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Essays on Labour Economics: The Case of Youth Unemployment

by

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ABSTRACT

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The integration of young people into the labour market should be an important objective all over the world. Economies generally have high youth unemployment compared to adult unemployment. Chapter 2 of this research replicates the annual pattern of unemployment among young people between 18 and 25 years old in USA using a two-sided direct search model, with heterogeneous agents whose that endogenously evolve between periods. Higher youth unemployment is jointly explained in this model by the fact that young people enter the labour market as unemployed, and also because a competition factor in which young people face a labour market inhabited by more skilled workers. Since direct search models are essentially efficient, the pattern of unemployment among young workers is a socially optimal equilibrium allocation. This research finds that policies, such as Active Labour Market Policies (ALMP), do not make people better off. In chapter 3 this research focuses on the theoretical effects of ALMPs on recruitment of young people, such as subsidies for training and hiring. To do this, this research uses the framework provided by search theory, in the context of heterogeneous agents with bias productivity. Following Shi (2002) and Shimer (2005), this research use direct search in the sense that firms first post wages and then workers subsequently apply for jobs. This study finds that active policies help reduce youth unemployment. However, there is a substitution effect of such policies on unskilled older workers. It should be mentioned that the equilibrium in this framework seems to be unstable and is influenced by the initial conditions assumed in the model which in turn limit the robustness of conclusions on the consequences of such policies. Finally Chapter 4 attempts to understand the factors that explain the evolution of unemployment rates for young workers in the United States at different levels of aggregation and its relationship with the business cycle. Using a dynamic model of business cycles, this research intends to elucidate the impact that hiring and separation flows have on the US unemployment variability. Results suggest a high heterogeneity in the inflow an outflow hazard rates when controlling by age and gender. This high heterogeneity compels us to avoid generalizations regarding the effect of different labour policies such as recruiting and unemployment subsidies, among on youth unemployment. Moreover, results suggest that finding rates seems to be the main factor in the youth unemployment dynamic, and therefore a crucial factor to be considered in the policy-making in youth unemployment.

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Abbreviations

ALMP: Active Labour Market Policies.

BLS: Bureau of Labor Statistics.

CES: Current Employment Statistics.

CMAR: Conditional and Missing at Random.

CPS: Current Population Survey.

GDP: Gross Domestic Product.

ILO: International Labour Organization.

JFR: Job Finding Rates.

JOLTS: Job Openings and Labor Turnover Survey.

LABSTAT: Bureau of Labor Statistics Data.

LRD: Longitudinal Research Datafile.

MAR: Missing at Random.

MIS: Month in the Sample.

NAICS: North American Industrial Classification System.

NBER: National Bureau of Economic Research.

OECD: Organisation for Economic Co-operation and Development.

PLMP: Passive Labour Market Policies.

SIPP: Income Program Participation.

SR: Separation Rate.

UI: Unemployment Insurance.

US: The United States of America.

UK: United Kingdom.

Declaration of Authorship

I, Marcos E. Gómez M., declare that the thesis entitled *Essays on Labour Economics: The Case of Youth Unemployment* and the work presented in the thesis are both my own, and have been generated by me as the result of my own original research.

I confirm that:

1. This work was done wholly or mainly while in candidature for a research degree at this University;

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5. I have acknowledged all main sources of help;

6. Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;

7. Either none of this work has been published before submission, or parts of this work have been published as:

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Date: ....

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1. Introduction

High unemployment amongst young people and their problems both to enter the labour market and to find stable jobs has been the subject of wide interest to economists, policy makers and international agencies. As their high unemployment rates compared to other age groups have been a worldwide persistent fact in most economies in the last forty years. In fact, young people who are just entering the labour market exhibit, in contrast to adult workers, high labour turnover, higher probability of being unemployed and more sensitivity to economic fluctuations. Moreover, attention to the problem of youth unemployment has increased after the 2008-2009 recession, as this episode generated a high cost in terms of employment - youth and adult - in developed economies.

Much debate about the causes for the high rate of youth unemployment and its possible solutions, have generated a rich literature in which multiple hypotheses either competitive or complementary have emerged to explain this phenomenon. Still, doubts and disputes still remain; studies on the causes of youth unemployment differ as their conclusions are generally influenced by the country samples used, the time period analysis, the variables of interest included in the analysis, the statistical and econometric techniques used, and the base assumptions used in the models.

However, certain consensus regarding the youth job scarcity problem strongly emerged in the economic literature in the United States during the 1970s (see Bell and Blanchflower, 2011 and Artner, 2013). This was due to both the economic crisis and the massive influx of baby-boomers into the labour market.

Clark and Summers (1978) and Freeman (1979) demonstrated that for the American case the weaker labour market behaved, the higher the youth employment was. Using data from the North American economy, Clark and Summers (1982) confirmed this view by showing that youth unemployment and participation strongly responded to macroeconomic conditions, and both - unemployment and participation - had a strong cyclical component. Lynch and Richardson (1982) confirmed these findings for the English case.

Freeman (1979), Welch (1979), Bloom et al. (1988) and Berger's (1989) researches kept their focus on the relationship between the size of a cohort and its performance in the labour market in terms of wages, worked hours and unemployment in the United States. These researches concluded that the massive influx of baby-boomers into the market substantially increased the job supply associated to that cohort, which worsened its performance in terms of employment.

In the 80s and 90s other dimensions of youth unemployment emerged. Heckman and Borjas (1980) and Lynch (1985) put emphasis on the state-dependence issue in the youth unemployment dynamics, that is, its influence on the probability a young finding a job when he has been unemployed in the previous period.

Furthermore, Freeman and Wise (1982) concluded that a downturn in economic activity affected negatively youth unemployment due to their inadequate skills and lack of knowledge of the labour market. Freeman and Wise also determined that higher minimum wages made young workers more expensive.

However a series of articles emerged in the United States that challenged the view - extended between 1960 and 1970 - that raising the minimum wage increased unemployment among adolescents (16-19 years of age) and young adults (20-24 years of age).

Based on a studies-sample Brown et al. (1982) concluded that a 10% increase in the minimum wage made the teen-employment rate fall between 1% and 3%, while having negligible effect for young adults. By expanding the years-sample of Brown et al. both Wellington (1991) as Klerman (1992) found that the effect on employment was lower at around 1%. Similarly, in an extensive meta-analysis Card and Krueger (1995) found that impacts were negligible or even marginally positive for workers between 16 and 24 years of age.

As a result from these challenging outcomes, some researchers approached this issue differently. Neumark and Wascher (1995) focused rather on the hours worked than on the young people employment levels. Through simulating changes in hours worked they demonstrated that an increase in the minimum wage caused a decrease in hours worked, thus confirming a negative effect on young people. When studying the retail sector in the State of California, Kim and Taylor (1995) also found that the minimum wage increase had a significant negative impact on youth employment.

The pre-great recession of 2008-2009 literature approached a number of issues and acquired a more global nature. Ryan (2001) centred the youth unemployment problem in the weakness of institutions that govern the transition from education to work. O'Higgins (2001) also emphasized that one of the main causes of youth unemployment lays in young people as lack of the appropriate skills as well as their poor attitudes towards work. Using an endogenous growth model, Mauro and Carmeci (2003) showed how unemployed youth lacked firm-specific human capital. Meanwhile Caroleo and Pastore (2007) explained that the key factor in youth unemployment lays in the lack of work experience rather than skills.

During this period, confirming the already mentioned finding for de 80s, a group of researchers focused on studying the relationship between labour institutions and their impact on employment levels. Nunziata (2002) and Neumark and Wascher (2003) studied the effects of the minimum wage-policy changes on employment for a sample of OECD countries. Both studies found that their estimates were consistent with the standard view that when the minimum wage levels were increased, youth unemployment increased, too. For a sample of European countries, Nunziata and Staffolani (2007) found evidence that fixed-term contracts were an effective tool to boost permanent job levels.

Presently, the agenda on youth labour market has been strongly influenced by the 2008-2009 recession as its impact on youth employment in Europe and in the United States, as well. Considerable attention has also been paid to the design, implementation and evaluation of Active Labour Market Policy (ALMPs) as this is considered - by an important group of economists and policy makers - an effective response to the instability of youth employment.

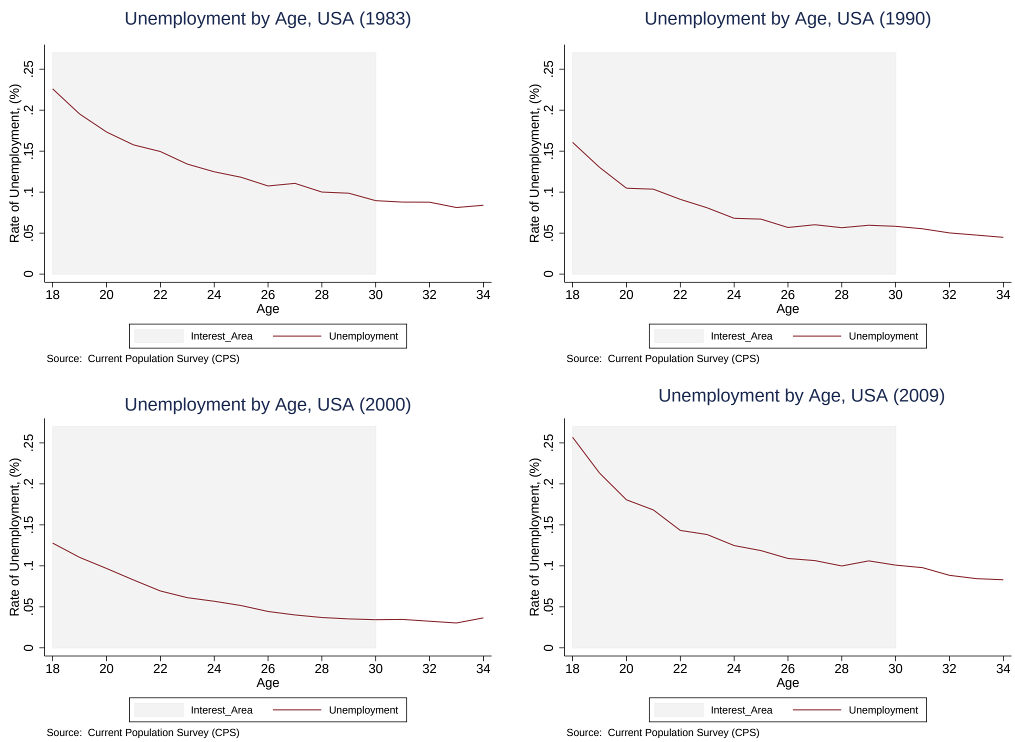
Verick (2009) analysed the impact of the economic crisis in the OECD countries. The author states his study confirms that youth employment is highly vulnerable, to some extent explained by the economic downturn and the collapse in aggregate demand. The author urges governments to actively intervene the youth labour market through subsidies and training schemes. Bell and Blanchflower (2010, 2011) obtained similar results for the United States and England. They questioned the effectiveness of the active policies in the youth labour market and held the controversial view that governments should commit to put more work available to young people as the only solution to this problem. Choudhry, Marelli, and Signorelli (2012) argue that under an economic crisis, governments must implement effective active labour market policies and therefore a smoother transition from education to work in order to reduce the risk of persistent youth unemployment. Quintini et al. (2007), Scarpetta et al. (2010), Dietrich (2012) and Artner (2013) also consider active policies as effective tools in the governments' hands to mitigate the effects of the crisis on younger people. However, there is a vast literature that questions the effectiveness of active labour market policies (see Card et al. (2010) for an excellent review of the literature).

The literature review presented below is not intended in any way to be exhaustive, but to establish, from the point of view of the economic literature, that there is no clear consensus neither on the reasons that explain youth unemployment nor on which are the most effective ways to prevent or reduce the job shortage in this market whether it is the strengthening of active policies, modifying work institutions, or the regulation or deregulation of this complex market.

However, the available literature provide a general set of factors that help explain youth unemployment: i. Changes in the population age structure, ii. Macroeconomic evolution and business cycle, iii. Institutional factors such as minimum wages, labour regulations and both passive (PPML) and active (PAML) policies in the labour market, iv. The education system, human capital formation and training, and v. Transition schemes between education and work.

In Chapter 2, this research uses a directed search model with heterogeneous agents who evolve endogenously over time, in order to analyse if the reason behind the high youth unemployment is the fact that young workers enter the market from unemployment to compete with more productive and experienced workers. It also aims to reveal whether the observed pattern in the unemployment rate for different ages groups, can be efficient from the point of view of the allocation of labour resources in an equilibrium context. In the USA, as in other economies, the following stylized facts can be observed (see Figure 1.1): i. The younger the individual, the higher is the level of unemployment, ii. As the age of an individual increase, unemployment decreases, iii. The unemployment rate stabilizes around the age of 28, and finally iv. These patterns hold during periods of economics growth as well as during recessions. Furthermore, the Chapter tries to clarify from a theoretical point of views if youth unemployment is an actual economic problem. This research models unemployment patterns by age in the United States for individuals aged 18 to 28. High youth unemployment is explained by the fact that they enter the labour market directly from unemployment to compete for jobs with more experienced

Figure 1.1: Unemployment by Age (US economy, selected years)



Based on own calculations using data from the CPS. Note that the years 1983 and 2000 correspond to periods of expansion in the American economy. In contrast, the 1990 and 2009 corresponds to recessions.

and productive workers. The model is also successful in explaining the fact that as young people get older their employability situation improves. Thus, the theoretical framework for the model presented in the Chapter 2 ensures that the unemployment pattern observed among young people is the most efficient allocation the market reaches for those entering the labour market.

In Chapter 3, this research uses a static model of direct search with heterogeneous agents to respond from a theoretical perspective whether the juvenile Active Labour Market Policies (ALMPs) actually benefit those workers the program aims to help (the youths, in this research). On particular question this chapter intends to answer is whether there is a labour substitution effect between the young workers benefited by a subsidy and those low-skilled adult workers, out of the program. This question, has been neglected in the literature despite its relevance from a social-optimum macroeconomic perspective. To analyse the effectiveness of ALMPs, this model will be calibrated to the US economy, so that to later perform numerical experiments that allow to assess the effects of the AMLPs, particularly the training and hiring subsidies, on employment levels for the different types of workers, and to find out if there are any displacement effects. This research confirms that the Active Labour Market Policies (ALMP) that intend to improve the position of young people in the labour market do not necessarily deliver the expected results policies produce a substitution effect between subsidized workers and those who are not benefited from it, as it happens with low-skilled adult workers. This research will also confirm that those young workers who may have access to the subsidies present a drop in their unemployment levels.

Chapter 2 and 3 lead to the following conclusions: the first conclusion is the fact that the high youth unemployment with respect to adult unemployment does not represent a real economic problem, as it is a socially optimal balance allocation. What follows is a second conclusion: that Active Labour Market Policies (ALMP) that attempt to improve the position of young workers in the labour market do not improve the economy and generate a welfare loss.

Additionally, it seems useful to study youth unemployment in USA by analyzing their labour flows between the different labour status. Indeed, there is an extensive literature regarding worker and job flow estimations in the U.S.

In an influential research by Darby, Haltiwanger, and Plant (1986), they analyze unemployment in terms of variation and distribution of people who become unemployed. They determine by ordering the population in different age groups (indexed by the index i) and specifying $\pi_{i,t}$ as the probability of leaving unemployment at the end of the month if unemployed at the beginning of the period (for a given group); that the level of unemployment in group i at time t ($s_{i,t}$) can be explained by the following relationship:

$$s_{i,t} = f_{i,t} + (1 - \pi_{i,t}) \cdot s_{i,t-1} \tag{1.0.1}$$

Where $f_{i,t}$ are the inflows toward unemployment for the group i at time t. Subsequently, they examine the unemployment rate variance decomposition expressing the equation (1.0.1) by the following relationship (see Darby et al., 1986, pp. 16-20):

$$u_t = \phi_t + \frac{1 - \pi_t}{1 + \gamma_t} \cdot u_{t-1} \tag{1.0.2}$$

Where u_t is the unemployment rate at aggregate level, γ_t is the workforce growth rate at t and ϕ_t is the job search rate. Then, working on Bureau of Labour Statistics Data (LABSTAT) from 1976:6 to 1985:1 they estimate and simulate the value of π_t and ϕ_t, finding that 90% of the unemployment total variance can be explained by variations in the flows and their distribution toward unemployment.

Working at firm-level in the U.S., Davis and Haltiwanger (1990) studied the magnitudes and behavior of the net employment creation, net work destruction and work reallocation based on manufacturing data (Annual Survey of Manufactures) from 1972 to 1986. To face the question whether the unemployment variation at business cycle level is driven by the work destruction or creation, they develop a methodology which decomposes the work reallocation within its idiosyncratic, sectoral and aggregate components. They also concluded that net destruction of employment plays a significant role in unemployment at activity-cycle level.

Blanchard and Diamond (1990) calculated the worker monthly flows among the different states of the workforce, disaggregated by age and gender, based on CPS data corrected by Abowd and Zellner (1985) to account for margin and classification errors. Additionally, Blanchard and Diamond (1990) estimated employment at the business flows level using manufacturing companies' data gathered by the Bureau of Labour Statistics and the Longitudinal Research Datafile (LRD) for the 1972-1985 period in order to compare their results at individual level obtained through the CPS. Although they are aware that the data sets correspond to different work flow definitions, they unequivocally get to the conclusion that the separation (that is, work loss) from employment is more volatile than the movements towards employment. They also concluded that the work destruction fluctuates more than the work creation at the business level.

Hall (2006) argues that the separation (that is, work loss) does not play an important role in the increase or unemployment during recessions. Hall states that unemployment increases because jobs are hard to find in those times and, in his own words: "... hiring decision as the central topic for understanding cyclical variation in unemployment ..." (p.101). Hall sustained his conclusions analysing that growth of unemployment did not experimented as counterpart an increasing in the separation rate during the recession in 2001 reported by the CPS according to different databases he studied, such as the Job Openings and Labor Turnover Survey (JOLTS), Current Population Survey (CPS) and Income Program Participation survey (SIPP). By contrast, when Hall (2006) analyzed the CPS data to study the job finding rate he found that it "has quite remarkable volatility" (p.117), which reinforced his idea that the work finding rate played a crucial role to understanding the employment fluctuations over the past fifty years in the U.S.

Shimer (2007), in his cited working paper (published in 2012) "Reassessing the Ins and Out of Unemployment", reached the same results as Hall (2006) when he calculated the probability that a worker became unemployed (separation probability) and the probability of finding an employment (job finding probability). Indeed, in this work Shimer concludes "the job finding probability is strongly procyclical while the employment exit probability explains only one-quarter of the fluctuations in the unemployment rate, and less during the last two decades" (p.4). Additionally, he states that the separation rate hardly contributes to the unemployment variability.

Shimer (2007) explains that previous studies, which estimate employment and hiring flows and their correlation with the economic cycle, will be biased since they do deal with the problem of time aggregation. According to Shimer, such problem arises when labour flows should be estimated in continuous time using the information available at discrete time, as it happens in the studies of labour flow case that use microeconomic data at individuals' level. This is because when work flows are calculated between two consecutive months (or quarters), any labour status changes arisen between the beginning and end of the period under analysis are not considered.

Nekarda (2008) confirms Shimer's findings when comparing weekly labour flows using the Income Program Participation survey (SIPP), and using monthly CPS. In his research Nekarda found that the flows estimated at monthly level systematically underestimate the number of transitions from one state to another at around 15-20%, as a result from the time aggregation problem.

To solve the time aggregation problem, Shimer (2007, 2012) incorporates the short-term unemployment in a two-state work model as a mechanism to adjust and capture state changes between periods. Then Shimer defines unemployment ($u_{t+\tau}$) and short-term unemployment ($u_t^s(\tau)$) in continuous time as:

$$\dot{u}_{t+\tau} = (1 - u_{t+\tau}) \cdot s_t - u_{t+\tau} \cdot f_t \tag{1.0.3}$$

$$\dot{u}_t^s(\tau) = (1 - u_{t+\tau}) \cdot s_t - u_t^s(\tau) \cdot f_t \tag{1.0.4}$$

Where s_t and f_t are the instantaneous separation and job finding rates. Besides, the sub index t refers to the analysis period (range between t and $t+1$) and τ delivers information regarding the time elapsed since the last measuring considered (τ between 0 and 1). Likewise, Shimer defines S_t and F_t as the probability of losing and getting a job during period t, measured by the traditional gross job flow traditional process at discrete time.

Subsequently he defines s_t (Inflow Hazard Rate) and f_t (Outflow Hazard Rate) as follows:

$$s_t \equiv -log(1 - S_t) \tag{1.0.5}$$

$$f_t \equiv -log(1 - F_t) \tag{1.0.6}$$

Solving the first differential equation, that is, equation (1.0.3), Shimer reaches the following result for the unemployment en $t + 1$:

$$u_{t+1} = \frac{s_t}{s_t + f_t} \cdot (1 - \exp(-s_t - f_t)) + u_t \cdot \exp(-s_t - f_t) \tag{1.0.7}$$

The importance of equation (1.0.7) is that shows that when a person loses a job she also has the probability of finding one within the same period of analysis. This relationship is lost when working with discrete time data, as is the case of traditional studies of gross flows. Shimer bases himself on this finding to establish that when traditional studies lose this short-term relationship they misunderstand the destruction and job creation measurements. It is worth saying that Shimer does not correct the series to account for margin and classification errors.

Elsby, Michaels, and Solon (2007) studied the unemployment cyclical component of inflows and outflows using the CPS in the U.S.. In particular, they replicate, reinterpret and extend Shimer's work. They confirm that the cyclical variation in unemployment can be attributed to the cyclical nature of the outflow hazard rates (ass opposed to papers that validate the separation vision). Unlike Shimer, although using his original data with slight changes in metrics, they manifest that inflows are also essential to understanding the evolution of unemployment when series are corrected by time aggregation.

Additionally, working on the steady-state unemployment approximation in terms of inflow and outflow (that is, $u_t^{ss} = \frac{s_t}{s_t+f_t}$), Elsby et al. (2007) develops significant linear decomposition that allows separating the unemployment changes as inflow and outflow's functions into two components, that is:

$$d\log(u_t^{ss}) \approx (1 - u_t^{ss}) \cdot [d\log(s_t) - d\log(f_t)] \tag{1.0.8}$$

This decomposition allows to understand the influence that inflows and outflows have at business cycle frequency over the variation of a steady state unemployment in an economy with two states of nature (employment and unemployment). Elsby et al. (2007), who explored this relationship, suggest that "a complete understanding of cyclical unemployment requires an explanation of countercyclical unemployment inflow rates as well as procyclical outflow rates" (p.23).

Fujita and Ramey also studied this separation and cyclical hiring behavior flow and its effect on the variability of unemployment through a series of working papers written during 2006 and 2007.

Actually, Fujita and Ramey (2006) concluded that "every measure of job loss and hiring, considered in terms of either total flows or transition hazard rates, exhibits high volatility at business cycle frequencies " (p .18). They built gross labour-flows data based on the CPS individual-level data for the period between 1976 and 2006. Fujita and Ramey corrected the series because of margin errors by an extended version of Missing at Random (MAR), developed by Abowd and Zellner (1985). They also corrected the series by time aggregation following Shimer's ideas.

In this paper the authors particularly solve the problem of time aggregation through a unified framework, allowing estimating the instantaneous separation and hiring rates (inflow (s_t) and outflow (f_t)) at continuous-time by calculating the separation and hiring rates at average discrete time (SR_t and JFR_t respectively) obtained via the CPS, as follows (this methodology and its finding are explained in detail step by step in Appendix

4.D of this research):

$$s_t = -\frac{SR_t}{SR_t + JFR_t} \cdot log(1 - SR_t - JFR_t) \tag{1.0.9}$$

$$f_t = -\frac{JFR_t}{SR_t + JFR_t} \cdot log(1 - SR_t - JFR_t) \tag{1.0.10}$$

After correcting series, they studied these flows' behavior in their cyclical components by filtering the series using the methodology developed by Baxter and King (1999), and its comovement with respect to the economic activity, measured by the industrial production index.

In terms of volatility, they found that separation rates (inflow) are about 30 percent more volatile than finding rates (outflow), but also that the separation rates (inflow) are about two times more volatile than the level of activity at business cycle frequency level, contradicting Hall (2006) and Shimer (2007, 2012)'s findings.

In order to contrast Shimer's findings (2007) in terms that separation rate scarcely contributes to the unemployment variability, Fujita and Ramey (2007) develop precise unemployment variability decomposition depending on the instantaneous separation and hiring rates. Basing on Elsby, Michaels, and Solon's findings (2007) they expand the underlying idea presented in equation (1.0.8), regarding that total unemployment variation can be expressed as the sum of both factors, where each factor is driven by the separation and hiring rate fluctuations. Then, assuming the following approximation occurs at steady state (see Appendix 4.C for calculation in the discrete case and continuous case):

$$u_t \approx \frac{s_t}{s_t + f_t} \equiv u_t^{ss} \tag{1.0.11}$$

Where u_t and u_t^{ss} represent unemployment and at steady-state unemployment respectively. So, according to equation (1.0.11) Fujita and Ramey (2007) approximate variations in unemployment because of the difference between the current (t) and past (t-1) steady state unemployment, that is $\Delta u_t = u_t^{ss} - u_{t-1}^{ss}$, which is expressed in terms of s_t, s_{t-1}, f_t, and f_{t-1} as (Appendix 4.E):

$$u_t = u_t^{ss} - u_{t-1}^{ss} = \frac{s_t}{s_t + f_t} - \frac{s_{t-1}}{s_{t-1} + f_{t-1}} \tag{1.0.12}$$

Then, by dividing by u_{t-1}^{ss} both sides of the equation (1.0.12) and performing an algebraic reasoning they determine that the percentage change in unemployment at steady state can be approximated by the following linear relationship (which includes an error term to maintain the mathematical equality):

$$du_t^{ss} = du_t^s + du_t^f + du_t^\varepsilon \tag{1.0.13}$$

Where:

- $du_t^{ss} = \frac{u_t^{ss} - u_{t-1}^{ss}}{u_{ss}^{t-1}}$

- $du_t^s = (1 - u_t^{ss}) \cdot \frac{s_t - s_{t-1}}{s_{t-1}}$

- $du_t^f = -(1 - u_{t-1}^{ss}) \cdot \frac{f_t - f_{t-1}}{f_{t-1}}$

- $du_t^\varepsilon = \varepsilon$

Finally, they make it possible to accurately calculate the unemployment decomposition variation by taking variance on both sides of equation (1.0.13):

$$Var(du_t^{ss}) = Cov(du_t^s, du_t^{ss}) + Cov(du_t^f, du_t^{ss}) + Cov(du_t^\varepsilon, du_t^{ss}) \tag{1.0.14}$$

Then, dividing equation (1.0.14) by $Var(du_t^{ss})$, Fujita and Ramey (2007) were able to separately understand the impact this separation and hiring rate fluctuations have over the percentage changed in unemployment at steady state. This is:

$$1 = \frac{Cov(du_t^s, du_t^{ss})}{Var(du_t^{ss})} + \frac{Cov(du_t^f, du_t^{ss})}{Var(du_t^{ss})} + \frac{Cov(du_t^\varepsilon, du_t^{ss})}{Var(du_t^{ss})} \tag{1.0.15}$$

Through this innovative methodology (see Appendix 4.E), which is ultimately expressed in Equation (1.0.15), Fujita and Ramey (2007) concluded that "separation rates make substantial contributions to the variability of the unemployment rate" (p.10). In fact, they report that depending on the size of the selected sample and the method by which the series trend component are eliminated, the effect of separation accounts for at least 40% of the percentage change in unemployment at steady state.

Then in the Chapter 4, this research intends to elucidate the degree of the impact hiring and separation flows have on the unemployment variability in the United States from a dynamic perspective and at business cycle frequency, controlling by age and sex. Particular attention will be paid to the employment situation of young people between 18 and 24 years-of-age (14.9% of the average workforce) in response to this heterogeneity problem and the growing literature that incorporates labour frictions to the general equilibrium stochastic models. Outputs will be contrasted with the premium age group, that is people aged between 25 and 54 years (68.1% of the average workforce). It is my opinion that for a better understanding of the factors that affect the unemployment of young people is crucial to obtain a more appropriate draft of the U.S.A. work current situation. In conclusion, it is found that the instantaneous flow separation and job creation in the United States in terms of correlation, volatility, comovement and their contribution to the unemployment volatility, result in a high heterogeneity when controlling by age and gender. This high heterogeneity found in microeconomic data compels us to avoid generalizations when trying to provide quantitative answers at macroeconomic level to labour policy issues by using general equilibrium dynamic models that incorporate labour -type friction. As a result, we can also conclude that regarding the target group case (young people aged between 18 and 24) finding hazard rates tends to be the main factor that would explain its unemployment dynamics. Therefore, they are a crucial factor to keep in mind when policies addressed to face the high unemployment levels in this population group, such as subsidies to recruiting, unemployment subsidies and subsidies for job training, among others. As mentioned earlier, this is not the case for premium-age population, where both factors (inflow and outflow) play an important role in explaining the unemployment dynamics.

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2. Why is Youth Unemployment so High? The role of competition between young and qualified workers.

Abstract

The integration of young people into the labour market should be an important objective all over the world. Economies generally have high youth unemployment compared to adult unemployment. This research replicates the annual pattern of unemployment among young people between 18 and 25 years old in USA using a two-sided direct search model, with heterogeneous agents whose that endogenously evolve between periods. Higher youth unemployment is jointly explained in this model by the fact that young people enter the labour market as unemployed, and also because a competition factor in which young people face a labour market inhabited by more skilled workers. Since direct search models are essentially efficient, the pattern of unemployment among young workers is a socially optimal equilibrium allocation. This research finds that policies, such as Active Labour Market Policies (ALMP), do not make people better off and that high youth unemployment does not represent a real economic problem, as its temporary nature is an outcome of the natural transition between education and work.

Keywords: Youth Unemployment, Two-Side Direct Search and Heterogeneous Agents.
JEL Classification: J64, E24, C78 and E13.

2.1 Introduction[1]

> *"The solution to the youth unemployment problem is simply put - more jobs for young people"* (Bell and Blanchflower, 2011, p.241)
> *"A consistent effort to keep the unemployment rate near its full employment level would do more to help young people find jobs than almost any other conceivable governmental policy"* (Clark and Summers, 1982, p.226)

High unemployment rates among young people and the problems they face to enter the labour market and to find stable jobs have been subject of wide interest to economists, policy makers and international agencies. Such interest has been growing as higher youth unemployment compared to other age groups has been a worldwide persistent fact in most economies for the last forty years (see Quintini et al., 2007; Perugini and Signorelli, 2010 and Bell and Blanchflower, 2011).

Attention on this issue has increased after the recession of 2008-2009, as this episode generated high costs in terms of employment - both for young and adults - in major developed economies (see Arpaia and Curci, 2010; Bell and Blanchflower, 2010 and O'Higgins, 2012). Spain, Greece and Italy have become paradigmatic cases. In 2010, youth unemployment was about 33% in Greece, 42% in Spain and 28% in Italy, but high rates were also found in Sweden (25%), France (23%), Portugal (22%), the UK (19%) and in the United States with a rate of 18%. In addition to these high unemployment rates, in most of these economies youth unemployment levels exceed more than 2.5 times the adult unemployment levels[2].

Literature on youth unemployment has opened much debate about the causes for this phenomenon and its possible solutions, generating multiple hypotheses that are either competitive or complementary (Rice, 1986). However, doubts and disputes still remain.

Studies on the causes of youth unemployment differ as their conclusions are generally influenced by the country samples being used, the analysis period considered, the variables of interest included in the analysis, the statistical and econometric techniques used, and the assumptions on which the models are based on (Gomez-Salvador and Leiner-

[1] My supervisor John Knowles taught me the theoretical model developed in this chapter. He dedicated many hours to this purpose. I am very grateful for their dedication. As usual, all errors, omissions and misinterpretations are my sole responsibility.

[2] Based on own calculation using the OECD Labour Market Statistics.

Killinger, 2008).

Despite the ongoing debate, it is possible to extract from the literature a key set of facts that help explain youth unemployment: i. changes in the population age structure ii. macroeconomic evolution and business cycle iii. institutional factors such as minimum wages, labour regulations and both Passive (PLMP) and Active (ALMP) Labour Market Policies, iv. the education system, human capital formation and training, and v. transition schemes between education and work. All these points have already been addressed in the introduction of this thesis.

Nevertheless, in this investigation the approach to this problem will be different to the existing literature. Indeed, this research will help to understand the youth unemployment problem by studying the job competition role in an intertemporal economic context, accounting for labour frictions, competitive/directed search and heterogeneous agents that evolve endogenously over time.

This research central hypothesis is the counterfactual argument that youth unemployment is not really a serious problem within the labour market, as an extensive literature present it to be. This study will consider that higher youth unemployment, can be largely explained by two facts that have been scarcely considered in the literature. First, young people access the labour market as an unemployed. Second, when young people enter the labour market they face strong competition for an employment position as most of them lack the adequate experience, productivity and skills required for some jobs, and sometimes they do not even know to "navigate" the labour market.

Both situations are natural problems every youngster should face once they get into the process of moving from the education system to the labour market, essentially transient and changing as one gets older. In the words of Choudhry et al. (2012) "the main reason for the generally worse youth labour market performance with respect to adults is related to the lower level of human capital, which - ceteris paribus - makes employers prefer adults to young people."

It is also worth noting that the labour market is full of labour frictions. It takes time and effort to workers, especially to younger ones, to find a suitable job. In addition, it is expensive for the companies to assess the productivity of a young worker applying for an employment position. Diamond (2011) clarifies this point as follows, "The primary friction

... is the need to spend time and effort to learn about opportunities - opportunities to buy or to sell, to hire or to be hired."(p.1045)

Undoubtedly this natural difficulty increases the more premature the entrance to the world of work is, as younger applicants will be less able to compete in terms of human capital, intellectual maturity and responsibility compared to older workers (see O'Higgins, 2001). In other words, difficulties for an individual of 18 years of age entering the labour market for the first time are not equivalent to those faced by to another individual who has completed the education system at 24 years old. This is because a 24-year-old individual has usually had more opportunities to acquire some level of firm-specific skills by the time he/she completes tertiary education (see Martin, 2009).

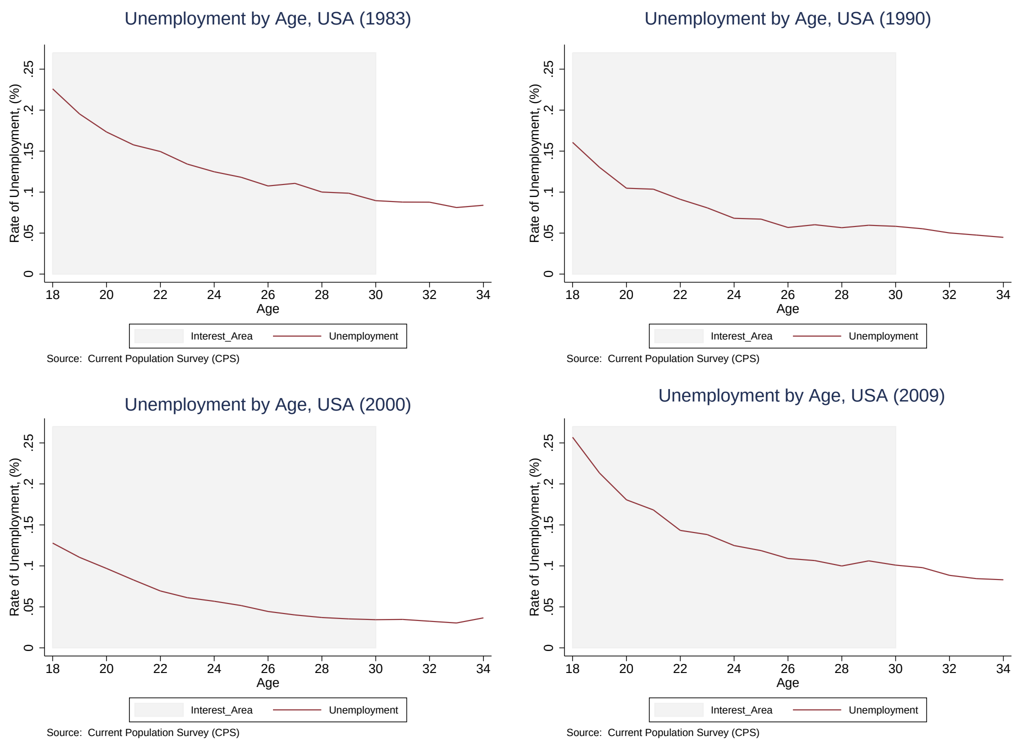
Figures 2.1 and 2.2 present unemployment-by-age patterns for a selected sample of years in the United States economy, calculated from the Current Population Survey (CPS).

Note that in Figure 2.1 the years 1983 and 2000 correspond to periods of expansion in the American economy. In contrast, the years 1990 and 2009 corresponds to recessions. Meanwhile, Figure 2.2 presents unemployment rates by age for the past 30 years in the American economy, where data is organised into decades: the 80s (graph A), 90s (graph B) and 2000 (graph C). Moreover, in graph D unemployment by age is generated by re-sampling 50,000 simulations through a Block Bootstrap technique. This statistical technique allows me to summarise 30 years of data in a single graph, showing the expected value of unemployment rates by age and their deviations from the average value at 95% confidence.

Regarding the patterns observed in Figures 2.1 and 2.2, the following stylized facts can be deduced, which are useful to support the counterfactual hypothesis in this research: i. the younger the individual, the higher the rate of unemployment, ii. As the age of an individual increases, unemployment decreases, iii. The unemployment rate stabilizes around the age of 28, and finally iv. these patterns are consistent for both expansions and recessions.

Note that these patterns appear in both industrialized and underdeveloped economies. It is important to highlight that the North American case will be used in this research with the unique purpose of illustrating the ability the model to be developed in the second section has in order to predict such stylized facts.

Figure 2.1: Unemployment by Age (US economy, selected years)

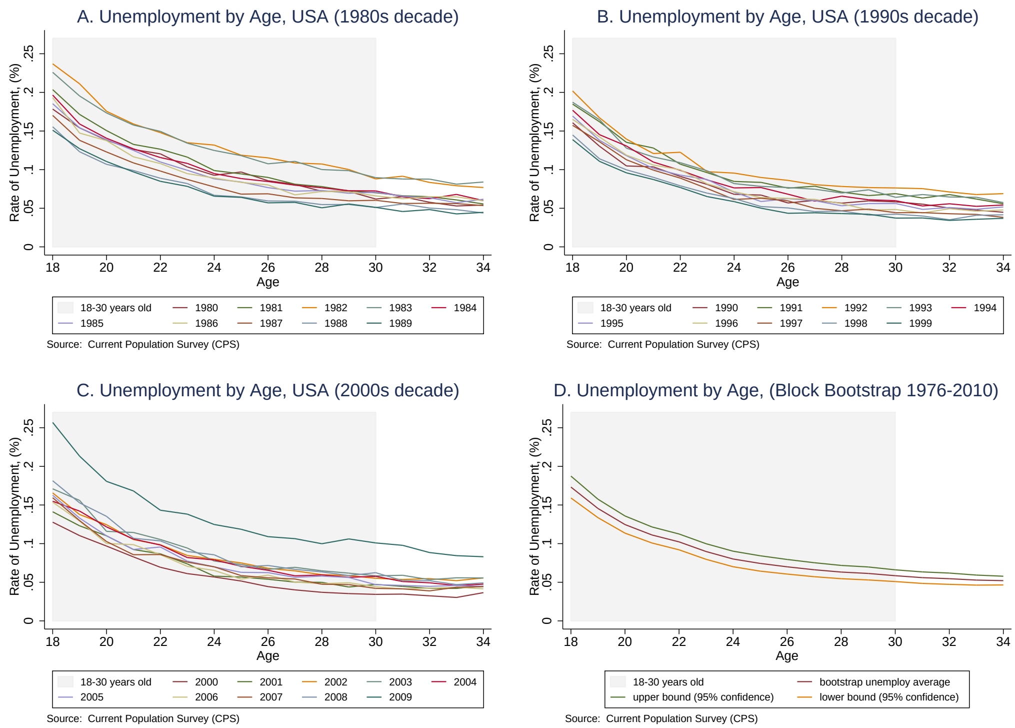


Based on own calculations using data from the CPS. Note that the years 1983 and 2000 correspond to periods of expansion in the American economy. In contrast, the 1990 and 2009 corresponds to recessions.

In summary, this research intends, from a theoretical perspective, to find out if the actual reason behind high youth unemployment is that young individuals entering the market have to compete with more productive and experienced workers. It also aims to reveal whether the pattern observed in Figures 2.1 and 2.2 can be efficient in terms of the allocation of labour resources in an equilibrium context. Finally, it tries to theoretically clarify if youth unemployment is a real economic problem or not.

The methodology used in this chapter follows those competitive search models developed by Moen (1997) and Shimer (1996). In such models, an agent (eg. a firms) posts a wage offer, after which an agent across the market (eg. a worker) directs his search to his best choice in a homogeneous agent context and in a single period of time. In such models, as opposed to random matching models, agents are allowed to use the price system to directly affect the frequency of matches. In essence, Moen (1997) finds that directed search

Figure 2.2: Unemployment by Age (US economy, last 30 years)



Based on own calculations using data from the CPS. The past 30 years of unemployment by age are presented for the American economy, divided into the decade of the 80s (graph A), 90s (graph B) and 2000 (graph C). Moreover, in graph D age unemployment generated by re-sampling with 50,000 simulations through Block Bootstrap technique is presented.

models are efficient as they allow the market to produce an efficient allocation of labour resources under the restriction of a matching technology[3]. In other words, Moen discovers that the natural rate of unemployment generated by the market is socially efficient. Also in this research, the contributions of Shi (2002) and Shimer (2005) are incorporated as they introduce heterogeneity in the agents.

In the specific model developed in this research two types of agents will be present: workers and firms. Workers will be classified as young and adults, being the latter more productive than young ones. This model also assumes stochastic aging. On the other hand, firms are classified as high and low productivity. In each period there are new unemployed workers and new firms with vacancy offerings. In each sub-markets, companies are posting

[3]See Moen (1997) as a seminal article, Rogerson et al. (2005) for an excellent review of the literature and Shi (2008) for an initial approach to this type of model.

the wages they want to pay to workers. At each point in time, the unemployed workers address their vacancy search toward sub-market where the most attractive wage can be found. Within each sub-market, the matching process between vacancies and the unemployed occurs through an "urn-ball" type random process. If a firm receives more than one application, the rule of recruiting the most productive worker is used first. Naturally, in this interaction unemployment and vacancies emerge together in competitive equilibrium conditions. This methodology allows this research to study the allocation of labour resources within a market with trade friction, applied to the case of young workers.

So far, there seems to be no models that address such labour issues from this perspective. However, the model developed here is similar to that developed by Knowles and Vandenbroucke (2013) to study marital and fertility preferences in France, with the necessary adaptations and simplifications that allow for its use in the labour market. Additionally, Menzio et al. (2012) develop a labour market life cycle model with direct search. However, unlike the research developed in this chapter, this study focuses on understanding transition patterns of workers between employment, unemployment across employers, for different ages; and it does not focus on the conditions of efficiency as in this investigation..

This research has been able to model unemployment patterns by age in the United States for individuals aged 18 to 28 years as observed in Figures 2.1 and 2.2. High youth unemployment is jointly explained by the facts that they enter the labour market - as previously mentioned - as unemployed and that they should compete for an employment position with more productive workers. The model is also successful in explaining the fact that as young people get older their employability situation improves. Finally, the model predicts that unemployment tend to converge by the age of 25, slightly different as the stylized facts show.

Thus, the theoretical framework for the model presented here, competitive/directed search, ensures that the unemployment pattern observed among young people is the most efficient allocation the market delivers once they enter the labour market. The policy prescriptions of this conclusion are two. The first one is that the Active Labour Market Policies (ALMP) that attempt to improve the position of youth in the labour market do not improve the economy and generate a welfare loss. The second, and still more powerful one, is that the high youth unemployment compared to adult does not represent a real economic problem, as it is a socially optimal equilibrium allocation.

This research is organized as follows. In the second section, a complete model characterization is presented. On the next section, the methodology used to solve the proposed model is explained. The procedure for calculating unemployment rates by age is explained in the fourth section. In the fifth section, numerical experimentation and results are presented. In the sixth section, lessons and economic policy issues are discussed. Finally, on the seventh section conclusions are presented.

