered by Quandt (1958). Later, Goldfeld and Quandt (1973) introduced the Markov switching model, a fundamentally non-linear model where the unobserved state variable, which controls the regime changes, follows a Markov chain\textsuperscript{2}. The Markov chain is a memoryless stochastic process and it has a mathematical structure which progresses from one state to another. It can be discrete or continuous, and can evolve through a finite or a fixed number of states. The state space\textsuperscript{3}, or as more often called the transition matrix, defines all possible probabilities of individual transitions including the

\textsuperscript{1}The fundamental difference between Bayesian and Frequentist approach is that the former exploits all available data and information, and conveys uncertainty based on priors, previous evidence as a probability distribution, and in such way explains the event under investigation. The latter sets hypotheses and may reject the null hypothesis based on an arbitrarily set p-value; however, the Frequentist approach does not explain the alternative hypothesis.

\textsuperscript{2}The history of the process is not relevant in a Markov chain, thus the chance of switching from one regime to another is solely determined by the present regime (Goldfeld and Quandt, 1973).

\textsuperscript{3}State spaces are frequently used to model time series, however they were first used in engineering by Kalman (1960). The system is made up by a time changing signal defined as the state which is unobserved and successive and a series of observations. As a whole, the state space describes the relationship between the states and observations, called the ‘measurement equation’. The ‘transition equation’ defines the dynamics of the system in the form of a first order difference equation. The most frequently adopted theoretical state space models are the linear dynamic systems, which robustly infer
Chapter 4 A Univariate Two-regime Switching Model to Detect Crises in the
Short-term Interbank Market

initial state through the state space (Hamilton, 1994a). It is assumed that the process
contains all possible states and transitions, furthermore the process is characterised by
a never-ending change from one state to another (Carter and Kohn, 1994). Originally,
regime switching models were built around some assumptions; one of them is that the
unobserved variable is deduced from mathematical models using variables which can be
observed.

An important concept in regime switching Markov models is the concept of uncertainty.
It is crucial at this stage to define the notion of probability, then to explain uncertainty.
Probability is a measure of uncertainty, based on subjective or objective interpretation
of a random event under study. The objective interpretation is mainly based on the
statistical measure of uncertainty, whereas the subjective measure is based on beliefs
about the observed event (Bijak, 2010). On the other hand, the notion of uncertainty,
as described by Bijak (2010, p. 61):

refers to the indeterminism or randomness of the phenomena under study,
which cannot be assessed or predicted using the present knowledge. There
is no general agreement in science as to whether uncertainty is an inherent
feature of the phenomena, or is merely a result of imperfect knowledge of
the deterministic rules that govern the world, as supported by, for example,
A. Einstein.

It has been widely established in the financial crises literature that in some cases the
Dickey-Fuller test (1979) is unreliable in detecting regime switches in unit root processes
(Nelson et al., 2001). There is a vast literature generated by Engle (1994) assessing
the power of Augmented Dickey-Fuller (ADF) test within a Markov volatility regime
changing structure. Kanas and Genius (2005) looked at the stationarity characteristic
of the US/UK exchange rates by implementing an extension of the ADF test, and
found that the autoregressive parameters vary in line with volatility regime changes.
Nikolsko-Rzhevskyy and Prodan (2012) improved Engle’s (1994) model by segmenting
non-stationary time series into a series of stochastic time trends. By analysing the out-
of-sample forecasting power of the Markov switching random walk model with drift, the
authors found that exchange rates are predictable in the long-run, however the fore-
casting power of the model decreases as the time period increases. Some (Perron and
Vogelsang, 1992; Dropsy, 1996; Siddique and Sweeney, 1998, among others) developed
models that allow a one-time change in the mean, arguing that, while accepting struc-
tural instability in time series, unit roots or near unit roots can be rejected. Contrarily,
Bergman and Hansson (2005) found that their stationary two-state AR(1) (autoregres-
sive of order 1) Markov chain model supports the fact that real exchange rates do have

the parameters of the model. However, real-life applications are best described by non-linear state space
models (Maddala and Kim, 1998).
unit roots. However, in the case of processes with near unit roots, the detection of regime changes becomes difficult (Granger and Swanson, 1997).

Hamilton (1989) developed a maximum likelihood (ML) based non-linear iterative filter\textsuperscript{4} and smoother\textsuperscript{5}, which helps determine the regimes with confidence. Therefore, the regimes are discovered by the interaction between the series and the Markov chain. The author argues that changes in regimes, which are actually AR(1) processes, are influenced by external factors and not by the series that are modelled. Furthermore, the author notes that structural break models (such as stochastic volatility models) are plagued by statistical biases, which produce misleading forecasts, among others. On the other hand, the parameters of the Markov regime-switching model vary randomly across regimes, and based on the examination of the behaviour of the series, one can draw optimal probabilistic inference on when a regime change occurred, and at the same time determine its persistence. Moreover, the parameters along with the variance of innovations are considered to be serially correlated. In fact, the AR(1) process tracks a non-linear stationary process rather than a linear stationary process. The parameters describe the dynamics of the states, which are serially dependent, and their detection is subject to non-linearities in the data. However, the ML platform was proven inefficient when dealing with a large number of parameters.

To improve the approximation procedure, a year later Hamilton (1990) introduced the Expectation Maximization (EM) algorithm with the aim of obtaining estimates for parameters that are subject to discrete shifts. The rationale behind the EM algorithm was to produce a computationally superior ML function for estimating large numbers of parameters, even when the starting values of the sampler are inadequately selected. The model improved the optimisation process by resulting in a faster convergence of the Markov chain Monte Carlo approximation. Nonetheless, Hamilton’s later models (1990; 1994) were in a sense restricted, for they did not allow time-varying transition probabilities. Filardo (1994) assesses business cycles and their relationship to regimes, by allowing time-varying probabilities in his models. Similarly, Hamilton and Susmel (1994) further improved on Hamilton’s (1989) model by allowing unexpected discrete moves in the values of ARCH parameters when applied to low-, moderate- and high-volatility regimes (where the high-volatility regime corresponds to a recession). Using the same dataset as Hamilton and Susmel (1994), Henneke et al. (2011) extend the model with the inclusion of ARMA-GARCH state dependent parameters computed on a Bayesian platform. The major difference from Hamilton and Susmel’s (1994) model is that Henneke et al. (2011) model the conditional variance as an amplified GARCH process, and the mean is specified as an Autoregressive Moving Average (ARMA) process.

\textsuperscript{4}Filtering is a sampling-based particle method which produces a set of samples that estimates the filtering distribution. Filters are estimations of time dependent $\beta$s grounded on information available up to time $t$ (Maddala and Kim, 1998).

\textsuperscript{5}As opposed to a filter, the smoother estimates the $\beta$s based on all information throughout the whole length of time (Maddala and Kim, 1998).
Contrarily to Hamilton and Susmel (1994), McCulloch and Tsay (1994) argue that the dynamics of two states (recession and expansion) differ significantly, and the difference between the two states is represented not only by a shift, but by a more complex process. The authors test the assumption that the process has a constant dynamic structure throughout the two states, and subsequently develop a model where each time period is assigned a prior probability that allows the parameters to vary from state to state, and examine whether the posterior probabilities of parameters are indeed well assumed by the corresponding prior probabilities (this is the so called sensitivity test).

To overcome Hamilton’s (1989) problem caused by a computationally demanding ML estimation, and using Bayesian inference to accurately generate the posterior joint density of the parameters, Harris (1999) extends the Markov switching model to Vector Autoregressive processes. These processes consist of individual discrete vector processes\textsuperscript{6} which are governed by different dynamics, and in such a way identify different parameter sets. If the dynamics of the vector processes change due to regime shifts, parameters will also be subject to shifts.

Hardy (2001) argues that independent log normal models offer realistic estimates over short-time time periods, however these models are not reliable when used on long time intervals. Therefore, such models fall short in capturing excessive price developments and stochastic fluctuation in the volatility parameter. The author includes stochastic volatility in his model, taking discrete values (for example regimes) and switching arbitrarily between those. Every regime is described by distinctive model parameters, and the process governing regime changes is assumed to be Markovian.

In order to find out regime changes in series that have both stationary and non-stationary sections, Fukuda (2005) divides the series into segments, and fits a stationary or a non-stationary autoregressive model to each segment correspondingly. To assess the goodness of fit of the global model which consisted of several local models (either two or triple regimes), as well as to reveal for each segment whether it prefers a stationary or a non-stationary model, simulations along with the Akaike Information criteria (AIC) and Bayesian Information criteria (BIC) are used. The model however is not assessed on real datasets.

So far in the field of economics, the regime switching literature primarily originated from Hamilton’s (1990) model. However, in the last 20 years, regime switching Markov chain applications became more popular particularly in the biomedical, geology and hydrology fields. These models follow a transparent and easily replicable structure. Following the previous literature which considered changes in regimes caused by external factors (exogenous regime changes), and while analysing river flow dynamics, Vasas et al. (2007) identify two discrete regimes: an ascending one representing shorter positive

\textsuperscript{6}In this setup, the vector processes/ regimes are assumed to be linear stationary, however as a result of discrete regime switches, the overall process becomes nonlinear stationary.
shocks to the system analogous to an independent gamma distributed process\(^7\), and a
descending regime equivalent to a Gaussian autoregression. The authors use the Gibbs
sampler\(^8\) from the posterior distribution to obtain estimates of the vector parameters.
The structural vector parameter obtained from the conditionals in each iteration of the
algorithm is being updated as the system dynamics dictates throughout the process.
The unobserved regime process is allowed to be by chance non-Markovian. However,
their model is not assessed in a multivariate setting. Ang and Bekaert (2002) use a
multivariate regime switching model to reveal the non-linearities found in interest rate
series and term spreads. Their focus is on the conditional variance and the difficulty of
modelling it in a regime switching setup. The authors argue that if there is a Granger
causality among the variables, it would be determined by the regime and render the
estimation of the interest rate innovations more reliable, as these innovations would
be correlated throughout the markets. To simplify their model, the authors disregard
state dependent probabilities and therefore the transition from one state to another is
constant. Due to the fact that the model is fitted with the unconditional moments of
the data, for these to converge, a huge number of simulations are needed, as well as to
make the estimates reliable. There are further shortcomings to their model; it does not
allow non-stationary processes; that is, their model is regime switching, but revealed
as a change in the autocorrelation structure only. Thus, the model does not explain
whether or not there is a random walk in the series.

Regime changes can also be modelled as a continuous behaviour of the independent
variables, and also by permitting direct interaction among all variables. These models
allow a deeper examination of the fundamentals of regimes in general. Originally, smooth
transition models were introduced by Teräsvirta and Anderson (1992), Teräsvirta (1994)
accommodating the time-varying volatility effects and the non-linearities of time series.
The authors argue that regimes and non-linearities drive the behaviour of time series
and not the ARCH effects.

Extending the volatility feedback model of Turner et al. (1989) and the independent
switching model of Maddala and Nelson (1975), Kim et al. (2008) use fewer assump-
tions and develop a Markov regime-switching model. The underlying assumption is that
the unobserved variable that drives regime changes is endogenous. The Gaussian model

\(^7\)The Gamma distribution is a stochastic process with positive shape and scale parameters, widely
used in Bayesian inference to detect waiting time events, for example.

\(^8\)The Gibbs sampler was introduced by Geman and Geman (1984); it is an updating process of an
iteration with values that have been drawn from a system. The sampler was designed for situations
when the set of full conditionals needs to be specified, for example in complex stochastic processes with
large number of variables. Implicitly, the individual full conditional distribution is governed solely by
some “neighborhood” subset of variables.
of endogenous Markov regime switching is supported by a probit\(^9\) condition for the realisation of the unobserved variable. Monte Carlo simulations are then run with different values of the endogenous estimators in order to result in accurate parameter estimates.

### 4.2.1 Conclusions and contributions to existing literature

The present investigation contributes to the empirical literature in several ways.

1. The literature investigating regime changes in financial markets is very limited. Nor there is a theory which discusses financial crises. Therefore, the present study fills in both the theoretical and empirical gaps by presenting a two-state regime-switching Markov chain model which is able to identify several crises in the money market spread;

2. The literature does not discuss models which are able to forecast financial crises ex-post;

3. The literature investigating financial crises looks at short time intervals, whereas the present study covers the period January 2002 to December 2011 in order to reveal several crises at the same time assessing their dynamics;

4. The novel regime switching Markov chain models presented in this chapter identify liquidity crashes with different thresholds. To my knowledge, there is no such study found in the financial crisis or interbank literature.

### 4.3 Research questions

The focus of this chapter is liquidity crash detection in the short-term interbank market and the analysis aims to answer the following research questions:

1. Can liquidity crises and the length of crisis and non-crisis periods be traced in the dynamics of the short-term interbank spread?

2. What are the empirical implications of estimating liquidity crises with different thresholds in the money market?

3. Are financial crises predictable by exploring the ex-post dynamics of the short-term interbank spread?

\(^9\)There is a vast literature which estimates the values of binary crisis indicators using a probit model in the field of Economics, particularly assessing recession and expansion periods within the business cycle, with the growth rate of GDP or term spread as the dependent variable (see Hamilton (1989), for example).
4.4 Data

The data set used in the present chapter is the daily LIBOR-OIS spread over a period of 10 years spanning from 1st January 2002 to 30th December 2011. A full description of the LIBOR-OIS spread and the rationale for investigating the time series can be found in Section 3.4.1.

4.5 Methodology

The model builds on an approach implemented by Martínez-Beneito et al. (2008) who developed a two-state regime-switching Markov chain model, which detects influenza epidemics at the moment of their onset. The model explores the probability of a financial liquidity crisis developing in the short-term interbank money market. The structure of the model allows for discrete shifts in the mean and for autoregressive dynamics to better illustrate possible structural breaks. In this way, the model is representative of a real set-up which describes economic and financial time series in turbulent times.

As opposed to Martínez-Beneito et al. (2008) who use first differences of the series, the analysis presented in this chapter uses the raw data and therefore it maintains the unique characteristic of the short-term interbank spread. The theoretical background for this investigation is provided by Kydland and Prescott (1982) who use the Business Cycle theory to assess the effect of real prices and policy changes on exogenous shocks (see Section 2.3.1).

4.5.1 Model specification for the univariate Markov regime switching model

Constructing a regime switching model allowing time-varying probabilities, results in a parsimonious model with very few assumptions. The structure is characterised by a hidden two-state Markov chain of order 1, meaning that the future state (or regime) will only depend on the present state. Hidden Markov models were pioneered by MacDonald and Zucchini (1997). Essentially, the data is modelled as partly observed, thus the states are latent and only the final output is observable, which is conditional on the state variable.

For simplicity, consider the following process with a deterministic transition, in which the intercept is changing dynamics in reference to a binary indicator variable \( I_t = 1, \ldots, K \).

The notation follows that of Perlin (2012).

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10MacDonald and Zucchini (1997) divided the time series into epidemic and non-epidemic periods. This had the benefit that the time series did not have to be segmented into periods; moreover, the model could be applied to any historical data. In simple Markov chains the only coefficients of the model are the transition coefficients.
Chapter 4: A Univariate Two-regime Switching Model to Detect Crises in the Short-term Interbank Market

Daily US LIBOR-OIS time series data

Prior distributions based on statistical tests and objective beliefs

Markov chain Monte Carlo simulations

Posterior parameter estimates to detect crisis and non-crisis states

Figure 4.1: The process of estimating crisis and non-crisis states and posterior parameters.
\[ Y_t = \mu_{I_t} + \epsilon_t \]  \hspace{1cm} (4.1)

where \( I_t \sim N(0, \sigma^2_{I_t}) \). In this scenario, the output will consist of \( K \) values of \( \mu \) and \( \sigma^2 \).

