Direct monitoring of fiber extension by correlation-based coherent optical time-domain reflectometry
Justin P W Hayward, Stuart J Russell, John P Dakin
Optoelectronics Research Centre, University of Southampton, Southampton, SO17 1BJ, UK.

ABSTRACT
We propose a novel method of strain sensing where the displacement is measured by correlation of the coherent Rayleigh backscatter trace from a reference section of fiber. The potential performance is considered analytically and the theoretical results compared with the results of numerical simulations.
Keywords: Coherent Rayleigh scattering, OTDR, strain, correlation.

1. INTRODUCTION
The optical time-domain reflectometer (OTDR) has found widespread use in measuring loss and reflections in optical fiber since it was first demonstrated by Barnoski and Jensen. For strain and temperature monitoring, various OTDR sensor systems have been proposed and demonstrated using many different parameters of the backscattered light to interrogate the fiber, e.g. intensity in micro-bend sensors, polarization in polarization-OTDR and the optical phase in the numerous interferometric sensors.
Geiger and Dakin demonstrated a quasi-distributed strain sensor using semi-reflective splices to provide discrete reflections when interrogated by an OTDR with pseudo-random encoding. By measuring the relative range to these points with high-resolution, direct strain-measurement over many independent separate sections was possible. The spatial resolution was improved beyond that set by the pulse width by using a quasi-continuous delay and correlation to locate each semi-reflective point.
In this paper we propose a novel method of strain measurement exploiting the power of modern data acquisition and processing systems to use coherent Rayleigh scattering to provide a unique pattern which can be monitored by correlation, thereby allowing range measurement to resolution below the pulse-width limit. Potential applications include monitoring the installation of fiber cables to prevent damage due to excessive strain and structural monitoring of, for example, bridges, buildings, roads or railways.

2. COHERENT RAYLEIGH SCATTERING
Scattering occurs in optical fibers due to the microscopic fluctuations in the refractive index, frozen into the glass on cooling. A theoretical analysis of Rayleigh scattering in single-mode fiber was first given by Brinkmeyer and later generalized by Hartog and Gold. When the coherence length of the source approaches the pulse length, we have to consider interference effects in determining the characteristics of the backscatter trace. Consider launching a pulse generated by gating a perfectly coherent source. The E-field of a pulse of temporal width τ can be described, as a function of distance along the fiber, z, and time, t, as:

\[ E_{\text{forward}}(z,t) = E_0 \left( t - \frac{z}{v_g} \right) e^{-\alpha z} \ e^{i (\omega t - kz)} \]

\[ \epsilon(t) = \begin{cases} 1 & 0 \leq t \leq \tau \\ 0 & \text{All other } t \end{cases} \]

where \( v_g \) is the group velocity, \( k \) is the wave-number, \( \alpha \) is the attenuation coefficient, \( \omega \) is the optical angular frequency and \( \epsilon(t) \) is the pulse envelope. The light returned to \( z = 0 \) from a single scattering site, \( m \), with scattering coefficient \( R_m \) located at \( z_m \) is thus described by:

\[ E_m(t) = E_0 R_m \left( t - \frac{z_m}{v_g} \right) e^{-2\alpha z_m} e^{i (\omega t - 2kz_m)}. \]

Considering the case where there are two scattering sites located within the spatial extent of the pulse, the light returned

\[ ^1 \text{jphw@orc.soton.ac.uk} \]
to the start of the fiber is the sum of the two contributions. Since we are only interested in the times when light is received from both points (i.e., $e = 1$ in each case), assuming that the scattering coefficients are equal for each site we obtain:

$$E_{i,j}(t) = E_0 R \left( e^{-2 \alpha t} e^{i(\omega t - \alpha t_1)} + e^{-2 \alpha t} e^{i(\omega t - \alpha t_2)} \right)$$

The intensity $I(t)$ at the detector is therefore:

$$I(t) = \left| E_{i,j}(t) E_{i,j}^*(t) \right|^2 = 2E_0^2 R \left( e^{-4 \alpha t} + e^{-4 \alpha t} \cos[2k(z_2 - z_1)] \right).$$

As expected, the intensity is determined by the optical phase-separation, $2k(z_2 - z_1)$, of the two scattering sites.