2.2 The Model

Consider an economy with an infinite succession of periods. Time is discrete and the discount rate between periods is equal β. In this economy there are two types of risk neutral agents, workers $\{M\}_{m=1}^{2}$ and firms $\{N\}_{n=1}^{2}$.

2.2.1 Agents

About the time t, workers are divided into two different categories, young workers ($m = 1$) and old workers ($m = 2$). In each time period, young workers become old with probability δ_1. Once a worker has become old this condition remains forever. In every time instant in this economy there is a constant flow of new young workers $f_{1,t}$ entering the labour market as unemployed. Workers who are unemployed can only apply to a single firm in each time period analyzed. If a worker finds a job he keeps working forever. In this model it is assumed that older workers are more productive than younger ones. Also, the worker-type is fully observable by firms. Note that in this model age is a proxy for productivity therefore young workers may also be called as low productivity workers. Thus, older workers may also be nominated as high productivity workers indistinctly.

Meanwhile, firms may be low ($n = 1$) and high technology ($n = 2$). This economy is nurtured by a constant flow $g_{n,t}$ of new vacancies in the market, where t indexes time and $n \in \{1, 2\}$. Firms want to hire a single employee and therefore in each period they open a single job position. If a firm fills a vacancy, this stays closed forever. It is assumed that the high-tech firms are more productive than low-tech firms. Similarly in the case of workers, the firm-type is fully observable by workers.

As standard in the competitive/directed search literature there is no relationship between the number of workers and the number of firms in this economy (see Shimer, 2005).

2.2.2 Value of a Match and Autarchy

In this economy workers and firms are matched in pairs. Let the current production be denoted as $x_t(m,n)$ when a worker type m is matched with a firm type n. Throughout this research the convention that in each ordered pair type (m,n) the first element will always refer to workers and the second to firms will remain. Also, $x_t(0,n) = 0$ will be assumed when a firm type n does not match with any worker. It will be assumed that $x_t(m,n)$ is increasing in both arguments, as usual in the literature.

Value of Match

As mentioned above when a worker matches with a firm in period t a level of production x_t originates. But since this is a discrete and an intertemporal model it is necessary to calculate the value that generates such successful matching. For this purpose both, what happens in period t as the expected value of a successful matching in period $t+1$, as well, should be considered. In other words, the matching value is given by the current production in t plus the present value of the expected matching value in the next period.

In order to make this matching value more intelligible the young worker case will be explained. If this worker successfully matches in period t, then the matching value will be equal to the current production ($x_t(1,n)$) plus the present value of both, the value of matching ($(1-\delta_1)\widetilde{x}_{t+1}(1,n)$) in $t+1$, in the event that the employee does not get old and the matching value ($\delta_1 \widetilde{x}_{t+1}(2,n)$) in $t+1$, if the worker becomes old; that is:

$$\widetilde{x}_t(1,n) = x_t(1,n) + \beta[(1-\delta_1)\widetilde{x}_{t+1}(1,n) + \delta_1 \widetilde{x}_{t+1}(2,n)] \tag{2.2.1}$$

where $n \in \{1,2\}$. Note that when a matching occurs between a worker and a firm this employment relationship is indestructible. The only change perceived is produced when the company that hired a young worker benefits from his aging as this aging process increases his productivity in the firm. In the adult worker case specification is relatively simple as there is no aging process. Then the matching value is determined by the following expression:

$$\widetilde{x}_t(2,n) = x_t(2,n) + \beta[\widetilde{x}_{t+1}(2,n)] \tag{2.2.2}$$

where $n \in \{1,2\}$. A recursive version will be used in the model for both equations (2.2.1) and (2.2.1). Due to the matching value as much as the current production are independent from the period of time considered, the company matching recursive value with an adult worker (equation (2.2.2)) is obtained by grouping the term $\widetilde{x}(2,n)$. Therefore in this case the matching value is determined by:

$$\widetilde{x}(2,n) = \frac{x(2,n)}{1-\beta} \tag{2.2.3}$$

where $n \in \{1,2\}$. For young workers, the recursive matching value is obtained by substituting equation (2.2.3) in equation (2.2.1) together with isolating the expression $\widetilde{x}(1,n)$, thus the value is:

$$\widetilde{x}(1,n) = \frac{x(1,n)}{1-\beta(1-\delta_1)} + \frac{\beta\delta_1 x(2,n)}{(1-\beta(1-\delta_1))(1-\beta)} \tag{2.2.4}$$

where $n \in \{1,2\}$.

Value in autarchy

Both agents - firms and workers - face a cost in this economy if they do not find a matching in period t. In the unemployed workers' case that cost can be assumed as an Unemployment Insurance (UI) of an amount of $z_{t,m}$, where $m \in \{1,2\}$. To simplify, it will be assumed constant for the whole period t, thus cost can be expressed as z_m. For firms the cost for maintaining a vacancy will be equal to $c_{t,n}$, where $n \in \{1,2\}$. This cost can also be considered as cost of hiring or filling a vacant. Similarly, it will be assumed constant for all period t, and it is expressed as c_n. To calculate the recursive value in autarky for both firms and workers, proceeds in the same manner as in the previous case (value of match). Then, the value of autarky for workers in the labour market is expressed by the following equations:

$$V_{w_1}^{np} = \frac{z_1}{1-\beta(1-\delta_1)} + \frac{\beta\delta_1 z_2}{(1-\beta(1-\delta_1))(1-\beta)} \tag{2.2.5}$$

$$V_{w_2}^{np} = \frac{z_2}{1-\beta} \tag{2.2.6}$$

where the equation (2.2.5) represents the situation of young workers, and equation (2.2.6) of adult workers'. For firms, the value of autarky is expressed by:

$$V_{f_n}^{np} = \frac{c_n}{1-\beta} \tag{2.2.7}$$

where $n \in \{1,2\}$.

2.2.3 Search and Matching Mechanism

As noted in the introduction to this chapter, the model implemented in this research is close in spirit to the competitive search models developed by Moen (1997) and Shimer (1996). In addition, in accordance with Shi (2002) and Shimer (2005) this research expands to a context of heterogeneous agents.

In the model presented here, firms post their wages $w_t(m, n)$ they want to pay workers in each sub-market and at each instant of time. Subsequently, each unemployed worker directs his vacancy search to a sub-market. He observes every available wages and applies for a position at the sub-market that seems more attractive. Those firms that receive at least one application only hire one worker. They pay the agreed wage and produce according to a committed value p_z, which is exogenous in the model. If a firm receives more than one application, it will use a lexicographic rule to decide which worker to hire, which will mean to hire the most productive worker as first priority.

Additionally in this study, the evolution of agents is endogenized over time equivalently to Knowles and Vandenbroucke's work (2013). Lining up with these authors, in this model it is assumed that a worker, who has not been hired in the current period, remains unemployed in the next. Similarly, if a firm does not fill a vacancy will remains open in the next period.

Finally, for mathematical tractability reasons separation rate has been assumed equal zero (see Shi, 2005, p.110).

Then let define $M_{m,t}$ as the total population of unemployed workers type m in t, with $m \in \{1, 2\}$. Furthermore, $N_{n,t}$ is the total number of unfilled vacancies in firm type n in t, with $n \in \{1, 2\}$. Additionally, at the beginning of the period it can also be determined $M_{m,t}(m, n)$ as worker mass type m applying for a work position at the sub-market type n. Note that at each instant of time these conditions must be satisfied:

$$M_{1,t} = M_{1,t}(1, 1) + M_{1,t}(1, 2) \tag{2.2.8}$$

$$M_{2,t} = M_{2,t}(2, 1) + M_{2,t}(2, 2) \tag{2.2.9}$$

Then let define the application queue length (also known as market rigidity) as the proportion of workers type m applying to the sub-market type n divided by the total number

Table 2.1: Queue, quantity of unemployed workers and unfilled vacancies

		Firms		
		Low-tech ($n = 1$)	High-tech ($n = 2$)	Total Workers (population)
Workers	Young ($m = 1$)	$q_t(1,1) = \frac{M_{1,t}(1,1)}{N_{1,t}}$	$q_t(1,2) = \frac{M_{1,t}(1,2)}{N_{2,t}}$	$M_{1,t}$
	Old ($m = 2$)	$q_t(2,1) = \frac{M_{2,t}(2,1)}{N_{1,t}}$	$q_t(2,2) = \frac{M_{2,t}(2,2)}{N_{2,t}}$	$M_{2,t}$
Total Vacancies		$N_{1,t}$	$N_{2,t}$	

of open positions in the sub-market n, this is:

$$q_t(m,n) = \frac{M_{1,t}(m,n)}{N_{n,t}} \tag{2.2.10}$$

where $m \in \{1,2\}$ and $n \in \{1,2\}$. The mathematical expressions of the application queue; the mass of unemployed workers and the mass of vacancies in each period t is summarized in Table 2.1.

Within each sub-market, matching process between vacancies and unemployed occurs through an "urn-ball" type random process. Following Shimer (2005), and Knowles and Vandenbroucke (2013), in this research the Poisson's distribution will be used $f(z,\lambda) = \frac{\lambda^z e^{-\lambda}}{z!}$ to calculate the distribution of probability that z workers of type m apply to a sub-market type n where λ is the application queue; that is:

$$f(z, q_t(m,n)) = \frac{q_t(m,n)^z e^{-q_t(m,n)}}{z!} \tag{2.2.11}$$

where $z \in \{0,1,2,2,...\}$, $m \in \{1,2\}$ and $n \in \{1,2\}$.

The matching probability depends on the application queue as follows (see Appendix 2.A for detailed calculation of these probabilities). The probability that a young worker (type $m = 1$) be hired within a sub-market type n is $p_z e^{-q(2,n)}\left(1 - e^{-q(1,n)}\right)$. This probability is determined by the product of the probability that no adult worker applies to the sub-market $e^{-q(2,n)}$ and the probability that at least one young worker applies to sub-market $\left(1 - e^{-q(1,n)}\right)$.

Note that the competitive factor in this model arises because we use the lexicographic

Table 2.2: Probability of Matching

		Firms	
		Low-tech ($n = 1$)	High-tech ($n = 2$)
Workers	Young ($m = 1$)	$p_z e^{-q_t(2,1)}\left(1 - e^{-q_t(1,1)}\right)$	$p_z e^{-q_t(2,2)}\left(1 - e^{-q_t(1,2)}\right)$
	Old ($m = 2$)	$p_z\left(1 - e^{-q_t(2,1)}\right)$	$p_z\left(1 - e^{-q_t(2,2)}\right)$

allocation rule. In fact, in the probability calculation that a young person is hired, the possibility that an adult applies to the same sub-market simultaneously has been excluded. This denotes the idea that young people have lower chance of finding a job compared to adults, since the latter are more productive.

Also, the probability that an old worker (type $m = 2$) is recruited in a sub-market type n is expressed as $\left(1 - e^{-q(2,n)}\right)$, where p_z is the probability the contract thrives after matching. Table 2.2 summarizes the probability of matching.

2.2.4 Rates of Employment and Filling Vacancies

The workers' unemployment and the firms' filling vacancy rate are calculated in this model section. Note that these calculations are made for the current period t.

Employment Rate in t

Young worker. First it is necessary to calculate the young worker mass that has been matched on a sub-market type n to calculate the employment rate at the end of the current period. Note that this must be determined by the end of period t when the matching process is complete. Then the mass of young workers ($E_t(1, n)$) employed in each sub-market is obtained by calculating the probability product that a young worker be matched and the mass of vacancies offered in that sub-market. Then $E_t(1, n)$ can be expressed by the following equation:

$$E_t(1, n) = p_z e^{-q_t(2,n)}\left(1 - e^{-q_t(1,n)}\right) N_{n,t} \tag{2.2.12}$$

where $n \in \{1, 2\}$. The employment rate $\Omega^E_{1,t}$ is calculated by dividing the total young worker mass employed in both sub-markets in the period t divided by the young worker mass in the same period ($M_{1,t}$), that is:

$$\Omega^E_{1,t} = \frac{\sum_{n=1}^{2} E_t(1,n)}{M_{1,t}} \tag{2.2.13}$$

Old worker. For adult workers identical procedure is followed. Then the adult workers mass ($E_t(2,n)$) matched in the sub-market type n and the employment rate $\Omega^E_{2,t}$ are calculated by the following equations:

$$E_t(2,n) = p_z\big(1 - e^{-q_t(2,n)}\big)N_{n,t} \tag{2.2.14}$$

$$\Omega^E_{2,t} = \frac{\sum_{n=1}^{2} E_t(2,n)}{M_{2,t}} \tag{2.2.15}$$

where $n \in \{1,2\}$.

Vacancies filling rate in t

In order to calculate the rate of vacancy filling at the end of the current period in a type n sub-market it is necessary first to calculate the mass of vacancies occupied by each worker type m. Note that the employed worker mass of each type (young and old) is obtained by calculating the product between the probability a worker is matched and the mass of vacancies offered in the sub-market type n (low and high technology). Then the total number of vacancies $V_{n,t}$ filled by both young and old workers in the sub-market type n can be expressed by the following equation:

$$V_{n,t} = p_z e^{-q_t(2,n)}\big(1 - e^{-q_t(1,n)}\big)N_{n,t} + p_z\big(1 - e^{-q_t(2,n)}\big)N_{n,t} \tag{2.2.16}$$

where $n \in \{1,2\}$. The vacancy filling rate $\Omega^V_{n,t}$ is calculated by dividing the total employed mass of workers by the mass of vacancies available ($N_{n,t}$) in the sub-market n, this is:

$$\Omega^V_{n,t} = \frac{V_{n,t}}{N_{n,t}} \tag{2.2.17}$$

where $n \in \{1,2\}$.

2.2.5 Transition Process between t and t+1

As mentioned above, the agents in this model evolve over time. It is therefore necessary to determine at the beginning of each period the mass of workers searching for jobs and the mass of vacancies offered by firms.

Workers

In this case young and old workers should be analyzed separately, as the laws that govern the evolution of each mass differ slightly. In the youth case, the volume of unemployed that start the job searching in the current period is determined by two sources. The first one is set by the flow of new young workers entering the labour market during the current period. The second one is set by those young workers that in the previous period found no employment and kept their young status. Then the motion law governing the evolution of young workers between period t and $t+1$ is expressed by the following equation:

$$M_{1,t+1} = f_{1,t+1} + M_{1,t}(1-\delta_1)(1-\Omega^E_{1,t}) \tag{2.2.18}$$

To be more precise the elements of the above equation are explained. Then let $M_{1,t+1}$ be defined as the mass of young workers who start period $t+1$ as unemployed workers. The expression $f_{1,t+1}$represents the flow of unemployed young workers entering the labour market at the beginning of period $t+1$. Besides, $\Omega^E_{1,t}$ represents the youth employment rate and $(1-\Omega^E_{1,t})$ the youth unemployment rate in the labour market at the end of period t. Finally, $M_{1,t}(1-\delta_1)(1-\Omega^E_{1,t})$ is equivalent to the number of unemployed young people who were not able to find a job during the period t and thus increasing the mass of young workers who start period $t+1$ as unemployed.

Similarly the law of motion that governs the evolution of older workers is determined by two sources. The first one is set by the mass of old workers who did not find job in the previous period; the second one for those young workers in the previous period that found no occupation and aged at the end of that very same period. Then the motion law between t and $t+1$ is expressed by the following equation:

$$M_{2,t+1} = M_{2,t}(1-\Omega^E_{2,t}) + M_{1,t}\delta_1(1-\Omega^E_{1,t}) \tag{2.2.19}$$

Then let $M_{2,t+1}$ be defined as the mass of unemployed adults at the beginning of period $t+1$. In this equation $\Omega^E_{2,t}$ expresses the adult employment rate and $(1-\Omega^E_{2,t})$ the rate of adults without occupation in the labour market at the end of period t. Likewise, $M_{2,t}(1-\Omega^E_{2,t})$ represents the mass of old unemployed workers that did not find a job in t, and that start as unemployed in the next period. Finally, $M_{1,t}\delta_1(1-\Omega^E_{1,t})$ expresses the number of unemployed youth who could not find a job during period t and aged during it, so that they increased the mass of old workers searching for employment at the beginning of period $t+1$.

Firms

Following the same procedure for workers' case, the law of motion that governs the vacancy evolution of low ($n=1$) and high ($n=2$) technology firms between the period t and $t+1$ is expressed by:

$$N_{1,t+1} = g_{1,t+1} + N_{1,t}(1-\Omega^V_{1,t}) \tag{2.2.20}$$

Then, $N_{n,t+1}$ is defined as the mass of vacancies available in the labour market at the beginning of period $t+1$. The value $g_{n,t+1}$ represents the flow of vacancies entering the labour market at the beginning of period $t+1$ for firms type n. Besides, $\Omega^V_{1,t}$ expresses the rate at which vacancies are filled in period t and $(1-\Omega^V_{1,t})$ the rate at which vacancies are not filled in the labour market at the end of the current period. Finally, $N_{1,t}(1-\Omega^V_{1,t})$ expresses the mass of vacancies that were not filled during period t and that are available in the market at the beginning of period $t+1$.

2.2.6 Decision of Workers in t

Unemployed workers apply to a sub-market just if they can get at least as similar wages as their expected market wage, also known as reservation wage. Let $\vartheta_{m,t}$ be defined as the best wage that workers can get from the best available alternative.

Lining up with Shimer (2005, p.1004) the conditions below should be met for each sub-market to which workers have applied to; thus the equation (2.2.21) will be for the case of youth and the equation (2.2.22) for the case of adults:

$$\vartheta_{1,t} = p_z e^{-q_t(2,n)} \frac{\left(1-e^{-q_t(1,n)}\right)}{q_t(1,n)} w_t(1,n) \tag{2.2.21}$$

$$\vartheta_{2,t} = p_z \frac{\left(1 - e^{-q_t(2,n)}\right)}{q_t(2,n)} w_t(2,n) \tag{2.2.22}$$

where $n \in \{1,2\}$. Note that $p_z e^{-q_t(2,n)} \frac{\left(1-e^{-q_t(1,n)}\right)}{q_t(1,n)}$ represents the probability that a young worker be hired. Likewise $p_z \frac{\left(1-e^{-q_t(2,n)}\right)}{q_t(2,n)}$ represents the probability that an adult worker be hired (see Shimer, 2005, p.1004).

2.2.7 Surplus

Let the surplus be defined as the difference between a matching value $\widetilde{x}(m,n)$ and the reservation value. In the case of firms the reservation value $Rv_{n,t}$ corresponds to the value of keeping an open vacancy in the current period. In the case of workers the reservation value $Ru_{m,t}$ is the value of staying as unemployed in t. Hence the surplus can be expressed as follows:

$$X_t(m,n) = \widetilde{x}(m,n) - Ru_{m,t} - Rv_{n,t} \tag{2.2.23}$$

where $m \in \{1,2\}$ and $n \in \{1,2\}$.

2.2.8 Value Function

Workers

Let $Vu_{m,t}$ be defined as the value of an unemployed worker type m that chooses to participate in the labour market; and $Ru_{m,t}$ as the value of a worker type m keeping unemployed. Likewise $Eu_{m,t}$ is defined as the expected gain when a worker type m gets a job position. Hence, it is defined that:

$$Vu_{m,t} = Ru_{m,t} + Eu_{m,t} \tag{2.2.24}$$

Subsequently, $Ru_{m,t}$ and $Eu_{m,t}$ values for the type m worker case will be determined. In this case it is known that worker type m will enjoy a return z_m if being unemployed in period t. Thus, $Ru_{m,t}$ can be expressed in a young worker case as $(m = 1)$:

$$Ru_{1,t} = z_1 + \beta[(1 - \delta_1)Vu_{1,t+1} + \delta_1 Vu_{2,t+1}] \tag{2.2.25}$$

Where $(1 - \delta_1)Vu_{1,t+1}$ is the participating value for a young worker in the labour market at beginning of period $t + 1$, as long as he does not age. Whereas, this value will be $\delta_1 Vu_{2,t+1}$ if this young worker becomes old at end of period t, as this worker is starting as an adult in period $t+1$. For the old worker, $Ru_{m,t}$ is denoted as (m=2):

$$Ru_{2,t} = z_2 + \beta[Vu_{2,t+1}] \tag{2.2.26}$$

Where $Vu_{2,t+1}$ is an old worker value at the beginning of period $t+1$ if workers keep working.

Also the expected gain, this is, $Eu_{m,t}$ is equal to the worker reservation wage $\vartheta_{m,t}$. This is because the expected income for a worker type m is identical in both, low as high technology markets. Let denote p as the probability that a worker type m enters a low-tech market and $1-p$ for the probability of applying to another market. Then, $Eu_{m,t}$ is expressed as:

$$Eu_{m,t} = p \cdot \{\vartheta_{m,t}\} + (1-p) \cdot \{\vartheta_{m,t}\} = v_{m,t} \tag{2.2.27}$$

which shows that $Eu_{m,t} = \vartheta_{m,t}$ where $m \in \{1,2\}$ and $n \in \{1,2\}$. With equations (2.2.25), (2.2.26) and (2.2.27) the value function (equation (2.2.24)) is determined for both types of workers.

Firms

Let now define $Vv_{m,t}$ as the value of a firm type n that chooses to participate in the labour market. Besides, let $Rv_{m,t}$ be the value of keeping a vacancy unfilled in a firm type n. Also, $Eu_{n,t}$ is defined as the expected gain when filling a vacancy in a firm type n. Then we have that:

$$Vv_{m,t} = Rv_{m,t} + Ev_{m,t} \tag{2.2.28}$$

Likewise to the worker case, $Rv_{n,t}$ and $Ev_{n,t}$ values for a firm type n will be next determined. In this case it is known that a firm type n faces a cost c_n if a vacancy is not filled in t. Then $Rv_{n,t}$ in the case of a firm type n can be expressed as follows:

$$Rv_{n,t} = c_n + \beta Vv_{n,t+1} \tag{2.2.29}$$

Where $Vv_{n,t+1}$ is the vacancy filling value for a firm type n at the beginning of period $t+1$.

The expected gain, that is, $Eu_{n,t}$ represents the firm net profit. This value is determined by a firm maximizing process subject to the workers' participation in the market. These values will be determined in the next model section.

2.2.9 Profit for Firm in t

In period t firms optimize their expected profit by deciding their posted wage $w_t(m,n)$ to the market. Therefore the expected profit for a firm type n is determined by the probability a worker type m be hired, multiplied by the gain the firm gets hiring such worker, summed across all types of workers in the market.

For a worker type 1 (young) and a firm type n the probability is determined by $p_z e^{-q_t(2,n)}(1-e^{-q_t(1,n)})$ and the gain obtained by the firm is expressed by $(X_t(1,n) - w_t(1,n))$. For a worker type 2 (old), the probability is identical to $(1 - e^{-q_t(2,n)})$ and the gain reported by the firm type n for this recruitment is determined by $(X_t(1,n) - w_t(1,n))$. For a low firm technology this optimization process is expressed as:

$$\begin{aligned}\max_{\{w_t(1,1),w_t(2,1)\}} \Pi_{1,t} = {} & p_z e^{-q_t(2,1)}\big(1 - e^{-q_t(1,1)}\big)\{X_t(1,1) - w_t(1,1)\} \\ & + p_z\big(1 - e^{-q_t(2,1)}\big)\{X_t(2,1) - w_t(2,1)\}\end{aligned} \tag{2.2.30}$$

subject to the participation condition of workers given in the sixth section of this model. Whereas for the case of a high-tech firm this optimization process is as follows:

$$\begin{aligned}\max_{\{w_t(1,2),w_t(2,2)\}} \Pi_{2,t} = {} & p_z e^{-q_t(2,2)}\big(1 - e^{-q_t(1,2)}\big)\{X_t(1,2) - w_t(1,2)\} \\ & + p_z\big(1 - e^{-q_t(2,2)}\big)\{X_t(2,2) - w_t(2,2)\}\end{aligned} \tag{2.2.31}$$

also subject to the participation condition of workers.

2.2.10 Definition of Equilibrium and efficiency

Next the model with a formal definition of a recursive competitive equilibrium is summarized.

Definition 3.1. A recursive competitive equilibrium direct search in the labour market consists of the following objects: an application queue $q(m,n)$, a mass of young unemployed workers M_m, a mass of vacancies N_n, wage $w(m,n)$ offered to workers by firms and the law of motion of workers $N_m^{'} = T(M_m, f_m, \delta_m, \Omega_m^E)$ and vacancies $N_n^d = T(N_n, g_n, \Omega_n^V)$, where $m \in \{1,2\}$ y $n \in \{1,2\}$. This list of objects must satisfy the following conditions:

1. Economic agents are optimizing:

   a. Workers get at least as much as their expected reservation wage, hence, equations (2.2.21) and (2.2.22) are satisfied.

b. Firms maximize profits, hence, problems posed in (2.2.30) and (2.2.31) are solved.

2. Application queues satisfy feasibility constraint, that is, satisfy the equation (2.2.8) and (2.2.9).

3. The mass of unemployed and vacancies satisfy the laws of motion described in equations (2.2.18), (2.2.19) and (2.2.20).

4. The markets clear.

Efficiency.

In this research an intertemporal model of direct search has been developed. In essence, at each instant of time a static problem - similar in spirit to that developed by Shimer (2005) - is solved, with the difference that in this model the unemployed workers and vacancies mass evolve endogenously between periods, as in Knowles and Vandenbroucke (2013). Shimer proves that his static model is efficient by demonstrating that the competitive solution is identical to that of a planner who wants to maximize the production value in economy. Those conclusions operate on the model developed in this research.

In essence, in the model developed here a static problem is solved at each instant of time, as mentioned above, and the market solution is similar to Shimer's social planner's solution. So if both solutions are equivalent, the mass of agents transiting from one period to another is also identical for both the competitive case as for a social planner case. Then it is possible to deduce that the planner's equilibrium and the equilibrium competing will always be equivalent, regardless the time period considered. Finally, this leads us to conclude that equilibrium in a steady- state must match in both, in a planning as much as in a competition context, which ensures that the model developed is efficient.

2.3 Strategy for Solving the Model

The main strategy to solve the model proposed in the previous section is to determine the steady state by an iterative process. For this purpose, early in the process the mass of unemployed workers M_m, the mass of vacancies N_n, the values of remaining unemployed Ru_m and the unfilled vacancy value Rv_n are guessed through a random value distributed independently. With this information, then proceeds to solve the static equilibrium and to obtain an optimal application queue (q^*). Static equilibrium is quite similar to that described by Shimer (2005) and consists of solving the following equation set:

$$\max_{\{w_t(1,1),w_t(2,1)\}} \Pi_{1,t} = p_z e^{-q_t(2,1)}\left(1-e^{-q_t(1,1)}\right)\{X_t(1,1)-w_t(1,1)\} \\ +p_z\left(1-e^{-q_t(2,1)}\right)\{X_t(2,1)-w_t(2,1)\} \tag{2.3.1}$$

$$\max_{\{w_t(1,2),w_t(2,2)\}} \Pi_{2,t} = p_z e^{-q_t(2,2)}\left(1-e^{-q_t(1,2)}\right)\{X_t(1,2)-w_t(1,2)\} \\ +p_z\left(1-e^{-q_t(2,2)}\right)\{X_t(2,2)-w_t(2,2)\} \tag{2.3.2}$$

$$\vartheta_{1,t} = p_z e^{-q_t(2,n)}\frac{\left(1-e^{-q_t(1,n)}\right)}{q_t(1,n)}w_t(1,n) \tag{2.3.3}$$

$$\vartheta_{2,t} = p_z \frac{\left(1-e^{-q_t(2,n)}\right)}{q_t(2,n)}w_t(2,n) \tag{2.3.4}$$

$$M_{1,t} = M_{1,t}(1,1) + M_{1,t}(1,2) \tag{2.3.5}$$

$$M_{2,t} = M_{2,t}(2,1) + M_{2,t}(2,2) \tag{2.3.6}$$

A detailed explanation of how to solve this set of equations is provided in Appendix 2.B. This Appendix explains the results for both the interior solution and for each the possible corner solutions.

With optimal queue obtained q^*, the proceeds to update the guessed values and to solve a static problem again so that to find a new optimal queue q^{**}. If the difference between q^* and q^{**} are smaller than an arbitrary value and small enough (ε), that is $|q^* - q^{**}| < \varepsilon$, the steady state is found. If not, the process is repeated until converging.

2.4 Unemployment Calculation for Age

After obtaining the steady-state equilibrium proceeds to calculate unemployment rates by age. Let then define the following vector, $\lambda^{\tau}_{w,ss} = [S^{\tau}_{1,ss}, S^{\tau}_{2,ss}, Emp^{\tau}_{w,ss}]$ which describes the number of individuals of age τ in different states of nature: i. unemployed with low productivity ii. unemployed with high productivity, and iii. employed. Then $S^{\tau}_{1,ss}$ refers to the number of unemployed workers of age τ with low productivity. $S^{\tau}_{2,ss}$ represents the number of unemployed workers of age τ with high productivity. Finally, $Emp^{\tau}_{w,ss}$

determines the number of workers aged τ who are employed. Then in the steady state, the law of motion which models the evolution of the mass of workers between the age τ and $\tau+1$ can be described by the following mathematical expression:

$$\lambda_{w,ss}^{\tau+1} = \lambda_{w,ss}^{\tau} \cdot \Phi_{w,ss} \tag{2.4.1}$$

Where $\Phi_{w,ss}$ is a stochastic matrix derived from the transition patterns of workers between different periods of time. This matrix is described below:

$$\Phi_{w,ss} = \begin{bmatrix} (1-\delta_1)(1-\Omega_{1,ss}^E) & \delta_1(1-\Omega_{1,ss}^E) & \Omega_{1,ss}^E \\ 0 & (1-\Omega_{2,ss}^E) & \Omega_{2,ss}^E \\ 0 & 0 & 1 \end{bmatrix} \tag{2.4.2}$$

where each element of the matrix determines the probability that an agent is in such state of nature. Remember $\Omega_{1,ss}^E$ and $\Omega_{2,ss}^E$ represent the employability rate for each type of worker. Note that the first row denotes the situation of low productivity workers in the steady state. Where $(1-\delta_1)(1-\Omega_{1,ss}^E)$ represents the probability that this type of worker be unemployed and that his productivity status has not changed. Also $\delta_1(1-\Omega_{1,ss}^E)$ represents the probability that a low productivity worker has changed status but keeps without finding a job. Finally, $\Omega_{1,ss}^E$ represents the probability that this type of worker be working. Note that these values like any stochastic matrix must sum the unity. The second row of the matrix represents the situation of workers of high productivity in a steady state. Here $(1-\Omega_{2,ss}^E)$ represents the probability that a productive worker be unemployed; $\Omega_{2,ss}^E$ otherwise represents the probability that be employed. Again these values sum the unity.

In the process described in equation (2.4.1) 18 years of age is considered as an initial period. More specifically, vector $\lambda_{w,ss}^{18} = [f_1, 0, 0]$ describes the starting point of our iterative process, where f_1 refers to the flow of 18-year-old entering as unemployed the labour market and therefore is present only in the first state of the nature. Then unemployment rate is calculated for workers of age τ at steady state through the following expression:

$$Ur_{ss}^{\tau} = 1 - \left[\frac{S_{1,ss}^{\tau}}{S_{1,ss}^{\tau} + S_{2,ss}^{\tau}} \Omega_{1,ss}^E + \frac{S_{2,ss}^{\tau}}{S_{1,ss}^{\tau} + S_{2,ss}^{\tau}} \Omega_{2,ss}^E \right] \tag{2.4.3}$$

2.5 Model Calibration

For a better understanding of the results emanating from the model developed in this research a numerical experiment will be performed to search if it is able to replicate the

U.S. patterns of unemployment by age for individuals aged 18 to 30 years old, showed in Figures 2.1 and 2.2. To carry out this study, first the steady state is solved for. Then with this steady state unemployment value equation (2.4.1) is calculated recursively. Finally by equation (2.4.3) the unemployment rates by age are estimated.

The value of the parameters in this experiment will be determined in two ways. First, some of the values of the parameters to be determined a priori are taken from the standard literature and some values calculated in this chapter. As is standard in the macroeconomic literature an annual discount factor β of 0.96 is used, a value that is consistent with an annual risk free interest rate of 4% annual. While the production function will be assumed to have constant returns to scale and it will be of the Cobb-Douglas type, $x_t(m,n) = m^{\alpha}n^{(1-\alpha)}$, where α is an exogenous parameter in the model with a value of 0.7, representing the labour income share[4]. The values of Unemployment Insurance (UI), for both young (between 18 and 24 years) to old (over 25 years of age), proceed to calculate using BLS Wage Data. The young and old average weekly wage (in 2013 dollars) between the years 2004 to 2013 was $468 and $831, respectively. Whereas in general the UI payment is 40% of previous wage and the affected is covered by 24 weeks, then can be estimated that the young and old receive UI annually by around $z_1 = \$4,495$ and $z_2 = \$7,973$, respectively. Then the ratio of unemployment insurance between young and old is $\frac{z_1}{z_2} = 0.56$.

Estimating the parameters of the cost of hiring and filling a job vacancy is a difficult task. According to Blatter et al. (2012) few empirical studies contain data on these estimates. However Dube et al. (2010) report for the U.S. economy, on average, an annual recruitment cost of $4,327 (in 2013 dollars). So knowing that the average weekly wage for the American economy was $796 in 2013, it is possible to estimate that the cost of hiring workers for a company is equivalent to the wage of 5.4 working weeks. The next step is to calculate the average weekly wage for workers in low and high tech firms in U.S., using the payroll and average weekly earnings data, at the establishment level, from Current Employment Statistics (CES). Then firms are classified according to the methodology used by the TechAmerica Foundation that is based on the North American Industrial Classification System (NAICS) for defining technology industries in USA. With this procedure it was possible to estimate that by 2013 the average weekly wage for high and low tech companies was $1,439 and $780, respectively. Finally, the estimated cost of hiring is calculated by multiplying the above values by 5.4 weeks which gives an estimate cost of hiring of $c_1 = \$7,825$ and $c_2 = \$4,243$ for a high and low tech firms respectively. Then the ratio

[4] Source: Bureau of Labour Statistics (BLS)

of cost the of hiring between low and high tech firms is $\frac{c_1}{c_2} = 0.54$. Finally, the degree of commitment, p_z, will be set to 1. This means that 100% of matching workers and firms agree to work together.

The remaining parameter values are chosen so that the model generates steady-state age unemployment that matches those in the data for the USA economy. The probability that a young worker becomes old in each period will be set at 0.53. Also, for the young workers flow will fit to 1.89 in each period. For the low-tech firms flow will be equivalent to 0.99 and for high-tech firms it will be equivalent to 1. In Table 2.3 all the parameters that feed the model are shown.

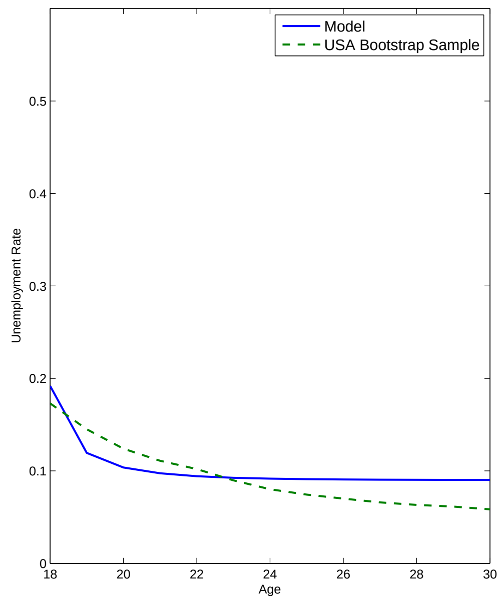
Table 2.3: Parameter Values for Benchmark Model

Parameters	Value	Role
β	0.96	Discount factor
α	0.70	Labour income share
z_1	\$4,495	Young unemployment Insurance (annually)
z_2	\$7,973	Old unemployment Insurance (annually)
$\frac{z_1}{z_2}$	0.56	Ratio of UI between young and old
c_1	\$4,243	Cost of open/fill vacancy for low-tech firm
c_2	\$7,825	Cost of open/fill vacancy for high-tech firm
$\frac{c_1}{c_2}$	0.54	Ratio of open/fill vacancy between low and hihg tech firm
p_z	1.00	Probability of commitment given a match
δ_1	0.53	The probability a young worker becomes old
f_1	1.89	Inflow of young
g_1	0.99	Inflow of low-tech firms
g_2	1.00	Inflow of high-tech firms

Figure 2.3 shows the evolution of unemployment levels by age generated by the model and as a reference, contrasted with data of unemployment levels by age reported by CPS for the U.S. economy for the Block Bootstrap sample from 1976-2010.

As shown in Figure 2.3 the model is able to predict the patterns of unemployment by age generated by re-sampling with 50,000 simulations through Block Bootstrap technique in the United States for individuals aged 18 to 30 years old from 1976 to 2010. The model shows that the high youth unemployment is explained jointly by the fact that they enter the labour market as unemployed because they have to compete for an employment

Figure 2.3: Unemployment Rate by Age τ (Model vs Data)



position with more productive workers. The model is successful in explaining the fact that as young people become old (age is a productivity proxy in the model) their employability situation improve, decreasing their unemployment levels. However, the model just predicts that unemployment tend to converge by the age of 25, slightly different as the stylized facts show. This difference in the ages of convergence of the unemployment rate can be explained by the assumption that the separation rate equal zero in the model. With this assumption, the model tends to underestimate the unemployment rate to early ages. Because young people tend to enter and exit the labour market more frequently than adults, with the assumption that the rate of separation is identical for both types of workers makes the model imperfectly predict the ages of convergence.

2.6 Discussion of Results and Policy Lessons

Results obtained in the previous section are interesting from the point of view of policies designed to reduce youth unemployment. As mentioned above, the model has been built following a competitive searching model, where wage posting and direct search is combined with heterogeneous agents. Moen (1997) and Shimer (2005) have shown that such models are efficient for both homogeneous and heterogeneous agent cases, respectively. In other words, under the theoretical framework developed in this research, the optimal market solution and the social planner's optimal solution are identical.

The theoretical framework developed above predicts that the pattern observed in youth unemployment is not the result of labour market inefficiencies. That is a common argument present in the literature that supports the idea that the Active Policies on the Youth Labour Market (ALMP) can improve the allocation of resources in this market (see O'Higgins (2012) and Andersen and Svarer (2012)). Actually, for Bell and Blanchflower (2010) the current situation of lack of employment for young people is a "strong case for policy intervention" (p.1). Also for Choudhry et al. (2012), and given the young people situation, it "seems appropriate to adopt effective active labour market policy" (p88) and that such policies are a fundamental tool because of "high risks of persistence and possible transformation of short-term unemployment into structural (long-term) unemployment" (P.88).

On the contrary, in our research, the actual youth unemployment pattern is the most efficient allocation (social optimum) the market delivers as young people enters process the labour market.

The results leads to two conclusion. The first one says that ALMPs that intend to improve the youth position in the labour market do not make the economy better and generate a welfare loss, as opposed to what many economists, policy makers and international agencies tend to suggest. This is because young people benefited by those programs can displace adults who have not participated from the labour market (see Dahlberg and Forslund (2005)). The second and more powerful one, is that the high youth unemployment compared to adult rates does not represent a real economic problem, as this is simply due labour friction, essentially temporal, and represents the natural cost of transition between education and work.

These results support the idea that what policymakers effectively posses to face the high youth unemployment is to find the way to reduce the overall level of unemployment in the economy. As Clark and Summers (1982) noted in their seminal research a "consistent effort to keep the unemployment rate near its full employment level would do more to help young people find jobs than almost any other conceivable governmental policy" (p.226). And this is not achieved through policies directed towards a particular group within the labour market, as ALMP proponents suggest (see Bell and Blanchflower (2011), Andersen and Svarer (2012) and Choudhry et al. (2012). On the contrary, it is required to encourage major flexibility in the labour markets. Botero et al. (2004) find that for a sample of 85 countries a strong price regulation in the labour market has adverse consequences on the level of participation in the labour force and unemployment levels, especially among younger workers.

In this direction, Shimer (2012) argues that the slow recovery of economic activity and employment following the recent recession of 2008-2009 is explained by the increase in wage rigidities amplifying and spreading the shocks on the economy. Nickell (1997) finds that high unemployment is associated with a generous system of unemployment benefits. Also Elmeskov et al. (1998), Nunziata (2002) and Furceri and Mourougane (2012), analyzing OECD countries, found that there is strong evidence that high structural unemployment is associated to institutional systems that provide generous and extended unemployment benefits. Finally, Bassanini and Duval (2006) and Bernal-Verdugo et al. (2012) have concluded that the more flexible the labour market is, the higher is the job creation rate, and as a positive consequence, there is a lower unemployment level at aggregate level.

2.7 Conclusion

Through a competitive direct search model, this research is able to successfully model the patterns of unemployment by age in the United States for individuals aged 18 to 28 years old. Also the theoretical model developed sheds light on the causes for high youth unemployment. High youth unemployment is jointly explained by the fact that they enter the labour market as unemployed and because they have to compete for a job position with more productive workers.

The model is also successful in explaining the fact that as young people get older their employability situation improves, reducing their unemployment levels. Additionally, the model is able to predict roughly - as the stylized facts show - that by the 28 years of age unemployment tends to converge. The result of this research is interesting from the economic policy point of view. The model shows that the pattern observed in youth unemployment is the most efficient market allocation for the young people entering the labour market.

Given the above, this research theoretically discards that active policies in the labour market for young people, make the economy better, as instead generate a welfare loss. Ultimately, this research concludes that the high youth unemployment compared to adult unemployment does not represent a real economic problem, as it is in essence, a temporary problems following the natural transition between education and work.

Appendix

2.A Probability of Matching

Let z_m^n be defined as the number of type m workers arrive at sub-market n. Remember that in this research the Poisson's distribution will be used $f(z,\lambda) = \frac{\lambda^z e^{-\lambda}}{z!}$ to calculate de distribution of probability that z_m^n workers apply to a sub-market type n where λ is the application queue $(q(m,n))$, with $m \in \{1,2\}$ and $n \in \{1,2\}$.

