Hierarchical modulation (HM) is widely employed in the telecommunication industry, which may be beneficially invoked for upgrading diverse telecommunication services. The system using HM is capable of superimposing diverse new services layer by layer, while maintaining backward compatibility [1, 2] with the original base-line system. In this way, the original devices may still be supported by the upgraded broadcast system, whilst delivering new additional services. The performance of multi-layer HM schemes was characterised for example in [3–5]. The layered structure of HM ensures that the most important information can indeed be flawlessly received, while the less important layers may be discarded without undue degradation, in case of network congestion or when the signal-to-noise ratio (SNR) is low. This unequal-error-protection (UEP) capability of HM has drawn substantial research interests [6–9]. More specifically, the authors of [10–12] invoked a HM scheme for providing UEP for image encoding, where the information bits are mapped to specific protection layers according to their error-sensitivity-based priority. Moreover, HM has also been combined with sophisticated channel coding schemes in [11, 12] for the sake of protecting the most important information. The simulation results of [11, 12] have shown that receiving the information having the highest priority requires a lower receive SNR than that of the conventional modulation schemes at a given target bit-error-ratio (BER) performance. However, the improved performance of the high-priority layers achieved by the UEP scheme inevitably leads to a degraded performance for the lower-priority layer. Hence, the SNR required for receiving the whole hierarchical modulated signal may be higher than that of the conventional modulation schemes, especially in the absence of relaying.

When evaluating the performance of a system, typically, the BER performance is the salient metric [1–4, 9]. The Gaussian $Q$-function and the Euclidean distances among the constellation points may be used for quantifying the lower or upper bound of the system. However, when considering the spatial diversity benefits of cooperative communications or the time-diversity gain of channel coding schemes, the $Q$-function is not applicable. First, the relay node (RN) would lend a path gain to the entire system. Second, as a benefit of channel coding, the BER performance will no longer be solely decided by the Euclidean distance of the constellation maps. Hence, characterising cooperative communication systems relying on HM and channel coding becomes more of a challenge.

Against this background, we proposed a cooperative communication system, which is assisted by a pair of decode-and-forward (DF) RNS intrinsically amalgamated with triple-layer HM. The design goal of our system is to reduce the power consumption of the entire cooperative network, while guaranteeing that all benefits of HM are retained, and additionally ensuring that all the associated transmission links have the same performance. To calculate the lower performance bound of the system, the coding scheme we employed is assumed to be a 'perfect' code. More specifically, the relevant discrete-input continuous-output memoryless channel (DCMC) capacity is used for analysing the cooperative system, relying on the optimised RN positions.

The main contributions of this paper are as follows:

- A triple-layer HM-64QAM scheme is designed for DF cooperative communications.
- The DCMC capacity is derived for characterising the achievable performance (the minimum received SNR required for achieving the target capacity) of the individual HM layers, when assuming that a 'perfect' channel coding scheme is employed.
- On the basis of our DCMC capacity analysis, a coded triple-layer HM scheme-based cooperative communication system is proposed. More explicitly, both the HM constellations and the positions of the RNs are taken into consideration, when deriving the lowest possible power consumption of the entire system.

The organisation of the paper is as follows. Section 2 introduces the system model. The specific HM-64QAM mapping rule designed for cooperative communication is detailed in Section 3. In Section 4, both the DCMC capacity and our optimisation strategy are characterised, while our performance results are discussed in Section 5. Our conclusions and future research ideas are provided in Section 6.

## 2 System model

Our HM-aided DF RN-based cooperative communication system is illustrated in Fig. 1, where the link is considered to be an uncorrelated Rayleigh fading channel, and the entire system benefits from 100% channel state information. During the first transmission time slot (TS), the source node (SN) will broadcast a
sequence of HM symbols \{x_1\} to both RN_1 and RN_2 as well as to destination node (DN). In the following two TSs, RN_1 will transmit a signal frame \{x_2\} to the DN and another signal frame \{x_3\} will be sent to the DN by the RN_2. Again, the entire system would require three TSs for conveying the triple-layer HM-64QAM-based signal frame \{x_1\} to DN.