In reality an optical pulse will interact simultaneously with a large number, $N$, of scattering sites and the resultant electric field will be given by the sum:

$$E_{\text{total}}(t) = \sum_{m=1}^{N} E_m R_m \left( e^{-2 \alpha t} e^{i(\omega t - \alpha t_m)} \right).$$

This argument tacitly assumes a single polarization state that gives perfect interference of light from each scatter site. In reality, where the polarization is not restricted, the presence of birefringence in the fiber and non-isotropic scattering can complicate matters; however, the general behavior is not affected.

3. PROPOSED SENSOR

The proposed sensor, shown in Figure 1, measures changes in the optical path length (OPL) of the gauge sections by correlating the initial coherent backscatter trace from each reference section with that from subsequent pulses, allowing changes in the OPL from the point of launch to each reference section to be measured. After correlating the sampled data, application of a suitable algorithm allows the change in OPL to be measured with sub-sample pitch resolution.

If the application of strain causes a significant change in the birefringence of the gauge sections, then the coherent backscatter pattern from the reference sections may be modified. In the case of small changes, the effect is to modify the relative intensity of peaks in the trace, without changing the overall structure of the pattern; in this case correlation should still be possible, albeit with a less well-defined peak. In the case of larger changes, where the character of the trace is altered beyond accurate correlation, two possible solutions may be adopted: polarization control may be used to ensure the state of polarization (SOP) entering each reference section remains unchanged; or, more elegantly, the launch SOP may be rotated over the whole surface of the Poincaré sphere and the resulting traces processed by “peak hold” or similar to define a polarization-independent backscatter characteristic of the reference. Clearly it is important that the reference section is sufficiently stable that its coherent backscatter pattern does not evolve significantly over the measurement period. Practical experience with coherent OTDR systems suggests that, without special measures, the trace remains stable for $-10 \text{ s}$; however, suitable mechanical and thermal controls should be capable of increasing this time significantly.

The potential resolution of such a sensor can be estimated by simple calculation. The feature size of the coherent backscatter trace will not be less than the limit set by the conventional OTDR spatial resolution, determined by the pulse width, so we shall assume, for the purposes of this estimate, that the intensity returned from a range $z$, $I(z)$ is:

![Figure 1: Layout of the proposed sensor.](image1)

![Figure 2: Sub-element estimation of peak location.](image2)
\[ I(x) = I_0 \sin \left( \frac{2\pi x}{\Lambda} \right) \]  

where \( \Lambda \) is the spatial period of the signal, approximated by the pulse length. Differentiating and taking the root mean square (RMS):

\[ \left( \frac{dI}{dx} \right)_{\text{rms}} = \frac{\sqrt{2\pi} I_0}{\Lambda} \]  

Now consider estimating the delay between traces by matching the intensity signal at a single point. The RMS error in range \( \delta z \) associated with an amplitude error \( \delta I \) can be approximated as:

\[ \delta z = \frac{\Lambda}{\sqrt{2\pi I_0}} \delta I \]  

Considering the improvement from carrying out the matching process at \( N \) points and rewriting in terms of the optical signal-to-noise power ratio, \( S = I_0 / \delta I \):

\[ \delta z = \frac{\Lambda}{\sqrt{2N\pi S}} \]  

As we would expect, improving the SNR, reducing the sample pitch and considering more points improves the resolution which can be achieved. Note, however, that this simple analysis is based on an idealized method of matching the reference and subsequent trace, and does not fully consider the nature of the correlation process, leading to the misleading result that a noiseless signal can be correlated with no uncertainty. Jacovitti & Scarano analyze the process of cross-correlation of discrete-time sampled data more fully from a statistical standpoint, deriving the standard deviation (SD) from the true value:

\[ \sigma_z = \sqrt{\frac{3 \Lambda_s^2}{\pi^2 N} \left( 1 + \frac{1+2S}{S^2} \right)} \]  

where \( \Lambda_s \) is the sampling pitch. For high SNR (\( S >> 10 \)) the SD approximates the asymptote:

\[ \sigma_z(S \to \infty) = \sqrt{\frac{3 \Lambda_s^2}{\pi^2 N}} \]  

representing the fundamental uncertainty due to discrete-time sampling. For a 1 km reference section with 2 m sample pitch (\( N = 500 \)) this suggests a standard deviation from the true extension of 49 mm. By comparison, the predicted resolution from (9), assuming a pulse width of 20 m and SNR of 20 dB, is only 2.0 mm.