2.A.1 Young Workers

Denote as A the set determines that at least one young worker $(m = 1)$ arrives at sub-market type n, that is $A = \{z_1^n \in \mathbb{N}_0 : z_1^n > 0\}$. Also denote as B the set determines that no adult worker arrives at the sub-market type n, that is $B = \{z_1^n \in \mathbb{N}_0 : z_2^n = 0\}$. Note also that A and B are two independent events and a young worker is hired only if no worker applies for the firm. Then the probability that a type n firm hires a worker type 1, that is $\mathbb{P}(A \cap B)$, is calculated by the following procedure:

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B) \tag{2.A.1}$$

where,

$$\mathbb{P}(A) = \mathbb{P}(z_1^n > 0) = 1 - \mathbb{P}(z_1^n = 0) = 1 - \left(\frac{1}{0!}q(1,n)^0 e^{-q(1,n)}\right) = 1 - e^{-q(1,n)} \tag{2.A.2}$$

$$\mathbb{P}(B) = \mathbb{P}(z_2^n = 0) = \left(\frac{1}{0!}q(2,n)^0 e^{-q(2,n)}\right) = e^{-q(2,n)} \tag{2.A.3}$$

So by replacing the results of equations (2.A.2) and (2.A.3) in equation (2.A.1) is possible to find the probability that a firm type n hires a worker type 1:

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B) = e^{-q(2,n)}\left(1 - e^{-q(1,n)}\right) \tag{2.A.4}$$

2.A.2 Old Workers

Denote as A the set determines that at least one old worker ($m = 2$) arrives at sub-market type n, that is $C = \{z_2^n \in \mathbb{N}_0 : z_2^n > 0\}$. Then the probability that a type n firm hires a worker type 2, that is $\mathbb{P}(C)$, is calculated by the following procedure:

$$\mathbb{P}(C) = \mathbb{P}(z_2^n > 0) = 1 - \mathbb{P}(z_2^n = 0) = 1 - \left(\frac{1}{0!}q(2,n)^0 e^{-q(2,n)}\right) = 1 - e^{-q(2,n)} \quad (2.A.5)$$

2.B Static Matching Problem

- Low-tech firms

Low-tech firms maximise the following objective function:

$$\max_{\{w_t(1,1), w_t(2,1)\}} \Pi_{1,t} = p_z e^{-q_t(2,1)}\big(1 - e^{-q_t(1,1)}\big)\{X_t(1,1) - w_t(1,1)\} \\ + p_z\big(1 - e^{-q_t(2,1)}\big)\{X_t(2,1) - w_t(2,1)\} \quad (2.B.1)$$

But I know that:

$$\vartheta_{1,t} q_t(1,1) = p_z e^{-q_t(2,1)}\big(1 - e^{-q_t(1,1)}\big) w_t(1,1) \quad (2.B.2)$$

$$\vartheta_{2,t} q_t(2,1) = p_z\big(1 - e^{-q_t(2,1)}\big) w_t(2,1) \quad (2.B.3)$$

Then the maximisation problem for a low-tech firm can be expressed as:

$$\max_{\{q_t(1,1), q_t(2,1)\}} \Pi_{1,t} = p_z e^{-q_t(2,1)}\big(1 - e^{-q_t(1,1)}\big) X_t(1,1) - \vartheta_{1,t} q_t(1,1) \\ + p_z\big(1 - e^{-q_t(2,1)}\big) X_t(2,1) - \vartheta_{2,t} q_t(2,1) \quad (2.B.4)$$

- High-tech firms

High-tech firms maximise the following objective function:

$$\max_{\{w_t(1,2), w_t(2,2)\}} \Pi_{2,t} = p_z e^{-q_t(2,2)}\big(1 - e^{-q_t(1,2)}\big)\{X_t(1,2) - w_t(1,2)\} \\ + p_z\big(1 - e^{-q_t(2,2)}\big)\{X_t(2,2) - w_t(2,2)\} \quad (2.B.5)$$

But I know that:

$$\vartheta_{1,t} q_t(1,2) = p_z e^{-q_t(2,2)} \big(1 - e^{-q_t(1,2)}\big) w_t(1,2) \tag{2.B.6}$$

$$\vartheta_{2,t} q_t(2,2) = p_z \big(1 - e^{-q_t(2,2)}\big) w_t(2,2) \tag{2.B.7}$$

Then the maximisation problems for a low-tech firm can be expressed as:

$$\begin{aligned} \max_{\{q_t(1,2), q_t(2,2)\}} \Pi_{2,t} = p_z e^{-q_t(2,2)} \big(1 - e^{-q_t(1,2)}\big) X_t(1,2) - \vartheta_{1,t} q_t(1,2) \\ + p_z \big(1 - e^{-q_t(2,2)}\big) X_t(2,2) - \vartheta_{2,t} q_t(2,2) \end{aligned} \tag{2.B.8}$$

2.B.1 Interior Solution

- Low-tech firms

Low-tech firms maximise:

$$\begin{aligned} \max_{\{q_t(1,1), q_t(2,1)\}} \Pi_{1,t} = p_z e^{-q_t(2,1)} \big(1 - e^{-q_t(1,1)}\big) X_t(1,1) - \vartheta_{1,t} q_t(1,1) \\ + p_z \big(1 - e^{-q_t(2,1)}\big) X_t(2,1) - \vartheta_{2,t} q_t(2,1) \end{aligned} \tag{2.B.9}$$

Then the first order conditions are:

$$\frac{\partial \Pi_{1,t}}{\partial q_t(1,1)} = p_z e^{-q_t(1,1) - q_t(2,1)} X_t(1,1) - \vartheta_{1,t} = 0 \tag{2.B.10}$$

$$\begin{aligned} \frac{\partial \Pi_{1,t}}{\partial q_t(2,1)} = -p_z e^{-q_t(2,1)} \big(1 - e^{-q_t(1,1)}\big) X_t(1,1) + \\ p_z e^{-q_t(2,1)} X_t(2,1) - \vartheta_{2,t} = 0 \end{aligned} \tag{2.B.11}$$

- High-tech firms

High-tech firms maximise:

$$\begin{aligned} \max_{\{q_t(1,2), q_t(2,2)\}} \Pi_{2,t} = p_z e^{-q_t(2,2)} \big(1 - e^{-q_t(1,2)}\big) X_t(1,2) - \vartheta_{1,t} q_t(1,2) \\ + p_z \big(1 - e^{-q_t(2,2)}\big) X_t(2,2) - \vartheta_{2,t} q_t(2,2) \end{aligned} \tag{2.B.12}$$

Then the first order conditions are:

$$\frac{\partial \Pi_{2,t}}{\partial q_t(1,2)} = p_z e^{-q_t(1,2)-q_t(2,2)} X_t(1,2) - \vartheta_{1,t} = 0 \tag{2.B.13}$$

$$\begin{aligned}\frac{\partial \Pi_{2,t}}{\partial q_t(2,2)} &= -p_z e^{-q_t(2,2)}\big(1-e^{-q_t(1,2)}\big) X_t(1,2) \\ &+p_z e^{-q_t(2,2)} X_t(2,2) - \vartheta_{2,t} = 0\end{aligned} \tag{2.B.14}$$

- Nonlinear Static Equilibrium

Then the static equilibrium in t must satisfy the following six equations:

$$p_z e^{-q_t(1,1)-q_t(2,1)} X_t(1,1) = \vartheta_{1,t} \tag{2.B.15}$$

$$-p_z e^{-q_t(2,1)}\big(1-e^{-q_t(1,1)}\big) X_t(1,1) + p_z e^{-q_t(2,1)} X_t(2,1) = \vartheta_{2,t} \tag{2.B.16}$$

$$p_z e^{-q_t(1,2)-q_t(2,2)} X_t(1,2) = \vartheta_{1,t} \tag{2.B.17}$$

$$-p_z e^{-q_t(2,2)}\big(1-e^{-q_t(1,2)}\big) X_t(1,2) + p_z e^{-q_t(2,2)} X_t(2,2) = \vartheta_{2,t} \tag{2.B.18}$$

$$q_t(1,1)N_{1,t} + q_t(1,2)N_{2,t} = M_{1,t} \tag{2.B.19}$$

$$q_t(2,1)N_{1,t} + q_t(2,2)N_{2,t} = M_{2,t} \tag{2.B.20}$$

- Linear Static Equilibrium

By matching the first and third equation in the nonlinear equilibrium, I have:

$$\begin{aligned} e^{-q_t(1,1)-q_t(2,1)} X_t(1,1) &= e^{-q_t(1,2)-q_t(2,2)} X_t(1,2) \\ e^{-q_t(1,1)-q_t(2,1)+q_t(1,2)+q_t(2,2)} &= \frac{X_t(1,2)}{X_t(1,1)} \end{aligned}$$

then applying logarithms on both sides of the equation:

$$-q_t(1,1) - q_t(2,1) + q_t(1,2) + q_t(2,2) = ln\Big(\frac{X_t(1,2)}{X_t(1,1)}\Big) \tag{2.B.21}$$

By matching the second and fourth equations in the nonlinear equilibrium, I have:

$$\begin{aligned} &-e^{-q_t(2,1)}\big(1-e^{-q_t(1,1)}\big) X_t(1,1) + e^{-q_t(2,1)} X_t(2,1) = \\ &\quad -e^{-q_t(2,2)}\big(1-e^{-q_t(1,2)}\big) X_t(1,2) + e^{-q_t(2,2)} X_t(2,2) \end{aligned}$$

expanding the brackets:

$$
\begin{aligned}
&-e^{-q_t(2,1)}X_t(1,1) + e^{-q_t(1,1)-q_t(2,1)}X_t(1,1) + e^{-q_t(2,1)}X_t(2,1) = \\
&\quad -e^{-q_t(2,2)}X_t(1,2) + e^{-q_t(1,2)-q_t(2,2)}X_t(1,2) + e^{-q_t(2,2)}X_t(2,2)
\end{aligned}
$$

simplifying, I have:

$$
-e^{-q_t(2,1)}X_t(1,1) + e^{-q_t(2,1)}X_t(2,1) = -e^{-q_t(2,2)}X_t(1,2) + e^{-q_t(2,2)}X_t(2,2)
$$

and grouping terms:

$$
\begin{aligned}
e^{-q_t(2,1)}(X_t(2,1) - X_t(1,1)) &= e^{-q_t(2,2)}(X_t(2,2) - X_t(1,2)) \\
e^{-q_t(2,1)+q_t(2,2)} &= \frac{(X_t(2,2) - X_t(1,2))}{(X_t(2,1) - X_t(1,1))} \qquad (2.B.22)
\end{aligned}
$$

then applying logarithms on both sides of the above equation:

$$
-q_t(2,1) + q_t(2,2) = ln\Big(\frac{X_t(2,2) - X_t(1,2)}{X_t(2,1) - X_t(1,1)}\Big) \qquad (2.B.23)
$$

The linear system is:

$$
\begin{aligned}
-q_t(1,1) - q_t(2,1) + q_t(1,2) + q_t(2,2) &= ln\Big(\frac{X_t(1,2)}{X_t(1,1)}\Big) && (2.B.24) \\
-q_t(2,1) + q_t(2,2) &= ln\Big(\frac{X_t(2,2) - X_t(1,2)}{X_t(2,1) - X_t(1,1)}\Big) && (2.B.25) \\
q_t(1,1)N_{1,t} + q_t(1,2)N_{2,t} &= M_{1,t} && (2.B.26) \\
q_t(2,1)N_{1,t} + q_t(2,2)N_{2,t} &= M_{2,t} && (2.B.27)
\end{aligned}
$$

in matrix form:

$$
\begin{bmatrix} -1 & -1 & 1 & 1 \\ 0 & -1 & 0 & 1 \\ N_{1,t} & 0 & N_{2,t} & 0 \\ 0 & N_{1,t} & 0 & N_{2,t} \end{bmatrix} \cdot \begin{bmatrix} q_t(1,1) \\ q_t(2,1) \\ q_t(1,2) \\ q_t(2,2) \end{bmatrix} = \begin{bmatrix} ln\Big(\frac{X_t(1,2)}{X_t(1,1)}\Big) \\ ln\Big(\frac{X_t(2,2)-X_t(1,2)}{X_t(2,1)-X_t(1,1)}\Big) \\ M_{1,t} \\ M_{2,t} \end{bmatrix} \qquad (2.B.28)
$$

2.B.2 Corner Solution with One Queue Equal to Zero

Case $q_t(1,1) = 0$

If $q_t(1,1) = 0$ then young workers do not apply to low-tech firms, then by using the resource constraint equation I have that $q_t^*(1,2) = \frac{M_{1,t}}{N_{2,t}}$. Also I can conclude that $\vartheta_{1,t} = 0$, $w_t(1,1) = 0$ and $w_t(1,2) = 0$. The new profit optimisation for low-tech and high tech firms and static equilibrium can be expressed as follow.

- Low-tech firms

  Low-tech firms maximise:

$$\max_{\{q_t(2,1)\}} \Pi_{1,t} = p_z\big(1 - e^{-q_t(2,1)}\big)X_t(2,1) - \vartheta_{2,t}q_t(2,1) \tag{2.B.29}$$

  Then the first order condition is:

$$\frac{\partial \Pi_{1,t}}{\partial q_t(2,1)} = p_z e^{-q_t(2,1)} X_t(2,1) - \vartheta_{2,t} = 0 \tag{2.B.30}$$

- High-tech firms

  High-tech firms maximise:

$$\begin{aligned}\max_{\{q_t(2,2)\}} \Pi_{2,t} = {} & p_z e^{-q_t(2,2)}\big(1 - e^{-q_t^*(1,2)}\big)X_t(1,2) - 0q_t^*(1,2) \\ & + p_z\big(1 - e^{-q_t(2,2)}\big)X_t(2,2) - \vartheta_{2,t}q_t(2,2)\end{aligned} \tag{2.B.31}$$

  Then the first order condition is:

$$\begin{aligned}\frac{\partial \Pi_{2,t}}{\partial q_t(2,2)} = {} & -p_z e^{-q_t(2,2)}\big(1 - e^{-q_t^*(1,2)}\big)X_t(1,2) \\ & + p_z e^{-q_t(2,2)} X_t(2,2) - \vartheta_{2,t} = 0\end{aligned} \tag{2.B.32}$$

- Nonlinear Static Equilibrium

Then the static equilibrium in t must satisfy the following three equations:

$$p_z e^{-q_t(2,1)} X_t(2,1) = \vartheta_{2,t} \tag{2.B.33}$$
$$-p_z e^{-q_t(2,2)}\big(1 - e^{-q_t^*(1,2)}\big) X_t(1,2) + p_z e^{-q_t(2,2)} X_t(2,2) = \vartheta_{2,t} \tag{2.B.34}$$
$$q_t(2,1) N_{1,t} + q_t(2,2) N_{2,t} = M_{2,t} \tag{2.B.35}$$

- Linear Static Equilibrium

By matching the first and second equations in the nonlinear equilibrium, I have:

$$\begin{aligned} e^{-q_t(2,1)} X_t(2,1) &= -e^{-q_t(2,2)}(1 - e^{-q_t^*(1,2)}) X_t(1,2) + e^{-q_t(2,2)} X_t(2,2) \\ e^{-q_t(2,1)} X_t(2,1) &= e^{-q_t(2,2)}\big((e^{-q_t^*(1,2)} - 1) X_t(1,2) + X_t(2,2)\big) \\ e^{-q_t(2,1)+q_t(2,2)} &= \Big(\frac{(e^{-q_t^*(1,2)} - 1) X_t(1,2)}{X_t(2,1)} + \frac{X_t(2,2)}{X_t(2,1)}\Big) \end{aligned}$$

then applying logarithms on both sides of the equation:

$$-q_t(2,1) + q_t(2,2) = ln\Big(\frac{(e^{-q_t^*(1,2)} - 1) X_t(1,2)}{X_t(2,1)} + \frac{X_t(2,2)}{X_t(2,1)}\Big)$$

The linear system is:

$$-q_t(2,1) + q_t(2,2) = ln\Big(\frac{(e^{-q_t^*(1,2)} - 1) X_t(1,2)}{X_t(2,1)} + \frac{X_t(2,2)}{X_t(2,1)}\Big) \tag{2.B.36}$$
$$q_t(2,1) N_{1,t} + q_t(2,2) N_{2,t} = M_{2,t} \tag{2.B.37}$$

in matrix form:

$$\begin{bmatrix} -1 & 1 \\ N_{1,t} & N_{2,t} \end{bmatrix} \cdot \begin{bmatrix} q_t(2,1) \\ q_t(2,2) \end{bmatrix} = \begin{bmatrix} ln\Big(\frac{(e^{-q_t^*(1,2)} - 1) x_t(1,2)}{X_t(2,1)} + \frac{X_t(2,2)}{X_t(2,1)}\Big) \\ M_{2,t} \end{bmatrix} \tag{2.B.38}$$

Case $q_t(1,2) = 0$

If $q_t(1,2) = 0$ then young workers do not apply to high-tech firms, then by using the resource constraint equation I have that $q_t^*(1,1) = \frac{M_{1,t}}{N_{1,t}}$. Also I can conclude that $\vartheta_{1,t} = 0$, $w_t(1,1) = 0$ and $w_t(1,2) = 0$. The new profit optimisation for low-tech and high tech firms and static equilibrium can be expressed as follows.

- Low-tech firms

Low-tech firms maximise:

$$\max_{\{q_t(2,1)\}} \Pi_{1,t} = p_z e^{-q_t(2,1)}\big(1 - e^{-q_t^*(1,1)}\big)X_t(1,1) - 0q_t^*(1,1) \\ + p_z\big(1 - e^{-q_t(2,1)}\big)X_t(2,1) - \vartheta_{2,t}q_t(2,1) \tag{2.B.39}$$

Then the first order condition is:

$$\frac{\partial \Pi_{1,t}}{\partial q_t(2,1)} = -p_z e^{-q_t(2,1)}\big(1 - e^{-q_t^*(1,1)}\big)X_t(1,1) \\ + p_z e^{-q_t(2,1)}X_t(2,1) - \vartheta_{2,t} = 0 \tag{2.B.40}$$

- High-tech firms

High-tech firms maximise:

$$\max_{\{q_t(2,2)\}} \Pi_{2,t} = p_z\big(1 - e^{-q_t(2,2)}\big)X_t(2,2) - \vartheta_{2,t}q_t(2,2) \tag{2.B.41}$$

Then the first order condition is:

$$\frac{\partial \Pi_{2,t}}{\partial q_t(2,2)} = p_z e^{-q_t(2,2)}X_t(2,2) - \vartheta_{2,t} = 0 \tag{2.B.42}$$

- Nonlinear Static Equilibrium

Then the static equilibrium in t must satisfy the following three equations:

$$-p_z e^{-q_t(2,1)}\big(1 - e^{-q_t^*(1,1)}\big)X_t(1,1) + p_z e^{-q_t(2,1)}X_t(2,1) = \vartheta_{2,t} \tag{2.B.43}$$

$$p_z e^{-q_t(2,2)}X_t(2,2) = \vartheta_{2,t} \tag{2.B.44}$$

$$q_t(2,1)N_{1,t} + q_t(2,2)N_{2,t} = M_{2,t} \tag{2.B.45}$$

- Linear Static Equilibrium

By matching the first and second equations in the nonlinear equilibrium, I have:

$$\begin{aligned}
-e^{-q_t(2,1)}(1-e^{-q_t^*(1,1)})X_t(1,1)+e^{-q_t(2,1)}X_t(2,1) &= e^{-q_t(2,2)}X_t(2,2)\\
e^{-q_t(2,1)}\big((e^{-q_t^*(1,1)}-1)X_t(1,1)+X_t(2,1)\big) &= e^{-q_t(2,2)}X_t(2,2)\\
e^{-q_t(2,1)+q_t(2,2)} &= \frac{X_t(2,2)}{(e^{-q_t^*(1,1)}-1)X_t(1,1)+X_t(2,1)}
\end{aligned}$$

then applying logarithms on both sides of the equation:

$$-q_t(2,1)+q_t(2,2)=ln\Big(\frac{X_t(2,2)}{(e^{-q_t^*(1,1)}-1)X_t(1,1)+X_t(2,1)}\Big)$$

and multiplying both sides of the equation by -1:

$$q_t(2,1)-q_t(2,2)=ln\Big(\frac{(e^{-q_t^*(1,1)}-1)X_t(1,1)+X_t(2,1)}{X_t(2,2)}\Big)$$

The linear system is:

$$q_t(2,1)-q_t(2,2)=ln\Big(\frac{(e^{-q_t^*(1,1)}-1)X_t(1,1)}{X_t(2,2)}+\frac{X_t(2,1)}{X_t(2,2)}\Big) \tag{2.B.46}$$

$$q_t(2,1)N_{1,t}+q_t(2,2)N_{2,t}=M_{2,t} \tag{2.B.47}$$

in matrix form:

$$\begin{bmatrix} 1 & -1 \\ N_{1,t} & N_{2,t} \end{bmatrix} \cdot \begin{bmatrix} q_t(2,1) \\ q_t(2,2) \end{bmatrix} = \begin{bmatrix} ln\Big(\frac{(e^{-q_t^*(1,1)}-1)X_t(1,1)}{X_t(2,2)}+\frac{X_t(2,1)}{X_t(2,2)}\Big) \\ M_{2,t} \end{bmatrix} \tag{2.B.48}$$

Case $q_t(2,1)=0$

If $q_t(2,1)=0$ then old workers do not apply to low-tech firms, then by using the resource constraint equation I have that $q_t^*(2,2)=\frac{M_{2,t}}{N_{2,t}}$. Also I can conclude that $\vartheta_{2,t}=0$, $w_t(2,1)=0$ and $w_t(2,2)=0$. The new profit optimisation for low-tech and high tech firms and static equilibrium can be expressed as follows.

- Low-tech firms

Low-tech firms maximise:

$$\max_{\{q_t(1,1)\}} \Pi_{1,t}=p_z\big(1-e^{-q_t(1,1)}\big)X_t(1,1)-\vartheta_{1,t}q_t(1,1) \tag{2.B.49}$$

Then the first order condition is:

$$\frac{\partial \Pi_{1,t}}{\partial q_t(1,1)} = p_z e^{-q_t(1,1)} X_t(1,1) - \vartheta_{1,t} = 0 \tag{2.B.50}$$

- High-tech firms

High-tech firms maximise:

$$\begin{aligned} \max_{\{q_t(1,2)\}} \Pi_{2,t} = p_z e^{-q_t^*(2,2)} \big(1 - e^{-q_t(1,2)}\big) X_t(1,2) - \vartheta_{1,t} q_t(1,2) \\ + p_z \big(1 - e^{-q_t^*(2,2)}\big) X_t(2,2) - 0 q_t^*(2,2) \end{aligned} \tag{2.B.51}$$

Then the first order condition is:

$$\frac{\partial \Pi_{2,t}}{\partial q_t(1,2)} = p_z e^{-q_t(1,2) - q_t^*(2,2)} X_t(1,2) - \vartheta_{1,t} = 0 \tag{2.B.52}$$

- Nonlinear Static Equilibrium

Then the static equilibrium in t must satisfy the following three equations:

$$p_z e^{-q_t(1,1)} X_t(1,1) = \vartheta_{1,t} \tag{2.B.53}$$

$$p_z e^{-q_t(1,2) - q_t^*(2,2)} X_t(1,2) = \vartheta_{1,t} \tag{2.B.54}$$

$$q_t(1,1) N_{1,t} + q_t(1,2) N_{2,t} = M_{1,t} \tag{2.B.55}$$

- Linear Static Equilibrium

By matching the first and second equations in the nonlinear equilibrium, I have:

$$\begin{aligned} e^{-q_t(1,1)} X_t(1,1) &= e^{-q_t(1,2) - q_t^*(2,2)} X_t(1,2) \\ e^{-q_t(1,1) + q_t(1,2)} X_t(1,1) &= e^{-q_t^*(2,2)} X_t(1,2) \\ e^{-q_t(1,1) + q_t(1,2)} &= e^{-q_t^*(2,2)} \frac{X_t(1,2)}{X_t(1,1)} \end{aligned}$$

then applying logarithms on both sides of the equation:

$$-q_t(1,1) + q_t(1,2) = -q_t^*(2,2) + ln\Big(\frac{X_t(1,2)}{X_t(1,1)}\Big)$$

The linear system is:

$$-q_t(1,1) + q_t(1,2) = -q_t^*(2,2) + ln\Big(\frac{X_t(1,2)}{X_t(1,1)}\Big) \tag{2.B.56}$$

$$q_t(1,1)N_{1,t} + q_t(1,2)N_{2,t} = M_{1,t} \tag{2.B.57}$$

in matrix form:

$$\begin{bmatrix} -1 & 1 \\ N_{1,t} & N_{2,t} \end{bmatrix} \cdot \begin{bmatrix} q_t(1,1) \\ q_t(1,2) \end{bmatrix} = \begin{bmatrix} -q_t^*(2,2) + ln\Big(\frac{X_t(1,2)}{X_t(1,1)}\Big) \\ M_{1,t} \end{bmatrix} \tag{2.B.58}$$

Or [5]:

$$\begin{bmatrix} 1 & -1 \\ N_{1,t} & N_{2,t} \end{bmatrix} \cdot \begin{bmatrix} q_t(1,1) \\ q_t(1,2) \end{bmatrix} = \begin{bmatrix} q_t^*(2,2) + ln\Big(\frac{X_t(1,1)}{X_t(1,2)}\Big) \\ M_{1,t} \end{bmatrix} \tag{2.B.59}$$

Case $q_t(2,2) = 0$

If $q_t(2,2) = 0$ then old workers do not apply to high-tech firms, then by using the resource constraint equation I have that $q_t^*(2,1) = \frac{M_{2,t}}{N_{1,t}}$. Also I can conclude that $\vartheta_{2,t} = 0$, $w_t(2,1) = 0$ and $w_t(2,2) = 0$. The new profit optimisation for low-tech and high tech firms and static equilibrium can be expressed as follows.

- Low-tech firms

  Low-tech firms maximise:

$$\begin{aligned} \max_{\{q_t(1,1)\}} \Pi_{1,t} = p_z e^{-q_t^*(2,1)}\big(1 - e^{-q_t(1,1)}\big)X_t(1,1) - \vartheta_{1,t}q_t(1,1) \\ +p_z\big(1 - e^{-q_t^*(2,1)}\big)X_t(2,1) - 0q_t^*(2,1) \end{aligned} \tag{2.B.60}$$

  Then the first order condition:

$$\frac{\partial \Pi_{1,t}}{\partial q_t(1,1)} = p_z e^{-q_t(1,1)-q_t^*(2,1)}X_t(1,1) - \vartheta_{1,t} = 0 \tag{2.B.61}$$

- High-tech firms

[5] Note that the system (2.B.58) is identical to the system (2.B.59)

High-tech firms maximise:

$$\max_{\{q_t(1,2)\}} \Pi_{2,t} = p_z\big(1 - e^{-q_t(1,2)}\big)X_t(1,2) - \vartheta_{1,t}q_t(1,2) \tag{2.B.62}$$

Then the first order condition:

$$\frac{\partial \Pi_{2,t}}{\partial q_t(1,2)} = p_z e^{-q_t(1,2)}X_t(1,2) - \vartheta_{1,t} = 0 \tag{2.B.63}$$

- Nonlinear Static Equilibrium

Then the static equilibrium in t must satisfy the following three equations:

$$p_z e^{-q_t(1,1)-q_t^*(2,1)}X_t(1,1) = \vartheta_{1,t} \tag{2.B.64}$$

$$p_z e^{-q_t(1,2)}X_t(1,2) = \vartheta_{1,t} \tag{2.B.65}$$

$$q_t(1,1)N_{1,t} + q_t(1,2)N_{2,t} = M_{1,t} \tag{2.B.66}$$

- Linear Static Equilibrium

By matching the first and second equations in the nonlinear equilibrium, I have:

$$\begin{aligned} e^{-q_t(1,1)-q_t^*(2,1)}X_t(1,1) &= e^{-q_t(1,2)}X_t(1,2) \\ e^{-q_t(1,1)+q_t(1,2)}X_t(1,1) &= e^{q_t^*(2,1)}X_t(1,2) \\ e^{-q_t(1,1)+q_t(1,2)} &= e^{q_t^*(2,1)}\frac{X_t(1,2)}{X_t(1,1)} \end{aligned}$$

then applying logarithms on both sides of the equation:

$$-q_t(1,1) + q_t(1,2) = q_t^*(2,1) + ln\Big(\frac{X_t(1,2)}{X_t(1,1)}\Big)$$

The linear system is:

$$-q_t(1,1) + q_t(1,2) = q_t^*(2,1) + ln\Big(\frac{X_t(1,2)}{X_t(1,1)}\Big) \tag{2.B.67}$$

$$q_t(1,1)N_{1,t} + q_t(1,2)N_{2,t} = M_{1,t} \tag{2.B.68}$$

in matrix form:

$$\begin{bmatrix} -1 & 1 \\ N_{1,t} & N_{2,t} \end{bmatrix} \cdot \begin{bmatrix} q_t(1,1) \\ q_t(1,2) \end{bmatrix} = \begin{bmatrix} q_t^*(2,1) + ln\left(\frac{X_t(1,2)}{X_t(1,1)}\right) \\ M_{1,t} \end{bmatrix} \tag{2.B.69}$$

2.B.3 Corner Solution with Two Queues Equal to Zero

Case $q_t(1,1) = 0$ and $q_t(1,2) = 0$: Young workers do not participate.

If $q_t(1,1) = 0$ and $q_t(1,2) = 0$ then young workers do not apply to any firms. Also I can conclude that $\vartheta_{1,t} = 0$, $w_t(1,1) = 0$ and $w_t(1,2) = 0$. The new profit optimisation for low-tech and high tech firms and static equilibrium can be expressed as follows.

- Low-tech firms

  Low-tech firms maximise:

$$\max_{\{q_t(2,1)\}} \Pi_{1,t} = p_z\left(1 - e^{-q_t(2,1)}\right) X_t(2,1) - \vartheta_{2,t} q_t(2,1) \tag{2.B.70}$$

  Then the first order condition:

$$\frac{\partial \Pi_{1,t}}{\partial q_t(2,1)} = p_z e^{-q_t(2,1)} X_t(2,1) - \vartheta_{2,t} = 0 \tag{2.B.71}$$

- High-tech firms

  High-tech firms maximise:

$$\max_{\{q_t(2,2)\}} \Pi_{2,t} = p_z\left(1 - e^{-q_t(2,2)}\right) X_t(2,2) - \vartheta_{2,t} q_t(2,2) \tag{2.B.72}$$

  Then the first order condition:

$$\frac{\partial \Pi_{2,t}}{\partial q_t(2,2)} = p_z e^{-q_t(2,2)} X_t(2,2) - \vartheta_{2,t} = 0 \tag{2.B.73}$$

- Nonlinear Static Equilibrium

Then the static equilibrium in t must satisfy the following three equations:

$$p_z e^{-q_t(2,1)} X_t(2,1) = \vartheta_{2,t} \tag{2.B.74}$$
$$p_z e^{-q_t(2,2)} X_t(2,2) = \vartheta_{2,t} \tag{2.B.75}$$
$$q_t(2,1)N_{1,t} + q_t(2,2)N_{2,t} = M_{2,t} \tag{2.B.76}$$

- Linear Static Equilibrium

By matching the first and second equations in the nonlinear equilibrium, I have:

$$e^{-q_t(2,1)} X_t(2,1) = e^{-q_t(2,2)} X_t(2,2)$$
$$e^{-q_t(2,1)+q_t(2,2)} = \frac{X_t(2,2)}{X_t(2,1)}$$

then applying logarithms on both sides of the equation:

$$-q_t(2,1) + q_t(2,2) = ln\Big(\frac{X_t(2,2)}{X_t(2,1)}\Big)$$

The linear system is:

$$-q_t(2,1) + q_t(2,2) = ln\Big(\frac{X_t(2,2)}{X_t(2,1)}\Big) \tag{2.B.77}$$
$$q_t(2,1)N_{1,t} + q_t(2,2)N_{2,t} = M_{2,t} \tag{2.B.78}$$

in matrix form:

$$\begin{bmatrix} -1 & 1 \\ N_{1,t} & N_{2,t} \end{bmatrix} \cdot \begin{bmatrix} q_t(2,1) \\ q_t(2,2) \end{bmatrix} = \begin{bmatrix} ln\Big(\frac{X_t(2,2)}{X_t(2,1)}\Big) \\ M_{2,t} \end{bmatrix} \tag{2.B.79}$$

Case $q_t(2,1) = 0$ and $q_t(2,2) = 0$: Old workers do not participate.

If $q_t(2,1) = 0$ and $q_t(2,2) = 0$ then old workers do not apply to any firms. Also I can conclude that $\vartheta_{2,t} = 0$, $w_t(2,1) = 0$ and $w_t(2,2) = 0$. The new profit optimisation for low-tech and high tech firms and static equilibrium can be expressed as follows.

- Low-tech firms

Low-tech firms maximise:

$$\max_{\{q_t(1,1)\}} \Pi_{1,t} = p_z\big(1 - e^{-q_t(1,1)}\big) X_t(1,1) - \vartheta_{1,t} q_t(1,1) \tag{2.B.80}$$

Then the first order condition:

$$\frac{\partial \Pi_{1,t}}{\partial q_t(1,1)} = p_z e^{-q_t(1,1)} X_t(1,1) - \vartheta_{1,t} = 0 \tag{2.B.81}$$

- High-tech firms

High-tech firms maximise:

$$\max_{\{q_t(1,2)\}} \Pi_{2,t} = p_z\big(1 - e^{-q_t(1,2)}\big) X_t(1,2) - \vartheta_{1,t} q_t(1,2) \tag{2.B.82}$$

Then the first order condition:

$$\frac{\partial \Pi_{2,t}}{\partial q_t(1,2)} = p_z e^{-q_t(1,2)} X_t(1,2) - \vartheta_{1,t} = 0 \tag{2.B.83}$$

- Nonlinear Static Equilibrium

Then the static equilibrium in t must satisfy the following three equations:

$$p_z e^{-q_t(1,1)} X_t(1,1) = \vartheta_{1,t} \tag{2.B.84}$$
$$p_z e^{-q_t(1,2)} X_t(1,2) = \vartheta_{1,t} \tag{2.B.85}$$
$$q_t(1,1) N_{1,t} + q_t(1,2) N_{2,t} = M_{1,t} \tag{2.B.86}$$

- Linear Static Equilibrium

By matching the first and second equations in the nonlinear equilibrium, I have:

$$e^{-q_t(1,1)} X_t(1,1) = e^{-q_t(1,2)} X_t(1,2)$$
$$e^{-q_t(1,1)+q_t(1,2)} = \frac{X_t(1,2)}{X_t(1,1)}$$

then applying logarithm on both sides of the equation:

$$-q_t(1,1) + q_t(1,2) = ln\Big(\frac{X_t(1,2)}{X_t(1,1)}\Big)$$

The linear system is:

$$-q_t(1,1) + q_t(1,2) = ln\Big(\frac{X_t(1,2)}{X_t(1,1)}\Big) \tag{2.B.87}$$

$$q_t(1,1)N_{1,t} + q_t(1,2)N_{2,t} = M_{1,t} \tag{2.B.88}$$

in matrix form:

$$\begin{bmatrix} -1 & 1 \\ N_{1,t} & N_{2,t} \end{bmatrix} \cdot \begin{bmatrix} q_t(1,1) \\ q_t(1,2) \end{bmatrix} = \begin{bmatrix} ln\Big(\frac{X_t(1,2)}{X_t(1,1)}\Big) \\ M_{1,t} \end{bmatrix} \tag{2.B.89}$$

Case $q_t(1,1) = 0$ and $q_t(2,1) = 0$: No workers apply to low-tech firms.

If $q_t(1,1) = 0$ and $q_t(2,1) = 0$ then no workers apply to low-tech firms. Then for the resource constraint I have that $q^*_t(1,2) = \frac{M_{1,t}}{N_{2,t}}$ and $q^*_t(2,2) = \frac{M_{2,t}}{N_{2,t}}$.

Case $q_t(1,2) = 0$ and $q_t(2,2) = 0$: No workers apply to high-tech firms.

If $q_t(1,2) = 0$ and $q_t(2,2) = 0$ mean no worker apply to low-tech firms. Then for the resource constraint I have that $q^*_t(1,1) = \frac{M_{1,t}}{N_{1,t}}$ and $q^*_t(2,1) = \frac{M_{2,t}}{N_{1,t}}$.

Case $q_t(1,1) = 0$ and $q_t(2,2) = 0$: Young workers only participate in the high-tech market and old workers only participate in low-tech market.

If $q_t(1,1) = 0$ and $q_t(2,2) = 0$ then young worker only participate in the high-tech market and old workers only participate in the low-tech market. Then for the resource constraint I have that if $q_t(1,1) = 0$ then $q^*_t(1,2) = \frac{M_{1,t}}{N_{2,t}}$. Moreover, if I have that $q_t(2,2) = 0$ then for the resource constraint I have that $q^*_t(2,1) = \frac{M_{2,t}}{N_{1,t}}$.

Case $q_t(1,2) = 0$ and $q_t(2,1) = 0$: Young workers only participate in the low-tech market and old workers only participate in the high-tech market.

If $q_t(1,2) = 0$ and $q_t(2,1) = 0$ then young workers only participate in the low-tech market and old workers only participate in the high-tech market. Then for the resource constraint I have that if $q_t(1,2) = 0$ then $q^*_t(1,1) = \frac{M_{1,t}}{N_{1,t}}$. Moreover, if I have that $q_t(2,1) = 0$ then for the resource constraint I have that $q^*_t(2,2) = \frac{M_{2,t}}{N_{2,t}}$.

2.B.4 Corner Solution with only One Active Market

Case $q_t(1,1) \neq 0$ and all other queues are equal to zero

By the resource constraint I have that $q_t^*(1,1) = \frac{M_{1,t}}{N_{1,t}}$.

Case $q_t(1,2) \neq 0$ and all other queues are equal to zero

By the resource constraint I have that $q_t^*(1,2) = \frac{M_{1,t}}{N_{2,t}}$.

Case $q_t(2,1) \neq 0$ and all other queues are equal to zero

By the resource constraint I have that $q_t^*(2,1) = \frac{M_{2,t}}{N_{1,t}}$.

Case $q_t(2,2) \neq 0$ and all other queues are equal to zero

By the resource constraint I have that $q_t^*(2,2) = \frac{M_{2,t}}{N_{2,t}}$.

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3. Effects of Active Labour Market Policies for Young People in the context of Direct Search

Abstract

According to the International Labour Organization (ILO) around 75 million young people are unemployed. Governments, Agencies, and policy-makers promote Active Labour Market Policies (ALMPs) to increase youth employment. In this context, the purpose of this research is to theoretically study the effect of ALMPs, such as subsidies for training and hiring, on recruitment of young people. To do this, the search theory framework of is used in this research, allowing of heterogeneous agents with productivity bias. As Shi (2002) and Shimer (2005), this research use direct search in the sense that firms first post wages and workers subsequently apply for jobs. This study finds that active policies reduce youth unemployment. However, there is a substitution effect of such policies on unskilled older workers. It should be mentioned that the equilibrium in this framework seems to be unstable and is influenced by the initial conditions assumed in the model which limit the robustness of conclusions found on the consequences of such policies.

Keywords: Youth Unemployment, Active Labour Policies, Direct Search and heterogeneous agents.
JEL Classification: J64, E24, J08, C78, and H53.

3.1 Introduction

> *"One of the great mistakes is to judge policies and programs by their intentions rather than their results."* (Friedman, 1975)

Youth unemployment is now considered one of the most urgent problems that both developed and developing economies face (see Bell and Blanchflower, 2011 and Artner, 2013). In general, young people who are just entering the labour market exhibit, in contrast to adult workers, high labour turnover, higher probability of being unemployed and their job situation seems to be more sensitive to macro economic fluctuations (see Caliendo et al., 2011 and Verick, 2011).

According to the International Labour Organization (ILO) (2012) there are about 75 million unemployed youths aged between 15 and 24 worldwide. And this is not just a problem of developing countries. In Europe (ILO (2012)), 5.5 million youths are unemployed, representing 22% per cent of the total unemployment, more than double the rate for adults. Also, long-term unemployment affects nearly 30% per cent of the unemployed youths in Europe. Even worse, some 14 million young people, or more than 15% of European youths aged between 15 and 29, are NEETs (Neither in Education, Employment or Training); this has also become a problem in the United States. At the highest point of the recession that hit the US economy from 2007 to 2009, unemployment among young Americans grew to 20%. Additionally, the Organization for Economic Co-operation and Development (OECD) has declared that the global economic crisis hit youths very hard and youth unemployment rates had reached record-high levels for the last 25 years (OECD (2010)).

The high incidence of youth unemployment and the strong impact of the recent economic crisis on this sector have prompted international agencies, governments and policymakers to promote Active Labour Market Policies (ALMPs) in order to reduce the high persistence of youth unemployment, and also as a way to increase youth participation in the labour market. Among the most recommended policies are the subsidies for recruitment and training. In fact, in words of the OECD Jobs Study "active labour market policies improve access to the labour market and jobs; develop job-related skills and promote more efficient labour market" (OECD (1994)). Additionally, ILO chief Guy Ryder has called "for less austerity and more investment to promote a jobs recovery at a time when the youth employment crisis threatens to scar the very fabric of our societies". Bell and Blanchflower (2011) are even more categorical. They state that "...the conventional wisdom on youth employment policy has turned out to be largely irrelevant during this recession. The solution to the youth unemployment problems is simply: put more jobs for young people" (p. 241). In the recent years, consequently, subsidy policies for employment and training of young workers have been implemented in many countries such as Germany, Chile, Czech Republic, Poland, South Africa, Sweden, UK and others (Bravo and Rau (2013)).

However, theoretical and empirical studies are inconclusive on the effectiveness of ALMPs on labour markets and youth labour markets, in particular (see Heckman and Smith, 1999; Martin and Grubb, 2001; Calmfors et al., 2002; Boone and Van Ours, 2004; Blundell et al., 2004; Card et al., 2010 and Kluve, 2010). The central debate in the literature is focused on assessing how appropriate and effective these policies are in reducing unemployment within the target group; to determine whether these programs are profitable from a cost-effectiveness perspective and if they are socially desirable (Brown and Koettl (2012)). Additionally, it is debated whether these policies produce a displacement effect on employment, that is, that contracted workers who do not have the benefit be displaced by subsidized workers (Dahlberg and Forslund (2005)). This last point is important regarding policies that seek to improve the position of young people in the labour market. In this case, ALMPs can provide an incentive to employers to replace (move) adult low-skill workers by non-experienced young workers who carry a government subsidy, as this could represent an appreciable reduction in the company labour costs.

The purpose of this research is to respond, from a theoretical perspective, whether the juvenile Active Labour Market Policies (ALMPs) actually benefit those workers to whom the program is addressed to (the youths, in this research). In addition, we analyse the presence of any labour substitution effect between young workers benefiting from a subsidy and those low-skilled adult workers, out of the program. This problem, according to Dahlberg and Forslund (2005), has been neglected in the literature but is quite relevant from a social-optimum macroeconomic perspective. Two are the main hypothesis in this research: i. The ALMPs reduce youth unemployment, and ii. Young workers, bearers of any kind of subsidy, substitute or displace the low-skilled adult workers.

The methodology developed in this chapter it as been inspired by the competitive search models developed by Moen (1997) and Shimer (1996). In these models, agents (eg. Firms) post wage offers, and agents across the market (eg. workers) search for their best alternative. In these kinds of models, opposed to random matching models, agents are permitted to use the price system to directly affect the frequency of pairings, allowing the market to produce an efficient allocation of labour resources. Also in this research, Shi (2002) and Shimer's contributions (2005) are incorporated as to introduce heterogeneity on both sides of the market.

Specifically, the model developed in this chapter is a one-period static model in which unemployment equilibrium is studied in the context of a biased-productivity direct search. There are two types of agents: workers and firms. Workers will be divided into three categories: high and low skilled adult workers and young workers, whereas companies will be divided into low and high productivity firms. In line with models of direct search, workers and firms will be matched in pairs. Also, the output produced will depend on the skills of workers and the type of technology of the firms they match with. Adult workers are assumed more productive than younger workers and among adult workers productivity will be determined by the type of firm in which they have been hired. In this sense this research follows the production function used by Shi (2002) where there are heterogeneous agents and skill-biased technology (two-workers and two-firms model) but we extended it to incorporate a low-productivity young worker (three-workers and two-firms model).

In this economy and in a first stage, firms entering the market simultaneously post the wages they undertake to pay for each type of worker. In a second stage, workers consider the wages posted by firms and choose which company to apply to. At the end of the second stage of this game, if a firm receives more than one application the rule of hiring the most productive worker first is used, paying the agreed wage. Naturally, in this interaction unemployment and vacancies jointly emerge in competitive equilibrium conditions. This methodology allows studying the allocation of labour resources in a market with trade frictions, regarding the young people case when Active Labour Market Policies are applied.

Thus, in this environment the active labour market policy effects on youth are studied through a comparative static analysis, which contrasts the competitive equilibrium in absence of any intervention with an equilibrium altered by the government intervention in favour of young workers. This methodology allows analyzing the intervention impact on youth unemployment level and assessing the substitution or displacement effect on low-skilled adults. In this sense this research has considered two active policies in the youth labour market that would increase the productivity of young people: a hiring subsidy and a training subsidy. Likewise, two sources of funding for government policies are considered: a financing exogenous to the model (it might be regarded as a tax on food market) and endogenous financing by incorporating an ad-valorem tax on the companies profits.

To analyze the effectiveness of ALMPs this model will be calibrated to the US economy, so that to perform later numerical experiments that permit to assess the effects of the AMLPs; particularly the training and hiring subsidies on employment levels for different types of workers in order to find out if there are any displacement effects.

To my knowledge, there are no models that analyse these labour issues from this perspective. However there is an extensive literature that has analyzed, from a theoretical perspective, the effects of ALPMs on labour variables for adult workers. For example, Layard et al. (1991), Calmfors and Lang (1995) and Calmfors et al. (2002) have analyzed this problem using standard labour market models. On the other hand, Calmfors (1994), Boone and Van Ours (2004), Van der Linden (2005) and Bucher (2010) have approached this issue by using a modelling random search focusing, aligned with the models developed by Mortensen-Pissarides, also for adult people. Meanwhile, Orszag and Snower (1999) study this problem using a life cycle model. Therefore, the aim of this research is to add to this extensive literature a different view of the ALMPs effects on young people, through a direct search model with heterogeneous agents.

