Ultralow loss and wide bandwidth
hollow-core photonic bandgap fibres
for telecom applications

by

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ABSTRACT

FACULTY OF PHYSICAL SCIENCE AND ENGINEERING
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ULTRALOW LOSS AND WIDE BANDWIDTH HOLLOW-CORE PHOTONIC BANDGAP FIBRES FOR TELECOM APPLICATIONS

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Light guidance in air has significant potential in diverse photonics applications, one of which is optical communications where it may be key to achieving lower attenuation and optical nonlinearity than in conventional silica fibres. This thesis presents research conducted as part of the EU FP7 project MODEGAP, and is concerned with the design of hollow-core photonic bandgap fibres (HC-PBGFs) with low loss and wide bandwidths suitable for high capacity data transmission.

In these fibres, loss is dominated by scattering from surface roughness and is subject to the design of the fibre cross-section. Using the criterion of reduced guided mode-field intensity at the interfaces, we conduct detailed finite element simulations which allow to identify preferable structures. A theory of light scattering from surface roughness in HC-PBGFs is then derived, and expressions are obtained which combine statistical information on the roughness and the guided mode-field overlap with scattering surfaces to predict the far-field scattering pattern and the fibre loss.

A model based on mass conservation is proposed to predict the properties of HC-PBGFs from knowledge of the preforms from which they are made, and this in turn allows optimizing the design of such preforms. A method allowing accurate modelling of fabricated HC-PBGFs from scanning electron micrographs of their cross-sections is devised and such simulations indicate that structural distortions in the fibre cross-section cause higher field intensities near the interfaces, and hence higher losses. Systematic studies of distortions are then conducted, and it is found that not all distortions are equally detrimental. Combining these findings and using realistic estimates, a HC-PBGF design with 37 cell core defect and loss as low as $0.2\,\text{dB/km}$ over $580\,\text{nm}$ of bandwidth near the wavelength of $2\,\mu\text{m}$ is presented.
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Declaration of Authorship

I, Eric R. Numkam Fokoua, declare that the thesis entitled *Ultralow loss and wide bandwidth hollow-core photonic bandgap fibres for telecom applications* and the work presented in the thesis are both my own, and have been generated by me as the result of my own original research. I confirm that:

- this work was done wholly or mainly while in candidature for a research degree at this University;
- where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated;
- where I have consulted the published work of others, this is always clearly attributed;
- where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;
- I have acknowledged all main sources of help;
- where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;
- parts of this work have been published in the journal articles and conference proceedings reported in the List of Publications at the end of the thesis.

Signed: ...............................................................................................

Date: ...............................................................................................
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A word of gratitude is due to my colleagues who worked on fibre fabrication as interaction with them sparked many of the ideas presented in the thesis.

Finally, I want to thank my wife without whom this journey may have never started.
## Nomenclature

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<td>19c</td>
<td>Nineteen cell</td>
</tr>
<tr>
<td>37c</td>
<td>Thirty-seven cell</td>
</tr>
<tr>
<td>7c</td>
<td>Seven cell</td>
</tr>
<tr>
<td>AFM</td>
<td>Atomic force microscope</td>
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<tr>
<td>ARROW</td>
<td>Antiresonnant reflective optical waveguide</td>
</tr>
<tr>
<td>ARS</td>
<td>Angularly resolved scattering</td>
</tr>
<tr>
<td>BW</td>
<td>Bandwidth</td>
</tr>
<tr>
<td>DGD</td>
<td>Differential group delay</td>
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<tr>
<td>FEM</td>
<td>Finite element method</td>
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<tr>
<td>FM</td>
<td>Fundamental mode</td>
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<tr>
<td>GVD</td>
<td>Group velocity dispersion</td>
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<tr>
<td>HC-PBGF</td>
<td>Hollow-core photonic bandgap fibre</td>
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<tr>
<td>HOM</td>
<td>Higher order mode</td>
</tr>
<tr>
<td>ID</td>
<td>Inner diameter</td>
</tr>
<tr>
<td>IP</td>
<td>Internet protocol</td>
</tr>
<tr>
<td>MGDM</td>
<td>Mode group division multiplexing</td>
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<tr>
<td>MIMO</td>
<td>Multiple-input multiple-output</td>
</tr>
<tr>
<td>MODEGAP</td>
<td>Multi-mode capacity enhancement with photonic bandgap fibres</td>
</tr>
<tr>
<td>MOF</td>
<td>Microstructured optical fibre</td>
</tr>
<tr>
<td>OD</td>
<td>Outer diameter</td>
</tr>
<tr>
<td>PBG</td>
<td>Photonic bandgap</td>
</tr>
<tr>
<td>PML</td>
<td>Perfectly matched layer</td>
</tr>
<tr>
<td>PSD</td>
<td>Power spectral density</td>
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<tr>
<td>RSM</td>
<td>Rod surface mode</td>
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<tr>
<td>SCW</td>
<td>Surface capillary wave</td>
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<tr>
<td>SDM</td>
<td>Spatial division multiplexing</td>
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<tr>
<td>SEM</td>
<td>Scanning electron micrograph</td>
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SL  Square lattice
SM  Surface mode
SMF Single mode fibre
SSM Strut surface mode
TLH triangular lattice of holes
TLR triangular lattice of rods
WDM Wavelength division multiplexing
ZDW zero dispersion wavelength
Chapter 1

Introduction

1.1 Motivation

Our ability to confine and guide lightwaves through optical fibres over thousands of kilometers has revolutionized the way we communicate, leading to an explosive growth in global data traffic. So phenomenal has been the growth that it is now estimated global IP traffic will reach 1.1 zettabytes\footnote{1 zettabyte = \(10^{21}\) bytes} per year by the end of 2016\cite{1}. That is the equivalent of 200 billion 5 gigabytes DVD discs worth of data exchanged in a single year.

This revolution was made possible in part by the unprecedented control over the purity of silica glass which is used to make the single mode optical fibres shuttling most of the data across the globe in the form of light pulses. It was indeed the recognition by Charles Kao that loss in optical fibres was dominated by impurity scattering and that its fundamental limit was probably below 20\(dB/km\) that sparked global interest and research into their practicality for optical communications \cite{2}. Subsequently, these research activities rapidly bore fruit, leading to the loss in optical fibres reduced to 0.2\(dB/km\) in just over a decade \cite{3, 4}. The ability to produce very long lengths of fibres with such low losses and the high optical frequencies of the carriers have established single mode fibres as the dominant long-haul telecom medium. In parallel, developments in light sources and detectors as well as suitable optical amplifiers completed the essentials of the physical layer on which modern telecommunication systems now rely.
It is increasingly emerging however, that current systems may not be able to cope with the explosive growth rate of $\sim 40\%$ year-on-year in the demand for bandwidth and capacity [1, 5, 6]. Indeed, as more bandwidth-hungry applications such as social networking, high definition video streaming, real-time video conferencing, etc... are being developed and increasingly incorporated into daily life, it is predicted that the current systems will soon reach their fundamental capacity limit estimated to be $\sim 35Tbit/s$ for a $2000km$ reach [7]. This limit is imposed on the one hand by the optical signal to noise ratio (OSNR) at the receiver, and on the other by another intrinsic property of the silica glass itself, namely its instantaneous optical Kerr nonlinearity [8, 9]. Although sometimes useful and exploited for the purposes of spectral engineering, phase sensitive amplification schemes and all-optical signal processing, the presence of optical nonlinearity is detrimental in data-carrying fibres as it leads to interaction between optical waves propagating in the same fibre. The net effect is a restriction on the signal powers that can be launched into a fibre, thereby limiting the achievable OSNR, spectral efficiencies and ultimately the transmission capacity [10].

As laboratory experiments and field trials are demonstrating results close to the capacity limit of current fibre technology, it is clear that new fibre links must be deployed in order to avert the eventual capacity gridlock, and this is a major opportunity for innovation both at the physical layer and on information coding schemes. Many ideas have been put forth in this sense and are being actively investigated by many research groups worldwide. The most prominent among these is the use of spatial division multiplexing (SDM), whereby a single fibre may incorporate a number $N$ of cores rather than a single one, resulting in an $N$-fold increase in capacity, or alternatively where data is transmitted over each of the guided modes of a multimode fibre [11]. Although several works have demonstrated unprecedented capacities and spectral efficiencies using SDM, the technology is yet to mature and is challenging in many respects as it requires devising means of exciting and detecting signals from individual cores or guided modes, mitigating intermodal or inter-core cross-talk and developing amplifiers capable of providing equal gain to all the cores or modes.

Fundamental improvements in the fibre itself including further reduction in fibre loss, development of broader bandwidth amplifiers and suppression or mitigation of optical nonlinearity will be of significant advantage to current systems or future ones exploiting SDM. As no further improvement in the loss or nonlinearity is realistically possible in conventional silica fibres, it is imperative that alternative
fibre materials or means of completely re-engineering fibre properties become key research objectives.

Interestingly, a new class of optical fibres commonly known as microstructured optical fibres (MOFs) have emerged over the past two decades. Over their relatively short history, microstructured fibres have proved capable of extending and surpassing properties and characteristics such as chromatic dispersion, effective optical nonlinearity, etc, achievable in conventional optical fibres [12, 13]. This class of fibres owe their name to the fact that in their transverse cross-section, they most commonly consist of a wavelength-scale arrangement of one material (usually air) embedded into a host one (usually silica). In such structures, light guidance can be fundamentally different from the total internal reflection of SMFs. Indeed, Birks et al. [14] predicted theoretically in 1995 that air-silica microstructured fibres could be designed to make possible light guidance in an air core via the photonic bandgap effect. Just four years later, the first hollow-core photonic bandgap fibre (HC-PBGF) in which the cladding contained a periodic arrangement of air holes in a silica matrix was successfully demonstrated by Cregan et al. [15]. Since then, HC-PBGFs have attracted tremendous research interest, fueled by the prospect of reducing their propagation loss to levels below conventional SMFs (which stands at $\sim 0.15\, \text{dB/km}$, see [16, 17]). The fact that over 99% of the optical power carried by the modes in HC-PBGFs is guided in air results in ultra-low optical nonlinearity, which together with the prospect of low loss and the potential for SDM would make these fibres ideal candidates as next-generation data-carriers.

Although the physical mechanisms giving rise to loss in conventional fibres such as Rayleigh scattering and multiphonon absorption are either absent or expected to play a negligible role in the attenuation of HC-PBGFs, the record low-loss reported in 2005 is still an order of magnitude higher than in SMFs [18, 19]. While relatively high attenuation levels have not prevented HC-PBGFs from finding application in various areas of photonics such as optical pulse compression or high power short pulse delivery [20, 21], gas-based nonlinear optics [22, 23] and metrology, they have proved too high for direct use in long-haul optical data transmission links. Scientists in the field soon realized that because of their unique features, loss in HC-PBGFs was limited by yet another scattering mechanism, this time from the many air-glass interfaces present in the structure [19]. These surfaces, as it turns out, possess an intrinsic nanoscale roughness which cannot be entirely suppressed because of its thermodynamic origin. Surface capillary waves present
on the surface of molten glass as the fibres are manufactured inevitably freeze in when the glass solidifies and give rise to this inherent roughness which then causes the scattering \[24, 25\]. In \[19\], Roberts et al. have shown that losses of $1.7 \sim 1.2\, \text{dB/km}$ are already limited by scattering from surface roughness, despite the roughness mean amplitude being as small as $0.1\, \text{nm}$.

Fortunately, the flexibility in tailoring the properties of HC-PBGFs means that surface roughness scattering can be substantially reduced through careful fibre design. Despite the lack of a thorough understanding of the scattering process in HC-PBGFs, existing works in the literature document very well that enlarging the size of the hollow-core or incorporating antiresonant features at its boundary to expel the guided mode field from the surfaces are all valid avenues through which the fibre loss can be further reduced \[20\]. While successful in lowering loss values, these approaches suffer from the drawbacks of introducing an impractically large number of air-guided modes or a severe reduction in the usable bandwidth due to the presence of modes guided in features on the glass surround of the core defect.

**Research aims**

Nearly four years ago, the University of Southampton in conjunction with seven other international organisations started collaborative work on an EU-funded FP7 project titled "Multi-mode capacity enhancement with photonic bandgap fibres" or MODEGAP for short. This ambitious project targeted a 100-fold enhancement in the capacity of broadband core networks with an order of magnitude increase resulting from the development of SDM over the fibre’s multiple guided modes and another order of magnitude increase resulting from ultralow loss, increased bandwidth, and ultralow nonlinearity of HC-PBGFs.

Specifically, MODEGAP aimed to demonstrate HC-PBGFs with loss values of $\sim 0.1\, \text{dB/km}$ at appropriate wavelengths, a target necessary to position HC-PBGFs as a credible alternative to conventional fibres in long-haul data transmission. To this end, it was clear that a breakthrough in the understanding of the surface scattering process in HC-PBGFs and the identification of physical parameters which critically influence the loss were essential. Attaining these objectives required developing theoretical models capable of accurately describing light propagation phenomena in these fibres. This task is made difficult by the complexity
and very fine size of the features present within the fibre cross-section, as well as the large differences in optical properties of the materials involved.

Undertaken within the framework of the MODEGAP project, this doctoral thesis sought to develop theoretical tools capable of accurately describing light propagation and loss mechanisms in HC-PBGFS, with the end goal of identifying realistically feasible designs for ultra-low loss and wide bandwidth operation. Therefore, while it aimed at answering basic scientific questions about the physics of light guidance in HC-PBGFs, this work was carried out with clear objectives in mind along the steps of what may be seen as an engineering design loop. These specifically included:

- Using existing criteria and methods to identify fibre designs with potential for low-loss and wide bandwidth,
- Develop a comprehensive theory for accurate description of light scattering from surface roughness, perform comparisons with existing methods and evaluate its implications in terms of the designs identified earlier,
- Provide theoretical guidance to colleagues working on fabrication and suggest methods through which optimum designs may be attained in practice,
- Develop a platform capable of accurate modelling of the properties of fabricated fibre samples, analyse their loss and bandwidth properties and compare with earlier predictions,
- Combine all the findings to propose improved fibre designs with potential to meet the project requirements.

Accomplishing these objectives has required interaction and collaboration with several researchers within the Microstructures Optical Fibre group here at the Optoelectronics Research Centre and their contribution to the work presented in this thesis is explicitly acknowledged where appropriate.

1.2 Thesis outline

Chapter 2 covers basic background information about HC-PBGFs. Our current understanding of photonic bandgap guidance in these fibres is discussed at length and
their basic key properties clearly introduced. We discuss in particular the mechanisms giving rise to loss in HC-PBGFs with an emphasis on surface roughness scattering. A critical quantity known as the normalized interface field intensity which has been widely adopted to describe roughness scattering in HC-PBGFs and differentiate between fibre designs is derived, and a heuristic calibration of this quantity to obtain the observed losses suggested. An introduction to the numerical modelling of HC-PBGFs is also covered with a discussion of the specific issues faced when simulating the properties of these fibres. A brief overview of the available numerical tools for such studies is presented, followed by a parametric description of idealised HC-PBGFs and the specific steps involved in modelling their properties with the finite element method, which we chose for all the work in the thesis.

In Chapter 3 a broad discussion on how the key properties of HC-PBGFs are impacted by structural parameters is presented. This is done by distinguishing between the impact of the periodic cladding and that of the core defect on the overall optical performance of the fibres. Results of detailed simulations performed on idealised fibres are presented in the form of contour maps which provide a visual and straightforward means of identifying the region of the parameter space to target for optimum operation. The criterion used to define low-loss here is that of reduced field intensity at the air-glass interfaces. The impact of the core defect size and boundary design is then examined.

Chapter 4 presents a theory of light scattering from surface roughness in HC-PBGFs. Starting from dipole radiation, expressions are derived which combine statistical information on the roughness, mode field distribution and fibre geometry to predict both the far-field scattering distribution and the fibre’s loss. The proposed model shows good qualitative agreement with reported experimental data and correctly predicts the wavelength dependence of the scattering loss. Loss prediction with the assumption that roughness is dominated by frozen-in surface capillary waves with an imposed frequency cut-off are shown to be within a factor of 2 of measured ones. A detailed discussion of the impact of the roughness power spectral density is then presented.

In Chapter 5 a simple theory aimed at predicting a fibre’s structural parameters from those of the preforms from which it is made is described. With the knowledge of two simple parameters that are easily tracked during fibre manufacture, the model relies on mass conservation to provide estimates of strut thicknesses and node size in the resulting fibres. This information is then combined with the
property maps of Chapter 3 to quickly estimate the spectral position and width
of the photonic bandgap of the fabricated fibres. This has the potential for real-
time prediction of the properties of fabricated fibres and eliminates the need to
perform on-site transmission measurements during fibre manufacture in order to
adjust fabrication parameters.

Chapter 6 presents a novel method allowing one to model the performance of fab-
ricated fibre samples. The properties of HC-PBGFs are known to be influenced
by structural details beyond the resolution of scanning electron microscope (SEM)
images. Our approach circumvents this problem by developing a method of re-
producing the fibre geometry from SEM images that makes use of two adjustable
parameters. Simulation results achieve for the first time a remarkable agreement
with measured loss and surface mode position. This leads to the realization that
the structural distortions present in the fibres cross-section have a significant im-
 pact on the fibres’ loss.

Chapter 7 presents systematic studies of the impact of structural distortions in HC-
PBGFs. We find that although mostly detrimental to the loss, some distortions
such as enlarged core defects and a rearrangement of glass nodes on the core
boundary may lead to further loss reduction. These findings are combined to
propose new and realistically feasible designs which in addition to low-loss and
wide bandwidths are also more resilient to the introduction of surface modes.

Finally, Chapter 8 addresses the conclusions from all this work and suggests av-
 enues for future investigations.
Chapter 2

Background: Introduction to hollow-core photonic bandgap fibres

Microstructured optical fibres (MOFs) are arguably one of the most exciting developments in optical fibre technology over the last two decades, reviving a line of scientific inquiry within a field industry perceived as mature with little left to discover or innovate upon. Although early signs of this class of optical waveguides date as far back as the 1970s with the single material fibre from Kaiser et al. [27] or the Bragg fibre of Yeh et al. [28], the development of MOFs truly took off in the early 1990s when the then recent ideas of photonic crystals and bandgaps [29, 30] inspired proposals for optical fibres with a photonic crystal cladding. Several years were needed to turn these ideas into practical devices [31] and since, fabrication techniques for microstructured fibres have become very sophisticated, allowing the fabrication of air-silica structures with remarkable complexity and accuracy. The wide variety of fibre structures achievable using these fabrication techniques, some examples of which are shown in Fig. 2.1, have offered an unprecedented flexibility in engineering key optical properties such as dispersion, optical nonlinearity, and birefringence. If these can be viewed as an extension of the range of properties of standard optical waveguides, microstructured fibres also brought about radically new and unique possibilities such as the guidance of light in a hollow-core via the photonic bandgap effect.

This possibility of light guidance in a hollow-core is particularly exciting as it implies a number of attractive properties such as high power handling capabilities, ultralow optical nonlinearities, near vacuum latency data transmission and above all the prospect of ultralow transmission losses. Fuelled by this prospect, which is
justified by the mechanisms limiting loss in solid fibres being either absent or highly suppressed in an air-filled fibre core, an important part of the research on hollow-core photonic bandgap fibres has sought to reduce the propagation losses to levels comparable or below that of conventional fibres which now stand at $\sim 0.15\,dB/km$ \[16, 17\]. The MODEGAP project, which this doctoral thesis was part of, sought to minimize the losses in HC-PBGFs, which together with the thousandfold reduction in optical nonlinearity would make them an ideal contender for next generation data-carrying fibres. Before presenting the main findings from this work, I start in this chapter by reviewing the current understanding of the physics of light guidance in HC-PBGFs and introduce the main optical properties of these fibres. A brief overview of the numerical tools available for their study, with particular focus on the finite element method which is used for the work, is then presented.

Figure 2.1: Sample MOFs reported in literature. (a) Index guiding endlessly single mode fibre [31] (b) Highly birefringent MOF [32] (c) Six air-holes suspended core holey fibre [33] (d) High numerical aperture air-clad fibre (e) Honeycomb photonic bandgap fibre [34, 35] (f) Low-loss triangular lattice hollow-core photonic bandgap fibre [36] (g) Air-silica Bragg fibre [37] and (h) Square lattice hollow-core photonic bandgap fibre [38]
2.1 Guidance and modes in conventional fibres

Before embarking on the description of our current understanding of photonic bandgap guidance, it is important to first review guidance in more conventional fibre types. The most common fibre type is the so-called step-index fibre which consists basically of a thin, very long and very flexible cylindrical strand of layered material, usually glass. Figure 2.2 shows an illustration of the structure of a typical step-index fibre. At the centre is a high refractive index core whose diameter typically ranges from a few to a few tens of micrometres depending on the intended application. The core is immediately surrounded by a lower index cladding. These are then enclosed in a plastic coating layer added for mechanical protection.

Light guidance is achieved in a step-index fibre by virtue of total internal reflection. Since the core material has a refractive index higher than that of the cladding, a meridional ray of light (i.e in a plane containing the fibre axis) incident upon the core cladding interface is completely reflected back into the core if the angle of incidence ($\theta_i$) is larger than the critical angle defined from Snell’s law as:

$$n_{co} \sin (\theta_c) = n_{cl} \sin \left( \frac{\pi}{2} \right)$$

$$\theta_c = \arcsin \left( \frac{n_{cl}}{n_{co}} \right)$$

(2.1)

where $n_{co}$ and $n_{cl}$ are the core and cladding refractive indices respectively.

![Illustration of the structure of a conventional step index fibre](image)

Figure 2.2: Illustration of the structure of a conventional step index fibre: Light is trapped inside the core via total internal reflection.
It may appear as though an infinite number of rays with angles $\theta_i \geq \theta_c$ are able to be guided in the core, but it is not so. As rays propagating at the same angle $\theta_i$ follow the zigzag path down the fibre, all associated wavefronts must remain in phase. In other words, the electromagnetic field must form a standing wave in the plane perpendicular to the fibre axis. This requirement results in the angle $\theta_i$ accepting only a finite number of discrete values which are dictated by the fibre’s dimensions (core radius $a$), refractive indices and the wavelength ($\lambda$) of the incident lightwave. Rays associated with each discrete value form a mode. A mode is essentially a spatial distribution of the electromagnetic field which except for a phase factor, remains constant as it propagates down the fibre. The number of guided modes is obtained from the normalised frequency parameter as \[2.2\]:

$$N \approx \frac{V^2}{2}, \quad \text{with} \quad V = \frac{2\pi a}{\lambda} \sqrt{n_{co}^2 - n_{cl}^2}. \quad (2.2)$$

The ray picture we have used so far is inappropriate for a full description of the modes guided in an optical fibre, and a full electromagnetic analysis is thus in order. Each guided mode of the fibre is completely described by the three spacial components of the electric and magnetic field vectors – $E_r, E_\theta, E_z$ and $H_r, H_\theta, H_z$ where $r, \theta, z$ form the natural cylindrical coordinates of the fibre – as well as the associated propagation constant in the axis or $z-$direction ($\beta = \frac{2\pi n_{co} \sin(\theta_i)}{\lambda}$). This complete information is obtained by solving Maxwell’s equations in the waveguide with the appropriate boundary conditions as explained in section 2.4. The mode with the highest $\beta$ (or largest $\theta_i$) is usually referred to as the fundamental mode. Total internal reflection requires that discrete modes guided in the fibre core have propagation constants that satisfy:

$$kn_{cl} \leq \beta \leq kn_{co} \quad (2.3)$$

In other words, they form states of the electromagnetic waves that may exist in the core, but not in the cladding.

The modes guided in an optical fibre are usually named depending on their polarization properties and the pattern of the field distributions across the fibre cross-section. Transverse electric (TE) modes are modes which have no electric field component in the fibre axis direction ($E_z = 0$) while transverse magnetic (TM) modes have zero axial magnetic field component ($H_z = 0$). In addition to these are the hybrid modes in which no component of either field vanishes. These are divided into $HE$ modes where the axial electric field $E_z$ is significant compared to
$E_r$ or $E_\theta$ and $EH$ modes where $H_z$ is significant compared to $H_r$ or $H_\theta$ [41]. Aside from this distinction, two numbers are further needed to fully describe a mode. For TE and TM modes, the radial and azimuthal components of the electric field vanishes respectively. Since their non-vanishing electric field component has no azimuthal dependence, they are therefore labelled $TM_{0n}$ and $TE_{0n}$ respectively, where $n$ is the number of maxima (or minima) in the radial direction. Hybrid modes are labelled $HE_{mn}$ and $EH_{mn}$ with azimuthal mode number $m$ and radial mode number $n$ respectively. $m$ represents half the total number of maxima and minima of either $E_r$ or $E_\theta$ along the azimuthal direction while $n$ second is the number of maxima in the radial direction. While the $TE_{0n}$ and $TM_{0n}$ modes are non-degenerate, the $HE_{mn}$ and $EH_{mn}$ are two-fold degenerate, meaning that they each are constituted of two submodes which share the same propagation constant. Other naming conventions exist in literature. Marcatili et al. [42] use a single $EH$ notation, but assign a negative $m$ to obtain the conventional $HE$ modes.

For the most common fibres currently used in many applications, the refractive index of the cladding is very close to that of the core, resulting in what is known as weak guidance [40]. The weak guidance condition greatly simplifies the characteristic equation for the modes of the fiber, introduces further degeneracies which lead to the concept of linearly polarized (LP) modes. As the name indicates, LP modes which are constructed as superpositions of true fiber modes (i.e. TE, TM, HE and EH modes), have their transverse electric field aligned along one single direction in the plane of the fibre cross-section. This is very important as these are the modes most likely to be excited by polarized laser light. They are typically constructed by using the following rules [41, 43]:

$$LP_{mn} = \begin{cases} 
HE_{1n} & \text{if } m = 0, \text{ two-fold degenerate} \\
TE_{0n} \text{ or } TM_{0n} + HE_{2n} & \text{if } m = 1, \text{ four-fold degenerate} \\
EH_{(m-1)n} + HE_{(m+1)n} & \text{if } m \geq 2, \text{ four-fold degenerate}
\end{cases} \quad (2.4)$$

It follows from these rules that for the $LP_{mn}$ mode, $n$ describes the number of radial oscillations of the field in the core, whereas $m$ is the number of radial variations of either $E_x$ or $E_y$ along the azimuthal direction. Figure 2.3 shows field distribution for the first five LP mode groups along with the true fiber modes within each group. As only $E_x$ is displayed, the other degenerate $HE_{1n}$ mode is not shown as it is polarized along the y-axis. Traditionally, fibres used in telecommunications
<table>
<thead>
<tr>
<th>LP designation and power distribution</th>
<th>Traditional designation and $E_x$ distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP(_{01})</td>
<td>HE(_{11})</td>
</tr>
<tr>
<td>LP(_{11})</td>
<td>TM(<em>{01})  TE(</em>{01})  HE(<em>{21})  HE(</em>{21})</td>
</tr>
<tr>
<td>LP(_{21})</td>
<td>EH(<em>{11})  EH(</em>{11})  HE(<em>{31})  HE(</em>{31})</td>
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<tr>
<td>LP(_{02})</td>
<td>HE(_{12})</td>
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<tr>
<td>LP(_{21})</td>
<td>EH(<em>{21})  EH(</em>{21})  HE(<em>{41})  HE(</em>{41})</td>
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Figure 2.3: Designations of modes guided in a conventional step-index fibre. The column on the left shows the LP designations and spatial distribution of the Poynting vector in the axial direction. The right column displays the different degenerate modes which form the LP-group along with the electric field component of each mode along the $x-$axis. Red is positive $E_x$ while blue is negative and the arrows show the total electric field lines.
have been designed to support only the two polarizations of the fundamental mode and are therefore called *single mode fibres*.

### 2.2 Photonic bandgap guidance

In hollow core photonic bandgap fibres, light is confined and guided in the central core defect by virtue of the photonic bandgap created by the periodic structure surrounding it. A photonic bandgap can be thought of as the optical analog of the electronic bandgap in that it describes a collection of states of the electromagnetic wave \((\omega, \mathbf{k})\) which are forbidden to exist within a given structure. The possibility of using a photonic bandgap to control light was first put forth independently by Yablonovitch [29] and John [30]. They proposed that dielectric structures incorporating three-dimensional wavelength-scale periodic refractive index distribution could completely forbid spontaneous emission due to the absence of allowed photonic states at certain values of \((\omega, \mathbf{k})\), or allow for a strong localization of light within a defect. Such photonic crystals immediately attracted much attention because of the exotic properties they would possess, and the many device possibilities that they could lead to. The requirement of high refractive index contrast in the plane of propagation meant that semiconductors were the material of choice for the first attempts at fabricating photonic crystals at optical frequencies [44]. As the fabrication of three-dimensional periodic structures proved a formidable challenge, the attention turned to planar devices which would rely on total internal reflection for confinement in the third dimension but which would be more easily fabricated [45]. Since, a myriad of devices based on these so-called photonic crystal slabs have been successfully incorporated on photonic integrated chips [46].

Inspired by these developments in periodic structures, Russell proposed a new optical fibre in which a periodic distribution of air holes running along the fibre length would form the cladding and provide confinement through a photonic bandgap, bypassing the usual total internal reflection in step-index fibres [13, 47, 48]. This *out-of-plane* propagation meant that the ratio between the wavevector components in the plane of periodicity for the two dielectric materials could be controlled simply by tuning the angle of propagation in the third dimension and thus achieving a bandgap effect for any two arbitrary materials. Such a development implied in
fact that light could be guided in a hollow-core surrounded by an air-glass microstructure for which the refractive index contrast is as low as $1 : 1.45$ (as is the case for air and silica [14, 49]).

It is interesting to point out that the first attempts at fabricating the newly proposed fibres did not produce fibres guiding light via a bandgap effect, but which instead relied on a modified form of total internal reflection. These fibres turned out however, to have unique properties of their own such as being able to guide only a single mode for all wavelengths, flexible dispersion and birefringence properties, etc, which would help extend the range of optical properties achievable in conventional fibres [31, 50–52]. Their development was a crucial step towards the first realization of a fibre guiding light in an air-core via a photonic bandgap effect [15].

Early theoretical studies into photonic bandgap guidance borrowed from solid-state physics tools used to calculate electronic bands and did not permit the study of the optical modes supported inside a core defect [14, 53, 54]. Several mathematical tools and numerical methods were subsequently developed for this purpose, some derived specifically for the study of MOFs while others readapted general methods such as finite element or finite difference methods [55]. One laments however that in many cases, deeper insight and thorough physical understanding of the origin of the photonic bandgap is obscured by the use of such brute force numerical methods.

To understand the physics of out-of-plane photonic bandgaps from a fibre-optic perspective, one may examine the simpler structure of all-solid bandgap fibres which consist of high index cylindrical inclusions in a low index background material and which have been found to support a photonic bandgap for an index contrast as low as $1\%$ [56]. Analyzing these fibres with the ARROW (antiresonant reflective optical waveguide) model, Litchinister et al. [58, 59] found that their transmission bands are primarily determined by the modal properties of the inclusions, with their exact arrangement playing a very minor role. Birks et al. [57] went a step further and proposed an elegant semi-analytical model that provides an intuitive understanding of bandgap formation in all-solid bandgap fibres. Indeed, simple equations were derived that describe the coupling between the modes of individual inclusions as they are brought together to form the cladding lattice. They showed that by regarding a cladding unit cell as a waveguide of its own, imposing the condition that the field of a specified $LP$ mode vanishes at the cell boundary will yield a characteristic dispersion equation for a symmetric mode of
the structure. Similarly, imposing the condition that the derivative of the field vanishes at the cell boundary yields a characteristic dispersion equation for an antisymmetric mode of the structure. All the states between these two modes are intermediate supermodes supported by the periodic cladding. If the dispersion of all the symmetric and antisymmetric supermodes are computed and superposed as done in Fig. 2.4 one realises at once that regions where no cladding mode has an effective index lower than that of the low index background material now exist. It is these gaps between the bands that constitute the photonic bandgaps of the periodic cladding.