When considering the reduced path loss introduced by the RN, in order to simplify the system model and the related discussions, we employ the simplified path-loss model of [13] and set the path-loss exponent to 3, which is usually used in the simulation of urban Q2 areas. Then, the reduced path loss of the two SR links is

\[ G_{SR} = \left( \frac{d_{SD}}{d_{SR}} \right)^3, \]  

and similarly, the reduced path loss of the two RD links are

\[ G_{RD} = \left( \frac{d_{SD}}{d_{RD}} \right)^3. \]  

The path loss between the SN and DN is normalised to be 0 dB. If we set the transmit power to \( p_t \), the receive power to \( p_r \) and the additive white Gaussian noise (AWGN) noise power to \( N_0 \), the transmit SNR \( \text{SNR}_t \) (The definition of transmit SNR (SNR) was proposed in [14], which is convenient for simplifying the discussions, although this is not a physically measurable quantity, because it relates the power at the transmitter to the noise at the receivers.) will be \( \frac{p_t}{N_0} \) and the receive SNR (SNR) will be \( \frac{p_r}{N_0} \). Since we assume that \( \text{SNR}_r \) and \( \text{SNR}_t \) share the same noise power \( N_0 \), \( \text{SNR}_r \) and \( \text{SNR}_t \) may indicate the strength of the transmit power and of the receive power. Note that by adding the path loss between the SN–DN link and considering a realistic noise power, our system model may be directly converted into a realistic wireless communication system model [15]. If the transmissions between the SN and DN take place on a frame-by-frame basis over an uncorrelated Rayleigh fading channel, the average received SNR at DN (SNR_{DN}) may be formulated as

\[ \text{SNR}_{DN} = E(|h|^2)\text{SNR}_r = E(|h|^2)\text{SNR}_t^2, \]  

where the \( \text{SNR}_t^2 \) may be expressed as

\[ \text{SNR}_t^2 = \frac{E(|x|^2)}{N_0} = \frac{1}{N_0}, \]  

where \( E(|x|^2) = 1 \). Furthermore, since the uncorrelated Rayleigh fading parameter \( h \) is generated by the complex-valued Gaussian distribution with a zero mean and a unit variance, when the number of uncorrelated Rayleigh fading components we generated is large, we have [16, 17]

\[ E(|h|^2) = \frac{\exp(-|h|^2)}{\gamma} \approx 1. \]  

Note that the distribution of \( |h|^2 \) obeys \( f(|h|^2) = \exp(-|h|^2) \), as detailed in [17]. Hence, for a large frame size of \( N \) symbols, we may assume that \( \text{SNR}_{DN} \) is equal to \( \frac{p_t}{N_0} \), or equivalently that we have \( \text{SNR}_{DN} = \text{SNR}_t^2 \).

The block diagram of our cooperative communication system is illustrated in Fig. 2. Three rate-1/2 encoders are employed by the SN, and the outputs \( c_1, c_2, c_3 \) of the three encoders may be merged together to form HM-64QAM signals. We stipulate that \( c_1 \) is in the base layer \( (L_1) \) which is the layer with the highest level of protection, \( c_2 \) is in the second layer \( (L_2) \) and \( c_3 \) is in the third layer \( (L_3) \), which has the lowest protection level. The SNR_{SN} is assumed to be only sufficient for the DN to receive \( L_1 \), where \( L_2 \) and \( L_3 \) will be forwarded to the DN separately by the two RNs. Therefore, the entire system may require three TSs to complete the transmissions. Note that in order to successfully receive \( L_2 \) and \( L_3 \) at the two RNs, the reduced path loss achieved by the two RNs should be sufficiently high.

### 3 Triple-layer HM modulation

Our triple-layer HM-64QAM constellations seen in Fig. 3 was originally introduced in [18] and detailed in [17]. Our system proposed in [17] is assisted by turbo-trellis coded modulation,
where set-partition-based bit-to-symbol mapping would give a better performance compared with Gray mapping. We also use set-partitioning-based mapping in this paper. Note, however, that the type of mapping used does not affect the DCMC capacity, as shown in Fig. 3.