Let’s assume that there are two states, such that \( I_t \in [0, 1] \). The two states can be described by the two equations, as follows:

\[ Y_t = \mu_1 + \epsilon_t \]  \hspace{1cm} (4.2)

and

\[ Y_t = \mu_2 + \epsilon_t \]  \hspace{1cm} (4.3)

The volatility of the error terms may be described by the two processes such as in Equation 4.8 and 4.9. The innovations describe the level of uncertainty (in terms of volatility) concerning the two states of the space. Depending on the values of \( \mu_1 \) and \( \mu_2 \), empirically the two processes describe the expected value of \( Y_t \) in times of tranquility and financial distress, for example. It is assumed that state 1 is true when observations in an exogenous time series \( Z_t \) cross some threshold, or otherwise. This reduces the system to a regression governed by a dummy variable, such that:

\[ Y_t = D_t(\mu_1 + \epsilon_{1,t}) + (1 - D_t) (\mu_2 + \epsilon_{2,t}) \]  \hspace{1cm} (4.4)

In the exogenous Markov regime switching model presented below the transition governing the regime switch from a non-crisis to a crisis state is stochastic, however presumed to be constant over time. The difference between constant and continuous transition is the elements of the transition matrix; the first one has transition probabilities, whereas the second has transition intensity elements (MacDonald and Zucchini, 1997). The main objective is to determine when breaks occurred in the time series, and subsequently approximate posterior parameters that describe the two states, while discovering the probability law for the evolution between the two states.

The following equation describes the dependent variable (which is the LIBOR-OIS spread), and also summarizes the assumed data generating process. The dependent variable \( Y_t \) corresponds to a stochastic process, however it is known that the process also contains an autoregressive component (as proved by the VAR model earlier in Chapter 4), and therefore the model can be written as:
Chapter 4 A Univariate Two-regime Switching Model to Detect Crises in the Short-term Interbank Market

\[ Y_t = \rho * Y_{t-1} I_t + \epsilon_t \]  

(4.5)

where \( t = 1, 2, \ldots, T \) represents the number of observations in the series, in this case 2609, and \( \rho \) is the autoregressive coefficient. Furthermore, it is assumed that:

\[ Y_t \sim N(\mu_t, \tau_t) \]  

(4.6)

where \( \tau \) is the precision parameter and corresponds to \( \tau = 1/\sigma^2 \). Moreover, \( \tau \) is both time and state dependent.

The \( I_t \) is the binary (dummy) indicator which can take the following values:

\[ I_t = \begin{cases} 
1, & \text{if there is crisis in the short term money market;} \\
0, & \text{if there is no crisis in the short term money market.} 
\end{cases} \]  

(4.7)

Furthermore, it is also assumed that the LIBOR-OIS spread is directly observable and its dynamics change in line with the value of the crisis variable, that is with \( I_t \). Fundamentally, the change from one regime to another will be a function of the LIBOR-OIS spread and its lag, as indicators. The state variable \( (I_t) \) is latent and is supposed to progress corresponding to a first-order Markov chain with unknown transition probabilities which are controlled by a set of covariance-stationary exogenous variables\(^{11}\) (Gelman et al., 2013).

There are two errors in the system of equations, one corresponding to the crisis and the other to the non-crisis equation. The error terms must satisfy the following:

\[ \epsilon_{0t} \sim iid \ N(0, \sigma^2_{0,t}) \]  

(4.8)

\[ \epsilon_{1t} \sim iid \ N(0, \sigma^2_{1,t} - \sigma^2_{0,t}) \]  

(4.9)

with

\(^{11}\)The exogenous variable can take the form of an external shock in the like of a new macroeconomic policy rule, for example.
\[ \sigma^2_{1,t} - \sigma^2_{0,t} > 0 \] (4.10)

This last assumption is owing to the sheer fact of random variations in the data (Martínez-Beneito et al., 2008), and not to the effect of a crisis, for example. Also, it is assumed that the two error terms are not correlated. At the moment, the model is concerned with regime changes in the mean, however in the case of a multinomial state space representation for example, the variance of the error terms can be exposed to regime switches, such that \( \sigma^2_{S_t} = \sigma^2_k \) if \( S_t = k \). Alternatively, the variance may be governed by a variable which is not dependent on the state variable and drives the vector of coefficients; in this case the transition process is endogenous \( \Theta \) (Kaufmann, 2011).

The model makes posterior inference about the ‘true’ crisis time, \( \theta \). Theta is paired with a set of explanatory variables \( X_1, X_2, \ldots X_T \) (in this case \( \mu, \tau, \lambda_1, \lambda_2 \) and \( I \)) with the aim of stating the link function and the ultimate structure of the model (Kim and Nelson, 1999). The model estimates the posterior parameter values that enclose both priors for all the parameters of the model and observed data information as shown in Figure 4.1.

Now assume a normally distributed stochastic process with \( \Theta \) being the set of parameters to be inferred, as follows:

\[ Y \sim N(\Theta) \] (4.11)

and

\[ \Theta = h(\theta, X^1, X^2, \ldots, X^T) \] (4.12)

where \( X^1, X^2, \ldots, X^T \) are iterations of the coefficients to be inferred.

Let \( \theta \) be the event that the times series of interest is in a state of crisis. Values of \( \theta \) fall between 0 and 1. The Likelihood Function\(^{12}\) comprises all the information fed by the sample and is determined by the equation:

\(^{12}\)The concept of likelihood was first formulated by Thomas Bayes and Pierre-Simon Laplace in the 18th century, however it was introduced by Fisher in 1921, one of the most prominent frequentists of the 20th century.
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\[ p(Y|\theta) = \theta^Y (1-\theta)^{n-Y} \]  
\[ (4.13) \]

with \( Y = 0, 1, \ldots, n \). The Evidence is determined by:

\[ p(Y) = \int p(Y|\theta) \, p(\theta|Y) \, d\theta \]  
\[ (4.14) \]

Therefore, the posterior distribution can be written as follows:

\[ p(\theta|Y) = \frac{p(Y|\theta) \, p(\theta)}{p(Y)} \]  
\[ (4.15) \]

In other words, it now follows from Bayes’ theorem that:

\[ p(\text{crisis}|\text{data}) = \frac{p(\text{data}|\text{crisis}) \, p(\text{crisis})}{p(\text{data})} \]  
\[ (4.16) \]

As explained earlier, the system consists of the state space, the combined parameter vector, the regime matrix describing the probabilities of individual transitions and an initial state. The aim is to infer the posterior distribution given the LIBOR-OIS time series spread. It is assumed that the initial state of the system is a non-crisis one. As \( t \to \infty \), the unconditional distribution of \( X_{t+1} \) converges to an exclusive stationary distribution. The marginal distributions, parameters and two regimes (crisis and normal times) are estimated using the Gibbs sampler or algorithm, which is fundamentally Markovian (randomised); it is a system where iterations are being run and are continuously revised. The algorithm is behind the Markov Chain Monte Carlo method (MCMC) and generates random samples from the joint posterior distribution. Thus, the MCMC method finds a sample based representation of the filtering distribution (Kim and Nelson, 1999). The rationale of adopting the Gibbs sampling is the appealing fact that convergence is achieved accurately and rather fast, regardless of the dimension of the coefficient vector which is estimated, as opposed to the particle filter (or sequential Monte Carlo filter, for example) which works well in non-linear settings and does not require any assumptions (Gelman et al., 2013). Moreover, the latter does not prove efficient when the vector of parameters to be estimated becomes bigger, such as in a complex multilevel hierarchical system, for example. Following the specification of an arbitrary set of starting values - which is not compulsory - and running a fairly large number of iterations, marginal distributions are estimated by convergence to the true
joint density value. All distributions depend on the observed data and the marginal distributions transform into marginal posteriors used for Bayesian estimation (Gelfand and Smith, 1990; Gelfand et al., 1990). Essentially, the ultimate aim in Bayesian statistics is not to condense results into a joint posterior distribution, but to review the marginal distribution for every single coefficient/parameter of interest.

Following the notation of Vasas et al. (2007), the transition probabilities (the probabilities related to various state changes) used in this analysis are determined by:

\[ P_{k,l} = P(I_{t+1} = l | I_t = k) \]  

where \( k, l \in \{0, 1\}, t = 0, 1, \ldots, T \). The equation tells us that the probability that the state variable \( I \), for example, is in crisis in period \( t + 1 \) given that \( I \) was equal to \( k \) in the previous period. Consequently, the model has four transition probabilities, which can be written in a matrix form, as follows:

\[ P = \begin{pmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{pmatrix} \]

The conditional probability distribution of the data given crisis and non-crisis intervals, that is, the probability distribution of the data when crisis is known to take the value of 0 or 1, are defined as follows:

\[ Y_t | I_t = 0 \sim N(0, \sigma^2_{0,t}) \]  

\[ Y_t | I_t = 1 \sim N(\rho Y_{t-1}, \sigma^2_{1,t}) \]

The full conditional distribution is the distribution of the coefficients dependent on the known information and all the other coefficients (Lunn et al., 2013). In the non-crisis state (\( I = 0 \)) the dependent variable follows a standard normal distribution with constant variance, whereas in the crisis state (\( I = 1 \)) the variance changes persistently and increases with time. Furthermore, it is assumed that in crisis time, the dependent variable follows an autoregressive process of order 1 with first-order autoregressive coefficient \( \rho \).
4.5.2 Priors, likelihood estimation and Gibbs sampling

The model is a typical hierarchical model (see Figure 4.2). The parameters of the vector $\Theta$ follow prior distributions, therefore after obtaining posterior distributions, one is able to explain correlations among the coefficients. Prior probability distributions are set with the aim to handle uncertainties attached to predictions of crisis and non-crisis states. More precisely, prior probabilities quantify the uncertainty of a particular variable before any evidence is taken into consideration (Gelman, 2006).

The prior distribution should contain all likely values of parameters which are to be estimated Kim and Nelson (1999). There are two major classes of priors: informative and noninformative\footnote{Among others, the weakly informative priors, flat priors, Jeffreys priors and reference priors are classified into this category.} priors. The former takes previous information (for example expert knowledge, data and results) into account when updating the model, and is consequently based on subjective knowledge. The latter is characterised by a flat, vague and/or dispersed density, so statistical inference is not influenced by information which is external to the data being analysed (Gelfand et al., 1990). Moreover, the distribution is not expected to be focused around the real or ‘true’ value, considering that information about the latent parameters comprised in the data will outshadow any rational prior probability specification (Gelman et al., 2013). Determining the correct prior distributions for the parameters of the model is crucial, for ultimately it influences the posterior inference (more precisely the posterior distribution). Among other factors, mathematical convenience, lack of or abundance of information, and sample size must be taken into consideration while choosing the prior for the parameters of interest (Gelman, 2006; Lunn et al., 2013).

Fundamentally, the prior summarises the knowledge on the latent state variable before any data is observed (Kim and Nelson, 1999). Moreover, the prior mean offers a prior point approximation for the parameter of concern, whereas the variance (with its two hierarchical components $\lambda_1$ and $\lambda_2$) describes the ambiguity regarding this approximation (Ntzoufras, 2009). To discriminate between the parameters of the model (the vector $\Theta$ which comprises all the parameters) and the parameters of the priors, the latter are called hyperparameters (Li and Tobias, 2011).

The latent state variable $\theta$ has categorical distribution, which is the generalisation of the Bernoulli distribution, and can take two values: 1 if there is a crisis state and 0 if there isn’t.

$$\theta \sim \text{Cat}(P)$$ (4.21)
Figure 4.2: Hierarchical structure of the two-state Markov regime switching model.
A variable that can be described by the uniform distribution, as part of the continuous probability distribution, fundamentally can take any value in a specific interval. The uniform (or noninformative or symmetric) distribution\(^\text{14}\) of the latent variable \(\rho\) (which represents the autocorrelation coefficient), for example, is found within some boundaries as \(a \rightarrow -\infty, b \rightarrow \infty\), and its notation is as follows (Gelman et al., 2013):

\[
\rho \sim U(a, b) \tag{4.22}
\]

where \(a\) and \(b\) are its minimum and maximum values respectively, and the condition \(b > a\) must be satisfied. The probability density function is determined by:

\[
p(\rho) = \frac{1}{b-a}, \text{ for } a \leq \rho \leq b, \rho \in [a, b] \tag{4.23}
\]

The transition probabilities \(P_{11}\) and \(P_{00}\) are assigned the Beta distribution with values in the range of \([0, 1]\).

\[
P_{11} \sim \text{Beta}(0.5, 0.5) \tag{4.24}
\]

\[
P_{00} \sim \text{Beta}(0.5, 0.5) \tag{4.25}
\]

The density function is given by:

\[
P(\theta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \Gamma(\beta)} \theta^{\alpha-1} (1 - \theta)^{\beta-1} \tag{4.27}
\]

To the remaining parameters of the model a uniform prior distribution is attributed over a range of values determined by thresholds, as follows:

\(^{14}\)The expected mean and variance of the latent variable \(\theta\) within the uniform distribution is given by \(E(\theta) = \frac{\alpha + \beta}{2}\) and \(\text{var}(\theta) = \frac{(\beta - \alpha)^2}{12}\) (Gelman et al., 2013). In the model presented in this chapter, \(\alpha\) corresponds to \(a\) and \(\beta\) corresponds to \(b\).
$\rho \sim U(0, 1.5)$

$\lambda_1 \sim U(\theta_{\text{inf}}, \theta_{\text{med}1})$  \hspace{1cm} (4.29)

$\lambda_2 \sim U(\theta_{\text{med}2}, \theta_{\text{sup}})$  \hspace{1cm} (4.30)

The prior distributions for the four thresholds are defined as follows:

$\theta_{\text{low}} \sim U(a, b)$  \hspace{1cm} (4.32)

$\theta_{\text{med}1} \sim U(\theta_{\text{low}}, b)$  \hspace{1cm} (4.33)

$\theta_{\text{med}2} \sim U(\theta_{\text{med}1}, b)$  \hspace{1cm} (4.34)

$\theta_{\text{sup}} \sim U(\theta_{\text{med}2}, b)$  \hspace{1cm} (4.35)

For example, the threshold parameter $\theta_{\text{low}}$ has a prior which is uniformly distributed with the lower and upper limit $a$ and $b$ as the hyperparameters. The uniform prior, also called an uninformative prior, is characterised by the smallest amount of subjectivity. This translates into less bias being introduced into the inference. It seems to be appealing to chose the uniform prior distribution for the above parameters for the distribution expresses uncertainty exclusively in terms of the observable values of $Y_t$ and $t$. Laplace reasoned that a less informative prior is fitting if not much is known about the $\theta$ and called it the ‘principle of insufficient reason’ (Gelman et al., 2013).

Considering the crisis and non-crisis problem, the values of the hyper-parameters $a$ and $b$ must be determined. $a$ can be set to the lowest value of all observations (the minimum and maximum values for the LIBOR-OIS series is 1.93 and 364.43, correspondingly), however for simplicity it is set to 0, while the value of $b$ is set to 370. The random variables are thus equally likely to take any values between $a$ and $b$. The probability of being in crisis or tranquil state is 50%.

The parameters of the model can be estimated in two ways. One can use the ML estimation or sampling based on a Bayesian procedure. Broadly speaking, inference is accomplished via filtering and prediction. Similarly, there are several avenues one can pursue when it comes to filtering. If the dynamic system is linear, one efficient filter is the Kalman filter which has a functional form, whereas the extended Kalman filter can be applied to non-linear systems as well. The Bayesian optimal filter works in a Bayesian setting, while the sequential Monte Carlo filter does not require assumptions, it does not work well in high-dimmensional or hierarchial systems (Gelman et al., 2013).
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The Gibbs sampling is a very popular and efficient way to generate Markov chains; it is going to be used in this study. The aim behind the MCMC algorithm is to draw probabilistic inference of the posterior mean and variance of coefficients. The algorithm begins with arbitrary initial values (for the elements of the parameter vector Θ) set by the system itself. Another option would be to set the initial values of the algorithm, however in this way more subjectivity is introduced into the sampling procedure.

The Gibbs sampler which was proposed by Geman and Geman (1984), generates and predicts an array of or forecasts from the joint density which then classifies the data into crisis and non-crisis periods. It is imperative that all prior probabilities are selected wisely and are proper, that is they integrate to a finite number, preferably to unity (Li and Tobias, 2011). Based on the fact that the \( Y_t \) process is explosive due to varying dynamics of the data, the crisis state is modelled as an autoregressive process. In the model specification, the values of \( \rho \) are set to have a lower boundary of zero and upper boundary with the minimum value of 1.1 (there are going to be several model specifications). The non-crisis state is described by a Gaussian white noise process and the noise terms \( \epsilon_t \) can take values between 0 and 370 (these values are the minimum and maximum values in the actual dataset, as explained earlier). Similarly, the standard deviation of the random effects (the two \( \sigma s' \) in this case) follow a uniform distribution and provides with inference on the boundaries of the uniform distribution (Gelman, 2006). Thus, one can also assess the amount of deviation in both the crisis and tranquil periods.