Although the picture is more complex in HC-PBGFs, the bandgap has the same origin, which is explained with the helps of Fig. 2.5 below. If we consider a single glass rod of radius \( r \) in air, it is well known from waveguide theory that it will support a discrete number of confined and guided modes with effective indices higher than that of air, and the whole structure supporting a continuum of modes with effective indices lower or equal to that of air. Typical dispersion curves for the guided modes can be computed numerically and an example is plotted in Fig. 2.4.
2.5(A) as \((\beta^2 - k_0^2)r^2\) versus the normalised frequency \(v = k_0 r \sqrt{n^2 - 1}\). Here, \(n\) is the glass refractive index, \(k_0\) the free space wavenumber and \(\beta\) the propagation constant in the \(z\) direction.

Figure 2.5: Origin of the photonic bandgap in a HC-PBGF cladding. (A) a single glass rod in air supports discrete vector modes with the grey area showing the continuum of modes supported in air. (B) Bringing identical rods sufficiently close together leads to the overlap of the individual mode fields near cut-off frequencies and produces multiple photonic bandgaps extending below the air line. (C) Further reducing the separation between the rods leads to narrower and deeper bandgaps and (D) the addition of the interconnecting struts raises the effective index of the rods surroundings and all but the fundamental photonic bandgap are shifted above the air line. (Courtesy of Dr. Francesco Poletti)
Bringing sufficiently close together a number of rods fundamentally changes these dispersion relations, especially near modal cut-offs. Indeed, as \( v \) gets close to the modal cut-off, the mode field of a single rod extends so much in the cladding that it inevitably overlaps with that of neighbouring rods. Each mode effectively broadens to become a band of supermodes of the entire structure, and as a result regions where guidance is forbidden in the structure now extend below the air line as can be seen from Fig. 2.5(B). Reducing the spacing between the glass rods leads to even stronger overlap between the fields of their individual modes, effectively narrowing the width of the forbidden guidance regions. Additionally, as a consequence of the narrower air gaps between the glass rods, the photonic bandgaps now extend further below the air-line (Fig. 2.5(C)). Finally, the addition of interconnecting glass struts necessary in real HC-PBGFs fundamentally alters the dispersion characteristics of the supermodes, with the most prominent effect being shifting many of the higher order gaps above the air line (Fig. 2.5(D)). This is because the effective index of the network of thin struts shown in green has a strong wavelength dependence. Thus, as the supermodes have a higher fraction of their power in the struts at higher frequencies, their effective index is inevitably raised above the air-line. This happens even for the thinnest feasible struts. HC-PBGFs therefore typically show a unique transmission band in the range \( 1 \leq v \leq 2 \).

Interestingly, apart from the main gaps that cross the air line, the band plots in Figs. 2.5(A) to (C) show distinct ‘holes’ well below the air line. This is in contrast with the earlier case we considered in Fig. 2.4 and arises primarily because of the higher refractive index contrast between air and glass. These holes in the band plots occur when the modes at the edges of the bands undergo an avoided crossing event.

From this simple intuitive model, the following conclusions can be drawn:

- the size of the glass nodes primarily determine the position of the photonic bandgap while its width is affected by the separation between them. It therefore suffices to change the rod size to scale the position of the bandgap accordingly so that \( r/\lambda \) is constant.

- The interconnecting silica struts have a detrimental effect on the photonic bandgap. It is therefore important to always minimize their thickness, and

- While periodicity may be beneficial in ensuring perhaps the widest bandgaps, it is not a strict requirement.
For further understanding of the photonic bandgap and properties of modes that may be guided in a defect, it is instructing to examine the modes near the gap edges, which are illustrated in Fig. 2.6. Such modes are traditionally studied by numerically computing their properties using a single unit cell of the periodic cladding and imposing periodic boundary conditions. The adopted terminology is therefore similar to that of solid-state physics. The inset of the figure shows the the primitive cell of the reciprocal lattice and the irreducible Brillouin zone shaded in black, with the symmetry points $\Gamma$ (centre), $M$ (face) and $K$ (corner) highlighted. This zone illustrates all the possible values of the in-plane component of the wave-vector.

One sees that at short wavelengths, the modes guided in each individual rod are well confined and their dispersion akin to that of a single rod suspended in air. As the wavelength increases, the mode fields spread into the cladding and overlap with those of the neighbouring rods, giving rise to bands of supermodes supported by the cladding, as described above. We see from the figure that symmetric and antisymmetric modes form the edges of the $LP_{01}$ band, with the long wavelength edge of the bandgap being the antisymmetric rod mode. As the $LP_{11}$ mode is more prone to overlapping with the struts, it is markedly affected by the cladding symmetry. The $\Gamma -$ mode which is primarily located in the struts is crossed by the $K -$ mode which is predominantly located in the air and forms the bottom edge of the photonic bandgap. This description of the modes at the bandgap edges is in good agreement with the experimental observations from Couny et al. [60] and is very important when studying the origin of surface states in the fibre [61] (see also sections 2.3.4 and 3.3.2).

As HC-PBGFs only guide at wavelengths within the photonic bandgap, significant work over the years has been devoted to identifying configurations that result in the widest bandgaps. The main conclusion from early studies were that, to a first order approximation, the width of the bandgap scales as the air-filling fraction [62]. In light of the previous analysis, we can understand why this is the case: high air-filling fractions often imply thinner glass struts and therefore weaker overlap between the modes of individual rods.

Another strand of activity sought to identify which lattice arrangement supports the widest possible bandgaps. Besides the most commonly encountered triangular lattice of air holes (TLH), both square (SL) and triangular lattices of rods (TLR) in which the rods rather than the air holes are arranged in a triangular lattice (see Fig. 2.7) have been investigated and shown to be capable of producing wider
photonic bandgaps [63–65]. This arises from the fact that if the node spacing is made the same for all three lattices, then further fixing the crucial ratio of strut thickness $t$ to inter-hole spacing $\Lambda$ results in struts that are $\sqrt{3}$ and 3 times thinner in the square and TLR lattices, respectively, compared to those in the triangular lattice [65]. As a result, the square and TLH lattices are capable of providing wider photonic bandgaps than the TLH, expanding up to an octave for a TLR with realistically achievable $t/\Lambda$ ratio.

In practice, HC-PBGFs are most commonly made by stacking similar circular capillaries, leading to some additional difficulty in producing lattice arrangements
other than the TLH. Furthermore, numerical studies have revealed that both the square lattice (SL) and TLR claddings require a larger number of ‘rings’ of air holes outside the core defect to effectively reduce leakage loss, making the stacked structure extremely large and time consuming to produce [63, 65]. Although no successful fabricated TLR fibre has been reported to date, a square lattice fibre with struts as thin as $\sim 30\text{nm}$ was reported by Dong et al. and was shown to have a normalized bandwidth as wide as 44% [38]. Unless otherwise mentioned, this thesis will solely focus on the study of fibres with a TLH cladding.

2.3 Photonic bandgap fibres and their properties

Having described the origin of photonic bandgap guidance in a periodic cladding, we now turn our attention to a waveguide obtained by removing a number of unit cells from such a periodic structure to form a core defect. Typically, the core defect is formed in a TLH by removing 7, 19 or 37 unit cells (hereafter referred to as 7$c$, 19$c$ and 37$c$ respectively), although fibres with 3 and 4 cell defects have been reported for single mode and high birefringence operation, respectively [66, 67]. Light launched into such a core defect is confined and guided along the longitudinal direction when the wavelength lies within the cladding’s photonic bandgap. The first successful fabrication of such a fibre was reported by Cregan et al. in 1999 [15]. Although this work only achieved transmission over a few centimetres due to high propagation loss, it unequivocally demonstrated light guidance in an air-core. Since then, tremendous progress in both understanding the properties of these fibres and their fabrication technology has been achieved. In this section,
we give a brief overview of the main optical properties of these fibres, focusing in particular on:

- transmission properties and typical fractions of power guided in the core defect,
- modal content, modal group delay and chromatic dispersion,
- optical nonlinearities and
- propagation loss, which is further divided into its main contributions, namely the leakage and the scattering from surface roughness.

To illustrate the typical values of these different properties, we use an example 7c fibre and discuss at length the effect of the core defect and other structural parameters in the next Chapter.

Figure 2.8 shows a plot of the fraction of guided power in the core for a 7c fibre as a function of wavelength, which is akin to the fibre’s transmission across the photonic bandgap. The inset shows a scanning electron micrograph (SEM) image of such a 7c fibre and the computed power distribution for the fundamental guided mode in such a fibre. At wavelengths outside the bandgap, the periodic cladding supports modes with the same \((\omega, \beta)\) as the core modes, and therefore, the core modes can easily couple into these cladding modes, making low-loss guidance impossible. At wavelengths within the bandgap however, the fundamental mode of 7c fibres with sufficiently high air-filling fractions carries more than 97% of its guided power within the core defect. This typically increases to 99.2% and 99.7% for 19c and 37c fibres, respectively.

2.3.1 Modal content, group delay and dispersion

The core defect in HC-PBGFs is formed by omitting 7, 19 or 37c from the centre of an otherwise periodic structure and typically supports several core-guided modes. If we regard the waveguide thus formed as a perfectly reflecting hollow cylinder, the properties of these modes may be studied by the very simple analytical model from Marcatili and Schmeltzer [42]. Numerical calculations are required however, if more accurate modelling is to be performed. Figure 2.9 shows the dispersion of the core-guided modes in a 7c photonic bandgap fibre. The fibre guides as
many as 12 modes, distributed in four nearly-degenerate mode groups with power
distributions similar to the LP modes in conventional fibres: $LP_{01}$, $LP_{11}$, $LP_{21}$
and $LP_{02}$. Typically in 7c fibres, modes of order higher than the $LP_{02}$ have their
propagation constant lower than that of the bottom of the photonic bandgap and
are therefore not guided since they easily couple to cladding modes.

The effective indices of the core guided modes are below unity, indicating that the
phase velocity is greater than the speed of light in vacuum. This does not violate
general relativity because the group index defined by:

$$n_g = c \left( \frac{\partial \beta}{\partial \omega} \right) = n_{eff} + \omega \frac{\partial n_{eff}}{\partial \omega}$$

is slightly higher than 1, implying that information carried by these guided modes
travels with a group velocity very close to (but slower than) the speed of light in vacuum. HC-PBGFs are therefore capable of data transmission at very low
latencies, a feature that is increasingly emerging as advantageous in various applica-
tions such as high speed trading and data centres. We plot in Fig. 2.10 the group
index as a function of wavelength for the fundamental mode across the photonic
bandgap and it can be appreciated that away from the bandgap edges, the low
group index implies that information can be transmitted at $v = c/n_g = c/1.005$
or 99.5% the speed of light in vacuum. We note that the achievable group delay

```
Figure 2.8: Transmission properties and modal power distribution in a 7c
HC-PBGF.
```
is subject to fibre design and in particular is sensitive to the core size and core
surround design, as will be shown in the coming chapters.

In HC-PBGFs, the chromatic dispersion which is a measure of how a pulse of light
sent through a mode of the fibre spreads in time and which is quantified through
the dispersion parameter:

\[
D = -\frac{2\pi c \partial^2 \beta}{\lambda^2 \partial \omega^2}
\]

(2.6)
typically takes the form shown in Fig. 2.10. This dispersion parameter \( D \) is
a measure of pulse spread per unit length of transmission per unit bandwidth.
The chromatic dispersion traditionally comprises material and waveguide contribu-
tions. The material dispersion arises from the fact that most optical materials
have refractive indices that change with wavelength. For most optical materials,
the refractive index change with wavelength is such that waves at shorter wave-
lengths typically see a higher refractive index than longer ones and therefore travel
slower. This is thus referred to as normal dispersion, and the opposite case where
waves at longer wavelengths travel slower than those at shorter ones is termed

Figure 2.9: Dispersion curves for the core-guided modes in a 7c HC-PBGFs.
Such a fibre supports up to twelve modes akin to the "LP" modes of con-
tentional fibres and distributed into four nearly degenerate mode groups:
\( LP_{01}, LP_{11}, LP_{21} \) and \( LP_{02} \).
anomalous dispersion. Waveguide dispersion results from the wavelength dependence of the confinement of the mode field within the waveguide. Since most of the light guided in HC-PBGFs is carried in the air core which has negligible material dispersion, their chromatic dispersion is dominated by waveguide contributions. The core size, shape and surround, as well as the cladding’s bandgap are crucial for managing the dispersion and modifying them provides some possibility of dispersion engineering [68]. Typically, the dispersion is very high and normal at the short wavelength edge, then quickly crosses zero near the short wavelength edge and remains anomalous across the bandgap before quickly increasing to very high values near the long wavelength of the photonic bandgap. In comparison, for hollow dielectric waveguides, Marcatili and Schmeltzer give the real part of the propagation constant for the $EH_{nm}$ mode as [42]:

$$\text{Re}(\beta) = \frac{2\pi}{\lambda} \left[ 1 - \frac{1}{2} \left( \frac{u_{nm}\lambda}{2\pi a} \right)^2 \right]$$

(2.7)

where $\lambda$ is the free-space wavelength, $a$ the hollow core’s radius and $u_{nm}$ is the
Chapter 2 Background

$m^{th}$ zero of the Bessel function of the first kind $J_{n-1}(x)$. The chromatic dispersion parameter is obtained from this expression as:

$$D = -\frac{2\pi c}{\lambda^2} \frac{\beta^2}{\omega^2} = \frac{1}{c} \left( \frac{u_{nm}}{2\pi a} \right)^2 \lambda,$$

which can be seen to be anomalous and changes as $\lambda/a^2$. It can be concluded therefore that well within the bandgap, the chromatic dispersion parameter generally increases with the mode order (increasing $u_{nm}$) and the wavelength, but decreases with the core size.

2.3.2 Optical nonlinearities in HC-PBGFs

In the presence of strong electric field densities, the electron orbit in the atoms of the dielectric medium is often strongly distorted, leading to a macroscopic response that no longer scales linearly with the incident field, but which also has components scaling as the square or third power of the field [69, 70]. This gives rise to a wealth of optical nonlinear effects that may be useful or create impediments depending on the application. In standard silica-based fibres, nonlinear optical effects arise due to the optical Kerr effect which is a modification the refractive index in response to the strong field intensity. Under the Kerr effect, the modified refractive index takes the form $n_0 + n_2 I$ where $n_0$ is the linear refractive index, $n_2$ the Kerr coefficient and $I$ the electric field intensity. In telecommunication applications, nonlinear effects often have detrimental implications for the data transmission capacity of the fibres as they typically lead to the generation of excess noise in the data. They are significant despite the rather small Kerr coefficient of silica primarily because of the tight field confinement which creates high power densities and also the long interaction lengths involved [70].

An additional benefit of guiding light in air is the virtual elimination of these nonlinear optical effects which lead to signal deterioration [9]. Nonlinear effects are usually described through an effective nonlinear coefficient $\gamma_{NL}$ defined as [70]:

$$\gamma_{NL} = \frac{2\pi \overline{n}_2}{\lambda A_{eff}}$$

where $\overline{n}_2$ is the average material Kerr coefficient and $A_{eff}$ the effective modal area. $A_{eff}$ which depends both on the fibre’s cross-section and mode-field distribution
is defined as \[70\]:

\[
A_{\text{eff}} = \frac{\left( \iint |F(x,y)|^2 \,dA \right)^2}{\iint |F(x,y)|^4 \,dA}
\] (2.10)

where \(F(x,y)\) is the field distribution and where the integrals are evaluated over the entire cross-section. For composite structures such as HC-PBGFs, we adopt the Heinberger’s definition of the average Kerr coefficient, that is \[71\]:

\[
\bar{n}_2 = \frac{\iint n_2(x,y) |F(x,y)|^4 \,dA}{\iint |F(x,y)|^4 \,dA}
\] (2.11)

with \(n_2(x,y)\) being the position dependent material Kerr coefficient. For standard silica single mode fibres \(n_2 = 2.2 \times 10^{-20} m^2/W\), \(A_{\text{eff}} \sim 80 \mu m^2\), the effective nonlinear coefficient is of the order of \(\gamma \sim 1 W \cdot km^{-1}\). In comparison, we plot in Fig.2.11 the calculated nonlinear coefficient for the fundamental mode of a 7c HC-PBGF.

![Figure 2.11: Nonlinear coefficient \(\gamma\) as a function of wavelength across the photonic bandgap for a 7c HC-PBGF. The effective nonlinearity is nearly three orders of magnitude below that of SMFs.](image)

It can be seen from the graph that at wavelengths within the bandgap and away from the edges, the fact that most of the modal power is carried in air which has a much lower Kerr coefficient \(n_2 \sim 3 \times 10^{-23} m^2/W\) results in a 7c HC-PBGF having a nonlinear coefficient \(\gamma \sim 1.5 \times 10^{-3} W \cdot km^{-1}\), nearly three orders of magnitudes
lower than that of the conventional single mode fibre. This dramatic reduction in optical nonlinearity is an important advantage that HC-PBGFs possess over conventional fibres in data transmission applications.

### 2.3.3 Loss in HC-PBGFs

One of the most important requirements for optical waveguides, especially those to be employed in data transmission applications, is that their attenuation or loss be kept at the very minimum. Very generally, three classes of mechanisms are at the origin of loss in optical waveguides: absorption, scattering and radiation. In spectral regions where the bulk material is highly transparent, absorption plays a negligible role and the loss is dominated by scattering as is the case for standard silica fibres. Radiation on the other hand, becomes significant when the waveguide goes through a bend or when it suffers from leakage, as is the case for HC-PBGFs.

Scattering can further be subdivided into bulk or volume scattering and surface scattering. It is well known that bulk or Rayleigh scattering which arises from density fluctuations or contaminant species and crystalline defects is responsible for the fundamental limit of attenuation in standard silica fibres \[2\]. In HC-PBGFs, only a very small fraction of the guided power propagates in glass (see Fig. 2.12), raising the prospect of reducing the attenuation losses to levels below that of standard single mode fibres.

However, one distinct feature of the HC-PBGF is the presence in its cross-section of a large number of air-glass interfaces, which though very smooth are nonetheless prone to display irregularities along the fibre axis. Surface scattering is therefore expected to be of particular importance in this class of fibres \[19\]. Denoting by \(\xi\) the fraction of modal power that propagates in the glass regions of the microstructure, the total loss of the HC-PBGF can be described by the expression:

\[
\alpha_{HC-PBGF} = \xi \left( \frac{A}{\lambda^3} + C_1 e^{\frac{C_2}{\lambda}} + D_1 e^{-\frac{D_2}{\lambda}} + E(\lambda) \right) + \alpha_S(\lambda) + I(\lambda) + C(\lambda). \tag{2.12}
\]

The terms between brackets, weighed by the fraction of power guided in glass \(\xi\), are the Rayleigh scattering, UV and IR absorption and scattering and absorption from impurities within bulk glass, respectively. \(A\) is the Rayleigh coefficient for silica, \(C_1\) and \(C_2\) are the amplitude and decay rate of the UV absorption or Urbach edge \[72\] and, \(D_1\) and \(D_2\) are the amplitude and slope of the infrared absorption
These loss mechanisms in bulk silica glass, pure or doped, have been studied extensively (see for example [73, 74]), but are expected to play a negligible role in HC-PBGFs. This is because the coefficient $\xi$ which defines the magnitude of the contribution of these loss mechanisms is typically very small in HC-PBGFs, as shown in Fig. 2.12. At wavelengths within the bandgap, $\xi$ is well below 1% for 7c fibres, and can be further reduced through careful fibre design and simple measures such as enlarging the core defect to 19c or 37c. This indicates that the glass’s contribution to loss is highly suppressed and becomes negligible for wavelengths where silica is highly transparent. However, at wavelengths beyond 2µm where infrared absorption becomes significant in silica, glass absorption becomes the dominant contribution to loss [75].

The term $\alpha_s(\lambda)$ in Eq. (2.12) represents the overall attenuation from the glass interfaces and embodies both surface roughness scattering and possible absorption from impurities adsorbed thereon. The term $I(\lambda)$ represents imperfection loss, which arises because of structural inhomogeneities and scattering or absorption centres along the fibre length. Such inhomogeneities can be introduced during any step of the fibre fabrication. Imperfection loss in HC-PBGFs is only starting to be studied systematically [76], and it appears that these fibres are prone to have discrete defects along their length. We assume however, that such inhomogeneities

![Figure 2.12: Fraction of modal Power carried in glass for a 7c HC-PBGF. At wavelengths within the photonic bandgap, less than 1% of power is guided in glass.](image-url)
along the fibre length can be effectively eliminated by improving the fabrication process. Finally, $C(\lambda)$ is the confinement or leakage loss. Confinement loss and scattering from surface roughness which are of particular significance in HC-PBGFs are further discussed below.

### 2.3.3.1 Confinement loss

In any microstructured optical fibre, the modes guided within the core always leaky as they lose some power to radiation while propagating along the fibre length. To understand why this is the case, a ray optics picture is helpful. In conventional fibres where light is trapped and guided in the core by total internal reflection, the core-cladding interface acts as a perfect mirror (i.e 100% reflectivity) when the incident ray strikes the interface at an angle greater than the critical angle. As it is perfectly reflected, the field in the cladding is evanescent and no energy is carried away from the core. Therefore, away from cut-off, the guided modes in conventional step-index fibres are well confined and not leaky. In microstructured fibres with periodic claddings however, the 'interface' between core and cladding will be considered a perfect mirror if and only if the extent of the cladding is infinite, that is, if the cladding is made up of an infinite number of periods. Since this is impractical, a light ray following a zigzag path as it travels in the core will always see some of its power carried away from the core into the cladding (thus lost to radiation) upon each reflection, regardless of its incidence angle. In solid-core single material solid core structures, this leakage has been likened to tunneling to the outer jacket which has the same refractive index as the core. As a result of this leakage, the guided mode assumes a complex propagation constant $\beta$, the imaginary part of which captures the loss of power due to leakage as the mode propagates down the fiber \[77\]. This leakage or confinement loss is thus defined mathematically as:

$$C(\lambda) = \frac{20}{\ln(10)} \text{Im} (\beta(\lambda)) \text{ in } [dB/m] \quad (2.13)$$

where $\text{Im}$ stands for imaginary part. Confinement loss in MOFs has been studied extensively, reaching the trivial but important result that additional rings of air holes around the core can effectively reduce it to negligible values\[78\].

Figure 2.13 shows the calculated confinement loss for a 7$c$ fibre with six rings of air holes surrounding the core defect. For HC-PBGFs, the confinement loss has
been found to obey the same rule mentioned above: the addition of further rings of air holes ultimately reduce it to negligible levels (see Fig. 3.1(a)). However, as

\[ \text{Figure 2.13: Confinement loss for the fundamental mode of a } 7c \text{ HC-PBGF with six rings of air holes around the core defect. At high air filling fractions, six rings of air holes suffice to keep the confinement loss below } 0.01 \text{dB/km across most of the photonic bandgap.} \]

described in greater detail in the following chapter, the confinement loss in these fibres has a strong dependence on the cladding structural parameters as well as on the core boundary definition [52, 79]. Saitoh et al. show in Ref. [52, 79] that the confinement loss decreases sharply with thinner cladding struts and that sharp angles in the core surround cause a stronger local overlap of the guided mode field, leading to higher confinement losses. We see in the next chapter that, in general, confinement loss is automatically minimized when the photonic bandgap is widest. This simultaneous optimisation is very important as it implies no design trade-off as far as these two performance metrics (wide bandwidth and low confinement loss) are concerned. Note that higher order modes are generally subject to higher confinement loss values, as may be deduced again from the analytical expressions for the simple hollow-dielectric waveguide [42].

### 2.3.3.2 Surface scattering loss

In equation (2.12), \( \alpha_s(\lambda) \) represents the overall attenuation that can be attributed to both impurity absorption and scattering from the glass-air interfaces within the
HC-PBGF. Assuming that absorption from surfaces can be effectively eliminated through careful storage and cleaning during fibre fabrication, we will consider $\alpha_s$ to describe surface roughness scattering only.

One fundamental mechanism giving rise to intrinsic roughness on air-glass interface in HC-PBGFs is that of frozen-in surface capillary waves [24]. As is the case on the surface of any liquid, thermally excited capillary waves exist on the surface of molten glass under the restoring force of surface tension. As molten glass goes through solidification, these surface capillary waves freeze in, giving rise to an intrinsic surface roughness. In [19], Roberts et al. argue that it is scattering from this intrinsic surface roughness that imposes a fundamental limit on loss in HC-PBGFs.

While a rigorous description of this scattering process is presented in Chapter 4, some insight may be gained by considering the simpler approach below. We assume that the rough inner cylindrical surface of a cladding hole surface can be described statistically by using the root-mean square height $h_{\text{rms}}$ of the roughness and its correlation lengths $L_z$ and $L_\phi$ along the fibre axis and azimuthal direction respectively. This interface can then be thought of as a collection of independent scattering elements radiating away part of the light incident upon them. Following reference [80], a typical scatterer on this surface has a volume $dV = h_{\text{rms}} L_z L_\phi$, and its induced dipole moment is:

$$p = \Delta\varepsilon dV \cdot E_0$$  \hspace{1cm} (2.14)

In this equation, $\Delta\varepsilon$ is the dielectric constant difference between air and silica and $E_0$ is the electric field strength at the scatterer location. In a free space approximation, the power radiated by this dipole is given by:

$$P_{sc} = \frac{c}{12\pi\varepsilon_0} \left( \frac{\omega}{c} \right)^4 |p|^2 = \frac{1}{12\pi} \left( \frac{\omega}{c} \right)^4 \frac{\Delta\varepsilon^2}{\varepsilon_0^2} h_{\text{rms}}^2 L_z^2 L_\phi^2 \left( \frac{\varepsilon_0}{\mu_0} \right)^{\frac{1}{2}} |E_0|^2.$$  \hspace{1cm} (2.15)

Since the number of scatterers per unit area is proportional to $\frac{1}{L_z L_\phi}$, the total radiated power from a section of length $L$ of the fibre is therefore:

$$P_{\text{rad}} = \frac{1}{12\pi} \left( \frac{\omega}{c} \right)^4 \frac{\Delta\varepsilon^2}{\varepsilon_0^2} h_{\text{rms}}^2 L_z L_\phi L \left( \frac{\varepsilon_0}{\mu_0} \right)^{\frac{1}{2}} \int_{\text{holes perimeter}} |E_0|^2 \, dl.$$  \hspace{1cm} (2.16)
If we now express the total power carried by the mode as a Poynting flux across the entire fibre cross-section, the surface scattering loss per unit length is:

$$\alpha_{sc} = \frac{P_{rad}}{P_0 L} \sim \frac{1}{6\pi} \left(\frac{\omega}{c}\right)^4 \Delta\varepsilon^2 h_{rms}^2 L z L_{\phi} \left(\frac{\varepsilon}{\mu_0}\right)^{\frac{3}{2}} \oint_{\text{holes perimeter}} \oint_{\text{cross-section}} \frac{|E_0|^2 dl}{E_0 \times H_0^* dS} \quad (2.17)$$

where the unitless quantity $\Delta\varepsilon = \frac{\Delta\varepsilon}{\varepsilon_0}$ has been introduced. From this equation we can isolate geometry and mode field-dependent factors from those that depend on the roughness. The geometry and mode-field dependence is captured by the following parameter:

$$F = \left(\frac{\varepsilon_0}{\mu_0}\right)^{\frac{3}{2}} \oint_{\text{holes perimeter}} \oint_{\text{cross-section}} \frac{|E_0|^2 dl}{E_0 \times H_0^* dS} \quad (2.18)$$

This is the normalized interface field intensity, which is routinely used to compare the scattering loss performance of HC-PBGF designs, since one can expect that the lower the value of $F$, the lower surface scattering loss will be.

Note that despite the simple assumptions involved in deriving equation (2.17), notably the oversimplified description of roughness and uncorrelated scatterers assumption, we have found through numerous examples that calibrating the value of $F$ can provide a good estimate of the loss in fabricated HC-PBGFs. For several fabricated fibres for which we performed simulations and present the results in Chapter 6, we have found that for operation around 1.55\,\mu m, the loss can expressed as

$$\alpha_{sc} \,(dB/km) = \eta \times F(\mu m^{-1}) \quad (2.19)$$

with the proportionality coefficient taking the value $\eta = 300$. In the absence of accurate knowledge of the surface roughness in HC-PBGFs, this simple but valuable calibration has consistently provided fairly accurate loss estimates.

Figure 2.14 shows a plot of the normalized interface field intensity $F$ as a function of wavelength across the photonic bandgap and it can be appreciated that it takes very high values outside the photonic bandgap, but quickly drops for wavelengths well within the bandgap where low-loss guidance is possible. By its very definition, the achievable value of $F$ is subject to the design of the fibre. It depends on the cladding and its photonic bandgap, and most importantly on the core size and
core boundary termination. Understanding this dependence on the geometry of the fibre is key to reducing the fibre loss, which is why the discussion is recurring throughout the thesis.

2.3.4 Defects and surface states

In HC-PBGFs, the termination of the periodic microstructured cladding near the core defect may be able to support modes of its own at wavelengths within the photonic bandgap. These are commonly referred to as surface modes, in analogy with surface states in solid-state physics. These surface modes have their power guided in glass regions around the core surround and are more prone to leakage, scattering and coupling to lossy cladding modes. Typically, the dispersion curves for surface modes have a different slope from that of the core guided modes, and as a result, there are wavelengths within the photonic bandgap at which core and surface modes are nearly degenerate. When the surface and core modes belong to different symmetry classes (more on this below), we speak of a crossing event at the wavelength where they share the same propagation constant. When they belong to the same symmetry class, an effective hybridisation of the
mode field patterns occurs at the wavelength of near degeneracy and we speak of anti-crossing. Due to the fact that surface modes have a significant overlap with core-guided modes and have nearly degenerate propagation constants at crossing or anti-crossing wavelengths, power is easily coupled from the guided to surface modes, therefore providing an additional avenue for loss.

Because surface modes introduce high loss peaks at wavelengths within the photonic bandgap, eliminating their presence is of paramount importance to achieving a wide operational bandwidth for HC-PBGFs. But doing so requires a careful design and accurate control of the core boundary. A number of studies in the literature have sought to identify design rules for avoiding the presence of surface modes within the photonic bandgap \[81-83\]. The most effective of these recipes was the suggestion by Amezcua-Correa et al. \[84, 85\] that the use of a core wall half as thick as the cladding struts would effectively eliminate the surface modes present within the bandgap. Following this suggestion, a 7c with low-loss and wide bandwidth was subsequently demonstrated in \[86\] and more recently, our own work in the framework of MODEGAP has also produced low-loss and wide bandwidth 19c fibres and demonstrated their suitability for high capacity data transmission \[87, 88\].

Although the \( \frac{1}{2} \) core surround thickness rule has proved effective in eliminating the guidance of surface modes within the bandgap, no physical understanding as to why this is the case has hitherto been provided. Such physical insight can be gained by reexamining the origin of the photonic bandgap, especially the modes that form its edges as shown in Fig. 2.6. For an infinitely periodic cladding, the long wavelength edge of the photonic bandgap is determined by the dispersion of the antisymmetric or odd "\( LP_{01} \)" rod mode whereas the short wavelength edge is determined by that of the symmetric or even "\( LP_{11} \)" mode which is predominantly localized in the silica struts, as illustrated in Fig.2.15

When terminating the periodic structure to form the core boundary, differences in rod size, strut thickness and strut length with respect to those in the cladding may lead to the core boundary supporting local modes (i.e. surface modes) with their fields located in the struts (SSM) or in the rods (RSM). It is evident that if the rods on the core boundary are smaller than those in the cladding, they will support modes with a lower effective index than the cladding mode forming the long wavelength edge of the bandgap. These modes have their fields predominantly located in the rods on the core boundary and are supported at wavelengths within the photonic bandgap. When the struts on the core boundary are different in
both length or thickness from those in the cladding, they may support modes with higher effective index than the cladding mode forming the short wavelength edge, which appear as strut localized surface modes.