This research confirms that the Active Labour Market Policies (ALMP) that intend to improve the position of young people in the labour market do not necessarily increase employment overall as these policies produce a substitution effect between subsidized workers and those who are not benefited from it, as it happens with low-skilled adult workers. This research will also confirm that those young workers who may have access to the subsidy present a decrease in their unemployment levels. But these results may be affected depending on the way these active policies are financed. It is observed that in the endogenous-to-the-model tax case the effects on the reduction of unemployment are lower compared to the exogenous financing case.

This research is organized as follows. The second section presents and develops the model. In section third equilibrium is resolved. In section fourth the unemployment rate for the different types of workers is calculated. In section fifth the comparative statics and the effects of the active policy on the equilibrium are developed. In the sixth section the model is calibrated and numerical experiments are performed. Finally, conclusion are drewn and policy implications are discussed.

3.2 The Model[1]

3.2.1 Participants and Production

The model developed in this section is an extension of the work of Shi (2002). Consider a one-period economy with full commitment. In this economy, two types of agents neutral to risk coexist: workers and firms. Let N be defined as the total number of workers in this economy and M as the total number of firms; where N is considered an exogenous variable and M an endogenous variable. Without losing generality, let also assume that N and M belong to the natural-numbers set.

In this economy, workers are divided into three different categories: adult high-skilled workers (s), adult low-skilled workers (u) and young workers (j). Values s, u and j represent the exogenous proportion of each worker type within total of the population, and thus $s + u + j = 1$ is satisfied. It is easy to see that the total number of high productivity workers, low productivity workers and youth are sN, uN and jN, respectively. In this model it is assumed that each type of worker is fully observable by firms and that within their type each of them is identical in productivity.

Each firm in this economy has probability $(1 - h)$ to enter this market as a low-tech firm (L) and probability h to enter this market as a high-tech firm (H). This model assumes that h is an exogenous variable. The total high-tech and low-tech number of firms are hM and $(1 - h)M$, respectively. Likewise, the high-tech firm and the low-tech firm cost of entering the market is K_L and K_H respectively, and it is assumed that $K_H > K_L$. Firms enter with free-entry privilege and each of them opens only one single job position. In addition, each firm obtains null net profit from the creation of an employment position. Similarly to workers, the assumption that the type of firms is fully observable and that within their type each of them is identical is maintained.

In this type of economy workers and firms are matched in pairs. The output produced depends on the skills of workers and the type of the firm technology they match with. If a firm does not match with the worker, the firm produces zero. Likewise, if a worker does not match with a firm, the worker will be assumed to obtain zero return. Additionally, let assume that adult workers are more productive than young workers, and that productivity within adult workers is determined by the type of company they match with. Let also

[1]This model is based on work done by Shi (2002) and Shimer (2005).

assume that y will be the economy overall productivity level which will be considered here as an exogenous variable.

When a high-tech firm (H) is paired with an adult high-skilled worker (s) the economy produces $\theta_1 y$, with $\theta_1 > 1$. Likewise, the overall productivity of the economy (y)is generated when a low-tech company (L) is paired with an adult high-skilled worker (s). In the case of an adult low-ability worker (u) the economy generates the same level of productivity, regardless the type of company which the worker is paired with. Finally, the economy generates $\theta_2 y$ for a young worker (j) regardless the type of company that has hired such worker, with $\theta_2 < 1$. Note that high-skilled workers enjoy of a positive productivity bias when they are hired by a high-tech firm ($\theta_1 > 1$). By contrast, young workers have a negative productivity bias regardless the type of firm that hires them ($\theta_2 < 1$). On Table 3.1 this economy production function of is summarized.

Table 3.1: Production function

Output	Low-tech firm $(1-h)M$	High-tech firm hM
Skilled workers (sN)	y	$\theta_1 y$
Unskilled worker (uN)	y	y
Young workers (jN)	$\theta_2 y$	$\theta_2 y$

In this model, it is assumed that $K_L < \theta_2$ y and that $K_H < \theta_1 y$, permitting both conditions to ensure that low-tech and high-tech firms can enter the market. Also let index worker type as T, with $T \in \{s, u, j\}$ y for the rest of Chapter, with the type of firm as F, with $F \in \{L, H\}$. Finally, let also define $n = \frac{N}{M}$ as the workers-to-firms ratio, where n and M will be determined in equilibrium.

3.2.2 Direct Search and Posting

Assume that N and M are considered large enough. Under this large-numbers condition the study will only focus on symmetric equilibrium situations. In a symmetric equilibrium, workers and firms of similar type apply the same strategy, and the coordination among agents is not allowed. However, mixed strategies are accepted in the decisions of workers. In this market agents play a two-stage game. In the first stage, firms that simultaneously enter the market publish the wages they undertake to pay for each type of worker. Then, let define the wages that firms are willing to pay as $w_{H,s}$, $w_{H,u}$ and $w_{H,j}$ for each hM high-tech companies if the worker is high-skilled, low skilled and a young worker, respectively. Likewise, let define as $w_{L,s}$, $w_{L,u}$ y $w_{L,j}$ the wages that firms are willing to pay for each of the $(1-h)M$ low-tech companies if the worker is high-skilled, low-skilled and a young worker, respectively. Note that unlike high-tech firm, low-tech companies must offer the same wages to high and low skilled workers, as the impact on production is identical, therefore for the rest of this research it is assumed that $w_{L,s} = w_{L,u}$. Thus, in this economy the vector of wages can be represented as follows:

$$W = (\underbrace{(w_{L,s}, w_{L,u}, w_{L,j})\dots\ \dots\ \dots}_{(1-h)M\ \ Low-techfirm};\underbrace{(w_{H,s}, w_{H,u}, w_{H,j})\dots\ \dots\ \dots}_{hM\ \ High-techfirm}) \tag{3.2.1}$$

On the second stage, workers choose which company to apply to after observing the wages posted by firms using a mixed strategy and making a randomly choice over the set of positions they prefer. A type T skilled-worker strategy is determined by the following application vector probabilities:

$$P_T = (\underbrace{(p_{L,s}, p_{L,u}, p_{L,j})\dots\ \dots\ \dots}_{(1-h)M\ \ Low-techfirm};\underbrace{(p_{H,s}, p_{H,u}, p_{H,j})\dots\ \dots\ \dots}_{hM\ \ High-techfirm}) \tag{3.2.2}$$

Where $p_{L,T}$ and $p_{H,T}$ are the probabilities that a worker type T applies to a high and low-tech firm, respectively, with $T \in \{s, u, j\}$. As application probabilities must sum one, the following condition for a worker type T is satisfied as follows:

$$\underbrace{p_{L,T} + p_{L,T} + \dots + p_{L,T}}_{(1-h)M\ \ Low-techfirm} + \underbrace{p_{H,T} + p_{H,T} + \dots + p_{H,T}}_{hM\ \ High-techfirm}) = 1 \tag{3.2.3}$$

where $T \in \{s, u, j\}$. Note that equation (3.2.3) may be rewritten as follows:

$$(1-h)Mp_{L,T} + hMp_{H,T} = 1 \tag{3.2.4}$$

At the end of the second step of this game, if a firm receives at least one labour application, one worker will be hired and will be paid the agreed wage. The selection criterion is as follows. If the firm receives applications from just one type of workers, the hired worker will be selected randomly. In contrast, if the firm receives applications of more than one type of worker, the selection rule will operate as follows. High-tech firms will always prefer high-skilled workers available as first choice and upon that group they will randomly make the selection (see Shi (2002)). Low-tech firms are indifferent to high or low skilled workers as both will generate the same output; therefore they will use the simplifying rule of preferring to hire the first-priority productive worker. Note that young workers will only have the chance of being hired if firms have not received applications of adult workers, either high or low skilled.

It is worth highlighting that the interaction between workers and firms is consistent with a direct search model, as agents here (firms) used the price system (wage posting) to affect directly the frequency of pairing with the workers. In turn, these workers face a trade-off between wages and the probability of being hired. But also, in this two-stage game, the process of matching workers and firms may be costly, as some firms may not receive applications, while some workers may not get a job apposition if more than one worker is applying to the same firm. Therefore in this model vacancy and unemployment may coexist together in an equilibrium condition within a context of direct search.

3.2.3 Application Queue

The application process for workers to a firm follows a binomial distribution. In general terms, let assume K as the total number of workers and k the number of possible applications to a firm, with $k \in \{0, 1, 2, , K\}$. Consequently, it is possible to calculate the probability that k workers may simultaneously apply to the same firm by means of the following formula:

$$p(k) = \binom{K}{k} p^k (1-p)^{K-k} \tag{3.2.5}$$

where p is the probability that a worker applies to the firm and $(1-p)$ is the probability that the worker do not apply. Thus, it is possible to calculate the expected number of applications to a firm by using the binomial distribution properties. Consequently it is well known that the expected value is calculated as follows:

$$\mathbb{E}(k) = \sum_{k=0}^{K} k \binom{K}{k} p^k (1-p)^{K-k} \tag{3.2.6}$$

Following Shi (2002), the application queue is defined as the expected number of workers who apply to a firm. This definition is consistent with the usual market rigidity definitions used in the direct search models; this is due to the number of available vacancies is equal to the unity in this model. Then, by using the formula (3.2.6) it is possible to calculate the application queue or the market rigidity for each type of worker and for each type of firm (see Appendix 3.A). Let $K = TN$ be defined as the total number of workers of type T and $p = p_{F,T}$ as the probability that a worker type T applies to a firm type F with $T \in \{s, u, j\}$ and $F \in \{L, H\}$. Then the application queue or market rigidity ($q_{F,T}$) for a worker type T and a firm F is determined by the following expression:

$$q_{F,T} = TNp_{F,T} \tag{3.2.7}$$

with $T \in \{s, u, j\}$ and $F \in \{L, H\}$. Table 3.2 summarizes the application queues for the nine possible cases. With the definition of the application queue and the workers-to-firms

Table 3.2: Queue of applications

Queue	Low-tech firm $(1-h)M$	High-tech firm hM
Skilled workers (sN)	$q_{L,s} = sNp_{L,s}$	$q_{H,s} = sNp_{H,s}$
Unskilled worker (uN)	$q_{L,u} = uNp_{L,u}$	$q_{H,u} = uNp_{H,u}$
Young workers (jN)	$q_{L,j} = jNp_{L,j}$	$q_{H,j} = jNp_{H,j}$

ratio, equation (3.2.4) can conveniently be rewritten as follows:

$$(1-h)Mp_{L,T} + hMp_{H,T} = 1 \qquad / \cdot TN$$

$$(1-h)MTNp_{L,T} + hMTNp_{H,T} = TN \qquad / \div M$$

$$(1-h)TNp_{L,T} + hTNp_{H,T} = Tn$$

$$(1-h)q_{L,T} + hq_{H,T} = Tn \tag{3.2.8}$$

with $T \in \{s, u, j\}$.

3.2.4 Probability of Hiring

To calculate the hiring probabilities let assume for a moment that N and M are fixed. Consequently, the probability that an adult high-skilled worker ($\rho_{F,s}$) is hired by a firm type $F \in \{H, L\}$ is equal to the probability that at least one adult high-skilled worker provides an application, divided by the expected number of workers of the same type that provide an application to the firm, that is:

$$\rho_{F,s} = \frac{P(k_{F,s} > 0)}{\mathbb{E}(k_{F,s})} = \frac{1 - P(k_{F,s} = 0)}{\mathbb{E}(k_{F,s})} = \frac{1 - (1 - p_{F,s})^{sN}}{sNp_{F,s}} \tag{3.2.9}$$

with $F \in \{H, L\}$. The expression (3.2.9) was determined in Appendix 3.B.1 using conveniently the binomial distribution properties applied to adult high-skilled workers. Note that the hiring probability of adult workers seems not to be affected by the application of less productive workers as it has been assumed that the first type is strictly preferred over the second one.

For adult low-skilled workers, the probability ($\rho_{F,u}$) of being hired by a firm type $F \in \{H, L\}$ is equal to the probability that no adult high skilled-worker applies to the firm, multiplied by the probability that at least one adult low-skilled worker provides an application, divided by the expected number of adult low-skilled workers that submit one application to the firm, that is:

$$\rho_{F,u} = \frac{P(k_{F,s} = 0) \cdot P(k_{F,u} > 0)}{\mathbb{E}(k_{F,u})} = \frac{P(k_{F,s} = 0) \cdot (1 - P(k_{F,u} = 0))}{\mathbb{E}(k_{F,u})}$$

$$\rho_{F,u} = \frac{(1 - p_{F,s})^{sN} \cdot (1 - (1 - p_{F,u})^{uN})}{uNp_{F,u}} \tag{3.2.10}$$

with $F\{\in H, L\}$. Similarly to the previous case, the expression (3.2.10) was determined in Appendix 3.B.2 using the binomial distribution properties.

For young workers, the probability ($\rho_{F,j}$) of being hired by a firm type $F \in \{H, L\}$ is equal to the probability that no adult high-skilled or low-skilled worker applies to the firm, multiplied by the probability that at least one young worker submit an application, divided by the expected young worker number applying to the firm, that is (see Appendix 3.C.1):

$$\rho_{F,j} = \frac{P(k_{F,s} = 0) \cdot P(k_{F,u} = 0) \cdot P(k_{F,j} > 0)}{\mathbb{E}(k_{F,j})}$$

$$\rho_{F,j} = \frac{P(k_{F,s} = 0) \cdot P(k_{F,u} = 0) \cdot (1 - P(k_{F,j} = 0))}{\mathbb{E}(k_{F,j})}$$

$$\rho_{F,j} = \frac{(1 - p_{F,s})^{sN} \cdot (1 - p_{F,u})^{uN} \cdot (1 - (1 - p_{F,j})^{jn})}{jNp_{F,j}} \tag{3.2.11}$$

with $F \in \{H, L\}$. Similarly to the previous case, the expression (3.2.11) was determined in Appendix 3.B.3 using the binomial distribution properties.

However, in this research it is assumed that N and M are large enough, therefore probabilities must be recalculated assuming that $N, M \to \infty$. Appendix 3.C shows the procedure in detail, step by step to calculate these probabilities when N and M tend to infinity, basically benefiting from the limit property and the exponential function definition. Nevertheless, with the intention of clarifying the procedure this method is explained for the case of adult high-skilled workers, next. Thus, the probability that a high-skilled worker is hired by firm type F when $N, M \to \infty$ is equal to:

$$P_{F,s} = \lim_{N,M\to\infty} \rho_{F,s} = \lim_{N,M\to\infty} \frac{1-(1-p_{F,s})^{sN}}{sNp_{F,s}} \tag{3.2.12}$$

Developing appropriately the above equation can be obtained:

$$P_{F,s} = \lim_{N,M\to\infty} \frac{1-\left(1-\frac{sNp_{F,s}}{sN}\right)^{sN}}{sNp_{F,s}} \tag{3.2.13}$$

Notwithstanding, by equation (3.2.7) it is known that $q_{F,s} = sNp_{F,s}$, thus, by substituting this result in the equation (3.2.13) and using consequently the property of limits, this equation can be rewritten as follows:

$$P_{F,s} = \lim_{N,M\to\infty} \frac{1-\left(1-\frac{sNp_{F,s}}{sN}\right)^{sN}}{sNp_{F,s}} = \frac{1-\lim_{N,M\to\infty}\left(1+\frac{-q_{F,s}}{sN}\right)^{sN}}{q_{F,s}} \tag{3.2.14}$$

However, it is also well known that the exponential function can be expressed as $e^x = \lim_{m\to\infty}\left(1+\frac{x}{m}\right)^m$; thus by assuming that $x = -q_{F,s}$ and $m = sN$ it is obtained that $e^{-q_{F,s}} = \lim_{N,M\to\infty}\left(1+\frac{-q_{F,s}}{sN}\right)^{sN}$. Consequently, by substituting this result in the equation (3.2.14) the following expression is obtained for the hiring probability of adult workers, when $N, M \to \infty$:

$$P_{F,s} = \frac{1-e^{-q_{F,s}}}{q_{F,s}} \tag{3.2.15}$$

with $F \in \{H, L\}$. With similar procedure an adult low skilled-worker probability of being hired by a firm type F can be obtained, when $N, M \to \infty$, that is (see Appendix 3.C.2 for details):

$$P_{F,u} = e^{-q_{F,s}}\frac{1-e^{-q_{F,u}}}{q_{F,u}} \tag{3.2.16}$$

with $F \in \{H, L\}$. Finally, the probability that a young worker be hired by a firm type F is (see Appendix 3.C.3 for details):

$$P_{F,j} = e^{-q_{F,s}-q_{F,u}} \frac{1-e^{-q_{F,j}}}{q_{F,j}} \tag{3.2.17}$$

with $F \in \{H, L\}$.

3.2.5 Decision of Workers

Workers of any kind apply to a firm just if they will be able to earn at least as much as the expected market salary, also known as reservation wage. Let define U_T, with $T \in \{s, u, j\}$ as the best wage that workers can get from their best available alternative in equilibrium. Following Shi (2002) and Shimer (2005) a worker type T applies to a firm type F with positive probability just and only if the expected wage to be obtained from the firm is at least as good as the reservation wage, otherwise the worker will not apply. Consequently, the following expression summarizes the decision of the workers:

$$q_{T,F} = \begin{cases} \in (0, \infty) & if \;\; P_{F,T} w_{F,T} = U_T \\ 0 & if \;\; P_{F,T} w_{F,T} < U_T \end{cases} \tag{3.2.18}$$

with $T \in \{s, u, j\}$ and $F \in \{L, H\}$.

3.2.6 Decision of Firms

Companies optimize their expected profit by deciding on wages $w_{F,T}$ they are posting to the market, with $T \in s, u, j$ and $F \in \{L, H\}$. Consequently, the expected profit for a firm type F is determined by the probability of hiring a worker type T multiplied by the benefit the firm obtains by hiring such worker, summed up across all types of market workers. For a low-tech firm this optimization process is expressed as:

$$\begin{aligned} \max_{\{w_{L,s}, w_{L,u}, w_{L,j}\}} \Pi_L = (1-e^{-q_{L,s}})\{y - w_{L,s}\} + e^{-q_{L,s}}(1-e^{-q_{L,u}})\{y - w_{L,u}\} \\ + e^{-q_{L,s}-q_{L,u}}(1-e^{-q_{L,j}})\{\theta_2 y - w_{L,j}\} \end{aligned} \tag{3.2.19}$$

Subject to the workers participation condition stated in the fifth section of this model. Meanwhile, for a high-tech firm case this optimization process is as follows:

$$\max_{\{w_{H,s},w_{H,u},w_{H,j}\}} \Pi_H = (1-e^{-q_{H,s}})\{\theta_1 y - w_{H,s}\} + e^{-q_{H,s}}(1-e^{-q_{H,u}})\{y - w_{H,u}\} + e^{-q_{H,s}-q_{H,u}}(1-e^{-q_{H,j}})\{\theta_2 y - w_{H,j}\} \quad (3.2.20)$$

also subject to workers participation.

3.2.7 Equilibrium Definition

The model with a formal definition of a competitive equilibrium is summarized next.

Definition. A symmetric equilibrium consists of a set of expected wages U_s, U_u and U_j, of workers-to-firms ratios (n), of workers and firms strategies, in such a way that:

(a) Given worker expected salaries, firms solves problems (3.2.19) and (3.2.20).

(b) b. The participation condition must be satisfied, that is:

$$q_{a,b} = \begin{cases} \in (0,\infty) & \text{if } P_{F,T}w_{F,T} = U_T \\ 0 & \text{if } P_{F,T}w_{F,T} < U_T \end{cases}$$

with $T \in \{s,u,j\}$ and $F \in \{H,L\}$

(c) The queue satisfies equation (3.2.8).

(d) The n ratio satisfies free entry condition, that is $\Pi_H = K_H$ and $\Pi_L = K_L$.

3.3 Solving the Equilibrium

3.3.1 Queue Calculation in the Equilibrium

At equilibrium, adult high-skilled workers have no incentive to apply for low-tech firms due to this type of companies offer the same expected wage, either high or low skilled, as for a firm these kinds of workers generate the same production level (Lema 1 in Shi (2002)). Therefore, it is easy to conclude that at equilibrium the adult high-skilled workers application queue to low-tech market is equal to zero ($q^*_{L,s} = 0$). This result has implications at competitive equilibrium, for both in the calculations of the probabilities of hiring other workers in the low-tech market as in the net profit function determination of such kind of companies, as well.

So assuming that an adult high-skilled worker will never apply to a low-tech company, consequently the equilibrium is found out by jointly solving the following benefit maximization set of problems subject to worker participation in the labour market:

$$\max \Pi_L = (1 - e^{-q_{L,u}})\{y - w_{L,u}\} + e^{-q_{L,u}}(1 - e^{-q_{L,j}})\{\theta_2 y - w_{L,j}\} \tag{3.3.1}$$

s.t:

$$w_{L,u} = \frac{U_u q_{L,s}}{(1 - e^{-q_{L,s}})} \tag{3.3.2}$$

$$w_{L,j} = \frac{U_j q_{L,j}}{e^{-q_{L,u}}(1 - e^{-q_{L,j}})} \tag{3.3.3}$$

$$\begin{aligned}\max \Pi_H = {} & (1 - e^{-q_{H,s}})\{\theta_1 y - w_{H,s}\} + e^{-q_{H,s}}(1 - e^{-q_{H,u}})\{y - w_{H,u}\} \\ & + e^{-q_{H,s} - q_{H,u}}(1 - e^{-q_{H,j}})\{\theta_2 y - w_{H,j}\}\end{aligned} \tag{3.3.4}$$

s.t:

$$w_{H,s} = \frac{U_s q_{H,s}}{1 - e^{-q_{H,s}}} \tag{3.3.5}$$

$$w_{H,u} = \frac{U_u q_{H,u}}{e^{-q_{H,s}}(1 - e^{-q_{H,u}})} \tag{3.3.6}$$

$$w_{H,j} = \frac{U_j q_{H,j}}{e^{-q_{H,s} - q_{H,u}}(1 - e^{-q_{H,j}})} \tag{3.3.7}$$

By properly solving the above system, the application queue is determined in equilibrium as function of y, θ_1, θ_2, U_s, U_u and U_j, as solved in the Appendix 3.D.1. Thus the application queues in equilibrium are identical to the following expressions:

$$\begin{aligned}
q_{H,s} &= Ln\Big(\frac{y(\theta_1-1)}{U_s-U_u}\Big) && (3.3.8)\\
q_{H,u} &= Ln\Big(\frac{1-\theta_2}{\theta_1-1}\frac{U_s-U_u}{U_u-U_j}\Big) && (3.3.9)\\
q_{H,j} &= Ln\Big(\frac{\theta_2}{1-\theta_2}\frac{U_u-U_j}{U_j}\Big) && (3.3.10)\\
q_{L,s} &= 0 && (3.3.11)\\
q_{L,u} &= Ln\Big(\frac{y(1-\theta_2)}{U_u-U_j}\Big) && (3.3.12)\\
q_{L,j} &= Ln\Big(\frac{\theta_2}{1-\theta_2}\frac{U_u-U_j}{U_j}\Big) && (3.3.13)
\end{aligned}$$

In addition, it is also possible to calculate the application queues in equilibrium as function of the exogenous variables s, u, j, h and of the endogenous variable n by appropriately using equation (3.2.8) and the equalities found in the above maximization process, that is that $q_{H,s}+q_{H,u}=q_{L,u}$ (see Appendix 3.D.1, equation (3.D.29)) and that $q_{H,j}=q_{L,j}$ (see Appendix 3.D.1, equation (3.D.30)). Consequently, the application queues can be rewritten as follows (see the algebraic calculation details on Appendix 3.D.2):

$$\begin{aligned}
q_{H,s} &= \frac{ns}{h} && (3.3.14)\\
q_{H,u} &= (u+s)n-\frac{sn}{h} && (3.3.15)\\
q_{H,j} &= jn && (3.3.16)\\
q_{L,s} &= 0 && (3.3.17)\\
q_{L,u} &= (u+s)n && (3.3.18)\\
q_{L,j} &= jn && (3.3.19)
\end{aligned}$$

Finally, it is feasible to calculate values of U_s, U_u and U_j in equilibrium as function of the exogenous y, s, u, j and h, and of the endogenous variable n by appropriately combining equations (3.3.14)-(3.3.19) as executed on Appendix 3.D.3. Thus, reservation wages in equilibrium can be rewritten as follows:

$$\begin{aligned}
U_s &= y(\theta_1-1)e^{-\frac{sn}{h}}+y(1-\theta_2)e^{-(u+s)n}+\theta_2 ye^{-n} && (3.3.20)\\
U_u &= y(1-\theta_2)e^{-(u+s)n}+\theta_2 ye^{-n} && (3.3.21)\\
U_j &= y\theta_2 e^{-n} && (3.3.22)
\end{aligned}$$

3.3.2 Profit when the Free Entry Condition is met

The objective of this section is to determine the profits obtained in equilibrium by both high or low-tech firms in conditions of free entry. For low-tech firms, by substituting equation (3.3.2) and equation (3.3.3) within the objective function (3.3.1) can be determined the gross profit function in terms of exogenous variables, application queues and reservation wages U_s, U_u y U_j, as resolved on Appendix 3.D.4. Consequently, the function of profits is written as follows:

$$\Pi_L = y - ye^{-q_{L,u}} - U_u q_{L,u} + \theta_2 ye^{-q_{L,u}} - \theta_2 ye^{-q_{L,j}-q_{L,u}} - U_j q_{L,j} \tag{3.3.23}$$

But in terms of the model exogenous variables and of workers-to-firms ratio this equation can be rewritten by substituting the equilibrium values for $q_{L,u}$, $q_{L,j}$, U_u and U_j found in the equations (3.3.18), (3.3.19), (3.3.21) and (3.3.22) respectively. Consequently the low-tech firm gross profits in equilibrium can be stated by the following expression (see Appendix 3.D.4, equation (3.D.50)):

$$\begin{aligned}\Pi_L = y - (y + y(1-\theta_2)(u+s)n - \theta_2 y)e^{-(u+s)n} \\ -(\theta_2 y(u+s)n + \theta_2 y + \theta_2 yjn)e^{-n}\end{aligned} \tag{3.3.24}$$

Finally, by imposing the free entry condition, that is, profits equal to zero ($\Pi_L = K_L$) in equilibrium it is satisfied that:

$$\begin{aligned}K_L = y - (y + y(1-\theta_2)(u+s)n - \theta_2 y)e^{-(u+s)n} \\ -(\theta_2 y(u+s)n + \theta_2 y + \theta_2 yjn)e^{-n}\end{aligned} \tag{3.3.25}$$

This equation can be rearranged and rewritten as follows (see Appendix 3.D.4, equation (3.D.51)):

$$\begin{aligned}\frac{K_L}{y} = 1 - (1 + (1-\theta_2)(u+s)n - \theta_2)e^{-(u+s)n} \\ -(\theta_2(u+s)n + \theta_2 + \theta_2 jn)e^{-n}.\end{aligned} \tag{3.3.26}$$

The same procedure will be used for the high-tech firm case. Thus, by substituting equations (3.3.5), (3.3.6) and (3.3.7) within the objective function (3.3.4) the gross profit function is obtained in terms of exogenous variables, application queues and reservation wages U_s, U_u y U_j, (see Appendix 3.D.4 for calculation details in the high-tech section).

Consequently, gross profits for high-tech firms are:

$$\Pi_H = (1 - e^{-q_{H,s}})\theta_1 y - U_s q_{H,s} + e^{-q_{H,s}}(1 - e^{-q_{H,u}})y - U_u q_{H,u} \\ + e^{-q_{H,s} - q_{H,u}}(1 - e^{-q_{H,j}})\theta_2 y - U_j q_{H,j} \tag{3.3.27}$$

But in terms of the model exogenous variables and of workers-to-firms ratio this equation can be rewritten by substituting the equilibrium values for $q_{H,s}$, $q_{H,u}$, $q_{H,j}$, U_s, U_u, and U_j stated in the equations (3.3.14), (3.3.15), (3.3.16), (3.3.20), (3.3.21) and (3.3.22) respectively. Consequently the high-tech firm gross profits in equilibrium can be expressed as follows (see Appendix 3.D.4 equation (3.D.54)):

$$\Pi_H = y(\theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}}) \\ - y(1 + (1 - \theta_2)(u + s)n - \theta_2)e^{-(u+s)n} \\ - y(\theta_2(u + s)n + \theta_2 + \theta_2 yn)e^{-n}. \tag{3.3.28}$$

Finally, by imposing the free entry condition, that is, profits equal to zero ($\Pi_H = K_H$) in equilibrium it is satisfied that:

$$K_H = y(\theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}}) \\ - y(1 + (1 - \theta_2)(u + s)n - \theta_2)e^{-(u+s)n} \\ - y(\theta_2(u + s)n + \theta_2 + \theta_2 yn)e^{-n}. \tag{3.3.29}$$

This equation can be rearranged and rewritten as follows (see Appendix 3.D.4 equation (3.D.56)):

$$\frac{K_H}{y} = (\theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}}) \\ - \{(1 + (1 - \theta_2)(u + s)n - \theta_2)e^{-(u+s)n} + (\theta_2(u + s)n + \theta_2 + \theta_2 yn)e^{-n}\}. \tag{3.3.30}$$

Note that the expression in curly bracket is identical to $\left(\frac{y - K_L}{y}\right)$ (see equation (3.3.26)). Therefore, by substituting this outcome in equation (3.3.30) and consequently rearranging appropriately the algebraic terms the equilibrium expression with free entry for high-tech firms can be rewritten as follows (see Appendix 3.D.4 for details):

$$\frac{K_H - K_L}{(\theta_1 - 1)y} = 1 - e^{\frac{-sn}{h}} - e^{-\frac{sn}{h}}\frac{sn}{h}. \tag{3.3.31}$$

3.3.3 System of Equations that Solves of Symmetric Equilibrium

In this economy, given the value of the following exogenous variables y, θ_1, θ_2, s, u, j, K_H, K_L, and h, the equilibrium value n^* is obtained by resolving the following equation system:

$$\begin{aligned}\frac{K_L}{y} &= 1-(1+(1-\theta_2)(u+s)n-\theta_2)e^{-(u+s)n} \\ &\quad -(\theta_2(u+s)n+\theta_2+\theta_2 jn)e^{-n}; \\ \frac{K_H-K_L}{(\theta_1-1)y} &= 1-e^{-\frac{sn}{h}}\left(1+\frac{sn}{h}\right).\end{aligned} \tag{3.3.32}$$

Consequently, once value n^* is obtained by solving the above system, the equilibrium values for the profits Π_H^*, Π_L^*, for the expected wages of U_s^*, U_u^* and U_j^*, and for the application queues are determined next by substituting value n^* in the equations (3.3.24), (3.3.28), (3.3.20) to (3.3.22) and from (3.3.14) to (3.3.19), respectively.

3.4 The Unemployment Rate

The unemployment rate of workers in competitive equilibrium is calculated in this section. To proceed to this calculation, the high and low-skilled adult and young worker mass-in-equilibrium hired at low and high tech market should be calculated first.

The number of workers hired at high-tech firms in the case of adult high-skilled workers ($N_{H,s}$) is obtained by multiplying the number of vacancies offered (hM) by the probability that this type of worker be hired in this market $\left(1-e^{-n^*\frac{s}{h}}\right)$. As this kind of worker hardly applies to a low-tech firm the number of workers hired at the high-tech market is identical to the total of high-skilled workers hired in economy (N_s). Thus, N_s can be expressed as follows:

$$N_s = N_{H,s} = hM\left(1-e^{-n^*\frac{s}{h}}\right) \tag{3.4.1}$$

Regarding adult low-skilled workers, they can be hired in both markets. The number of workers in equilibrium hired at the high-tech market ($N_{H,u}$) is obtained by multiplying the number of vacancies offered (hM) by the probability that this kind of worker be hired in this market $\left(e^{-n^*\frac{s}{h}}-e^{-(u+s)n^*}\right)$. Whereas, the number of adult low-skilled workers hired at the low-tech market ($N_{L,u}$) is obtained by multiplying the number of vacancies offered ($(1-h)M$) by the probability that this type of worker be hired $\left(1-e^{-(u+s)n^*}\right)$. Thus, the total number of such workers hired (N_u) in the market can be expressed by the following equation:

$$N_u = N_{H,u} + N_{L,u} = hM\left(1 - e^{-n^*\frac{s}{h}}\right) + (1-h)M\left(1 - e^{-(u+s)n^*}\right) \tag{3.4.2}$$

For young workers the same procedure is executed. The number of young workers hired in equilibrium at the high-tech market ($N_{H,j}$) is obtained by multiplying the number of vacancies offered (hM) by the probability that this kind of worker be hired in such market $\left(e^{-(u+s)n^*} - e^{-n^*}\right)$. Whereas, the number of young workers hired at the low-tech market ($N_{L,j}$) is obtained by multiplying the number of vacancies offered ($(1-h)M$) by the probability that this kind of worker be hired $\left(e^{-(u+s)n^*} - e^{-n^*}\right)$. Thus, the total number of such workers (N_j) hired in the market can be expressed by the following equation:

$$N_j = N_{H,j} + N_{L,j} = hM\left(e^{-(u+s)n^*} - e^{-n^*}\right) + (1-h)M\left(e^{-(u+s)n^*} - e^{-n^*}\right) \tag{3.4.3}$$

The employment rate (Em_T) for each type of worker is calculated by dividing the total mass of workers that have been hired in both markets by the mass of workers of the same type, available in period (TN), that is:

$$Em_T = \frac{N_T}{TN} \tag{3.4.4}$$

with $T \in \{s, u, j\}$. More accurately, the employment rate of adult high-skilled workers is expressed by the following formula:

$$Em_s = \frac{N_s}{sN} = \frac{hM\left(1 - e^{-n^*\frac{s}{h}}\right)}{sN} = \frac{h\left(1 - e^{-n^*\frac{s}{h}}\right)}{sn} \tag{3.4.5}$$

Likewise, the unemployment rate of adult low-skilled workers can be formulated as follows:

$$Em_u = \frac{N_u}{uN} = \frac{N_{H,u} + N_{L,u}}{uN} = \frac{h\left(1 - e^{-n^*\frac{s}{h}}\right) + (1-h)\left(1 - e^{-(u+s)n^*}\right)}{un} \tag{3.4.6}$$

Finally, for young workers the unemployment rate is obtained as follows:

$$Em_j = \frac{N_j}{jN} = \frac{N_{H,j} + N_{L,j}}{jN} = \frac{\left(e^{-(u+s)n^*} - e^{-n^*}\right)}{jn} \tag{3.4.7}$$

As known, the employment rate (Em_T) plus the unemployment rate (Ur_T) must be equal to one, that is, $Em_T + Ur_T = 1$. Therefore, the unemployment rate for a worker type T

is obtained by isolate (Ur_T) in the above equation as follows:

$$Ur_T = 1 - Em_T \tag{3.4.8}$$

with $T \in \{s, u, j\}$. Finally, the aggregate unemployment in this economy (U_{ag}) in the competitive equilibrium is obtained by the following mathematical expression:

$$U_{ag} = sUr_s + uUr_u + jUr_j \tag{3.4.9}$$

with $T \in \{s, u, j\}$.

3.5 Comparative Statics: Active Labour Market Policies for Young workers

The model developed in the previous section is a one-period static model where the equilibrium with unemployment is studied within a productivity bias direct search context for three different types of workers: adult high-productive workers (s), adult low-productive workers (u) and Young (j). In this environment, an active government policy effects on the young labour market will be studied through a comparative static analysis which contrasts the competitive equilibrium in absence of any intervention with an equilibrium situation altered by the intervention of the government.

As previously explained, young workers can only get a job to the extent that no adult worker applies to the firm. This is because young workers are located in the lowest productivity level in the market. Therefore, an active government policy design which intends to improve the youngs options should consider their productivity increasing to turn them more attractive to firms. Note that the productivity difference between a young worker and an adult low-productivity is of $y - \theta_2 y$ regardless the type of firm considered. Thus, an active policy in this model should increase the young worker productivity capacity in $y - \theta_2 y$ so that the business considers their hiring with evenly chances when an adult low-skilled worker applies to the firm. In this way, two active policies in this research are considered in the young labour market that increases productivity on young in to the extent of $(y - \theta_2 y)$. The first one is a hiring subsidy, and the second one a training subsidy prior to the job search. This methodology will allow to analyze the intervention impact over the young workers unemployment and to assess the substitution or displacement effects over the adult low-skilled workers, thus comparing the base case with a government intervention event.

3.5.1 Active Policy: Hiring Subsidies

A hiring subsidy operates as follows in this investigation model development. The government advertises to every firm that a proportion of δ young workers will receive a subsidy voucher that will cover the difference between a young worker and adult low-skilled worker productivity by an amount of $y - \theta_2 y$. This productivity-Voucher difference will be paid provided the subsidized young worker is hired by the firm. Otherwise this voucher will lose its validity and value. Assuming that firms are able to detect at zero cost if a young worker is beneficiary of this voucher, it can be deduced that they will regard a young worker as productive as an adult low-skilled worker as the production gap will be covered by the government policy. In such circumstances, firms will react indifferently when hiring either a young or an adult low-skilled worker. Thus with this type of government intervention, the proportion of workers in this economy is changed as now there are young workers who are regarded as more productive with respect of the base scenario (see table 3.3). Therefore, in this post government intervention economy, s will be the adult high-skilled proportion and $u_{wp} = u + \delta j$ will be the adult low-skilled workers and young workers funded with the benefit of the voucher. Finally, $j_{wp} = (1 - \delta) j$ will be the proportion of young workers who were not able to benefit from the subsidy. Subsequently, the equilibrium situation when a hiring subsidy is funded by imposing an ad valorem of amount t tax, exogenous and endogenous to the model, will be obtained.

Table 3.3: Production function

Output	Low-tech firm $(1-h)M$	High-tech firm hM
Skilled workers (sN)	y	$\theta_1 y$
Unskilled and young with voucher $((u + \delta j)N)$	y	y
Young workers without voucher $(((1-\delta)j)N)$	$\theta_2 y$	$\theta_2 y$

Equilibrium with Exogenous Financing

If the government intervention is funded outside the model, as for example through a value-added tax, the equilibrium structure will not be altered except by the worker proportion in each productivity group with respect of the base-case without intervention. Consequently, in this economy the equilibrium value n^*_{wp}, when given the following exogenous variables y, θ_1, θ_2, s, u_{wp}, j_{wp}, K_H, K_L and h is obtained by solving the following system of equations:

$$\begin{aligned} \frac{K_L}{y} &= 1-(1+(1-\theta_2)(u_{wp}+s)n^*_{wp}-\theta_2)e^{-(u_{wp}+s)n^*_{wp}} \\ &\quad -(\theta_2(u_{wp}+s)n^*_{wp}+\theta_2+\theta_2 j_{wp}n^*_{wp})e^{-n^*_{wp}}; \\ \frac{K_H-K_L}{(\theta_1-1)y} &= 1-e^{-\frac{sn^*_{wp}}{h}}\left(1+\frac{sn^*_{wp}}{h}\right). \end{aligned} \tag{3.5.1}$$

Then, once the value n^*_{wp} is obtained by solving the above system, the equilibrium values for the high and low-tech firm profits; the expected wages for the three types of workers and the application queues are determined applying the same procedure used for the base case. However, the unemployment rates should be recalculated with respect to the base scenario as now with the government intervention we find a fraction of young workers (δ) who have higher chances to be hired as they can accede to the government voucher. Consequently, the adult high-skilled worker unemployment rate is equal to:

$$Ur^{wp}_s = 1-\frac{h\left(1-e^{-n^*_{wp}\frac{s}{h}}\right)}{sn^*_{wp}} \tag{3.5.2}$$

Likewise, the adult low-skilled workers are expressed as:

$$Ur^{wp}_u = 1-\frac{h\left(e^{-n^*_{wp}\frac{s}{h}}-e^{-(u_{wp}+s)n^*_{wp}}\right)+(1-h)\left(1-e^{-(u_{wp}+s)n^*_{wp}}\right)}{u_{wp}n^*_{wp}} \tag{3.5.3}$$

Finally, Young unemployment rate differs to that calculated in the base without intervention scenario as now Young subsidized with government vouchers have higher chances to be hired than those unsubsidized ones. Thus, the Young unemployment is equal to:

$$Ur^{wp}_j = (1-\delta)\left(1-\frac{\left(e^{-(u_{wp+s})n^*_{wp}-e^{-n^*_{wp}}}\right)}{j_{wp}n^*_{wp}}\right)+\delta Ur^{wp}_u \tag{3.5.4}$$

The aggregate unemployment in equilibrium with intervention is rewritten as follows:

$$Uag^{wp} = sUr^{wp}_s + uUr^{wp}_u + (1-\delta)Ur^{wp}_j + \delta Ur^{wp}_u \tag{3.5.5}$$

Equilibrium with Endogenous Financing

Subsequently, the situation when the government is funded within this model is analyzed. Basically the hiring subsidy will be funded by an ad valorem rate t tax imposed to the high and low-tech firm profits. A detailed procedure explanation to calculate the new equilibrium is found in Appendix 3.E. Thus, the free entry condition with endogenous tax must satisfy the following conditions:

$$K_H = (1-t)\Pi_H \tag{3.5.6}$$

$$K_L = (1-t)\Pi_L \tag{3.5.7}$$

In this economy the equilibrium value n^*_{wp} and t, given the following exogenous variables y, θ_1, θ_2, s, u_{wp}, j_{wp}, K_H, K_L, and h is obtained by solving the following system of equations (see Appendix 3.E and equation (3.E.10)):

$$\begin{aligned}
\frac{K_L}{(1-t)y} &= 1-(1+(1-\theta_2)(u_{wp}+s)n^*_{wp}-\theta_2)e^{-(u_{wp}+s)n^*_{wp}} \\
&\quad -(\theta_2(u_{wp}+s)n^*_{wp}+\theta_2+\theta_2 j_{wp}n^*_{wp})e^{-n^*_{wp}} \\
\frac{K_H-K_L}{(\theta_1-1)(1-t)y} &= 1-e^{-sn^*_{wp}/h}\left(1+\frac{sn^*_{wp}}{h}\right) \\
\left(\frac{t}{1-t}\right)(K_H+K_L) &= n^*_{wp}(\delta j)Em^{wp}_j(y-\theta_2 y)
\end{aligned} \tag{3.5.8}$$

In contrast to the exogenous funding situation, the equilibrium structure changes with respect to the base model because in addition to incorporating the tax to the firm optimization structure, a new equation which includes the government budget constraint should be added. Note that this subsidy operates only if a worker with a voucher is hired by the firm. Consequently, in the budget constraint the variable Em^{wp}_j represents the employment rate of Young who were able to accede to the subsidy and hired by the firms, that is:

$$Em^{wp}_j = \frac{h\left(e^{-n^*_{wp}\frac{s}{h}}-e^{-(u_{wp}+s)n^*_{wp}}\right)+(1-h)\left(1-e^{-(u_{wp}+s)n^*_{wp}}\right)}{u_{wp}n^*_{wp}} \tag{3.5.9}$$

Then, once the value n^*_{wp} is obtained by solving the above system, the equilibrium values for the high and low-tech firm profits; the expected wages for the three types of workers and the application queues are determined applying the same procedure used for the base case. The unemployment rates are calculated using the equations (3.5.2)-(3.5.5).

3.5.2 Active Policy: Training

A training subsidy operates as follows in this investigation model development: The government advertises to every firm that a proportion of δ young workers will receive an instant training prior to the job search increasing the Young productivity in $y - \theta_2 y$, and that it will be funded in full by the government. Likewise the hiring subsidy case, firms will react indifferently when hiring either an adult low-skilled worker or a Young worker who acceded to training. Also similarly to the hiring subsidy situation, the government intervention will modify the worker proportion as some Young will have gained more productivity. Then, s will be the adult high-skilled proportion, $u_{tp} = u + \delta j$ will be the adult low-skilled and Young with the training benefit proportion and finally, $j_{tp} = (1 - \delta)j$ will be the young workers who were not benefited with training. As the previous case, the exogenous and endogenous situation will be studied.

Equilibrium with Exogenous Financing

The equilibrium with training subsidy is similar to the equilibrium with hiring subsidy when funding is exogenous to the model. The unique difference between both policies lies in the fact that the training subsidy is applied to the whole benefited Young proportion whereas for the hiring subsidy, this works only if the young worker is hired by the firm. But the difference in the subsidy amount the government should provide is only relevant when the funding is not exogenous but endogenous to the model. Given the above, to obtain the equilibrium when the training subsidy is funded by an exogenous tax just rewrite the equations (3.5.1)-(3.5.5), modifying subscript w by t.