We denote the six bits of a HM-64QAM symbol as $(b_5 b_4 b_3 b_2 b_1 b_0)$, where $L_1$ is occupied by $(b_5 b_4)$, while $(b_3 b_2)$ belong to $L_2$ and $(b_1 b_0)$ are contained in $L_3$. The generation rule of the triple-layer HM-64QAM symbols may be expressed as

$$S_{HM-64QAM} = S_{4QAM} + \sqrt{d_1}e^{\pm j\theta_1} + \sqrt{d_2}e^{\pm j\theta_2}.$$  

(6)

A HM ratio pair of $(R_1 = d_1/d_0, R_2 = d_3/d_2)$ is defined along with the control parameters $\beta$, $\delta_1$ and $\delta_2$. The relationships among these parameters are

$$\beta = 1/\sqrt{1 + 2\delta_1^2 + 2\delta_2^2}. $$

(7)

$$\delta_1 = \frac{1}{\sqrt{2(1 + R_1)}},$$

(8)

$$\delta_2 = \frac{R_1 - R_2}{\sqrt{2(1 + R_1)(1 + R_2)}}.$$  

(9)

The HM ratio pair is used for controlling the formation of the constellation map and the restrictions imposed on the HM ratio pair are

$$\begin{cases} 
0 < R_2 < R_1 & \text{if } R_1 < 1 \\
\frac{1}{2}(R_1 - 1) < R_2 < R_1 & \text{if } R_1 > 1.
\end{cases}$$

(10)

The related derivations of $\delta_1$, $\delta_2$ as well as the restrictions of $R_1$ and $R_2$ are detailed in [17]. In the simulations, different HM ratio pairs will be tested and we will optimise the average SNR ($\text{SNR}_t$) of the system based both on the HM ratio pair $(R_1, R_2)$ and on the positions of the two RNs.
4 DCMC capacity-based system analysis

On the basis of the capacity bounds of the full-duplex relay channels proposed in [19], the general upper bound on the continuous-output memoryless channel capacity of a half-duplex relaying system has been investigated in [20]. By contrast, the DCMC capacity was detailed in [21], which is more pertinent for the design of channel-coded modulation. The DCMC capacity bounds of a practical half-duplex relaying system were further investigated in [22, 23]. In this paper, the DCMC capacity will be used to calculate the bound of our HM-aided cooperative communication system, as well as the minimum receive SNR required for receiving $L_2$, $L_3$, $L_4$ from the triple-layer HM-64QAM symbols, where assuming that a 'perfect' channel code is employed.

The input to the DCMC channel is $X = \{x_0, x_1, \ldots, x_M-1\}$, where $M$ is the constellation size and $x_i$ is the complex-valued modulated symbol. The corresponding output symbols are $Y = \{y_0, y_1, \ldots, y_M-1\}$. The transition probability of receiving $y$ given that $x_k$ is transmitted is expressed as [21]

$$p(y|x_k) = \frac{1}{\pi N_0} \exp \left( -\frac{|y - h x_k|^2}{N_0} \right),$$  \hfill (11)

where we have

$$p(y) = \sum_{k=0}^{M-1} p(y|x_k)p(x_k).$$  \hfill (12)

The mutual information of receiving $y$ when $x_i$ is transmitted is given by

$$I(X; Y) = \sum_{i=0}^{M-1} \log \left( \frac{p(y|x_i)p(x_i)}{p(y|x_j)p(x_j)} \right) dy,$$  \hfill (13)

So the DCMC capacity [Some authors refer to this modulation-dependent DCMC capacity as the achievable rate.] $C$ can be formulated as

$$C_{\text{DCMC}} = \max_{p(x)} I(X; Y),$$  \hfill (14)

where ML stands for maximum likelihood, $I(X; Y)$ is maximised when we have $p(x_i) = 1/M$ $(i \in \{0: M-1\})$ and (14) may be simplified as [21]

$$C_{\text{DCMC}} = \log_2(M) - \frac{1}{M} \sum_{k=0}^{M-1} E \left[ \log_2 \left( \frac{p(y|x_k)}{p(y|x_{k+1})} \right) \right],$$  \hfill (15)

where the unit of $C$ is bits per symbol (bps). $E[|x_i|]$ is the expectation of $A$ conditioned on $x_i$, whereas the term $\Phi_{i,k}$ may expressed similar to that in [21]

$$\Phi_{i,k} = -\sqrt{Gh} (x_i - x_k)^2 + |n|^2.$$  \hfill (16)

where $h$ is the fading coefficient, $G$ is the path gain and $n$ is the AWGN at the receiver.