It appears therefore that in principle, a core surround design in which node size, node spacing and strut thickness are uniform with those in the cladding should eliminate surface modes. This is not realistically achievable in TLH fibres with 7, 19 or 37c core defects because the perimeter of the core surround is always longer than an integer number of the cladding rod spacing. The next alternative which we present in Chapter 7 is one in which the nodes on the core boundary are made equidistant and the strut thickness on the core boundary minimized.
2.3.5 Summary

In this section, we used the illustrative example of a 7c HC-PBGF to describe the main optical properties of this class of fibres in general. As most of the optical power of the guided modes is carried in air, HC-PBGFs offer low optical nonlinearities and ultra-low latencies. Although this should in principle also result in low propagation loss, HC-PBGFs are particular prone to lose the guided optical power to radiation via scattering from surface roughness. The presence of surface modes not only provides an additional avenue through which this loss is exacerbated, it also severely reduces the operational bandwidth.

All of these optical properties of HC-PBGFs can and usually are engineered simply by modifying the geometry of their cross-section. It becomes imperative to identify through numerical simulations, which structural geometries can achieve desired optical properties while eliminating unwanted ones such as surface modes. In the following section, we introduce the problem of modelling the properties of HC-PBGFs with particular focus on the method I used to perform simulations throughout this thesis.

2.4 Numerical modelling of HC-PBGFs

Having introduced HC-PBGFs and their main properties, we now turn our attention to describing means of accurately and reliably model their properties. Accurate modelling of the fibre’s modal properties is critical in fibre design and is an essential step in identifying structures possessing desired optical properties. In our particular case, these designs are those with the potential for low attenuation and wide bandwidth. To introduce how such modelling is usually performed, we start with a brief overview of the macroscopic Maxwell equations which are the foundation of the mathematical problem that describes light propagation in all media. A brief survey of the numerical methods encountered in the relevant scientific literature is then presented before turning to a detailed parametric description of realistic fibres and the steps involved in modelling them with the finite element method, which has been adopted and used throughout this thesis.
2.4.1 Basic equations

The mathematical treatment of light propagation in HC-PBGF starts with the Maxwell equations which govern all macroscopic electromagnetism. In mixed dielectric media free of electric charges and current sources, these are given in SI units as [43, 46, 89]:

\[
\begin{align*}
\nabla \times \mathbf{E}(\mathbf{r}, t) &= -\frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t} \\
\nabla \times \mathbf{H}(\mathbf{r}, t) &= \frac{\partial \mathbf{D}(\mathbf{r}, t)}{\partial t} \\
\nabla \cdot \mathbf{B}(\mathbf{r}, t) &= 0 \\
\nabla \cdot \mathbf{D}(\mathbf{r}, t) &= 0
\end{align*}
\] (2.20)

where \( \mathbf{E} \) and \( \mathbf{H} \) are the electric and magnetic fields respectively and \( \mathbf{B} \) and \( \mathbf{D} \) the magnetic induction and electric displacement respectively. Restricting the dielectric materials involved to be isotropic and the light intensities small enough for nonlinear effects to be negligible, constitutive relations linking \( \mathbf{D} \) to \( \mathbf{E} \) and \( \mathbf{B} \) to \( \mathbf{H} \) for the materials can be cast as:

\[
\begin{align*}
\mathbf{D}(\mathbf{r}, t) &= \varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r}, t) \\
\mathbf{B}(\mathbf{r}, t) &= \mu_0 \mathbf{E}(\mathbf{r}, t)
\end{align*}
\] (2.21)

Here, \( \varepsilon(\mathbf{r}) \) is the space-dependent electric permittivity and \( \mu_0 \) the free space permeability which is used here under the assumption that the materials involved are magnetically impermeable. For time harmonic problems, the fields change periodically with time and are usually expressed out of mathematical convenience as the real part of complex-valued functions:

\[
\begin{align*}
\mathbf{E}(\mathbf{r}, t) &= \text{Re} \left( \mathcal{E}(\mathbf{r}) \exp(-j\omega t) \right) \\
\mathbf{H}(\mathbf{r}, t) &= \text{Re} \left( \mathcal{H}(\mathbf{r}) \exp(-j\omega t) \right)
\end{align*}
\] (2.22)

where \( \text{Re} \) stands for the real part. This is no real restriction as arbitrary functions of time can be decomposed into harmonics via Fourier analysis. However, it eliminates the time dependence from the Maxwell’s equations, which by making use
of the constitutive relations can be written as:

\[
\begin{align*}
\nabla \times \mathcal{E}(\mathbf{r}) &= j\omega \mu_0 \mathcal{H}(\mathbf{r}) \quad (2.23a) \\
\nabla \times \mathcal{H}(\mathbf{r}) &= -j\omega \varepsilon(\mathbf{r}) \mathcal{E}(\mathbf{r}) \quad (2.23b) \\
\nabla \cdot \mathcal{H}(\mathbf{r}) &= 0 \quad (2.23c) \\
\n\nabla \cdot (\varepsilon(\mathbf{r}) \mathcal{E}(\mathbf{r})) &= 0 \quad (2.23d)
\end{align*}
\]

By dividing both sides of Eq. (2.23b) by \( \varepsilon(\mathbf{r}) \), taking the curl on both sides and substituting Eq. (2.23a), one obtains:

\[
\nabla \times \left( \frac{1}{\varepsilon_r(\mathbf{r})} \nabla \times \mathcal{H}(\mathbf{r}) \right) = \left( \frac{\omega}{c} \right)^2 \mathcal{H}(\mathbf{r}). \tag{2.24}
\]

Following similar steps, the equation for \( \mathcal{E} \) is derived as:

\[
\nabla \times \nabla \times \mathcal{E}(\mathbf{r}) = \left( \frac{\omega}{c} \right)^2 \varepsilon_r(\mathbf{r}) \mathcal{E}(\mathbf{r}). \tag{2.25}
\]

where \( c = 1/\sqrt{\mu_0 \varepsilon_0} \) is the speed of light in vacuum and \( \varepsilon_r(\mathbf{r}) = \varepsilon(\mathbf{r})/\varepsilon_0 \) the space dependent relative permittivity. The two equations (2.24) and (2.25) are source-free vector wave equations in terms of the magnetic and electric fields respectively and take the form of generalized eigenvalue problems with linear operators. The standard approach consists of solving either of the two equations while enforcing the divergence conditions of Eq. (2.23c) or (2.23d) and using

\[
\mathcal{E}(\mathbf{r}) = \frac{1}{j\omega \varepsilon(\mathbf{r})} \nabla \times \mathcal{H}(\mathbf{r}) \tag{2.26}
\]

or

\[
\mathcal{H}(\mathbf{r}) = -\frac{1}{j\omega \mu_0} \nabla \times \mathcal{E}(\mathbf{r}) \tag{2.27}
\]

to obtain the other field. Doing so provides all the information about the fields and completes the solution for our problem. In cases of translationally invariant structures, for example in the \( z \) direction, the magnetic field may be written as:

\[
\mathcal{H}(\mathbf{r}) = \mathcal{H}(x, y) \exp(-j\beta z) \tag{2.28}
\]

where \( \beta \) is the propagation constant along the \( z \) direction, from which we can define the effective index as \( n_{\text{eff}} = \beta c/\omega \). Substitution of this into Eq. 2.24 allows
us to obtain:

\[
\left( \nabla_t^2 + \left( \frac{\omega}{c} \right)^2 \varepsilon_r - \beta^2 \right) \mathcal{H}_t = \left( \nabla_t \times \mathcal{H}_t \right) \times \nabla_t \ln \varepsilon_r \tag{2.29a}
\]

\[
\left( \nabla_t^2 + \left( \frac{\omega}{c} \right)^2 \varepsilon_r - \beta^2 \right) \mathcal{H}_z = \left( \nabla_t \mathcal{H}_z - j \beta \mathcal{H}_t \right) \cdot \nabla_t \ln \varepsilon_r \tag{2.29b}
\]

where we have dropped the explicit spatial dependence and where the subscripts \( t \) and \( z \) denote the transverse and longitudinal components, respectively. Because Eq. (2.29a) does not contain the longitudinal field component, it is usually the one solved by most numerical methods since the number of unknown field components is reduced from 3 to 2 (i.e., \( \mathcal{H}_x \) and \( \mathcal{H}_y \)). It can be rearranged to take the standard eigenvalue problem form

\[
\mathcal{L} \mathcal{H}_t = \beta^2 \mathcal{H}_t \tag{2.30}
\]

where the operator \( \mathcal{L} = \nabla_t^2 + \left( \frac{\omega}{c} \right)^2 \varepsilon_r + \nabla_t \ln \varepsilon_r \times (\nabla_t \times) \). Again, a similar equation may be obtained for the electric field. When one chooses that approach, extra care must be taken when using certain numerical tools as the divergence equation (2.23d) is not employed at all in the derivation. This leads to artificial solutions or spurious modes which must be discarded before further analysis.

The solution of Eq. (2.30) provides both the eigenvalues \( \beta \) or \( n_{\text{eff}} \) and the associated eigenvectors \( \mathcal{H} \). \( \beta \) is a key parameter that determines properties of the modes supported by the fibre such as phase and group velocities, birefringence, confinement loss, etc. The eigenvector describes the field distribution or mode profile which is important in evaluating loss due to roughness, nonlinear optical effects, etc. When a single homogenous material is involved, the relative permittivity has no spatial dependence and Eq. (2.29a) reduces to the simpler scalar Helmholtz equation, which may be solved analytically. When more than one homogeneous material are involved, an analytical solution may still be found if the geometry in the cross-section is simple enough that the boundary conditions may be expressed in closed form. This is the case for conventional single or multimode silica fibres and simple hollow dielectric waveguides \([42, 90, 91]\). For HC-PBGFs, the geometry of the cross-section is complex and no analytical solution can be found. One therefore has to turn to numerical methods for the solution of the eigenvalue problem in such structures.
2.4.2 Overview of numerical methods

Since the early proposals of MOFs, several numerical methods have been developed to solve for Maxwell’s equations and derive the optical properties of these fibres. Here we present a brief summary of these methods, outlining their capabilities and shortcomings, focussing especially on those suitable for the study of HC-PBGFs.

One of the very first methods employed for the analysis of periodic structures with photonic bandgaps was the plane wave expansion (PWE) method \[92, 93\]. The essence of this method is to enforce periodic boundary conditions on a single unit cell of the structure and express the eigenvector solutions of Maxwell’s equations as well as the permittivity profile as an infinite sum of sinusoidal functions (plane waves). The eigenvalue problem is therefore transformed into a linear matrix eigenvalue problem. As a result, the method is very efficient in finding the bands (dispersion curves for supermodes) and the bandgaps of perfectly periodic structures. With little modification, the PWE can calculate the band structure for out-of-plane propagation in periodic structures such as HC-PBGFs \[54\]. Defect modes can also be analysed with so-called supercells whereby the full structure comprising the defect is regarded as a unit cell and periodic boundary conditions enforced on its boundaries. This greatly increases the number of plane waves required and the computation time. A drawback for the method is that in addition to the permittivity profile \(\varepsilon_r(\mathbf{r})\), it requires the propagation constant \(\beta\) as input and solves for a corresponding wavelength \(\lambda\), making difficult the study of materials whose permittivity depends on the wavelength. The study of such dispersive materials though possible requires the extra complexity of multiple runs. The freely available implementation from Johnson and Joannopoulos \[94\] made the method very popular and it has been modified and used for several studies on HC-PBGFs \[83, 95–97\].

Other general numerical methods have also been be tailored for solving Maxwell’s equations in HC-PBGFs. These include the beam propagation methods \[98, 99\], finite difference \[100, 101\] and finite element methods \[102, 104\]. These very general numerical methods work under the broad principle of discretizing the waveguide geometry into a mesh of smaller elements (usually triangles or quadrilaterals). The wave equation is then enforced separately on each element of the mesh and the final modal solution obtained by summing up the collective contributions from all the mesh elements. Each of these numerical methods has its own merits and
disadvantages and selection of one over the other depends on the nature of the problem to solve and the usual trade-off between speed and accuracy.

For the simulation work carried out throughout this thesis, we selected the finite element method (FEM) as it appears to offer the most flexibility and is currently the numerical method of choice for the study of HC-PBGFs. It is a fully vectorial method that can effectively exploit the symmetry properties of the structure under study, and that can solve the wave equation for arbitrary refractive index distributions. Furthermore, the FEM easily allows for the incorporation of material dispersion when solving the problem as the eigenvalue is the propagation constant. More importantly, choosing the FEM as a platform allows us to focus entirely on understanding the properties of HC-PBGFs and optimizing their design, relying instead on the external expertise of commercial FEM software providers to implement and update the solver algorithms and other required numerical routines.

The commercial package used throughout this work is COMSOL MULTIPHYSICS\footnote{http://www.comsol.com}, which is a general finite element modeling tool for a wide range of physical modeling problems and arbitrary user-defined partial differential equations. Together with the general computing software MATLAB, they form a powerful and flexible tool that makes easy the description, solution and postprocessing of the wave propagation problem through use of a scripting language.

### 2.4.3 Modelling realistic HC-PBGFs with the FEM

Having made the case for our selection of the finite element method for the study of HC-PBGFs, we now describe the practical steps involved in simulating the modal properties of these fibres with the COMSOL MULTIPHYSICS software.

#### 2.4.3.1 Parametric description of idealized HC-PBGFs

The study of the optical properties of realistic HC-PBGFs starts with the ability to accurately describe the geometry of their cross-sections. As we seek to identify structures with optimized loss and bandwidth performance, a parametrization of the geometry is necessary in order to assess the impact of geometry changes on the desired performance. Figure 2.16 illustrates a portion of the cross-section of a typical 7c HC-PBGF with a TLH cladding. Following previous work on HC-PBGFs
Figure 2.16: Idealized HC-PBGF cross-section used in simulations, along with the eight-parameter used for its description. The number of rings $N$ of air holes outside the core is the final parameter.

[62, 84, 105], we use eight structural parameters to describe the geometry of the cross-section. The cladding is a TLH with air holes spacing or pitch $\Lambda$. To mimic the shape of the air holes in real fibres, they are described as rounded hexagons of side-to-side (diameter) $d$, the corners of which are filleted with a circle of diameter $D_c$ (see [62]). This leaves in place glass struts with thickness $t = (1 - d/\Lambda)\Lambda$. It follows from the analysis of section 2.2 that $\Lambda$ and the dimensionless quantities $\frac{d}{\Lambda}, \frac{D_c}{\Lambda}$ are the 3 essential parameters which determine the spectral position, width and depth of the cladding photonic bandgap since they completely define the strut thickness and the spacing and size of the glass nodes.

In the first ring of holes surrounding the core defect, six air holes at the corners (hereafter referred to as *corner holes*) have five neighboring air holes and therefore maintain a hexagonal shape, while the others have only four neighbours and are forced as a result of surface tension and differential pressures during fibre draw, to assume a pentagonal shape. The corners of these pentagonal air holes near the core boundary are filleted with circles of diameter $D_1$.

The core boundary assumes the shape of a dodecagon with nominal radius $R_c$ and whose corners, mimicking the effect of surface tension, are filleted with circles of diameter $D_2$. The core boundary has its own characteristic thickness $t_c$ which
allows to model the effect of the inclusion of a core tube with arbitrary thickness in the fibre preforms. The final parameter (not shown in the figure) is the number $N$ of rings of air holes surrounding the core defect and is important in determining the leakage loss suffered by the guided modes of the fibre.

### 2.4.3.2 Modelling steps

The practical steps involved in modelling HC-PBGFs are illustrated in Fig. 2.17 and can be summarized as follows:

- **Geometry definition**
  The first step in modelling the fibre’s modal properties is to define the geometry of the cross-section, making use of the parameters described above. The **COMSOL Multiphysics** provides an extensive library of 2D geometry functions, in particular, polygons of arbitrary shape can be built provided the $(x, y)$ coordinates of their vertices are known. In addition, the corners of such a polygon can be filleted with circles of arbitrary radii. Although Fig. 2.17(a) shows the geometry for a full cross-section of the fibre, it is customary to exploit the symmetry in the problem which allows reducing the computational domain to a $\pi/2$ sector of the fibre. Doing so greatly reduces computation time and allows the study of guided modes belonging to a selected symmetry group, which is determined by the boundary conditions applied to the edges of the chosen fibre sector.

- **Physics settings**
  The next step consists of setting the physics of the problem we desire to solve. This involves stating the equations we need to solve either by specifying a user-defined partial differential equation or by selecting one of the built-in modules of the COMSOL software. Then we assign the relevant material properties to each of the subdomains making up the geometry. In particular, we assume a wavelength independent refractive index of unity for the air regions, and use a Sellmeier equation to compute the wavelength dependent refractive index of silica, that is [106]:

$$n_g^2 = 1 + \sum_{i=1}^{n} \frac{B_i \omega_i^2}{\omega - \omega_i^2}$$  \hspace{1cm} (2.31)
Figure 2.17: Steps involved in a typical FEM simulation using the FEM package Comsol Multiphysics. In (a), the geometry is defined according to the previous section. In (b) Material properties are assigned to each subdomain, and the boundary conditions applied to each boundary line. In this particular case, continuous boundary conditions are used everywhere and scattering conditions applied to the outermost boundaries. In (c) a mesh is calculated with user-specified parameters. In (d), the problem is solved and the obtained solution post-processed.

where \( n \) is the total number of absorption resonances in the glass, \( \omega_i = 2\pi/\lambda_i \) the \( i^{th} \) resonant frequency and \( B_i \) the corresponding resonance strength. In practice we have used \( n = 3 \) and \((B_1, B_2, B_3) = (0.6961663, 0.4079426, 0.8974794)\) and \((\lambda_1, \lambda_2, \lambda_3) = (0.0684043, 0.1162414, 9.896161) \mu m \) \cite{70}. As seen from Fig. 2.17, the microstructure is enclosed by Perfectly Matched Layers (PMLs) which are deployed to truncate what would be an infinite computational domain into a finite one. In essence, PMLs effectively absorb any radiation incident upon them regardless of the incidence angle, wavelength or polarization. The PMLs formulation we adopt is the one by Sacks et al. \cite{107} in
which the PML are made of anisotropic absorbing materials whose permittivity and permeability are described as two complex tensors:

\[
\begin{align*}
\varepsilon_r &= \varepsilon_r \Lambda \\
\mu_r &= \mu_0 \Lambda 
\end{align*}
\]  

(2.32a)  

(2.32b)

where the diagonal tensor \( \Lambda \) assumes different values depending on the layer orientation. For vertical layers (2 and 4) one has:

\[
\Lambda = \begin{bmatrix}
\frac{1}{s} & 0 & 0 \\
0 & s & 0 \\
0 & 0 & s \\
\end{bmatrix}
\]  

(2.33)

and for the horizontal layers (1 and 3):

\[
\Lambda = \begin{bmatrix}
s & 0 & 0 \\
0 & \frac{1}{s} & 0 \\
0 & 0 & s \\
\end{bmatrix}
\]  

(2.34)

Finally, for the PML regions at the corners (5-8), the \( \Lambda \) tensor is obtained by multiplying the corresponding tensors for neighboring vertical and horizontal layers and therefore:

\[
\Lambda = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & s^2 \\
\end{bmatrix}
\]  

(2.35)

\( s \) is a complex number whose real part attenuates incident evanescent waves and whose imaginary part dampens incident propagating waves. Generally:

\[
s = b - j\alpha_{\text{max}} \left( \frac{\rho}{d} \right)^m
\]  

(2.36)

where \( b, \alpha_{\text{max}} \) and \( m \) are free parameters, \( \rho \) the normal distance from the point of interest in the PML to the PML edge and \( d \) the width of the PML which impose to be the same for horizontal and vertical layers. The COMSOL MULTIPHYSICS provides a built-in implementation of the PML, which provides accurate simulation results with the default values \( b = 1, m = 2 \).
Once material properties have been assigned to the subdomains, the boundary conditions have to be defined. When modelling a full cross-section, we apply scattering boundary conditions to the four outermost boundaries. When simulating a $\pi/2$ sector instead, either Dirichlet or Neumann conditions maybe applied to the boundaries as shown in Fig.2.18, resulting in a selection of a class of modes according to the definition of McIsaac [108, 109].

![Figure 2.18](http://www.cise.ufl.edu/research/sparse/umfpack)

Figure 2.18: Boundary conditions and mode classes when modelling a quarter HC-PBGF. Straight lines indicate perfect electric conductors or Dirichlet conditions while dashed lines indicate perfect magnetic conductors or Neumann condition.

- **Meshing**
  
  In this step, the computational domain is discretized into a large number of small triangular elements over which the eigenvalue problem is effectively enforced. COMSOL provides powerful mesh generating algorithms with several free parameters which are chosen depending on the nature of the problem at hand. These algorithms automatically make the mesh denser near curved boundaries where a more rapid change in the field is expected to occur, but also allow for the option of making the mesh denser or less in arbitrary subdomains. We have always opted for example for denser meshes in the core region where the guided mode fields are predominantly concentrated.

  Generally, the denser the mesh, the more accurate the solution, but also the longer the computation time. As a result, a mesh convergence test designed to find the minimum number of mesh elements that still provide an accurate solution is usually performed for any given structure.

- **Solving the problem**
  
  COMSOL MULTIPHYSICS provides a range of linear solvers to choose from, with a number of control parameters that may be specified for time and accuracy. The direct solver UMFPACK\(^2\) was found to provide the fastest

\(^2\)http://www.cise.ufl.edu/research/sparse/umfpack
solution selected for the simulations presented in this thesis. When solving for guided modes in a HC-PBGF, the two most crucial parameters to provide are a guess value for the eigenvalue and the desired number of eigenmodes to solve for.

• Postprocessing

With the eigenvalues and corresponding eigenvectors solved for, an extensive library of postprocessing functions are available to allow extracting valuable information regarding the optical properties of the fibre. The first step in processing this data is the automatic selection of the mode of interest from the returned eigenvectors. As our main interest lied in optimizing the properties of the fundamental mode of the fibre, i.e. the $LP_{01}$-like mode, we designed a number of criteria allowing to select it from the eigenvectors. These typically include selecting the mode with the highest fraction of power in the core, the power of which has a near maximum at the core centre, or the mode with lowest confinement loss. Once the desired mode is selected, its field distribution can be plotted and the relevant properties such as $F$, confinement loss, fraction of power guided in the glass and dispersion computed.

Programs and scripts based on the procedure just highlighted had been developed at the ORC prior to my beginning work on this thesis, and this greatly sped up the learning process. These scripts (provided to me courtesy of Dr. Francesco Poletti) were used with little modification for the work on idealized fibres presented in Chapter 3 and the remainder of the simulation work in the thesis was performed with scripts developed solely by myself.

2.5 Summary

In this chapter, we have presented a broad introduction to HC-PBGFs, from the physical origin of the photonic bandgap to their main properties and features. We have discussed their low optical nonlinearity, ultra low transmission latency, and analysed the major mechanisms that give rise to loss in these fibres. We have described surface modes and their detrimental effect on bandwidth and fibre loss. Finally, numerical methods used to simulate the properties of HC-PBGFs were presented with a special emphasis on the finite element method which has been used throughout this thesis.
In the next chapter, we present the results of extensive numerical simulations that have allowed us to identify fibre designs with low interface field intensity as well as wide transmission bandwidths, while also continuing with a review of existing literature in the pursuit of low-loss HC-PBGFs.
Chapter 3

Optimizing the properties of idealized HC-PBGFs

After the introduction to HC-PBGFs and the brief description of the approach adopted throughout this thesis to simulate their properties, I proceed in this chapter to present results of extensive and detailed numerical simulations which sought to identify the optimal structural parameters required for broad bandwidth and low-loss operation. To this aim, the impact of the cladding and the core defect itself are studied separately. For the cladding, I present the results in the form of two-dimensional contour maps, which provide an effective and highly visual way of identifying preferred regions of operation in the parameter space. The effect of the size and surround of the core defect on loss and bandwidth is then discussed at length. To target fibres with potential for low-loss operation, I rely here on reducing the normalised interface field intensity which as discussed in the previous chapter is routinely used to estimate the fibres’ roughness scattering loss. Understanding how loss and bandwidth are affected by structural parameters and identifying in the first instance which structures ought to be targeted was a crucial first step in loss reduction efforts.

3.1 Impact of the number of rings of air holes

One critical aspect to consider before designing low-loss HC-PBGFs is the impact of the number of rings of air holes on the overall loss and guidance properties of the fibre. As HC-PBGFs preforms are most commonly made by stacking together
hundreds of circular capillaries, it is important in order to avoid time-consuming labour and speed up the development cycle, to determine beforehand the smallest number of capillaries to be used that will ensure proper operation of the fibres. As mentioned in the previous chapter, it is well known for example and follows logically from the nature of bandgap guidance that the confinement loss of the fundamental core-guided mode is very sensitive to the number of rings of air holes surrounding the core defect. Figure 3.1 shows plots of the confinement loss and interface field intensity $F$ as a function of wavelength for a $7c$ fibre surrounded by 1 to 8 rings of air holes.

While the confinement loss is strongly dependent on the number $N$ of rings of air holes as expected, the interface field intensity $F$ does not change appreciably within the bandgap when more than two rings of air holes are included. This suggests that most of the roughness scattering occurs at the surfaces within the first two rings near the core defect (and indeed within the first ring only for wavelengths away from the bandgap edges) and that adding further rings of air holes does not significantly reduce or increase the scattering loss of the fibre. Further detailed study on the interface field intensity reveals in fact that for the fundamental core-guided mode, scattering from the inner and outer surfaces surrounding the core account for as much as 90% of the scattering.

These findings clearly indicate that in deciding the number of elements to stack when making fibre preforms, minimizing the confinement loss should be the primary concern. They also show that in order to study low scattering loss designs, time-consuming simulations with many rings of air holes are not necessary. For the parameters of Figure 3.1 six rings of air holes would suffice to keep the confinement loss below $10^{-3}$ dB/km throughout the bandgap. Note however, that the number of rings required to keep the confinement loss at negligible values is also subject to other design parameters as will be further discussed shortly.

A closer look at Figure 3.1 reveals that although similar in trend, the dependence of $F$ and confinement loss across the bandgap are slightly different. Confinement loss drops sharply from the short wavelength edge of the bandgap but is already increasing near the center of the bandgap especially in the case of fewer rings of air holes are, whereas the curve for $F$ is more symmetric. This is because the propagation constant whose imaginary part gives the confinement loss is more sensitive to changes in the size of the geometry with respect to the wavelength, whereas the field distribution and its normalized value at the interface depends more on the properties of the photonic bandgap.
Figure 3.1: (a) Confinement Loss dependence on number of rings of air holes. Above 7 rings, numerical inaccuracies give rise to the apparent fluctuations. (b) F-factor dependence on the number of rings of air holes around the core. Data obtained with a 7-cell core, $\Lambda = 4.7 \mu m, d/\Lambda = 0.99$ and $D_c/d = 0.6$. Whereas the confinement loss decreases by more than an order of magnitude for each additional ring, $F$ does not change appreciably when more than 2 rings are included.
3.2 Fibre properties and Cladding design

The position and width of the photonic bandgap for an idealized cladding are determined by the the parameters $\Lambda$, $d/\Lambda$ and $D_c/d$ (see Fig. 2.16) and the refractive indices of air and glass. As it is customary to take the air index as unity and constant at all wavelengths, the most important material properties to consider are those of the silica. Because silica’s index changes very little over the range of wavelengths of operation targeted (e.g. $\Delta n/n = 0.8\%$ between 1µm and 2µm for silica), it can be considered effectively constant for the broader purpose of analysing the properties of the bandgap. Under such approximation, the scale-free nature of Maxwell’s equations ensures that the central wavelength $\lambda_c$ of the photonic bandgap scales linearly with the pitch $\Lambda$ whereas the normalized bandwidth $\Delta\lambda/\lambda_c$ is invariant for a given combination of the parameters $d/\Lambda$ and $D_c/d$ `[46]`. We therefore seek to identify combinations of these parameters which yield the widest possible photonic bandgaps and investigate their impact on fibre loss.

3.2.1 Bandgap position

Predicting the central wavelength of the bandgap would be straightforward if analytical expressions describing the dispersion curves of the three modes forming the bandgap edges can be derived (see the discussion in section 2.2 and Fig. 2.6). However, the shape and spatial distribution of the nodes and the network of silica struts make such a task prohibitively complex for HC-PBGF claddings. Early efforts to predict the width and position of the bandgap have shown that both wavelength edges of the bandgap shifted to shorter wavelengths with increasing air-filling fraction while the bandwidth increased exponentially `[62]`. While this is correct in a first-order approximation, the air-filling fraction which according to our description of section 2.4.3.1 is given by:

$$ f = \left( \frac{d}{\Lambda} \right)^2 \left[ 1 - \left( 1 - \frac{\pi}{2\sqrt{3}} \right) \left( \frac{D_c}{d} \right)^2 \right] $$

(3.1)

cannot fully account for all the bandgap properties as discussed in section 2.2 (see also `[110`, pp. 148–153]).

To obtain a complete picture, we have carried out a large number of numerical simulations using our FEM platform, for several structural parameter combinations
yielding a photonic bandgap centered around $1.55\mu m$. To simultaneously capture all the relevant properties of the fibre, we did not use the unit cell of a perfectly periodic cladding but performed the calculations on full cross-sections of 19c HC-PBGFs with six rings of air holes surrounding the core. For these simulations, $d/\Lambda$ was varied between 0.95 and 0.99 in 0.01 steps, while $D_c/d$ increased from 0.2 to 0.95 in steps of 0.05 as illustrated in Fig. 3.2. The pitch $\Lambda$ was then scaled accordingly so that all these different fibres would have their photonic bandgap centered near $\lambda_c = 1.55\mu m$.

Figure 3.2: Illustration of sample fibre cross-sections used to study the impact of cladding. In total, 80 fibres with different $(\Lambda, d/\Lambda, D_c/d)$ combinations and six rings of air holes (rather than the 3 rings shown here for illustration only) were simulated.

Figure 3.3 shows the dependence of the normalized central wavelength on the two cladding parameters $d/\Lambda$ and $D_c/d$. Two main observations may be made from examining this contour map. First, for a fixed value of $d/\Lambda$, the central wavelength increases with $D_c/d$. This is expected because if the pitch $\Lambda$ is fixed at a constant value, an increase in $D_c/d$ (thereby making the holes take on a more circular shape) would accumulate more silica at the nodes. As a result, the modes at both edges of the bandgap would now cross the air line at longer wavelengths, leading to the center wavelength shifting to longer wavelengths accordingly. As
Figure 3.3: Contour plot of the normalized central wavelength $\lambda_c/\Lambda$ as a function of cladding parameters for silica-air HC-PBGFs. The plot was obtained by modelling 80 fibres with different $(\Lambda, d/\Lambda, D_c/d)$ parameters yielding photonic bandgaps centred around 1.55$\mu$m.

an example, for $d/\Lambda = 0.97$, the normalized central wavelength increases from $\approx 0.33$ to $\approx 0.49$ when $D_c/d$ is increased from 0.2 to 0.95, that is, when the air holes change from nearly perfect hexagons to nearly circular shapes. Secondly, the central wavelength decreases with increasing $d/\Lambda$ if the ratio $D_c/d$ is kept constant. Again, this results from the simultaneous reduced amount of glass at the nodes and thinner struts which shift the photonic bandgap to shorter wavelengths. Our results regarding the the central wavelength of the photonic bandgap are therefore in agreement with those of Mortensen et al. [62] as the fibres with the highest air-filling fractions (those at the top left corner of the map) are also the ones with the smallest $\lambda_c/\Lambda$.