Posterior distribution of the parameters is obtained by combining the prior distributions with the likelihood function (Kaufmann, 2011). The Gibbs sampler uses the conditional distributions to estimate the joint and marginal distributions. To obtain a sample from the posterior distribution, \( J \) number of iterations are run. Convergence is achieved fast as the number of iterations goes to infinity (assuming \( J \) is a fairly large number).

First, the state variable is sampled, such as \( P(\theta|Y) \), by implementing state-identifying restrictions (similar to Martínez-Beneito et al. (2008)) outlined from Equation 4.29 to 4.35:

1. One could define starting or initial values for the coefficients to be estimated;
2. Sample \( X_1^1 \) from \( f(X_1|X_1^0, \ldots, X_k^0) \);
3. Sample \( X_1^2 \) from \( f(X_2|X_1^1, X_1^0, \ldots, X_k^0) \);
4. Sample \( X_1^3 \) from \( f(X_3|X_1^1, X_2^1, X_4^0, \ldots, X_k^0) \);
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To end one iteration, draw $X_k^1$ from $f(X_k^1 | X_{1:k}^1, X_{2:k}^1, X_{4:k}^1, \ldots, X_{k}^0)$.

The above process should be repeated $J$ times until convergence is achieved (Geman and Geman, 1984). Next, the transition parameters and the remaining parameters which are conditional on both the state parameter and data are sampled using the above steps.

1. Sample the remaining coefficients of the model using the steps from 1 → $n$;
2. Monitor convergence by plotting the posterior estimates.

Another way of estimating the parameters of the model is to use the Metropolis-Hastings algorithm\(^\text{15}\). However, this procedure involves reject/accept decisions by first drawing samples from a ‘proposal’ distribution, which then estimates the full conditional. The method is less efficient and more time consuming as it requires tuning and it is more suited to high dimension hierarchical models (Lunn et al., 2013).

4.5.3 Model validation and selection

In traditional statistic analyses, one would use hypothesis tests based on the likelihood ratio, such as the the AIC, to choose the correct model. In a Bayesian setting, one is rather expected to center a comparative scale on the posterior distribution of the deviance, or to use Bayes factors, namely the BIC which is a classical test in both statistical and Bayesian analyses; it differs from the AIC in the sense that the likelihood is integrated and not maximised throughout the parameter space. The Deviance Information Criteria (DIC) is a test of absolute fit, and is consequently used to validate and select the best model out of the four to be presented later (Lunn et al., 2013). It has been successfully applied in various fields since it was developed by Spiegelhalter et al. (2002). Essentially, the DIC is a generalisation of the AIC; it is best used in models where the posterior distribution of parameters is normal. The DIC is calculated by the following expression (given in Ntzoufras (2009)):

$$DIC(m) = 2D(\theta_m, m) - D(\bar{\theta}_m, m) = D(\bar{\theta}_m, m) + 2p_m$$  \hspace{1cm} (4.36)

where $D(\theta_m, m)$ is the deviance measure (as a function of $\theta$) given by:

$$D(\theta_m) = -2\log p(Y|\theta)$$  \hspace{1cm} (4.37)

\(^{15}\)The foundation of the algorithm was laid down by Metropolis et al. (1953) and further extended by Hastings (1970).
\( \bar{\theta}_m \) is the posterior mean of the coefficients of the model \( m \), and \( p \) represents the number of efficient parameters in the model, given by:

\[
p_m = D(\theta_m, m) - D(\bar{\theta}_m, m) \tag{4.38}
\]

In the present investigation, the use of the DIC is fairly simple. For the reason that all four models are univariate, one does not have to perform stepwise elimination of variables from the regression equations. The efficient number of parameters to be estimated depends on the structure of the model and on the data available. On the other hand, the posterior distribution of the parameters will depend on the information conveyed by the data and priors. After running 10,000 iterations, estimated posterior distributions of parameters are reported and the DIC is obtained. A low DIC is preferred over a high value.

To infer the posterior parameter values, the WinBUGS\(^{16}\) (Lunn et al., 2000) software is utilised, while the econometric analysis of the output data is performed using the Stata (StataCorp., 2013) software.

### 4.6 Empirical results

As opposed to simple Markov chains where the states are observable, in this analysis the crisis and tranquil states are latent. By implementing a two-state Markov regime switching method, the models presented below segment the LIBOR-OIS spread into crisis and non-crisis intervals. Let \( Y = \{ Y_t; t= 1, \ldots, 2609 \} \) with the series running from 1st January 2002 to 30th December 2011. In the next subsections, four univariate autoregressive models are presented. As a prerequisite for latent Markov chain models, the starting state must be determined from onset. Therefore, for all four cases, it is assumed that at \( t = 1 \) the system is in a non-crisis state. To forecast the crisis and non-crisis periods, a binary state indicator variable \( I_t \) is included in the regression equations. For all the models, the first 1,000 iterations (corresponding to the burn-in period) are discarded, then further 10,000 samples are run. The resulting projected densities are plotted and visually inspected. Interestingly, after only 2000 samples, the desired marginal distributions converged to their expected stationary values.

\(^{16}\)WinBUGS is a programming language used for Bayesian inference and Markov Chain Monte Carlo analysis using the Gibbs sampler. The statistician first specifies the model (which contains both logical and distribution components) in syntax similar to \( R \) or \( Splus \), loads and compiles the data and specifies or generates initial values of the chain. By updating the sampler with \( n \) number of samples and simultaneously monitoring the nodes (the parameters of the model), the tool obtains posterior distributions for the parameters. Convergence can be graphically examined by tracing the plots of the estimated coefficients (Lunn et al., 2000).
Table 4.1 summarises the four models and their characteristics. The parameters are allowed to vary in the specified intervals.

Table 4.1: The four models and their characteristics.

<table>
<thead>
<tr>
<th>Variables</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime change</td>
<td>mean</td>
<td>const. &amp; mean</td>
<td>const. &amp; mean</td>
<td>const. * τ_t &amp; mean</td>
</tr>
<tr>
<td>State param.</td>
<td>[0,1]</td>
<td>[0,1]</td>
<td>[0,1]</td>
<td>[0,1]</td>
</tr>
<tr>
<td>ρ</td>
<td>[0,1.2]</td>
<td>[0,1.1]</td>
<td>[0,1.5]</td>
<td>[0,1.5]</td>
</tr>
<tr>
<td>const.</td>
<td>0</td>
<td>[-40,40]</td>
<td>[-400,400]</td>
<td>[-400,400]</td>
</tr>
<tr>
<td>Hyper param.</td>
<td>[0.370]</td>
<td>[0.370]</td>
<td>[0.370]</td>
<td>[0.370]</td>
</tr>
</tbody>
</table>

4.6.1 Model 1

The model below assumes that the regime switch occurs only in the mean and there are 12 parameters to be estimated. As explained earlier, the innovations follow a Gaussian normal distribution with zero mean and variance $\sigma^2_{0,t}$ and $\sigma^2_{1,t}$, respectively. The explanatory variable is the observed autoregressive component which is state dependent. There is no constant included in the equation. The state parameter follows a categorical distribution and can take two values, 0 and 1. The transition parameters follow a Beta distribution, whereas the remaining parameters follow a univariate probability distribution.

For values $t = 2, \ldots, 2609$, the expected mean is determined as follows:

$$E(Y_t) = \rho Y_{t-1} I_{t,2} + \epsilon_t \quad (4.39)$$

where $\epsilon_t$ is a white noise error. Considering that the process is assumed explosive, the autoregressive coefficient $\rho$ is arbitrarily set to take any value between $[0,1.2]$. $\tau$ is the precision parameter satisfying $\tau = \frac{1}{\sigma^2}$. Thus, $\tau$ is the inverse of the variance.

The hyper-parameter values $a$ and $b$ in Equations 4.29 to 4.35 are fixed at values 0 and 370 respectively. Table 3.1 shows that the minimum value of the spread is at 1.91 and the maximum is at 364.43. Martínez-Beneito et al. (2008) set the $a$ value at the minimum value of their data set, and the $b$ value was set to be the difference between the maximum and minimum value. However, for simplicity the values are set to $a = 0$ and $b = 370$. 
4.6.2 Model 2

A constant (intercept) is included in the equation, however it is not state dependent. The interval of the autocorrelation coefficient $\rho$ is limited to vary between $[0,1.1]$ and the constant is allowed to vary in the interval $[-40,40]$.

$$E(Y_t) = \alpha + \rho Y_{t-1} I_{t,2} + \epsilon_t$$  \hspace{1cm} (4.40)

4.6.3 Model 3

The difference between Model 2 and Model 3, is that the autoregressive coefficient and the constant are allowed to vary in the interval $[0,1.5]$ and $[-400,400]$ respectively. It is expected that larger intervals for the autoregressive coefficient and constant would better support the model in identifying crises more accurately over the 10 year period of analysis.

$$E(Y_t) = \alpha + \rho Y_{t-1} I_{t,2} + \epsilon_t$$  \hspace{1cm} (4.41)

4.6.4 Model 4

In this model, changes in the level are attributed to changes in the variance, as follows:

$$E(Y_t) = \alpha \tau_t + \rho Y_{t-1} I_{t,2} + \epsilon_t$$  \hspace{1cm} (4.42)

Both the time dependent constant and noise terms for the two regimes follow an uninformative distribution.

Table 4.2 presents the posterior parameter estimates and their 95% credible interval for the four models described above. A single chain of simulations with 10,000 iterations was run, as this proved sufficient for the posterior parameters to converge. Figure 4.3 and Figure 4.4 present the posterior mean values with the two periods (crisis and non-crisis) of the four distinct models. The simplest model, represented by Model 1, identifies eight crises. Whereas, if changes in the levels are allowed to be affected by the variance, such as in Model 4, 12 crises are identified for the period ranging from 1st January 2002 to 30st December 2011. As can be seen in Figure 4.6, where Model 2’s crisis output is mapped, the probabilities are close to or equal to 1 - in the case of a tranquil phase -,
and they are close to or equal to 2 - indicating a crisis phase. All values equal to or above 1.5 are considered to correspond to a crisis state.

Table 4.2: Estimated parameter values and regime change probabilities.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{inf}$</td>
<td>1.69</td>
<td>1.665</td>
<td>1.923</td>
<td>0.802</td>
</tr>
<tr>
<td>95% interv.</td>
<td>[0.1316,2.707]</td>
<td>[0.1281,2.783]</td>
<td>[0.1382,2.98]</td>
<td>[0.05109,2.223]</td>
</tr>
<tr>
<td>$\theta_{med1}$</td>
<td>4.015</td>
<td>3.897</td>
<td>4.323</td>
<td>1.828</td>
</tr>
<tr>
<td>95% interv.</td>
<td>[2.694,7.192]</td>
<td>[2.454,7.014]</td>
<td>[2.622,7.218]</td>
<td>[0.9262,3.621]</td>
</tr>
<tr>
<td>$\theta_{med2}$</td>
<td>6.462</td>
<td>6.382</td>
<td>7.046</td>
<td>2.985</td>
</tr>
<tr>
<td>95% interv.</td>
<td>[3.378,8.562]</td>
<td>[3.342,8.534]</td>
<td>[3.427,8.61]</td>
<td>[1.407,4.685]</td>
</tr>
<tr>
<td>$\theta_{sup}$</td>
<td>75.79</td>
<td>72.43</td>
<td>73.25</td>
<td>63.08</td>
</tr>
<tr>
<td>95% interv.</td>
<td>[8.674,330.6]</td>
<td>[8.594,325.3]</td>
<td>[8.6,327.1]</td>
<td>[3.924,320.5]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.4291</td>
<td>0.9049</td>
<td>0.9048</td>
<td>0.9959</td>
</tr>
<tr>
<td>$P_{0,0}$</td>
<td>0.9948</td>
<td>0.9948</td>
<td>0.9951</td>
<td>0.9821</td>
</tr>
<tr>
<td>$P_{0,1}$</td>
<td>0.0051</td>
<td>0.005186</td>
<td>0.004942</td>
<td>0.0179</td>
</tr>
<tr>
<td>$P_{1,0}$</td>
<td>0.00838</td>
<td>0.008771</td>
<td>0.008257</td>
<td>0.006261</td>
</tr>
<tr>
<td>$P_{1,1}$</td>
<td>0.9916</td>
<td>0.9912</td>
<td>0.9917</td>
<td>0.9937</td>
</tr>
</tbody>
</table>

The DIC is lowest for Model 2 with a value of 1716. Model 1, 3 and 4 had values of 1798, 1767 and 1881, respectively. Thus, it can be concluded that Model 2 performed best. Visual inspection of Figures 4.3 and 4.4 corroborate the findings and show that Model 2 identifies the crises periods more accurately than the other three models. To further support the results, the identified nine crisis periods are linked to real events which are believed to have led to financial crises (see Figure 4.6 and Table 4.3).

The estimated parameters inferred by the Gibbs sampler efficiently characterise the dynamics of the two states, and perfectly correspond to the dating of identified crisis and non-crisis periods, as shown in Figure 4.6. By defining the transition probabilities and thresholds $\theta_{inf}$, $\theta_{mid1}$, $\theta_{mid2}$ and $\theta_{sup}$, the model also explains the likely mean values of spreads being in an inferior-, lower-mid-, upper-mid- and superior phase. The thresholds are not fixed, and therefore the models define crises in the LIBOR-OIS spread with two different intensities; for example, as seen in Table 4.2, the 95% credible interval for $\theta_{mid2}$ corresponds to a low intensity crisis, whereas $\theta_{sup}$ corresponds to a high intensity crisis period. With a 95% confidence level, the posterior mean representing crisis level $\theta_{sup}$ for Model 3, has a value of 73.25 and falls within the interval [8.6, 327.1]. Thus,
Figure 4.3: State prediction for Model 1 and Model 2 versus time for the period 1st January 2002 to 30th December 2011.

Figure 4.4: State prediction for Model 3 and Model 4 versus time for the period 1st January 2002 to 30th December 2011.
Chapter 4 A Univariate Two-regime Switching Model to Detect Crises in the Short-term Interbank Market

Figure 4.5: The LIBOR-OIS spread and the expected crisis mean values for the four models versus time for the period 1st January 2002 to 30th December 2011.

observations that take values within this interval are assumed to correspond to a crisis period.

The autoregressive coefficient has near unity value for Models 2, 3 and 4. The approximated posterior mean values of the four models were projected against the LIBOR-OIS spread (see Figure 4.5). The financial crisis of 2007 had an impact on the parameter estimates since it translates into explosive jumps in the spread, and for the period August 2007 to June 2009 the estimated mean values are continuously in the crisis state. Thus, the main finding is that explosive spread rates are associated with crisis time, and low spread values with non-crisis periods. There are clear breaks between the crisis and non-crisis phases as seen in Figures 4.3 and 4.4, and these correspond to regime switches. For all four models, the transition parameter estimates show that there is a high probability to be in a non-crisis state if the previous state was also a non-crisis one (see the $P_{0,0}$ values, for example in Table 4.2), exhibiting 99% probability for the first three models, and 98% probability for the fourth model. Similarly, a crisis state is more likely to be followed by a crisis state, as picked up by the transition estimates $P_{1,1}$, for example with 99% probability for all four models; this is consistent with the volatility clustering phenomenon seen in time-series during turbulent times.
Figure 4.6: Timeline with the crises identified by the two-state regime switching model.

Table 4.3: Time periods classified as crises according to Model 2 and ex-post identification of significant events.