Equilibrium with Endogenous Financing

Subsequently, the situation when the government intervention is funded within the model is analyzed. Similarly to the hiring subsidy, this training subsidy will be funded by an ad valorem rate t tax imposed to high and low-tech firm profits, being the unique difference between both cases the government constraint as previously was said the training is conducted to all the benefited Young prior to their job search. Consequently, in this economy the equilibrium value n^*_{tp} and t, given the following exogenous variables y, θ_1, θ_2, s, u_{tp}, j_{tp}, K_H, K_L, and h is obtained by solving the following system of equations (see Appendix

3.E and equation (3.E.10)):

$$\begin{aligned}\frac{K_L}{(1-t)y} &= 1-(1+(1-\theta_2)(u_{tp}+s)n^*_{tp}-\theta_2)e^{-(u_{tp}+s)n^*_{tp}}\\ &\quad -(\theta_2(u_{tp}+s)n^*_{tp}+\theta_2+\theta_2 j_{tp}n^*_{tp})e^{-n^*_{tp}}\\ \frac{K_H-K_L}{(\theta_1-1)(1-t)y} &= 1-e^{-sn^*_{tp}/h}\left(1+\frac{sn^*_{tp}}{h}\right)\\ \left(\frac{t}{1-t}\right)(K_H+K_L) &= n^*_{tp}(\delta j)(y-\theta_2 y)\end{aligned} \tag{3.5.10}$$

Consequently, once the value n^*_{wp} is obtained by solving the above system, the equilibrium values for the high and low-tech firm profits; the expected wages for the three types of workers and the application queues are determined following the same procedure used for the base case. The unemployment rates are calculated using the equations (3.5.2)-(3.5.5), but substituting the subscript w by t.

3.6 Calibration and Numerical Experiments

The goal of this chapter is to evaluate the impact of active policies on the Young unemployment level and to analyze the substitution or displacement effects on low-skilled adult workers for the American economy. For this purpose, the model for this economy will be calibrated first, being the calibration target the unemployment rate for both, at the aggregate level and for each type of worker for the United States economy during 2013. Consequently, once the developed model predictive ability is tested the training active policy and wage subsidy effects over the unemployment levels of Young with and without subsidy shall be simulated. Also, in this simulation the displacement effects on the adult low-skilled workers will be analyzed.

To calibrate this model it is necessary to specify a number of parameters. These parameters may be divided in two types: fixed parameters that can be obtained directly through empirical observations, and free parameters chosen so as to minimize the distance between the American unemployment rate, both at aggregate level and by type of worker and the unemployment rates predicted by the developed model.

Then the calculations of fixed parameters is presented. Note that the parameters s, u and j representing the proportion of workers in the economy can be determined through the 2013 CPS. Assuming all those workers over 24 years of age with some university study level as high-productivity workers it can be determined basing on the CPS that this group represents 23.8% of the labour force ($s = 0.238$). Likewise, if all those workers over 24 years of age with no further education than secondary studies are considered as adult low-productivity workers, using the CPS this group represents the 61.3% of the working population ($u = 0.613$). Finally also using the CPS, it can be estimated that young workers represent the 14.9% of the labour force ($j = 0.149$). To determine θ_1, which represents the productivity positive bias between the adult high-skilled and low-skilled workers, both average wage levels obtained by CPS are divided. It is assumed here that wages represent an adequate approximation to the productivity of individuals. Therefore, according to the CPS data from 2013 adult high-skilled workers obtained an average salary of US$ 1233.33, while low-skilled ones obtained an averaged US$ 623.37 per month. Thus θ_1 can be estimated in 1.98. The same procedure is followed to determine the productivity negative bias between young workers and adult low-skilled workers, this is θ_2. However in this case the wages earned by young workers is divided by the wages earned by low-skilled adult workers. According to CPS data for 2013, adults low-skilled workers obtained an average salary of US$ 623.37 while young workers obtained an average wage of US$ 454.0 per month, so θ_2 can be calculated at 0.72.

Finally the free parameters are presented. The overall productivity of the economy and the costs of companies of low and high technology K_L and K_H to entry the market, besides the proportion of high-tech companies h will be considered free parameters and are fixed at 10, 3.15, 2.15 and 0.5 respectively. The Table 3.4 summarizes all the model parameters.

Table 3.4: Parameter Values for Benchmark Model (USA 2013)

Parameters	Value	Role
s	0.238	Proportion of adult high-productivity workers
u	0.613	Proportion of adult low-productivity workers
j	0.149	Proportion of young workers
θ_1	1.98	Productivity positive bias of adult high- productivity workers
θ_2	0.72	Productivity negative bias of Young workers
y	10.00	Overall productivity of economy
K_H	3.15	High-Tech Firm entry cost
K_L	2.15	Low-Tech Firm entry cost
h	0.5	High-Tech Firm Proportion

In Table 3.5 the model calibration process results can be seen. Outcomes are satisfactory and show a good model predictive fit to data of the American economy for Year 2013. Therefore, the model demonstrated to be empirically appropriate to study the active policy effects on Youth Unemployment Level.

Table 3.5: Calibration Results

Unemployment rate	Models Unemployment	USA Unemployment (2013)
Aggregate	8.08%	7.04%
Adult high-skilled workers	4.43%	5.21%
Adult low-skilled workers	8.18%	7.00%
Young workers	13.30%	14.00%

Then two numerical experiments are performed (see Table 3.6) to analyse the behavior of the model developed. For both experiment are determined a set of values for the relevant parameters of the model. To obtain a set of equilibrium susceptible to be plotted in experiment A is moved the value of h (the probability of a firm to enter the market as high-tech) between 0.4 and 0.6. However, for the experiment B is kept fixed the value of h, but now is moved the value of theta 1 between 1.75 and 1.90. For experiment A and B is analysed the effects of active policies for the case that young beneficiaries are 3% (case 1) and for the case where the young beneficiaries are 5% (case 2). It should be noted that both the A and B experiments analyse active policies of subsidizing training and a recruitment subsidy (voucher), both when it is funded out of the model, and when it is financed internally in the model.

Table 3.6: Numerical Experiment

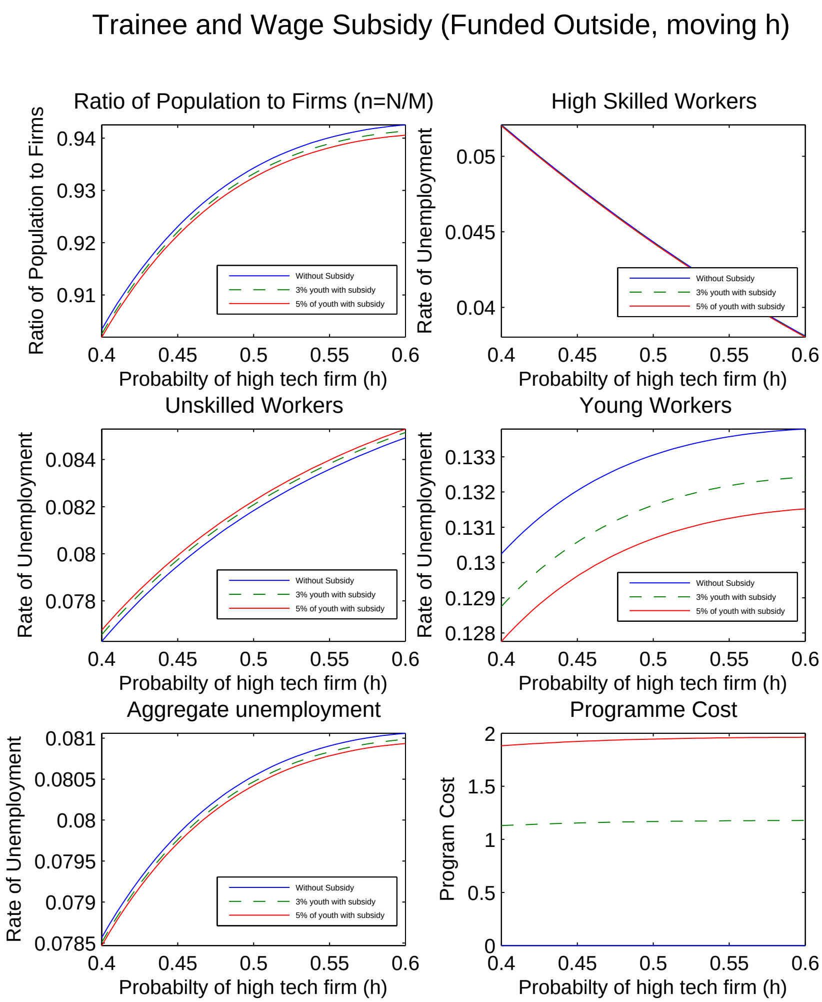
Numerical Experiment A

Parameters	Value
Output	$y = 10$
Positively biased productivity	$\theta_1 = 1.98$
Negatively biased productivity	$\theta_2 = 0.72$
Entry cost for high-tech Firms	$K_H = 3.15$
Entry cost for low-tech Firms	$K_L = 2.15$
Proportion skilled workers	$s = 0.238$
Proportion unskilled workers	$u = 0.613$
Proportion young workers	$j = 0.149$
Of entering probability as high-tech firm	$h \in (0.4, 0.6)$
Case 1: percentage of young with the benefit	$\delta = 0.03$ (3%)
Case 2: percentage of young with the benefit	$\delta = 0.05$ (5%)

Numerical Experiment B

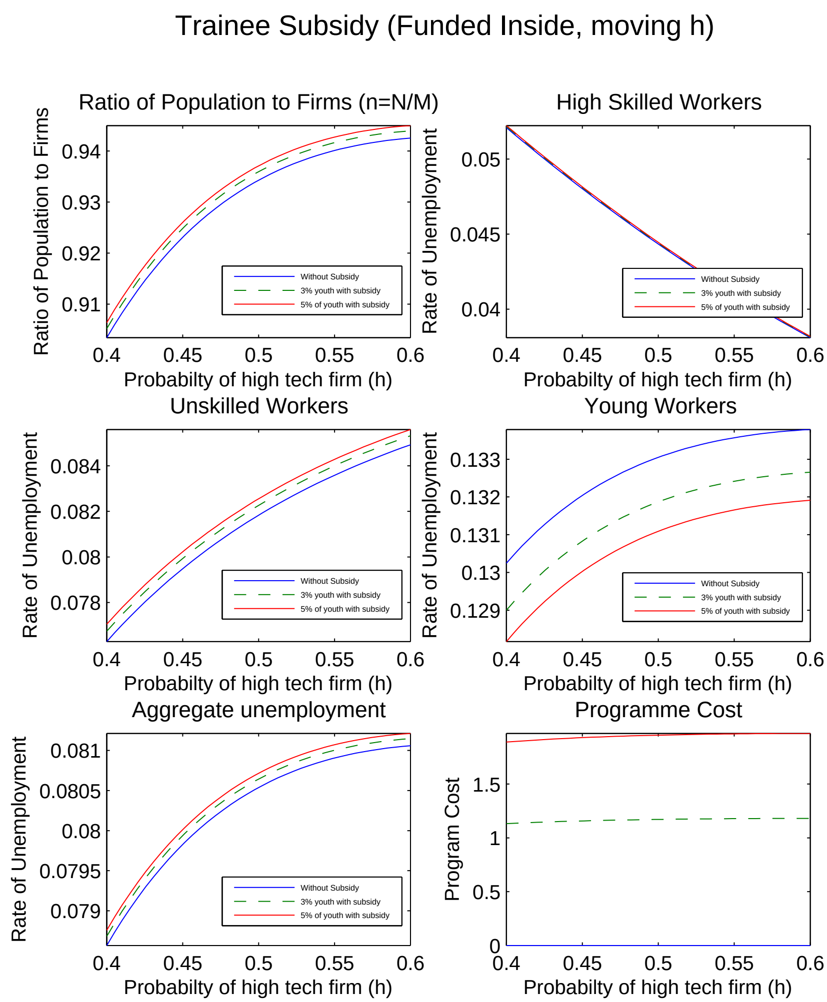
Parameters	Value
Output	$y = 10$
Negatively biased productivity	$\theta_2 = 0.72$
Entry cost for high-tech Firms	$K_H = 3.15$
Entry cost for low-tech Firms	$K_L = 2.15$
Proportion skilled workers	$s = 0.238$
Proportion unskilled workers	$u = 0.613$
Proportion young workers	$j = 0.149$
Of entering probability as high-tech firm	$h = 0.5$
Positively biased productivity	$\theta_1 \in (1.90, 2.10)$
Case 1: percentage of young with the benefit	$\delta = 0.03$ (3%)
Case 2: percentage of young with the benefit	$\delta = 0.05$ (5%)

Numerical Experiments A



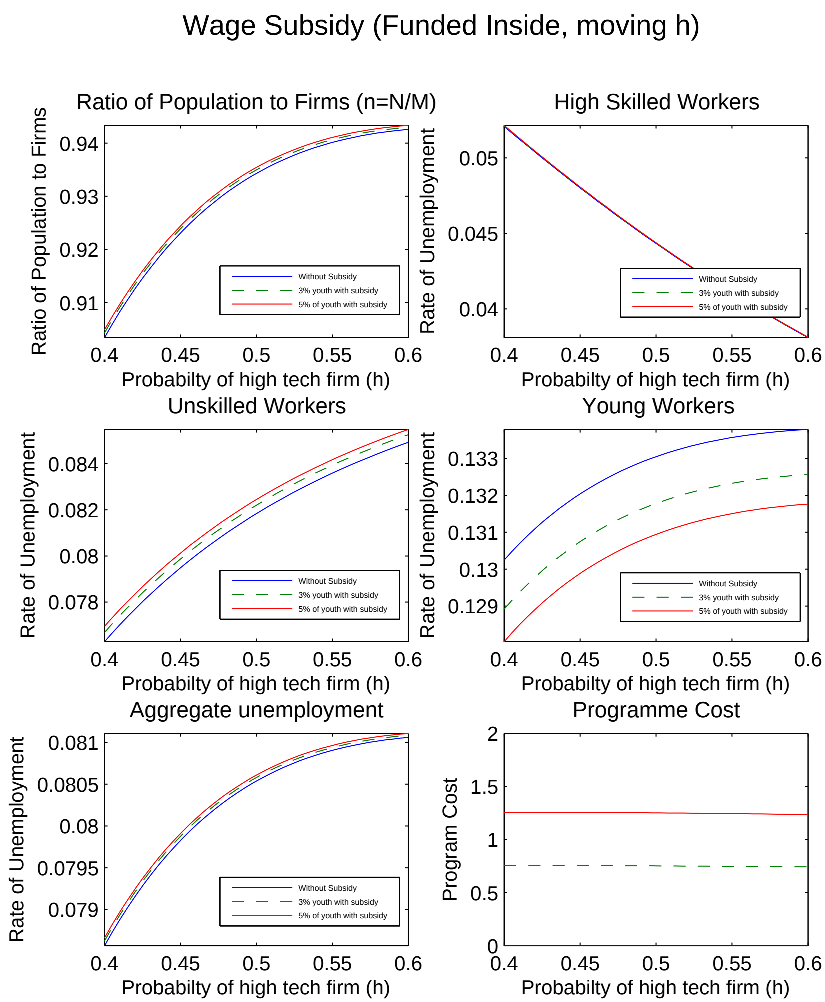
Note: Blue line is the case without intervention. Green line is the case with intervention (3% of young worker). Red line is the case with intervention (5% of young worker).

Figure 3.1: Numerical Experiment A. Funded outside the model. Applied to the case of subsidised training and recruitment.



Note: Blue line is the case without intervention. Green line is the case with intervention (3% of young worker). Red line is the case with intervention (5% of young worker).

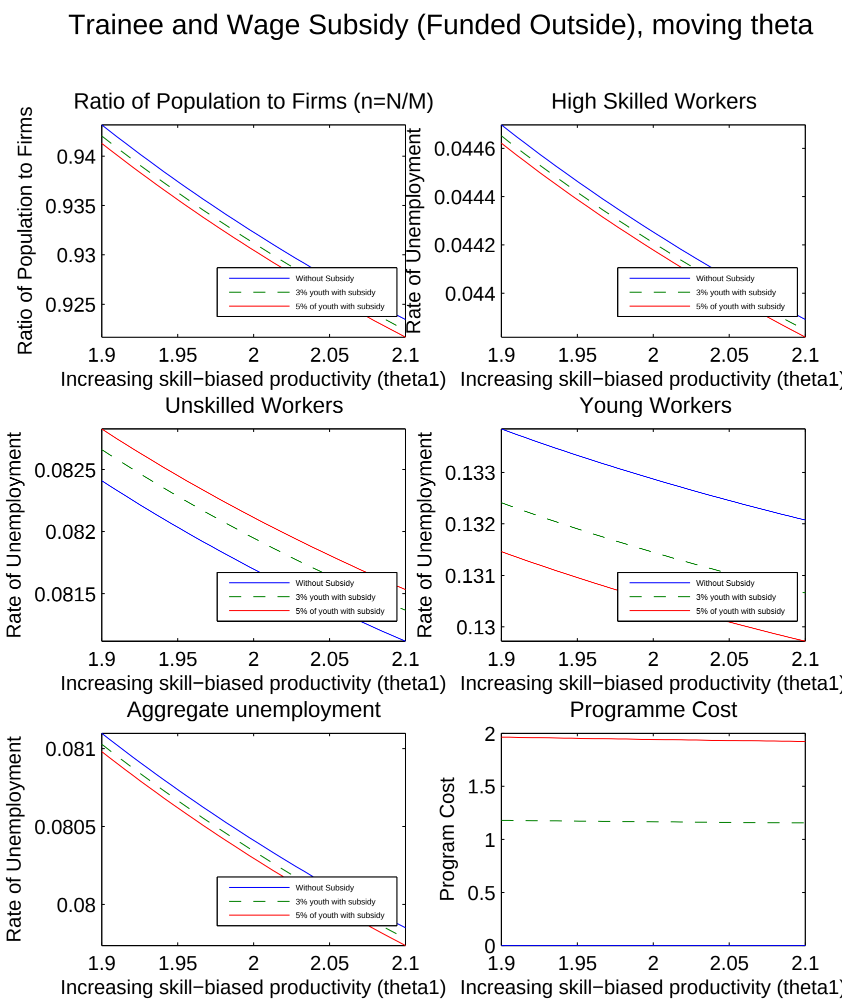
Figure 3.2: Numerical Experiment A. With an endogenous tax (funded inside the model). Applied to the case of subsidised training.



Note: Blue line is the case without intervention. Green line is the case with intervention (3% of young worker). Red line is the case with intervention (5% of young worker).

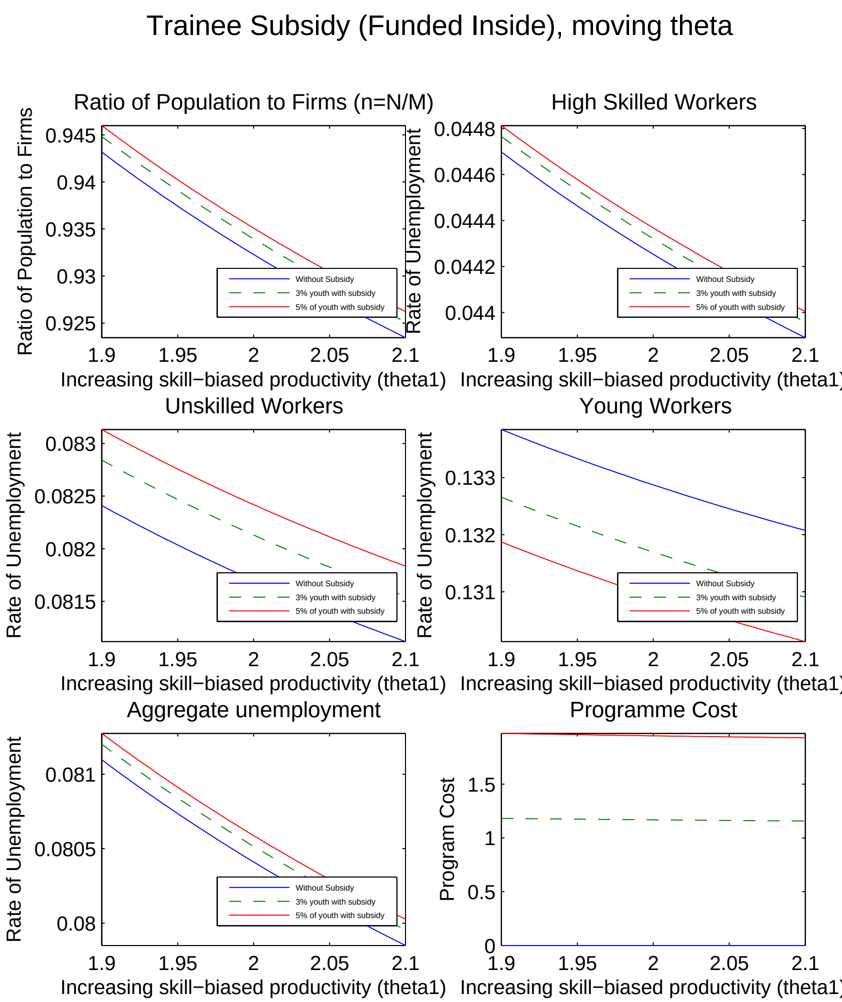
Figure 3.3: Numerical Experiment A. With an endogenous tax (funded inside the model). Applied to the case of subsidised recruitment.

Numerical Experiments B



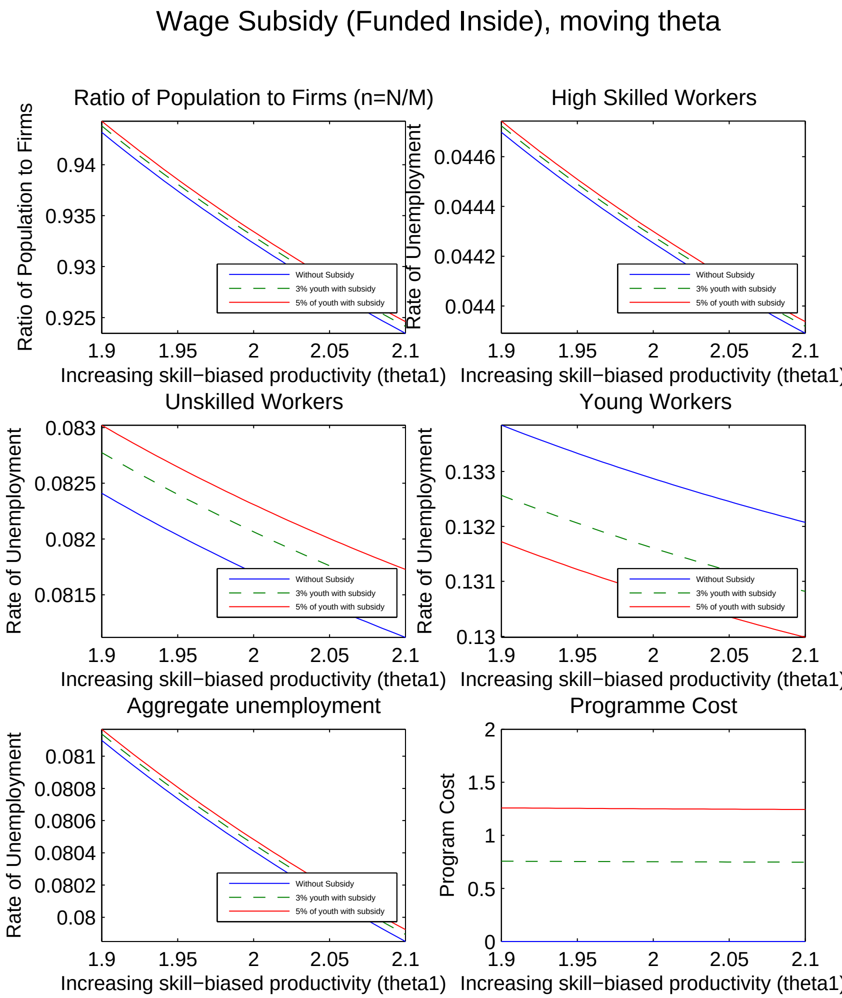
Note: Blue line is the case without intervention. Green line is the case with intervention (3% of young worker). Red line is the case with intervention (5% of young worker).

Figure 3.4: Numerical Experiment B. Funded outside the model. Applied to the case of subsidised training and recruitment.



Note: Blue line is the case without intervention. Green line is the case with intervention (3% of young worker). Red line is the case with intervention (5% of young worker).

Figure 3.5: Numerical Experiment B. With an endogenous tax (funded inside the model). Applied to the case of subsidised training.



Note: Blue line is the case without intervention. Green line is the case with intervention (3% of young worker). Red line is the case with intervention (5% of young worker).

Figure 3.6: Numerical Experiment B. With an endogenous tax (funded inside the model). Applied to the case of subsidised recruitment.

In developed experiments can be seen that active policies in the youth labor market are effective in reducing unemployment. However, its effectiveness depends on whether funding is considered exogenous or endogenous to the labor market. Naturally, experiments confirm that when funding is considered exogenous, its impact is greater. Additionally, this research shows that if there is a substitution effect. Adult less skilled workers are displaced by young a subsidised workers, the subsidy being in the form of training or salary.

3.7 Conclusion

In Chapter 3, this research uses a static model of direct search with heterogeneous agents to respond from, a theoretical perspective, whether the juvenile Active Labour Market Policies (ALMPs) actually benefit those workers to whom the program is addressed to (the youths, in this research). At the same time, the model allows us to analyse if there are any labour substitution effect between young workers benefited by a subsidy and those low-skilled adult workers, who are not part of the programme.

Presence of an eventual labour substitution effect has been neglected in the literature despite its relevance from a social-optimum macroeconomic perspective. To analyse the effectiveness of ALMPs this model will be calibrated to the US economy, so that to perform later numerical experiments that allow to assess the effects of the AMLPs; particularly the effect of training and hiring subsidies on employment levels for different types of workers and to find out if there are displacement effects.

This research confirms that the Active Labour Market Policies (ALMP) which intend to improve the position of young people in the labour market do not necessarily increase employment overall as these policies produce a substitution effect between subsidized workers and those who are not benefited from it, as it happens to low-skilled adult workers. This research will also confirm that those young workers who may have access to the subsidy experience a decrease in their unemployment levels.

Appendix

3.A The Application Queue.

Let K_T be defined as the total number of workers of type T in the economy, with $T \in \{s, u, j\}$. Also let $k_{F,T}$ be defined as, with $k_{F,T} \in \{0, 1, 2, ..., K_T\}$, the numbers of workers of type T that apply to the firm of type F, whit $T \in \{s, u, j\}$ and $F \in \{L, H\}$. Then the probability of $k_{F,T}$ workers of type T applying simultaneously to the same firm F is given by the following Binomial distribution:

$$\mathbb{P}(k_{F,T}) = \binom{K_T}{k_{F,T}} (p_{F,T})^{k_{F,T}} (1 - p_{F,T})^{K_T - k_{F,T}} \tag{3.A.1}$$

where $p_{F,T}$ is the probability that a worker of type T applies to the firm of type F and $(1 - p_{F,T})$ is the probability that worker does not apply.

Let the application queue, $q_{F,T}$, be defined as the expected number of applications of workers of type T who apply to the firm type F. Then by use the property the binomial distribution that the expected value is equal to the number of trial (which in the model is $k_{F,T}$) times the success probability in each trial (which in the model is $k_{F,T}$), the application queue can be calculated as follows:

$$q_{F,T} = \mathbb{E}(k_{F,T}) = \sum_{k_{F,T}=0}^{K_T} k_{F,T} \binom{K_T}{k_{F,T}} (p_{F,T})^{k_{F,T}} (1 - p_{F,T})^{K_T - k_{F,T}} = p_{F,T} K_T \tag{3.A.2}$$

whit $T \in \{s, u, j\}$ and $F \in \{L, H\}$.

Then the expected number of applications to a firm for each type of worker can be calculating for using the equation (3.A.2), method to be applied in the following section.

3.A.1 The Application Queue of a Skilled Worker

The number of skilled workers in this economy is known as sN, where s is the proportion of skilled workers and N is the total population of the economy. Then the application queue of skilled workers who apply to the high-tech firms is as follows:

$$q_{H,s} = \mathbb{E}(k_{H,s}) = p_{H,s}sN \tag{3.A.3}$$

where $p_{H,s}$ is the probability that a worker of type s applies to the firm of type H. When high-skilled workers apply to the low-tech firm, the application queue is as follows:

$$q_{L,s} = \mathbb{E}(k_{L,s}) = p_{L,s}sN \tag{3.A.4}$$

where $p_{L,s}$ is the probability that the worker of type s applies to the firm type L.

3.A.2 The Application Queue of an Unskilled Worker

The number of unskilled workers in this economy is known as uN, where u is the proportion of skilled workers and N is the total population of the economy. Then the application queue of skilled workers who apply to the high-tech firms is as follows:

$$q_{H,u} = \mathbb{E}(k_{H,u}) = p_{H,u}uN \tag{3.A.5}$$

where $p_{H,u}$ is the probability that the worker of type u applies to the firm of type H. When high-skilled workers apply to the low-tech firm, the application queue is as follows:

$$q_{L,u} = \mathbb{E}(k_{L,u}) = p_{L,u}sN \tag{3.A.6}$$

where $p_{L,u}$ is the probability that the worker of type u applies to the firm type L.

3.A.3 The Application Queue of a Young Worker

The number of young workers in this economy is known as jN, where j is the proportion of skilled worker and N is the total population of the economy. Then the application queue of skilled workers who apply to the high-tech firms is as follows:

$$q_{H,j} = \mathbb{E}(k_{H,j}) = p_{H,j}jN \tag{3.A.7}$$

where $p_{H,j}$ is the probability that the worker of type j applies to the firm type H. When high-skilled workers apply to the low-tech firm, the applications queue is as follows:

$$q_{L,j} = \mathbb{E}(k_{L,j}) = p_{L,j} jN \tag{3.A.8}$$

where $p_{L,j}$ is the probability that the worker of type j applies to the firm type L.

3.B Probability Calculation when N, M are Finite.

3.B.1 The Case of the Skilled Worker

The probability, $\rho_{F,s}$, that a skilled worker is hired by a firm of type F is equal to the probability that at least one skilled worker submits an application divided by the expected number of number of skilled workers' applications, and that is:

$$\rho_{F,s} = \frac{\mathbb{P}\left(k_{F,s} > 0\right)}{\mathbb{E}\left(k_{F,s}\right)} \tag{3.B.1}$$

where $F \in \{H, L\}$. However it is known that $\mathbb{P}\left(k_{F,s} > 0\right) = 1 - \mathbb{P}\left(k_{F,s} = 0\right)$ and by equation (3.A.1) also it is known that $\mathbb{P}\left(k_{F,s} = 0\right)$ is equal to:

$$\mathbb{P}(k_{F,s} = 0) = \binom{K_s}{0} (p_{F,s})^0 (1 - p_{F,s})^{K_s - 0} = (1 - p_{F,s})^{K_s} = (1 - p_{F,s})^{sN}. \tag{3.B.2}$$

Given that $K_s = sN$. Then $\mathbb{P}\left(k_{F,s} > 0\right)$ is equal to:

$$\mathbb{P}(k_{F,s} > 0) = 1 - (1 - p_{F,s})^{sN}. \tag{3.B.3}$$

Then by substituting equation (3.B.3) into equation (3.B.1), $\rho_{F,s}$ is equal to:

$$\rho_{F,s} = \frac{1 - (1 - p_{F,s})^{sN}}{\mathbb{E}\left(k_{F,s}\right)}. \tag{3.B.4}$$

Finally, it is known that $\mathbb{E}\left(k_{F,s}\right) = p_{F,s} sN$ by equations (3.A.3) and (3.A.4), then $\rho_{F,s}$ can be rewrite as follows:

$$\rho_{F,s} = \frac{1 - (1 - p_{F,s})^{sN}}{p_{F,s} sN} \tag{3.B.5}$$

where $F \in \{H, L\}$.

3.B.2 The Case of the Unskilled Worker

The probability, $\rho_{F,u}$, that an unskilled worker is hired by firm of type F is equal to the probability that any skilled worker arrive to the firm multiplied by the probability that at least one unskilled worker submits an application divided by the expected number of number of unskilled workers' applications, and that is:

$$\rho_{F,u} = \mathbb{P}\left(k_{F,s} = 0\right) \frac{\mathbb{P}\left(k_{F,u} > 0\right)}{\mathbb{E}\left(k_{F,u}\right)} \tag{3.B.6}$$

where $F \in \{H, L\}$. However it is know that $\mathbb{P}\left(k_{F,u} > 0\right) = 1 - \mathbb{P}\left(k_{F,u} = 0\right)$ and by equation (3.A.1) also it is known that $\mathbb{P}\left(k_{F,u} = 0\right)$ is equal to:

$$\mathbb{P}(k_{F,u} = 0) = \binom{K_u}{0}(p_{F,u})^0(1 - p_{F,u})^{K_u - 0} = (1 - p_{F,u})^{K_u} = (1 - p_{F,u})^{uN}. \tag{3.B.7}$$

Given that $K_u = uN$. Then $\mathbb{P}\left(k_{F,u} > 0\right)$ is equal to:

$$\mathbb{P}(k_{F,u} > 0) = 1 - (1 - p_{F,u})^{uN}. \tag{3.B.8}$$

Then by substituting equation (3.B.2) and (3.B.8) in equation (3.B.6), $\rho_{F,u}$ is equal to:

$$\rho_{F,u} = (1 - p_{F,s})^{sN} \frac{1 - (1 - p_{F,u})^{uN}}{\mathbb{E}\left(k_{F,u}\right)}. \tag{3.B.9}$$

Finally, it is known that $\mathbb{E}\left(k_{F,u}\right) = p_{F,u}uN$ by equations (3.A.5) and (3.A.6), then $\rho_{F,u}$ can be rewrite as follows:

$$\rho_{F,u} = (1 - p_{F,s})^{sN} \frac{1 - (1 - p_{F,u})^{uN}}{p_{F,u}uN} \tag{3.B.10}$$

where $F \in \{H, L\}$.

3.B.3 The Case of the Young Worker

The probability, $\rho_{F,j}$, that a young worker is hired by firm type F is equal to the probability that any skilled and unskilled worker arrives to the firm multiplied by the probability that at least one young worker submits an application divided by the expected number of young workers' applications, and that is:

$$\rho_{F,j} = \mathbb{P}\left(k_{F,s} = 0\right) \mathbb{P}\left(k_{F,u} = 0\right) \frac{\mathbb{P}\left(k_{F,j} > 0\right)}{\mathbb{E}\left(k_{F,j}\right)} \tag{3.B.11}$$

where $F \in \{H, L\}$. However it is known that $\mathbb{P}\left(k_{F,j} > 0\right) = 1 - \mathbb{P}\left(k_{F,j} = 0\right)$ and by equation (3.A.1) also it is known that $\mathbb{P}\left(k_{F,j} = 0\right)$ is equal to:

$$\mathbb{P}(k_{F,j} = 0) = \binom{K_j}{0}(p_{F,j})^0(1 - p_{F,j})^{K_j - 0} = (1 - p_{F,j})^{K_j} = (1 - p_{F,j})^{jN}. \tag{3.B.12}$$

Given that $K_j = jN$. Then $\mathbb{P}\left(k_{F,j} > 0\right)$ is equal to:

$$\mathbb{P}(k_{F,j} > 0) = 1 - (1 - p_{F,j})^{jN}. \tag{3.B.13}$$

Then by substituting equation (3.B.2), (3.B.7) and (3.B.13) in equation (3.B.11), $\rho_{F,j}$ is equal to:

$$\rho_{F,j} = (1 - p_{F,s})^{sN}(1 - p_{F,u})^{uN}\frac{1 - (1 - p_{F,j})^{jN}}{\mathbb{E}\left(k_{F,j}.\right)} \tag{3.B.14}$$

Finally, $\mathbb{E}\left(k_{F,j}\right) = p_{F,j}jN$ is known by equations (3.A.7) and (3.A.8) then $\rho_{F,j}$ can be rewrite as follows:

$$\rho_{F,j} = (1 - p_{F,s})^{sN}(1 - p_{F,u})^{uN}\frac{1 - (1 - p_{F,j})^{jN}}{p_{F,j}jN} \tag{3.B.15}$$

where $F \in \{H, L\}$.

3.C Probability Calculation when $N,\ M \mapsto \infty$.

In this section is calculated the probability of a type T worker being hired by a firm of type F when N and M are infinite, whit $T \in \{s, u, j\}$ and $F \in \{L, H\}$.

3.C.1 The Case of a Skilled Worker

The probability that a skilled worker is hired by a firm of type F when $N,\ M \mapsto \infty$ and $F \in \{L, H\}$ is equal to:

$$P_{F,s} = \lim_{N,M \to \infty} \rho_{F,s} = \lim_{N,M \to \infty} \frac{1 - (1 - p_{F,s})^{sN}}{sNp_{F,s}}. \tag{3.C.1}$$

Developing the above equation is obtained that:

$$P_{F,s} = \lim_{N,M \to \infty} \frac{1 - (1 - \frac{sNp_{F,s}}{sN})^{sN}}{sNp_{F,s}}. \tag{3.C.2}$$

However it is known that $q_{F,s} = sNp_{F,s}$, then by substituting this value in equation (3.C.2) and subsequently by using the properties of limits, it can be rewritten as follows:

$$P_{F,s} = \lim_{N,M\to\infty} \frac{1-(1-\frac{q_{F,s}}{sN})^{sN}}{q_{F,s}} = \frac{1-\lim_{N,M\to\infty}(1+\frac{-q_{F,s}}{sN})^{sN}}{q_{F,s}}. \tag{3.C.3}$$

Also it is known that by the definition that $e^x = \lim_{x\to\infty}(1+\frac{x}{n})^n$ then by assuming that $x = -q_{F,s}$ and $n = sN$ is obtained that $e^{-q_{F,s}} = \lim_{N,M\to\infty}(1+\frac{-q_{F,s}}{sN})^{sN}$. So by submitting this result in equation (3.C.3) the following expression for the probability $P_{F,s}$ is obtained, as follows:

$$P_{F,s} = \frac{1-e^{-q_{F,s}}}{q_{F,s}} \tag{3.C.4}$$

with $F \in \{L, H\}$.

3.C.2 The Case of a Unskill Worker

The probability that an unskilled worker is hired by a firm of type F when $N,\ M \mapsto \infty$ and $F \in \{L, H\}$ is equal to:

$$P_{F,u} = \lim_{N,M\to\infty} \rho_{F,u} = \lim_{N,M\to\infty} (1-p_{F,s})^{sN} \frac{1-(1-p_{F,u})^{uN}}{uNp_{F,u}}. \tag{3.C.5}$$

Developing the above equation is obtained that:

$$P_{F,u} = \lim_{N,M\to\infty} \left(1-\frac{sNp_{F,s}}{sN}\right)^{sN} \left(\frac{1-(1-\frac{uNp_{F,u}}{uN})^{uN}}{uNp_{F,u}}\right). \tag{3.C.6}$$

However it is known that $q_{F,s} = sNp_{F,s}$ and $q_{F,u} = uNp_{F,u}$, then by substituting these values in equation (3.C.6) and subsequently by using the properties of limits, it can be rewritten as follows:

$$P_{F,u} = \lim_{N,M\to\infty} \left(1-\frac{q_{F,s}}{sN}\right)^{sN} \left(\frac{1-(1-\frac{q_{F,u}}{uN})^{uN}}{q_{F,u}}\right)$$

$$P_{F,u} = \left(\lim_{N,M\to\infty} \left(1+\frac{-q_{F,s}}{sN}\right)^{sN}\right) \left(\frac{1-\lim_{N,M\to\infty}(1+\frac{-q_{F,u}}{uN})^{uN}}{q_{F,u}}\right). \tag{3.C.7}$$

Also it is known that by the definition that $e^x = \lim_{x\to\infty}(1+\frac{x}{n})^n$ then by assuming that $x = -q_{F,s}$ and $n = sN$ is obtained that $e^{-q_{F,s}} = \lim_{N,M\to\infty}(1+\frac{-q_{F,s}}{sN})^{sN}$. Additionally by assuming that $x = -q_{F,u}$ and $n = uN$ is obtained that $e^{-q_{F,u}} = \lim_{N,M\to\infty}(1+$

$\frac{-q_{F,u}}{uN})^{uN}$. So by substituting these results in equation (3.C.7) the following expression for the probability $P_{F,u}$ is obtained:

$$P_{F,u} = e^{-q_{F,s}} \left(\frac{1 - e^{-q_{F,u}}}{q_{F,u}} \right) \tag{3.C.8}$$

with $F \in \{L, H\}$.

3.C.3 The Case of Young Worker

The probability that a young worker is hired by a firm of type F when $N,\ M \mapsto \infty$ and $F \in \{L, H\}$ is equal to:

$$P_{F,j} = \lim_{N,M \to \infty} \rho_{F,j} = \lim_{N,M \to \infty} (1 - p_{F,s})^{sN} (1 - p_{F,u})^{uN} \left(\frac{1 - (1 - p_{F,j})^{jN}}{jNp_{F,j}} \right). \tag{3.C.9}$$

Developing the above equation is obtained that:

$$P_{F,j} = \lim_{N,M \to \infty} \left(1 - \frac{sNp_{F,s}}{sN}\right)^{sN} \left(1 - \frac{uNp_{F,u}}{uN}\right)^{uN} \left(\frac{1 - (1 - \frac{jNp_{F,j}}{jN})^{jN}}{jNp_{F,j}} \right). \tag{3.C.10}$$

However it is know that $q_{F,s} = sNp_{F,s}$, $q_{F,u} = uNp_{F,u}$ and $q_{F,j} = uNp_{F,j}$, then by substituting these values in equation (3.C.10) and subsequently by using the properties of limits, it can be rewritten as follows:

$$P_{F,j} = \lim_{N,M \to \infty} \left(1 - \frac{q_{F,s}}{sN}\right)^{sN} \left(1 - \frac{q_{F,u}}{uN}\right)^{uN} \left(\frac{1 - (1 - \frac{q_{F,j}}{jN})^{jN}}{q_{F,j}} \right)$$

$$P_{F,j} = \left(\lim_{N,M \to \infty} \left(1 + \frac{-q_{F,s}}{sN}\right)^{sN} \right) \left(\lim_{N,M \to \infty} \left(1 + \frac{-q_{F,u}}{uN}\right)^{uN} \right) \left(\frac{1 - \lim_{N,M \to \infty} (1 + \frac{-q_{F,j}}{jN})^{jN}}{q_{F,j}} \right). \tag{3.C.11}$$

Also it is known that by the definition that $e^x = \lim_{x \to \infty} (1 + \frac{x}{n})^n$, then by assuming that $x = -q_{F,s}$ and $n = sN$ is obtained that $e^{-q_{F,s}} = \lim_{N,M \to \infty} (1 + \frac{-q_{F,s}}{sN})^{sN}$. Additionally by assuming that $x = -q_{F,u}$ and $n = uN$ is obtained that $e^{-q_{F,u}} = \lim_{N,M \to \infty} (1 + \frac{-q_{F,u}}{uN})^{uN}$. Finally, by assuming that $x = -q_{F,j}$ and $n = jN$ is obtained that $e^{-q_{F,j}} = \lim_{N,M \to \infty} (1 + \frac{-q_{F,j}}{jN})^{jN}$. So by substituting these results in equation (3.C.11) the following expression for the probability $P_{F,j}$ is obtained:

$$P_{F,u} = e^{-q_{F,s}-q_{F,u}} \left(\frac{1-e^{-q_{F,j}}}{q_{F,j}} \right) \tag{3.C.12}$$

with $F \in \{L, H\}$.