### 3.2.2 Bandwidth

For most practical applications in which HC-PBGFs may be used, a wide operational bandwidth is often desirable. This is particularly the case for long and short-haul data transmission where wavelength division multiplexing technology may be implemented over much broader spectral ranges than currently exploited.
This need for broader bandwidths was recognized since the very early days of HC-PBGFs and several theoretical investigations sought to find out the cladding lattice arrangements, the structural and material parameters and the simple rules that could help the identification of designs with the widest photonic bandgaps [62, 63, 111–114]. It emerged from these early studies that to a first approximation, the width of the photonic bandgap was deeply impacted by the cladding’s air-filling fraction, with other structural details playing only a secondary role [62, 79].

As already discussed in section 2.2, when claddings with TLH, square lattice and TLR configurations share the same node size, spacing and \( d/\Lambda \), the TLH cladding possesses the thickest glass struts and therefore the narrowest photonic bandgap. Despite this seeming disadvantage, practical considerations from a fabrication point of view still make the TLH the lattice of choice. Indeed, both the SL and TLR require the stacking of several hundreds elements more than the TLH to keep leakage loss at negligible values [63, 65]. Detailed optimization of the bandgap width for this lattice and understanding of how it is affected by the cladding parameters is therefore of significant relevance.

Figure 3.4 shows a contour plot summarizing the range of normalized bandwidth values \( \Delta \lambda/\lambda_c \) that can be expected from idealised HC-PBGFs (with TLH lattice) as a function of cladding parameters \( d/\Lambda \) and \( D_c/d \). As before, the data was obtained by simulating eighty different parameter combinations yielding a photonic bandgap centred at 1.55 µm, and interpolating the results at those points not explicitly computed.

From this contour map, we note at first that for a constant \( D_c/d \), increasing \( d/\Lambda \) values (i.e. making the struts thinner) always results in wider photonic bandgaps. Indeed, with thinner silica struts, individual modes of the individual cladding nodes overlap less strongly, leading to the observed wider photonic bandgaps. In going from \( d/\Lambda = 0.95 \) to 0.99, the normalized bandgap width more than doubles from 0.2 to 0.42 when the air holes are such that \( D_c/d = 0.6 \). Therefore, as a general rule, fibres with glass struts as thin as possible should be targeted in practice.

When \( d/\Lambda \) is fixed on the other hand, we observe that there exists an optimum value of \( D_c/d \) for which the photonic bandgap is the widest. For example, for \( d/\Lambda = 0.98 \), the widest photonic bandgap is obtained when \( D_c/d = 0.7 \). This optimum value is clearly function of \( d/\Lambda \) as can be observed form the contour
Chapter 3 Optimizing idealized HC-PBGF

Figure 3.4: Contour plot of the normalized Bandwidth $\Delta\lambda/\lambda_c$ as a function of cladding parameters for silica-air HC-PBGFs. The plot was obtained by modelling 80 fibres with different $(\Lambda, d/\Lambda, D_c/d)$ parameters yielding photonic bandgaps centred around 1.55$\mu$m.

Figure 3.5: Illustration of unit cells with different $D_c/d$, hence different node size. The unit cells share the same $d/\Lambda = 0.98$ and are drawn to scale so that the resulting bandgap is centered at the same wavelength.

As $D_c/d$ changes from 0 to 1, the air hole shape goes from a perfect hexagon to...
a perfect circle. The resulting accumulation of glass at the node requires that
the pitch be adjusted to produce photonic bandgaps centred around the same
wavelength. This inevitably results in a severe reduction in the air-filling fraction
which goes from \((d/\Lambda)^2\) to \(\pi/2\sqrt{3}(d/\Lambda)^2\) for \(D_c/d = 0\) and \(D_c/d = 1\) respectively.

However, the photonic bandgap for the fibre with the highest air-filling fraction
\((D_c/d = 0.2\) in this case) is only half as wide as that of the fibre with the op-
timum \(D_c/d\) ratio or 0.7. This suggests that in contrast to what was believed
from previous works [62, 115], the highest air-filling fractions do not result in the
widest bandgaps, but rather, optimizing the cladding details can be of significant
importance. When \(D_c/d\) is smaller than the optimum value, the bandwidth de-
creases very quickly as \(D_c/d\) goes towards 0. In contrast, the bandwidth decreases
at a much slower rate as \(D_c/d\) is further increased beyond the optimum value.
We conclude therefore that when limited by the \(d/\Lambda\) ratio achievable in practice,
fibres with lower rather than higher air-filling fractions may be desirable in order
to optimize the operational bandwidth.

3.2.3 Confinement loss

Before scattering from surface roughness was identified as imposing a fundamental
limit on attenuation in HC-PBGFs, it was well known that the fibres suffered
leakage or tunneling loss which could be reduced by incorporating many rings of
air-holes around the core defect as shown in section 3.1 [79].

Besides its dependence on the number of rings of air holes, the confinement loss in
HC-PBGFs is also intimately linked to the exact cladding structural parameters
as well as the core boundary definition [52, 79].

Seeking to identify the region of the parameter space where the confinement loss
would be maintained at negligible values with the incorporation of as few rings of
air holes as possible, we simulated a large number of 19c fibres with six rings of air
holes as previously described with their bandgap centred around 1.55\(\mu\)m. Figure
3.6 shows a contour map of the minimum confinement loss for the \(x\)-polarized
fundamental mode for each fibre.

A quick comparison with the normalized bandwidth contour of Fig. 3.4 shows
that the confinement loss follows a similar trend as the the width of the bandgap.
We conclude therefore that achieving the widest possible bandwidth also results
in simultaneous achievement of very low confinement loss value. This somewhat
expected result is significant as it implies no design trade-off between bandwidth and confinement loss in HC-PBGFs.

### 3.2.4 Interface field intensity

The case was made in the previous chapter that the normalized interface field intensity $F$ is a valuable quantity that is relatively easy to evaluate and has proved to provide good estimates of scattering loss when calibrated as in Eq. (2.19). As a result, we assume here and further discuss in greater detail in chapter 4 that fibres with minimized interface field intensity also have lower scattering loss.

Earlier seminal work from the group at the university of Bath [19, 26, 80, 116, 117] sought to achieve the objective of fibres with lower interface field through engineering antiresonant features on the core surround. While this was effective in reducing the loss, it also introduced several surface modes and therefore the greatly reduced the transmission bandwidth. Subsequent work conducted here at

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**Figure 3.6:** Contour plot of the minimum confinement loss as a function of cladding parameters for 19c silica-air HC-PBGFs with six rings of air holes. The plot was obtained by modelling 80 fibres with different $(\Lambda, d/\Lambda, D_c/d)$ parameters yielding photonic bandgaps centred around 1.55µm. Above the purple line, leakage loss is below 1dB/km.
the ORC revealed that removing core tubes and antiresonant features on the core surround would enable the achievement of a wide usable bandwidth \cite{118}.

With the aim of identifying low-loss and surface mode free fibre designs, we investigated the impact of cladding structural parameters on the normalized interface field intensity. Figure \ref{fig:3.7} shows a contour map of the minimum interface field intensity for idealized 19c fibres with their core wall half as thick as the cladding struts, and again, all having their photonic bandgaps centered around 1.55\mu m. For clarity, we have calibrated the interface field intensity with a factor $\eta = 300$ as we have found this simple scaling to provide an accurate description of the loss in fabricated fibres (see chapter \ref{chap:6}). The first observation from this contour map is that the minimum interface field intensity is greatly affected by the choice of cladding parameters. For example, if $D_c/d$ is fixed at a value of 0.6, then going from $d/\Lambda = 0.95$ to $d/\Lambda = 0.99$ reduces the interface field intensity by a factor of more than 3. On the other hand, optimizing the node size may also prove important. When $d/\Lambda$ is fixed at 0.98, operating with the optimal value of $D_c/d = 0.7$ reduces

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3_7.png}
\caption{Contour plot of the minimum interface field intensity $F$ as a function of cladding parameters for 19c silica-air HC-PBGFs. The plot was obtained by modelling 80 fibres with different $(\Lambda, d/\Lambda, D_c/d)$ parameters yielding photonic bandgaps centred around 1.55\mu m. The map shows $\eta \times F$ with $\eta = 300$, corresponding to the normalization factor observed to yield a good match with fabricated fibres. (See chapter \ref{chap:6}).}
\end{figure}
Chapter 3 Optimizing idealized HC-PBGF

the interface field intensity by 30% when compared to operation at $D_c/d = 0.3$. Operating at the optimal rod size is even more crucial when the values of $d/\Lambda$ are lower. Note that as observed before for the normalized bandwidth, the optimum node size changes with $d/\Lambda$. It is interesting, but also somewhat expected that the normalized interface field intensity largely follows a similar trend to that of the bandwidth and confinement loss (see Figs. 3.4 and 3.6), even though they are not optimized for exactly the same parameter values.

Although it may have been suspected, this dependence of scattering loss on cladding parameters has not been previously recognized. These results show that in the pursuit of low-loss HC-PBGFs, optimization the microstructured cladding should not be ignored.

Finally, we note that with the normalisation adopted, that is setting $\eta = 300$ in Eq. (2.19) as it correctly gives the loss in current fibres, it appears from the map of Fig. 3.7 that the minimum achievable loss in 19c HC-PBGFs at the wavelength of 1.55$\mu$m is as high as 2.1$dB/km$ (for $d/\Lambda = 0.99$, $D_c/d = 0.55$). This can be reduced to about 1.2$dB/km$ after further optimisation of the structure, as discussed in Chapter 7.

3.2.5 Summary

Through the contour maps shown in this section, we have demonstrated that in addition to directly determining the position and width of the photonic bandgap, the cladding parameters have a profound impact on both confinement and scattering loss of HC-PBGFs. Although these maps were calculated for fibres guiding around 1.55$\mu$m, we have made the case that the operating wavelength scales with the pitch $\Lambda$ while the normalized bandwidth remains unchanged under such rigid scaling. In addition, when scaled to operate at longer wavelengths, it is expected that the normalized interface field intensity should decrease slightly. These results obtained for fibres at 1.55$\mu$m are therefore of broad generality.

The identification of optimized designs is greatly simplified by the fact that the three performance metrics, namely bandwidth, leakage loss and interface field intensity follow a similar dependence on the cladding parameters. As a result, as far as optimizing the cladding is concerned, we will focus only on the most important of the three, namely the interface field intensity. Under these premises, achieving the lowest scattering loss will always guarantee wide bandwidth and
low confinement loss. We illustrate this in Fig. 3.8 where the preferred region of operation in the parameter space for fibres operating at 1.55\(\mu\)m is highlighted. The design constraints we use as an example are that the 19c fibres should have confinement loss below 0.1\(dB/km\) with six rings of air holes, and have a pitch \(\Lambda \leq 5\mu m\) to avoid fibres with too large diameters (small fibre diameters are desirable when splicing to SMFs). The example constraints also require a bandwidth at least as wide as 25\% of the central wavelength. In practice, other design constraints may include the highest achievable \(d/\Lambda\) value or the smallest strut thickness that may be attained during the draw. When this is the case, the node size must be chosen in accordance.

We have seen through the maps have shown that fibres with the highest air-filling fractions do not yield the widest bandgaps or the lowest loss as may be intuitively expected. For a given strut thickness value, a corresponding node size exists that maximizes the width of the photonic bandgap while nearly minimizing the loss.
3.3 Optimizing the core defect

The core defect and its thin (or thick) glass boundary in HC-PBGFs are of paramount importance, as they are directly responsible for many of the fibre’s properties such as propagation loss, number of core-guided modes, presence or absence of surface modes, modal dispersion, etc. It is well known for example that enlarging the core defect is an effective method of reducing the fibre loss, although at the expense of a higher number of core-guided modes. It is also well established that the introduction of antiresonant features on the core boundary is a viable way to further reduce fibre loss, to engineer its dispersion, birefringence and other modal properties, though inevitably resulting in the segmentation of the transmission into narrower bands due to the introduction of surface modes. Deciding which core size and boundary design to employ is therefore subject to both the intended application and the practicality of the fabrication of the attained design.

For long and short-haul data transmission specifically, low loss and wide transmission bandwidths are of primary importance. Additional requirements for transmission fibres in which many guided modes may be exploited for data transmission are low differential group delay between the guided modes and low intermodal power coupling or cross-talk. Contrary to the impact of the periodic cladding where reducing the loss also leaves in place wide bandwidth, these requirements cannot be all met by a unique core design and some tradeoffs are always necessary. In this section, we investigate the role of the core size and boundary.

3.3.1 Core size

Traditionally, the core in HC-PBGFs is formed by omitting a number of unit cells from an otherwise perfectly periodic cladding. In addition to the more conventional and six-fold symmetric 7, 19, 37 cell core defects that are of primary importance to the present work, fibres with 3 and 4 cell cores targeted at single mode and high group birefringence operation respectively have also been reported, as illustrated in Fig. 3.9. The fact that HC-PBGFs cores can be made to nearly arbitrary shape and size is important for many practical applications, but complicates their theoretical analysis.

Because of its non-circular shape (often dodecagonal, especially in idealised fibres) and the presence of glass nodes on its surround, no analytical treatment of
light guidance in the HC-PBGF core can realistically be provided and therefore, accurate scaling laws with the core size can only be obtained empirically from numerical simulations or experiments. Valuable insight and approximate scaling laws can readily be obtained however, if the core is regarded in a first order approximation as a circular hollow glass waveguide of radius $R$. Such a circular hollow waveguide is a cylindrical air hole surrounded by an infinite glass cladding. In this simpler case, the fields in the air core, and outer glass region can be expressed as combinations of cylindrical functions [28, 42]. For the fundamental $x$–polarized $HE_{11}$ mode, the nearly transverse electric field inside a hollow waveguide is approximated as [42]:

$$E_x(r) = AJ_0\left(\frac{u_{11}r}{R}\right)$$

and the general $HE_{nm}$ mode has field components expressed in terms of Bessel functions of the first kind $AJ_{n-1}\left(\frac{u_{nm}r}{R}\right)$. Here $A$ is a constant, $J_n$ is the Bessel function of the first kind of order $n$, $u_{nm}$ the $m^{th}$ zero of $J_{n-1}$ and $r$ is the radial distance from the circular core’s centre. We see therefore, that the field is vanishingly small near the core boundary. Assuming that the mode is well confined so that the core essentially carries all of the guided power, the total power carried by
the fundamental mode is:

\[ P = \int_{\theta=0}^{2\pi} \int_{r=0}^{R} E \times H^* r dr d\theta \propto \int_{0}^{R} |E_z^2(r)| r dr = \frac{|A|^2}{2} R^2 J_1^2(u_{11}). \] (3.3)

On the other hand, as the field nearly vanishes near the core boundary, the field at radial position \( r = R - \delta \) where \( \delta \) is arbitrarily small, may be obtained through a Taylor expansion as:

\[ E_x(R - \delta) = -\frac{1}{1!} \frac{dE_x}{dr} \delta + \frac{1}{2!} \frac{d^2E_x}{dr^2} \delta^2 - ... = A \frac{u_{11}}{R} J_1(u_{11}) \delta + \mathcal{O}(\delta^2). \] (3.4)

Combining these expressions, one obtains the normalized interface field intensity defined in Eq. (2.18) as:

\[ F = \left( \frac{\epsilon_0}{\mu_0} \right)^{\frac{1}{2}} \frac{2 \pi R |E_x(R - \delta)|^2}{P} \propto \frac{1}{R^3}. \] (3.5)

It follows therefore that in this simpler case, scattering from surface roughness scales with the core radius as \( R^{-3} \). In [42], Marcatili et al. show that the confinement loss follows the same scaling rule and Johnson et al. [121] confirm that it also holds for hollow Bragg fibres with perfect circular symmetry.

Although HC-PBGFs are structurally more complex, the loss contributions follow a similar trend when the core is enlarged. It is important when discussing the impact of the core size in HC-PBGFs to distinguish between core size control via the deliberate removal of more or less unit cells from the stacked preform which guarantee a uniform cladding, or via differential pressure imbalance during fibre draw, in which case the core is expanded or compressed at the expense of the cladding. Here, we focus on the former case and treat the latter in Chapter 7 when we discuss the impact of structural distortions. Figure 3.10 shows calculated dispersion curves for the core-guided modes in HC-PBGFs with 7, 19 and 37 core defects, along with plots of the normalized interface field intensity for the first twelve guided modes.

In order to avoid surface modes, we have in these simulations, made the struts on the core boundary half as thick as those in the cladding [86]. Although the loss decreases with increasing core size, it can also be appreciated that the number of guided modes increases rapidly. Indeed, waveguides with larger core sizes are prompt to support more guided modes, with the number of modes typically
increasing as $R^2 \ [91, 121]$. This increase in the number of guided modes leads to 
a decrease in the separation between their propagation constants, and as a result 
modes in larger core fibres are more prone to intermodal coupling, which may 
worsen the cross-talk in data transmission applications exploiting SDM. Figure 
3.11 summarizes these simulation results by plotting the minimum interface field intensity and number of guided modes as a function of core radius. It is to be appreciated that the loss follows $R^{-p}$ with $p = 2.84$ in this particular case while the number of guided modes increases as $R^2$ as expected. Although in HC-PBGFs, $p$ will generally depend on the exact cladding parameters as well, we conclude that the added complexity of the core shape and surround does not greatly affect the scaling rules with core size, and as a result the $R^{-3}$ rule may always be used as a first approximation.

It can also be observed from Fig. 3.10 that higher order modes suffer from higher
losses than the fundamental mode. This is a consequence of higher order modes having higher field amplitudes near the interfaces, as may be seen by calculating the corresponding $E(R - \delta)$ with the expressions from Marcatili’s paper [42]. For the simpler case of a hollow dielectric waveguide, it can be deduced from the simple field expressions that the ratio of the loss for the $EH_{nm}$ to that of the fundamental mode is approximately (assuming loss ratio is also ratio between $F$ values):

$$
\zeta = - \left( \frac{u_{nm}}{u_{11}} \right)^2 \frac{[J_{n-2}(u_{nm}) - J_n(u_{nm})]^2}{4J_{n-2}(u_{nm})J_n(u_{nm})}.
$$

(3.6)

This means that modes in the ‘$LP_{11}$’ group (made of the $TE_{01}$, $TM_{01}$ and the two $HE_{21}$ modes) are at least 2.5 times lossier than the fundamental mode, those in the ‘$LP_{21}$’ mode group 4.6 times lossier and the ‘$LP_{02}$’ modes 5.3 times lossier, etc. It can be seen from Fig 3.10 and table 3.1 that this also holds approximately for the first few modes of HC-PBGFs, but is quickly less accurate as the mode order increases and the guided mode effective index becomes closer to the bottom of the bandgap. This differential loss mechanism poses another challenge to the exploitation of higher order modes for data transmission as it requires amplifiers capable of providing higher gains to higher order modes.

Other potential disadvantages in employing large cores are mostly practical: their mode areas as may be derived from the expressions of ref. [42] are larger, suggesting
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<table>
<thead>
<tr>
<th>Mode</th>
<th>Hollow waveguide</th>
<th>7c PBGF</th>
<th>19c PBGF</th>
<th>37c PBGF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LP_{11}$</td>
<td>2.5</td>
<td>2.4</td>
<td>2.46</td>
<td>2.38</td>
</tr>
<tr>
<td>$LP_{21}$</td>
<td>4.6</td>
<td>5.3</td>
<td>5.3</td>
<td>4.9</td>
</tr>
<tr>
<td>$LP_{02}$</td>
<td>5.3</td>
<td>9.35</td>
<td>5.72</td>
<td>5.4</td>
</tr>
</tbody>
</table>

Table 3.1: Normalised differential loss with respect to the fundamental mode in a hollow waveguide and HC-PBGFs with 7, 19 and 37c core defects. Note how the $LP_{02}$ mode is very lossy in the 7c fibre which is a consequence of it not being well confined in such a small core.

A higher tendency for macrobending losses and the control of the integrity of the structure during fabrication is known to be far more challenging when the core defects are very large.

All of the above factors should therefore be carefully evaluated before choosing the size of the core defect. It is of interest to point out that the recent advent of self-interferometric spatial and spectral ($s^2$) imaging technique has offered a glimpse into the modal content of fabricated HC-PBGFs [122, 123]. As a result of the inevitable distortions imparted by the fibre drawing process, real HC-PBGFs always support some surface modes which render the analysis of the modal properties more complex. Nonetheless, it has been demonstrated experimentally that at wavelengths sufficiently far from surface modes, individual modes can be selectively excited and propagated over significant fibre lengths [119, 124]. This indicates the intermodal coupling induced cross-talk might be manageable and that the first few modes of the fibres can effectively be used in spatial division multiplexed data transmission [119, 124]. Since loss reduction were the primary target for the MODEGAP project, the focus has been to optimize fibres with 19c and 37c core defects.

### 3.3.2 Core surround design and surface states

We discussed in section 2.3.4 that unless designed properly, the core boundary may be able to support surface modes which have detrimental effects on both the loss and operational bandwidth of HC-PBGFs. With the analysis presented there, it is interesting to examine in hindsight some of the landmark HC-PBGFs reported in the literature and whose core defects are illustrated in Fig. 3.12. The 7c fibre reported by Smith et al [36] and shown in (a) had 6 glass ‘bulbs’ on the core boundary in the middle of the strut of each pentagonal air-hole and as a result supported strut surface modes right in the middle of the bandgap. The
antiresonant core surround of Mangan et al [18] shown in (b) was so thick that it supported not one but 3 to 4 strut surface modes of increasing order and equally spaced within the photonic bandgap and which hence considerably reduced the usable bandwidth to only 20nm. With the core wall half as thick as the cladding struts, the 7c fibre by Amezcu-Correa et al [86] did not appear to support any surface mode and therefore featured a broad bandwidth. Its 19c counterpart shown in (d) and made during the MODEGAP project [88] did feature a wide bandwidth. However, both rod and strut surface modes were present near the bandgap edges as a result of structural distortions such as the six over-expanded corner holes around the core which made the pentagonal holes edges shorter and thicker (see section 6.5.1).

From our understanding of the origin of surface modes presented in section 2.3.4 a general rule for avoiding surface modes is for the struts on the core boundary to achieve uniformity both in thickness and length with those in the cladding, and for the core nodes to be of similar size as those in the cladding. This is not possible in idealised fibres with a TLH cladding because the presence of the pentagonal air-holes make the core perimeter longer than an integer multiple of the cladding node separation. Therefore, the struts on the core wall cannot be of equal length as those in the cladding. The next best option is to make the glass nodes on the core boundary equidistant, and we show indeed in section 7.3.3 that the fundamental mode of a nineteen cell with a core boundary as thick as the cladding struts and equidistant nodes maintains a low loss with a negligible bandwidth penalty with respect to the idealized fibre.
3.4 Conclusion

In this chapter, we have presented results from extensive simulations on idealized HC-PBGFs that have produced maps allowing a simple reading of optimal cladding parameters to target. We have shown that an optimum cladding design is able to simultaneously provide a wide bandwidth and a low loss value. The trade-offs to consider when choosing a size for the core defect were then examined in detail. While larger core sizes have the potential for achieving lower losses, they support more core-guided modes and their structure is more challenging to control during fabrication. We also discussed the impact of surface modes and general guidelines for suppressing them. Specifically, to support fabrication activities under the MODEGAp project, work presented in this chapter determined that:

- Efforts should focus on 19c and 37c fibres which have the potential for low-loss operation while their modal content can still be manageable.

- Six rings of air holes surrounding the core defect in a 19c fibre would suffice to keep confinement loss at negligible values, provided modest to high air-filling fractions can be achieved. Fewer rings would be required for a suitable choice of cladding parameters. This is important in speeding up the fabrication cycle.

- The air filling fraction alone is not an accurate guide for low-loss and wide bandwidth operation. Rather, the amount of glass at the nodes can play a significant role in optimizing these both bandwidth and loss.

So far, scattering loss has been studied through a very simple approximate model that does not take into account the physics of the scattering process, nor the specific roughness profiles one might get in practice. In the next chapter, I proceed to present a theory to describe light scattering from surface roughness in HC-PBGFs. Such an endeavour is necessary since a thorough understanding of the scattering process is a prerequisite to develop fabrication recipes for effective loss reduction.
Chapter 4

Light Scattering in HC-PBGFs

As mentioned in section 2.3.3.2, it is currently believed that frozen-in surface capillary waves gives rise to an intrinsic nanoscale roughness on the air-glass interfaces in HC-PBGFs which cannot be eliminated by improvements in fabrication technology because of its thermodynamic origin. The effect of these irregularities and deviations from a perfect waveguide is to scatter some of the power carried by an incident guided mode into other guided modes of the waveguide or into radiation modes, effectively causing loss.

Recognizing that scattering from surface roughness is the dominant loss contribution in HC-PBGFs, we relied in the previous chapter on the normalized interface field intensity $F$ to quantify it, and hence identify fibre designs with the potential for low-loss operation. Although $F$ is a useful quantity for such purposes, the assumptions made in its derivation are too simplistic and do not describe the scattering process accurately. This is because it assumes uncorrelated scatterers and fails to capture the complexity of the surface roughness and its power spectral density.

Scattering from boundary irregularities was recognized very early in the development of optical fibres for telecommunication applications. A logical first step in tackling the issue is arguably the mathematical description and physical understanding of the process. In step-index fibres, a variety of theoretical tools including coupled-mode theory \cite{125,127} and volume-source methods \cite{128} have been used to describe both the coupling between bound waveguides modes and the coupling to radiation modes. Although the direct application of these methods to the analysis of roughness in microstructured fibres is problematic in several respects, Roberts

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et al. [129] formulated a description along the lines of coupled mode theory for scattering in solid-core holey fibres. Their treatment however involves complex mathematical formalism based on Green functions which can be evaluated only for perfectly circular air holes, and for these reasons cannot be extended to realistic HC-PBGFs.

In this chapter, a theory based on dipole radiation is proposed to describe roughness scattering. The derived expressions combine statistical information about surface roughness, mode field distribution and fibre geometry to accurately describe the far-field distribution of scattered light and the loss in HC-PBGFs with an arbitrary cross-sectional distribution of air holes with no restriction on their shape.

4.1 Background

As with all waveguides, a first step in developing effective loss reduction strategies in HC-PBGFs shall be the physical understanding and mathematical description of the mechanisms contributing to their loss. It has been shown by Roberts et al. [26] that scattering from the intrinsic nanoscale surface roughness imposes a fundamental limit on loss in HC-PBGFs. Before embarking on our own mathematical description of the process, it is useful to examine existing approaches to the problem in much simpler optical waveguide structures. Without loss of generality, we briefly review them here by considering the simple planar structure of Fig. 4.1 which consists of a high index core layer sandwiched between infinitely thick cladding layers of a lower refractive index.

In an ideal waveguide such as the one shown in Fig. 4.1(a), the guided modes would not experience scattering from boundary imperfections throughout the structure. In the presence of imperfections such as waveguide wall roughness shown in Fig. 4.1(b), part of the mode field incident on the core-cladding interface is scattered away and coupled to other guided modes when they are supported, or lost to highly lossy radiation modes. The main mathematical tools available for tackling such a problem fall into three categories: coupled-mode theory, volume-current methods or induced source methods to which belong the simpler approach of dipole radiation we adopt for our study of HC-PBGFs. These methods are briefly discussed in the following subsections.
4.1.1 Coupled-mode theory

The formalism of coupled-mode theory was formulated in the early 1950s for microwave devices and was expanded for the description of optical waveguides in the early 1970s [125–127, 130, 131]. It has since proved to be a powerful tool for many problems, for example the analysis of power redistribution among the bound modes of a waveguide, the analysis of loss due to boundary imperfections and the design of devices such as directional couplers.

In a perfect waveguide such as the one shown in Fig. 4.1 (a), The solution to Maxwell’s equation under the appropriate boundary conditions consists of the discrete guided modes $E_n$ as well as the infinite continuum of radiation modes. Mathematically, these form a complete and orthogonal set [90] which may be used to express the field of a perturbed waveguide.

The essence of coupled mode theory for a single waveguide consists of expanding the field of the perturbed waveguide in terms of the guided and radiation modes of the corresponding ideal structure. As such, when applicable, the theory yields very useful results by simple analytical means. This theory is in principle exact as it is a recast of Maxwell’s equations, but it requires expressions for the guided and radiation modes of the unperturbed waveguide, as well as normalization and orthogonality.

Direct application of such a treatment to HC-PBGFs is problematic in two respects. First, the modes in a HC-PBGF are not truly guided but always lose some power to leakage as their propagation constants always have an imaginary part (giving rise to confinement loss). As such, they can neither be normalized since the mode fields do not decay to zero at a position infinitely far from the waveguide core, nor do they strictly form an orthogonal set. Although these limitations
may be overlooked in a first order approximation by neglecting leakage loss and approximating the modes to truly guided ones (as done by Roberts et al. in [129]), it remains a daunting task to find the infinite radiation modes of HC-PBGFs. An added complication which renders the problem all the more complex is the presence of a large number of boundaries, often of very different shapes, between the two dielectric media.

4.1.2 Volume-current methods

The principle in these methods is to consider the perturbation arising from roughness as a source term in Maxwell’s equations. Consider the perturbed waveguide in Fig.4.1 (b) above, the scalar wave equation

$$\nabla^2 E_x(y,z) + k_0^2 n^2 E_x(y,z) = 0$$

which holds everywhere can be rewritten as:

$$\nabla^2 E_x(y,z) + \left( k_0^2 n_1^2 + k_0^2 (n_0^2 - n_1^2) U(a + f(z) - |y|) \right) E_x(y,z) = 0$$

where $U$ is the heaviside step function, and the $f(z)$ represents the random distortion of the waveguide walls which is uncorrelated from one boundary to the other but have the same spectral distribution. Equation (4.2) can be rearranged as:

$$\nabla^2 E_x(y,z) + k_0^2 n_1^2 E_x(y,z) = k_0^2 (n_1^2 - n_0^2) U(a + f(z) - |y|) E_x(y,z)$$

The approximation in the volume-current method consists of replacing the field $E_x(y,z)$ on the right hand side of equation (4.3) with the known solution of the unperturbed waveguide. The equation therefore clearly becomes a wave propagation problem in the presence of current source at the waveguide boundaries. The method was pioneered by Snyder [91, 128] in dealing with radius variations in cylindrically symmetric optical fibres, but later perfected and adapted to other waveguides by various authors (see [132, 133]). Payne and Lacey [134] provided a systematic solution to such a problem in planar waveguides with a closed-form expression for the scattering loss, and the relative simplicity of their approach has made it somewhat of a standard. The volume-current method can in principle be applied to HC-PBGFs, but as in the approach of Payne and Lacey, it requires
the calculation of the appropriate Green functions, which is challenging given the complexity of the geometry of HC-PBGFs.

### 4.1.3 Dipole radiation method

The essence of this method is to consider any irregularity such as roughness on the boundaries as a dipole which upon excitation by the incident modal field radiates power in all directions. Some of the light radiated is recaptured by the waveguide and some is lost to radiation. Rawson [135–137] pioneered scattering loss calculation using this technique, coherently adding up the scattered fields contributions from various dipoles and giving expressions for the total scattered power in the far-field as well as loss coefficients. In the coming sections, we build on this method and derive expressions describing roughness scattering in HC-PBGFs or in any other microstructured waveguide geometry.

### 4.2 Model formulation

As shown in Fig. 4.2, the optical field of the core-guided modes in HC-PBGFs always has some unavoidable overlap with the air-glass interfaces within the fibre cross-section. As these interfaces are intrinsically rough, the overlap gives rise to scattered radiation and hence results in loss. If we consider a single air hole labelled \(i\) such as the one highlighted in Fig.4.2, its boundary imperfections due to the roughness can be described by a random function \(f(s_i, z)\) where \(s_i\) is the curvilinear coordinate along the hole perimeter.