<table>
<thead>
<tr>
<th>Time period</th>
<th>Significant events</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 January 2002 - 11 January 2002</td>
<td>Declaration of War on Terror, USA recession</td>
</tr>
<tr>
<td>18 March 2002 - 18 April 2002</td>
<td>Israeli-Palestinian conflict</td>
</tr>
<tr>
<td>19 Sept. 2002 - 14 January 2003</td>
<td>President Bush to seek approval to attack Iraq</td>
</tr>
<tr>
<td>28 October 2003 - 26 December 2003</td>
<td>Historical high unemployment rate in the USA</td>
</tr>
<tr>
<td>3 August 2007 - 3 September 2009</td>
<td>Global economic crisis</td>
</tr>
<tr>
<td>6 May 2010 - 20 August 2010</td>
<td>European debt crisis</td>
</tr>
<tr>
<td>2 February 2011 - 27 May 2011</td>
<td>Arab Spring and Japanese Tsunami</td>
</tr>
</tbody>
</table>
4.6.5 Out-of-sample forecasting for detecting liquidity crises

In line with the previous analysis, the Bayesian paradigm is used to predict the future values of the LIBOR-OIS spread. Let $Y$ represent the LIBOR-OIS time series of $T$ sample realisations of a random process for the period $t = 1, 2, \ldots, 1399$; $Y = [y_1, y_2, \ldots, y_{1399}]^\top$. Therefore, the first 1399 observations of the LIBOR-OIS spread series are used to forecast the subsequent period, which includes the financial crisis of 2007-08 and the Eurozone crisis that followed. The last observation of $t$ (on the timescale 1, 2, \ldots, 1399) corresponds to 11th of May 2007, when there were no apparent signs (such as official announcements) of a liquidity crash in the short-term interbank market in the US nor in the Eurozone market. Now let $Y^p$ represent a vector of future observations of the LIBOR-OIS time series; $Y^p = [y_{T+1}, y_{T+2}, \ldots, y_{T+q}]^\top$. When providing the predictive distribution of $Y^p$, Zellner’s (1971) notation is followed.

\[ p(Y^p|Y) = \int_{\Theta} p(Y^p, \theta|Y) \, d\theta = \int_{\Theta} p(Y^p|\theta, Y) \, p(\theta|Y) \, d\theta \quad \text{(4.43)} \]

Essentially, the predictive probability distribution of $Y^p$ is conditional on the 1399 observations. The model forecasts the next 1210 observations using the Gibbs sampler. Figure 4.7 maps the forecasted values to the LIBOR-OIS series. The conclusion is that the forecasts accurately match the real financial events for the period May 2007 - December 2011.

4.7 Discussion of how the predicted crises are evident in other financial markets

The ex-post predicted liquidity crises identified in the long-run behaviour of the US LIBOR-OIS spread might have originated from other financial markets (stock market, for example), or the crises spilled over from the short-term interbank market to other financial markets. The crash of the stock market and later the insurance market caused by the September 11th, 2001 terrorist attack for example, coincides with the first liquidity crisis identified (see Figure 4.6) by the univariate autoregressive regime switching Markov model. Posner and Vermeule (2009) argue that the market meltdown following the attacks of 9/11 was similar in certain ways to the financial crisis of 2007-08. Using probabilistic measures, Straetmans et al. (2008) find that the event of 9/11 significantly altered US stock market indices values. Opposed to this view, Mishkin and White (2002) argue that the Dow Jones, S&P500 and NASDAQ indices experienced sharp declines between August 2000 and December 2001, however this did not affect the whole financial system and therefore did not translate into widening of interest rate spreads.
Chapter 4 A Univariate Two-regime Switching Model to Detect Crises in the Short-term Interbank Market

The stock market crash of 2002, also known as the dot-com bubble crash, was also mirrored in fluctuations perceived in the US LIBOR-OIS spread. Wang (2007) finds that the dot-com stock index was severely affected by the high number of dot-com companies exiting the market. The univariate regime switching model presented in this Chapter identifies the financial crisis of 2007-08 as starting in July 2007. The Dow Jones Industrial Average index reached a level above 14,000 in October 2007, however it started declining in the first quarter of 2008 until October when it dropped significantly (Hudomiet et al., 2011). Using an extensive data set on the Asset Backed Securities indices (ABX), Longstaff (2010) finds evidence that originally, the financial crisis spread from the ABX market to other financial markets as identified by the forecasting power of the ABX index. Frank et al. (2008) argue that the S&P 500 index and the liquidity of Treasury bonds were affected by the risk originating from the US stock markets during the financial crisis of 2007-08.

4.8 Conclusions, contributions and limitations

In this study the investigation relies on a univariate autoregressive Markov regime-switching model to trace liquidity crises in the US LIBOR-OIS spread (which fundamentally represents the short-term interbank market), and to determine the length of
such turbulent periods. The above regime switching model has only a mean specification and assesses the period of 1st January 2002 to 30th December 2011. Gibbs sampling, the algorithm behind the Markov chain Monte Carlo method, is used to estimate posterior mean probabilities of regime changes. The posterior distribution is the foundation for statistical interpretation and decision-making. After every single draw, the conditionals of the parameters ($\beta|y$, for example) were updated. 10,000 identically and independently distributed samples were obtained. Figures 4.3 and 4.4 provided evidence on the properties of probabilistic inference of posterior parameters, clearly depicting crisis and non-crisis phases in the time series. By using thresholds ($\theta_{\text{linf}}, \theta_{\text{med1}}, \theta_{\text{med2}}$ and $\theta_{\text{sup}}$) the model can clearly delimit the two states in the time series. Detecting financial crises while they develop is essential, for contagion rapidly propagates liquidity shocks across interconnected financial markets. The consequences of such phenomena are manifested by the recent financial crisis, which had a devastating effect on several economies, triggering a prolonged and painful recession.

The model can predict the moment a liquidity crisis is about to hit with a high probability, and it is able to measure the persistence of crisis and non-crisis periods. The results fully support the fact that the interbank money market was in a financial distress at least eight times for the analysed period including the financial crisis and the Eurozone crisis that followed. Compared to the approach of Kaminsky and Reinhart (2000) for example, who use the three standard deviations above the mean to classify the observations into crisis and non-crisis periods, this analysis uses a much more realistic approach to gauge the turbulent times for the analysed period. The mean and its standard deviation can only be representative of the data if that is normally distributed. However, as it has been proved many times in previous investigations, asset prices and economic and financial rates are characterised by extreme lows and highs in times of financial or economic distress and therefore in such circumstances one cannot use standard econometric models to investigate market fundamentals. Thus, the model developed in this chapter can surely benefit policy makers and institutional players alike.

Bayesian inference proved to be a landmark in estimating crisis and non-crisis regimes in short-term financial series. The main significance of the findings is that they can be used as a basis to develop an early warning system to detect liquidity shocks within the interbank market. The univariate setting proved powerful, however a multivariate regime-switching model would detect crises in several markets (such as the currency and bond market), while tracing the propagation of liquidity shocks between these markets. Moreover, one could consider a model with regime change in the constant and/or in the variance term.
Chapter 5

A Multivariate Endogenous Regime Switching Markov Chain Model to Trace Liquidity Shock Propagation within the Interbank Market

5.1 Introduction

This chapter studies the foundations of fluctuations in interbank rates and spreads, as well as the propagation of liquidity shocks within the short-term interbank market. The aim is to yield predictive distributions for the money market spreads and interest rates, and at the same time analyse the effects of liquidity shocks on the interbank market overall.

The concept of liquidity risk and contagion is one of the most important in finance. Financial crises and their destabilising effect on economies and the functionality of the financial system has been the focus of recent research. However, there is no financial model which is able to describe the propagation of liquidity shocks in the interbank market or to predict liquidity crashes. Multivariate volatility models do exist, however these are based on the GARCH representation \(^1\) (Engle, 1982; Bollerslev, 1986), which in a sense are limited for these only work well in linear settings\(^1\). Linear models cannot describe financial crises, for drastic changes in price levels or interest rates, which last for prolonged periods are fundamentally structural breaks or regime changes. Some, such

\(^1\)Engle and Ng (1993) developed a non-linear GARCH model, however this does not consider the different dynamics of volatility in different regimes.
as Hamilton and Susmel (1994) and Henneke et al. (2011) for example, have developed regime changing GARCH models or MCMC estimated financial risk contagion models, such as Zhou and He (2012) for example, however these would not explain what exactly drives such volatile time periods and how liquidity risk propagates within the interbank market.

Endogenous dis-equilibrating forces or shocks are owed to the interconnectedness of financial markets (Danielsson, 2011). Therefore, linkages between short-term interbank rates and spreads enable us to understand how liquidity risk/shocks propagate in times of financial crises (Minsky, 1992). The majority of models described in the financial risk literature assume that risk itself is exogenous, arising from shocks which originate outside the system being modelled. Thus, variations in asset prices and interest rates are external to the influence of market players. Such external forces, which are fundamental in the development of bubbles, are for example market expectations, innovations in technology, market participants dropping out, etc. (Allen and Gale, 2000a). Danielsson (2011) argues that market participants, who use and rely on these risk models, must be price takers. Every single market participant through its trade influences movements observed in asset prices and interest rates. This view originates from Danielsson and Saltoğlu (2003) who suggest the term ‘endogenous risk’ as being the one generated and intensified from inside the financial system. The authors emphasise the significance of interactions among market players in shaping outcomes, such as prices and rates.

In reality, both exogenous and endogenous risk affects the smooth functioning of financial markets; however, the later has a more pronounced and at times devastating effect. Danielsson (2011) and Brunnermeier and Oehmke (2012), among others, argue that financial crises are primarily the result of endogenous financial risk, owing to the fact that endogenous risk is harder to model. If risk models do not incorporate endogenous risk, these are flawed and consequently unreliable from financial stability point of view. Danielsson (2011) stresses that the Basel Accords regulations do not aid the smooth functioning of the financial market, but rather contribute to the accumulation of endogenous risk. A remarkable analogy is the representation of the feedback loop in the Millennium Bridge case.

Danielsson (2011) uses the Millennium Bridge and its imminent closure right after it opened to illustrate the devastating effect of endogenous risk. Under the feet of spectators present at the opening ceremony, the bridge started to swing. Immediately, all individuals present on the bridge adjusted their stance to regain stability. This synchronised movement caused the bridge to rock even more. This further prompted individuals to adjust their stance even more radically. Essentially, this was a mutually reinforcing phenomenon, and ultimately led to the closure of the bridge right at the event of its opening.
lead to insolvency and bankruptcy among the liquidity affected market players. The first shock paving the way to the financial crisis of 2007-08 was felt in February 2007, when the Federal Home Loan Mortgage Corporation revealed that it would no longer purchase asset-backed securities known as collateralised debt obligations. The crisis in the interbank financial market was self-sustaining as a result of institutions responses to the initial liquidity shocks. On the 9th of August 2007, the BNP Paribas bank announced that it stopped trading as a result that its complex assets could not be valued. Even though the initial shock from February 2007 had passed, the reactions of financial institutions continued to rock the financial system. Danielsson (2011) argues that this a perfect illustration of a force which is produced and intensified inside of the system. The response is endogenous, and is very distinct from a shock which originates externally to the system. Moreover, Danielsson (2011, p.195) argues that “endogenous risk is most likely to arise when there is a prevailing consensus concerning the direction of market outcomes”.

Kim et al. (2008) argue that agents do not discern the states (of the state space), however they draw inference grounded on a particular information set. Yet, the elements of the information set are unidentified by the econometrician. If the actual state is used to proxy for inference, this subsequently leads to a regression with measurement error in the covariates, and hence endogeneity in the estimated model. In essence, endogeneity implies that in the regression equation, the independent variables are correlated with the unobserved nuisance term. One way to correct for endogeneity is to use instrumental variables when estimating the parameters of the model.

To address the issues discussed above, the present study proposes a new liquidity risk contagion model (also called an early warning system to signal interbank liquidity crisis and contagion). There are several advantages of using an endogenous and therefore transitional regime switching model. The model presented below assumes that the rules which govern the changes from one state to another are not independent from the rest of the system. By using an endogenous model, one is able to obtain more efficient estimates via extra information contained within the endogenous system. Moreover, in this endogenous regime switching model the Markov chain is affected by the shocks or innovations of the system, and not only by the previous regime state. Therefore, the transition from one state to another is stochastic over time.

The study implements a first-order Markov chain process with a time and state dependent transition probability matrix, which essentially drives the evolution of crisis and non-crisis states. Simple volatility models (such as GARCH models) do not consider the time-varying nature of parameter estimates in turbulent times, therefore these yield

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3 There are various names allocated to the same concept. Measurement error, random error, or observational error is the difference between the calculated (or actual) value of a quantity and the value acquired by some measurement (Anderson et al., 2011).

4 A variable is said to be endogenous whose value is decided on or controlled by one or more of the explanatory variables containing itself (Danielsson, 2011).
unreliable results. By using a binary indicator, the model detects a finite number of structural changes in the time series. Also, the two identified states will have different dynamics. Therefore, the new model shows that shifts in the level, means and variance are different for the two regimes. Thus, the dynamic linear (or non-linear) system is described by a crisis and a non-crisis state\(^5\).

Empirically, the following study estimates the parameters of a liquidity contagion model, in which the daily three-month German-US bond spread depends on the LIBOR-OIS spread, the Euro-dollar currency swap, and on the daily EONIA rate. The study investigates 10 years of daily data, including the recent financial crisis, thus it covers the period 1st January 2002 to 30th December 2011. The short-term interbank time spreads and rates used in the analysis are representative of the interbank financial system and consequently capture well the dynamics of the market. ML is used to approximate the true value of the observed parameters which are allowed to change over time.

The remainder of the chapter is organised as follows. The literature review which examines endogenous univariate and multivariate analyses is presented in Section 5.2. Section 5.3 presents the research questions. The data sets used in this study are described in Section 5.4. The methodology, detailing all the methods used, is revealed in Section 5.5. Empirical results are shown in Section 5.7 and conclusions, limitations and future work are presented in Section 5.8.

### 5.2 Literature Review

A summary of the regime switching literature along with the literature on regime switching with constant transition probabilities was presented in Section 4.2. A survey of the literature discussing regime switching models with time-varying (endogenous) transition probabilities (TVTP) is presented below.

Before looking at the relevant literature, it is important to define endogenous and exogenous variable. Maddala and Lahiri (1992) gives the classic definition of the terms. Broadly speaking, endogenous variables, or jointly determined variables are defined by the economic model, whereas exogenous variables, or predetermined variables are governed external to the model, and consequently are independent of the innovations of the model. Lancaster (2006, p. 313) gives the following (more specific) definition for an endogenous variable:

\begin{quote}
A variable (...) that appears on the right hand side of an econometric equation system (so it is a causal variable in the theorist’s underlying deterministic model), and that is presumed to be correlated with the errors in the model is called endogenous variable.
\end{quote}

\(^5\)The state, as the unobserved variable, is a fundamental element of the state space (Hamilton, 1994a).
Chapter 5 A Multivariate Endogenous Regime Switching Markov Chain Model to Trace Liquidity Shock Propagation within the Interbank Market

The regime switching literature is significantly diverse. This chapter presents the two broad strands in the literature which discuss endogenous regime switching models. The former is the state space approach to the business cycle, and the latter generally lies within the macroeconomic policy context. However, before discussing the literature, the theoretical underpinnings of endogenous regime switching models are laid down.

5.2.1 Theoretical underpinnings

Figure 5.1 displays the fundamental feedback-loop of a liquidity shock hitting the interbank market. The figure visualises how single shocks to the system intensify and result in extreme market behaviour, which ultimately lead to financial turmoil in the interbank market. As explained in the introduction section of this chapter, the turmoil of the financial market is a self-sustaining process as a result of agents’ responses to an initial liquidity shock. Even after the initial shock has dissipated, the process continues to intensify due to an ‘energy’ that is produced and magnified within the financial system. As a perfect analogy to the recent financial crisis, an initial decrease in the value of assets - as outcomes of market players’ responses to the Federal Home Loan Mortgage Corporation’s and credit rating agencies’ actions - further triggered actions within the interbank market, while at the same time initial liquidity shocks continued to magnify.

At this point it is essential to define the notion of exogenous shocks and exogenous regime switching. There are two types of effects originating from exogenous shocks or innovations. The first ones are ‘direct effects’, and these are typical impacts of shocks which arise when market players place zero probability on a regime change. The second type of effects are the ‘expectation formation effects’ when it is assumed that market players’ rational expectations of potential regime change prompts them to revise their expectation functions (Davig and Leeper, 2006). When shocks are exogenous, the analysis cannot identify whether these produce asymmetric effects or not; moreover, the investigation cannot yield quantitatively significant results, nor one can formulate policy responses or advise policy makers (Kim, 2004). In the exogenous regime switching scenario, a stochastic process drives the dynamics of the system. Both rules and instruments of the system are state dependent, however the Markov chain which drives the transition from one state to another evolves independently from the other parts of the system (Kaufmann, 2011). The only thing that matters in deciding the future state is the present state, therefore the decision process is not influenced by the realisations of the time series which enter the regression equation (Kim, 2009). This view is not realistic considering that in most financial and economic processes it is something from within the system that defines future states, and not factors external to it (Branch and Evans, 2007; Dănuță, 2011; Chaouachi et al., 2013). If the states were independent
and rules were declared for some thresholds, one would get the smooth transition model of Teräsvirta and Anderson (1992)\(^6\).