3.D Solving for the Symmetric Equilibrium

3.D.1 Solving the Optimisation Process for the Equilibrium

By solving the two constrained profit maximisation problems and by combining their results it is possible to obtain the values of queues in equilibrium as a function of y, θ_1, θ_2, U_s, U_u and U_j as follows:

In the case of the High-Tech firm:

$$\begin{aligned} \max_{\{w_{H,s}, w_{H,u}, w_{H,j}\}} \Pi_H = (1-e^{-q_{H,s}})\{\theta_1 y - w_{H,s}\} + e^{-q_{H,s}}(1-e^{-q_{H,u}})\{y - w_{H,u}\} \\ + e^{-q_{H,s}-q_{H,u}}(1-e^{-q_{H,j}})\{\theta_2 y - w_{H,j}\} \end{aligned} \tag{3.D.1}$$

s.t.:

$$w_{H,s} = \frac{U_s q_{H,s}}{1-e^{-q_{H,s}}} \tag{3.D.2}$$

$$w_{H,u} = \frac{U_u q_{H,u}}{e^{-q_{H,s}}(1-e^{-q_{H,u}})} \tag{3.D.3}$$

$$w_{H,j} = \frac{U_j q_{H,j}}{e^{-q_{H,s}-q_{H,u}}(1-e^{-q_{H,j}})}. \tag{3.D.4}$$

By substituting equations (3.D.2), (3.D.3) and (3.D.4) into (3.D.1), the objective function is obtained as a function of the application queues, as follows:

$$\begin{aligned} \max_{\{q_{H,s}, q_{H,u}, q_{H,j}\}} \Pi_H = (1-e^{-q_{H,s}}) \left(\theta_1 y - \frac{U_s q_{H,s}}{1-e^{-q_{H,s}}} \right) \\ + e^{-q_{H,s}}(1-e^{-q_{H,u}}) \left(y - \frac{U_u q_{H,u}}{e^{-q_{H,s}}(1-e^{-q_{H,u}})} \right) \\ + e^{-q_{H,s}-q_{H,u}}(1-e^{-q_{H,j}}) \left(\theta_2 y - \frac{U_j q_{H,j}}{e^{-q_{H,s}-q_{H,u}}(1-e^{-q_{H,j}})} \right). \end{aligned} \tag{3.D.5}$$

Then by developing the objective function in (3.D.5) the maximisation process is equivalent to:

$$\max_{\{q_{H,s},q_{H,u},q_{H,j}\}} \Pi_H = (1-e^{-q_{H,s}})\theta_1 y - U_s q_{H,s} + e^{-q_{H,s}}(1-e^{-q_{H,u}})y - U_u q_{H,u} \\ + e^{-q_{H,s}-q_{H,u}}(1-e^{-q_{H,j}})\theta_2 y - U_j q_{H,j}. \quad (3.D.6)$$

Where the first order conditions are:

$$\frac{\partial \Pi_H}{\partial q_{H,s}} = y(\theta_1 - 1)e^{-q_{H,s}} + y(1-\theta_2)e^{-q_{H,s}-q_{H,u}} + \theta_2 y e^{-q_{H,s}-q_{H,u}-q_{H,j}} - U_s = 0 \quad (3.D.7)$$

$$\frac{\partial \Pi_H}{\partial q_{H,u}} = y(1-\theta_2)e^{-q_{H,s}-q_{H,u}} + \theta_2 y e^{-q_{H,s}-q_{H,u}-q_{H,j}} - U_u = 0 \quad (3.D.8)$$

$$\frac{\partial \Pi_H}{\partial q_{H,j}} = \theta_2 y e^{-q_{H,s}-q_{H,u}-q_{H,j}} - U_j = 0. \quad (3.D.9)$$

Then using equation (3.D.9) exogenous variables and the variable U_j are isolated. After doing this, logarithms of both sides of equation are taken, this procedure gives:

$$\theta_2 y e^{-q_{H,s}-q_{H,u}-q_{H,j}} = U_j$$

$$e^{-q_{H,s}-q_{H,u}-q_{H,j}} = \frac{U_j}{\theta_2 y}$$

$$e^{q_{H,s}q_{H,u}q_{H,j}} = \frac{\theta_2 y}{U_j} \qquad / \ Ln()$$

$$q_{H,s} + q_{H,u} + q_{H,j} = Ln\left(\frac{\theta_2 y}{U_j}\right). \quad (3.D.10)$$

Further, by substituting the equation (3.D.10) into the equation (3.D.8), this equation can be rewritten as:

$$y(1-\theta_2)e^{-q_{H,s}-q_{H,u}} + \theta_2 y e^{-Ln\left(\frac{\theta_2 y}{U_j}\right)} - U_u = 0$$

$$y(1-\theta_2)e^{-q_{H,s}-q_{H,u}} + \theta_2 y e^{Ln\left(\frac{U_j}{\theta_2 y}\right)} - U_u = 0$$

$$y(1-\theta_2)e^{-q_{H,s}-q_{H,u}} + U_j - U_u = 0. \quad (3.D.11)$$

Again by means of isolating the exogenous variables, the variables U_j and U_u, and by taking logarithms of both sides of equation (3.D.11) is obtained that:

$$q_{H,s} + q_{H,u} = Ln\left(\frac{(1-\theta_2)y}{U_u - U_j}\right). \quad (3.D.12)$$

Then by subsituting $q_{H,s} + q_{H,u}$ in equation (3.D.12) into equation (3.D.10) is found that $q_{H,j}$ is equal to:

$$q_{H,j} = Ln\left(\frac{\theta_2}{(1-\theta_2)}\frac{U_u - U_j}{U_j}.\right) \tag{3.D.13}$$

Also, to get the value of the queue, $q_{H,s}$, it is necessary to substitute equation (3.D.10) and equation (3.D.12) into equation (3.D.7), as follows:

$$\begin{gathered}
y(\theta_1 - 1)e^{-q_{H,s}} + y(1-\theta_2)e^{-Ln\left(\frac{(1-\theta_2)y}{U_u - U_j}\right)} + \theta 2ye^{-Ln\left(\frac{\theta_2 y}{U_j}\right)} - U_s = 0 \\
y(\theta_1 - 1)e^{-q_{H,s}} + y(1-\theta_2)e^{Ln\left(\frac{U_u - U_j}{(1-\theta_2)y}\right)} + \theta 2ye^{Ln\left(\frac{U_j}{\theta_2 y}\right)} - U_s = 0 \\
y(\theta_1 - 1)e^{-q_{H,s}} + U_u - U_j + U_j - U_s = 0 \\
y(\theta_1 - 1)e^{-q_{H,s}} + U_u - U_s = 0. \qquad (3.D.14)
\end{gathered}$$

Again by means of isolating the exogenous variables, the variables U_u and U_s, and by taking logarithms of both sides of equation (3.D.14), $q_{H,s}$ is equal to:

$$q_{H,s} = Ln\left(\frac{y(\theta_1 - 1)}{U_s - U_u}\right). \tag{3.D.15}$$

The value of $q_{H,u}$ is obtained by substituting $q_{H,s}$ calculated in equation (3.D.15) into equation (3.D.12), as follows:

$$\begin{aligned}
q_{H,u} &= Ln\left(\frac{(1-\theta_2)y}{U_u - U_j}\right) - Ln\left(\frac{y(\theta_1 - 1)}{U_s - U_u}\right) \\
q_{H,u} &= Ln\left(\frac{1-\theta_2}{\theta_1 - 1}\frac{U_s - U_u}{U_u - U_j}\right). \qquad (3.D.16)
\end{aligned}$$

In the case of the Low-Tech firm:

$$\max_{\{w_{L,u}, w_{L,j}\}} \Pi_L = (1 - e^{-q_{L,u}})(y - w_{L,u}) + e^{-q_{L,u}}(1 - e^{-q_{L,j}})(\theta_2 y - w_{L,j}) \tag{3.D.17}$$

s.t.:

$$\begin{aligned}
w_{L,u} &= \frac{U_u q_{L,u}}{(1 - e^{-q_{L,u}})} \qquad (3.D.18) \\
w_{L,j} &= \frac{U_j q_{L,j}}{e^{-q_{L,u}(1-e^{-q_{L,j}})}}. \qquad (3.D.19)
\end{aligned}$$

By substituting equations (3.D.18) and (3.D.19) into (3.D.17), the objective function is obtained as a function of the application queues, as follows:

$$\max_{\{q_{L,u},q_{L,j}\}} \Pi_L = (1-e^{-q_{L,u}})\left(y - \frac{U_u q_{L,u}}{(1-e^{-q_{L,u}})}\right) + e^{-q_{L,u}}(1-e^{-q_{L,j}})\left(\theta_2 y - \frac{U_j q_{L,j}}{e^{-q_{L,u}}(1-e^{-q_{L,j}})}\right) \tag{3.D.20}$$

Then by developing the objective function in (3.D.20)the maximization process is equivalent to:

$$\max_{\{q_{L,u},q_{L,j}\}} \Pi_L = (1-e^{-q_{L,u}})y - U_u q_{L,u} + e^{-q_{L,u}}(1-e^{-q_{L,j}})\theta_2 y - U_j q_{L,j} \tag{3.D.21}$$

The first order conditions are:

$$\frac{\partial \Pi_L}{\partial q_{L,u}} = ye^{-q_{L,u}} - \theta_2 ye^{-q_{L,u}} + \theta_2 ye^{-q_{L,u}-q_{L,j}} - U_u = 0 \tag{3.D.22}$$

$$\frac{\partial \Pi_L}{\partial q_{L,j}} = \theta_2 ye^{-q_{L,u}-q_{L,j}} - U_j = 0. \tag{3.D.23}$$

Then using equation (3.D.23) exogenous variables and the variable U_j are isolated. After doing this, logarithms of both sides of the equation are taken, this procedure gives:

$$q_{L,u} + q_{L,j} = Ln\left(\frac{\theta_2 y}{U_j}\right). \tag{3.D.24}$$

To get the value of the queue, $q_{L,u}$, is necessary to substituting equation (3.D.24) into equation (3.D.22), as follows:

$$ye^{-q_{L,u}} - \theta_2 ye^{-q_{L,u}} + \theta_2 ye^{-Ln\left(\frac{\theta_2 y}{U_j}\right)} - U_u = 0 \tag{3.D.25}$$

$$ye^{-q_{L,u}} - \theta_2 ye^{-q_{L,u}} + \theta_2 ye^{Ln\left(\frac{U_j}{\theta_2 y}\right)} - U_u = 0.$$

$$ye^{-q_{L,u}} - \theta_2 ye^{-q_{L,u}} + U_j - U_u = 0. \tag{3.D.26}$$

Through the means of isolating the exogenous variables, the variables U_j and U_u, and by taking logarithms of both sides of equation (3.D.26) is obtained that $q_{L,u}$ is equal to:

$$q_{L,u} = Ln\left(\frac{y(1-\theta_2)}{U_u - U_j}\right). \tag{3.D.27}$$

The value of $q_{L,j}$ is calculated by substituting $q_{L,u}$ obtained in equation (3.D.27) into equation (3.D.24) as follows:

$$\begin{aligned} q_{L,j} &= Ln\left(\frac{\theta_2 y}{U_j}\right) - Ln\left(\frac{y(1-\theta_2)}{U_u - U_j}\right) \\ q_{L,j} &= Ln\left(\frac{\theta_2}{1-\theta_2}\frac{U_u - U_j}{U_j}\right). \end{aligned} \tag{3.D.28}$$

Note also that by equations (3.D.12) and (3.D.27) can be concluded that in equilibrium the following is satisfied:

$$q_{H,s} + q_{H,u} = q_{L,u}. \tag{3.D.29}$$

Furthermore, by equations (3.D.13) and (3.D.28) in the equilibrium the following is satisfied:

$$q_{H,j} = q_{L,j}. \tag{3.D.30}$$

3.D.2 The Queue as a Function of s, u, j, h and n in the equilibrium.

In this section the value of the queues in the equilibrium will be calculated as a function of exogenous variables s, u, j, h and the endogenous variable n, by using the equality constraints (**??**)-(**??**), by using the equalities (3.D.29) and (3.D.30) found in the process of maximising profits and by Lemma 3.1 in Shi (2002).

By equation (**??**) it is known that:

$$hq_{H,s} + (1-h)q_{L,s} = sn. \tag{3.D.31}$$

Moreover, it is known by Lemma 3.1 in Shi (2002) that:

$$q_{L,s} = 0. \tag{3.D.32}$$

Then by substituting the above result in equation (3.D.31) the value of $q_{H,s}$ is found:

$$q_{H,s} = \frac{sn}{h}. \tag{3.D.33}$$

Also, by equation (**??**) it it know that:

$$hq_{H,j} + (1-h)q_{L,j} = jn. \tag{3.D.34}$$

As well by (3.D.30) it is known that $q_{H,j} = q_{L,j}$, then by substituting this result in (3.D.34) can be obtained that:

$$q_{H,j} = q_{L,j} = jn. \tag{3.D.35}$$

Moreover, by equation (**??**) we have that:

$$hq_{H,u} + (1-h)q_{L,u} = un \tag{3.D.36}$$

Besides by equations (3.D.29) and (3.D.33) it is known that $q_{H,s} + q_{H,u} = q_{L,u}$ and $q_{H,s} = \frac{sn}{h}$, respectively. Then by combining these two equations that the value of $q_{H,u}$ is found:

$$q_{H,u} = q_{L,u} - \frac{sn}{h}. \tag{3.D.37}$$

Then, by substituting this result in equation (3.D.36) an expression for $q_{L,u}$ is obtained, as follows:

$$h(q_{L,u} - \frac{sn}{h}) + (1-h)q_{L,u} = un$$

$$hq_{L,u} - sn + (1-h)q_{L,u} = un$$

$$q_{L,u} = (u+s)n. \tag{3.D.38}$$

Finally, the expression for $q_{H,u}$ is obtained by substituting (3.D.38) into equation (3.D.37) as follows:

$$q_{H,u} = (u+s)n - \frac{sn}{h}. \tag{3.D.39}$$

3.D.3 Calculation of U_s, U_u and U_j in the Equilibrium.

In this subsection the values of U_s, U_u and U_j will be calculated in equilibrium as function of the exogenous variables y, s, u, j, θ_1 and θ_2 and the endogenous variable n, by means of applying the findings in subsections 3.D.1 and 3.D.2 of this appendix.

Calculating U_j

To obtain an expression for U_j, $U_u - U_j$ will be isolated in equation (3.D.27) as follows:

$$q_{L,u} = Ln\left(\frac{y(1-\theta_2)}{U_u - U_j}\right)$$

$$e^{q_{L,u}} = \frac{y(1-\theta_2)}{U_u - U_j}$$

$$U_u - U_j = \frac{y(1-\theta_2)}{e^{q_{L,u}}}. \tag{3.D.40}$$

Then substitute the expression $U_u - U_j$ obtained in the equation (3.D.40) into equation (3.D.28) and proceed by isolating U_j as follows:

$$q_{L,j} = Ln\left(\frac{\theta_2}{1-\theta_2}\frac{U_u - U_j}{U_j}\right) = Ln\left(\frac{\theta_2}{1-\theta_2}\frac{\frac{y(1-\theta_2)}{e^{q_{L,u}}}}{U_j}\right)$$

$$q_{L,j} = Ln\left(\frac{\theta_2 y}{e^{q_{L,u}}U_j}\right)$$

$$e^{q_{L,j}} = \frac{\theta_2 y}{e^{q_{L,u}}U_j}$$

$$U_j = \frac{\theta_2 y}{e^{q_{L,u}+q_{L,j}}}$$

$$U_j = \theta_2 y e^{-q_{L,u}-q_{L,j}}. \tag{3.D.41}$$

It is known that $q_{L,j} = jn$ (equation (3.D.35)) and that $q_{L,u} = (u+s)n$ (equation (3.D.38)), then U_j can be rewritten as:

$$U_j = \theta_2 y e^{-un-sn-jn} = \theta_2 y e^{-n}. \tag{3.D.42}$$

Calculating U_u

To obtain an expression for U_u, U_j will be isolated in equation (3.D.27), as follows:

$$q_{L,u} = Ln\left(\frac{y(1-\theta_2)}{U_u - U_j}\right)$$

$$e^{q_{L,u}} = \frac{y(1-\theta_2)}{U_u - U_j}$$

$$e^{q_{L,u}}U_u = y(1-\theta_2) + e^{q_{L,u}}U_j$$

$$U_u = y(1-\theta_2)e^{-q_{L,u}} + U_j. \tag{3.D.43}$$

It is known that $q_{L,u} = (u+s)n$ (equation (3.D.38)) and that $U_j = \theta_2 y e^{-n}$ (equation (3.D.42)) and by substituting these expressions into equation (3.D.43) an expression for U_u is found, as follows:

$$U_u = y(1-\theta_2)e^{-(u+s)n} + \theta_2 y e^{-n} \tag{3.D.44}$$

Calculation for U_s

To obtain an expression for U_s, U_s will be isolated in equation (3.D.15), as follows:

$$q_{H,s} = Ln\left(\frac{y(\theta_1 - 1)}{U_s - U_u}\right)$$

$$e^{q_{H,s}} = \frac{y(\theta_1 - 1)}{U_s - U_u}$$

$$e^{q_{H,s}}U_s = y(\theta_1 - 1) + e^{q_{H,u}}U_u$$

$$U_s = y(\theta_1 - 1)e^{-q_{H,s}} + U_u \tag{3.D.45}$$

It is known that $q_{H,s} = \frac{sn}{h}$ (equation (3.D.33)). Also it is know U_u in the equation (3.D.44). Then by substituting these results into equation (3.D.45) an expression for U_s is calculated, as follows:

$$U_s = y(\theta_1 - 1)e^{-\frac{sn}{h}} + y(1 - \theta_2)e^{-(u+s)n} + \theta_2 ye^{-n}. \tag{3.D.46}$$

3.D.4 Profits of the Firms and Free Entry Conditions in equilibrium.

Low Tech Firm

The profits of the Low-Tech firm, Π_L, without discounting the cost of entry K_L, is determined by the objective function in (3.D.21), that is:

$$\Pi_L = (1 - e^{-q_{L,u}})y - U_u q_{L,u} + e^{-q_{L,u}}(1 - e^{-q_{L,j}})\theta_2 y - U_j q_{L,j}. \tag{3.D.47}$$

The expressions for $q_{L,j}$, $q_{L,u}$, U_j and U_u are known in the equilibrium through of the following equations (3.D.35), (3.D.38), (3.D.42) and (3.D.44), respectively. Then substituting these equations into equation (3.D.47), Π_L can be expressed as follows:

$$\begin{aligned}\Pi_L = (1 - e^{-(u+s)n})y - (y(1 - \theta_2)e^{-(u+s)n} + \theta_2 ye^{-n})(u + s)n \\ +e^{-(u+s)n}(1 - e^{-jn})\theta_2 y - (\theta_2 ye^{-n})jn.\end{aligned} \tag{3.D.48}$$

Rearranging terms in the above equation, Π_L can be rewritten as follows:

$$\begin{aligned}\Pi_L = y - (y + y(1 - \theta_2)(u + s)n - \theta_2 y)e^{-(u+s)n} \\ -(\theta_2 y(u + s)n + \theta_2 y + \theta_2 yjn)e^{-n}.\end{aligned} \tag{3.D.49}$$

Assuming the free entry condition holds, that is $\Pi_L = K_L$, the following expression is obtained (note that K_L is an exogenous variable):

$$\begin{aligned} K_L = y - (y + y(1-\theta_2)(u+s)n - \theta_2 y)e^{-(u+s)n} \\ -(\theta_2 y(u+s)n + \theta_2 y + \theta_2 yjn)e^{-n} \end{aligned} \tag{3.D.50}$$

Taking y as a common factor and then dividing both sides of the equation by y is obtained the first of the equations, from which the equilibrium for this economy will be obtained. This is:

$$\begin{aligned} \frac{K_L}{y} = 1 - (1 + (1-\theta_2)(u+s)n - \theta_2)e^{-(u+s)n} \\ -(\theta_2(u+s)n + \theta_2 + \theta_2 jn)e^{-n}. \end{aligned} \tag{3.D.51}$$

High Tech Firm

The profits of the High-Tech firm, Π_H, without discounting the cost of entry K_H, is determined by the objective function in (3.D.6), that is:

$$\begin{aligned} \Pi_H = (1 - e^{-q_{H,s}})\theta_1 y - U_s q_{H,s} + e^{-q_{H,s}}(1 - e^{-q_{H,u}})y - U_u q_{H,u} \\ + e^{-q_{H,s} - q_{H,u}}(1 - e^{-q_{H,j}})\theta_2 y - U_j q_{H,j}. \end{aligned} \tag{3.D.52}$$

The expression for $q_{H,s}$, $q_{H,j}$, $q_{H,u}$, U_j, U_u and U_s are known in the equilibrium through of the following equations (3.D.33), (3.D.35), (3.D.39), (3.D.42), (3.D.44) and (3.D.46), respectively. Then substituting these equations into equation (3.D.52), Π_H can be expressed as follows:

$$\begin{aligned} \Pi_H = (1 - e^{-\frac{sn}{h}})\theta_1 y - (y(\theta_1 - 1)e^{-\frac{sn}{h}} + y(1-\theta_2)e^{-(u+s)n} + \theta_2 y e^{-n})\frac{sn}{h} \\ + e^{-\frac{sn}{h}}(1 - e^{-(u+s)n - \frac{sn}{h}})y - (y(1-\theta_2)e^{-(u+s)n} + \theta_2 y e^{-n})((u+s)n - \frac{sn}{h}) \\ + e^{-\frac{sn}{h} - ((u+s)n - \frac{sn}{h})}(1 - e^{-jn})\theta_2 y - (\theta_2 y e^{-n})jn. \end{aligned} \tag{3.D.53}$$

By rearranging terms in the above equation it can be rewritten as:

$$\begin{aligned} \Pi_H = y(\theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}}) \\ -y(1 + (1-\theta_2)(u+s)n - \theta_2)e^{-(u+s)n} \\ -y(\theta_2(u+s)n + \theta_2 + \theta_2 yn)e^{-n}. \end{aligned} \tag{3.D.54}$$

Assuming the free entry condition holds, that is $\Pi_H = K_H$, the following expression is obtained (note that K_H is an exogenous variable):

$$\begin{aligned} K_H = y(\theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}}) \\ -y(1 + (1-\theta_2)(u+s)n - \theta_2)e^{-(u+s)n} \\ -y(\theta_2(u+s)n + \theta_2 + \theta_2 yn)e^{-n}. \end{aligned} \tag{3.D.55}$$

By dividing both sides of the equation by y, the following expression is obtained:

$$\begin{aligned} \frac{K_H}{y} = (\theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}}) \\ -\{(1 + (1-\theta_2)(u+s)n - \theta_2)e^{-(u+s)n} + (\theta_2(u+s)n + \theta_2 + \theta_2 yn)e^{-n}\}. \end{aligned} \tag{3.D.56}$$

Note that the expression in curly brackets in the above equation is identical to $\frac{y-K_L}{y}$ (see equation (3.D.51)). Then by using this value, equation (3.D.56) can be expressed as:

$$\frac{K_H}{y} = (\theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}}) - \left(\frac{y - K_L}{y}\right). \tag{3.D.57}$$

By rearranging terms, the second of the equations from which the equilibrium for this economy is obtained. This is:

$$\begin{aligned} \frac{K_H}{y} &= (\theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}}) - 1 + \frac{K_L}{y} \\ \frac{K_H - K_L}{y} &= -1 + \theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}} \\ \frac{K_H - K_L}{y} &= (\theta_1 - 1) - (\theta_1 - 1)e^{\frac{-sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} \\ \frac{K_H - K_L}{(\theta_1 - 1)y} &= 1 - e^{\frac{-sn}{h}} - e^{-\frac{sn}{h}}\frac{sn}{h}. \end{aligned} \tag{3.D.58}$$

3.D.5 The System of Equations that solves of symmetric equilibrium.

The ratio of workers to firms $n = \frac{N}{M}$ that solves for the symmetric equilibrium must satisfy the following two equations:

$$\begin{aligned} \frac{K_L}{y} = 1 - (1 + (1-\theta_2)(u+s)n - \theta_2)e^{-(u+s)n} - (\theta_2(u+s)n + \theta_2 + \theta_2 jn)e^{-n}, \\ \frac{K_H - K_L}{(\theta_1 - 1)y} = 1 - e^{\frac{-sn}{h}}\left(1 + \frac{sn}{h}\right). \end{aligned} \tag{3.D.59}$$

Where the following exogenous variables are taken as given y, K_H, K_L, θ_1, θ_2, s, u, j and N.

3.E Solving for the Symmetric Equilibrium with Taxation.

3.E.1 Low-Tech Firm

From equation (3.D.49) the profit before both tax and the free entry condition is calculated for the Low-Tech firm as follow:

$$\Pi_L = y(1 - (1 + (1-\theta_2)(u+s)n - \theta_2)e^{-(u+s)n} - (\theta_2(u+s)n + \theta_2 + \theta_2 jn)e^{-n}). \quad (3.E.1)$$

Assuming an entrance cost of K_L and a payable amount of tax, t, the free entry condition is defined for Low-Tech firm as follows:

$$K_L = (1-t)\Pi_L. \quad (3.E.2)$$

By substituting the free entry condition with tax into equation (3.E.1), it can be rewritten as follows:

$$\frac{K_L}{(1-t)y} = 1 - (1 + (1-\theta_2)(u+s)n - \theta_2)e^{-(u+s)n} - (\theta_2(u+s)n + \theta_2 + \theta_2 jn)e^{-n}. \quad (3.E.3)$$

3.E.2 High-Tech Firm

From equation (3.D.54) the profit before both tax and the free entry condition is calculated for the High-Tech firm as follows:

$$\begin{aligned} \Pi_H &= y(\theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}}) \\ &-y(1 + (1-\theta_2)(u+s)n - \theta_2)e^{-(u+s)n} - y(\theta_2(u+s)n + \theta_2 + \theta_2 yn)e^{-n}. \end{aligned} \quad (3.E.4)$$

A common factor $-y$ can be taken in the second and third terms of the above equation. Then Π_H can be rewritten as follows:

$$\begin{aligned} \Pi_H &= y(\theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}}) \\ &-y\{(1 + (1-\theta_2)(u+s)n - \theta_2)e^{-(u+s)n} + (\theta_2(u+s)n + \theta_2 + \theta_2 yn)e^{-n}\}. \end{aligned} \quad (3.E.5)$$

Note that part in curly brackets in the above equation is identical to $1-\frac{K_L}{(1-t)y}$ (see equation (3.E.3)). Using this, the equation (3.E.5) can be rewritten as follows:

$$\Pi_H = y(\theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}}) - y\left(1 - \frac{K_L}{(1-t)y}\right)$$

$$\Pi_H = y(\theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}}) - y + \frac{K_L}{(1-t)}. \tag{3.E.6}$$

Assuming an entrance cost of K_H and payable amount of tax, t, the free entry condition is defined for High-Tech firm as follows:

$$K_H = (1-t)\Pi_H \tag{3.E.7}$$

By substituting the free entry condition with tax into the equation (3.E.6), it can be rewritten as follows:

$$\frac{K_H}{1-t} = y(\theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}}) - y + \frac{K_L}{(1-t)}. \tag{3.E.8}$$

By rearranging terms, it can be written as follows:

$$\frac{K_H - K_L}{(1-t)} = -y + y(\theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}})$$

$$\frac{K_H - K_L}{(1-t)y} = -1 + \theta_1 - \theta_1 e^{-\frac{sn}{h}} - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} + e^{-\frac{sn}{h}}$$

$$\frac{K_H - K_L}{(1-t)y} = (\theta_1 - 1) - (\theta_1 - 1)e^{-\frac{sn}{h}}\frac{sn}{h} - (\theta_1 - 1)e^{-\frac{sn}{h}}$$

$$\frac{K_H - K_L}{(1-t)(\theta_1 - 1)y} = 1 - e^{-\frac{sn}{h}}\left(1 + \frac{sn}{h}\right). \tag{3.E.9}$$

3.E.3 The System of Equations that solves for the symmetric equilibrium with Tax.

Then the ratio of workers to firms $n = \frac{N}{M}$ and t that solves for the symmetric equilibrium with tax must satisfy the following two equations:

$$\begin{aligned} \frac{K_L}{(1-t)y} &= 1 - (1 + (1-\theta_2)(u+s)n - \theta_2)e^{-(u+s)n} \\ &\quad -(\theta_2(u+s)n + \theta_2 + \theta_2 jn)e^{-n}, \\ \frac{K_H - K_L}{(1-t)(\theta_1 - 1)y} &= 1 - e^{-\frac{sn}{h}}\left(1 + \frac{sn}{h}\right). \end{aligned} \tag{3.E.10}$$

Where the following exogenous variables y, K_H, K_L, θ_1, θ_2, s, u, j, and N are taken as given.

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4. Cyclical Unemployment in U.S.?: The Case of Young People

Abstract

While it is widely recognized that the unemployment level is negatively correlated with the economic activity, it is not so widely known that its impact differs in level and persistence when considering demographic factors, such as gender and age. Total US unemployment has more than doubled from the fourth quarter of 2007 (4.9%) to the first quarter of 2010 (9.8%). In the case of youth unemployment, it has more than doubled the unemployment for premium-age population, reaching 17.5% during the first half of 2010. Therefore, to dimension the impact of unemployment on welfare, it is imperative to understand the factors that explain the evolution of unemployment rates in the United States, either at aggregate, age or gender levels, and its relationship with the business cycle. Using a dynamic model of business cycle, we intend to elucidate the impact that hiring and separation flows have on the US unemployment variability. Results suggest a high heterogeneity in the inflow an outflow hazard rates when controlling by age and gender. This high heterogeneity compels us to avoid generalizations when trying to provide quantitative answers at macroeconomic level to labour policy issues, such as recruiting and unemployment subsidies, among others. Moreover, results suggest that finding rates (outflows) seems to be the main factor in the youth unemployment dynamic, and therefore a crucial element to be considered in the policy-making process to address youth unemployment.

Keywords: Youth Unemployment, Separation Rate, Job Finding Rate, Business Cycle.
JEL Classification: J64, E24 and E32.

4.1 Introduction

The unemployment rate is one of the main economic indicators to analyze an economy health and the welfare of its population (Hibbs, 1979). As we know, the unemployment level is negatively correlated with the economic activity. However, the way in which changes in Gross Domestic Product (GDP) impact on unemployment differs in level and persistence when considering gender, age, education level, race, work experience and other disaggregation. (Elsby et al., 2010 and Katz, 2010)

In fact, during the recent economic crisis in the United States (US), unemployment rate at aggregate level more than doubled from 4.9% in the fourth quarter of 2007 to 9.8% in the first quarter of 2010. However, unemployment for young people between 18 and 24 years of age reached 17.5% during the first half of 2010. Even for young people between 16 and 17 years of age, the unemployment rate was of 28.5% for the same period, showing a dramatic gap between youth and adult unemployment. Furthermore, when analysing a longer period of time, from 1976 to 2011 it can be observed that compared for young people aged between 18 and 24, unemployment more than doubled in respect to the prime-age population unemployment[1], that is, 11.9% and 5.2% respectively[2]. (see table 4.27)

As for persistence, after 4 years from the US financial crisis, unemployment rates on January 2012 reached 8.3%, two percentual points higher than its average over the past 40 years (6.4%); while unemployment among young people between 16 and 19 years of age remained above 20.0%. Katz (2010) in his testimony to Congress states that "the past two and a half years have been particularly trying ones for American workers and their families" (p.1). Therefore, to measure the impact of unemployment on welfare, it is imperative to understand the factors that explain the evolution of unemployment rates in the US, at aggregate level, as well as disaggregated by age and gender, and its connection with the business cycle.

To analyze the unemployment dynamics through worker and job flows[3] across the different work states[4] an individual may find himself in over time is an interesting way to

[1] People between 24 and 54 years of age.

[2] Calculation based on own elaboration applying Current Population Survey (CPS).

[3] Flows of workers are analyzed in some studies and employment flows in others, differing basically with regard to the database used for each analysis. For example, if the database is at individual level we speak of workers' flows, whereas if the database is at firm level we speak of employment flows. As CPS is at individual level; therefore in this case it corresponds to refer in terms of workers' flow.

[4] Employment (E), Unemployment (U), and Inactive (N)

investigate this problem in US. The central idea is to analyze how these separation and creation flows (of workers and jobs) explain the unemployment evolution, both in terms of activity as a business cycle frequency.

After decades of research in this area, findings are still contradictory. This view is supported on the work of Yashiv (2007), who studying the dynamics of the U.S. labour market and its implications for the study of business cycles concluded that there are still opaque and contradictory areas that do not allow a clear view of the dynamics involved. Fujita and Ramey (2008) who also share this view, sating that "the empirical behavior of U.S. job loss and hiring over the business cycle remains an elusive and controversial subject, despite decades of research" (p.1). Such controversies are still open to research and debate.

As a matter of fact, Darby et al. (1986), Davis and Haltiwanger (1990, 1992), Blanchard and Diamond (1990), and Bleakley et al. (1999), among others pioneering works on gross flows of workers and jobs, conclude with the central idea that fluctuations in flows into unemployment (also known as separation) are broader than the fluctuations of flows into employment (also known as hiring flows). This would mean that the jobs destruction play a key role in understanding the unemployment variations, especially during economic downturns.

Hall (2006) and Shimer (2007, 2012) challenge this view. For them, the key to understand the fluctuations in unemployment is the creation - not destruction - of jobs since destruction, according to their research, would have little variability over the economic cycle. In particular, Shimer (2007, 2012) supports the idea that previous research do not consider the time aggregation problems and therefore studies at worker gross flow levels would be biased leading to wrong conclusions.

However, working on the Current Population Survey (CPS) Elsby, Michaels, and Solon (2007) contradict Shimers inference. Elsby et al. (2007) concludes that "...a complete understanding of cyclical unemployment requires an explanation of countercyclical unemployment inflow rates as well as procyclical outflow rates" (p.23). Likewise, Fujita and Ramey (2006) also note on their CPS research "that all worker flows and transition hazard rates are highly volatile at business cycle frequencies" (p.20). Additionally, Fujita and Ramey (2007) concluded that separation rate makes a substantial contribution to the unemployment variability.

Given the high level of disagreement in the literature, this research intends to shed light on the impact that hiring and separation flows have on the unemployment variability in the US from a dynamic perspective and at business cycle frequency, when controlling by age and sex groups.

Our central hypothesis states that these results are heterogeneous when controlling by age and sex; therefore forcing us to avoid generalizations when trying to provide quantitative answers at macroeconomic level to labour policy issues by using general equilibrium dynamic models. As Browning et al. (1999) point out "Accounting for Heterogeneity is required to calibrate dynamic models to Microeconomic evidence" (p.5), which compels inputs should be rigorously justified in these types of models (for example separation and hiring rates) from a microeconomic perspective.

Particular attention will be paid in this Chapter to the employment situation of young people between 18 and 24 years of age (14.9% of the average workforce) in response to their heterogeneity problem and the growing literature that incorporates labour frictions to the general equilibrium stochastic models. Outputs will be compared to those of the prime-age group, that is people aged between 25 and 54 years (68.1% of the average workforce). In our opinion, for a better understanding of the factors that affect the unemployment of young people it is crucial to obtain a more accurate picture of the US labour market. Additionally, understanding this heterogeneity will help designing appropriate macroeconomic models which in turn help to improve public-policies designed for that specific sector, particularly bearing in mind the high unemployment rate in this population group.

Thus, working on the CPS database this research identifies worker's gross flows for each month from 1976 to 2011 for three labour force states (E, U and N) both at an aggregate level, as well as disaggregated by age and sex. Once this is done, attention will be focused only on two labour force states: Employment (E) and Unemployment (U). Separation Rate ($SR_t = \frac{EU_t}{E_{t-1}}$) and the Job Finding Rate ($JFR_t = \frac{UE_t}{U_{t-1}}$) will be calculated at discrete time and on a monthly basis. This is because unemployment level and variation at a steady state for the American case are entirely explained by these flows and it does not seem necessary to consider the flows from and to labour inactivity in such analysis. (Hall, 2006; Shimer, 2007 and Fujita and Ramey, 2008)

To avoid the well-known problems of margin errors that arise when working with samples that rotate over time (Abowd and Zellner, 1985, pp. 254-255) The relative flows constructed are corrected using the technique of Conditional and Missing Random (CMAR) (Nekarda, 2008, 2009). Subsequently, the flows related to unemployment ($\frac{EU_t}{E_{t-1}}$) and employment ($\frac{UE_t}{U_{t-1}}$) are corrected following Fujita and Ramey (2008)'s methodology, and inflow s_t and outflow f_t hazard rates are built. Aggregation problems arise when one wants to estimate time continuing relationships using information just available at discrete time. Such is the case of the labour flow estimation when using CPS. (Nekarda, 2009 and Shimer, 2007, 2012)

Consequently, the cyclical properties of outflow and inflow hazard rates are studied on quarterly basis controlling by age and sex. It is worth saying that this study is carried out at business cycle frequency, that is, all interest variables were seasonally adjusted (by the moving average technique) and filtered from its central tendency (through the Hodrick-Pescott's filter, with 1600 parameter) prior to analysis.

Finally, the outflow and inflow hazard rates effect over the steady state unemployment percentage variation are calculated, applying the methodology developed by Elsby, Michaels, and Solon (2007, 2009), and Petrongolo and Pissarides (2008). Consequently, the variance decomposition suggested by Fujita and Ramey (2007) is applied and the steady-state unemployment percentage change variation is accurately determined - at business cycle level - originated by both the flow separation and the job creation, disaggregated again by age and gender.

The analysis of correlations at business cycle frequency shows that the inflow hazard rate at the aggregate level - men and women over 16 years of age - is counter-cyclical with respect to GDP. Whereas when controlling by age and gender, it is found that correlation is highly heterogeneous, and counter-cyclicality as a common factor. As for the outflow hazard rate - also at business cycle - it is observed that these flows are highly pro-cyclical at aggregate level with respect to GDP. In this case, unlike the inflow hazard rate, enough homogeneity is found when controlling by age and gender, therefore, in general terms it means a high proportion of procyclicality.

In terms of variability at activity cycle frequency, inflow hazard rates at aggregate level are highly volatile; which means about 4 times the GDP volatility. However, when controlling by age and gender groups, a large scatter is found in the data, ranging GDP from twice (in the case of a youngster between 18-24 years of age) up to seven times its value (in the case of an adult in retirement age).

In the case of the outflow hazard rate it is also found a high volatility at the aggregate level (around 5 times the GDP volatility). When controlling by age and gender, results are homogeneous around 5 times the GDP dispersion value. In relative terms, outflow presents major volatility than inflow, both at aggregate level as, well as for the different groups disaggregated by age and gender.

Analysing the comovement through the cross correlation between inflow hazard rate and GDP at business cycle level, it can be concluded that inflow follow the GDP with a lag of 1 to 2 quarters (depending on age and gender group). Otherwise, the outflow hazard rates anticipate the GDP around a quarter in its cyclical component, being this value consistent for the different age and gender groups. In addition, studying the cross-correlation between the inflow and outflow hazard rates it can be concluded that inflow follow the outflow around 2 to 3 semesters at cycle level depending on the age and sex group.

By studying the steady state unemployment percentage variation caused by both, flow separation (inflow hazard rate), a for job creating (outflow hazard rate), it is determined that - at aggregate level - around 46% of the unemployment variation is explained by the variations in the inflow hazard rate, and about the 51% by the outflow's (the remaining 3% is explained by an error component).

As for our interest group, that is young people between 18 to 24 years of age, it is noted that inflow hazard rate has a counter-cyclical behavior with respect to GDP (-0.70) at business cycle frequency, with a relative standard deviation to GDP of 2.5 times. As for the outflow's, it presents a pro-cyclical hazard rate in respect to GDP (0.87) with relative standard deviation to GDP of 5.2. In terms of comovement in relation to GDP, the inflow and outflow hazard rates maintain a similar behavior as they do at aggregate level. That is, a lag of one or two quarters in the inflows, and an advance of a quarter with respect to the GDP. As for the inflow and outflow impacts over the steady state unemployment variation, results show that, unlike the aggregate and prime-age cases, 62% on youth unemployment variation is explained by the finding hazard rates (outflow).

This conclusion is relevant for the general equilibrium stochastic model designed with frictions labour which intends to provide quantitative answers to labour-policy issues, as results show that inflow and outflow hazard rates are heterogeneous at microeconomic estimation level when controlling by age and gender. There is also evidence of this heterogeneity at the business cycle frequency. For example, the inability to find employment is the main factor behind the high unemployment of young people. This is not the case for prime-age population, because the unemployment percentage variation at a steady state is explained in greater proportion by the flow separation at the business cycle frequency (52% is explained by inflows and 45% by outflows). These results generally hold when controlling by gender.

The remainder of this research proceeds as follows. In the second section, the database and the methodology for determining the flow of the workplace between the different states of work are described; and the necessary improvements for proper interpretation of data are made. In the third section work-flows cyclical properties are discussed and the contribution of hazard rates to the volatility of unemployment are calculated. It is worth mentioning that in three preceding sections calculations shall be made for the different age and gender groups, focusing on this research's interest groups: young and prime-aged. Finally, on the fourth section conclusions are presented.

4.2 From CPS Data Base to Calculation of Hazard Rates

This section proceeds as follows. First the CPS survey and the mechanism to determining gross labour flows are briefly explained. Later on, the classification errors that arise when studying a survey like the CPS are explained. On the third part the margin errors and the methodology developed by Nekarda (2008, 2009) are examined to approach them. Subsequently, the time aggregation problem and the methodology developed by Fujita and Ramey (2006) to solve it in an integrated manner are examined. Finally, on the fifth part how the data series were built for this research is explained in detail.

4.2.1 CPS Data Base and Gross Flow Data

The CPS is a monthly cross-sectional survey, which studies the labour force distribution - at household level - of individuals at working age (over 16 years of age) within the following status: employee (E), unemployed (U), and inactive (N). Besides, the survey consults about work hours, income and other demographic characteristics. The sample of households is randomly selected and each of them is surveyed during eight months in a period of sixteen months, which means rotating the sample during that period.

The rotation procedure consists of a first round of interviews for four months; then that household is left aside of the sample during 8 months to be finally interviewed again for another 4 months[4]. Approximately 60,000 households are eligible to be interviewed each month and about 50,000 households are surveyed, and job information of 112,000 people over 16 years of age is obtained each month.

The idea of rotating the sample comes up as a statistic mechanism to improve the monthly estimation by using past information; approaching in a better way the bias problems that arise in non-rotation surveys, and obtaining more information of the population without increasing the size sample (Woodruff, 1963, p.454). Therefore, given to this survey rotational feature, longitudinal information of around 75% of households interviewed can be theoretically obtained in two consecutive months[5].

Benefiting from this rotation process, researchers have studied labour flows individually by identifying identical individuals during two consecutive months and finding out what has happened with their employment status. For this purpose, typically 9 gross labour flows (non adjusted), in the literature are constructed such as EE, EU, EN, UE, UU, UN, NE, NU and NN, where the first letter represents the employment status in the previous month and the second letter represents the status of the current month.

On Table 4.1, the 9 possible cases of gross labour flows between month $t-1$ and month t are shown in a convenient matrix representation. For instance, the EU ($Flow_{UE}$) flow means the number of individuals who were unemployed the past month (at $t-1$) and now are currently employed (t). It is important to state out that when adding by rows labour stock values calculated through labour flows are in theory found. For example, in the case of employment at $t-1$, this could be calculated by adding the following flows, EE, EU and NE and that is: $E_{t-1} = Flow_{EE} + Flow_{EU} + Flow_{EN}$. Similar procedure could be followed to calculate U_{t-1} and N_{t-1}.

However, it is well known in literature that this methodology bears a number of statistical limitations as when generating these flows researchers are faced to problems of classification errors (see Abowd and Zellner, 1985 and Poterba and Summers, 1986); margin errors (see Abowd and Zellner, 1985 and Nekarda, 2008) and time aggregation errors (see Shimer,

[4]CPS defines a variable coded as MIS (Month in Sample) which takes values from 1 to 8 to identify in the sample the month in which the household is being surveyed.

[5]Longitudinal information on the MIS 2, 3, 4, 6, 7 and 8 is found in the CPS. For example, for households with MIS equal to 2 longitudinal data is found as the same household is found in MIS equal 1. However for the MIS 1 and 5 is impossible to find longitudinal data.

Table 4.1: Transition Flow Matrix

		Month (t)		
		E_t	U_t	N_t
	E_{t-1}	$Flow_{EE}$	$Flow_{EU}$	$Flow_{EN}$
Month (t-1)	U_{t-1}	$Flow_{UE}$	$Flow_{UU}$	$Flow_{UN}$
	N_{t-1}	$Flow_{NE}$	$Flow_{NU}$	$Flow_{NN}$

2007 and Nekarda, 2009) which affects the effectiveness of such measurements to express accurately the dynamics in the labour market.