Any local departure from the ideal boundary can be treated as a dipole of volume \(dV = |f(s, z)|dsdz\) which is excited by the incident modal field. This approximation is justified by the fact that the \(\text{rms}\) roughness resulting from frozen-in SCWs is of the order of \(0.1\text{nm}\), i.e. much smaller than the operating wavelength in most cases of interest. Since continuity holds strictly for the normal component of the electric displacement \(D\) and the parallel component of the electric field \(E\) at all interfaces, the induced dipole moment is rigorously given as:

\[
p = \alpha ||dVE|| + \gamma \perp dVD \perp
\]
where $\alpha_{\parallel}$ and $\gamma_{\perp}$ are the polarizability tensors [138]. In general, these two quantities will depend on the exact shape of the dipole and the refractive index contrast between the two dielectric media. In [138], Johnson et al. have computed the polarizabilities $\alpha$ and $\gamma$ for dipoles of spherical and cubical shapes and for different values of the index contrast. From their results, neglecting the exact shape of the dipole results in an error of less than 1% for silica-air interfaces at infrared wavelengths ($\varepsilon_{\text{air}} : \varepsilon_{\text{glass}} = 1 : 2.1$). Further neglecting the effects of the discontinuity of the normal component of the electric field also results in a negligible error. The polarizability additionally depends on the sign of the roughness defect. For example, when the perturbation is such that glass protrudes into air we can adopt the convention of ‘positive’ roughness, otherwise, the roughness is said to be ‘negative’. We take the polarizability $\alpha_+$ of a positive roughness “bump” as that of a glass sphere suspended in air and that of a negative one $\alpha_-$ as the polarizability of an air sphere in a glass background. The two differ in both sign and magnitude [89, 138], but since $f$ is random with zero average, we will neglect all the effects above and write the induced dipole moment in terms of the incident electric field $E_0$ at the interface as:

$$p = \alpha_0 \text{sgn}(f(s_i, z))dV E_0 = \alpha_0 f(s_i, z)ds_i dz E_0$$

(4.5)

where $\alpha_0 = \varepsilon_0 (|\alpha_+| + |\alpha_-|)/2 \approx 0.715\varepsilon_0$ for silica and air, $\varepsilon_0$ being the free-space permittivity.
We now wish to calculate the far-field distribution of light scattered out of a section of length $2L$ of the perturbed waveguide. This section is chosen such that it contains all the roughness spectral components of interest, but also short enough so that the incident field $E_0$ is approximately constant throughout. We consider an observing point $P$ located on a distant sphere of radius $R$. This point $P$ is located in a plane which contains the $z$–axis and makes an angle $\vartheta$ with the $yz$–plane. Furthermore, $P$ lies in a direction $\phi$ with respect to the $z$–axis, see Figure 4.3.

From dipole radiation theory, the scattered field at $P$ from a single dipole located on the waveguide wall at $(\theta, z)$ is given by [89]:

$$dE_s(\phi, \vartheta) = \frac{1}{4\pi \varepsilon_0} \frac{k_0^2}{R} (r \times p) \times r.$$  \hspace{1cm} (4.6)

where $k_0$ is the free-space propagation constant. The total scattered field at $P$ is obtained by adding coherently the contributions from all the radiating dipoles, that is by taking into account their relative phases. Taking as reference a ray that would be scattered from the origin, the phase difference for a ray scattered from
position \((s_i, z)\) can be calculated with the help of Fig.4.4 as:

\[
\Phi(\vartheta, \phi, s_i, z) = \beta z - k_0 z \cos \phi - k_0 y' \sin \phi
\] (4.7)

In this expression, \(\beta z\) represents the phase build up of the incident mode field between \(z = 0\) and \(z\), while \(k_0 z \cos \phi + k_0 y' \sin \phi\) is the phase accumulated by the reference ray in the same time. The underlying assumption in reaching this expression is that all the scattered rays propagate essentially through air, i.e., we have neglected the phase build-up when the rays go through the thin silica struts of the cladding. \(y' = y \cos \vartheta - x \sin \vartheta\) is the new \(y\) coordinate of the radiating dipole in the frame obtained by rotating the original \(xy\) axes by an angle \(\vartheta\) around the \(z\)-axis.

Combining Eqs. (4.6) and (4.7), the total scattered field at \(P\) from the \(i\)-th air hole is

\[
E_{s,i}(\vartheta, \phi) = \int_{z=-L}^{L} \oint dE_s(\vartheta, \phi) \exp(-j\Phi)
\] (4.8)

where the closed path integral is along the hole boundary.

When multiple air holes are present such as in HC-PBGFs, we assume that the
random distortions on each surface are identically distributed but uncorrelated. Using \( i \) as an index to label each air hole, the total scattered field is therefore obtained by a summation over all surfaces, leading to:

\[
\mathbf{E}_s(\theta, \phi) = \sum_i \int_{z=+L}^{z=-L} \mathbf{E}_s(\theta, \phi) \, d\mathbf{E}_s(\theta, \phi) \exp(-j\Phi)
\]

\[
= \sum_i \frac{k_0^2 \alpha_0}{4\pi \varepsilon_0 R} \int_{z=+L}^{z=-L} \int \mathbf{f}(s_i, z)(\mathbf{r} \times \mathbf{E}_0) \times \mathbf{r} \exp(-j\Phi) \, ds_i \, dz
\] (4.9)

The boundary distortion function \( \mathbf{f}(s_i, z) \) in Eq. (4.9) is so far of very general form. In simple structures such as those with perfect circular symmetry, the roughness can be separated into axial and azimuthal parts, and the solution to Eq. (4.9) follows along the lines of the work by Mazumder et al. [139]. For most cases of interest, it is reasonable to assume that the most relevant features of the distortion function \( \mathbf{f} \) those that are responsible for scattering light are those along the fibre axis. Therefore, restricting \( \mathbf{f}(s_i, z) \) to \( \mathbf{f}(z) \) only, the double integral of equation (4.9) is separable and by expanding \( \Phi \) through Eq. (4.7), its \( z \)-dependent part can be rewritten as:

\[
\int_{-L}^{L} f(z)e^{-j(\beta-k_0 \cos \phi)z} dz \simeq \tilde{F}(\beta-k_0 \cos \phi)
\] (4.10)

where \( \tilde{F} \) is the Fourier transform of the distortion function \( f \). This approximation holds under the stated assumption that the section \( 2L \) contains all the relevant frequency components of the distortion.

To obtain the integral along the air-hole boundary, it is necessary to compute the vector product by expanding the incident field at the scattering interface in terms of its components perpendicular \( (E_{0\perp}) \) and parallel \( (E_{0\parallel}) \) to the \( Pz \) plane. Doing so results in Eq. (4.9) rewritten as:

\[
\mathbf{E}_s(\theta, \phi) = \frac{k_0^2 \alpha_0}{4\pi \varepsilon_0 R} \tilde{F}(\beta-k_0 \cos \phi)
\]

\[
\times \sum_i \int \mathbf{E}_0 \left[ \mathbf{E}_{0\perp} \nu + (E_{0\parallel} \sin \phi - E_{0\perp} \cos \phi) \mu \right] ds_i
\]

\[
= \frac{k_0^2 \alpha_0}{4\pi \varepsilon_0 R} \tilde{F}(\beta-k_0 \cos \phi) \times \left[ E_{su} \mathbf{u} + E_{sv} \mathbf{v} \right]
\] (4.11)

where \( \mathbf{u} \) and \( \mathbf{v} \) are the unit vectors shown in Fig. 4.3. This result shows that the far-field distribution of scattered light is the product of a component depending
on the roughness spectrum and a quantity that depends on the incident electric field at the interfaces within the cross-section of the fibre. Equation (4.11) is a significant result as it states that each spatial frequency component $\kappa$ of the distortion scatters light in a specific direction $\phi$ such that $\cos(\phi) = (\beta - \kappa)/k_0$, in strict conformity with the more complicated analysis of coupled-mode theory or volume-current methods [90, 91]. This also implies that only the spatial frequencies satisfying $\beta - k_0 \leq \kappa \leq \beta + k_0$ contribute to radiation loss, since the scattering angle $\phi$ can only assume values from 0 to $\pi$.

Equation (4.11) contains all the details of the scattered field at every point on a distant sphere of radius $R$. But often, detailed knowledge of the scattered field is not necessary and the main quantity of interest is the average angular distribution of scattered power with respect to $\phi$, such as what is typically measured in angularly resolved scattering (ARS) experiments [19, 129, 136, 139]. The total power scattered into a hollow cone with half angle $\phi$ and angular spread $d\phi$ is simply given by:

$$P(\phi) = \frac{1}{2\pi} \int_{\theta=0}^{2\pi} \frac{1}{2} \text{cn} \epsilon_0 |E_s(\vartheta, \phi)|^2 R^2 \sin \phi d\vartheta.$$

(4.12)

As the distortion function $f$ is random, a more appropriate expression for $P(\phi)$ is by considering it as an ensemble average over a large set of equivalent waveguides. Equation (4.12) is therefore rewritten as:

$$P(\phi) = \frac{1}{2\pi} \int_{\theta=0}^{2\pi} \frac{1}{2} \text{cn} \epsilon_0 \langle |E_s(\vartheta, \phi)|^2 \rangle R^2 \sin \phi d\vartheta \quad (4.13)$$

where $\langle ... \rangle$ is the ensemble average over a large set of waveguides with equivalent roughness statistics. Defining the roughness power spectral density of $f(z)$ as [140]:

$$S(\kappa) = \left\langle \lim_{L \to \infty} \left[ \frac{1}{2L} \left| \int_{-L}^{L} f(z)e^{-i\kappa z}dz \right|^2 \right] \right\rangle \quad (4.14)$$

one deduces that the the factor $\left\langle |\tilde{F}(\kappa)|^2 \right\rangle$ is:

$$\left\langle |\tilde{F}(\kappa)|^2 \right\rangle = \left\langle \lim_{L \to \infty} \left[ \int_{-L}^{L} f(z)e^{-i\kappa z}dz \right]^2 \right\rangle = 2LS(\kappa). \quad (4.15)$$
By inserting Eq. (4.11) into Eq. (4.12), we obtain with the help of Eq. (4.15):

\[
P(\phi) = \frac{2L}{2\pi} \left( \frac{k_0^2 \alpha_0}{4\pi \epsilon_0} \right)^2 S(\beta - k_0 \cos \phi) 
\times \int_{\theta=0}^{2\pi} \frac{1}{2} \epsilon_0 c \left[ |E_{su}|^2 + |E_{sv}|^2 \right] \sin \phi d\theta.
\]  

(4.16)

Other quantities of utmost importance are the total power lost to radiation and the loss coefficient arising from scattering. To evaluate these, it is important to account for the fact that some of the scattered light may be recaptured by the waveguide, for example if scattered at an angle falling within the total internal reflection or the photonic bandgap acceptance cones. To do so, we denote by \( L(\phi) \) the fraction of light scattered in the polar direction \( \phi \) that is actually lost. For simplicity, in the example of step-index fibres below, we have assumed \( L(\phi) \) to vanish for angles below the critical angle and to be unity for all other values. In HC-PBGFs, we have assumed all scattered light is lost. The total power lost to radiation is therefore written as:

\[
P_{sc} = 2\pi \int_0^\pi P(\phi)L(\phi)d\phi.
\]  

(4.17)

If \( P_0 \) is the total optical power carried by an incident guided mode, the exponential loss coefficient is obtained as:

\[
\alpha_{sc} = -\frac{1}{2L} \ln \left( 1 - \frac{P_{sc}}{P_0} \right) \approx \frac{1}{2L} \frac{P_{sc}}{P_0}.
\]  

(4.18)

Note that in deriving Eq. (4.16), we have assumed that the scattered light propagates in air, which is a reasonable approximation within the microstructured area. In all HC-PBGFs however, the microstructure is surrounded by a silica outer jacket, and in ARS experiments an index-matched fluid is often used between the fibre and an external detector. In this case, the scattered light is refracted at the interface between the microstructured cladding and the glass outer jacket with refractive index \( n_{gl} \). The measured scattering angle \( \phi_m \) is therefore obtained through Snell’s law \( n_{gl} \cos \phi_m = \cos \phi \).
4.3 Validation: sinusoidal radius variations in single mode fibres

A direct verification of the theory formulated in the previous section would ideally be performed by implementing an angularly-resolved scattering setup and comparing experiments and simulations. While efforts are still ongoing to do so, we validate the model here by indirect means, that is by comparing our predictions to numerical or experimental results obtained by other authors. We consider here the classical problem of radiation loss due to scattering from a sinusoidal diameter variation in a standard single-mode fibre. This problem has been solved using both coupled-mode theory and the volume-current method [90, 128, 141]. The geometry in this case is illustrated in the inset of Fig. 4.5 and consists of a step index fibre with core and cladding indices $n_1$ and $n_2$ respectively, with the core radius changing sinusoidally as $r = a + b\sin(2\pi z/\Lambda_s)$.

When the sinusoidal perturbation is small enough to satisfy the assumptions in our treatment ($b \ll \lambda$), the scattering loss suffered by the $x$-polarized fundamental $HE_{11}$ mode can be calculated using Eq. (4.16). Since the cladding is no longer air, the relative phase difference of Eq. (4.7) needs to be adjusted by replacing $k_0$ with $k_0n_2$. For a pure sinusoidal perturbation over a long fibre section ($L \gg \Lambda_s$), the power spectral density is

$$S(\kappa) = \frac{b^2}{4} \delta \left( \kappa - \frac{2\pi}{\Lambda_s} \right)$$

(4.19)

where $\delta$ is the Dirac delta function. This implies that if the section $2L$ of the fibre is sufficiently long ($2L \gg \Lambda$) scattering by a pure sinusoidal distortion occurs predominantly in a single direction at an angle which is obtained from the phase matching relation:

$$\beta - k_0n_2\cos\phi - \frac{2\pi}{\Lambda_s} = 0 \leftrightarrow \cos\phi = \frac{\beta - 2\pi/\Lambda_s}{k_0n_2}.$$  (4.20)

The angular distribution of scattered power is therefore sharply peaked around the resonance angle. Rather than showing such a peak, it is more useful to use equations (4.16) and (4.18) to compute the exponential loss coefficient as function of the resonance angle. Such an example is shown in Fig. 4.5. In the example the refractive indices of the core and cladding are $n_1 = 1.46$ and $n_2 = 1.458$ respectively. The fibre has radius $a = 5\mu m$ and the wavelength chosen is $\lambda = 1\mu m$. 

Figure 4.5: Radiation loss of the fundamental mode due to sinusoidal core diameter perturbation in a single mode fibre with the parameters shown in the inset. The loss is plotted as a function of the escaping angle $\phi$ which is given by $\phi = \cos^{-1}((\beta - 2\pi/\Lambda_s)/n_2k_0)$. The result plotted here and obtained by solving Eq. (4.16) is identical to that obtained by coupled-mode theory (see Ref. [90], page 159) so that the waveguide is just below the cut-off of the first higher-order mode ($V = 2.4$).

To show the compatibility of our method with common numerical tools, the field components of the fundamental mode were calculated using our fully vectorial finite element method, and Eq (4.11) was evaluated to obtain the far-field scattering distribution. Imposing that $L(\phi) = 1$ for all angles larger than the critical angle ($\phi_c \simeq 3^\circ$) and $L(\phi) = 0$ otherwise, the results of Figure 4.5 are identical to those obtained by Marcuse using coupled mode theory (see Fig.4.4.2 on page 159 in [90]). Results obtained by our method also reproduce exactly those of the volume-current method [128, 141].

Clearly, the peaks and dips of Figure 4.5 are indicative of an interference phenomenon. For some frequencies of the sinusoidal distortion, the scattered light interferes destructively in the resonant angle direction, resulting in very minimal scattering loss.
For this simpler waveguide geometry, our formulation based on dipole radiation therefore clearly reproduces the results obtained by more established methods. The simple expression of Eq. (4.11) yields the same result as the complex formalism involving finding all the radiation modes and computing coupling coefficients as is the case in coupled-mode theory. Another advantage of our simpler approach lies in the fact that the integral in Eq. (4.11) can be accurately calculated numerically using post-processing functions provided by our finite element method package. This makes our model suitable for the treatment of surface scattering in waveguides with complex cross-section geometry such as HC-PBGFs.

4.4 Application to idealized HC-PBGFs

As no analytical solution for wave propagation in HC-PBGFs can be obtained, our general approach evaluates scattering from surface roughness in HC-PBGFs by first solving for the field components at the interfaces of the guided modes using our fully vectorial finite element method. This information is then used to compute the integral in Eq. (4.11). A realistic model of the surface roughness then completes the computation of the far-field distribution of the scattered light obtained from Eq. (4.11). We therefore begin this section with a discussion on the roughness in HC-PBGFs.

4.4.1 Surface roughness due to frozen-in surface capillary waves

It is currently believed that thermally excited surface capillary waves on the surface of molten glass freeze in as the fibre cools and the glass solidifies during the fibre draw and give rise to an intrinsic roughness on the air-glass interfaces in HC-PBGFs. Jäckle and Kawasaki used a linear-response model to show that on a flat glass surface, the intrinsic roughness that results from these frozen-in surface capillary waves has a power spectral density given by [24]:

$$ S(\kappa) = \frac{k_B T_g}{\rho g + \gamma |\kappa|^2} = \frac{k_B T_g}{\rho g + \gamma (\kappa_x^2 + \kappa_z^2)}, $$

(4.21)

where $\kappa$ is the two dimensional surface wavevector (with components $\kappa_x$ and $\kappa_z$), $\rho$ the glass density and $g$ the gravity constant. $k_B$ is Boltzmann’s constant, $T_g$ the
glass transition temperature and $\gamma$ the surface tension of the glass. For pure silica, the typical accepted values for $T_g$ and $\gamma$ are $\sim 1500K$ and $\sim 0.3J.m^{-2}$ respectively [142, 143]. However, it is unclear as of yet whether the multiple dehydration steps taken during fibre fabrication ultimately increase the value of surface tension, or more generally affect the ratio $T_g/\gamma$. Roberts et al. argue that this may be the case and the roughness measurement they report in [19] is well fitted with the value $\gamma = 1J/m^2$. Unless otherwise mentioned, we employ this value of surface tension hereafter. We note that expression 4.21 for the power spectral density is such that $\frac{S(\kappa)}{d\kappa} = \frac{2\pi}{k_BT_g} \frac{k_BT_g}{4\pi\gamma} \int_0^\kappa d\kappa \left[ \ln(\rho g + \gamma \kappa^2) \right]_0^\infty$

must be a finite number. As a result, the spectrum is only valid until a high frequency cut-off $\kappa_u$. On the other hand, gravity imposes another lower frequency cut-off determined in the case of flat surfaces by the capillary length $l_c = \sqrt{\gamma/\rho g}$, which is of the order of $\sim 4mm$ [145].

If one neglects the influence of the weight of the glass, since the fibres and surfaces which interest us are made under an imposed vertical flow, then by normalizing so that the mean square is now given by $S(\kappa) d\kappa$, one may integrate Eq.[4.21] along one of the two directions in a plane to obtain the 1-D form of the roughness spectral density as:

$$S(\kappa_z) = \int_{\kappa_z} k_BT_g \frac{k_BT_g}{4\pi\gamma \kappa_z} d\kappa_x.$$  \hspace{1cm} (4.23)

If the flat surface is of finite width $W$ and is closed to form a cylindrical hole, then one of the wavevector components becomes quantized and take values $2\pi m/W$. 

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where \( m \) takes all integer values. The integral of Eq. (4.23) may then be replaced by an infinite sum, and the expression for the roughness PSD along the axial direction is obtained as:

\[
S_z(\kappa) = \frac{k_B T_g}{4\pi^2 \gamma} \sum_{m=-\infty}^{\infty} \frac{1}{(2\pi m)^2 + \kappa_z^2 W} \frac{2\pi}{W}
\]

\[
= \frac{k_B T_g}{4\pi\gamma\kappa_z} \pi \sum_{m=-\infty}^{\infty} \frac{W\kappa_z}{\kappa_z^2 + \left(\frac{W\kappa_z}{2}\right)^2} \frac{1}{\pi^2 m^2 + \left(\frac{W\kappa_z}{2}\right)^2}
\]

\[
= \frac{k_B T_g}{4\pi\gamma\kappa_z} \coth\left(\frac{W\kappa_z}{2}\right) \]

(4.24)

In addition to a high frequency cut-off, Eqs (4.24) and (4.23) also require the existence of a low spatial frequency cut-offs to be physically acceptable. Since the high frequency cut-off is estimated to be of the order of a few molecular lengths (\( \sim 0.5 \text{nm} \)), it is of no particular concern here by virtue of being higher than the highest frequency \( \beta + k_0 \) contributing to scattering. In HC-PBGFs, the origins of a low-frequency cut-off are not yet known, neither has the nature of the roughness at very low spatial frequencies ever been studied experimentally or theoretically. This is primarily due to the difficulty in accessing and measuring the roughness on significant lengths of the thin struts in HC-PBGFs. Two such measurements have been reported in literature, but stand in stark contrast. Roberts et al. [19] used an atomic force microscope to measure an \( \text{rms} \) roughness of \( \sim 0.1 \text{nm} \) and a PSD consistent with Eqs. (4.23) and (4.24) between \( \kappa = 0.3 \text{\mu m}^{-1} \) and \( \kappa = 30 \text{\mu m}^{-1} \) in a HC-PBGF. In contrast, Phan-Huy et al. [146] measured a completely different spectrum with an \( \text{rms} \) roughness as high as \( 7 \text{nm} \) in a small core holey fibre with the same technique. Assuming both set of measurements were performed on uncontaminated surfaces, it remains unclear what may be at the origin of the discrepancy. Recently, there has been a suggestion that the thickness of the glass struts in HC-PBGFs may severely impact the roughness which they support [147], however, this has not yet been validated experimentally.

1we exploit the identity \( \coth(x) = \sum_{m=-\infty}^{\infty} \frac{x}{x^2 + m^2 \pi^2} \), which itself is proved by applying Cauchy’s theorem to the complex function \( f(z) = \pi \cot(\pi z) \frac{x}{x^2 + \pi^2 z^2} \) in the complex plane.
While the surface roughness properties at low spatial frequencies remains to be elucidated, in the remainder of the chapter, we will assume as a working hypothesis that the roughness PSD in HC-PBGFs is adequately described by Eq (4.23) with a low frequency cut-off point at $\kappa_c = 0.1 \mu m^{-1}$ (smaller than the AFM measuring range), below which the PSD is flat and takes the value $S(\kappa_c)$.

### 4.4.2 Scattered field distribution and loss

We now apply the theory derived above to a full HC-PBGF assuming a roughness spectrum given by Eq.(4.23) with the spatial cut-off frequency just discussed. The fibre used here for illustration purposes is an idealized HC-PBGF with average structural parameters matching those measured in the fabricated fibre reported in [148]. The core is a 19c defect and the cladding air holes are hexagons with rounded corners (as described in section 2.4.3.1) with parameters $\Lambda \sim 4.4 \mu m$, $d/\Lambda \sim 0.975$ and $D_c/d = 0.42$.

Figure 4.6 shows the calculated scattered power distribution $P$ from the fundamental mode on a distant sphere as a function of $(\phi, \vartheta)$. As expected, the symmetry of the structure has a strong impact on the scattered field distribution, the scattered power is not evenly distributed among all the $\vartheta$ from 0 to $2\pi$. As a result, an experiment studying the full far-field distribution of the scattered power shall allow not only an assessment of the loss due to roughness, but may also be a valid method for interrogating the integrity of the microstructure.

The implication for simplified measurements such as in angularly resolved scattering [129, 136, 139] experiments where the scattered power is measured as a function of $\phi$ for a single value of $\vartheta$ is that care must be taken to average the distribution over all $\vartheta$ for the implied loss values to be accurate. Roberts et al. [129] do so for example by twisting the fibre under test and making sure that the scattered power is evenly distributed in the azimuthal direction.

As the roughness from frozen-in SCWs have very large components at low spatial frequencies, scattering occurs predominantly in the forward direction. This is reflected in the graph of Fig. 4.7, where we plot the average distribution of scattered power $P(\phi_m)/(2LP_0)$ as a function of scattering angle $\phi_m$ (measured in glass) for the first lower order modes guided within the fibre. The calculations assume that all scattered light is lost, or $L(\phi) = 1$ for all $\phi$. The curve for the fundamental mode shows good qualitative agreement with the measured ARS data.
Figure 4.6: Normalized far-field distribution of scattered light from the fundamental mode of the HC-PBGF. Note how the scattering pattern reflects the sixfold symmetry of the structure. The scattering is very directional as the roughness power is high at low spatial frequencies from Roberts et al. performed on a not too dissimilar fibre design and reproduced in Fig. 4.8 [19]. Our model goes one step further in allowing us to fully evaluate the differential scattering loss between the guided modes of the fibre, which is reported in the legend and can be seen to be non-negligible as previously discussed in section 3.3.1.

Figure 4.9 shows a plot of the calculated roughness scattering loss and confinement loss across the photonic bandgap for the fundamental mode of the fibre, along with the measured attenuation spectrum. As can be seen, the total loss is dominated by the contribution from roughness scattering near the middle of the bandgap, while the bandgap edges are determined by confinement loss. The calculated scattering loss values are within a factor of 2 of the measured ones, which is an excellent agreement considering the simplicity of our method and the assumptions it relies on. The underestimate in loss from the simulation may arise from various sources. For example, the assumed roughness profile may not necessarily hold at low spatial
Figure 4.7: Average angular distribution of scattered power from frozen-in SCWs roughness for a few guided modes of the HC-PBGF. Higher order modes overlap more strongly with scattering surfaces and are therefore subject to higher losses.

Figure 4.8: Measured scattered power distribution in a 19c hollow-core photonic bandgap fibre from Roberts et al. [19]

frequencies (more on this below) and the structural distortions present in the fibre cross-section may cause a stronger overlap of the field with scattering surfaces.
4.4.3 The impact of the surface roughness spectrum

It is clear from Eq. (4.16) that accurate knowledge of the roughness statistics and its PSD in particular is of utmost importance for the loss properties of HC-PBGFs. However, the practical limitations of AFM measurements have made it so far impossible to obtain information about the very important low spatial frequency region of the spectrum, and alternative techniques are yet to be implemented. We have assumed in the previous sections that the frozen-in surface capillary waves leads to a PSD given by Eq. (4.23) with an arbitrary spatial frequency cut-off $\kappa_c = 0.1\mu m^{-1}$. It is interesting to conduct a deeper investigation on the impact of the roughness PSD by studying the effect of changing this arbitrary frequency cut-off. In Fig. 4.10 we plot the computed scattering loss for the example HC-PBGF already studied as a function of the cut-off frequency at the wavelength of $\lambda = 1.55\mu m$. It can be seen that $\kappa_c$ has a significant impact on loss, especially at small values. As it increases, the PSD takes the lower value $S(\kappa_c)$ over a broader range of frequencies and the loss decreases accordingly.

Figure 4.11 shows the total loss across the photonic bandgap for three values of $\kappa_c$. A smaller value of $\kappa_c$ implies a higher PSD at the short spatial frequencies. This does not appear to impact all wavelengths equally as can be seen from the change in
Figure 4.10: Scattering loss as a function of spatial frequency cut-off at a wavelength of 1.55μm.

the slope of the loss curve for \( \kappa_c = 0.007 \mu m^{-1} \). This effect originates in part from the fact that for longer wavelengths, the spectral region \( 0 \leq \kappa \leq \kappa_c \) corresponds to a slightly wider scattering angle spread. As the PSD is significantly higher for low spatial frequencies, scattering at longer wavelengths is more impacted by a decrease in \( \kappa_c \) giving rise to a slightly slanted loss curve within the photonic bandgap.

SCWs may not be the only mechanism generating surface roughness in a HC-PBGF. While they have been found to dominate the roughness spectrum in HC-PBGFs between the spatial frequencies of 0.2 and 30μm\(^{-1}\), little is known for the spectral region below 0.3μm\(^{-1}\). Roughness from other sources may in principle be present at frequencies below those practically measurable by AFM. It is well known for example that most realistic surfaces have roughness statistics that present one or more of three basic components: a long-range waviness, short-range random roughness and periodicity [140]. These additional roughness components may originate from drawing parameter fluctuations during fibre fabrication and will ultimately have an effect on the overall loss [149]. Shall advances be made in determining the exact nature of roughness at these low spatial frequencies, the simple expressions derived here will remain valid for estimating the loss resulting from scattering.
4.4.4 Scattering loss and wavelength scaling

A small ambiguity often arises when discussing the wavelength dependence of loss in HC-PBGF. It is important to distinguish the wavelength dependence across the photonic bandgap of a single fibre from the wavelength dependence of the minimum loss for fibres with similar design but with different dimensions so that their photonic bandgaps are centered at proportionally scaled wavelengths. In the first case, as seen in Fig. 4.9 the wavelength dependence is clearly dictated by the photonic bandgap. The scattering loss is high near the long and short wavelength edges where the mode-field is less confined, and reaches a minimum towards the center of the bandgap. The more interesting case is the wavelength dependence when the fibre is scaled. It is crucial to understand this dependence in order to locate the spectral regions in which HC-PBGF loss is minimized.

As discussed in section 3.2, the negligible change in the refractive index of silica between the wavelengths of 1 and 2µm results in the central operating wavelength \( \lambda_c \) of HC-PBGFs scaling linearly with its transverse dimension (i.e. the pitch \( \Lambda \)) when the cross section is kept unchanged [46]. Roberts et al. [19] showed experimentally for 7c fibres of similar cross-section design but drawn to different diameters that the scaling results in the minimum loss decreasing as \( \lambda_c^{-3} \), where \( \lambda_c \) is the wavelength at which the minimum loss occurs. Our group has recently
confirmed that a similar wavelength dependence holds for 19c fibres and the total loss decreases until a wavelength around 2µm, at which point the loss contribution from infrared absorption in silica becomes significant and starts to dominate [150]. These results for 19c fibres are summarized in Fig. 4.12.

A physical understanding of the dependence of the scattering loss is not straightforward. In [19], the authors argue through dimensional analysis that since the power scattered in a particular direction scales with the square of the roughness amplitude of the corresponding frequency component, the loss must scale with $\lambda^{-3}$ for it to maintain an inverse length dimension. While the argument is correct, it strictly applies here because for the SCW-induced roughness, the square of the amplitude ($S(\kappa)\delta\kappa$) is independent of wavelength in the absence of a frequency cut-off. When a cut-off is imposed, the roughness spectrum deviates from the $1/\kappa$ form and the $\lambda^{-3}$ scaling ceases to hold strictly.

Figure 4.13 shows the wavelength dependence of the loss as predicted by our
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Figure 4.13: Wavelength dependence of the scattering loss as a function of the imposed cut-off frequency. The data was generated by evaluating Eq. (4.16) at the central wavelength for idealised fibres with the same cross section design \((d/\Lambda, D_c/d)\) but scaled to different dimensions. The roughness PSD for surface capillary waves was then employed with the different cut-off values shown and material dispersion was ignored.

model for HC-PBGFs with the parameters described in the previous section. As can be seen, the loss decreases indeed as \(\lambda^{-\nu}\), where \(\nu\) depends on the roughness PSD. One could conclude after closer scrutiny of equation (4.16) that \(\nu\) must indeed take values around 3 since the frozen-in SCW spectrum of Eq. (4.23) changes as \(S \propto 1/k_0 \propto \lambda_c\) above the imposed frequency cut-off and therefore \(k_0^4 S(\beta - k_0 \cos \phi)\) scales as \(\lambda_c^{-3}\). If one assumes a constant PSD for all spatial frequencies (in other words a point defect or a single dipole), the wavelength dependence for scaled fibres is akin to the scattering from a single dipole with the typical \(\lambda^{-4}\) dependence as shown in Fig. 4.13. This suggests that for higher cut-off frequencies, \(a\) gets closer to 4 and decreases as the cut-off takes lower values, ultimately reaching the value of 3 when the cut-off frequency is very small. This is confirmed by the simulations shown in Fig. 4.13 which illustrates the wavelength dependence for a few values of the cut-off frequency and from which the slopes are \(\nu = 3.003, 3.011, 3.25, 3.25\) and 3.56 for \(\kappa_c = 10^{-7}, 10^{-4}, 0.007, 0.1\) and \(1\mu m^{-1}\), respectively. It can be seen indeed that imposing a cut-off on the SCW spectrum implies that the coefficient \(\nu\) becomes higher than 3, but no more than 4.
We conclude therefore, that the wavelength dependence of the scattering loss for scaled fibres is primarily dominated by the nature of the roughness spectrum. The fact that it scales approximately as $\lambda^{-3}$ in HC-PBGFs testifies to the roughness being dominated by frozen-in SCWs. The agreement between experimental data and calculations performed using our simple method confirms the validity of our approach and also indicates that our choice of cut-off frequency $\kappa_c = 0.1\mu m^{-1}$ is a plausible one as it leads to an acceptable value of the loss slope when fibres are scaled.