One of the fundamental assumptions of the OLS is that variables in the linear equation are not correlated with the error term, such that \(E[\epsilon|x_1, x_2, \ldots, x_k] = 0\) (Wooldridge, 2012). To reveal what exactly drives the processes in the cointegrated relationships, the exogeneity assumption can be relaxed, thus it is assumed that one or some of the variables of interest are endogeneous, such that \(\text{Cov}[x_j, \epsilon] \neq 0\), where \(j = 1, 2, 3\) is the number of variables and \(\epsilon\) is the error term. To make it more specific, for example in the system of equations, the path of some endogeneous covariates are controlled or influenced by each other and some exogenous variables.

To follow on from the findings in Chapter 3, the two identified long-run cointegrating equilibrium equations (without constants), are as follows:

\(^6\)In the transition matrix the coefficients of the independent variables are restricted to zero. It is also assumed that the innovations of the system are also exogenous. Moreover, the process driving the state change initiates a new source of disturbance.
where the first equation corresponds to a restricted constant equilibrium model and the second to a no constant and no trend equilibrium model. In both equations the differenced German-US Bond spread level - as the regressand - depends on the currency swap rate- and the US LIBOR-OIS spread levels, as regressors. In the GerUS3M and EUSWEC space, the liquidity curve\footnote{Due to the fact that in this paper liquidity risk, crash and contagion in the short-term interbank market are the focus, the regression curve represents an equilibrium liquidity relationship.} slopes upward, whereas in the GerUS3M and LIBOR-OIS space the curve slopes downward. This is, however, a minimalistic or restricted view of the relationships. There must be certain information comprised in some data coming from micro level, which is not added to the above relationship. Liquidity risk affects the realisations of the independent variables and the state variable; however, liquidity risk (as a variable) is missing from the equation, owing to the fact that actually it is not observed. Consequently, the state variable (a binary indicator variable in this case) and the innovations are correlated (Kang, 2014). If one adds the innovation term $\epsilon_t$ (which is a segment of the prediction error) to the two equations above, the estimated regression coefficients will be misspecified. This is because a shock or innovation to the GerUS3M spread will alter both the equilibrium level of the currency swap and that of the LIBOR-OIS spread. Theoretically, endogeneity in the regime switching state-space configuration is similar regarding the missing variable problem.

In the endogenous Markov regime switching scenario, the coefficients driving the level of adjustment of the short-term bond rate for example, to financial interbank variables, are themselves a function of the state. Therefore, the dynamics of the transition from one state to the other is influenced by both the realisation of the time series of the model, and also by past and current states (Kaufmann, 2011). This is in sharp contrast to exogenous Markov regime switching models, where only the past state influences the transition dynamics. It is also assumed that the unobserved variables, be that the state variable or any of the covariates, are correlated to past shocks and innovations, and as a result any shock and innovation affecting the system influences the transition probabilities (Kim et al., 2008). Considering that some of the unobserved factors can be autoregressive and therefore have the pretext of a unit root, the transition can be non-stationary and in such case the coefficient of the unobserved autoregressive factor would drive the persistance of state changes (Chib and Dueker, 2004). Due to the fact that extra information goes into the estimation process, the estimates and transition
probabilities are more precise and valid as endogeneity is accounted for (Danielsson, 2011).

Essentially, the rule of the system is a function of the lagged endogenous state variable (Davig and Leeper, 2006), such that:

$$y_t = \beta_1(x_{t-1}) + \beta_2(x_{t-1})x_t + \beta_3z_t + \epsilon_t$$  \hspace{1cm} (5.3)

where endogenous switching can render $\beta_1(\cdot)$ and $\beta_2(\cdot)$ either time dependent or stochastic. The model presented in this study uses the so-called centered parametrisation, where there are no rules for thresholds. However, this would not obstruct the identification of the time-changing effect of the endogenous covariate. Kaufmann (2011) shows that a threshold different from the mean of any of the independent variables can be estimated by utilising the part that the independent variables play in the time-varying transition distribution. In the univariate regime switching model presented in Chapter 4 it was assumed that the initial state is non-crisis. If the purpose of the analysis was to forecast liquidity crisis and contagion between markets, an ordering of the states would be imperative.

Regime switching models are extremely useful in modeling recurring regime changes in time series. Hamilton’s (1989) Markov switching model is restricted, for the transition probabilities are constant over time and thus the effect of exogenous shocks in the form of new information for example, macroeconomic policies and the economy’s internal transmission mechanism (among other effects) will not affect the prediction of the length of a particular state (Filardo and Gordon, 1998). To overcome this constraint, Diebold et al. (1994) developed an Expectation Maximisation (EM) algorithm which enables the modelling of time-varying probabilities. According to this new method, the behaviour or more precisely the dynamics of economic fundamentals depend on time as well as on other factors, such as the $\beta$’s, for example. Similarly to the constant transition case, the states of the space are not observed and neither the complete data set. The EM algorithm maximises the likelihood function, with the steps of the algorithm as follows (Diebold et al., 1994):

1. Set initial values for the parameters of $\Theta^{(0)}$, however this is not compulsory;
2. Calculate:

$^8$While using this method, Kaufmann (2011, p.2) notes that “the threshold level is defined as the level at which the divergence between the persistence probabilities of states is minimised”.

$^9$If this wasn’t the case, one could implement the random permutation sampler of Frühwirth-Schnatter (2001) to initiate the state ordering. The sampler estimates the unconstrained posterior distribution of the identified model, from which unique state-identifying restrictions are later estimated.
\[ P(s_t = 1|y_t, x_t; \Theta(0)) \forall t, \] (5.4)
\[ P(s_t = 0|y_t, x_t; \Theta(0)) \forall t, \] (5.5)
\[ P(s_t = 1, s_{t-1} = 1|y_t, x_t; \Theta(0)) \forall t, \] (5.6)
\[ P(s_t = 0, s_{t-1} = 1|y_t, x_t; \Theta(0)) \forall t, \] (5.7)
\[ P(s_t = 1, s_{t-1} = 0|y_t, x_t; \Theta(0)) \forall t, \] (5.8)
\[ P(s_t = 0, s_{t-1} = 0|y_t, x_t; \Theta(0)) \forall t; \] (5.9)

Now the expected value of \( \log f(y_t, s_t|x_t; \Theta(0)) \) can be estimated:

3. Set \( \Theta^{(1)} = \arg \max E[\log f(y_t, s_t|x_t; \Theta(0))] \);

4. Iterate until the parameters converge to the true probability distribution.

In the first step of the process, an initial guess of the parameter vector \( \Theta \) launches the algorithm. In step 2, which is called the estimation step, conditional on \( \Theta^{(0)} \), the smoothed state probabilities are computed (after they were initially filtered). By computing the smoothed joint state probabilities, one gets the smoothed marginal state probabilities. Conditional on the coefficients of the model, the smoothed marginal state probabilities are then maximised in step 3 by generating an updated parameter vector \( \Theta^{(1)} \). A relatively huge number of iterations would ensure that convergence of the EM algorithm to the true distribution has been achieved. The resultant parameter estimates are contained in \( \hat{\Theta} \). The bigger the sample size, the better the estimates are going to be. Diebold et al. (1994) find that the time-varying transition probability model (TVTP) outperforms Hamilton’s (1989) model. The approximated parameter values estimate the transition probabilities well, while the mean-state errors are lower compared to those of the constant transition model. However, in the presence of autoregressive components, the EM algorithm does not behave well in the maximisation step. For computational reasons, Filardo (1994) develops a time-varying transition model based on ML estimation. Fundamentally, the technique is similar to that of the Kalman filter (1960).

Linear models, such as the ARIMA are not able to reliably approximate asymmetric shocks. On the other hand, TVTP models capture well characteristics of interbank spreads. There are three reasons why such models outperform constant transition models. First, transition probabilities are flexible, and therefore they can rise or fall before a crisis or a non-crisis period occurs. Second, such models track well the progressive persistence of states and this can be attributed to an autoregressive component present in the model. Third, the estimated length of states is time-dependent across the length of the series, which is integral to the Markov switching structure (Filardo, 1994).
To overcome the issue of dealing with an increased parameter space when applying the EM or the ML method\(^\text{10}\), Filardo and Gordon (1998) implement Bayesian methods to estimate the time-varying transition probability parameters. In some respect, the Gibbs sampler outperforms the EM and ML estimation method, and consequently the parameters of the model and the uncertainty which surrounds them can be estimated with high precision. In the classic econometric model, first the parameters are approximated, then the duration of the states is estimated. However, the TVTP model allows the joint estimation of the parameters as well as the conditional development of the states. Besides, this method requires the identification of precise priors. By specifying the transition probabilities, one can detect and forecast whether there is a crisis looming. This is a result of changes in the transition probabilities creating changes in the expected length of crisis and non-crisis periods.

5.2.2 Empirical literature on regime switching models with time-varying transition probabilities

A string of financial crises during the last decade prompted the development of models and analyses on early warning systems. Abiad (2003) and Berg et al. (2005) survey such models and note that in most cases these perform better in forecasting financial crises than credit ratings, bond spreads-, and estimates of credit and liquidity risk (which are measures of financial vulnerability) did in the past. Abiad (2003) identifies several problems associated with EWSs. The main issues are related to knowing \textit{a priori} the timing of the crises, the determination of sample-dependent threshold levels (which are set arbitrarily, and vary from \(1.5 \times \sigma\) to \(3 \times \sigma\)), the selection of 'exclusion windows' (which are set arbitrarily and may vary between 3 -18 months) and issues associated with the transformation of continuous variables into binary ones (in which case important dynamics are removed from the variables). To avoid all these pitfalls, Abiad (2003) proposes a Markov switching model with time-varying transition probabilities to forecast ex-post currency crises. Using macroeconomic, capital flows and financial fragility indicators, the model identifies crisis periods as an output of crisis forecast probabilities based on a ML platform. The author acknowledges that EWSs are not able to successfully forecast crises (as his model only identifies two-thirds of the crisis periods), however, it is an improvement to threshold dating methods based on binary signals, such as the classic indicator variable model.

Using a latent and an endogenous autoregressive variable, Chib and Dueker (2004) implement a new non-Markovian regime switching method to model the GDP growth and its association to strength of regime lengths for the period 1960Q1 to 2003Q4.

\(^{10}\)Filardo and Gordon (1998) argue that the EM and ML methods are computationally intensive due to the fact that during the estimation process all possible permutations of the latent binary variables must be taken into consideration. For example, when the sample size is \(T = 2609\), the possible configurations for the crisis and non-crisis phases will be \(2^{2609}\).
The aim is to explain links among regimes, recessions determined by NBER and the time-dependent estimated lengths of regimes. Estimates are categorised via the Kalman filter\(^{11}\) which identifies an effective density for the unobserved state variable. Similarly to autoregressive models, where the dependent variable responds better to their own lagged values, regime switches are more often governed by an endogenous rather than by an exogenous variable; or, it is assumed that the force for a regime change forms progressively through time. For example, a downward impulse driving the business cycle may be positively or negatively correlated to a downward impulse to the observed GDP growth. The probability of the state is determined by both the previous state and by the continuous evaluation of the strength of the former regime. Regime lengths are time-varying even when there are no independent variables in the regime equation. Chib and Dueker (2004) use the extended Kalman filter to infer the parameters of the data generating process, and later the estimates are compared to MCMC computer-generated estimates of some simulated data. The main finding is that the regime strength variable is driven by its past value (1st lag) and not by the lagged value of GDP growth; however, the covariance parameter \(\rho\) does not have a significant positive value. If the lagged GDP growth is replaced with the lagged change in the index of leading indicators, that is the time-varying strength of the regime in this case, the probability interval for the covariance coefficient \(\rho\) improves somehow. Consequently, to some extent, the degree of cyclical strength (or the weakness of the economy) has forecasting power. Moreover, the estimated GDP growth rates are found to be correlated with the data, and therefore the pressure for a change in regime builds gradually across time.

The macroeconomic policy literature in an endogenous switching setting stems from Hamilton’s (1989) work. Modelling an unknown structural break point, Kim (2004) transforms Hamilton’s decision filter\(^{12}\) (Hamilton, 1989) to a quasi maximum likelihood filter to model endogenous regime switching in the forward-looking monetary policy rule for the period 1960Q1 to 1996Q4. Fundamentally, if covariates are correlated with the disturbance term, ML estimation is not a valid tool to infer the model parameters. Cholesky factorisation decomposes the covariance matrix into a vector of independent shocks; consequently, the regressors and error terms will not be correlated. In the first step of the ML approach, the estimated instrumental coefficients are approximated using OLS. Second, maximising the log likelihood and calculating the transition probabilities by implementing the Hamilton filter, the model parameters are inferred. A Monte Carlo experiment is implemented to estimate the two models - with and without the bias error correction terms - and subsequently reveal the data generating process and coefficient values. Then the model with the bias error correction term is fitted to the dataset.

---

\(^{11}\) The method was pioneered by the process engineer Kalman (1960). The filter algorithm solves the state space problem in a linear dynamic setting only. The extended Kalman filter however, works well in non-linear settings. In such case, non-linear functions are linearised and thus state space inference is accomplished.

\(^{12}\) Hamilton (1989) developed a nonlinear iterative filter which allowed the econometrician to infer the coefficients of the model based on a ML principle. In spite of the fact that he filter is computationally intensive, it proves successful in forecasting future parameter values.
Kim (2004) argues that the inflation variable becomes endogenous after the date of a structural break has been identified, and therefore in the 1970s, the Federal Reserve’s reaction to inflation might not have been progressive. Kim’s (2004) model however, is in a sense restricted, for the parameters of the first-order autoregressive process are assumed constant, and therefore are independent of progression of time. While implementing a joint estimation method along with a two-step method, Kim (2009) builds on his (2004) model and permits for hypothetically serially correlated or independent regime switching variables to induce the regime changes.

In macroeconomics it is often reasonable to assume that the variable driving regime changes is endogenous, and consequently it is correlated with the business cycle. Using the data set of Kim (2004), Kim et al. (2008) develop a model based on a probit specification, while expanding the volatility feedback model of Turner et al. (1989). The authors relax the assumption that the unobserved state variable, which drives the regime change, is exogenous. The coefficients of the model are estimated via ML and Monte Carlo experiments are run to test the sensitivity of the estimates.

Similarly, Davig and Leeper (2006) analyse monetary policy change behaviour in an endogenous setting. According to Taylor’s rule\textsuperscript{13}, policy changes respond systematically to variations in the environment. However, the authors argue that modelling money growth from an exogenous perspective can hardly propose any realistic policy recommendation. Implementing a time-varying threshold regime switching Markov model based on the self-exciting threshold autoregressive method, Davig and Leeper (2006) address the conflict between the two views (that is the endogenous and exogenous switching). According to the flexible price model of inflation determination, when some target variables set by the Central Bank cross some specified threshold, the policy regulation changes accordingly. The authors aim to show that exogenous switching causes the disproportionate propagation of shocks (meaning that some symmetric impulse responses will have an asymmetric influence) as well as the expectation formation effects which govern the impact of monetary policy and ultimately cause the distribution of the parameters to be skewed.

Branch and Evans (2007) show that regime changes in volatility may arise endogenously due to model uncertainty, estimates of underparameterised models or dynamic forecasting model choices. In their proposed model, which is based on a standard Lucas-type monetary model\textsuperscript{14}, the inflation’s path is determined by some causal stochastic activity in the log GDP and inflation, and by agents’ inflation outlook; the latter in turn depends on agents’ beliefs, and this might be the reason why the authors are inclined to underparameterise their predicting model.

\textsuperscript{13}In achieving inflation and output targets as means of accomplishing price stability and full employment, central banks follow the Taylor rule in changing the nominal interest rate. The rule was proposed by the monetary economist John B. Taylor in 1993 (Taylor, 1993).