4.1 Channel capacity of the SN-DN link

The detection rules of the triple-layer HM-64QAM symbols are detailed in [17]. Note that the assumption of the system is that SNR$^SN$ is set to be relatively low so that the SNR may or may not be capable of receiving $L_1$. Hence, even though the signal broadcast by the SN during the first TS is a HM-64QAM symbol, the DN may treat it as those 4QAM symbols, where the soft information derived by the De-Mapper at the DN may be expressed as

$$p(y_{SD}\mid \lambda_{SD}) = p(y_{SD}\mid \nu_{SD}) = \frac{1}{\pi N_0} \exp \left( -\frac{|y_{SD} - G_{SD}\nu_{SD}|^2}{N_0} \right),$$

$$\lambda_{SD} = \begin{bmatrix} \beta e^{j\pi/4}, \beta e^{j3\pi/4}, \beta e^{-j\pi/4}, \beta e^{-j3\pi/4} \end{bmatrix},$$  \hfill (17)

where we have $i \in \{0, 1, 2, 3\}$ and $q \in \{1, 2, \ldots, q\}$, $q$ is the order of the symbol in the received signal frame, while $\eta$ is the block size of the soft decoder. The $q \times 4$-element soft information matrix $p(y_{SD}\mid \nu_{SD})$ may then be sent to decoder 1 for detecting the information contained in $L_1$. Therefore, the layer $L_1$ may be received by the DN, while the information in $L_2$ and $L_3$ may be discarded because of having an insufficient receive SNR. Upon substituting (17) into (13), the DCMC capacity of only receiving $L_1$ from the coded HM-64QAM signal at DN may be expressed as

$$C_{\text{HM-64QAM}} = -\frac{1}{4} \sum_{i=0}^{4} E \left[ \log_2 \left( \frac{p(y|x_i)}{p(y|x_{i+1})} \right) \right].$$  \hfill (18)

4.2 Channel capacity of the SN-RN$_i$ link

For decoding the information contained in layer $L_2$ of the HM-64QAM symbols, RN$_i$ would detect the signal frame $\{x_i\}$ as HM-16QAM symbols, which is formulated as

$$p(y_{SR}\mid \nu_{SR}) = \frac{1}{\pi N_0} \exp \left( -\frac{|y_{SR} - G_{SR}\nu_{SR}|^2}{N_0} \right),$$

$$\lambda_{SR} = \begin{bmatrix} \beta e^{j\pi/4}, \beta e^{j3\pi/4}, \beta e^{-j\pi/4}, \beta e^{-j3\pi/4} \end{bmatrix},$$  \hfill (19)

where we have $i \in \{0, 1, \ldots, 15\}$, and then the log-likelihood ratio (LLR) converter 1 (as shown in Fig. 2) will calculate the soft information of $L_2$ from the HM-64QAM symbols we received, which may be expressed as

$$p(y_{SR}\mid \lambda_{SR}) = p(y_{SR}\mid \nu_{SR}) + p(y_{SR}\mid \nu_{SR}^{16}) + p(y_{SR}\mid \nu_{SR}^{12}),$$