4.2.2 Classification Errors

When labour flows are estimated using the CPS these are biased because of classification problems as it fails to identify the real employment status of a respondent. This problem arises because of the interviewer coding errors; or by the possible randomness of responses and/or because of misunderstanding of the actual employment status of the individual who answered the survey. However it is also known in literature that these classification problems affect the flow estimating but not the analysis at working level, as the unemployment and labour force calculation. Indeed, as Poterba and Summers (1986), "while these errors may largely cancel in tabulations of the unemployment rate or other labour market aggregates, estimated flow rates between labour market states may be extremely sensitive to them" (p.1320).

Diverse methodologies have been developed to overcome this problem. Abowd and Zellner (1985) and Poterba and Summers (1986) solve it by correcting the data using the CPS Reinterview Survey and its reconciliation process. The central idea is to estimate the impact of the classification errors in the study of the gross labour flows, comparing the estimated flows in the original survey with the flows obtained in the survey collated by the process of revisiting. This is because during the process of revisiting a population sample, past responses can be reviewed and the employment status be adjusted if differences are found through an information reconciliation process.

Abowd and Zellner (1985) state that their correction process shows that the unadjusted worker flows differ around 1% with the adjusted flows through the process of adjusting the revisiting information. Similarly, Poterba and Summers (1986) conclude that data corrected by their methodology show that labour-flows data are less dynamic than those normally assumed. However, both authors point out that this conclusion should be regarded with caution as it is based on the assumption that the generating of errors on responses follows a stochastic process (p. 1336).

Chua and Fuller (1987) calculate the estimated-flows classification errors on CPS data using not reconciled data. Nevertheless, the strong statistical assumptions assumed in the model, the low revisited sample representativeness and its non-reconciliation may make that their results may not adequately represent the CPS as a whole.

As the reconciled data obtained through the revisit process are no longer available, Fujita and Ramey (2006) faced this problem as they used the fixed error rates estimated by Poterba and Summers (1986) finding that "the classification error correction has a negligible effect on our results concerning the cyclical behavior of EU and UE flows and transition rates" (p.17).

4.2.3 Margin Error

As we know, longitudinal data can be found in the 75% of the CPS sample. Nevertheless, not all information on employment status related to individuals who theoretically may be met into two consecutive months is available[6]. This might occur because a family changes of home address (CPS does not follow families that change address) or the individual may not be present on the day of the interview, or even because respondents refuse to answer questions about their employment status. Thus, in the absence of complete information regarding an individual's employment status, margin-errors problems originate when labour flows are estimated because of the loss of transitions between one to another labour status.

According to Abowd and Zellner (1985), about 7.5% of the sample presents such errors, which implies that only the 67.5% of the total sample is in conditions to be analyzed toward building labour flows, but estimating biases due to the missing values. Besides, according to Frazis et al. (2005), the margin-errors fact originate the main source of discrepancy between the changes on labour stock information - level of employment, unemployment and participation - and the changes in the same labour stocks estimated through flows by using the CPS (p.4).

As Fujita and Ramey (2006) pointed out, the most common technique for solving this

[6]In the CPS only individuals encoded with MIS equal to 2, 3, 4, 6, 7 y 8 can be gathered in two consecutive months.

problem is to assume the missing data randomly and subsequently to correct the labour flows by the Missing at Random Technique (MAR) (p. 6). This technique to estimating labour flows is used by omitting the missing observations and re-weighing the transitions by assigning the population distribution to the missing values. However, Abowd and Zellner (1985), Fujita and Ramey (2006), Frazis et al. (2005) and Nekarda (2008, 2009) agree that using the MAR technique generates inconsistencies in the labour stocks built through labour flows with respect to official statistics released by the Bureau of Labor Statistics (BLS)[7].

To correct the labour flows because of margin errors Abowd and Zellner (1985) and Fujita and Ramey (2006) use a similar methodology. They try to determine weights by minimizing the squared difference between the labour stocks directly determined by the CPS, and the stocks calculated through labour flows by adding transitions converging to labour states at period t. As the CPS varies over time, Fujita and Ramey perform regressions of 10 years on samples so that the estimators may capture the workforce variation in the total sample, unlike to Abowd and Zellner's study (1985) that uses a parameter fix assignment. In both articles the authors state that their methodology improves the estimations in comparison with those studies that only adjust by MAR.

Nekarda (2008, 2009) makes corrections applying the Conditional Missing at Random Technique (CMAR). This technique assumes that a person with lost employment status at $t-1$, and with work status known at t is taken at random from the group of people with reported employment status at t[8]. To modify the flows, Nekarda builds a correction rate obtaining the quotient among the aggregate flows toward an employment status at t; and the aggregate flows including missing observations at $t-1$ toward the same work state at t.

Nekarda methodology proceeds as follows: X is defined as work status of an individual at $t-1$ (where $X \in \{E, U, N\}$) and Y as employment status at t (where $Y \in \{E, U, N\}$). Then XY_{srt} represents the number of individuals with sex $s \in \{M, F\}$ and race $r \in \{W, NW\}$ who having work status X in $t-1$ transit to work status Y at t; where W is a white-race individual and NW is non-white. Then CMAR correction ratio is determined as (see

[7]One should expect a reasonable variability estimation of the employment at t (measured at the stock level), when adding transitions from employment, unemployment and inactivity at $t-1$ toward the employment level at t. But margin errors affect this estimation as they bias results.

[8]A person is conditional missing at random for Nekarda when having a known work status at t and lost status at $t-1$.

Nekarda, 2008, 2009):

$$R^{Y}_{srt} = \frac{EY_{srt} + UY_{srt} + NY_{srt}}{EY_{srt} + UY_{srt} + NY_{srt} + MY_{srt}} \tag{4.2.1}$$

Where MY_t is the number of individuals with known employment status at t but lost at $t-1$, of sex s and race r. Then the corrected work flow for sex and race from state X to state Y is determined by the following expression (Nekarda, 2008, p.31):

$$\widetilde{XY}_{rst} = \frac{XY_{srt}}{R^{Y}_{srt}} \tag{4.2.2}$$

Then, the margin error flow is found through adding the flow corrected by the equation (4.2.2) by sex and race considering all categories, as follows:

$$\widetilde{Flow}_{XY_t} = \sum_{s}\sum_{r}\widetilde{XY}_{rst} = \sum_{s}\sum_{r}\left\{\frac{XY_{srt}}{R^{Y}_{srt}}\right\} \tag{4.2.3}$$

This proceeding corrects the series caused by margin error problems. It is worth noting that Nekarda (2008) incorporates gender and race in his characteristic correction process as, according BLS, this provides better performance using this type of control in his adjusting (p. 30).

4.2.4 Time Aggregation Error

Time aggregation problems arise when relationships at continuous time by discrete-time data available are estimated, as Kaitz (1970) and Petersen (1991), among others, state. Indeed, researchers often use continuous time hazard rate models to estimate the amount of time a person, company or organization spends in a space state of nature. It is usually assumed as supposition in this type of analysis that the target to study - as workers - can enter or leave a state of nature at any moment. However, this information is unknown, as, in general, it is available only at time intervals (as weeks, months or years), which tends to bias estimations by time aggregation in these model types (Petersen, 1991, p.264).

The problem of biasing by time aggregation clearly emerges in the models that try to calculate the probability that a worker becomes unemployed or employed through labour flows calculation using information available in the CPS. In fact, when workflows are calculated on monthly basis there is a high probability of biasing the outcomes as there is a group of individuals moving from one job state to another within the analysis period and that, because of the availability of data, cannot be calculated.

As discussed in the literature review, Shimer (2007, 2012) approached this problem avoiding bias by incorporating short-term unemployment information available in the CPS up to 1994. However, this approach seems limited since that information was eliminated during the CPS redesigning made in 1994, and the inferences that might be obtained about short-term unemployment for the recent years do not appear to be too strong. In this sense, the methodology developed by Fujita and Ramey seems appropriate as it faces the time aggregation problem in a unified framework and so it can be applied independently of the database the researcher studies to estimate hazard rates.

Fujita and Ramey (2006) develop their integrated methodology as follows (see in Appendix 4.D a detailed explanation of this methodology). They assume index continuous time workforce z and workforce normalized to unit. Besides, they assume two work states (employment and unemployment)[9] and defining s_t and f_t as the arrival rate for the transitions of workers to and from unemployment in the time interval $t-1$ and t. Then they assume that the unemployment dynamics can be modeled by the following differential equation, where $u(z)$ is the unemployment level in z:

$$\dot{u}(z) = s_t \cdot (1 - u(z)) - f_t \cdot u(z) \tag{4.2.4}$$

Fujita and Ramey when solving this differential equation found that the continuous-time unemployment can be determined as:

$$u(z) = \frac{s_t}{s_t + f_t} + (u_{t-1} - \frac{s_t}{s_t + f_t}) \cdot \exp[-(z - t + 1) \cdot (s_t + f_t)] \tag{4.2.5}$$

Where the number of unemployed workers at the end of month t is $u_t = u(t)$. Likewise, they define e_t^n as the number of workers employed at the end of period (at t), who may have had the employed or unemployed status at the beginning of the same period, with the following expression:

$$e_t^n = \int_t^{t-1} f_t \cdot u(z) \cdot \exp[-s_t \cdot (t - z)]dz \tag{4.2.6}$$

[9] Fujita and Ramey (2006) extend their analysis to three working states. But for the purposes of this research we will refer to the methodology developed for two periods only.

Then, replacing the equation (4.2.5) on equation (4.2.6), and solving it they get that (see Appendix 4.D):

$$e_t^n = \left(\frac{f_t}{s_t + f_t}\right) \cdot (1 - \exp[-s_t]) + \left(u_{t-1} - \frac{s_t}{s_t + f_t}\right) \cdot (1 - \exp[-f_t]) \tag{4.2.7}$$

Alternatively, Fujita and Ramey define e_t^n in terms of transition rates, that is, Separation Rate (SR) and Job Finding Rate (JFR) at discrete level, as:

$$e_t^n = u_{t-1} \cdot JFR_t + (1 - u_{t-1}) \cdot (1 - SR_t - \exp[s_t]) \tag{4.2.8}$$

Then, matching equations (4.2.7) and (4.2.8) coefficients and performing a suitable algebraic handling expressions are obtained for inflow (s_t) and outflow (f_t) hazard rates and on JFR_t and SR_t, basis, which allows correcting with an integrated methodology by time aggregation as follows:

$$s_t = -\frac{SR_t}{SR_t + JFR_t} \cdot log(1 - SR_t - JFR_t) \tag{4.2.9}$$

$$f_t = -\frac{JFR_t}{SR_t + JFR_t} \cdot log(1 - SR_t - JFR_t) \tag{4.2.10}$$

In short, both identities allow Fujita and Ramey (2006) to estimate the instantaneous separation and hiring rates in continuous time by calculating separation and hiring obtained from the CPS in discrete-time, simultaneously fixing the time aggregation problem.

4.2.5 Construction of the Data Series (Hazard Rates)

CPS database has been analyzed in this research from January 1976 to August 2011 to calculate the workers' flows. As this research main purpose is to analyze the unemployment heterogeneity at business cycle frequencies (with emphasis on the group of people between 18 and 24 years old), it is necessary to work with raw files available on the National Bureau of Economic Research website (NBER). This is because the BLS only provides information on labour flows disaggregated by gender and only for individuals over 16 years of age.

Once raw files have been extracted we have used Shimer's code (with modifications)[10] to obtain longitudinal information in respect of status and labour flows between two con-

[10] The code is available in: `http://sites.google.com/site/robertshimer/research/flows`. We modified the code as Shimer does not correct the data for errors in the margin. To make this correction we need to find job status missing values for gender and race.

secutive months for the entire regarded period. To obtain this information it is necessary to find identical individuals during two consecutive months[11]. For this purpose, household serial number should be compared along with other demographic variables as age, gender and race to ensure that a very same individual is being compared. Once this has been done, his employment status is identified.

It is defined as X an individual's situation in month t-1 to identify the job status, where X can take the following values $\{E, U, N \ and \ M\}$. Likewise, the same individual's job status is defined as Y in month t, where Y can take the following values $\{E, U, N\}$. It should be noted that we are interested in finding missing values for job status lost values (M) in the previous month, that is in month $t-1$, as unlike in Shimer (2005), it is aimed to correct biases by margin errors.

As we know, gross flows calculation involves a series of errors explained in the previous section. Therefore, we perform appropriate adjustments when we find it necessary. In this sense, we do not correct by classification error as there is evidence that it not affect estimations significantly (Fujita and Ramey, 2006). Nekarda's CMAR technique (2008; 2009) is used to correct margin errors adjusting by gender and race. Next, flows are identified regarded by sex and race and subsequently Nekarda's correction ratio is established (see equation (4.2.1)). Then, flow correction by equation (4.2.2) is determined. Finally, labour flows corrected because of margin error, for each time t instant through equation (4.2.3) are obtained. Once finished this, the same procedure is performed with series disaggregated by sex and age groups.

As for age, 5 groups are determined in this investigation (see Table 4.24 shows the distributed workforce by age group). Group 1 includes people between 16 and 17 years of age (2.4% of the workforce). Whereas, Group 2 includes people aged between 18 and 24 (14.8% of the workforce). Group 3 includes adults between 25 and 54 years old (68.3% of the workforce), who we called prime-aged individuals. Also, a Group 4 is defined with adults between 55 and 64 years old (11.3% of the workforce). Finally, Group 5 with adults mostly removed from the workforce aged over 64 years (3.2% of the workforce) is defined. Each group is also disaggregated by gender. (see Tables 4.25 and 4.26 for sex-disaggregated data)

[11] To do this we must remove the sample MIS 1 and 5 (since theoretically can not be gathered) and all individuals under 16 years of age.

Thus, we redefine the definition of worker flow $Flow_{XY_{gst}}$ as the number of individuals of age group g (where $g \in \{G0, G1, G2, G3, G4\ and\ G5\}$) and gender s (where $s \in \{A, F, M\}$) corrected because of margin error who are passing from job status X in $t-1$ to in job status Y at time t. Note that the Group 0 is the group that regards aggregate level, that is, people over 16 years of age representing the 100% of the workforce, and the gender aggregate variable (A) that includes men and women.

Due to problems with the household identifier in the CPS database transitions for 8 months during the regarded period were lost[12]. To resolve this problem, the missing values for linear interpolation were determined by using the Stata© command ipolate©. Likewise and as it is recognized in the literature, monthly flows are subject to seasonal changes, therefore seasonal adjustment is applied to all series by using the moving average technique with uniform weights.

Once realized the margin error correction, the interpolation process and the seasonal adjustment, the construction of the separation rate $\widetilde{SR}$ and the job finding rate $\widetilde{JFR}$ for the aggregate level, by age group (g) and gender desegregation (s) at time t is performed as follows:

$$\widetilde{SR}_{sgt} = \frac{\widetilde{Flow}_{EU_{sgt}}}{\widetilde{Flow}_{EE_{sgt}} + \widetilde{Flow}_{EU_{sgt}} + \widetilde{Flow}_{EN_{sgt}}} \tag{4.2.11}$$

$$\widetilde{JFR}_{sgt} = \frac{\widetilde{Flow}_{UE_{sgt}}}{\widetilde{Flow}_{UE_{sgt}} + \widetilde{Flow}_{UU_{sgt}} + \widetilde{Flow}_{UN_{sgt}}} \tag{4.2.12}$$

As noted above, these flows face time aggregation problems. This is because the separation rate and the job finding rate do not capture the transitions within the period under review. To resolve this problem it is used Fujita and Ramey's method (2006), and the inflow and outflow hazard rates are calculated (see equation (4.2.9) and (4.2.10)) for the aggregate level, by age group (g) and sex (s) during the instant at time t:

$$\widetilde{s}_{sgt} = -\frac{\widetilde{SR}_{sgt}}{\widetilde{SR}_{sgt} + \widetilde{JFR}_{sgt}} \cdot log\left[1 - \widetilde{SR}_{sgt} - \widetilde{JFR}_{sgt}\right] \tag{4.2.13}$$

$$\widetilde{f}_{sgt} = -\frac{\widetilde{JFR}_{sgt}}{\widetilde{SR}_{sgt} + \widetilde{JFR}_{sgt}} \cdot log\left[1 - \widetilde{SR}_{sgt} - \widetilde{JFR}_{sgt}\right] \tag{4.2.14}$$

[12] The missed periods are: 1978:01, 1985:07, 1985:10, 1994:01, 1995:06, 1995:07, 1995:08 and 1995:09.

Inflow and outflow hazard rates values at quarterly level are calculated by averaging the monthly values obtained within the relevant quarter. From Table 4.5 to Table 4.10 we show the first and second moment calculated for the two data series generated on quarterly basis by age and gender group, adjusted because margin error and time aggregation.

As seen in Table 4.5, the inflow hazard rates are more than twice for youth between 18 and 24 years of age than the prime-aged people value, that is 3.6% compared to 1.5%. We may also see greater volatility for youth (0.7%) than for older adults in prime-age (0.4%), pattern that keeps on when controlling by sex (see Tables 4.7 and 4.9). As for the outflow hazard rates it is observed that the mean value and the standard deviation tends to be relatively more homogeneous in the aggregate, by age and sex levels with respect to the inflow hazard rates (see Tables 4.6, 4.8 and 4.10). For young people between 18 and 24 years of age the outflow is 34% and for prime-aged people is 30%, being 5.5% and 4.8% standard deviations respectively. (see Tables 4.6)

Only two workforce states are assumed in this research, understanding that in the American work market job flows can explain unemployment at a steady state and this very state also explain the common unemployment. On Appendix 4.C unemployment dynamics is described in terms of labour flows for both continuous and discrete cases obtaining the following equations $u^{ss} = \frac{SR_t}{SR_t + JFR_t}$ and $u^{ss} = \frac{s_t}{s_t + f_t}$ for each case respectively.

Then, calculating unemployment levels for the aggregate level and by age group directly through CPS the recruitment is proceeded if the assumption of working with only two job states is adequate. For this purpose, the averages for the two unemployment measures and their correlation for both the aggregate level and for different age groups (see Table 4.23) are compared. The high correlation found, around 98% average, suggests that the U.S. job market approaching by analyzing only two job states is plausible.

4.3 Cyclical Behavior of Hazard Rates

The next section will analyze the cyclical behavior of hazard rates computed with respect to GDP. It is necessary to remove the trend component from the series to analyze so that to work at business cycle frequency. For this purpose, the Hodrick-Pescott's filter with parameter 1600 was used. The section proceeds as follows: The first section will analyze the results with respect to correlation, variability and comovement of hazard rates relative to GDP. The second section will explore the effect of hazard rates on the variability of the percentage change in unemployment level at business cycle frequency.

4.3.1 Correlation, Variability and Hazard Rates Comovement

Correlation Between Hazard Rates and Economic Activity

In terms of business cycle, we find that Inflow Hazard Rates at aggregate level (men and women over 16 years of age) are highly counter-cyclical with respect to the GDP, that is, a correlation of -0.82 (see Table 4.14). When controlled by age and gender, it is found that the correlation between Inflow Hazard Rates and GDP at of business cycle level are highly heterogeneous, maintaining its against cyclicality as common factor, in some cases weakly (-0.16 in young people between 16 and 17 years old) and other cases more strongly (-0.83, that is people between 25 to 54 years old). As a common pattern it is also found that women present a weaker correlation with respect to the correlation obtained in men's case, for different age groups. As for interest groups, that is, for population aged 18-24 and 25-54 years of age it is found that correlation coefficients are -0.70 (-0.69 for men and -0.42 for women) and -0.83 (- 0.84 for men and -0.65 for women) respectively. (see Tables 4.16 and 4.18)

As for the outflow hazard rates - also at business cycle - it is found that at aggregate level (men and women over 16 years old) such flows are highly pro-cyclical with respect to the GDP, with 0.84 correlations (see Table 4.15). In this case, unlike inflow hazard rates, quite homogeneity is found when it is controlled by different age and sex groups, and this is a high pro-cyclicality in general terms. In particular for the interest groups, that is for population aged 18 to 24 and 25 to 54 years it is found that the correlation coefficients are 0.87 (0.86 in men and 0.82 in women) and 0.81 (0.82 in men and 0.78 in women) respectively. (see Tables 4.17 and 4.19)

Variability Between Hazard Rates and Economic Activity

In terms of activity cycle, the inflow and outflow hazard rate variability is studied by calculating the quotient between the analyzed flow standard deviation and the GDP standard deviation (called relative standard deviation). Regarding inflow hazard rates, it is found that at the aggregate level (men and women over 16 years of age) variability is highly volatile, around 4 times the GDP volatility (see table 4.14). However, high data dispersion is found when controlling by age, ranging from twice the GDP in the case of a young man aged between 18 and 24, up to seven times its value in the case of an adult at retirement age. As for groups of interest, that is people aged between 18 and 24, and between 25 and 54, it is found that the relative standard deviation for the inflow hazard rates are 2.5 (3.1 for men and 2.7 for women) and 4.7 (5.9 for men and 3.2 for women) respectively. (see Tables 4.16 and 4.18)

It has been also found that for the outflow hazard rates case it is also highly volatile at aggregate level (men and women aged over 16) (see Table 4.15). This is about 5 times the GDP volatility. However, when controlling by age and sex groups fairly homogeneous results are found compared with the inflow hazard rate results obtained, around 5 times the GDP dispersion value. For interest groups, that is people aged between 18 and 24, and 25 and 54 it is found that for the case of outflow hazard rates relative standard deviation are 5.2 (5.8 for men and 4.7 for women) and 5.3 (5.9 for men and 5.1 for women) respectively. (see Tables 4.17 and 4.19)

Comparing the volatility with respect to the inflow and outflow hazard rate GDP it can be roughly stated that outflows present higher volatility than inflows in relative terms, both at the aggregate level and for different age and sex groups.

Comovements Between Hazard Rates and Economic Activity

Investigating the cross-correlation between the inflow hazard rate and GDP at business cycle frequency it can be concluded that inflows keep a lag of 1 or 2 quarters with respect of the GDP, depending on the age and sex groups (see Figure 4.3). Regarding the outflow hazard rates, cyclical component lead the GDP around a quarter, and this value keeps consistent for the different age and sex groups (see Figure 4.4). Studying the cross-correlation between the inflow and outflow hazard Rates it can be concluded that, at cycle-level, inflows follow outflows around 2 and 3 semesters depending on the age and sex groups regarded for our analysis. (see Figure 4.5)

4.3.2 Contribution of Hazard Rates to the Volatility of Unemployment

Applying the novel methodology developed by Fujita and Ramey (2007) the unemployment percentage change variation at steady state caused by both, the flow separation (inflow hazard rate), and the work creation (outflow hazard rate) at business cycle are studied. This methodology was explained in the review of the literature, and in detail step by step in Appendix 4.E of this research. The series du_t^{ss} , du_t^s and du_t^f are built as follows:

$$du_t^{ss} = \frac{u_t^{ss} - u_{t-1}^{ss}}{u_{t-1}^{ss}} \tag{4.3.1}$$

$$du_t^s = (1 - u_t^{ss}) \cdot \frac{s_t - s_{t-1}}{s_{t-1}} \tag{4.3.2}$$

$$du_t^f = -(1 - u_{t-1}^{ss}) \cdot \frac{f_t - f_{t-1}}{f_{t-1}} \tag{4.3.3}$$

Considering that u_t^{ss} is the steady-state unemployment at time t (obtained through equation (1.0.11)). Besides s_t and f_t are the inflow and outflow hazard rates at t. Note that (and include an error term to maintain the mathematical equality):

$$du_t^{ss} = du_t^s + du_t^f + du_t^\varepsilon \tag{4.3.4}$$

As it is operated at business cycle frequency, the tendency is eliminated when applying the Hodrick-Pescott filter to all elements of equation (4.3.4) (with parameter 1600). Now such relationship can be expressed as follows (the variable top-line indicates that this variable is the cyclical component of the original series):

$$\overline{du_t}^{ss} = \overline{du_t}^s + \overline{du_t}^f + \overline{du_t}^\varepsilon \tag{4.3.5}$$

Then, by taking variance on both sides of equation (4.3.5) and dividing by $Var(\overline{du_t}^{ss})$ it is obtained that (see the algebraic process to obtain this expression on Appendix 4.E):

$$1 = \frac{Cov(\overline{du_t}^{ss}, \overline{du_t}^s)}{Var(\overline{du_t}^{ss})} + \frac{Cov(\overline{du_t}^{ss}, \overline{du_t}^f)}{Var(\overline{du_t}^{ss})} + \frac{Cov(\overline{du_t}^{ss}, \overline{du_t}^\varepsilon)}{Var(\overline{du_t}^{ss})} \tag{4.3.6}$$

Following Fujita and Ramey (2006), the finance beta concept is used to define the unemployment variation total portion attributed to hazard rates and to an error term at business cycle frequency, as follows:

$$\overline{\beta}^s = \frac{Cov(\overline{du_t}^{ss}, \overline{du_t}^s)}{Var(\overline{du_t}^{ss})} \tag{4.3.7}$$

$$\overline{\beta}^f = \frac{Cov(\overline{du_t}^{ss}, \overline{du_t}^f)}{Var(\overline{du_t}^{ss})} \tag{4.3.8}$$

$$\overline{\beta}^\varepsilon = \frac{Cov(\overline{du_t}^{ss}, \overline{du_t}^\varepsilon)}{Var(\overline{du_t}^{ss})} \tag{4.3.9}$$

So by replacing the values of equation (4.3.7), (4.3.8) and (4.3.9) in equation (4.3.6) we have:

$$1 = \overline{\beta}^{s} + \overline{\beta}^{f} + \overline{\beta}^{\varepsilon} \tag{4.3.10}$$

Consequently, it is determined that around the 46% of changes in unemployment in men and women over 16 years of age at aggregate level is explained by the inflow hazard rate variations, and about 51% by the outflow's. But controlling by age, it is found that in the case of young people aged between 18 and 24, around of 62% of unemployment variation is explained by the outflows (see Table 4.20). This fact is important for the designing of public policies as it points out that the inability to find employment is one of the main factors behind the high unemployment of young people. Such is not for prime-age population (people aged between 25 and 54) as the unemployment percentage variation at steady state is explained in major proportion by the flow separation, that is, around 52% by the inflow's and 45% by outflow's. These results in general are maintained when it is checked by sex. (see Tables 4.21 and 4.22)

4.4 Conclusion

In conclusion, after analysing the instantaneous flow separation and job creation in the U.S. in terms of correlation, volatility, comovement and their contribution to the unemployment volatility, a high heterogeneity of results arise when controlling by age and gender. This high heterogeneity found in the microdata compels us to avoid generalizations when trying to provide quantitative answers at the macroeconomic level to labour policy issues by using general equilibrium dynamic models that incorporate labour-type friction. As a result, we can conclude that regarding our target group (young people aged between 18 and 24) findings rates tends to be the main factor that would explain its unemployment dynamics, becoming a crucial factor to keep in mind when designing policies to reduce unemployment in this group. As mentioned earlier, this is not the case for prime-age population, where both factors (inflow and outflow) play an important role in explaining the unemployment dynamics.

Appendix

4.A Tables

4.A.1 Unemployment Rates

Table 4.2: Unemployment Rate (u_t): First and Second Moments

Age	Mean (μ_{u_t})	Standard Deviation (σ_{u_t})
over 16 year old	0.0642	0.0155
16 - 17 year old	0.2020	0.0302
18 - 24 year old	0.1182	0.0229
25 - 54 year old	0.0519	0.0141
55 - 64 year old	0.0390	0.0109
over 64 year old	0.0358	0.0095

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and calculation include Male and Female. All calculations are performed for each age group. Source: Own elaboration in base CPS.

Table 4.3: Male Unemployment Rate (u_t): First and Second Moments

Age	Mean (μ_{u_t})	Standard Deviation (σ_{u_t})
over 16 year old	0.0643	0.0171
16 - 17 year old	0.2149	0.0330
18 - 24 year old	0.1249	0.0273
25 - 54 year old	0.0509	0.0163
55 - 64 year old	0.0410	0.0126
over 64 year old	0.0360	0.0106

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and calculation include Male. All calculations are performed for each age group. Source: Own elaboration in base CPS.

Table 4.4: Female Unemployment Rate (u_t): First and Second Moments

Age	Mean (μ_{u_t})	Standard Deviation (σ_{u_t})
over 16 year old	0.0643	0.0149
16 - 17 year old	0.1890	0.0295
18 - 24 year old	0.1108	0.0195
25 - 54 year old	0.0533	0.0126
55 - 64 year old	0.0367	0.0095
over 64 year old	0.0354	0.0085

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and calculation include Female. All calculations are performed for each age group. Source: Own elaboration in base CPS.

4.A.2 Inflows and Outflow Hazard Rates

Table 4.5: Inflow Hazard Rate (s_t): First and Second Moments

Age	Mean (μ_{s_t})	Standard Deviation (σ_{s_t})
over 16 year old	0.01767	0.00258
16 - 17 year old	0.04150	0.00704
18 - 24 year old	0.03644	0.00384
25 - 54 year old	0.01483	0.00219
55 - 64 year old	0.00971	0.00133
over 64 year old	0.00758	0.00191

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and with interpolation for missing value. Inflow hazard rate is adjusted for time aggregation error. All calculations are performed for each age group. Source: Own elaboration in base CPS.

Table 4.6: Outflow Hazard Rate (f_t): First and Second Moments

Age	Mean (μ_{f_t})	Standard Deviation (σ_{f_t})
over 16 year old	0.30090	0.04765
16 - 17 year old	0.24854	0.04874
18 - 24 year old	0.34004	0.05492
25 - 54 year old	0.30067	0.04795
55 - 64 year old	0.24703	0.04279
over 64 year old	0.20166	0.04482

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and with interpolation for missing value. Outflow hazard rate is adjusted for time aggregation error. All calculations are performed for each age group. Source: Own elaboration in base CPS.

Table 4.7: Male Inflow Hazard Rate (s_t): First and Second Moments

Age	Mean (μ_{s_t})	Standard Deviation (σ_{s_t})
over 16 year old	0.01984	0.00321
16 - 17 year old	0.04562	0.00760
18 - 24 year old	0.04292	0.00528
25 - 54 year old	0.01681	0.00306
55 - 64 year old	0.01063	0.00177
over 64 year old	0.00763	0.00196

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and with interpolation for missing value. Inflow hazard rate is adjusted for time aggregation error. All calculations are performed for each age and gender group. Source: Own elaboration in base CPS.

Table 4.8: Male Outflow Hazard Rate (f_t): First and Second Moments

Age	Mean (μ_{f_t})	Standard Deviation (σ_{f_t})
over 16 year old	0.32344	0.05206
16 - 17 year old	0.24303	0.05282
18 - 24 year old	0.35962	0.06191
25 - 54 year old	0.33447	0.05526
55 - 64 year old	0.25285	0.04423
over 64 year old	0.19906	0.04668

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and with interpolation for missing value. Outflow hazard rate is adjusted for time aggregation error. All calculations are performed for each age and gender group. Source: Own elaboration in base CPS.

Table 4.9: Female Inflow Hazard Rate (s_t): First and Second Moments

Age	Mean (μ_{s_t})	Standard Deviation (σ_{s_t})
over 16 year old	0.01511	0.00200
16 - 17 year old	0.03724	0.00703
18 - 24 year old	0.02948	0.00261
25 - 54 year old	0.01253	0.00135
55 - 64 year old	0.00856	0.00120
over 64 year old	0.00760	0.00204

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and with interpolation for missing value. Inflow hazard rate is adjusted for time aggregation error. All calculations are performed for each age and gender group. Source: Own elaboration in base CPS.

Table 4.10: Female Outflow Hazard Rate (f_t): First and Second Moments

Age	Mean (μ_{f_t})	Standard Deviation (σ_{f_t})
over 16 year old	0.27509	0.04507
16 - 17 year old	0.25567	0.04687
18 - 24 year old	0.31784	0.05124
25 - 54 year old	0.26437	0.04443
55 - 64 year old	0.23952	0.04505
over 64 year old	0.20575	0.05496

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and with interpolation for missing value. Outflow hazard rate is adjusted for time aggregation error. All calculations are performed for each age and gender group. Source: Own elaboration in base CPS.

4.A.3 Business Cycles Properties: Unemployment

Table 4.11: Unemployment Rate u_t vs. GDP: Correlation and Relative Std.

Age	Correlation ($\rho_{u_t,gdp}$)	σ_{u_t}	σ_{gdp}	Std. Relative ($\frac{\sigma_{u_t}}{\sigma_{gdp}}$)
over 16 year old	-0.8731	0.1012	0.0148	6.8230
16 - 17 year old	-0.7772	0.0622	0.0148	4.1972
18 - 24 year old	-0.8880	0.0832	0.0148	5.6145
25 - 54 year old	-0.8751	0.1181	0.0148	7.9653
55 - 64 year old	-0.8306	0.1232	0.0148	8.3105
over 64 year old	-0.6183	0.0884	0.0148	5.9628

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and calculation include Male and Female. All variables are in logarithms and with HP filter (1600). All calculations are performed for each age group. Source: Own elaboration in base CPS.

Table 4.12: Unemployment Rate u_t vs. GDP: Correlation and Relative Std.

Age	Correlation ($\rho_{u_t,gdp}$)	σ_{u_t}	σ_{gdp}	Std. Relative ($\frac{\sigma_{u_t}}{\sigma_{gdp}}$)
over 16 year old	-0.8870	0.1185	0.0148	7.9925
16 - 17 year old	-0.7799	0.0688	0.0148	4.6382
18 - 24 year old	-0.8918	0.0958	0.0148	6.4638
25 - 54 year old	-0.8880	0.1426	0.0148	9.6150
55 - 64 year old	-0.8370	0.1370	0.0148	9.2418
over 64 year old	-0.5908	0.1039	0.0148	7.0070

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and calculation include Male. All variables are in logarithms and with HP filter (1600). All calculations are performed for each age group. Source: Own elaboration in base CPS.

Table 4.13: Unemployment Rate u_t vs. GDP: Correlation and Relative Std.

Age	Correlation ($\rho_{u_t,gdp}$)	σ_{u_t}	σ_{gdp}	Std. Relative ($\frac{\sigma_{u_t}}{\sigma_{gdp}}$)
over 16 year old	-0.8291	0.0817	0.0148	5.5091
16 - 17 year old	-0.7051	0.0597	0.0148	4.0272
18 - 24 year old	-0.8442	0.0706	0.0148	4.7613
25 - 54 year old	-0.8283	0.0917	0.0148	6.1831
55 - 64 year old	-0.7567	0.1121	0.0148	7.5581
over 64 year old	-0.4535	0.0960	0.0148	6.4768

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and calculation include Female. All variables are in logarithms and with HP filter (1600). All calculations are performed for each age group. Source: Own elaboration in base CPS.

4.A.4 Business Cycles Properties: Hazard Rates

Table 4.14: Inflow Hazard Rate s_t vs. GDP: Correlation and Relative Std.

Age	Correlation ($\rho_{s_t,gdp}$)	σ_{s_t}	σ_{gdp}	Std. Relative ($\frac{\sigma_{s_t}}{\sigma_{gdp}}$)
over 16 year old	-0.8247	0.0540	0.0148	3.6411
16 - 17 year old	-0.1609	0.0458	0.0148	3.0924
18 - 24 year old	-0.7042	0.0380	0.0148	2.5647
25 - 54 year old	-0.8377	0.0698	0.0148	4.7098
55 - 64 year old	-0.7600	0.0800	0.0148	5.3970
over 64 year old	-0.0251	0.1063	0.0148	7.1710

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and with interpolation for missing value. Inflow hazard rate is adjusted for time aggregation error. All variables are in logarithms and with HP filter (1600). All calculations are performed for each age group. Source: Own elaboration in base CPS.

Table 4.15: Outflow Hazard Rate f_t vs. GDP: Correlation and Relative Std.

Age	Correlation ($\rho_{s_t,gdp}$)	σ_{s_t}	σ_{gdp}	Std. Relative ($\frac{\sigma_{s_t}}{\sigma_{gdp}}$)
over 16 year old	0.8437	0.0763	0.0148	5.1457
16 - 17 year old	0.7895	0.0877	0.0148	5.9157
18 - 24 year old	0.8731	0.0771	0.0148	5.2001
25 - 54 year old	0.8158	0.0792	0.0148	5.3432
55 - 64 year old	0.5871	0.0910	0.0148	6.1399
over 64 year old	0.5767	0.1003	0.0148	6.7651

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and with interpolation for missing value. Outflow hazard rate is adjusted for time aggregation error. All variables are in logarithms and with HP filter (1600). All calculations are performed for each age group. Source: Own elaboration in base CPS.

Table 4.16: Inflow Hazard Rate s_t vs. GDP: Correlation and Relative Std.

Age	Correlation ($\rho_{s_t,gdp}$)	σ_{s_t}	σ_{gdp}	Std. Relative ($\frac{\sigma_{s_t}}{\sigma_{gdp}}$)
over 16 year old	-0.8359	0.0680	0.0148	4.5886
16 - 17 year old	-0.3213	0.0710	0.0148	4.7857
18 - 24 year old	-0.6999	0.0470	0.0148	3.1717
25 - 54 year old	-0.8451	0.0875	0.0148	5.9035
55 - 64 year old	-0.7864	0.0937	0.0148	6.3191
over 64 year old	-0.3489	0.1130	0.0148	7.6197

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and with interpolation for missing value. Inflow hazard rate is adjusted for time aggregation error. All variables are in logarithms and with HP filter (1600). All calculations are performed for each age and gender group. Source: Own elaboration in base CPS.

Table 4.17: Male, Outflow Hazard Rate f_t vs. GDP: Correlation and Relative Std.

Age	Correlation ($\rho_{s_t,gdp}$)	σ_{s_t}	σ_{gdp}	Std. Relative ($\frac{\sigma_{s_t}}{\sigma_{gdp}}$)
over 16 year old	0.8579	0.0823	0.0148	5.5521
16 - 17 year old	0.7617	0.0913	0.0148	6.1550
18 - 24 year old	0.8696	0.0870	0.0148	5.8664
25 - 54 year old	0.8227	0.0888	0.0148	5.9923
55 - 64 year old	0.6069	0.0958	0.0148	6.4593
over 64 year old	0.2786	0.1243	0.0148	8.3850

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and with interpolation for missing value. Outflow hazard rate is adjusted for time aggregation error. All variables are in logarithms and with HP filter (1600). All calculations are performed for each age and gender group. Source: Own elaboration in base CPS.

Table 4.18: Female, Inflow Hazard Rate s_t vs. GDP: Correlation and Relative Std.

Age	Correlation ($\rho_{s_t,gdp}$)	σ_{s_t}	σ_{gdp}	Std. Relative ($\frac{\sigma_{s_t}}{\sigma_{gdp}}$)
over 16 year old	-0.6517	0.0373	0.0148	2.5150
16 - 17 year old	0.1767	0.0688	0.0148	4.6383
18 - 24 year old	-0.4202	0.0410	0.0148	2.7658
25 - 54 year old	-0.6598	0.0480	0.0148	3.2359
55 - 64 year old	-0.4774	0.0875	0.0148	5.9034
over 64 year old	0.3148	0.1421	0.0148	9.5812

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and with interpolation for missing value. Inflow hazard rate is adjusted for time aggregation error. All variables are in logarithms and with HP filter (1600). All calculations are performed for each age and gender group. Source: Own elaboration in base CPS.

Table 4.19: Female, Outflow Hazard Rate f_t vs. GDP: Correlation and Relative Std.

Age	Correlation ($\rho_{s_t,gdp}$)	σ_{s_t}	σ_{gdp}	Std. Relative ($\frac{\sigma_{s_t}}{\sigma_{gdp}}$)
over 16 year old	0.8103	0.0732	0.0148	4.9401
16 - 17 year old	0.6946	0.0979	0.0148	6.6015
18 - 24 year old	0.8229	0.0707	0.0148	4.7674
25 - 54 year old	0.7845	0.0764	0.0148	5.1558
55 - 64 year old	0.4627	0.1006	0.0148	6.7871
over 64 year old	0.5583	0.1596	0.0148	10.7655

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Data seasonal adjustment and with interpolation for missing value. Outflow hazard rate is adjusted for time aggregation error. All variables are in logarithms and with HP filter (1600). All calculations are performed for each age and gender group. Source: Own elaboration in base CPS.

4.A.5 Contribution to Unemployment Volatility

Table 4.20: Unemployment Decompositions

Age	$Var(du_t^{ss})$	$Cov(du_t^{ss}, du_t^s)$	$Cov(du_t^{ss}, du_t^f)$	β^s	β^f	$\beta\varepsilon$
over 16 year old	0.0015	0.0007	0.0008	0.46	0.51	0.03
16 - 17 year old	0.0017	0.0006	0.0010	0.39	0.59	0.03
18 - 24 year old	0.0011	0.0004	0.0007	0.36	0.62	0.02
25 - 54 year old	0.0021	0.0011	0.0009	0.52	0.45	0.03
55 - 64 year old	0.0023	0.0011	0.0012	0.46	0.53	0.01
over 64 year old	0.0056	0.0033	0.0022	0.59	0.39	0.02

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Where du_t^{ss} is percentage change of unemployment rate in steady state. Also du_t^s, du_t^f and du_t^ε are measure of contribution of changes in s_t, f_t and ε into changes in steady state unemployment, respectively. Beta is the contribution of current changes in transition hazard rates to the variance of steady state unemployment, that is: $\beta^j = \frac{Cov(du_t^{ss}, du_t^j)}{Var(du_t^{ss})}$ with $j = s, f$. All variables with HP Filter (1600). Source: Own elaboration in base CPS.

Table 4.21: Male Unemployment Decompositions

Age	$Var(du_t^{ss})$	$Cov(du_t^{ss}, du_t^s)$	$Cov(du_t^{ss}, du_t^f)$	β^s	β^f	$\beta\varepsilon$
over 16 year old	0.0022	0.0011	0.0011	0.48	0.49	0.03
16 - 17 year old	0.0026	0.0014	0.0012	0.52	0.46	0.02
18 - 24 year old	0.0016	0.0006	0.0010	0.37	0.61	0.02
25 - 54 year old	0.0032	0.0016	0.0014	0.51	0.45	0.04
55 - 64 year old	0.0035	0.0017	0.0017	0.49	0.49	0.01
over 64 year old	0.0089	0.0041	0.0047	0.46	0.53	0.01

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Where du_t^{ss} is percentage change of unemployment rate in steady state. Also du_t^s, du_t^f and du_t^ε are measure of contribution of changes in s_t, f_t and ε into changes in steady state unemployment, respectively. Beta is the contribution of current changes in transition hazard rates to the variance of steady state unemployment, that is: $\beta^j = \frac{Cov(du_t^{ss}, du_t^j)}{Var(du_t^{ss})}$ with $j = s, f$. All variables with HP Filter (1600). Source: Own elaboration in base CPS.

Table 4.22: Female Unemployment Decompositions

Age	$Var(du_t^{ss})$	$Cov(du_t^{ss}, du_t^s)$	$Cov(du_t^{ss}, du_t^f)$	β^s	β^f	$\beta\varepsilon$
over 16 year old	0.001	0.000	0.001	0.45	0.52	0.02
16 - 17 year old	0.003	0.001	0.001	0.44	0.55	0.01
18 - 24 year old	0.001	0.001	0.001	0.50	0.48	0.02
25 - 54 year old	0.001	0.001	0.001	0.52	0.46	0.02
55 - 64 year old	0.003	0.001	0.002	0.49	0.51	0.00
over 64 year old	0.015	0.008	0.006	0.54	0.44	0.02

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Where du_t^{ss} is percentage change of unemployment rate in steady state. Also du_t^s, du_t^f and du_t^ε are measure of contribution of changes in s_t, f_t and ε into changes in steady state unemployment, respectively. Beta is the contribution of current changes in transition hazard rates to the variance of steady state unemployment, that is: $\beta^j = \frac{Cov(du_t^{ss}, du_t^j)}{Var(du_t^{ss})}$ with $j = s, f$. All variables with HP Filter (1600). Source: Own elaboration in base CPS.