To further investigate the performance that may be expected from such a system, a numerical simulation of a single-reference sensor has been written in C++. The simulation generates backscatter traces for a number of different strain profiles, the strain being equally applied over the whole gauge length and incremented each time. The fiber is divided into a large number (~\( 10^6 \)) elements, each of which is treated as a discrete scattering site having a random phase. A pulse propagation method is then used to generate the backscatter trace. After calculating the initial unstrained trace, for subsequent traces the strain-induced phase change between sites is calculated. This is then included in the backscatter calculation and the time-of-flight delay for each element due to the applied strain is output to file, along with the generated backscatter trace. The element-by-element traces are then sampled at fixed temporal intervals, using the time-of-flight delays and interpolation where required, to represent data acquired at fixed time intervals relative to the launch rather than at the fixed element locations, representing the operation of a discrete-time sampled OTDR. The initial reference section is then cross-correlated with the subsequent traces, using a simple estimation algorithm, described below, to measure the increase in OPL to sub-cell resolution.

Consider the sampled correlation function, \( \rho \), illustrated in Figure 2. Having located the peak of the sampled coherence function, \( \rho[j] \), the true peak location between samples \( j \) and \( j+1 \) can be estimated by assuming that the correlation peak is symmetrical and may be approximated as triangular with sides of slope \( k \), by inspection:
\[ k = \rho[j] - \rho[j - 1]; \ \rho[j] + k\Delta = \beta[j + 1] + (1 - \Delta)\beta \]  
(12)

Solving for the correction:

\[ \Delta = \frac{1}{2} \Lambda_s \frac{\rho[j + 1] - \rho[j - 1]}{\rho[j] - \rho[j - 1]}, \quad (\rho[j + 1] > \rho[j - 1]) \]  
(13)

Similarly, where the true peak lies before the peak element:

\[ \Delta = -\frac{1}{2} \Lambda_s \frac{\rho[j - 1] - \rho[j + 1]}{\rho[j] - \rho[j + 1]}, \quad (\rho[j + 1] < \rho[j - 1]) \]  
(14)

In order for the algorithm to locate the true peak, the correlation peak must contain at least 3 well-defined points with magnitude greater than the residual value of the correlation function. Using the simulated data, this condition was found to be satisfied whenever the sample rate is \( > 5 \) samples per pulse width and the reference length is \( > 100 \) samples.

Figure 3 shows the effect of the reference length on the standard deviation (SD) of the decoded extension relative to the applied value using a pulse-width of 20 m and a 2 m sample pitch. The gauge extension was applied in 0.1 m steps so as to give a maximum gauge-OPL increase of 2 m over 20 traces, equivalent to a peak extension of one sample pitch. The peak extension, equal to the sample pitch of 2 m, was chosen so that the SD could be calculated over the full range of the sub-sample pitch estimator to include any bias or sag in the estimation. As can be seen the simulated results relate well to the expected values at long reference lengths, but at shorter reference lengths there is poor agreement with a sudden transition to high SD. This is because the correlation peak is then no longer well defined; causing errors because the points either side of a peak may no longer exceed the residual noise of the correlation function. In the extreme the peak may be incorrectly located.

4. CONCLUSION

We have considered the predicted performance of a sensor which directly measures the change in OPL between the point of launch and any number of reference sections of fiber. We have shown that, where sufficient number of measurement samples are available, it is possible to improve on the conventional OTDR resolution by using correlation techniques and sub-element estimation. For example, with a 20 m interrogation pulse we have shown, by theory and simulation, that a gauge OPL followed by a 1 km reference section, sampled at 2 m pitch, can be located with an absolute accuracy \(< 0.1 \) m, independent of the actual gauge length. If the gauge length were to be a few km this corresponds to a strain sensitivity of a few 10s of micro-strain! Such a sensor could have many real-world applications provided the technical challenges of its practical implementation can be overcome.

5. REFERENCES