In (20), we defined $L_{15}^{(1)}$ as the pair of bits (00) in $L_2$, $L_{15}^{(2)}$ as (01), $L_{15}^{(3)}$ (10) and finally $L_{15}^{(4)}$ for (11). Then, RN$_i$ may decode the layer $L_2$ according to the $(\eta \times 4)$-element soft information matrix $p(y_{SR}\mid \lambda_{SR})$. Since RN$_i$ demaps the HM-64QAM symbol as a 16QAM signal, the DCMC capacity that we can calculate based on the output of the HM-16QAM De-Mapper block (as seen in Fig. 2) is the joint capacity of the two independent layers, namely of $L_1$ and $L_2$. The DCMC capacity of receiving $L_1$ and $L_2$ of HM-64QAM may be expressed as

$$C_{\text{HM-64QAM}} = 4 - \frac{1}{16} \sum_{i=0}^{15} E \left[ \log_2 \left( \frac{p(y|x_i)}{p(y|x_{i+1})} \right) \right],$$  \hfill (21)

where we have
We assume that the pair of bits contained in $L_1$ is $(b_3, b_4)$ of Fig. 3, while $(b_5, b_6)$ belong to $L_2$. Then, based on the chain rule of mutual information [24, 25], we arrive at

$$I(b_3, b_4, b_5, b_6 ; y) = I(b_3, b_4 ; y) + I(b_5, b_6 ; y | b_3, b_4).$$

(22)

Therefore, it can be stated that

$$I(b_3, b_4 ; y | b_5, b_6) = I(b_5, b_6 ; y | b_3, b_4),$$

(23)

where we have $C_{L_1,L_2}^{HM-64QAM} = \max \{I(b_3, b_4 ; b_5, b_6), I(b_3, b_4 ; y | b_5, b_6)\}$, which is the DCMC capacity of receiving both $L_1$ and $L_2$. Furthermore, we have $C_{L_1}^{HM-64QAM} = \max \{I(b_3, b_4 ; y)\}$, which is the DCMC capacity of receiving $L_1$ from the triple-layer HM-64QAM signal. When considering $I(b_3, b_4 ; b_5, b_6)$, we found that the reception of $L_2$ will not be totally independent of the information contained in $L_1$. The DCMC capacity of receiving $L_2$ will only be approached when $L_1$ is perfectly received. Hence we may define the DCMC capacity of receiving $L_2$ to be $C_{L_2}^{HM-64QAM} = \max \{I(b_3, b_4 ; y | b_5, b_6)\}$, where the DCMC capacity of the layer $L_2$ is given by

$$C_{L_2}^{HM-64QAM} = C_{L_1,L_2}^{HM-64QAM} - C_{L_1}^{HM-64QAM}.$$  

(24)

### 4.3 Channel capacity of the SN–RN$_2$ link

The RN$_2$ will retransmit the information of $L_1$ of the HM signal and the output of the HM-64QAM De-Mapper block in Fig. 2 will be a $(q \times 64)$-element soft information matrices

$$p(x^{(q)}_y) = \frac{1}{\pi N_0} \exp \left( - \frac{x^{(q)}_y - \sqrt{G_{SR}\frac{H_{SR}B_{SR}}{N_0}}}{N_0} \right)$$

(25)

where $i \in \{0, 1, \ldots, 63\}$. Let $L^{(i)}_3$ denote the pair of bits (00) in $L_3$, $L^{(1)}_3$ represent (01), $L^{(10)}_3$ (10) and finally $L^{(11)}_3$ for (11). Then the LLR converter 2 of Fig. 2 may produce the soft information matrix for decoding $L_2$

$$p(y^{(q)}_{SR}) = \frac{1}{15} \sum_{k=0}^{15} p(y^{(q)}_{SR})^{(4k+i)}.$$  

(26)

Since the RN$_2$ has to fully demap the HM-64QAM signal, the DCMC capacity of transmitting the HM-64QAM symbols may be expressed as

$$C_{HM-64QAM} = \frac{6}{64} \sum_{i=0}^{63} E \left[ \log_2 \sum_{k=0}^{63} \exp(\Phi_i)^{(q)} \right].$$  

(27)

where we have

$$\Phi_i = \left[ B_{SR}^{(q)} \pm \sqrt{2} \delta \epsilon^{\pm(i/4)} \right]$$

and $B$ is the normalisation parameter of the HM-64QAM symbols based on the current HM ratio. The pair of bits contained in the layers $L_1 = (b_3, b_4)$, $L_2 = (b_5, b_6)$ and $L_3 = (b_1, b_2)$ may be expressed according to the chain rule of mutual information [24, 25] as

$$R(b_1, b_2, b_3, b_4, b_5, b_6) = R(b_1, b_2 ; b_3, b_4 ; y) + R(b_1, b_2 ; y | b_3, b_4),$$