\textsuperscript{14}In such model, reduced aggregate demand and supply is determined by some exogenous autoregressive shocks.
Billio and Casarin (2010b) show that in terms of forecasting, stochastic transition models are more efficient as opposed to the constant or time-changing transition Markov switching models. Analysing positive growth and recession periods in the context of the business cycle, Billio and Casarin (2010a) suggest a novel Stochastic-Transition Markov switching model applied to Euro-zone data. The authors use an autoregressive configuration while introducing a residual term in their model to capture the unexplained changes in the length of duration of the various regimes. The model is validated against the constant transition model, where transition is not time dependent, and against the dynamic transition model, where the transition is stochastic and therefore changes with time. The three models yield different parameter estimates, and consequently the business cycle timings and durations do not match. However, for a particular time period, the stochastic transition model and the dynamic transition model are consistent with the reference business cycle.

Using a two-pillar Philips curve method implemented for the euro area, with the inflation rate as independent variable and low-frequency components of M3 growth and high-frequency component of the output gap as dependent variables, Kaufmann (2011) estimates a K-state Markov regime switching model. The model is parameterised using the multinomial logit function in order to identify the variables that drive the change from one state to another. The author builds on Billio and Casarin (2010a)'s model, however, the transition probabilities are assumed to be endogenous, such that the state probability is assumed to be induced by the endogenous variable, while the transition is independent of the past prevailing state. To obtain unbiased approximations of the identified model, filtering is achieved by the random permutation sampling of Frühwirth-Schnatter (2001).

### 5.2.3 Conclusions and contributions to the existing literature

The models presented in this study complement the empirical liquidity and contagion models as well as providing a theoretical explanation of the self-fulfilling behaviour of liquidity shocks and liquidity risk propagation within the short-term interbank market.

1. The limitation of classical econometric models applied in the finance literature are that they cannot jointly estimate the parameters of the model as well as the duration of states. By implementing a multivariate endogenous regime switching model based on a ML estimation, the model fills in the methodological gap identified in the liquidity risk contagion literature. Moreover, a non-parametric model estimates with precision the parameters when dealing with time series which evolve over turbulent times. The regime switching Markov model with time-varying transition probabilities presented below provides a framework to implement inferences about fundamental financial crisis occurrences.
2. The finance literature does not discuss the endogenous nature of liquidity shocks and their propagation within the short-term interbank market. This study fills the gap by showing that liquidity shocks originate from within the banking system, and not only from externally, as the majority of financial crisis literature assumes. Both these risks contribute to the development of financial crises.

3. The period under investigation covers a longer time interval (from January 2002 to December 2011) than the existing literature. Therefore, one can trace the dynamics of liquidity shocks over a longer period of time and subsequently better understand interbank liquidity risk and its effect on the whole financial system.

5.3 Research questions

Taking into consideration the gap identified in the literature and driven by the relevant economic and finance theory, the study aims to answer the following questions:

1. What drives the variability of short-term interbank spreads and rates in turbulent times? Is the driving force endogenous or exogenous to the system?

2. Why is it important to identify endogenous dis-equilibrating shocks originating from within the short-term interbank market?

3. Can the liquidity shortage in the short-term interbank system be traced back in shocks in the short-term interest rate spread, the bond and currency markets and the currency swap rate?

4. Is possible to efficiently and reliably forecast (ex-post) liquidity crashes in the short-term interbank market?

Therefore, with the novel model presented in the subsequent section, the aim is to identify which out of the four time series used in the analysis drive regime changes in turbulent times. Moreover, the new model aims to identify whether external or internal shocks are responsible for the dynamics (or more precisely the transitions) of the two states (crisis and non-crisis state). By identifying the nature of the shocks which influence the system and with the use of non-linear econometric tools, the ultimate objective is to trace the path of liquidity shocks within the money market.

5.4 Data

The data used in this chapter are the historical and closing daily spreads between the US LIBOR and overnight indexed swap (OIS) rate, the daily three-month German-US bond
spread, the daily Euro-US swap spread and the EONIA rate. The data spans the period from 1st January 2002 to 30th December 2011, and includes some significant events which occurred during the last decade, such as the recent financial crisis and Eurozone crisis. Most of the recent studies assessed shorter time periods, and mainly covered the first period of the financial crisis of 2007-08. Looking at longer time horizons allows one to capture various magnitudes of a financial distress caused by self-fulfilling liquidity risk, subsequently providing a foundation in developing an early warning system to signal liquidity shocks and contagion in the short-term interbank market. Moreover, the data sets used in the study are fundamental money market rates and spreads; their long-term movement define liquidity risk, and therefore describe the market they represent.

A full description of the LIBOR-OIS spread can be found in Section 3.4.1. A discussion on the daily three-month German-US spread can be found in Section 3.4.2. The daily Euro-dollar currency swap is discussed in 3.4.3.

5.4.1 The Euro Over-night Index Average rate

The Euro Over-night Index Average (EONIA) is the overnight interbank rate and is calculated as the weighted average interest rate at which a group of highly active European banks provide unsecured, euro denominated loans to one another. The rate, seen as the overnight Euribor rate\(^{15}\), is the benchmark rate for the euro short-term money market; it is published on daily basis by the European Central Bank. Essentially, the rate indicates the stance of ECBs monetary policy (Hassler and Nautz, 2008). The time series has 2609 observations spanning the interval 1st January 2002 to 30th December 2011.

Compared to the Federal Reserve Bank’s and the Bank of England’s interest rate target, the European Central Bank (ECB) does not have a clear target \textit{per se} for the overnight interbank rate. Nevertheless, according to the ECB guidelines, the rate should vary within a corridor set by the standing facilities rates (Soares and Rodrigues, 2013). In normal market conditions, the rate is very close to that of the ECB policy rate which is less than 5% basis points (Beirne, 2012). During the financial crisis of 2007-08, the ECB tightened the standing facilities corridor. Movements in the rate can be attributed primarily to liquidity conditions (Würtz, 2003; Linzert and Schmidt, 2011), market expectations of policy decisions (Quiros and Mendizabal, 2006) and calendar effects (Bindseil et al., 2003). Some (such as Linzert and Schmidt (2011) and Välimäki (2008)) argue that there is a positive correlation between the the overnight interbank rate and the structural liquidity deficit.

---

\(^{15}\)Using market standards, a panel of European banks determine the long-term deposit rate to be used within the Euro-zone. It is considered a benchmark rate and it is supervised by the Steering Committee (EMMI, 2010).
Beirne (2012) investigates the driving forces behind the EONIA spread widening and factors which contributed to increased volatility levels during the recent financial crisis. The main argument is that liquidity risk played a major role in the persistence of heightened volatility levels. For the period 2004-08, Soares and Rodrigues (2013) use a two-regime variance model, while for the period 2008-09 an Exponential GARCH (EGARCH) specification is implemented to model the volatility of the EONIA rate. The authors use the Rational Expectations Theory to explain fluctuations of the rate. Under ordinary market circumstances, the EONIA rate should vary around the main reference rate (the EONIA spread) set by the ECB. During the financial crisis of 2007-08, the dynamics which drove the movements in the spread were the liquidity premium (owing to the fact that the EONIA is computed from unsecured transactions) and calendar effects.

In turbulent market conditions, all the spreads used in this study are good indicators of risk premiums as a result of credit-, funding liquidity-, default-, forex-, and ultimately, systemic risk. Moreover, the spreads reflect movements in interest rates on the two significant geographical markets affected by the financial crisis of 2007-08, that is the Eurozone and US market. Using the above four data sets helps identify where liquidity crises start in the short-term interbank market and trace their contagion beyond regional borders.

5.5 Methodology

To assess the validity of different multivariate regime switching models in the context of liquidity risk contagion, two models are presented and assessed:

1. Regime switching model with constant transition probabilities;

2. Regime switching model with time dependent transition probabilities.

The Hamilton filter (Hamilton, 1989) estimates the transition probabilities, while the ML estimation infers the parameters of the models. The study employs the ‘centered parameterisation’ method (adopted from Teräsvirta and Anderson (1992)) to approximate the time-varying effect of the supposedly endogenous variable, regardless of a threshold. All of the covariates used in this study, as well as the error terms are regarded as random variables and processes, which is fundamental in regime-switching Markov processes. Both the constant and time-varying transition probabilities are estimated using the logit function, which provides the base for the two-state Markov regime switching model.

\[16\] The authors developed a smooth transition autoregressive model to estimate thresholds of regime changes within the business cycle context.
The state space consists of two states: a crisis and a non-crisis (tranquil) state. Small bold letters indicate vectors, whereas matrices are denoted by bold capital letters. $t$ represents current time, while $t_0$ indicates the time when the first observation starts. Observed covariates are denoted by $x_t$, $y_t$ and $z_t$ etc., and $\Sigma$ denotes the covariance matrix. $\epsilon_t|u_t$ is the estimate of $\epsilon_t$ given $u_t$.

To make the time-varying transition probabilities endogenous, one either correlates the variables that are expected to drive the transition from one state to another to the approximated state probabilities, or explicitly incorporates the assumed influencing variables in the transition probabilities. In both scenarios, the independent variable that drives the dynamics of the transition, implicitly via the state dependent $\beta_s$ influences the impact of a variable in $X_t$ (which a vector with the covariates) (Kaufmann, 2011).

The two types of models outlined below are applied to the four short-term interbank spreads and rates. Fundamentally, the regime switching method implies that the states of the world have an ordering. By analysing recessions and expansions, Hamilton (1989) proposes that the logarithm of the real income consists of a Markov trend and a random walk process. Using this analogy on the established Equations 5.1 and 5.2, it is assumed that the LIBOR-OIS spread is the sum of a Markov trend and a random walk process, as follows:

$$\{LIBOR - OIS_t\} = n_t + z_t$$

(5.10)

where $n_t$ is a Markov trend and $z_t$ is a random walk process with the following attributes:

$$n_t = n_{t-1} + \gamma_0 + \gamma_1 S_t$$

(5.11)

where $\gamma_0$ and $\gamma_1$ are coefficients, and $S_t$ is a latent binary variable which models crisis and non-crisis periods, and

$$z_t = z_{t-1} + \epsilon_t$$

(5.12)

where $\epsilon_t$ is the error term with zero mean and $\sigma^2$ variance. The binary variable $S_t$ follows a first-order two-state autoregressive Markov regime switching process, and can take the following values:
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\[ S_t = \begin{cases} 0, & \text{if there is no crisis in the interbank market;} \\ 1, & \text{if there is a crisis in the interbank market.} \end{cases} \] (5.13)

The posterior distribution of the parameter vector $\Theta$, which contains all the parameters of the model, is defined as follows:

\[ p(\Theta \mid y) \propto p(\Theta) p(y \mid \Theta) \] (5.14)

The above representation condenses the common process of Bayesian learning, since priors join together with the likelihood to produce posterior distributions for the model coefficients (Baum, 2006).

The transition probabilities are defined as follows (by following the notation of Hamilton (1989)):

\[ Pr[s_t = 0 \mid s_{t-1} = 0] = p, \, Pr[s_t = 1 \mid s_{t-1} = 0] = 1 - p \] (5.15)
\[ Pr[s_t = 1 \mid s_{t-1} = 1] = q, \, Pr[s_t = 0 \mid s_{t-1} = 1] = 1 - q \] (5.16)

Considering the two states, the transition matrix is defined as:

\[ P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \] (5.17)

For example, in column one and row two the probability $P_{12}$ denotes the probability of switching from crisis to non-crisis state. Each column must sum up to one due to the fact that they characterise full probabilities for both the crisis and non-crisis states. If $\psi_t$ represents new information available at some time $t$, then it can be assumed that the probability of the system being in either crisis or non-crisis state is 50%, therefore $Pr(S_0 = j) = 0.5$ where $j = 1, 2$. The steady-state probabilities are calculated as follows (Perlin, 2012):

\[ p(S_0 = 0 \mid \psi_0) = \frac{1 - P_{11}}{2 - P_{11} - P_{22}} \] (5.18)
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\[ p(S_0 = 1|\psi_0) = \frac{1 - p_{22}}{2 - p_{22} - p_{11}} \] (5.19)

Thus, at time \( t = 1 \), the state probability for both states up to time \( t - 1 \) is given by:

\[ p(S_t = j|\psi_{t-1}) = \sum_{i=1}^{2} p_{ji} \left( Pr(S_{t-1} = i|\psi_{t-1}) \right) \] (5.20)

As new information arrives at time \( t \), the coefficients of the model are being updated along with the transition probabilities. Fundamentally, this is the Hamilton filter (Hamilton, 1989). The updating process runs through all observations of the model, from \( t \) to \( T \) using the formula:

\[ p(S_t = j|\psi_t) = \frac{f(y_t|S_t = j, \psi_{t-1}) \ p(S_t = j, \psi_{t-1})}{\sum_{j=1}^{2} f(y_t|S_t = j, \psi_{t-1}) \ p(S_t = j, \psi_{t-1})} \] (5.21)

The focus is on the coefficients of the model contained in the parameter vector \( \Theta = (\beta, \theta, \text{etc.}) \). The parameters are estimated using the ML procedure.

As noted earlier, the states are unknown. Using Perlin’s (2012) notation, the likelihood function is determined by:

\[ f(y_t|S_t = j, \Theta) \] (5.22)

where \( j = 1, 2 \).

As \( t \to \infty \), the maximum likelihood estimate \( \hat{\theta} \), also called the sufficient statistic, provides all available information about \( \theta \) (which is the uncertainty parameter) attainable from the data. If the length of \( t \) is short, \( \hat{\theta} \) is not going to be efficient. Moreover, if the number of parameters of the model increases and the chosen priors are non-informative, the consistency of the estimates become ambiguous and questionable (Gelman et al., 2013). It is assumed that the distribution inputed in the ML estimation is normal. There are various ways of performing ML estimation based on either:

- analytic methods, such as differentiation with respect to the coefficient vector and solving the system of equations to find the maximum;
• on grid search by trying out various estimates and see which one yields with the largest likelihood or
• on some iterative algorithm; start the iteration at some particular value until the parameters of interest converge to their true value.

Besides, a ML does not exists in all cases, however in most cases, such as in normally distributed processes, a closed form solution can be applied.

For the two states, the model is estimated by maximising the following equation with respect to all coefficients contained in $\Theta$:

$$
\ln L = \sum_{t=1}^{T} \ln \sum_{j=1}^{2} \left( f(y_t|S_t = j, \Theta) \right) \ p(S_t = j|\psi_t))
$$

(5.23)

The estimated output is inferred by filtering the coefficients to the data. Thus, the filtered probabilities are the probabilities of $S_t = j$ conditional on $\psi_t$ (which is the information at time $t$). The algorithm for the smoothed probabilities or forecasts conditional on the data at time $t$ was designed by Kim (1994). The notation follows that of Hamilton (1994b):

$$
\hat{\xi}_{t|T} = \hat{\xi}_{t|t} \odot \{ P_{T} [ \hat{\xi}_{t+1|T} \div \hat{\xi}_{t+1|t} ] \}
$$

(5.24)

where $t = 1, 2, \ldots, T$ and $P$ is the transition matrix.

Due to the fact that the spreads of interest are cointegrated, as shown in Chapter 3, the relationship between the GerUS3M spread and the LIBOR-OIS spread can be described by the following equation:

$$
\{\text{GerUS3M}_t\} = \kappa_0 + \kappa_1 s_t + \beta_1 \{\text{LIBOR} - \text{OIS}_t\} + \beta_2 \{\text{EUSWEC}_t\} + \beta_3 \{\text{EONIA}_t\} + \nu_t
$$

(5.25)

where $\nu_t$ is the error term (which has two different dynamics in the two different states). Substituting for the $\{\text{LIBOR} - \text{OIS}_t\}$ with Equation 5.10, one gets:

$$
\{\text{GerUS3M}_t\} = \kappa_0 + \kappa_1 s_t + \beta_1 n_t + \beta_1 z_t + \beta_2 \{\text{EUSWEC}_t\} + \beta_3 \{\text{EONIA}_t\} + \nu_t
$$

(5.26)
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This last equation states that during crisis and non-crisis periods, the GerUS3M spread’s behaviour is also described by a Markov trend and a random walk process.

5.5.1 A Multivariate Regime Switching System with constant transition probabilities

As a benchmark model, a multivariate regime switching model is presented where the transition from one state to another is assumed constant. In such case, the effect of the explanatory variables is irrelevant and consequently only information related to the previous state is being taken into consideration when estimating the filtered transition probabilities.