4.5 Conclusions

Starting from the theory of dipole radiation, we have derived mathematical expressions describing light scattering from surface roughness in HC-PBGFs. We have shown that the theory thus derived is in excellent agreement with more complex treatments of the same problem in simpler waveguide structures. Our approach allows to calculate the complete distribution of scattered light in the far-field, and therefore the average angular distribution of scattered light such as measured in angularly resolved scattering experiments. With the reasonable assumptions made on the roughness power spectral density from frozen-surface capillaries (namely the cut-off frequency $\kappa_c$ and the ratio $T_g/\gamma$), the method predicts the value of the scattering loss within a factor of 2 of measured values for a state-of-the-art HC-PBGF. The underestimate in the loss value may arise possibly from structural distortions present in the fabricated fibre and the inaccurate knowledge of the roughness at low-spatial frequencies. Precise information on the roughness, either from measurements or from roughness models and especially in the spectral region not accessible by current roughness measurement methods should lead to more accurate loss predictions.

Our approach allows one to calculate scattering losses from any microstructured fibre of arbitrary geometry for which the power spectral density of the roughness is known, and it can be used in combination with any numerical method to calculate the field intensity of the guided mode at the interfaces.

Although the accuracy of the model presented here comes at the price of lengthy and time consuming calculations, we have shown that the normalized interface field intensity which is considerably faster to calculate always predicts the correct
loss trend. It therefore reliable benchmark to compare fibre designs and choose the ones with the lowest loss.
Chapter 5

Preform and cane design

5.1 Introduction

In Chapter 3, we identified through optical maps, which regions of the parameter space shall be preferable for both low-loss and wide bandwidth operation. In this chapter, we aim to provide some elementary guidance to fabrication efforts aimed at producing fibres with those structural parameters and desirable properties.

The fabrication process for HC-PBGFs differs considerably from that routinely used to produce all solid step or graded-index fibres. While both require first producing a macroscopic preform and drawing it down desired dimensions, the process is more complex for HC-PBGFs due to the presence of the many hollow regions they contain. A wide variety of techniques have been reported for the production of preforms for microstructured fibres in general, among which extrusion [151–153], sol-gel casting [154] and more commonly the stacking of capillaries and rods [31]. The most widely used approach for HC-PBGFs in particular is the two-step stack and draw method [66]. In a first-step, high purity silica capillaries with an outer diameter $\sim 1\text{mm}$ are stacked to form the desired lattice arrangement. These are drawn from a starting fused silica tube, and differential pressure controls allow to obtain capillaries with inner/outer diameter ratio typically ranging from 0.3 to 0.9. The stack is then inserted in a jacketing tube and the assembly fused and drawn to obtain several meter-long and a few millimeter diameter second-stage preforms or canes. In a second step, the cane is also inserted in a jacketing tube and the assembly mounted on the fibre tower and drawn into fibre.
The application and accurate control of pressure differentials in the cladding holes during the second-stage draw is key not just to counterbalance the collapsing effect of surface tension, but also to achieve substantial expansion of the holey region required to obtain the desirable high air-filling fraction fibres. As a result of the complex fluid dynamics of the drawing process, the structure of the final fibre often differs considerably from that of the starting cane. This leaves the fibre fabricators to rely on complex on-site transmission measurements to gauge the photonic bandgap position and adjust the draw parameters accordingly.

The work presented in this chapter was carried out with the aim of assisting microstructured fibres fabricators in predicting beforehand from the structure of a fabricated cane, which drawing parameters they need to target in order to achieve fibres with desired properties. Coincidentally, achieving this aim also enables the reverse process whereby canes and preforms can be engineered to facilitate the production of fibres with structural parameters (hence optical properties) lying in the region of the parameter space identified as optimal in the previous chapters.

The theoretical description and modeling of the complex dynamics of the drawing process which involves an intricate interplay between the high temperature viscous flow, surface tension and applied pressure differentials has proved to be a challenging task. The research literature abounds with theoretical treatments of the problem for one single capillary [155], and more recently, detailed numerical studies have been reported for fibres with a few air holes in the cross-section [156–158]. A comprehensive analytical or numerical treatment of the problem for structures as complicated as HC-PBGFs is yet to be reported. While it would the most rigorous approach to achieve the aims above, such endeavour is clearly outside the scope of this work. Rather, the approach I have adopted assumes that regardless of the complexity of the process itself, no material flow occurs in the transverse direction during the fibre draw. As a result of this assumption, simple expressions can be derived to predict the average glass node size and strut thickness in the resulting fibres. Once these parameters are known, optical properties such as photonic bandgap position and width are quickly extrapolated from the optical maps presented in Chapter 3. Note however, that the potential structural distortions introduced by the drawing process still require post-draw analysis via SEM images to achieve a full correlation with other more crucial properties such as loss, modal dispersion and surface modes. This will be subject of investigation in the following chapters.
5.2 Model Formulation

Figure 5.1 illustrate the second step draw process for HC-PBGFs where the cane and jacketing tube assembly is mounted on the fibre tower and drawn into fibre. In a structure with several hundreds of air holes, the effects of the interplay between the drawing parameters such as temperature distribution in the hot zone inside the furnace, draw and feed speeds, draw tension, pressure differentials and material properties such as surface tension and glass viscosity on the integrity of the structure during the draw are not yet well understood or well described. It is well known from empirical tests however, that the application of active pressure differentials to the cladding holes provides some control over the final fibre structure and air-filling fraction that can be achieved, as illustrated in Fig. 5.2.

Overlooking the complexity of the physics involved and concerning myself only with what happens to a single unit cell in the cladding and not the core defect during fabrication, the second stage of the fibre drawing process is regarded here as a transverse isotropic expansion of the structure, followed by drawdown to a desired fibre diameter. I assume that all the air holes in the cane are identical and that control over the applied pressure differentials ensures to a first order approximation, that all the air holes except for the core defect which is not considered here are equally expanded. In addition, I assume that no material flows between the microstructured region and the outer silica jacket. Since all the cladding holes are subject to equal pressure, I further assume that no material flow occurs between adjacent unit cells, nor does any glass flow from the nodes to the struts and vice versa. In essence, the amount of glass present in each of these regions of the cane is redistributed along the fibre length and the principle of mass conservation can be invoked to find the node size and strut thickness in the resulting fibre.

With these assumptions and the additional knowledge of (i) the fibre outer diameter (or drawdown ratio) and (ii) the expansion ratio (defined as the ratio between the diameter of the microstructure and the outer diameter of the fibre, see Fig. 5.2), I proceed to estimate the structural parameters of the fibres from those of the starting cane. The fibre outer diameter is commonly monitored and logged in real-time during the fibre draw while the expansion ratio can be quickly and accurately estimated from optical microscope images of the fibre cross-section. Non-destructive schemes based on interferometry may also be developed to measure the microstructure diameter.
Figure 5.1: Illustration of the second step stack and draw technique. Control over the applied pressure differentials determines the expansion and structural properties of the resulting fibres.

We now suppose that a given cane and jacket tube ensemble for which the parameters $(\Lambda, \bar{d}/\Lambda, D_c/d)$ and expansion ratio $e_1$ are known is drawn into a fibre with an outer diameter $OD$. From a mass conservation point of view, the drawing process can be seen as the sequence of two separate steps: a drawdown of the fibre to the desired OD (drawdown ratio $f$) conserving the initial cane expansion $e_1$, followed
by an expansion of the microstructure to its final value \( e_2 \) at a constant \( OD \). As no glass flows between the silica jacket and the microstructure, the volume of glass in a section of length \( l_1 \) of the unexpanded fibre shall yield a final fibre of length \( l_2 \) such that:

\[
\frac{\pi OD^2}{4} (1 - e_1^2) l_1 = \frac{\pi OD^2}{4} (1 - e_2^2) l_2 \\
\Rightarrow l_2 = \frac{1 - e_1^2}{1 - e_2^2} l_1.
\]

(5.1)

Here, the subscript 1 denotes the hypothetical fibre which is simply a scaled down version of the cane and jacket assembly and for which all the structural parameters are known, while the subscript 2 denotes the fibre of interest, with unknown parameters. Equation (5.1) implies that for a fixed fibre OD, higher expansion ratios yield longer fibres, a fact that is both intuitive and well-known to fibre fabricators. The microstrutured area will adapt itself to match the elongation imposed by the outer jacket. Since all the holes are assumed to expand equally, it follows that the

Figure 5.2: Cross-sections of a cane and three fibres drawn from it with increasing expansion ratios and air-filling fractions. The expansion ratio is defined as \( e = \frac{id}{od} \)
pitch will increase in proportion to the expansion ratio, or:

\[
\frac{\Lambda_1}{e_1} = \frac{\Lambda_2}{e_2} \iff \Lambda_2 = \frac{e_2}{e_1} \Lambda_1,
\]

where \( \Lambda_1 = f \times \Lambda \).

The glass node is defined as the area enclosed by the three circular arcs used to fillet the three closest air holes corners and highlighted in Fig. 5.3. The reason for this choice is that we have found in simulations that the photonic bandgap of unconnected dielectric rods of the same area always overlap very well with that of the full fibre. With the parametric definition adopted so far and illustrated in Fig. 5.3, this area is:

\[
A_r = \left( \sqrt{3} - \frac{\pi}{2} \right) \left( \frac{D_c}{2d} \right)^2 \left( \frac{d}{\Lambda} \right)^2 \Lambda^2 \\
+ \sqrt{3} \frac{D_c}{2d} \left( 1 - \frac{d}{\Lambda} \right) \Lambda^2 + \frac{\sqrt{3}}{4} \left( 1 - \frac{d}{\Lambda} \right)^2 \Lambda^2
\]

(5.3)

Figure 5.3: Idealized structural parameters in a cane. A hexagonal unit cell as well as the node areas are highlighted.

It follows that the total area occupied by glass in a unit cell is

\[
A_t = 2A_r + 3 \left( \frac{1 - D_c/d}{\sqrt{3}} \right) \left( \frac{d}{\Lambda} \right) \left( 1 - \frac{d}{\Lambda} \right) \Lambda^2
\]

(5.4)

where the second contribution is the area of the thin glass struts. As the glass in each unit cell and in each glass node is assumed to redistribute longitudinally, we
Chapter 5 Preform and cane design

obtain the following:

\[
\begin{align*}
A_{r2} \times l_2 &= A_{r1} \times l_1 \\
A_{t2} \times l_2 &= A_{t1} \times l_1.
\end{align*}
\]

(5.5)

Since the parameters \(\Lambda_1\), \((d/\Lambda)_1\) and \((D_c/d)_1\) are known from those in the cane and jacket tube assembly, knowledge of the expansion ratio and outer diameter of the final fibre makes possible the calculation the structural parameters \(\Lambda_2\), \((d/\Lambda)_2\) and \((D_c/d)_2\) by solving equations (5.5). With the help of (5.1) and (5.2), Eqs. (5.5) essentially becomes a system of two equations with two unknowns. After some algebraic manipulation, one arrives at:

\[
\frac{\pi}{8} u^4 - \frac{\pi}{4} u^3 + \left(\frac{\pi}{4} C + B - \frac{2\sqrt{3} - \pi}{8}\right) u^2 + \frac{2\sqrt{3} - \pi}{4} Cu - \frac{2\sqrt{3} - \pi}{8} C^2 = 0
\]

(5.6a)

\[
\left(\frac{D_c}{d}\right)_2 = \left(1 - \frac{C}{u(1-u)}\right)
\]

(5.6b)

where \(u = 1 - (d/\Lambda)_2\), and the coefficients \(C\) and \(B\) are given by:

\[
B = \frac{1 - e_2^2 e_1^2}{1 - e_1^2 e_2^2} \times \frac{A_{r1}}{A_{r1}^2}
\]

(5.7)

\[
C = \frac{1 - e_2^2 e_1^2}{1 - e_1^2 e_2^2} \left(1 - \left(\frac{D_c}{d}\right)_1\right) \left(\frac{d}{\Lambda}\right)_1 \left(1 - \left(\frac{d}{\Lambda}\right)_1\right)
\]

(5.8)

The fourth degree polynomial (5.6a) is solved for to find the resulting \((d/\Lambda)_2\) when the structure is expanded. Of the four possible roots, two are either negative or larger than one. Of the remaining two, only one yields a value of \((D_c/d)_2\) satisfying \(0 \leq D_c/d \leq 1\) when replaced in equation (5.6b). This completes the solution of the initial problem and gives a first approximation as to how these parameters change during fibre draw.

5.3 Influence of cane parameters

Designing canes with potential of producing fibres with optimal properties requires the ability to predict the structure of the fibre from that of the cane. To do so, we investigate how fibre parameters change as a function of their counterpart in
the cane. Unless otherwise stated, the canes considered here have the standard dimensions of those commonly fabricated in the course of this project. They feature an outer diameter of 3\(\text{mm}\) with microstructure region having a diameter of 2.5\(\text{mm}\). The jacket tube used has an inner diameter of 3.5\(\text{mm}\), an outer diameter of 10\(\text{mm}\) and this leads to a cane and jacket tube assembly that has an initial expansion ratio of \(e_1 = 0.2541\).

5.3.1 Impact of \(d/\Lambda\)

Equations (5.6a) and (5.6b) were solved for canes featuring different values of \(d/\Lambda\) but having otherwise the same \(D_c/d\). Figures 5.4 and 5.5 show the change in \(d/\Lambda\) and \(D_c/d\) in the resulting fibres as a function of increasing expansion ratio for three different initial canes with \(d/\Lambda\) of 0.8, 0.85 and 0.9 and similar initial value \(D_c/d = 0.6\). The results confirm the expected trend that canes with higher \(d/\Lambda\) values require less expansion to reach high air-filling fractions (Fig. 5.4). According to our parametric definition, canes with higher \(d/\Lambda\) values also have less amount of glass at the nodes. As a result, when expanded, the air holes in such canes quickly evolve towards hexagons with smaller fillet radii, i.e small values of \(D_c/d\) (see Fig. 5.5).

![Figure 5.4: Change in relative strut thickness \((d/\Lambda)\) with expansion when the starting canes have different \(d/\Lambda\) ratios. The three canes have the dimensions described above and their air holes are filleted such that \(D_c/d = 0.6\).](image-url)


Figure 5.5: Change in fillet radius ($D_c/d$) with expansion when the starting canes have different $d/\Lambda$ ratios. The three canes have the dimensions described above and their air holes are filleted such that $D_c/d = 0.6$.

5.3.2 Impact of $D_c/d$

Figures 5.6 and 5.7 plot changes in fibre parameters in the resulting fibres as a function of the expansion ratio for three starting canes with $d/\Lambda = 0.85$ but with $D_c/d = 0.4, 0.6$ and 0.8.

As can be seen, $d/\Lambda$ increases at a faster rate when the starting $D_c/d$ is higher, that is, when the nodes in the starting canes are larger and the struts shorter. In addition, for the same expansion ratio, mass conservation predicts that canes with

Figure 5.6: Change in relative strut thickness ($d/\Lambda$) with expansion when the starting canes have the same $d/\Lambda = 0.85$ ratio, but different fillet radii.
larger node size also yield fibres with in which the nodes are larger (Fig. 5.7). This finding is interesting as it indicates that preforms stacked with deliberate addition of glass at the nodes will allow to obtain fibres with high \( d/\Lambda \) values at modest expansion ratios. At an expansion ratio of 50\% for example, \( d/\Lambda = 0.968, 0.973 \) and 0.98 for the three canes considered above, respectively. This could be beneficial to reduce fibre distortions, as it is well known that the more expanded the fibre is, the more prone it will be to structural distortions.

### 5.3.3 Summary

The model thus derived predicts that higher expansion ratios inevitably produce thinner glass struts in the cladding (i.e. higher \( d/\Lambda \)) and more hexagonal air holes (i.e. smaller \( D_c/d \)), a trend that has been observed for example when inflating endlessly single mode fibres \[159\]. Because of our chosen definition of the node area to be conserved throughout the draw, canes with circular air holes have no struts and therefore lead to fibres with circular holes as well. This is in contradiction to what has been observed experimentally such as in the example by Birks et al. \[159\] and remains a limitation of this model. Nonetheless, in the majority of cases where the canes are drawn from a stack hollow capillaries with modest to high inner/outer diameter ratios, the present method has yielded very reliable and accurate predictions as will be shown in the next sections.
5.4 Application to fabricated samples

Figure 5.8 below shows a selection of canes produced here at the ORC in the course of the MODEGAP project. The vast majority of preforms were stacked with a $19c$ core defect and $6\frac{1}{2}$ rings of air holes outside the core, although $7c$ and $37c$ fibres were pursued as well. The $19c$ and $37c$ preforms were drawn into canes of $3mm$ diameter with a microstructure diameter of $2.5mm$. In some instances, the interstitial regions between the circular capillaries are not completely closed in the cane and will only collapse during the second stage draw. To reflect this, we subtract the estimated area of the open region from the node area of equation (5.3) for the starting cane.

To validate the model and test its accuracy, its application to two different canes is described below. The first cane features a $7c$ core defect and the other with a $19c$ core defect.

5.4.1 Example 1: Seven cell cane

An image of the cane under consideration is shown in Fig. 5.9. From a thorough analysis of the microscope image, the average structural parameters were $(\Lambda, d/\Lambda, D_c/d) \approx (80.7 \pm 1\mu m, 0.87 \pm 0.01, 0.8 \pm 0.05)$. This $7c$ cane had an
outer diameter of 1.76mm and was inserted in a jacket tube with $ID/OD = 4.8mm/10mm$. Figure 5.9 shows the predicted cladding parameters in potential fibres to be drawn from this cane as a function of expansion.

Three fibres A, B and C were drawn at expansion ratios 0.254, 0.276 and 0.282 and outer diameters 206.3, 214.3 and 218µm, respectively. The corresponding predicted cladding parameters are highlighted on the plots of Fig. 5.9. The optical transmission of idealised 7c fibres with these predicted structural parameters were then simulated. Figure 5.10 shows the calculated fraction of core-guided power, compared with the measured transmission over a short length of the tree fabricated fibres.

As can be appreciated, the simulated transmission from our simple model is in very good agreement with experimental data. The calculated photonic bandgaps are slightly wider and centered at a shorter wavelength than measured. These small discrepancies highlighted in table 5.1 are well within acceptable error margins and may result from both the inaccuracies in estimating cane parameters or small distortions in the fibre. The good agreement between simulation and experimental data is some indication that the assumptions made in deriving equation 5.6a are justifiable. The fairly accurate response of this simple model to small changes in fibre expansion and outer diameter shows that the method can reliably be used to predict fibre properties before they are drawn from canes.
Figure 5.10: SEM images and short length transmission measurements for three fibres drawn from the cane of Fig. 5.9. The transmission curves measured by predominantly exciting the fundamental mode via careful central launch are superposed with the simulated percentage of power in the core.

Table 5.1: Calculated and measured position and width of the photonic bandgap fibres for 7c fibre samples A, B and C drawn from the cane of Fig. 5.9.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Pred. $\lambda_c/\Lambda$</th>
<th>Meas. $\lambda_c/\Lambda$</th>
<th>error(%)</th>
<th>Pred. $\Delta \lambda/\lambda_c$</th>
<th>Meas. $\Delta \lambda/\lambda_c$</th>
<th>error(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.466</td>
<td>0.475</td>
<td>1.7</td>
<td>0.230</td>
<td>0.218</td>
<td>5.2</td>
</tr>
<tr>
<td>B</td>
<td>0.427</td>
<td>0.439</td>
<td>2.0</td>
<td>0.243</td>
<td>0.227</td>
<td>6.6</td>
</tr>
<tr>
<td>C</td>
<td>0.418</td>
<td>0.429</td>
<td>2.6</td>
<td>0.252</td>
<td>0.25</td>
<td>6.7</td>
</tr>
</tbody>
</table>

5.4.2 Example 2: Nineteen cell cane

The same procedure was employed to predict the properties of fibres drawn from the cane which cross-section is shown in Fig. 5.11 below. This cane featured a 19c core defect and had structural parameters $(\Lambda, d/\Lambda, D_c/d) \approx (145.9 \pm 1\mu m, 0.88 \pm 0.01, 0.6 \pm 0.1)$ with an OD of 3mm. The jacket tube employed for the draw had $ID/OD = 3.5mm/10mm$. Five fibre samples were fabricated from this cane, with expansion ratios of 0.437, 0.441, 0.458, 0.473 and 0.482 respectively. After solving for the structural parameters of the respective fibres, their optical properties are predicted by simple extrapolation on the optical maps presented in the previous chapter.
Figure 5.11: Predicted cladding parameters as a function of expansion ratio for the 19c cane described in the text.

Figure 5.12 shows the overlap of the $d/\Lambda$ vs. $D_c/d$ trajectory for all possible fibres that can be drawn from this 19c cane and the property maps from which the normalized central wavelength and bandwidth can be read. The stars on the plots are the predicted properties while the yellow dots correspond to measured ones. As before, the very good agreement between the predictions and measurement can be appreciated, with the relative errors highlighted in table 5.2 below.

These results can be generalized: possible fibres from a given cane always lie on a trajectory similar to that black curve of Fig. 5.12. This is illustrated by a further example shown as the white curve, which is the trajectory for an improved 19c cane made from a preform stacked with thicker capillaries. This cane is shown in Fig 5.2 and the reason for using thicker capillaries in the stack was to increase the amount of glass at the nodes. Its structural parameters were $(\Lambda, d/\Lambda, D_c/d) \approx (131.8 \pm 1 \mu m, 0.68 \pm 0.01, 0.6 \pm 0.1)$. The curve showing potential fibres that may be obtained from this cane is shifted towards higher $D_c/d$. However, since $d/\Lambda$ is much lower in the cane, higher expansion ratios are needed to obtain fibres with good optical properties. The three fibre samples X,Y and Z which cross-sections are shown in Fig. 5.2 were drawn from this cane at expansion ratios of 0.574, 0.638 and 0.726 respectively. Their measured and predicted optical properties are shown as red dots and green stars respectively, but the values for sample Z falls outside the range of contour plots as its predicted $d/\Lambda$ was 0.9906, slightly higher than the highest value used in simulations.
Chapter 5 Preform and cane design

Figure 5.12: Overlap of predicted fibre structural parameters and optical properties maps. The colormap shows the normalized bandwidth $\Delta \lambda / \lambda_c$ and the labelled solid contour lines are for the normalized bandgap position $\lambda_c / \Lambda$. The black Curve represents possible fibres from the 19c cane of Fig. 5.11. White stars are the predicted properties for samples 1 to 5 and magenta dots the corresponding measured ones. The white curve is the trajectory of possible fibres from the cane shown in Fig. 5.2 which was made with thicker capillaries. The stars and dots show predicted and measured properties for two fibre samples drawn from this cane.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Pred. $\lambda_c / \Lambda$</th>
<th>Meas. $\lambda_c / \Lambda$</th>
<th>error(%)</th>
<th>Pred. $\Delta \lambda / \lambda_c$</th>
<th>Meas. $\Delta \lambda / \lambda_c$</th>
<th>error(%)</th>
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<tr>
<td>①</td>
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</tr>
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<td>1.2</td>
<td>0.266</td>
<td>0.263</td>
<td>1.0</td>
</tr>
<tr>
<td>⑤</td>
<td>0.327</td>
<td>0.332</td>
<td>1.6</td>
<td>0.275</td>
<td>0.273</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Table 5.2: Calculated and measured position and width of the photonic bandgap fibres for 19c fibre samples 1 to 5 drawn from the cane of Fig. 5.11.
5.5 Preferred Cane designs

It emerges from the results presented above that according to our model, not all initial canes have the potential to yield fibres with optimum properties such as widest photonic bandgaps or potentially low loss. It is clear from Fig. 5.12 that the first 19c cane shown in Fig. 5.11 will not yield fibres in the very optimal operation region (predominantly in red), regardless of the expansion ratio. The choice of the initial cane is thus key to achieving fibres with desired properties.

Combining the findings presented in the previous section, our simple mass conservation model predicts that in order to target fibres with thin struts (high \(d/\Lambda\)) and optimal node sizes (see Chapter 3), the preferred cane designs should be made from thin wall capillaries, with increased amount of glass at the nodes. This will allow obtaining fibres with high \(d/\Lambda\) without the need of over-expansion which has the potential to introduce detrimental distortions. Fibres fabricated from such a cane

Figure 5.13: Overlap of predicted fibre structural parameters and optical properties maps for a preferred cane design. The colormap shows the normalized bandwidth \(\Delta \lambda/\lambda_c\) and the labelled contour lines are for the normalized bandgap position \(\lambda_c/\Lambda\). The white Curve represents possible fibres from the designed 19c cane and the oval highlights the spatial region where fibres possess optimal properties.
would fall on a trajectory that goes through the optimum region of the parameter space. Such canes may be achieved in practice for example via the introduction of packing rods in the starting stack. This is somewhat similar to the designs patented by Williams et al[160], although the authors were primarily concerned with an easier route to completely closing the interstitials areas.

One possible cane design made with this approach would feature parameters $d/\Lambda = 0.92$ and $D_c/d = 0.8$. Provided such a cane is drawn to standard dimensions (2.5\text{mm}/3\text{mm}) and the usual jacket tube with $ID/OD = 3.5\text{mm}/10\text{mm}$ is used, then the fibre structural parameters to expect are overlapped on the optical property maps and shown in Fig. 5.13. As can be seen, the fibres obtained from this cane fall more or less within the region we have identified as optimal. At an expansion ratio of 48% the fibre parameters are $d/\Lambda = 0.99$ and $D_c/d = 0.6$, with an estimated loss value around $2dB/km$ (estimate based on normalized interface field intensity) in the absence of distortions and surface modes.

5.6 Conclusion

In this chapter, we have presented a simple model aimed at predicting changes in fibre parameters during the fibre draw. Although clearly an oversimplification of the dynamics of the fibre draw, the assumptions made in deriving the model were justified in light of the agreement between predicted and measured data. Although a full understanding of how draw parameters and material properties affect the resulting fibre structure would require a more complex fluid dynamics based study, this simple model proposed here can already be employed to predict the position and width of the photonic bandgap fibres prior to or during the draw. In fact, it has been used successfully on many fibre draws and has been made an integral part of the tools used by fibre fabricators here at the ORC. Combining this simple model with the precalculated optical maps of Chapter 3 has allowed us to identify cane designs that would lead to HC-PBGFs with optimal properties such as wide bandwidth and possibly low loss.

Although this simple model has allowed us to accurately predict bandgap position and width, our attempts to predict the fibre loss from the map of Fig. 3.7 did not yield any results that correlate with experimental values. This is because loss and other properties such as the presence or absence of surface modes depend more crucially on the exact structure of the fibre and are more affected by the
structural distortions present in the fibre, especially near the core boundary. In the next chapter, we study fabricated fibres from scanning electron microscope images of their cross-sections and investigate the impact of structural distortions more broadly in chapter 7.
Chapter 6

Accurate Modelling of Fabricated HC-PBGFs

6.1 Introduction

The fibre structures analyzed so far have consisted of idealized representations of HC-PBGFs. They have all featured perfectly periodic claddings and nearly dodecagonal core defects. However, the complex physics of the fibre drawing process generally result in fibres that are visibly different from these idealized models. Although the idealized representation provides valuable insight into the operation of HC-PBGFs, it fails to reliably reproduce or predict those properties which are of paramount importance but which depend critically on small structural details of the fibre: attenuation, surface mode position, dispersion, etc. It is therefore imperative to develop tools that allow for the modelling of the cross-sections of fabricated samples. Besides the obvious objective of providing an explanation for experimentally observable fibre properties, accurate modeling of fabricated fibres is also useful in predicting those properties which are more challenging to assess experimentally. Besides, it also opens the door to studying the impact little changes in the structure of fabricated fibres may have on their properties and therefore should be useful for further optimization of HC-PBGF designs.

Traditionally, the modelling of fabricated HC-PBGFs has been performed by generating a refractive index profile of the fibre from scanning electron microscope (SEM) images of their cross-sections. Obtaining reliable and faithful results from this seemingly simple procedure has proved more challenging than one may expect.
In this chapter, a novel and more versatile approach to address these challenges and accurately simulate fabricated HC-PBGFs is presented. Our approach leads to simulation results in excellent agreement with experimental measurements and points to structural distortions as being detrimental to the fibre attenuation.

### 6.2 Challenges in modeling fabricated samples

Modeling the properties of fabricated fibres is a conceptually simple task. The most common procedure involves converting the usually grayscale scanning electron microscope image of the cross-section into a two-level black and white image. At this point, simple algorithms and routines can be employed to accurately detect the step edges between white (1) and black (0) pixels \[161\]. The coordinates of pixels on the detected edges are then used to build smooth curves, usually splines, which form the boundaries between the embedded air holes and the silica matrix. Although edge detection is very accurate in principle, previous attempts at modeling fabricated fibres using this simple procedure failed to reliably reproduce all the experimentally measurable fibre properties such as bandgap position and width, attenuation, dispersion or surface modes position (in case there were any present). This has been pinned down to many issues, among which are:

- **Required simulation time**
  
  When one relies on edge detection routines to select pixels lying on the boundaries of air-holes in SEM images of fabricated HC-PBGFs, a large number of smooth curves connecting all these boundary pixels (usually splines) must be joined together to faithfully represent the fibre geometry. When simulating the generated geometry, the large number of boundary curves requires a denser mesh and hence a larger number of matrix orthogonalisations and more computer memory to perform the calculations. This inevitably results in longer simulation times, especially when comparing with the idealized fibres used so far.

- **Impact of the metallic coating**
  
  Imaging the HC-PBGF cross-section using an SEM requires the application of a conductive metallic coating to prevent or alleviate charging effects. This coating layer significantly exaggerates the value of the strut thicknesses (typically, we have measured thicknesses of $130\text{nm}$ for silica struts estimated
to be $100\text{nm}$ or less, with a coating layer of $10\text{nm}$ or less) and therefore, SEM images typically show structures with thicker struts and lower air-filling fractions than in the actual fibres. Evidence of this is the fact that modelling performed on high quality and high resolution images always result in narrower photonic bandgaps centered at much longer wavelengths than experimentally measured $[115, 162]$.

- **Image quality and resolution**
  
  In $[162]$, Poletti et al. identify the size and resolution of the SEM images as crucial factors limiting simulation accuracy when modelling fabricated fibres. As the fibre properties depend on very fine details in the fibre cross-section, SEM images ought to simultaneously and reliably resolve the entire cross-section ($\sim 100\mu\text{m}$) and the smallest features such as the thin silica struts (typically $\leq 200\text{nm}$) within it. The limited size of the SEM images implies that silica struts in the cross-section are typically less than 2 pixels wide. Such small thicknesses often lead to the inaccuracy of edge detection routines when reproducing the true fibre cross section. This is because the decision to consider a "gray" pixel as white or black can significantly change the fibre’s air-filling fraction and hence shift its photonic bandgap considerably. Therefore, achieving a simple match between the bandgap position between simulation and experiment using this approach has proved very challenging. This is aggravated by the fact that SEM images are often collected with a tilt, sometimes show visible charging effects and/or display debris resulting from imperfect fibre cleaves, all of which may require retouching and altering the image before it can be used in simulations.