(28)

where we have $C_{HM-64QAM} = \max \{I(b_3, b_4 ; y | b_5, b_6)\}$, hence the DCMC capacity of receiving $L_3$ is $C_{HM-64QAM} = \max \{I(b_1, b_2 ; y | b_3, b_4)\}$, which is given by

$$C_{L_3}^{HM-64QAM} = C_{HM-64QAM} - C_{L_2}^{HM-64QAM}. $$  

(29)

### 4.4 Overall system optimisation

In our simulations, the same rate-1/2 encoder is employed by all three SN encoders. Hence we only focus our attention on the specific SNR values, where the DCMC capacity reaches 1 bps. Multiple values of the two HM ratios $R_1$ and $R_2$ had been tested. At a given HM ratio pair $(R_1, R_2)$, the minimum receive SNR required $SNR_{L_1}$ for decoding the $L_1$ at the RN, $SNR_{L_2}$ of $L_2$ at the RN$_1$ and $SNR_{L_3}$ of $L_3$ at RN$_2$ may be computed. The SNR differences among the three layers are

$$G_{SR1} = SNR_{L_1} - SNR_{L_2}$$

(30)

$$G_{SR2} = SNR_{L_2} - SNR_{L_3}$$

(31)

where $SNR_{SR_j}$ is the SNR difference between $SNR_{L_j}$ and $SNR_{L_j}$ for $j \in \{2, 3\}$. If we set $SNR_{SR_j}$ to be identical to the SNR required for receiving the information of $L_j$, namely to $SNR_{SNR_j} = SNR_{L_j}$, this would guarantee that the BER of decoding $L_j$ would reach an arbitrarily low value. In this situation, if we want the BER performance of receiving $L_2$ to become sufficiently low, the channel gain $G_{SR_1}$ of the SN–RN$_1$ link should satisfy

$$10 \log_{10} G_{SR_1} + SNR_{L_2} = SNR_{L_2}.$$  

(32)

If we use the distance ratio $d_{SR_1} / d_{SD}$ to represent the position of the RN, we arrive at

$$SNR_{SR_1} = 10 \log_{10} \left( \frac{d_{SR_1}}{d_{SD}} \right)^3,$$  

(33)

where $SNR_{SR_1}$ is given by (30) and hence we have

$$d_{SR_1} = \sqrt[3]{\frac{G_{SR_1}}{G_{SR_2}} d_{SD}}.$$  

(34)

Once the position of the RN is available, the path gain between the RN and DN link can be formulated as

$$G_{R_1,D} = \left( \frac{1 - d_{SR_1}}{d_{SD}} \right)^3.$$  

(35)

In the capacity analysis, we observe from (15) that a system employing a rate-1/2 channel coding scheme and 4QAM modulation for communication over uncorrelated Rayleigh fading channels requires $SNR_{r} = 1.81$ dB to reach a DCMC capacity of 1 bit per symbol. Hence the SNR of RN$_1$ $(SNR_{SNR_{R1}})$ to satisfy

$$SNR_{SNR_{R1}} = 1.81 - 10 \log \left( \frac{G_{R_1,D}}{d_{SD}} \right).$$  

(36)

Similarly, the SNR of RN$_2$ $(SNR_{SNR_{R2}})$ may be formulated as

$$SNR_{SNR_{R2}} = 1.81 - 10 \log G_{R_2,D},$$  

(37)

while the position of RN$_2$ is related to

$$d_{SR_2} = \sqrt[3]{\frac{G_{SR_2}}{G_{SR_1}} d_{SD}}.$$  

(38)

(38)
the triple-layer HM-64QAM symbol is expressed by (24) [or (29)] and Fig. 5 was generated. The DCMC capacity of receiving only the HM-64QAM symbol streams. HM-64QAM signal is determined by (21), while (27) formulates detection of $R_1$, shown in Fig. 5, where we can observe that the optimum position of RN1 was formulated to be 3. This illustrates that with the aid of a sufficient path gain, the detection of $L_2$ at RN3 and the detection of $L_1$ at RN2 are achieved together with the detection of $L_1$ at the DN for the same $SNR_{t}$ value of $-0.71$ dB. In order to find the optimum HM-64QAM ratio pair, multiple groups of ($R_1$, $R_2$) have been investigated in the same way and Fig. 5 was generated.