Now consider the following equation:

\[
GerUS3M_t = \beta_{1,S_t} LIBOROIS_t + \beta_{2,S_t} EUSWEC_t + \beta_{3,S_t} EONIA_t + \epsilon_{t,S_t}
\]  

(5.27)

where \( S_t \) is the state at time \( t \), with \( S_t = 1, 2 \). \( \epsilon_t \) is the vector of residuals and it is assumed to be normally distributed with zero mean and variance \( \sigma^2_{St} \). There is no constant term present in the model, for previous research (see Chapter 3) found that restricted models (with no constant) perform better. All the covariates are observed and the coefficients of interest are the \( \beta_{s} \)s and the state dependent variance \( \sigma^2_{St} \).

There are \( i = 1, 2 \) and \( j = 1, 2 \) probability cells in the transition matrix. In the case of constant transition probabilities, the transition matrix of the state variable is as follows (Filardo and Gordon, 1998):

\[
P(S_t = s_t | S_{t-1} = s_{t-1})
\]

(5.28)

\[
Q_t = \begin{pmatrix}
q_{11,t} & q_{12,t} \\
q_{21,t} & q_{22,t}
\end{pmatrix}
\]

(5.29)

where \( p = P(S_t = 1 | S_{t-1} = 1) \) and \( q = P(S_t = 0 | S_{t-1} = 0) \).

For illustration purposes, three models with constant transition probabilities are presented and compared:
Model 1: only the coefficient of the LIBOR-OIS spread \((\beta_{1,S_t})\) and the variance changes regimes. There are eight parameters to be estimated.

Model 2: it is assumed that the state dependent coefficients \(\beta_{1,S_t}, \beta_{2,S_t}\) and \(\beta_{3,S_t}\) govern the regime change in the mean equation. Moreover, it is expected that the vector of innovations is switching states too. The number of parameters to be estimated increases to ten.

Model 3: none of the coefficients of the independent variables change regimes, but only the variance. There are seven parameters to be estimated.

As a result of dealing with two states, for all three models two equations have to be solved, that is one for state 0 and one for state 1 (more precisely one for the non-crisis and one for the crisis state). Consequently, there are two variance terms with two different dynamics, such that in the non-crisis state the error term evolves according to \(\epsilon_t \sim N(0, \sigma^2_0)\) and in the crisis state \(\epsilon_t \sim N(0, \sigma^2_1)\), respectively. The higher the uncertainty in the model, the higher the variance will be.

To estimate the parameters and the transition probabilities of the three models, the Hamilton filter (Hamilton, 1989) is implemented by using the \texttt{MS_Regress} function (Perlin, 2012) in Matlab (MATLAB, 2014). The method describes changes in the coefficients of the models, named as ‘turning points’ of the time series of interest. The state-vector is non-linear and discrete. The benefit of using the Hamilton filter is that it allows approximation as well as hypothesis testing concerning the coefficients of the system. The filter uses two recursive equations (prediction and updating) when estimating the distribution of the state variable. Estimation is achieved via ML. Fundamentally, the filter calculates \(f(y|\Theta, y_{t-r+1}, \ldots, y_0)\) and maximises with respect to \(\Theta\), which is the parameter vector. Probabilistic inference about the unobserved state variable is achieved by passing of the maximum likelihood of \(\hat{\Theta}\) through the filter (Perlin, 2012).

5.5.2 A Multivariate Regime Switching System with continuous transition probabilities

Considering the nature and focus of this analysis, regime switching models with constant probabilities do not describe well interbank market behaviour. Thus, a more appropriate way of revealing the dynamics of such markets is to implement a time-varying transition probability model (TVTP). In this way, one can see whether a turning point in a particular time series is imminent, and such enables forecasting regime switches. Similarly to the constant transition probability case, the ML method is used to estimate the parameters of the model. The modified Hamilton filter (Ding, 2012) determines the time-varying transition probabilities.
In the endogenous setting, the state transitions are functions of some independent variables, which can be either the state variable or any of the covariates. In other words, the dynamics of the switching is determined and driven by the endogenous state variable or covariate (Kaufmann, 2011). The states of the space are latent and path-dependent, also they evolve according to a first-order Markov process. Similarly, the parameters of the model are time-dependent. For stronger predictive power, one could include lagged independent variables or state-dependent innovation processes. To calculate the transition probabilities, one can choose a logistic or probit specification. The logit function seems more adequate due to the fact that the logistic function has fatter tails and so do the time series of interest (Kim, 2004).

As opposed to the regime switching model with constant transition probabilities, in this setup the transition process may be driven by any of the covariates of the model. In fact, the state variables which influence the transition probabilities can be different for every probability. Moreover, the process evolves over time and the transition probabilities measure the persistance of crisis and non-crisis periods.

Assume there are two states: a crisis and a non-crisis, such that $S_t \in [1, 2]$. In the case of time-varying transition probabilities, the transition matrix calculated at each time $t$ can be estimated by a logit specification, and is written as follows:

$$P(S_t = s_t | S_{t-1} = s_{t-1}, z_t)$$ (5.30)

$$Q_t = \begin{pmatrix} q_{11,t}(z_t) & q_{12,t}(z_t) \\ q_{21,t}(z_t) & q_{22,t}(z_t) \end{pmatrix}$$ (5.31)

where the dynamics of the latent state variable $S^*_t$ will depend on variations of the information contained in crisis-indicator vector $z_t$ and on the past prevailing state. The state variable is described by the following equation:

$$S^*_t = \gamma_0 + \gamma_T z_t + \gamma_1 s_{t-1} + \epsilon_t$$ (5.32)

where $\gamma_z$ is a vector of parameters to be estimated, $z_t$ contains the covariate which is expected to influence the transition from one state to another and $\epsilon_t \sim iid N(0, \sigma^2)$. In this analysis, the covariate which is assumed to drive regime changes is the daily LIBOR-OIS spread. Following Ding’s (2012) notation, the probability function (which is not the ultimate probability) can be generated by:
\[ q_{ij,t} = \Phi(z_{ij,t} \gamma_z) \] (5.33)

where \( \Phi \) is the cumulative density function. Next, an ancillary matrix grounded on \( Q_t \) is created, as follows:

\[
R_t = \begin{pmatrix}
1 & 1 \\
1 - q_{11,t} & 1 - q_{12,t}
\end{pmatrix}
\] (5.34)

Finally, the transition probability matrix is constructed, as follows\(^\text{17}\):

\[
P_t = Q_t \cdot R_t = \begin{pmatrix}
p_{11,t} & p_{12,t} \\
p_{21,t} & p_{22,t}
\end{pmatrix}
\] (5.35)

Continuous transition probabilities guarantee that the influence of an increase in the LIBOR-OIS spread is to reduce the probability of remaining in the non-crisis state, and subsequently to increase the probability of staying in the crisis state (Perez-Quiros and Timmermann, 2000).

The three models with time-varying transition probabilities are as follows:

- **Model 1**: only the coefficient of the LIBOR-OIS spread (\( \beta_{1,S_t} \)) and the variance changes regimes. There are eight parameters to be estimated.

- **Model 2**: it is assumed that the state dependent coefficients \( \beta_{1,S_t} \), \( \beta_{2,S_t} \) and \( \beta_{3,S_t} \) govern the regime change in the mean equation. Moreover, it is expected that the vector of innovations is switching states too. The number of parameters to be estimated increases to ten.

- **Model 3**: none of the coefficients of the independent variables change regimes, but only the variance. There are seven parameters to be estimated.

### 5.6 Model comparison and model strength criteria

In this Chapter, the focus is on the multivariate endogenous regime-switching model with continuous transition probabilities. To measure the strength of such a model, a

\(^{17}\text{The Hadamart product of two matrices is constructed by elementwise multiplication. To identify the product, the two matrices must have the same dimension. Every column must sum up to 1.}\)
benchmark multivariate regime-switching model with constant transition probabilities is presented. The central idea is to highlight the importance of the endogenous covariate and its effect on transition probabilities in determining crisis and non-crisis states for the examined time interval. When deciding on the model strength, the main question asked is: Which model best explains the observed data considering the two types of transition probabilities, while also taking into consideration that for the continuous transition probabilities is assumed that an endogenous variable drives the dynamics of the system?

Individually for the two class of models, the measures critical to model outcomes are primarily based on hypothesis testing and the p-values (at 5% significance level) of the parameters under scrutiny. Under the null hypothesis, for all regression equations, the values of the parameters are assumed to be zero.

Other criteria for model strength are the transition probability values (the probability of being in a crisis-state if the previous state was a crisis one, for example) along with expected duration of regimes (in terms of days). Strength criteria will be also positioned on the interaction between the covariates of the models and parameter values.

5.7 Empirical results

The section below presents the summary statistics and the parameter estimates for the two cases outlined earlier.

5.7.1 Summary statistics

A detailed description of the summary statistics for the US LIBOR-OIS, the 3-month German-US bond spread and the euro-dollar currency swap was given in Section 3.6.1. There are 2609 observations for the LIBOR-OIS and German-US bond spreads, 2607 observations for the EUSWEC spread and 2609 observations for the EONIA rate. Table 5.1 presents the summary statistics for the four variables. The LIBOR-OIS spread presents the highest variability in the data, with a standard deviation of 40.65; observations vary between the interval $[1.91, 364.43]$. The spread is skewed to the right and has a peakedness level of 22.32. The GerUS3M, EUSWEC and EONIA spread have a standard deviation of 1.23. Similar to the LIBOR-OIS spread, in terms of long-term dynamics, the other three spreads show similar behaviour over time, as can be seen Figures 3.2 and 5.2. The width of the GerUS3M, EUSWEC and EONIA spreads however is not as large as that of the LIBOR-OIS spread. Also, the three spreads have a near 0 skewness and a level of kurtosis close to 2.
Table 5.1: Summary statistics of the LIBOR-OIS, GerUS3M, EUSWEC and EONIA spreads.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>USLIBOIS</td>
<td>28.50</td>
<td>40.65</td>
<td>1.91</td>
<td>364.43</td>
<td>3.80</td>
<td>22.32</td>
</tr>
<tr>
<td>GerUS3M</td>
<td>-0.27</td>
<td>1.23</td>
<td>-4.01</td>
<td>2.29</td>
<td>0.35</td>
<td>2.44</td>
</tr>
<tr>
<td>EUSWEC</td>
<td>2.25</td>
<td>1.23</td>
<td>0.35</td>
<td>4.35</td>
<td>-0.006</td>
<td>1.83</td>
</tr>
<tr>
<td>EONIA</td>
<td>2.23</td>
<td>1.23</td>
<td>0.295</td>
<td>4.601</td>
<td>-0.06</td>
<td>1.85</td>
</tr>
</tbody>
</table>

Figure 5.2: Behaviour of the daily EONIA rate for the period 1st January 2002 to 30th December 2011.

5.7.2 Parameter estimates of the Multivariate Regime Switching System with constant transition probabilities

Fundamentally, the regime switching model with constant transition probabilities presented in this section followed the methodology of Hamilton (1989). Figures 5.3, 5.4 and 5.5 depict the regime changes in the three models which were considered. The parameters of the models are presented in Table 5.2. The matrices with the transition probabilities and duration (measured in days as units) of states is shown in Table 5.3.

At 5% significance level, the parameters for Model 1 (when only the coefficient of the LIBOR-OIS spread and the variance switch regimes) and Model 3 (when only the variance switches regimes) are significant in identifying crisis and non-crisis periods. Thus, these two constant transition probability models identify relationships between the GerUS3M spread, LIBOR-OIS, EUSWEC and EONIA spreads. For Model 2, the coefficient of the currency swap is not significant. Compared to the non-crisis state for all three models, the Variance of the crisis state is much elevated with values 4.97,
Chapter 5 A Multivariate Endogenous Regime Switching Markov Chain Model to Trace Liquidity Shock Propagation within the Interbank Market

Figure 5.3: Model 1 - Estimation of crisis and non-crisis regimes in the GerUS3M spread. The multivariate model with constant transition probabilities switches in the coefficient of the LIBOR-OIS spread ($\beta_2$) and in the variance.

Figure 5.4: Model 2 - Estimation of crisis and non-crisis regimes in the GerUS3M spread. The multivariate model with constant transition probabilities switches in all the coefficients as well in the variance.
Table 5.2: Coeff. estimates for regime changes with constant transition probabilities.

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Non-switch. param.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EONIA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.1421</td>
<td></td>
<td>0.2807</td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.0237 (0.0000)</td>
<td>0.0353 (0.0000)</td>
<td></td>
</tr>
<tr>
<td>LIBOR-OIS</td>
<td></td>
<td>-0.0053</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td>0.0002 (0.0000)</td>
</tr>
<tr>
<td>EUSWEC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.3441</td>
<td>-0.3746</td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.0244 (0.0000)</td>
<td>0.0372 (0.0000)</td>
<td></td>
</tr>
<tr>
<td><strong>Distrib. param.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-crisis State</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model Variance</td>
<td>0.02179</td>
<td>0.0265</td>
<td>0.03565</td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.0011 (0.0000)</td>
<td>0.0010 (0.0000)</td>
<td>0.0020 (0.0000)</td>
</tr>
<tr>
<td>Crisis State</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model Variance</td>
<td>4.9683</td>
<td>0.7279</td>
<td>2.0629</td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.1912 (0.0000)</td>
<td>0.0333 (0.0000)</td>
<td>0.0710 (0.0000)</td>
</tr>
<tr>
<td><strong>Switch. param.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EONIA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-crisis State</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.1234</td>
<td></td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.0207 (0.0000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crisis state</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.5679</td>
<td></td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.2021 (0.0100)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LIBOR-OIS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-crisis State</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.0023</td>
<td>-0.0019</td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.0001 (0.0000)</td>
<td>0.0001 (0.0000)</td>
<td></td>
</tr>
<tr>
<td>Crisis State</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.0071</td>
<td>-0.0124</td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.0012 (0.0000)</td>
<td>0.0007 (0.0000)</td>
<td></td>
</tr>
<tr>
<td>EUSWEC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-crisis State</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.6215</td>
<td></td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.0212 (0.0000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crisis State</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.1629</td>
<td></td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.2002 (0.4200)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
0.73 and 2.06 respectively, denoting that actually the crisis state is much more volatile, thus reflecting uncertainty surrounding market fundamentals. Model 1 has the highest variance and Figure 5.3 displays some sensitivity with significant noise around the time interval 1600-1800 which corresponds to the interval February 2008 to mid November 2008. All transition probabilities suggest that there is at least 99% chance that a non-crisis phase will be followed by a non-crisis state; similarly, there is at least 99% chance that a crisis phase will be followed by a crisis state. To articulate this, if the interbank market is exiting a long-term crisis state, it is unlikely that the interbank market will fall back into a financial crisis. This is analogous to the notion of business cycles where recession and expansion states alternate within the economy.

The expected duration of states in terms of days varies significantly. Model 1 suggests that it is expected that non-crisis phases will last on average for 108 days, whereas crisis phases will last less, with an average of 63 days. Model 2 exhibits extreme values for the duration of states with an average of 865,655 days for the non-crisis state and 1,102 days for the crisis period. Model 3 exhibits a more balanced view of alternating states, with 208 days for the non-crisis phase and 355 days on average for the crisis period. In terms of real events which occured for the period January 2002 - December 2011 (see Table 4.3 and Figure 4.6 for comparison), the model with regime change in the mean of the LIBOR-OIS spread and variance (Model 1) identifies 7 crises. The model with
regime change in the mean of the EUSWEC, LIBOR-OIS, EONIA spread and in the variance (Model 2) identifies 3 crisis periods, whereas the model with regime change in the variance (Model 3) identifies short crisis periods around August-November 2004, November 2007, September 2008 - February 2011 and August-December 2011.

Table 5.3: Constant transition probability matrices for the three models.