Recent attempts at solving these challenges have sought to obtain very high resolution images by limiting the SEM’s field of view to the core and the first few rings surrounding it thereby forsaking the remainder of the cladding $[163]$. Although fibre properties such as dispersion and modal birefringence have been reproduced with good accuracy using this approach, it proved to be very complex. Rather than using edge detection routines, the authors opted to perform a large number of measurements of the features in the fibre cross-section and used them to manually reproduce the geometry.

In the following sections, a workaround to these identified challenges is presented and shown to allow for fast and accurate modelling of fabricated fibres.
6.3 Generating geometries from SEM images

The aim of this work is to extract from an SEM image a cross-section geometry that accurately matches that of the fabricated HC-PBGF under study while having less severe demands on the image quality or resolution. This is achieved by realizing that in high quality SEM images free of charging effects or other defects, the center positions of the air holes and those of the glass nodes on the images are accurately representative of the actual fibre, regardless of the image resolution and the effects of the coating. Once the locations of all the glass nodes are obtained, an accurate representation of the fibre can be generated provided that the node size and strut thicknesses are known. In practice, the procedure is carried out in two steps.

6.3.1 Step 1: Accurate detection of holes and nodes centre positions

The procedure that I have developed to pinpoint the centre locations of air holes and glass nodes is illustrated in Fig. 6.1. Once the original image is converted into a binary black and white, image processing routines such as those provided by MATLAB® are able to identify each of the connected areas as separate objects. Detailed properties of these objects including the centroid location can subsequently be extracted. After obtaining the centroid locations for each of the air holes, a dilation operation is applied to each of them with a structural element chosen such that all air holes are merged together, leaving in place isolated nodes, the centre positions of which can in turn be extracted. This automatic procedure generates an accurate representation of the fibre structure.

6.3.2 Step 2: Building the geometry with free parameters $t$ and $r_c$

With the centre position of each air hole and interstitial node accurately determined, a hexagon (or pentagon) is built around each air hole by finding the six (or five) nodes closest to its center. As shown in Fig. 6.2, each edge of the hexagon (pentagon) is then moved closer to the hole’s center by a distance corresponding to half the desired strut thickness $t$, which is set here as a free parameter. The other free parameter in our method is the radius $r_c$ of the circle used to round the
corners of the air holes. This parameter is crucial as it determines the node size which we know is critical in controlling the photonic bandgap position. An accurate estimate of both free parameters can be obtained by averaging measurements performed on highly magnified portions of the cross-section and adjusting for the effect of the coating. Alternatively, if the geometry parameters of the second-stage preform from which the fibre was drawn are known, then the model developed in chapter 5 can also give an accurate estimate of both $t$ and $r_c$.

The freedom to create struts of arbitrary thickness allows one to generate geometries that better match the actual fibre, avoiding the limitation posed by the finite SEM resolution. To make the generated geometry even more similar to that of the real fibre, mass conservation is applied to each silica strut by forcing its thickness to be inversely proportional to its length. In other words, if $\Lambda_r$ is the average node
Figure 6.2: Setting strut thicknesses and fillet radii. Given the coordinates of \( A, B \) and \( C \) and the thicknesses \( t_1 \) and \( t_2 \) of the struts \( AB \) and \( AC \), the location of \( A' \) can be easily computed. Each corner of the resulting hexagon is then filleted with a circle of radius \( r_c \).

spacing and \( t_r \) the average thickness, struts longer than \( \Lambda_r \) will be thinner than \( t \) and vice-versa so that the area of each strut is constant. This can have profound implications, particularly for the struts on the core boundary, which determine whether or not surface modes are supported within the photonic bandgap.

An example geometry obtained using this simple procedure is shown in Fig. 6.3. The agreement between the glass-air boundaries and the structure reconstructed with the proposed model and shown in red is remarkable. One notes that as no nodes lie beyond the outermost ring of air holes, this final ring cannot be reconstructed in the simulated geometry and the simulated leakage loss will be slightly overestimated. However, its omission has very little impact on the fibre loss, since leakage remains negligible and the total loss is clearly dominated by scattering and/or absorption as will be shown in the examples below.
6.4 Modelling the loss

In Chapter 4, we formulated a rigorous treatment of roughness scattering in HC-PBGFs and derived expressions for estimating the scattering loss when the roughness PSD in the fibres is known. Prior to that, we had relied on the normalized interface field intensity to identify designs with the potential for low-loss operation. It is important to draw some comparisons between the two approaches and gauge in particular which of the two can provide reliable loss estimates at the lowest computational cost. Regarding the computational cost, accurate numerical evaluation of the loss using Eq. (4.10) requires an angular resolution of at least half a degree in both $\phi$ and $\vartheta$. Since $\phi$ varies from 0 to $\pi$ and $\vartheta$ from 0 to $2\pi$, this implies that the integral of Eq. (4.9) must be evaluated at least $360 \times 720$ times over all the air-glass interfaces. Symmetries in the far-field radiation pattern may be exploited to reduce the number of integrals, but on a 4-core processor
with 24GB of memory and simulating only a quarter fibre structure, it still takes twenty minutes of post-processing to compute the loss for one single wavelength. In comparison, a single integral is required for evaluating $F$. Therefore, if a calibration of the normalized interface field intensity can reliably predict the loss and how it changes with fibre parameters, it may be advantageous to use this simpler approach. To this end, we compare here the loss as predicted by the rigorous theory and estimates based on $F$.

Figure 6.4 shows the computed scattering loss at the wavelength of 1.55$\mu$m as a function of $d/\Lambda$ for idealized fibres in which $D_c/d = 0.6$ but with different pitches $\Lambda$ so that their bandgaps remain centered around 1.55$\mu$m. Predicting fiber loss with our scattering theory of Chapter 4 resulted in lower values than measured experimentally, primarily due to the assumptions made on the roughness spectrum. On the other hand, estimating the loss from a simple scaling of the normalized interface field intensity via:

$$\alpha_{sc} = \eta \times F \quad (6.1)$$

with $\eta = 300$ for the 1.55$\mu$m wavelength achieved a very good agreement with measured loss values. In order to compare both methods, we therefore readjusted the roughness PSD by keeping the spacial frequency cut-off at 0.1$\mu$m$^{-1}$, but increasing the ratio $T_g/\gamma$ to 4400K/J$\cdot$m$^{-2}$. It can be appreciated from the resulting plots of Fig. 6.4 that although not strictly proportional, the loss predicted by both methods follows the same trend as the structural parameters are changed. We found indeed that all the structural parameters that give reduced field intensity at the interfaces also result in low-loss as obtained from Eq. (4.16).

These observations indicate that at a single wavelength, loss may be obtained from either methods provided the free parameters are chosen appropriately. However, we note that $F$ alone does not correctly predict the wavelength loss dependence when the fibres are rigidly scaled. Indeed, with the electric field at the interface changing roughly as $\sim \lambda/R$, $F$ changes roughly as $\lambda^2/R^3$. This implies that if material dispersion is ignored, then for rigidly scaled fibres $F$ decreases as $1/\lambda$. This is illustrated in Fig. 6.5 which shows a comparison between the wavelength dependence of the loss as predicted by the scattering theory and obtained by a simple calibration of the normalized interface field intensity. As before, the calibration factor for the normalized interface field intensity is $\eta = 300$ for all
wavelengths and the cut-off frequency for the roughness PSD is $0.1\mu m^{-1}$, and we have reverted to $T_g/\gamma = 1500 K / J \cdot m^{-2}$.

It can be seen that $F$ decreases indeed as $1/\lambda$ when the fibres are scaled rigidly (material dispersion ignored). On the other hand, the scattering model predicts the correct wavelength dependence of roughly $\sim \lambda^{-3}$. Because of this, we cannot expect a unique scaling factor $\eta$ to aptly reproduce the loss for fibres operating at different wavelengths. Since $F$ decreases approximately as $\lambda^{-1}$ and both measured loss from experiments and loss calculations from our rigorous theory are known to decrease as $\lambda^{-3}$, we choose to change the calibrating parameter $\eta$ with wavelength such that for fibres operating near central wavelength $\lambda_c$,

$$\eta(\lambda_c) = \left(\frac{1.55\mu m}{\lambda_c}\right)^2 \eta(1.55\mu m). \quad (6.2)$$

This ensures that for rigidly scaled fibres, loss from calibrating $F$ would decrease as $\lambda^{-3}$. We have therefore employed this simple scaling to estimate the loss as
Figure 6.5: Calculated scattering loss dependence on the central wavelength $\lambda_c$ in an idealized 19c HC-PBGFs ($\Lambda = 4.4\mu m, d/\Lambda = 0.975, D_c/d = 0.42$) when the fibre is rigidly scaled. With a spatial frequency cut-off at $\kappa_c = 0.1\mu m^{-1}$, the data fits very well to $\lambda^{-a}$, with $a = 3.25$. Note that though the normalized interface field intensity decreases, its slope is only $a_F = -0.9992$.

6.5 Modelling results

The procedure detailed above was extensively used to assess the properties of various fabricated fibre samples, examples of which are given in this section. The primary interest of our work lied in comparing simulations with the measurable fibre properties, in particular attenuation and bandwidth, although every simulation also gives properties which are not routinely measured or are more challenging presented in the following section. In the absence of consistent roughness measurements and a complete picture of small structural variations along the fibre length, this simple scaling hints at the reliability and repeatability of the fabrication process itself. Indeed, if the fabrication process is consistent and repeatable and with the assumption that roughness will not change with fibre dimensions, one single $\eta$ value should be characteristic of the process and loss variations between different fibre samples must explained solely by differences in the geometry of their cross-sections.
to evaluate experimentally such as dispersion, differential modal loss, etc. Loss contributions from roughness scattering and leakage are taken into account, which is why we revisit here the two major approaches we have described to estimate the loss.

### 6.5.1 Example 1: Low-loss fibre with no core tube

A sample of choice for numerical study is the fibre reported in [88] and shown in Fig. 6.6(a). The fibre was fabricated without a core tube, with the aim of achieving low-loss guidance over a broad bandwidth, and has been thoroughly characterized and used for low-latency data transmission experiments [88]. An analysis of the cross-section reveals an average hole-to-hole spacing of $\Lambda = 4.4 \pm 0.1 \mu m$, an average strut thickness $t = 110 \pm 10 nm$ and an average fillet radius $r_c/d = 0.21 \pm 0.05$ where $d$ was the average air hole diameter. These average values were used to reconstruct the profile superposed on the SEM image on Fig. 6.6(a). As before, this structure was simulated using the finite element method platform, for wavelengths between 1.2 and 1.8 $\mu m$ in 5 $nm$ steps. Figure 6.6(b) shows an example of the simulated fundamental mode profile at a low loss wavelength within the bandgap.

Figure 6.6(c) shows plots of measured short length transmission and simulated fraction of guided power in the core. The agreement between bandgap positions and widths is remarkable and is a good testimony of the accuracy of the method used to reproduce the fibre geometry. We note that this agreement was not achieved in the first simulation attempt but required three iterations in which the free parameters $t$ and $r_c$ were altered, before selecting the final average values reported above. In addition, the simulation also accurately reproduces the dips in transmission near both edges of the photonic bandgap due to coupling to surface modes. This is examined in more detail later on.

Shown on Fig. 6.6(d) is a comparison between measured and simulated fibre attenuation. The simulated loss was taken as an average of both polarizations of the fundamental mode. As can be appreciated, the agreement between computed and measured loss values is excellent. This suggests that the scaling rule of equation (6.1) can be used to estimate the attenuation of fibres produced using the same process.

The modal content of idealized HC-PBGFs can be already quite complex as shown in Fig. 3.10 and certainly becomes more so when distortions and surface modes
Figure 6.6: Transmission and loss measurement compared with simulation for a low-loss fibre made with no core tube (a) shows the overlap between the original SEM image and the reconstructed geometry. (b) is the fundamental mode profile at the lowest loss wavelength of 1.5\(\mu m\). (c) Short length transmission measurement (measured over 10\(m\)) and simulated power in the core(d) Cutback loss measurement and simulated fundamental mode loss. Loss is computed as the sum of contributions from scattering and leakage. Here \(Loss = \frac{1}{2} (loss(LP_{01x}) + loss(LP_{01y}))\)

are present [66, 122]. For SDM data transmission in HC-PBGF such as pursued throughout the MODEGAP project [119], it is important to have a thorough understanding of the modal properties of the fabricated fibre. The distortions present in most fabricated fibres profoundly impact their modal properties, which unfortunately are challenging to evaluate experimentally. The ability to accurately model the fibre thus becomes all the more important, and the technique developed here can be quite useful. As an illustration, the time average \(z\)-component of Poynting vector for the first five mode groups calculated at 1.5\(\mu m\) is shown in Fig.6.7. Whereas idealized and distortion-free HC-PBGFs support true vector modes (i.e \(HE_{11}, TM_{01}, TE_{01}, HE_{21}\) etc...), the modes of the fabricated fibre are predominantly linearly polarized. This arises because of the visible ellipticity of the core which cases the modes to deviate from those of idealized fibres [91].

The dispersion map showing the effective index trajectories for these modes across
Figure 6.7: Calculated time average of the $z$-component of the Poynting vector for the first 5 core-guided mode groups in the fabricated HC-PBGF of Fig. 6.6. The slight ellipticity of the core and the small numerical aperture of the fibre make the modes essentially linearly polarized.

The photonic bandgap is plotted in Fig. 6.8. One recognises at once that the reduction in transmission at both edges of the photonic bandgap in Fig. 6.6(c) is due to coupling to surface modes. The first group of surface modes near the short wavelength edge have their power located in the struts of the first ring air holes which appear to be shorter and thus thicker than average. The second group near the long wavelength edge have their power concentrated in glass nodes in the vicinity of the core which are smaller in size than those in the cladding [61]. It
Figure 6.8: Dispersion map for the low-loss HC-PBGF of Fig. 6.6. The group of surface modes at the short wavelength edge of the bandgap have their power guided in the struts of the first ring of air holes. Those at the long wavelength edge are primarily located in the nodes near the core as these are closer together than average.

is appreciated however, that despite the presence of these two groups of surface modes, not using a core tube in the preform has resulted in a thin core surround which leaves in place a wide operational bandwidth. Avoiding surface modes is crucial for loss reduction and operational bandwidth enhancement, and also for ensuring a slowly varying dispersion across the bandgap which is beneficial for data transmission.

The visible asymmetries of the fibre’s core cause some slight birefringence of the order of $\sim 5 \times 10^{-6}$ for the two polarizations of the fundamental mode, and also introduce a differential loss as high as $0.3\text{dB/km}$ between the two. As the mode order increases, the $LP_{mn}$ modes with no circular symmetry ($m > 0$) tend to split into two groups of two nearly degenerate modes $LP_{mna}$ and $LP_{mnb}$ with increasing difference in effective indices. For the $LP_{31}$ mode group for example, the index difference between the $a$ and $b$ subgroups is as high as $7 \times 10^{-4}$. The reason for this, Besides the core’s ellipticity, is also the presence of the six enlarged corner holes near the core boundary. One of the two field patterns has a lobes near the
pentagonal air holes and their slightly thicker walls whereas the other doesn’t and as a result, the difference in effective index is introduced.

Other important modal properties such as group index, differential modal group delay (DMGD) and dispersion are readily obtained from the effective index trajectories. The calculated group index for the fundamental modes (defined in Eq. (2.5)) is \( n_g = 1.002 \) and changes very little across the C-Band. This compares with the typical value \( \geq 1.46 \) for the single mode fibre and confirms the somewhat obvious but very important low-latency data transmission potential of HC-PBGFs.

The differential group delay between guided modes is an important parameter in mode division multiplexed data transmission as it determines the amount of memory to be used in the MIMO compensation algorithm at the receiver [11]. In HC-PBGFs however, the DMGD is very sensitive to small details in the fibre core surround. Figure 6.9 shows plots of the simulated differential group delay with respect to the \( y \)-polarized fundamental mode, together with values measured experimentally using a time-of-flight setup [88]. DMGD was measured by offset-launching linearly polarized light into the PBGF. This predominantly excites one lobe of the higher order modes, and as a result, only the two polarizations of the \( LP_{mn} \) or \( LP_{mnb} \) could be excited and detected with the same launching conditions.

As can be seen, DMGD in HC-PBGFs is several orders of magnitude higher than the values reported for the best solid fibres, such as the \( \sim -0.08 ps/m \) reported by Grüner-Nielsen in [164]. The very high DMGD in HC-PBGFs stems from the fact higher order modes which extend further into the cladding tend to have a fraction of their power carried in the first ring air holes. As a result, their effective indices decrease more prominently as the wavelength is increased towards the long wavelength edge of the bandgap. This results in the trajectories of \( n_{eff} vs. \lambda \) having significant slope differences and thus the higher DMGD. The implication for such high delay differences is that MIMO processing in SDM transmission systems using HC-PBGFs would be far more complex, unless coupling between the guided modes can be eliminated in which case no MIMO processing will be required as each mode will then be able to be excited and detected separately. The debate is ongoing, however, as to whether this will in the long run be an advantage in SDM systems. The calculated DMGD values can be seen to agree very well with the measured ones. The discrepancies for the \( LP_{02} \) are attributed to the remaining inaccuracies in determining the thickness of the core wall near the enlarged corner holes. The mode profiles of Fig. 6.7 suggest that small changes in the thickness of these boundaries will particularly impact the \( LP_{02} \) modes.
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Figure 6.9: Differential modal group delay in the HC-PBGF of Fig. 6.6. Colors represent each mode group. The markers show the measured data using a time of flight setup (courtesy of Dr. Radan Slavík) while the solid lines show simulated data.

Figure 6.10 shows plots of quantities that are more challenging to assess experimentally: the loss and dispersion for individual modes of the fibre. Because the modes overlap differently with the scattering glass surfaces, the differential modal loss within a mode group and between the mode groups is significant. This is an important consideration if HC-PBGF are to be used in SDM data transmission. In this case, accurate modelling of each of the individual modes is all the more important. The dispersion curves too can be quite different for the various modes. In general, both the zero dispersion wavelength and dispersion slope tend to increase with the mode order.

6.5.2 Example 2: Fibres produced with a core tube

As already discussed, prior work on idealized fibres established that core boundaries of equal thickness or thicker than struts in the cladding inevitably support surface modes, leading severe bandwidth and scattering loss penalties [85, 86, 118].
Therefore, fibres targeting wide bandwidth operation ought to be drawn from pre-forms stacked with no core tube, such as in the previous example. We have understood however (as will be explained in the next chapter), that provided the nodes
on the core boundary can be kept equidistant and with minimal structural distortions, fibres with a core wall as thick as the cladding struts suffers little bandwidth and loss penalties. We therefore advocated for preforms with core tubes as thick as cladding capillaries to be produced. Producing a preform with a core tube has the advantage of increased fibre yield because the plugs that are used at both ends to keep the stack in place in preforms without core tubes are unnecessary, and as a result, more canes and therefore fibres can be drawn from such a preform. In addition to this advantage of increased yield, the benefits of the introduction of a core tube also include easier control over the stability of the structure during the fibre draw.

A number of secondary preforms were assembled from virtually identical canes with a core tube and drawn into several fibre samples. Despite their very high air-filling fractions which from work on idealized fibres should have also resulted in wide transmission bandwidths and low attenuations, transmission measurements on these fibres showed that they supported several surface modes and suffered from high attenuation. For further analysis, we selected three fibre samples with increasing expansion ratios and air-filling fractions. Their cross-sections are shown in Fig.6.11 and their structural parameters summarized in Table 6.1.

![Figure 6.11: Three fibres with increasing air-filling fractions produced from a cane with a thin core tube.](image)

<table>
<thead>
<tr>
<th>Fibre</th>
<th>Expansion(%)</th>
<th>Core(\phi/5\Lambda)</th>
<th>(t)(nm)</th>
<th>(\lambda_c)((\mu m))</th>
<th>Loss (dB/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>57.35</td>
<td>0.9272</td>
<td>105.0</td>
<td>1.57</td>
<td>6.0</td>
</tr>
<tr>
<td>B</td>
<td>63.76</td>
<td>0.9546</td>
<td>85</td>
<td>1.44</td>
<td>8.5</td>
</tr>
<tr>
<td>C</td>
<td>72.60</td>
<td>0.8892</td>
<td>56</td>
<td>1.46</td>
<td>6.6</td>
</tr>
</tbody>
</table>

Table 6.1: Measured structural parameters and measured loss for three fibres drawn with core tube. Core\(\phi/5\Lambda\) is the ratio of the measured core diameter to that of an idealized fibre with the same average pitch. \(t\) the estimated average strut thickness and \(\lambda_c\) the central wavelength of the bandgap.
In sharp contrast with the fibre of section 6.5.1, the core of the three fibres are compressed as compared to that of an idealized fibre, as can be seen from table 6.1. The compression is most severe for Fibre C which has the highest air-filling fraction and a core more than 10% smaller than in the idealized case. Since we know that larger core sizes are beneficial for loss reduction, Fibre C is a priori not expected to give the lowest loss, despite its very high air-filling fraction. Initial modeling on these fibres with the assumption of mass conservation showed much lower losses and failed to reproduce the surface modes positions within the bandgap. Further examination of both the cane and fibres revealed that struts at the boundaries between core and pentagonal holes in the first ring are considerably longer. Although these boundaries have the same thickness in the preform, they are approximately 1.7 times longer than the average cladding struts but end up in the fibres being about 1.5 times shorter, as illustrated in Fig. 6.12.

By adjusting the core wall thicknesses in the reproduced geometries accordingly, fibres with considerably better agreement with the measurements were obtained. Figure 6.13 shows comparisons between simulated fraction of power in the core and measured short length transmission, and between measured and simulated loss values. The agreement between experiment and simulation is excellent, and the simulation methodology can be seen to accurately reproduce the surface modes positions.

Generally, the simulated width of the low-loss windows are wider than the measured ones. This is likely due to small variations that may occur along the fibre length.
Figure 6.13: Measured and simulated transmission and loss properties for fibres A, B and C drawn with a core tube. The solid black curves are measured values while the read curves show the simulated counterparts.

which will increase the confinement loss near the bandgap edges and near the surface modes. The simulated minimum loss values computed with the simple formula of Eq. (6.1) are 5.5, 6.1 and 5.6\,dB/km for fibres A, B and C respectively which compare very well to the measured values in table 6.1. The discrepancy is highest for Fibre B (2.4 \,dB/km difference), which may be indicative of small residual inaccuracies in regenerating the geometry, but may also indicate some longitudinal degradation along the fibre length.

The main conclusion from these simulation is that both the scattering dominated loss and the position of the surface modes are correctly explained by the geometry
of the fibres cross-sections. Therefore, surface modes can in principle be eliminated and the loss reduced if the identified structural distortions can be corrected. Through the illustration of Fig. 6.12 we can see that in many second stage preforms, the glass is not evenly distributed in all the struts near the core boundary. Unfortunately, when corner holes are enlarged during the fibre draw, the struts which are the longest in the cane end up being the shortest and thickest in the fibres and are therefore prone to support surface modes. The supported surface modes are strut surface modes of increasingly higher order, as can be seen in Fig. 6.14 where the power distributions for the surface modes of Fibre C are shown.

Figure 6.14: Power distribution of the surface modes supported in Fibre C. The fibre supports three groups of surface modes at 1.25, 1.46 and 1.62\(\mu m\) respectively. These surface modes have their power located in the thicker struts and their order as indicated by the number of lobes in the power distribution increases with shorter wavelengths.
One route to effectively address this problem should be to engineer the second stage preform in such a way that each strut on the core boundary possesses and equal amount of glass. In addition, steps should be taken during the second stage draw to ensure that the struts near the core boundary are approximately uniform in both length and thickness in the final fibre. Both theoretical and experimental work is underway to identify which effective recipes and steps during fabrication will help attain such objectives.

6.5.3 Example 3: Fibre guiding at \(2\mu m\)

Fibre samples operating at a central wavelength of \(2\mu m\) were also drawn from second stage preforms similar to that of Fig. 6.12 and featuring a core tube. Following the procedure highlighted above, we also simulated the properties of one such sample, the cross-section of which can be seen from the inset of Fig. 6.15. The fibre featured an expansion ratio of 61.7% and an outer diameter of 175\(\mu m\). The estimated strut thickness was \(t = 120\,nm\) and the fillet radius was \(r_c/d = 0.23\) with \(d\) being the average hole diameter. The pressure balance in this particular draw ensured the core was as large as in idealized case, although some distortions are clearly visible. Figure 6.15 shows comparisons between the simulated power in the core and measured transmission over 10\(m\) of fibre, and between the simulated and cutback measurement from 1.1\(km\) to 10\(m\).

As can be seen, the simulation performed on a geometry generated using our method and compensating for the thicker struts around the core is in good agreement with measurement. The position of the surface modes is accurately reproduced as well as the minimum loss value. It is interesting to note that the loss was estimated in this by also using Eq. (6.1), but this time by changing the scaling constant so that:

\[
\eta_{2\mu m} = \left(\frac{1.55\mu m}{2.0\mu m}\right)^2 \eta_{1.55\mu m} \tag{6.3}
\]

Using this scaling which is based on the fact that the scattering loss changes as \(\lambda_c^{-3}\) as discussed previously (see also ref. [165]) has yielded surprisingly good agreement between simulation and experiment as can be appreciated from Fig. 6.15. The minimum measured attenuation was 2.5\(dB/km\) at 2.092\(\mu m\) whereas the simulated minimum value of 2.6\(dB/km\) occurs at 2.1\(\mu m\). We see therefore that in absence of reliable roughness measurements and a theory capable of describing the PSD
of the roughness present in HC-PBGFs, the simple scaling presented here may be used in a first estimate of the loss and may also be a test of the reproducibility of the specific fabrication process used to make the fibres.

### 6.5.4 Example 4: mid-IR HC-PBGF

Besides the development of low-loss HC-PBGFs for telecom applications, fibres were also drawn for operation in the mid-IR region of the spectrum. The spectral
region above 3\(\mu m\) is interesting in particular for sensing applications since it covers strong vibrational lines of many molecular species. Fibres capable of guiding at those wavelengths with low-loss and low bend sensitivity may provide an avenue for high precision spectroscopy and compact gas sensors with fast response times. Since high purity synthetic silica effectively ceases to be transparent at wavelengths above 2.5\(\mu m\) (absorption due to OH is higher than 10000\(dB/km\), see [166]), HC-PBGFs are an attractive option for making durable, bend insensitive fibres with good power handling capability in that spectral region.

The 19c fibre which cross-section is shown in Fig. 6.3 was made specifically for operation near 3.3\(\mu m\). Its preform was stacked without a core tube, and operation at the desired wavelength implied increasing the fibre dimensions with respect to those of fibres operating in the near IR. The average pitch was \(\Lambda = 9.3 \pm 0.2\mu m\), the average strut thickness was \(t = 300 \pm 20nm\) and the fillet radius was \(r_c/d = 0.45\) with \(d = \Lambda - t\) being the average air hole diameter. The fibre’s outer diameter was measured to be 345 \(\pm 1\mu m\).

Having generated the matching geometry using the same process detailed above, we proceeded to simulate the fibre’s properties and performed comparisons with experiments. Figure 6.16 shows the comparison between measured transmission over a 5\(m\) section of the fibre and the simulated power in the core, and between the measured loss and simulated loss contributions.

It was determined that the visible loss peaks on the measured loss curve between 3.3 and 3.6\(\mu m\) was due to HCl which is present in the fibres because chlorine gas is used to dehydrate the bulk silica glass from which the fibre is made. These peaks correlated very well with the theoretical absorption lines of HCl [75] and were substantially reduced when the fibre was purged with argon for five days to remove the HCl. Purging also led to a decrease of the measured loss from 0.13 \(\pm 0.05dB/m\) to 0.05 \(\pm 0.03dB/m\). It is interesting to note the four peaks on which argon purging had no effect at all. These may be due to surface modes which are the peaks near the short wavelength edge of the bandgap, although simulating the reconstructed geometry did not achieve an exact match with the wavelengths at which they appear.

Simulations were performed both in the absence and presence of material absorption. In the first case, we considered silica to be effectively lossless, in which case loss would still be dominated by scattering, but would only amount to 0.2\(dB/km\) due to the long wavelength of operation, as shown by the magenta curve in Fig.
Figure 6.16: Measured and simulated properties for a 19c HCPBGF without a core tube operating near 3.3µm. In (a), the solid black curve corresponds to transmission over 5m while the red one shows the simulated power in core. In (b), the blue and green curves are the measured losses before and after the fibre was purged with argon gas to remove HCl gas residues. The magenta curve shows simulated loss in the absence of material absorption while the red curve shows the result for a simulation that takes material absorption into account (measurements, courtesy of Dr. Natalie Wheeler).

6.16(b). The wavelength-dependent absorption of suprasil F300 was then extracted from [166] and converted into an imaginary part of the silica refractive index as an input to the FEM solver. The inclusion of material absorption resulted in the minimum total loss value of 0.07dB/m, in good agreement with the loss measured
after the fibre was purged of HCl. Similar loss numbers were obtained when the simulated fraction of power in glass ($\sim 0.1\%$) was multiplied by the bulk absorption.

It can be appreciated that here again, even though the loss is limited by a different mechanism, our method of reconstruction of the geometry from SEM images leads to simulation results in excellent agreement with experiments.

### 6.6 Summary

This chapter has presented a new approach to simulating the properties of fabricated HC-PBGFs from SEM images. It has demonstrated that details beyond the SEM image resolution can still be reproduced to achieve a better match with the actual fabricated fibre. The tool developed here has shown remarkable accuracy as it enables simulations capable for the first time of accurately reproducing experimental measurements of loss and surface mode position within the photonic bandgap. It has been shown that such a tool can provide a valuable estimate for those fibre properties which are more challenging to assess experimentally such as modal differential loss and individual mode dispersion.

Simulations carried out on several fibre samples have confirmed that for fibres operating at $1.55$ and $2\mu m$, loss is still dominated by surface scattering, which is dictated by the geometry of the cross-section. The first implication of the agreement between simulations and measurements achieved here is therefore that for the simulated fibre samples, structural variations along their length is only playing a minor role in determining their loss, and they can be considered essentially invariant long their length. The second one is that there is scope to further reduce fibre losses provided that both preform design and fibre drawing processes can be controlled to produce fibres within which the observed distortions and uneven strut thicknesses are absent.

The example simulations presented here clearly indicate that the structural distortions present within the fibre cross section are at the origin of the observed loss values and surface modes. The following chapter presents a method to systematically study different classes of distortions in order to identify which ones are more detrimental and which ones may be more tolerable or perhaps beneficial for loss reduction.
Chapter 7

Impact of structural distortions in HC-PBGFs

We concluded in the last chapter from simulations performed on the cross-sections of fabricated HC-PBGFs that structural distortions might have a severe impact on the loss and other optical properties of the fibres. However, simulating individual fibre samples does not allow a systematic study of some particular classes of distortions. This chapter presents such systematic studies of distortions in HC-PBGFs. In particular, the impact of distortions within the first two rings of air holes near the core is assessed. It is found that enlarged ‘corner’ holes prompt the electric field to overlap more strongly with the scattering surfaces, thereby increasing the overall scattering loss. A trade-off between confinement and scattering loss is revealed for the first time when studying the impact of the core size. These findings are combined to propose designs which shall allow even lower losses at the expense of only a small penalty on the bandwidth. The possibility of fabricating such fibres with the incorporation of a core tube is then investigated by studying to which extent the resulting core surround in the fibres may support surface modes.

7.1 Modelling methodology

The approach we use here to generate fibre geometries with arbitrary distortions follows closely from the method presented in the previous chapter. Realistic distortions present in a fabricated fibre are reproduced by considering the cladding as made up of individual glass nodes interconnected by a web of thin struts. As
such, the glass nodes can be arbitrarily moved from their positions in the idealised fibre to generate any desired cladding distortion and later reconnected to form the transformed and distorted structure. With such a technique, a selected air hole may be enlarged, collapsed, rotated or have any of its corner points undergo a different transformation. Obviously, distorting one hole simultaneously affects some or all of its neighboring holes as well.