The resultant three-dimensional (3D) $SNR_{t}$ versus ($R_1$, $R_2$) plot is shown in Fig. 5, where we can observe that the optimum HM-64QAM ratio pair for our cooperative system is ($R_1 = 1.5$, $R_2 = 0.6$) and the minimum $SNR_{t}$ of $-0.71$ dB is considered to be the lower performance bound of our cooperative communication system. Hence, the lower performance bound of our cooperative communication system shown in Fig. 2 is $SNR_{t}$, while the throughputs of the system is 1 bps.

As mentioned in Section 4.4, for a single link system assisted by a rate-1/2 'perfect' channel coding scheme using conventional 4QAM mapping, which has the same throughput as our optimised system, the $SNR_{t}$ required for achieving the DCMC capacity of 1 bps is about 1.81 dB. This means that in order to transmit three independent symbol frames, the system requires three TSs and the $SNR_{t}$ necessitated is 1.81 dB, which is $1.81 + 0.71 = 2.52$ dB higher than that of our optimised and idealised cooperative communication system operating at the DCMC capacity. Meanwhile, Fig. 6 makes a comparison between our optimised triple-layer HM-64QAM scheme and the conventional 16QAM scheme. It can be observed that, if a rate-3/4 'perfect' channel coding scheme is employed by the conventional 16QAM, the required receive SNR for receiving the three information bits in the coded 16QAM symbol would be 12.35 dB. While, the required receive SNR for receiving the information bit contained in the coded 16QAM symbol are 2.64, 11.60 and 18.35 dB, respectively. Therefore, it can be observed that, in order to receive the total three-layer information in our optimised HM scheme, the required receive $SNR_{t}$ is about 18.35 – 12.35 = 6.0 dB higher than that of the coded conventional 16QAM scheme, which is considered to be the main drawback of the HM scheme. As a remedy, cooperative communications may

\begin{align}
SNR_{t} = SNR_{t}^{1}. &
\end{align}
Fig. 6 DCMC capacity against SNR of both our optimised triple-layer HM scheme and of the conventional 16QAM scheme. The HM-64QAM ratio pair is \((R_1 = 1.5, R_2 = 0.6)\). The simulations are based on (17)–(29), and the number of samples used for calculating the DCMC capacity is 100, 000. The channel is an uncorrelated Rayleigh fading channel.

be employed and we found that the transmit SNR, for the SN may be reduced to 2.64 dB. More explicitly, due to the reduced path loss, the two RNs may be able to receive \(L_2\) and \(L_1\) correctly and separately with a reduced transmission power at the SN. Furthermore, each RN only needs to forward a single layer of information to the DN, which further helps to reduce the required SNR at the two RNs. Hence, by employing the HM in cooperative communications, we may be able to reduce the SNR at each node in the cooperative networks. However, the improved power efficiency will lead to a decreased time efficiency, note that our optimised HM scheme requires three TSs to convey three information bits. By contrast, if the transmit SNR is higher enough, it only needs a single TS for the rate-3/4 encoder-aided conventional 16QAM scheme to deliver the same amount of information.

6 Conclusions

A HM-aided cooperative communication system was proposed in this paper. The DCMC capacity of our specifically designed HM schemes has been formulated and the DCMC capacity of each individual layer of our HM-64QAM scheme was also derived. Theoretically, if the system relies on a ‘perfect’ rate-1/2 channel coding scheme, our communication strategy becomes capable of reducing the SNR of the entire system investigated in Section 5 to \(-0.71\) dB, while the required SNR may be set to 2.64 dB. The results showed in this paper are mainly based on simulations. The theoretical analysis of the coded HM in cooperative communication may be considered in future work, although simplifying assumptions may be required for making the problem analytically tractable. On the basis of this DCMC capacity analysis, near-capacity HM may be designed for cooperative communication in our future research.

7 References

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