<table>
<thead>
<tr>
<th>Trans. prob.</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[0.99 0.02]</td>
<td>[1.00 0.00]</td>
<td>[1.00 0.00]</td>
</tr>
<tr>
<td></td>
<td>[0.01 0.98]</td>
<td>[0.00 1.00]</td>
<td>[0.00 1.00]</td>
</tr>
<tr>
<td>St.er, p. value</td>
<td>(0.10,0.00)</td>
<td>(0.10,0.88)</td>
<td>(0.58,0.09) (0.29,1.00) (0.17,0.00) (0.15,0.98)</td>
</tr>
<tr>
<td>Exp. duration</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-crisis State</td>
<td>108.26</td>
<td>865655.12</td>
<td>208.36</td>
</tr>
<tr>
<td>Crisis State</td>
<td>63.05</td>
<td>1101.85</td>
<td>354.92</td>
</tr>
</tbody>
</table>

In the non-crisis phase for Model 2 for example, when there is a regime switch in the mean of the LIBOR-OIS, EUSWEC and EONIA spreads as well as in the variance, the relationship between the German-US bond spread and the other three spreads can be written as:

$$GerUS3M_t = -0.0019 \, LIBOROIS_t - 0.6215 \, EUSWEC_t + 0.1234 \, EONIA_t + \epsilon_t$$

(5.36)

whereas for the crisis phase the relationship can be written as:

$$GerUS3M_t = -0.0124 \, LIBOROIS_t + 0.5679 \, EONIA_t + \epsilon_t$$

(5.37)

5.7.3 Parameter estimates of the Multivariate Regime Switching System with time-varying transition probabilities

The regime switching model with continuous transition probabilities presented in this section is based on the model of Diebold et al. (1994). The parameters of the models are presented in Table 5.4. The matrices with the transition probabilities and duration
(measured in days as units) of states is shown in Table 5.5. Fundamentally, the sequence of regimes is determined by the sequence of liquidity shocks affecting the interbank market and also by the correlation of liquidity shocks with some of the covariates. In all three models presented earlier, the LIBOR-OIS spread is the endogenous variable which is presumed to drive the transition from a non-crisis state to a crisis phase (and vice versa). When the LIBOR-OIS spread crosses some threshold, regime changes are triggered within the GerUS3M spread. Figures 5.6, 5.7 and 5.8 present the behaviour of the GerUS3M spread when some of the coefficients of the model are allowed to vary over time. At 5% significance level, the coefficients of Model 2 and 3 are significant, denoting that there is a relationship between the German-US bond spread and the LIBOR-OIS, EUSWEC and EONIA spreads. However, for Model 1, the coefficient of the EONIA spread is not significant at 5% critical level. All distribution parameters are significant at 5% significance level. The level of variation is much accentuated in crisis times, with values for the three models of 0.80, 0.06 and 7.44 respectively.

Compared to the constant transition models, the persistence of the states is more balanced and it is the longest for the regime switch in the mean of the LIBOR-OIS and variance model. For the three models, the crisis phase lasts on average for 525, 108 and 210 days respectively. Model 1 and Model 3 identify two crisis periods lasting on average for 525 and 210 days. Model 2 identifies 4 crisis states lasting on average for 108 days. The major difference between the regime switching models with constant transition and the three regime switching models with time-varying transition probabilities is that in the latter the endogenous covariate drives the transition between the states.

In Model 2, all of the coefficients of the model are allowed to switch regimes and the information contained in the LIBOR-OIS spread is able to successfully forecast the transition probabilities, which is then used to calculate the duration of regimes. The relationship between the German-US bond spread and the other three spreads can be written as:

$$ GerUS3M_t = -0.0084 \text{LIBOROIS}_t + 0.5003 \text{EUSWEC}_t - 0.6477 \text{EONIA}_t + \epsilon_t $$

(5.38)

whereas for the crisis phase the relationship can be written as:

$$ GerUS3M_t = -0.0188 \text{LIBOROIS}_t + 1.4053 \text{EUSWEC}_t - 0.6757 \text{EONIA}_t + \epsilon_t $$

(5.39)
Chapter 5 A Multivariate Endogenous Regime Switching Markov Chain Model to Trace Liquidity Shock Propagation within the Interbank Market

Figure 5.6: Model 1 - Estimation of crisis and non-crisis regimes in the GerUS3M spread. The multivariate model with time-varying transition probabilities switches in the coefficient of the LIBOR-OIS spread ($\beta_2$) and in the variance.

Figure 5.7: Model 2 - Estimation of crisis and non-crisis regimes in the GerUS3M spread. The multivariate model with time-varying transition probabilities switches in all the coefficients of the model as well in the variance.
Table 5.4: Coeff. estimates for regime changes with time-varying transition probabilities

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Non-switch. param.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>EONIA</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.0272</td>
<td>0.1731</td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.0592 (0.6450)</td>
<td>0.0189 (0.0000)</td>
<td></td>
</tr>
<tr>
<td><strong>LIBOR-OIS</strong></td>
<td></td>
<td>-0.0003</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td></td>
<td>0.0000 (0.0014)</td>
<td></td>
</tr>
<tr>
<td><strong>EUSWEC</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.4056</td>
<td>-0.6829</td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.0599 (0.0000)</td>
<td>0.0197 (0.0000)</td>
<td></td>
</tr>
<tr>
<td><strong>Distrib. param.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Non-crisis State</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model Variance</td>
<td>0.2809</td>
<td>0.9563</td>
<td>0.0425</td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.0074 (0.0000)</td>
<td>0.0374 (0.0000)</td>
<td>0.0009 (0.0000)</td>
</tr>
<tr>
<td><strong>Crisis State</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model Variance</td>
<td>0.8043</td>
<td>0.0573</td>
<td>7.4365</td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.0518 (0.0000)</td>
<td>0.0027 (0.0000)</td>
<td>0.7188 (0.0000)</td>
</tr>
<tr>
<td><strong>Switch. param.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>EONIA</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-crisis State</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>-0.6477</td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td></td>
<td>0.1150 (0.0000)</td>
<td></td>
</tr>
<tr>
<td>Crisis state</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>-0.6757</td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td></td>
<td>0.0618 (0.0000)</td>
<td></td>
</tr>
<tr>
<td><strong>LIBOR-OIS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-crisis State</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.0016</td>
<td>-0.0084</td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.0004 (0.0000)</td>
<td>0.0005 (0.0000)</td>
<td></td>
</tr>
<tr>
<td>Crisis state</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.2966</td>
<td>-0.0188</td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td>0.0051 (0.0000)</td>
<td>0.0003 (0.0000)</td>
<td></td>
</tr>
<tr>
<td><strong>EUSWEC</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-crisis State</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>0.5003</td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td></td>
<td>0.1137 (0.0000)</td>
<td></td>
</tr>
<tr>
<td>Crisis State</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>1.4053</td>
<td></td>
</tr>
<tr>
<td>St. error (p. value)</td>
<td></td>
<td>0.0630 (0.0000)</td>
<td></td>
</tr>
</tbody>
</table>
For all three models, the transition probabilities suggest that there is 100% chance that a non-crisis phase will be followed by a non-crisis state; similarly, there is 100% chance that a crisis phase will be followed by a crisis state.

5.8 Conclusions, contributions and limitations

Fundamentally, financial crises go hand in hand with amplified interest rates and elevated spread levels. According to the stylised facts, the variability of short-term interest rates is time dependent and one way to model time-varying volatility is to implement a regime switching model with time-varying transition probabilities (Kim, 2004). Such models outperform single regime models and volatility models (the family of GARCH models, for example) where the estimated parameters indicate explosive variances and are constant over time; therefore, these cannot capture the true dynamics of money market rates and spreads (Dahlquist and Gray, 2000).

However, increased variation in rates does not necessarily lead to a financial crisis. Policy decisions do disturb rates and asset prices, however the market generally quickly stabilises. What is specific to crisis periods is that they gather impetus from endogenous reactions of market participants (Danélsson, 2011). Consequently, this will be reflected
Table 5.5: Time-varying transition probability matrices and expected duration of regimes for the three models.

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>pa{1,1}</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.3019</td>
<td>0.2667</td>
<td>0.2906</td>
</tr>
<tr>
<td>St.er, (p. value)</td>
<td>0.0652 (0.0000)</td>
<td>0.0000 (0.0000)</td>
<td>0.0952 (0.0023)</td>
</tr>
<tr>
<td><strong>pa{1,2}</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.3450</td>
<td>-0.2469</td>
<td>-0.2871</td>
</tr>
<tr>
<td>St.er, (p. value)</td>
<td>0.0181 (0.0000)</td>
<td>0.0352 (0.0000)</td>
<td>0.0161 (0.0000)</td>
</tr>
<tr>
<td><strong>Trans. prob.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[1.00 0.0000]</td>
<td>[1.00 0.0000]</td>
<td>[1.00 0.0000]</td>
<td></td>
</tr>
<tr>
<td>[0.0000 1.00]</td>
<td>[0.0000 1.00]</td>
<td>[0.0000 1.00]</td>
<td></td>
</tr>
<tr>
<td><strong>Exp. duration</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-crisis State</td>
<td>267.23</td>
<td>149.81</td>
<td>222.35</td>
</tr>
<tr>
<td>Crisis State</td>
<td>525.33</td>
<td>107.79</td>
<td>209.88</td>
</tr>
</tbody>
</table>

in volatile rates and spreads. The financial crisis of 2007-08 is a perfect example of a self-fulfilling feedback mechanism that feeds on endogenous liquidity risk. In other words, liquidity shocks are meaningful and the state of the economy reacts to such endogenous behaviour. The above analysis reveals a specific characteristic of endogenous liquidity risk, namely that small liquidity shocks induce large variation in market rates and asset prices. The Markov process presented above shows that new information arriving into the market is smoothly incorporated into the realisation of shocks via a non-linear framework. This framework proved optimal compared to exogenous regime switching models and therefore the implications for Basel III are significant. Thus, the uncertainty that drives liquidity dynamics in crisis periods is better explained as being endogenous rather than exogenous.

Under normal market circumstances, movements in the LIBOR-OIS spread, German-US bond rate, Euro-dollar currency swap and EONIA rate should reflect market fundamentals. Such is the Rational Expectations Theory and the Business Cycle Theory, which explain the yield curve of these spreads.
All previous research in the field of interbank behaviour hypothesises on exogenous processes that affect financial behaviour. In such cases, analyses cannot propose practical policy advice. On the other hand, recent investigations in the field of macroeconomic policy acknowledge that policy change behaviours (such as shocks) are in fact endogenous, such that these respond methodically to variations in the macroeconomic environment (Davig and Leeper, 2006; Kaufmann, 2011). Two econometric methods were adopted to reveal regime changes in the short-term interbank market. In the first method, the moments of the model were constant, whereas in the second method the conditional distribution of money market rates were allowed to vary in line with the state of the economy. This model is able to jointly approximate the coefficients of the model and the conditional evolution of crisis and non-crisis states. The results of this study indicate that short-term interest rates and spreads fully reveal regime switching information. If transition probabilities are state and time dependent, regime changes will be a function of the level of the LIBOR-OIS spread. Moreover, changes in the mean and variance of the LIBOR-OIS spread has a drastic and everlasting effect on the short-term interbank time series; one can conclude that a change to the high-volatility state might be driven by an increased level of the LIBOR-OIS spread. In other words, liquidity shocks originating from movements of the LIBOR-OIS spread drives regime changes in the German-US bond spread.

Due to the fact that TVTP models are able to estimate crisis and non-crisis periods, policy makers and market players can predict when such periods are likely to end. Consequently, policy makers are able to adjust their ways of dealing with economic decisions.

Endogenous time-varying regime switching represents a new method by which the development of market expectations matters, specifically in regulating and managing the impacts of interbank liquidity shocks.

As an improvement, the model could have included autoregressive components. Moreover, a possible extension of the present study would be to determine thresholds with various intensities, so that when predetermined financial variables cross some limit, interbank policy rules would adjust and subsequently aid the uninterrupted functioning of the financial market.
Chapter 6

Conclusions

The Thesis consists of three distinct, yet linked investigations. The first study analysed causality, interdependence and equilibrium relationships among the times series of interest. The study revealed that there is causality running in both ways between the US LIBOR-OIS, German-US bond spread and Euro-dollar currency swap. Important long- and short-run relationships were established, however they were affected by structural breaks. Moreover, it was determined that the LIBOR-OIS spread is a leading variable, as it has proved to efficiently forecast the path of the German-US bond spread and that of the Euro-dollar currency swap (which are named lagging variables).

In the second study, a novel autoregressive Markov regime switching model was introduced to trace liquidity crises in the US LIBOR-OIS spread. By using various thresholds, the persistence of crisis and non-crisis intervals was established. Also, the model was successful in forecasting ex-post liquidity crises in the LIBOR-OIS spread. In this study we observed outcomes of a decision-making process governed by probability rules, which classified the outcomes into either a crisis or a non-crisis state. Yet, the model did not reveal the ‘internal decision’ process.

In the third analysis, an endogenous liquidity risk contagion model was introduced to determine what drives liquidity shock propagation within the short-term interbank market. It is acknowledged that modelling uncertainty is one of the complex tasks an econometrician may encounter. It is vital to recognise the underlying dynamics of the process of interest. In circumstances where situations similar to gambling are to be modelled, exogeneity as an assumption is sufficient. However, when the dynamics of the system imply self-fulfilling processes, one must take into consideration the endogenous nature of uncertainty, as endogenous shocks can cause system-wide effects. This study proved that for the interval January 2002 to December 2011, the identified financial crises had endogenous liquidity risk at their heart as opposed to present risk models which advocate that financial risk originates from outside the system, meaning it is exogenous. Moreover, in this study the transition probabilities revealed the decision rules, which
ultimately uncovered what drives the outcomes being ordered into crisis or non-crisis states.

The contribution of this Thesis is twofold. Empirically, there are several contributions:

1. The majority of the literature implement simple linear models to investigate liquidity risk and contagion (such as Hui et al. (2009), Frank et al. (2008), Hui et al. (2011), Barth et al. (2012), Gideon (2012) and Gerlach and Lewis (2014), among others). These models cannot describe financial crises, for such events are characterised by drastic changes in the mean or volatility, nor can they estimate the propagation of asymmetric shocks.

2. This study provides some macroeconomic foundation for financial spillovers as opposed to Allen and Gale (2000b) who model the microeconomic underpinnings of such events.

3. The analysis illustrates the aggregate implications of liquidity shocks propagation within the short-term interbank market.

4. The multivariate regime switching model is a dynamic representation of real life market outcomes. This is in contrast with the majority of literature which implements either a static non-Markovian approach or an exogenous regime switching approach (such as Bussiere and Fratzscher (2006) who developed an exogenous regime switching EWS).

5. The present analysis introduced the idea of endogenous regime switching in the short-term interbank market, which was non-existent in the finance and banking literature. Such regime switching models outperform random walk and constant transition probability models.

From a theoretical viewpoint, there are several contributions:

1. The study combined the two core characteristics of the Business Cycle Theory. First, the investigation depicted the causes of cycles (regimes) in form of liquidity shocks originating from the LIBOR-OIS spread, which had devastating effects on the other short-term interbank rates and spreads.

2. The multivariate regime switching model revealed that liquidity shocks originate from within the system (endogenously) due to a self-fulfilling feedback mechanism which amplifies the outcomes in the form of crisis and non-crisis intervals. Therefore, motivated by the analogy observed between business cycle dynamics and occurrences of financial crises, the study unequivocally justified the state dependence of liquidity shocks and their propagation within the interbank market.
3. The study demonstrated that financial contagion can be modelled as an equilibrium phenomenon, which is in line with the work of Allen and Gale (2000b) and Freixas et al. (2000). The multivariate regime switching model with time-varying transition probabilities is supported by the Dynamic Stochastic General Equilibrium Theory, in the sense that the time element and changes from one equilibrium to another are crucial. The German-US bond spread depends on variations of the LIBOR-OIS, EUSWEC and EONIA rate. The system is in equilibrium and the setup is stochastic, as random shocks originating from the LIBOR-OIS spread disturb the short-term interbank market.

6.1 Limitations and further research

There are several limitations of the present study and they can be addressed in future research:

- Various macroeconomic variables could have been added to the data sets;
- Forecasting liquidity crashes in the LIBOR-OIS spread for the future and not just ex-post;
- A Markov switching Stochastic Volatility model could have been adopted to powerfully capture volatility clusters in a regime switching setting;
- The development of an endogenous regime switching model with time-varying transition probabilities using the Gibbs sampler for more powerful parameter estimates;
- A possible extension of the endogenous regime switching model would be to determine thresholds with various intensities, so that when predetermined financial variables cross some limit, interbank policy rules would adjust and subsequently aid the uninterrupted functioning of the financial market.
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