The most relevant distorted structures for our study are those bearing close resemblance to fabricated fibres. Figure 7.1 shows how distorted structures that can be generated using this process are a more accurate representation of fabricated fibre samples that the idealised representation in which the cladding is perfectly periodic (see Fig. 7.1(c)). Although the idealised representation appears to work
Figure 7.2: Parameters used to generate distorted geometry. By parameterizing the structure, systematic studies on the impact of distortions can be conducted.

for 7c fibres which tend to be less distorted, it fails to do so for fibres with larger core sizes such as 19c or 37c HC-PBGFs. As can be seen in Fig. 7.1(d) where the routinely used idealised representation is overlapped with a realistic distorted profile, the most prominent and typical distortions are the enlarged core defect and the presence of six overinflated ‘corner’ holes.

As illustrated in Fig. 7.2, a number of structural parameters are necessary to generate distorted structures which resemble fabricated ones. First, the fibre core radius $R_c$, the number of rings of air holes around the core $N$ and the mean radius of the microstructured cladding $R_d$ (not shown) are specified. Secondly, the dimensions of the enlarged hexagonal corner holes $(L_1, L_2, h_1, h_2)$ along with those of the air holes on the diagonal $(W, H)$ need to be set (see Fig. 1 for their description). A tapering of the size of these air holes may be also incorporated if desired. The remaining nodes are then placed so that the cladding resembles that of fabricated fibres by focusing on one $\pi/3$ sector, and subsequently a $C_{6v}$ symmetry is imposed to generate the full structure. The parameters crucial to the spectral position of the photonic bandgap are the strut thickness $t$, the fillet radius $r_e$ (and $r_1$ for the corners on the core surround) and the average spacing
between the glass nodes [65]. As in the previous chapter, $t$ is set to be inversely proportional to the strut length. This has been shown to be essential in ensuring accurate results, especially for those struts on the core boundary which thicknesses can have profound impacts on the properties of the fibre.

Such parametrization makes possible the systematic study of the impact of each type of distortion and hence the identification of those which may be responsible for additional loss in the fibre. The large number of parameters however, makes it prohibitive to perform systematic studies on each of them individually. The focus in the remainder of the chapter will therefore be kept on the most prominent and realistic structural distortions. As in the previous chapter, equation (6.1) will be employed to estimate the scattering loss which will be combined with leakage loss contribution to give the total loss of the fibre.

7.2 Cladding distortions beyond second ring

The microstructure surrounding the core defect determines the spectral position and width of the photonic bandgap. The study on the impact of distortions therefore most fittingly begins with an investigation into the effects of non-uniformities far away from the core defect. There is arguably a myriad of possible distortions that may occur in the cladding, but as seen in Section 3.1 these will have very little impact on the scattering loss if located beyond the second ring of air holes outside the core defect. Here, the discussion is limited to the impact of a compression of the cladding beyond the second ring of air-holes.

As is well established, the position and width of the photonic bandgap are determined by the average node size, the average distance separating them and the strut thickness. Therefore, for a fixed core radius $R_c$, increasing the ratio $R_c/R_{cl}$ by compressing the microstructured cladding leads to a proportional decrease in the average distance between the cladding nodes, ultimately resulting in a narrower photonic bandgap shifted to slightly shorter wavelengths. By altering the cladding size only beyond the second ring outside the core, some insight can be gained into the effect of distortions in this region on the propagation loss of the fibre.

Figure 7.3 shows plots of loss vs. wavelength for three fibres in which the core and the first two rings of holes are kept the same, while the remainder of the cladding
Figure 7.3: Impact of cladding compression beyond the second ring. Fibre A in blue is close to an idealized and undistorted fibre; in Fibres B (green) and C (red) the cladding beyond the second ring is scaled down to 95% and 90% of its original size, respectively. The dotted lines indicate leakage loss only, while the solid lines show the total loss (i.e. leakage + scattering). The inset shows the total loss of the fibres on a linear scale between 0 and 10dB/km.

is progressively radially compressed. Fibre A shown in blue has a core radius $R_c = 13 \mu m$, a ratio between core and microstructured cladding diameter of 31% (29.4% in ideal fibres) and an average strut thickness of 110nm, which together with the fillet radius $r_c/W = 0.21$ (see Fig. 7.2) yield a photonic bandgap centered around $1.7 \mu m$. The outer four rings are then modified so that the cladding diameter is 95% smaller for Fibre B shown in green and 90% smaller for Fibre C shown in red. It can be observed that confinement loss, plotted in dotted lines, increases
by more than an order of magnitude for each progressive cladding compression, and contributes to a net reduction in the overall transmission bandwidth. The narrower bandwidth results from the more closely spaced nodes (narrowing the photonic bandgap) and a thinner holey air region surrounding the core (increasing the confinement loss). However, at wavelengths well within the bandgap, the three fibres have essentially the same value of total loss, confirming that loss is dominated by scattering from surface roughness, which is mostly unaffected by the structure beyond the second ring.

A first conclusion that can be drawn therefore is that structural defects beyond the second ring of air holes do not have damaging effects on the fibre loss, as long as the cladding is uniform enough to ensure low leakage. Efforts to reduce the loss should therefore focus on the optimization of the core defect and the first two rings immediately surrounding it.

### 7.3 Distortions within the first two rings

#### 7.3.1 Corner Holes

Another type of distortion frequently observed in fabricated HC-PBGFs such as those simulated in the previous chapter and those reported in \cite{88, 119, 167} is the presence of six oversized corner holes around the core defect (see Fig.7.1). These arise as a natural consequence of surface tension trying to create a circular core surround from an original hexagonal structure. To assess the impact of such enlarged corner holes, fibre geometries with parameters similar to those obtained from the SEM image of the real fibre shown in Fig.7.1(a) are simulated. The measured core diameter of $2R_c = 26 \mu m$ and core to cladding diameter ratio of $0.36$ with six rings of air holes are employed. The average thickness of the cladding struts was estimated to be $t = 110 nm$ in the cladding and $t/2$ on average on the core surround ring to account for the absence of a core tube in the fabrication of the fibre. In addition, conservation of the glass volume in the struts was imposed as before, resulting in struts longer than average being thinner than $t$ and vice-versa. The fillet radius used to round the holes’ corners was set to $r_c/W = 0.21$ and $r_1/W = 1/\sqrt{3}r_c/W$ was used to reflect the reduced amount of glass at the core nodes. In an idealized fibre with core radius $R_c$, the distance between two nodes everywhere in the cladding would be $l = 2R_c/5\sqrt{3}$ . In this study, the
corner holes side length $L_1$ was changed in six incremental steps from $1.4l$ where the nodes on the core wall are nearly equidistant, to $2.4l$ which is the structure that best resembles the fabricated fibre and is shown in Fig. 7.1(b). The mode profiles and fundamental mode loss for all wavelengths across the bandgap for the six resulting fibres were then computed and the results are shown in Fig. 7.4 below. Also superposed on the figure are the loss plots for the idealised fibre (Fig. 7.1(c)) and the cutback measurement for the fabricated fibre of Fig. 7.1(a).

As in the previous section, we found that the scattering loss contribution remains dominant when six rings of air holes surround the core. Although the leakage loss contribution in the fibre with the most enlarged corner holes (Fibre 6) is twice as much as for the fibre with the least enlarged ones (Fibre 1), it still only amounts to $0.035 dB/km$ - a small fraction of the total loss.
Chapter 7 Impact of structural distortions in HC-PBGFs

The loss plots of Fig. 7.4 indicate that fibres with larger corner holes suffer from higher losses. After some further studies, we concluded that larger corner holes lead to larger gaps between the glass nodes on the core boundary, and this prompts the electric field to overlap more strongly with the scattering surfaces, thereby generating a significantly higher scattering loss. To illustrate this effect, the time average power flow in the fibre axis direction for the fundamental mode is shown in Fig. 7.5 for fibres 1 and 6 having the least and most enlarged corner holes, respectively. The figure also shows the same plot for the idealised version of the fibre at the same wavelength of 1.5µm.

![Contour plots](image)

Figure 7.5: Contour plots of the fundamental mode time average power flow in the z-direction for distorted and ideal fibres. (a) Contour plot for fibre 6, (b) corresponding plot of fibre 1 and (c) contour plot for the idealized version of the fibre. The contour lines are over a 25dB range and are 5/3dB apart. Note how large gaps between nodes on the core boundary prompt the guided mode field to overlap more strongly with the air-glass interfaces.

It is remarkable to observe that the idealised and distortion free structure most routinely used in simulations and usually considered to be the optimum fibre design (see Fig. 7.1(c)), is the one providing the highest loss despite having a core surround designed to preserve the periodicity of the cladding. This high loss is due to two main reasons. First, the idealized fibre has a core diameter that is 1.18 times smaller than Fibres 1 to 6. We saw in section 3.3.1 that in idealised structures, if the core is enlarged while preserving the cladding periodicity, for example by removing 7, 19 or 37 unit cells, the loss decreases approximately as \( R_{c}^{-3} \). In the case of distorted fibres the core is enlarged at the expense of some compression of the cladding. If this compression is ignored and the \( R_{c}^{-3} \) rule is applied nonetheless, the idealised fibre loss is expected to be higher by a factor of \( 1.18^3 = 1.64 \) at most with respect to that of Fibre 1. However, the actual loss of the idealized fibre is 2.4 times higher than that of Fibre 1, indicating that
other differences in the fibre structure may be playing a significant role. Indeed, if the core size alone accounted for the loss differences, Fibres 1 to 6 would have essentially the same loss, but this is clearly not the case. Careful examination of the fibre structures points to the more even distribution of the nodes on the core boundary in Fibre 1 as being key to the much lower scattering loss value. An insight into why this may be the case is gained again by inspecting contour plots of the time average power flow of Fig. 7.5, where significantly lower overlap is seen between the mode field and the air-glass interfaces for the fibre in (b).

It therefore appears that the more regular the spacing of glass nodes on the core boundary, the more the electric field of the guided mode is prevented from overlapping with the scattering surfaces. An equal spacing of the glass nodes on the core boundary therefore shall be the optimal design for loss reduction purposes. In principle, achieving uniformity in terms of node and strut size on the core boundary should also lead to a design more robust in avoiding the introduction of surface modes within the bandgap, since the struts would be of equal length and equal thickness.

7.3.2 Impact of core size

We discussed in section 3.3.1 that increasing the size of the core defect by removing an increasing number of unit cells (7, 19, 37,...) leads to a reduction in the overlap of the guided mode with the scattering surfaces and hence to loss decreasing approximately as $R_c^{-3}$. A requirement for these scaling laws is that the structure of the cladding be maintained. This is not the case in fabricated HC-PBGFs where the core defect is expanded or compressed at the expense of distortions in the surrounding cladding such as seen in the previous chapter. In reference [84], the authors investigated the impact of the core defect size in 7-cell HC-PBGFs when it is expanded or compressed at the expense of the first ring of air holes surrounding it. In the rather restricted range of parameters examined, their results indicated very generally that the larger the core defect, the lower the scattering loss will be.

Here the same effect is investigated when the fibre core is enlarged or slightly compressed at the expense of the remainder of the cladding. By maintaining the same cladding radius $R_{cl}$ and imposing equidistant node spacing on the core boundary (see Fig. 7.6 below), six fibres with increasingly large core diameters were
simulated and analysed in terms of loss performance. To track the changes in fibre properties, it becomes important to define a normalized core diameter which is the core diameter of the fibre under study divided by that of an idealized fibre with the same cladding size and number of rings of air holes. In other words, for 19c fibres:

$$C = \frac{R_c}{5 \times R_{cl}/(2N + 5)}$$

(7.1)

where $N$ is the number of rings of air holes surrounding the core. Values of $C$ larger than 1 indicate enlarged core defects while those lower than 1 will be indicative of compressed core defects. For this study, the cladding parameters were chosen as $R_{Cl} = 74.8\mu m$, $r_c/W = 0.21$ and $t = 110nm$. $C$ was then changed from 0.986 to 1.326 in five steps. Mass conservation was assumed as before, and the core wall thickness was set to half the value in the cladding to mimic the absence of a core tube. Figure 7.6 summarizes the main findings.

As the core diameter increases, a red-shift and slight narrowing of the photonic bandgap can be observed. The minimum scattering loss decreases steadily with core size (Fig. 7.6(a)) as confinement loss increases instead very rapidly as a result of the thinner cladding region. The lowest achievable total loss occurs at $C = 1.258$ and beyond this point, the confinement loss contribution becomes significant. Although this is accompanied by a penalty in transmission bandwidth, the remaining bandwidth in excess of 200nm is wide enough for many applications including WDM data transmission. Enlarging the core defect in this way clearly indicates a trade-off between confinement and scattering loss. For fibres with enlarged corner holes, the same trends were observed, although the confinement loss was higher and led to a smaller optimum core size.

It is interesting to note from the results shown in Fig. 7.6(d) that the minimum scattering loss decreases approximately as $R_c^{-1.5}$ instead of $R_c^{-3}$ when more cells are removed to form the core while preserving a periodic cladding. This is somewhat expected as the cladding is also modified when changing the core size.

It is clear from these findings that in addition to even spacing of the core nodes on the core boundary, a slightly enlarged core size will also be of benefit to loss reduction efforts, especially since more rings of air holes can be included to further reduce the confinement loss.
7.3.3 Core wall thickness and surface modes

We have seen throughout this dissertation that a core wall on average half as thick as the cladding struts is effective in avoiding surface modes within the photonic bandgap. Despite the well-known benefits of introducing a core tube (see section 6.5.2), the previous chapter showed that when drawn to cane, preforms stacked with a core tube often yielded canes with uneven distribution of glass in the core struts. This uneven distribution when coupled with the distortions occurring during the second-stage draw led to some struts on the core wall being short and thick with the undesirable consequence of supporting a multitude of surface modes within the photonic bandgap.

Figure 7.6: Impact of changes in the core size on fibre (a) Scattering loss as function of wavelength for different normalized core diameter values (b) Confinement loss (c) total loss between 1 and 10dB/km and (d) minimum loss contributions and total loss as a function of normalized core diameter.
Having now established that a structure with equidistant node spacing on the core boundary is optimum for loss reduction, it becomes important to investigate to which extent introducing a core tube in such a structure may be detrimental to fibre properties. Assuming that canes can be produced in which each strut on the core boundary contains the same amount of glass and that no material flow will occur from between said struts in the second stage draw, the impact of the core wall thickness on the fibre properties is studied in two sets of fibres. Both sets of fibres have claddings with the structural parameters described in section 7.3.1. The first set of fibres have enlarged corner holes such as in Fibre 5 of Fig. 7.4 and the second features equidistant node spacing on the core boundary. The core wall thickness is then increased from \( t/2 \) where no core tube is present to \( t \) where a core tube as thick as a cladding capillary is used and finally \( 5t/4 \) where a slightly thicker core tube is incorporated. In other words, the normalized core wall thickness defined as

\[
T = \frac{\text{Core wall thickness}}{\text{cladding strut thickness}}
\]

(7.2)
is changed from \( 1/2 \) to \( 5/4 \).

Figure 7.7 shows the simulation results obtained for the first set of fibres in which the corner holes are enlarged, with the average fundamental mode loss and modal content plotted for each of them. When no core tube is incorporated, the fibre properties are similar to those of the fabricated fibre studied in section 6.5.1. The imposed \( C_{6v} \) symmetry maintains the degeneracy of the higher order mode groups. The bandgap is free of surface modes except for those near both edges and the minimum fundamental mode loss is \( 2.9\,\text{dB/km} \) over a \( 3\,\text{dB} \) bandwidth of \( 230\,\text{nm} \).

Doubling the core wall leads to the appearance of strut surface modes near the short edge of the photonic bandgap. This gives rise to the apparent step in the figure and is a mode primarily confined in the shorter segments of the core boundary. Though the fundamental mode loss increases to \( 4.6\,\text{dB/km} \) and the bandwidth narrows to \( 160\,\text{nm} \), the surface modes predominantly interact with higher order modes. It is interesting to compare these results to those obtained for fibres fabricated with core tubes in section 6.5.2. Even though neither the core defect nor the corner holes were as enlarged in those fibres, the surface modes in the middle of the bandgap are clearly due to the accumulation of glass in the shorter struts on the core boundary. It is therefore of utmost importance to devise
fabrication schemes which will allow the production of canes in which glass struts on the core boundary contain the same amount of glass.

Further increasing the core wall thickness has even more detrimental effects on the fibre performance. When $T = 5/4$, the surface modes now interact with the fundamental mode near the middle of the photonic bandgap. As a result, the bandwidth is severely reduced (100 nm) and the loss significantly increased (8 dB/km). Even more dramatic is the impact on higher order modes which have several crossing events with the different surface mode groups.

Figure 7.8 summarizes the findings for the second set of fibres in which the core nodes are equidistant. Here again, the loss is low and no surface modes except at the bandgap edges are present within the bandgap for a core wall half as thick
as the cladding struts. Interestingly, doubling the core wall thickness does not
tremendously affect too significantly the fundamental mode loss. This is due to
the surface mode introduced as a result of the increase in thickness crossing the
fundamental mode very close to the short wavelength edge of the bandgap. The
minimum loss is increased from $2$ to $2.4\, \text{dB/km}$ and the bandwidth reduced only
by $10\, \text{nm}$. This is an important finding as it indicates the possibility for fibres
made with core tubes to suffer very little loss and bandwidth penalty.

![Graphs showing the effect of increasing core wall thickness on the properties
of a fibre with equidistant core nodes spacing.](image)

Figure 7.8: The effect of increasing core wall thickness on the properties
of a fibre with equidistant core nodes spacing.

As before, further increasing the core wall thickness pushes surface modes to the
center of the bandgap, leading to further loss increase and bandwidth reduction.

In summary, the design in which the core nodes are equidistant and the core
slightly enlarged is more tolerant to increases in the core wall thickness. This is
an important conclusion as it indicates ultra-low loss and wide-bandwidth fibres may be produced with the inclusion of a core tube of adequate wall thickness in the preform.

7.4 Low-loss HC-PBGF designs

Without the intention of closing the debate on what is the lowest loss achievable in HC-PBGFs, it is appropriate in light of the findings presented in the previous sections and perhaps throughout this thesis, to attempt to assess what realistic loss values can be achieved in these fibres.

The main requirements in undertaking such an assessment is that the proposed fibre designs be amenable to fabrication using existing fabrication techniques and that this fabrication process will result in roughness and longitudinal distortions consistent with those in currently fabricated fibres. In other words, scattering loss is assumed to be correctly described with equation (6.1) with \( \eta = 300 \), the same value which we used in Chapter 6 to accurately estimate the loss in the current fibres. This is equivalent as discussed in section 6.4, to using the power spectral density of SCW roughness with a cut-off at \( 0.1 \mu m^{-1} \) and \( T_g/\gamma = 4400 K/J \cdot m^{-2} \).

Figure 7.9 shows simulated loss for four fibres with structural parameters identified as optimal. The first two operate around 1.55\( \mu m \), one with a nineteen cell core defect and the other with a thirty-seven cell one. The last two are similar fibres but scaled to operate around 2\( \mu m \). The cladding parameters in the first group of fibres are as follows: the average cladding strut thickness is \( t = 50 nm \) and the fillet radius is such that \( r_c/W = 0.3 \). The nineteen cell fibre core diameter is \( 2R_c = 28 \mu m \) and is expanded so that \( C = 1.235 \). With a core diameter of \( 2R_c = 41 \mu m \), the thirty seven cell fibre also has a normalized core diameter of \( C = 1.235 \). When scaled to operate at 2\( \mu m \), the strut thickness becomes \( t = 65 nm \) and the core diameters are 37 and 52\( \mu m \) respectively. It is assumed that the preforms can be assembled without a core tube and care taken so that the fibres have equidistant node spacing on the core boundary.

Table 7.1 summarizes the results for these different, optimum fibres. If no improvement is made to the current fabricated fibres, the minimum achievable loss at 1.55\( \mu m \) is of the order of 1.22\( dB/km \) for a nineteen cell fibre, dropping to
Chapter 7 Impact of structural distortions in HC-PBGFs

Figure 7.9: Simulated Loss properties for optimized fibre designs. Loss as a function of wavelength for 19c fibres is plotted in blue while the red curves correspond to 37c fibres. Scattering loss was calculated with eq. (6.1), though the rigorous scattering theory predicts lower loss values.

0.47dB/km if the core defect is enlarged to thirty-seven cell. When scaled to operate near 2µm, the loss values are reduced to 0.46 and 0.17dB/km respectively. This is very encouraging as these loss values are achieved with no penalty on the transmission bandwidth.

<table>
<thead>
<tr>
<th>λc</th>
<th>Core Size</th>
<th>min. Loss (η = 300)</th>
<th>Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.55µm</td>
<td>19c, C = 1.235</td>
<td>1.22</td>
<td>480nm</td>
</tr>
<tr>
<td></td>
<td>37c, C = 1.235</td>
<td>0.46</td>
<td>470nm</td>
</tr>
<tr>
<td>2.0µm</td>
<td>19c, C = 1.235</td>
<td>0.57</td>
<td>600nm</td>
</tr>
<tr>
<td></td>
<td>37c, C = 1.235</td>
<td>0.21</td>
<td>580nm</td>
</tr>
</tbody>
</table>

Table 7.1: Summary of loss and bandwidth properties of proposed HC-PBGF designs.

Shall the manufacturing process be significantly improved so that the normalizing coefficient η is significantly reduced (akin for example to reducing $T_o/\gamma$ in the SCW power spectral density), these loss values will be reduced accordingly. Such low loss values indicate the great potential of HC-PBGFs in telecom applications,
although as outlined, efforts in several aspects of fibre fabrication will have to be made for such low-loss fibres to become a reality.

7.5 Conclusion

In summary, this chapter has presented detailed simulations on the impact that structural distortions have on the loss performance of HC-PBGFs. A generic method for generating HC-PBGF geometries with arbitrary distortions in their cross section was devised and used to simulate fibres with distortions such as overexpanded or compressed core defects and enlarged air holes at the corners of the core defect.

Simulations on distorted profiles have confirmed here again, that scattering from surface roughness remains the dominant source of loss. As scattering occurs primarily within the first two rings of air holes, distortions beyond this region have minimal impact on the overall fibre loss, although they may lead to a penalty in the transmission bandwidth.

It was shown that enlarged ‘corner’ holes near the core defect resulting in an uneven spacing of the glass nodes on the core boundary prompts the guided mode field to overlap more prominently with scattering surfaces leading to much higher losses. We showed for example that fibre designs in which the core nodes were nearly equidistant had a 1.8 times lower loss than those with a similar core diameter, but with enlarged corner holes. This suggests that fibre designs in which the core nodes are more evenly spaced are preferable for loss reduction purposes.

Studying the impact of the core size, it was shown that when the core defect can be slightly enlarged so that it occupies a higher proportion of the fibre cross-section, the scattering loss contribution decreases while the leakage loss increases significantly. However, with a core defect 1.25 times larger than in idealised fibres, leakage can be maintained at very low values with the inclusion of six rings of air holes around the core, and the scattering loss decreases by a factor of 1.5, which is significant when reducing the loss to very low values.

The very practical issue of the inclusion of a core tube in the preform was then investigated and the results indicated that in fibres with enlarged cores and evenly spaced core nodes, a core tube as thick as the cladding capillaries results in very little loss and bandwidth penalty for the fundamental mode. The surface modes
introduced by the increase in core wall thickness cross the higher order modes near the middle of the bandgap and thereby are found to severely impacting their operational bandwidth. This could prove an effective method to control the number of modes effectively guided in the fibre.

Finally, combining all these findings, realistic fibre designs were proposed that will feature record low loss levels, with values as low as $0.17\,dB/km$ near $2\mu m$. As this is achieved over a bandwidth in excess of $500\,nm$, such a fibre would have great potential for optical communications.
Chapter 8

Conclusions

The body of research work reported in this dissertation was aligned with the objectives of the EU FP7 MODEGAP project. One of the key goals of this ambitious research endeavour targeted the development of HC-PBGFs with propagation losses comparable or lower than those of the conventional single mode silica fibre. Reaching such low-loss levels in fibres with optical nonlinearities as low as those of HC-PBGFs is imperative to complement SDM research activities and cope with the ever increasing demand for data transmission capacity. Although efforts aimed at fabricating HC-PBGFs with such low loss are still underway, this thesis has contributed to a deeper understanding of the physical phenomena that occur when light propagates in these fibres and how these are affected by their structural features. In the course of the project, the work presented herein has provided guidance to fabrication efforts which have led to landmark results such as the first demonstration of high capacity and ultra-low latency data transmission in a low-loss and wide bandwidth 19c HC-PBGF [88], and the subsequent demonstration of SDM over the first two mode groups of a 37c HC-PBGF [124], both presented as postdeadline papers at the annual Optical Fiber Communications Conference (OFC) in 2012 and 2013 respectively.

In summary, we have started by discussing in great detail in Chapter 2 the current understanding of photonic bandgap guidance in HC-PBGFs and described their attractive properties of low nonlinearity, low latency and their potential for low-loss. We then briefly described the challenges in modelling their properties and supported the choice of the finite element method as a versatile and accurate tool for simulating HC-PBGFs. Accurate modelling of these fibres not only provides
support for experimental activities, it also allows the understanding and realistic estimates for those fibre properties which are difficult to assess experimentally.

The finite element method was then used to analyse a large number of fibre designs according to an existing parametric description, and the results were presented in the form of optical property maps. These allow to quickly identify the region of the parameter space to target for optimal operation and also provide an initial assessment of the ultimate optical performances to be expected from fibres in which distortions are negligible. Analyzing the impact of the core defect, we have shown that while larger core sizes are beneficial for loss reduction, they also result in the fibres supporting an increasingly large number of modes. Furthermore, with large core defects, it is more challenging to ensure the structural uniformity and consistency of HC-PBGFs during the fibre draw. The impact of the core boundary was also investigated in the light of new understanding of the nature of surface modes.

Seeking to further understand light scattering from surface roughness which dominates the loss in HC-PBGFs, this thesis has proposed a theoretical description of the process based on dipole radiation. This approach leads to simple expressions which combine the roughness power spectral density with the overlap between the guided mode field and the air-glass interfaces to give the far-field scattering pattern and the loss. This theory replicates results from more complex treatments of the same problem in simpler cylindrically symmetric waveguide structures, and gives accurate loss predictions with reasonable assumptions on the roughness spectral density. This is further proof that loss in HC-PBGFs is already limited by this scattering phenomenon. The theory also provides greater insight into the wavelength dependence of the loss and should in principle be employed for loss predictions when comparing fibre designs. Unfortunately, the practical difficulties in measuring the roughness at low spatial frequencies has required that several assumptions be made on the roughness spectrum in our calculations. These include in particular imposing a low frequency cut-off on the roughness power spectral density. With improved measurements and better understanding of the roughness, our treatment should provide more accurate loss estimates for HC-PBGFs and other MOFs in general.

In the past, fibre fabricators have relied on transmission measurements during the fibre draw to obtain feedback and readjust draw parameters so that the fabricated fibres guided light at the desired wavelengths. In an attempt to circumvent this difficulty, this work proposes a simple model based on mass conservation which
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aims at predicting the node size and strut thickness in fibres made from second-stage preforms of known parameters. Combining the predictions of this model with the pre-calculated optical property maps, it is possible to predict both the position and width of the photonic bandgap with accuracy of 8 percent or better. Although this approach works well as far as these two quantities are concerned, it cannot predict other important properties of the fibre such as its loss or the presence or absence of surface modes which depend crucially on small structural details around the core defect. Despite its simplicity, this model has been repeatedly used by colleagues working on fibre fabrication to reliably predict the position and width of the bandgap. Furthermore, it establishes clear guidelines for the production of canes that may yield fibres with the widest bandgaps and potentially lowest losses.

As loss predictions using the property maps and the model above were significantly lower than the measured values, a detailed investigation of the impact of the exact structure of the fibres became necessary. This thesis therefore devised a novel method to accurately reproduce and simulate fibre geometries from scanning electron microscope images. The excellent agreement both in measured and calculated loss as well as surface mode positions for fibres made from different preforms confirms the validity of the approach and reveals that structural distortions, especially in the vicinity of the core defect are responsible for increasing the surface scattering loss.

Subsequently, the thesis presents a method to systematically study the most common types of structural distortions found in fabricated fibres. It is found that enlarged corner holes cause the guided mode-field to overlap more strongly with scattering surfaces and this leads us to conclude therefore that designs in which the nodes on the core boundary are kept equidistant should produce improved performance. It has also been shown that while an enlarged core defect as compared to the idealised case compresses the cladding and results in a modest narrowing of the photonic bandgap and an increase in confinement loss, it also helps reduce the scattering from surface roughness and it is therefore desirable in fabricated fibres when loss reduction is paramount.

Combining these findings on distorted fibre structures, the thesis proposes fibre designs with enlarged core defects and equidistant core nodes which are predicted to have losses of 1.2 and 0.5dB/km for 19c and 37c fibres at 1.55µm respectively, which are reduced to 0.45 and 0.17dB/km respectively at 2µm. These loss values are achieved over several hundreds nanometers of bandwidth and are estimated by assuming no reduction in the roughness present in currently produced fibres.
Future work

Although this work has laid the foundations for loss reduction in wide bandwidth HC-PBGFs, much remains to be done to make these fibres true competitors for next generation communication links. Above all is the task of fabricating fibres with the structures advocated herein. This is actively being pursued by my ORC colleagues and several ideas to modify the current fabrication process so that ultra-low loss geometries may be produced are being implemented. In support of these activities, a fluid dynamics model aimed at simulating the fibre draw and incorporating all the physical properties of the materials and fibre draw parameters is under development and shall provide helpful guidance in improving the fabrication process.

Moving forward, several theoretical studies will have to be undertaken on HC-PBGFs in order to further understand their properties and how these can be engineered for use in data transmission applications. First, it will be important to study the interaction between guided modes of the HC-PBGFs. The necessity of such a study arises since mode coupling has to be unraveled in MDM data transmission, and this appropriately starts with an accurate description of the coupling process. It is also crucial to investigate to which extent the rather large differential group delay and mode dependent loss as well as the chromatic dispersion in HC-PBGFs may be engineered by modifying the fibre structure. Another worthwhile undertaking would be to perform a thorough comparison between HC-PBGFs of different cladding lattice arrangements and to actively seek methods of fabricating these with the same quality as the current TLH fibres. There is some indication that both the square lattice and TLR have the potential to produce wider operational bandwidths [38, 65] and in principle, they should produce fibres with lower losses too.

On a less hopeful note, it might well be the case that inherent problems with fibre fabrication will take several years to overcome before producing structures with the ultra-low loss required for long-haul data transmission. With the current routinely achieved loss levels of a few $dB/km$ however, HC-PBGFs already have some potential in short-haul links and in applications where their low-latency and radiation hardness features are more important such as in data centers, high-speed trading applications or within high energy physics experiments.
Furthermore, our recent activity in applications other than data transmission has shown great promise. HC-PBGFs for operation at mid-IR wavelengths are interesting for sensing applications as many molecular species have their absorption bands in that spectral region. Compared to other fibres that may be used for the same purpose, HC-PBGFs hold the distinct advantage of suffering from very low bend losses, a feature important in making devices with very small footprints such as gas cells. Little is known yet however, as to why HC-PBGFs show such robustness to bends and an inquiry into this feature as well as purposefully designing fibres with highly suppressed absorption will be key to making mid-IR HC-PBGF-based sensing devices. Materials other than silica may also be of great interest and potential, especially since they may be more transparent than silica in the mid-IR. It remains to be seen however, if these can reliably be used to produce HC-PBGFs.
References


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List of Publications

Journal Papers (7)


N. V. Wheeler, A. M. Heidt, N. K. Baddela, E. Numkam Fokoua, J. R. Hayes, S. R. Sandoghchi, F. Poletti, M. N. Petrovich and D. J. Richardson,
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Conference proceedings (24)


F. Poletti, E. Numkam Fokoua, “Understanding the physical origin of surface modes and practical rules for their suppression,” ECOC 2013 London 22-26 Sep 2013 Tu.3A.4