4.A.6 Stock and Steady State Unemployment

Table 4.23: Correlation Stock vs. Steady State Unemployment

Age	$\rho_{u_t,u_t^{ss}}$
over 16 year old	0.98
16 - 17 year old	0.95
18 - 24 year old	0.98
25 - 54 year old	0.99
55 - 64 year old	0.98
over 64 year old	0.90

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Where Stock of unemployment (u_t) is calculated through CPS data base. Steady State unemployment (u_t^{ss})is calculated by the following formula: $u_t^{ss} = \frac{s_t}{s_t+f_t}$, where s_t and f_t are inflow and outflow hazard rates respectively. All calculations are performed for each age group. Source: Own elaboration in base CPS.

4.A.7 Labour Force and Working-Age Population

Table 4.24: Labour Force and Working-Age Population: Distribution by Age: All

Age	% in Labour Force	% Working-Age Population
over 16 year old	100.0%	100.0%
16 - 17 year old	2.4%	4.0%
18 - 24 year old	14.8%	13.5%
25 - 54 year old	68.3%	54.5%
55 - 64 year old	11.3%	12.4%
over 64 year old	3.2%	15.6%

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. All calculations are performed for each age group. Source: Own elaboration in base CPS.

Table 4.25: Labour Force and Working-Age Population: Distribution by Age: Male

Age	% in Labour Force	% Working-Age Population
over 16 year old	100.0%	100.0%
16 - 17 year old	2.3%	4.3%
18 - 24 year old	14.3%	13.9%
25 - 54 year old	68.6%	55.7%
55 - 64 year old	11.5%	12.4%
over 64 year old	3.4%	13.7%

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. All calculations are performed for each age and gender group. Source: Own elaboration in base CPS.

Table 4.26: Labour Force and Working-Age Population: Distribution by Age: Female

Age	% in Labor Force	% Working-Age Population
over 16 year old	100.0%	100.0%
16 - 17 year old	2.5%	3.8%
18 - 24 year old	15.5%	13.1%
25 - 54 year old	68.1%	53.3%
55 - 64 year old	11.0%	12.5%
over 64 year old	3.0%	17.4%

Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. All calculations are performed for each age and gender group. Source: Own elaboration in base CPS.

4.A.8 Ratio of Unemployment

Table 4.27: Average Unemployment, Population Distribution by Age

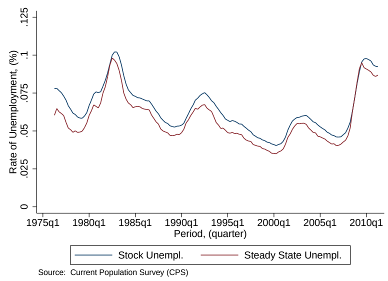
Age	% Unemployment	Ratio of Unemployment	% in Labour Force
over 16 year old	6.4%	1.00	100.0%
16 - 17 year old	20.2%	3.14	2.4%
18 - 24 year old	11.9%	1.84	14.9%
25 - 54 year old	5.2%	0.81	68.2%
55 - 64 year old	3.9%	0.61	11.2%
over 64 year old	3.6%	0.56	3.2%

Note: Sample covers U.S. data from 1976:01 to 2011:08. Source: Own elaboration in base CPS.

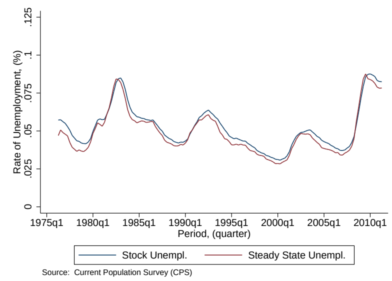
4.B Graphs

Figure 4.1: Stock and Steady Steady Unemployment

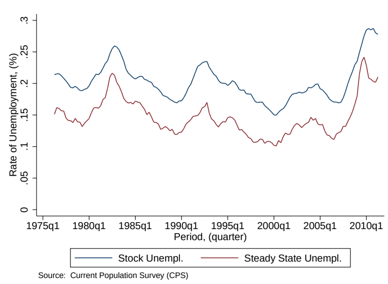
(a) People over 16 years old



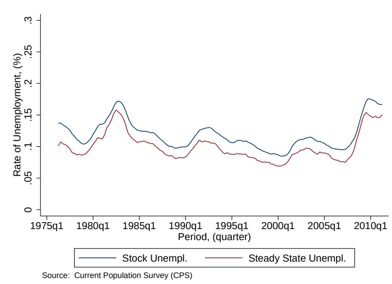
(b) People between 25-54 years old



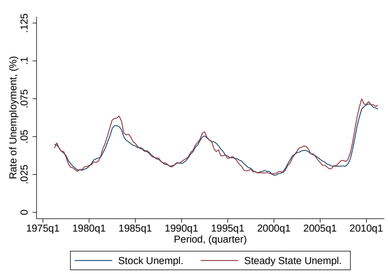
(c) People between 16-17 years old



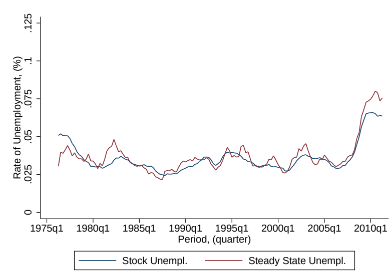
(d) People between 18-24 years old



(e) People between 55-64 years old



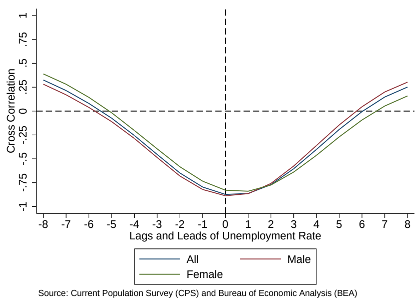
(f) People between over 64 years old



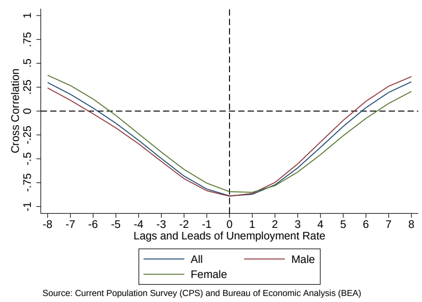
Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Unemployment stock is calculated through the CPS database, by dividing the level of unemployment with respect to the labour force. Steady state unemployment is constructed assuming that workers can only move between employment and unemployment, under the following equation: $u_{ss} = \frac{s_t}{s_t + f_t}$, where s_t and f_t are inflow and outflow hazard rates respectively. All calculations are performed for each age group.

Figure 4.2: Cross Correlation GDP and Unemployment Rate (cyclical components)

(a) People over 16 years old

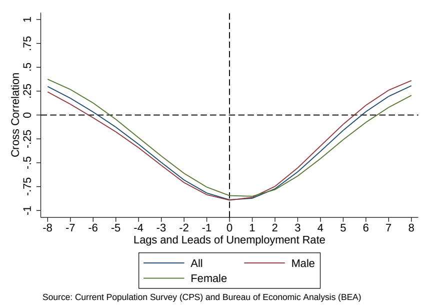


(b) People between 25-54 years old

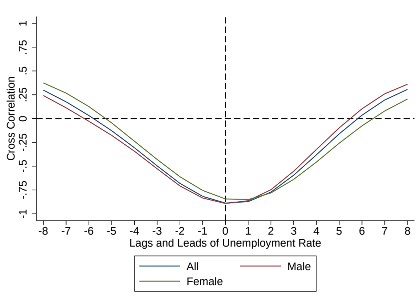


(c) People between 16-17 years old

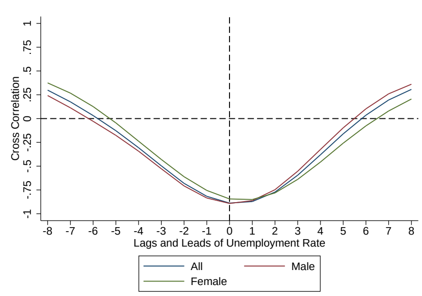
(d) People between 18-24 years old



(e) People between 55-64 years old



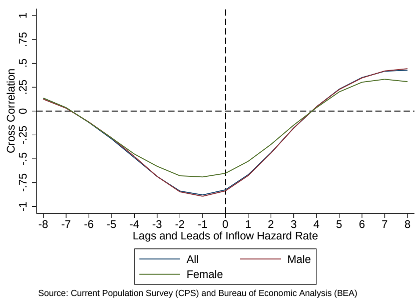
(f) People between over 64 years old



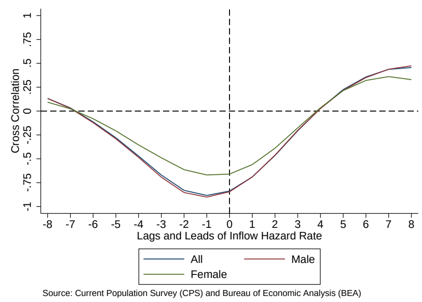
Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Unemployment rate and GDP with seasonal adjustment, in logarithms and application of Hodrick-Prescott filter with smoothing parameter of 1600. For the calculation of cross correlation the cyclical components of unemployment rate was lags and leads of eight periods. All calculations are performed for each age and gender group.

Figure 4.3: Cross Correlation GDP and Inflow Hazard Rate (cyclical components)

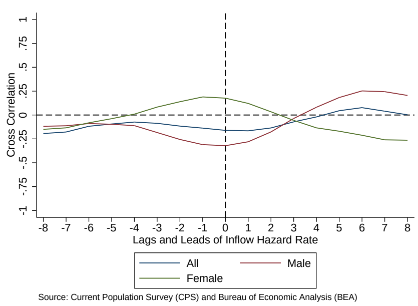
(a) People over 16 years old



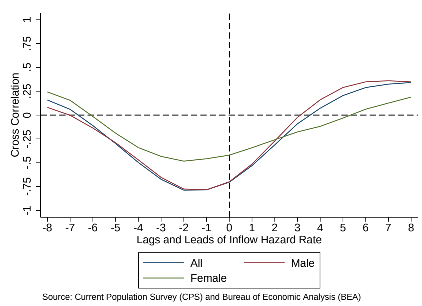
(b) People between 25-54 years old



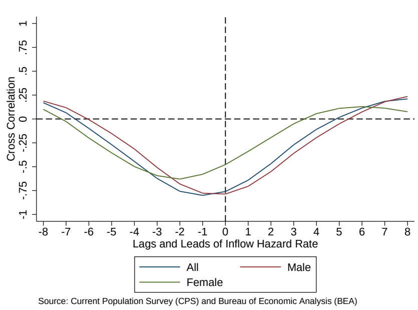
(c) People between 16-17 years old



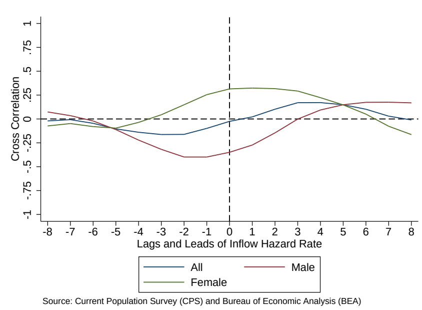
(d) People between 18-24 years old



(e) People between 55-64 years old



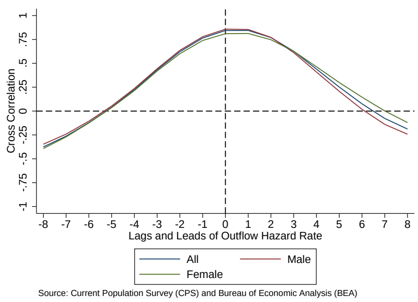
(f) People between over 64 years old



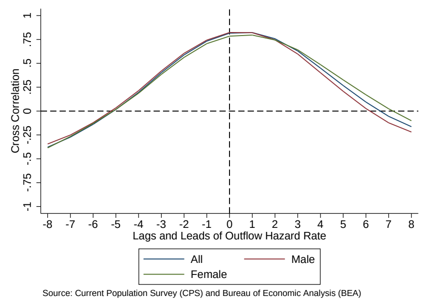
Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Inflow hazard rate and GDP with seasonal adjustment, in logarithms and application of Hodrick-Prescott filter with smoothing parameter of 1600. For the calculation of cross correlation the cyclical components of inflow hazard rate was lags and leads of eight periods. All calculations are performed for each age and gender group.

Figure 4.4: Cross Correlation GDP and Outflow Hazard Rate (cyclical components)

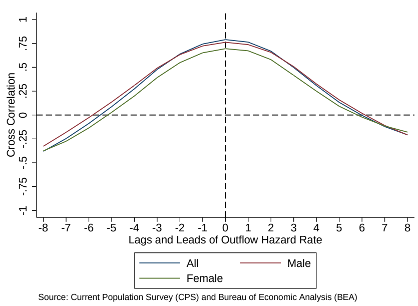
(a) people over 16 years old



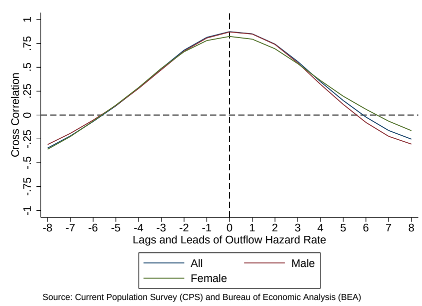
(b) people between 25-54 years old



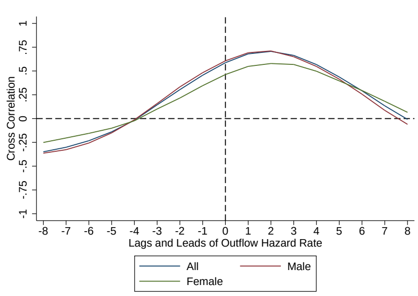
(c) people between 16-17 years old



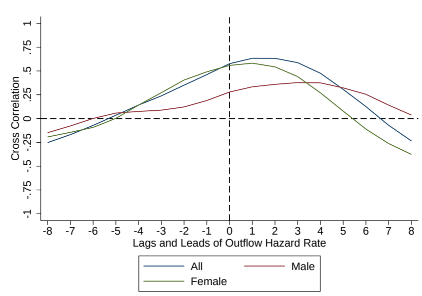
(d) people between 18-24 years old



(e) people between 55-64 years old



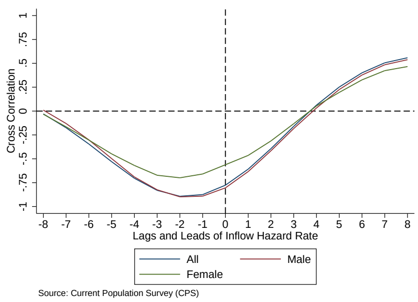
(f) people between over 64 years old



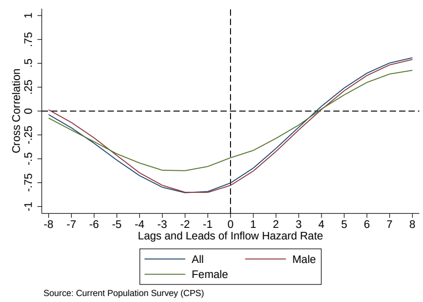
Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Outflow hazard rate and GDP with seasonal adjustment, in logarithms and application of Hodrick-Prescott filter with smoothing parameter of 1600. For the calculation of cross correlation the cyclical components of outflow hazard rate was lags and leads of eight periods. All calculations are performed for each age and gender group.

Figure 4.5: Cross Correlation Inflow and Outflow Hazard Rates (cyclical components)

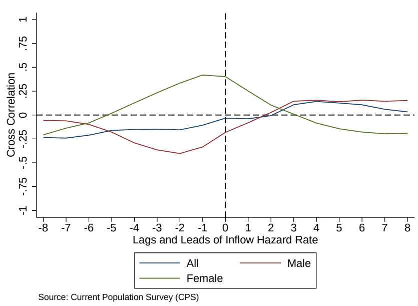
(a) People over 16 years old



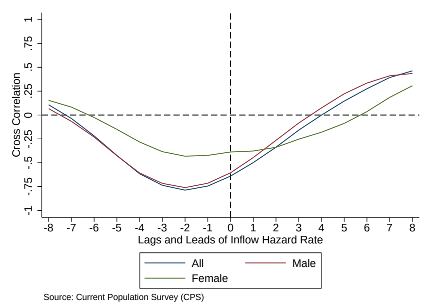
(b) People between 25-54 years old



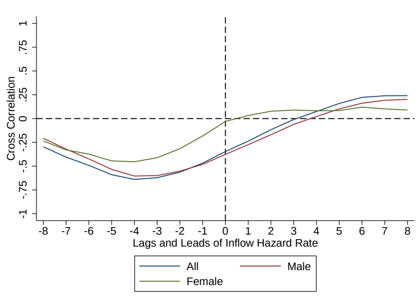
(c) People between 16-17 years old



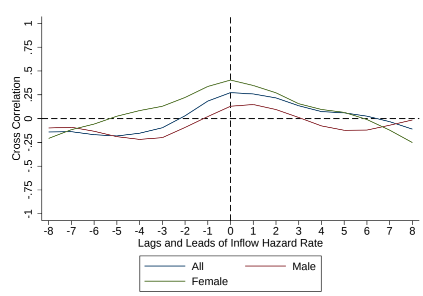
(d) People between 18-24 years old



(e) People between 55-64 years old



(f) People between over 64 years old



Note: Sample covers U.S. data from 1976:Q2 to 2011:Q2. Outflow hazard rate and inflow hazard rate with seasonal adjustment, in logarithms and application of Hodrick-Prescott filter with smoothing parameter of 1600. For the calculation of cross correlation the cyclical components of inflow hazard rate was lags and leads of eight periods. All calculations are performed for each age and gender group.

4.C Unemployment Dynamic

4.C.1 Discrete Case

We can determine the dynamics of the stock of unemployment in terms of inflows and outflows of unemployment, through the following mathematical relationship:

$$U_{t+1} = U_t - M_t^{UE} + S_t^{EU} + F_t^{IU} - F_t^{UI}, \qquad (4.C.1)$$

where:

- U_t: Stock of unemployment in t.

- M_t^{UE}: Gross hiring flows from unemployment (in $t-1$) to employment (in t).

- S_t^{EU}: Gross separation flows from employment (in $t-1$) to unemployment (in t).

- F_t^{IU}: Gross flows from inactive (in $t-1$) to unemployment (in t).

- F_t^{UI}: Gross flows from unemployment (in $t-1$) to inactive (in t).

We define separation rate and job finding rate as follow:

- Separation Rate in t: $SR_t = \frac{S_t^{EU}}{E_t}$,

- Job Finding Rate in t: $JFR_t = \frac{M_t^{UE}}{U_t}$,

where:

- E_t: Stock of unemployment in t.

Then developed the equation (4.C.1),

$$\begin{aligned} U_{t+1} &= U_t - \frac{M_t^{UE}}{U_t} \cdot U_t + \frac{S_t^{EU}}{E_t} \cdot E_t + F_t^{IU} - F_t^{UI} \\ U_{t+1} &= U_t - JFR_t \cdot U_t + SR_t \cdot E_t + F_t^{IU} - F_t^{UI} \\ U_{t+1} &= U_t(1 - JFR_t) + SR_t \cdot E_t + F_t^{IU} - F_t^{UI}, \end{aligned}$$

and by dividing this equation by U_t, we have that,

$$\begin{aligned}
\frac{U_{t+1}}{U_t} &= \frac{U_t}{U_t}(1-JFR_t) + SR_t \cdot \frac{E_t}{U_t} + \frac{F_t^{IU} - F_t^{UI}}{U_t} \\
\frac{U_{t+1}}{U_t} &= (1-JFR_t) + SR_t \cdot \frac{E_t}{U_t} + \frac{F_t^{IU} - F_t^{UI}}{U_t} \\
\frac{U_{t+1}}{U_t} - 1 &= -JFR_t + SR_t \cdot \frac{E_t}{U_t} \cdot \frac{L_t}{L_t} + \frac{F_t^{IU} - F_t^{UI}}{U_t} \cdot \frac{L_t}{L_t},
\end{aligned}$$

where L_t is defined as the work force, that is equal to the sum of the stock of employment and unemployment ($L_t = E_t + U_t$). Then if we apply this identity in the equation above, we have that:

$$\begin{aligned}
\frac{U_{t+1} - U_t}{U_t} &= -JFR_t + SR_t \cdot \frac{L_t - U_t}{L_t} \cdot \frac{L_t}{U_t} + \frac{F_t^{IU} - F_t^{UI}}{L_t} \cdot \frac{L_t}{U_t} \\
\frac{\Delta U_{t+1}}{U_t} &= -JFR_t + SR_t \cdot (1 - \frac{U_t}{L_t}) \cdot \frac{L_t}{U_t} + \frac{F_t^{IU} - F_t^{UI}}{L_t} \cdot \frac{L_t}{U_t}. \qquad (4.C.2)
\end{aligned}$$

Note that the unemployment rate is defined as:

- $u_t = \frac{Ut}{L_t} = \frac{Ut}{E_t + U_t}$.

Then equation (4.C.2) can be expresses as:

$$\frac{\Delta U_{t+1}}{U_t} = -JFR_t + SR_t \cdot (1 - u_t) \cdot \frac{1}{u_t} + \frac{F_t^{IU} - F_t^{UI}}{L_t} \cdot \frac{1}{u_t}. \qquad (4.C.3)$$

Assuming an economy with the following assumptions:

- Economy with only two states of nature (employment and unemployment), this mean that worker only move from employment to unemployment: $\frac{F_t^{IU} - F_t^{UI}}{L_t} = 0$.

- Economy in steady state, this mean that: $\frac{\Delta U_{t+1}}{U_t} = 0$ and $u^{ss} = u_t$.

With this assumptions the equation (4.C.3) can be simplified:

$$\begin{aligned}
0 &= -JFR + SR \cdot (1 - u^{ss}) \cdot \frac{1}{u^{ss}} \\
0 &= -JFR \cdot u^{ss} + SR - SR \cdot u^{ss} \\
u^{ss} \cdot (SR + JFR) &= SR \\
u^{ss} &= \frac{SR}{SR + JFR}. \qquad (4.C.4)
\end{aligned}$$

4.C.2 Continuous Case

For simplicity we assume an economy with only two states of nature, employment (e_t) and unemployment (u_t). These means that workers can not move into or out of inactivity. Also assume that in this economy the growth rate of labour force is zero, that means that we can normalize labour force to one ($e_t + u_t = 1$). Then if we defined s_t as the instantaneous unemployment inflow rates and f_t as the instantaneous unemployment outflow rates, the rate of unemployment ($u_t = \frac{U_t}{U_t+E_t}$) change according to the following differential equation:

$$\frac{du(t)}{dt} = \dot{u}_t = s_t \cdot e_t - f_t \cdot u_t. \tag{4.C.5}$$

In the steady state the unemployment is constant ($u_t = u_t^{ss}$) then the above equation (4.C.5) can be write as follow:

$$\begin{aligned} \dot{u}_t^{ss} &= s_t \cdot (1 - u_t^{ss}) - f_t \cdot u_t^{ss} \\ 0 &= s_t - s_t \cdot u_t^{ss} - f_t \cdot u_t^{ss} \\ (s_t + f_t) \cdot u_t^{ss} &= s_t \\ u_t^{ss} &= \frac{s_t}{s_t + f_t} \end{aligned} \tag{4.C.6}$$

4.D Hazard Rates and Total Flows

4.D.1 Worker Flows and Transition Hazard Rates

We use the methodology developed by Fujita and Ramey (2006) to solve the problems of time aggregation through a unified framework. The assumptions are:

- z: Continuous time, measurement at time $z = 1, 2, 3, ...$
- Two state of nature, employment and unemployment.
- $u(z)$: Number of unemployment workers at time z.
- $e(z) = 1 - u(z)$: Number of employment workers at time z.
- Labour force is normalized to unite.
- s_t : Instantaneous arrival rate for worker transition into unemployment for each z $\in [t-1, t)$.

- f_t : Instantaneous arrival rate for worker transition out unemployment for each z $\in [t-1, t)$.

With these assumptions the dynamics of unemployment can you model according to the following differential equation (see equation (4.C.5) of Appendix C):

$$\frac{du(z)}{dz} = s_t \cdot e(z) - f_t \cdot u(z). \tag{4.D.1}$$

Solving this differential equation:

$$\begin{aligned}
\frac{du(z)}{dz} &= s_t \cdot e(z) - f_t \cdot u(z) \\
\frac{du(z)}{dz} &= s_t \cdot (1 - u(z)) - f_t \cdot u(z) \\
\frac{du(z)}{dz} &= s_t - (s_t + f_t) \cdot u(z) \\
du(z) &= (s_t - (s_t + f_t) \cdot u(z)) \cdot dz \\
\frac{1}{(s_t - (s_t + f_t) \cdot u(z))} \cdot du(z) &= dz
\end{aligned} \tag{4.D.2}$$

Applying integration on both sides of the equation (4.D.2) and solving:

$$\begin{aligned}
\int \frac{1}{(s_t - (s_t + f_t) \cdot u(z))} \cdot du(z) &= \int dz \\
-\frac{ln(s_t - (s_t + f_t) \cdot u(z))}{s_t + f_t} &= z + c
\end{aligned} \tag{4.D.3}$$

As the initial condition is $s = t - 1$ then we have that $u(t-1) = u_{t-1}$. Then if we applied the initial condition in equation (4.D.3) we obtain the value of the constant c, as follow:

$$\begin{aligned}
-\frac{ln(s_t - (s_t + f_t) \cdot u(t-1))}{s_t + f_t} &= t - 1 + c \\
-\frac{ln(s_t - (s_t + f_t) \cdot u_{t-1})}{s_t + f_t} &= t - 1 + c \\
-\frac{ln(s_t - (s_t + f_t) \cdot u_{t-1})}{s_t + f_t} - t + 1 &= c
\end{aligned} \tag{4.D.4}$$

Then replacing equation (4.D.4) in equation (4.D.3) we have:

$$-\frac{ln(s_t - (s_t + f_t) \cdot u(z))}{s_t + f_t} = z + -\frac{ln(s_t - (s_t + f_t) \cdot u_{t-1})}{s_t + f_t} - t + 1$$

$$\frac{ln(s_t - (s_t + f_t) \cdot u(z))^{-1}}{s_t + f_t} + \frac{ln(s_t - (s_t + f_t) \cdot u_{t-1})}{s_t + f_t} = z - t + 1$$

$$ln(s_t - (s_t + f_t) \cdot u(z))^{-1} + ln(s_t - (s_t + f_t) \cdot u_{t-1}) = (z - t + 1)(s_t + f_t)$$

Applying exponential to both sides of above equation:

$$\begin{aligned}
\exp[ln(s_t - (s_t + f_t) \cdot u(z))^{-1} + ln(s_t - (s_t + f_t) \cdot u_{t-1})] &= \exp[(z - t + 1)(s_t + f_t)] \\
\exp[ln(s_t - (s_t + f_t) \cdot u(z))^{-1}] \cdot \exp[ln(s_t - (s_t + f_t) \cdot u_{t-1})] &= \exp[(z - t + 1)(s_t + f_t)] \\
(s_t - (s_t + f_t) \cdot u(z))^{-1} \cdot (s_t - (s_t + f_t) \cdot u_{t-1}) &= \exp[(z - t + 1)(s_t + f_t)] \\
\frac{(s_t - (s_t + f_t) \cdot u_{t-1})}{(s_t - (s_t + f_t) \cdot u(z))} &= \exp[(z - t + 1)(s_t + f_t)]
\end{aligned}$$

Then isolate $u(z)$ of above equation:

$$\begin{aligned}
\frac{(s_t - (s_t + f_t) \cdot u_{t-1})}{\exp[1 \cdot (z - t + 1)(s_t + f_t)]} &= (s_t - (s_t + f_t) \cdot u(z)) \\
(s_t - (s_t + f_t) \cdot u_{t-1}) \cdot \exp[-1 \cdot (z - t + 1)(s_t + f_t)] &= s_t - (s_t + f_t) \cdot u(z) \\
((s_t + f_t) \cdot u_{t-1} - s_t) \cdot \exp[-1 \cdot (z - t + 1)(s_t + f_t)] + s_t &= (s_t + f_t) \cdot u(z) \\
\left(u_{t-1} - \frac{s_t}{s_t + f_t}\right) \cdot \exp[-1 \cdot (z - t + 1)(s_t + f_t)] + \frac{s_t}{s_t + f_t} &= u(z)
\end{aligned}$$

Then unemployment in z can be expressed as:

$$u(z) = \frac{s_t}{s_t + f_t} + \left(u_{t-1} - \frac{s_t}{s_t + f_t}\right) \cdot \exp[-(z - t + 1)(s_t + f_t)] \tag{4.D.5}$$

Now we define e_t^n number of employment workers at the end of month t who were either unemployment or employment in different jobs at beginning of the month. Then e_t^n:

$$e_n^t = \int_{t-1}^{t} f_t \cdot u(z) \cdot \exp[-s_t(t - z)]dz \tag{4.D.6}$$

By replace equation (4.D.5) in equation (4.D.6) we have:

$$e_n^t = \int_{t-1}^{t} f_t \cdot \left(\frac{s_t}{s_t + f_t} + \left(u_{t-1} - \frac{s_t}{s_t + f_t}\right) \cdot \exp[-(z - t + 1)(s_t + f_t)]\right) \cdot \exp[-s_t(t - z)]dz$$

The above integral we can be divided into two components, A and B respectively:

$$A = \int_{t-1}^{t} f_t \cdot \left(\frac{s_t}{s_t + f_t}\right) \cdot \exp[-s_t(t-z)]dz, \tag{4.D.7}$$

$$B = \int_{t-1}^{t} f_t \Bigg(\left(u_{t-1} - \frac{s_t}{s_t + f_t}\right) \cdot \exp[-(z-t+1)(s_t+f_t)]\Bigg) \cdot \exp[-s_t(t-z)]dz, \tag{4.D.8}$$

where $e_n^t = A + B$.

Solving equation (4.D.7) we have:

$$\begin{aligned}
A &= \int_{t-1}^{t} f_t \cdot \left(\frac{s_t}{s_t + f_t}\right) \cdot \exp[-s_t(t-z)]dz \\
A &= \left(\frac{f_t s_t}{s_t + f_t}\right) \int_{t-1}^{t} \exp[-s_t(t-z)]dz \\
A &= \left(\frac{f_t s_t}{s_t + f_t}\right) \left\{\frac{\exp[s_t(z-t)]}{s_t}\right\}_{t-1}^{t} \\
A &= \left(\frac{f_t s_t}{s_t + f_t}\right) \left\{\frac{1}{s_t} - \frac{\exp[-s_t]}{s_t}\right\} \\
A &= \left(\frac{f_t}{s_t + f_t}\right) \Big\{1 - \exp[-s_t]\Big\}
\end{aligned} \tag{4.D.9}$$

Solving by equation (4.D.8) we have:

$$\begin{aligned}
B &= \int_{t-1}^{t} f_t \Bigg(\left(u_{t-1} - \frac{s_t}{s_t + f_t}\right) \cdot \exp[-(z-t+1)(s_t+f_t)]\Bigg) \cdot \exp[-s_t(t-z)]dz \\
B &= \int_{t-1}^{t} f_t \Bigg(\left(u_{t-1} - \frac{s_t}{s_t + f_t}\right) \cdot \exp[-f_t(z-t) - (s_t+f_t)]\Bigg)dz \\
B &= f_t\left(u_{t-1} - \frac{s_t}{s_t + f_t}\right) \int_{t-1}^{t} \exp[-f_t(z-t) - (s_t+f_t)]dz \\
B &= f_t\left(u_{t-1} - \frac{s_t}{s_t + f_t}\right) \left\{-\frac{\exp[f_t(t-z) - (f_t+s_t)]}{f_t}\right\}_{t-1}^{t} \\
B &= f_t\left(u_{t-1} - \frac{s_t}{s_t + f_t}\right) \left\{-\frac{\exp[-(f_t+s_t)]}{f_t} + \frac{\exp[-s_t]}{f_t}\right\} \\
B &= \left(u_{t-1} - \frac{s_t}{s_t + f_t}\right) \Big\{-\exp[-(f_t+s_t)] + \exp[-s_t]\Big\}
\end{aligned}$$

Then B can be expressed as:

$$B = \left(u_{t-1} - \frac{s_t}{s_t + f_t}\right) \exp[-s_t]\Big\{1 - \exp[-f_t]\Big\} \tag{4.D.10}$$

Then adding A (equation (4.D.9)) and B (equation (4.D.10)) we find the value of e_n^t as follows:

$$e_n^t = \left(\frac{f_t}{s_t+f_t}\right)\left\{1-\exp[-s_t]\right\} + \left(u_{t-1} - \frac{s_t}{s_t+f_t}\right)\exp[-s_t]\left\{1-\exp[-f_t]\right\} \quad (4.D.11)$$

Also e_t^n may be represented in terms of average transition, where SR_t and JFR_t are separation rate and job finding rate respectively, then we find that:

- $u_{t-1} \cdot JFR_t$: Quantity of workers that start as unemployment and finish in the period as employment.
- $(1-u_{t-1})(1-SR_t-\exp[-s_t])$: Quantity of workers who are employed at both the start and end of the month experienced spells of unemployment within the month.

Then, e_t^n can be expressed as:

$$e_n^t \quad = \quad u_{t-1}\cdot JFR_t + (1-u_{t-1})(1-SR_t-\exp[s_t]) \quad (4.D.12)$$

Then if we balance the equation (4.D.11) and equation (4.D.12) we can obtain the value of separation rate (SR_t) and job finding rate (JFR_t) as function of s_t and f_t respectively. Then development equation (4.D.11) we have:

$$e_n^t = \left(\frac{f_t}{s_t+f_t}\right)\left\{1-\exp[-s_t]\right\} + \left(u_{t-1} - \frac{s_t}{s_t+f_t}\right)\exp[-s_t]\left\{1-\exp[-f_t]\right\}$$
$$e_n^t = \left(1-\frac{s_t}{s_t+f_t}\right)(1-\exp[-s_t]) + \left(u_{t-1} - \frac{s_t}{s_t+f_t}\right)\exp[-s_t](1-\exp[-f_t])$$
$$e_n^t = 1-\exp[-s_t] - \left(\frac{s_t}{s_t+f_t}\right) - \left(\frac{s_t}{s_t+f_t}\right)\exp[-s_t] + u_{t-1}\exp[-s_t](1-\exp[-f_t])$$
$$-\frac{s_t}{s_t+f_t}\exp[-s_t](1-\exp[-f_t])$$

$$e_n^t = 1-\exp[-s_t] - \left(\frac{s_t}{s_t+f_t}\right) + u_{t-1}\exp[-s_t](1-\exp[-f_t])$$
$$-\frac{s_t}{s_t+f_t}\exp[-s_t](1-\exp[-f_t]-1)$$
$$e_n^t = 1-\exp[-s_t] - \left(\frac{s_t}{s_t+f_t}\right) + u_{t-1}\exp[-s_t](1-\exp[-f_t])$$
$$-\frac{s_t}{s_t+f_t}\exp[-s_t-f_t]$$
$$e_n^t = 1-\exp[-s_t] + u_{t-1}\exp[-s_t](1-\exp[-f_t]) - \frac{s_t}{s_t+f_t}(1-\exp[-s_t-f_t]) \quad (4.D.13)$$

Also, development equation (4.D.12) we have:

$$\begin{aligned}
e_n^t &= u_{t-1} \cdot JFR_t + (1-u_{t-1})(1-SR_t-\exp[-s_t]) \\
e_n^t &= u_{t-1} \cdot JFR_t + 1 - SR_t - \exp[-s_t] - u_{t-1}(1-SR_t-\exp[-s_t]) \\
e_n^t &= 1-\exp[-s_t] - u_{t-1}(1-JFR_t-SR_t-\exp[-s_t]) - SR_t \\
e_n^t &= 1-\exp[-s_t] + u_{t-1}(JFR_t-1+SR_t+\exp[-s_t]) - SR_t \qquad (4.D.14)
\end{aligned}$$

So if we equate the fourth coefficient of the equations (4.D.13) and (4.D.14) we find that:

$$-SR_t = -\frac{s_t}{s_t+f_t}(1-\exp[-s_t-f_t])$$

Then according above equation separation rate can be expressed as function of s_t and f_t, as follow:

$$SR_t = \frac{s_t}{s_t+f_t}(1-\exp[-s_t-f_t]) \qquad (4.D.15)$$

Now if we equate the third coefficient of the equations (4.D.13) and (4.D.14) we find that:

$$\begin{aligned}
u_{t-1}\exp[-s_t](1-\exp[-f_t]) &= u_{t-1}(JFR_t-1+SR_t+\exp[-s_t]) \\
\exp[-s_t](1-\exp[-f_t]) &= (JFR_t-1+SR_t+\exp[-s_t]) \\
\exp[-s_t]-\exp[-s_t-f_t]) &= JFR_t-1+SR_t+\exp[-s_t] \\
JFR_t &= 1-SR_t+\exp[-s_t-f_t]) \qquad (4.D.16)
\end{aligned}$$

By replacing equation (4.D.15) in equation (4.D.16), we have that job finding rate can be expressed as:

$$\begin{aligned}
JFR_t &= 1-SR_t+\exp[-s_t-f_t]) \\
JFR_t &= 1-\left(\frac{s_t}{s_t+f_t}(1-\exp[-s_t-f_t])\right)+\exp[-s_t-f_t]) \\
JFR_t &= 1-\frac{s_t}{s_t+f_t}-\left(1-\frac{s_t}{s_t+f_t}\right)\exp[-s_t-f_t] \\
JFR_t &= \left(1-\frac{s_t}{s_t+f_t}\right)(1-\exp[-s_t-f_t]) \\
JFR_t &= \frac{f_t}{s_t+f_t}(1-\exp[-s_t-f_t]) \qquad (4.D.17)
\end{aligned}$$

4.D.2 Equation for Time Aggregation

Working on equations (4.D.16) and (4.D.17) we can get s_t y f_t as a function of SR_t and JFR_t. Then we divide equation (4.D.16) and (4.D.17):

$$\frac{SR_t}{JFR_t} = \frac{\frac{s_t}{s_t+f_t}(1-\exp[-s_t-f_t])}{\frac{f_t}{s_t+f_t}(1-\exp[-s_t-f_t])} \tag{4.D.18}$$

Then:

$$\frac{SR_t}{JFR_t} = \frac{s_t}{f_t} \tag{4.D.19}$$

Then isolate separation rate of equation (4.D.19) and by replacing in equation (4.D.15), as follow:

$$\begin{aligned}
SR_t &= \frac{\frac{SR_t}{JFR_t}f_t}{\frac{SR_t}{JFR_t}f_t+f_t}(1-\exp[-\frac{SR_t}{JFR_t}f_t-f_t]) \\
SR_t &= \frac{SR_t}{SR_t+JFR_t}(1-\exp[-f_t\frac{(SR_t+JFR_t)}{JFR_t}]) \\
SR_t+JFR_t &= 1-\exp[-f_t\frac{(SR_t+JFR_t)}{JFR_t}] \\
\exp[-f_t\frac{(SR_t+JFR_t)}{JFR_t}] &= 1-SR_t-JFR_t
\end{aligned} \tag{4.D.20}$$

Applying logarithm to both sides of the equation (4.D.20) we find instantaneous unemployment outflow rates f_t as function of separation and job finding rate:

$$\begin{aligned}
-f_t\frac{(SR_t+JFR_t)}{JFR_t} &= log(1-SR_t-JFR_t) \\
f_t &= -\frac{JFR_t}{SR_t+JFR_t}\cdot log(1-SR_t-JFR_t)
\end{aligned} \tag{4.D.21}$$

Now if we isolate job finding rate of equation (4.D.19) and by replacing in equation (4.D.15), as follow:

$$\begin{aligned}
JFR_t &= \frac{\frac{JFR_t}{SR_t}s_t}{s_t+\frac{JFR_t}{SR_t}s_t}(1-\exp[-s_t-\frac{JFR_t}{SR_t}s_t]) \\
JFR_t &= \frac{JFR_t}{SR_t+JFR_t}(1-\exp[-s_t\frac{(SR_t+JFR_t)}{SR_t}])
\end{aligned}$$

$$\begin{aligned} SR_t + JFR_t &= 1 - \exp[-s_t \frac{(SR_t + JFR_t)}{SR_t}] \\ \exp[-s_t \frac{(SR_t + JFR_t)}{SR_t}] &= 1 - SR_t - JFR_t \end{aligned} \quad (4.D.22)$$

Applying logarithm to both sides of the equation (4.D.22) we find instantaneous unemployment inflow rates s_t as function of separation and job finding rate:

$$\begin{aligned} -s_t \frac{(SR_t + JFR_t)}{SR_t} &= log(1 - SR_t - JFR_t) \\ s_t &= -\frac{SR_t}{SR_t + JFR_t} \cdot log(1 - SR_t - JFR_t) \end{aligned} \quad (4.D.23)$$

4.E Decomposition and Contribution to Unemployment Volatility

4.E.1 Decomposition of Unemployment

According to Appendix C, we can represent the dynamics of unemployment (continuous case) as a function of instantaneous unemployment inflow and outflow rates, s_t and f_t respectively, as follow (see equation (4.C.6)):

$$u_t^{ss} = \frac{s_t}{s_t + f_t}. \quad (4.E.1)$$

If we take a lag (one) to the equation (4.E.1) we have the following expression:

$$u_{t-1}^{ss} = \frac{s_{t-1}}{s_{t-1} + f_{t-1}}. \quad (4.E.2)$$

Subtracting equation (4.E.2) from equation (4.E.1), we have:

$$\begin{aligned} u_t^{ss} - u_{t-1}^{ss} &= (\frac{s_t}{s_t + f_t}) - (\frac{s_{t-1}}{s_{t-1} + f_{t-1}}) \\ \triangle u_t^{ss} &= \frac{s_t(s_{t-1} + f_{t-1}) - s_{t-1}(s_t + f_t)}{(s_t + f_t)(s_{t-1} + f_{t-1})} \\ \triangle u_t^{ss} &= \frac{s_t f_{t-1} - s_{t-1} f_t}{(s_t + f_t)(s_{t-1} + f_{t-1})} \end{aligned} \quad (4.E.3)$$

So if we add and subtract $s_t f_t$ to the numerator of the equation (4.E.3) we have:

$$
\begin{aligned}
\triangle u_t^{ss} &= \frac{s_t f_{t-1} - s_{t-1} f_t + [s_t f_t - s_t f_t]}{(s_t + f_t)(s_{t-1} + f_{t-1})} \\
\triangle u_t^{ss} &= \frac{s_t f_{t-1} - s_t f_t + s_t f_t - s_{t-1} f_t}{(s_t + f_t)(s_{t-1} + f_{t-1})} \\
\triangle u_t^{ss} &= \frac{s_t f_{t-1} - s_t f_t + s_t f_t - s_{t-1} f_t}{(s_t + f_t)(s_{t-1} + f_{t-1})}
\end{aligned}
$$

Taking s_t and f_t as a common factor:

$$
\begin{aligned}
\triangle u_t^{ss} &= \frac{s_t(f_{t-1} - f_t) + f_t(s_t - s_{t-1})}{(s_t + f_t)(s_{t-1} + f_{t-1})} \\
\triangle u_t^{ss} &= \frac{f_t(s_t - s_{t-1}) - s_t(f_t - f_{t-1})}{(s_t + f_t)(s_{t-1} + f_{t-1})} \\
\triangle u_t^{ss} &= \frac{f_t(\triangle s_t) - s_t(\triangle f_t)}{(s_t + f_t)(s_{t-1} + f_{t-1})} \\
\triangle u_t^{ss} &= \frac{f_t(\triangle s_t)}{(s_t + f_t)(s_{t-1} + f_{t-1})} - \frac{s_t(\triangle f_t)}{(s_t + f_t)(s_{t-1} + f_{t-1})} \\
\triangle u_t^{ss} &= \frac{f_t}{s_t + f_t} \cdot \frac{\triangle s_t}{s_{t-1} + f_{t-1}} \cdot \frac{s_{t-1}}{s_{t-1}} - \frac{s_t}{s_t + f_t} \cdot \frac{\triangle f_t}{s_{t-1} + f_{t-1}} \cdot \frac{f_{t-1}}{f_{t-1}} \\
\triangle u_t^{ss} &= \frac{f_t}{s_t + f_t} \cdot \frac{s_{t-1}}{s_{t-1} + f_{t-1}} \cdot \frac{\triangle s_t}{s_{t-1}} - \frac{s_t}{s_t + f_t} \cdot \frac{f_{t-1}}{s_{t-1} + f_{t-1}} \cdot \frac{\triangle f_t}{f_{t-1}}. \qquad (4.E.4)
\end{aligned}
$$

Recall that:

$$
\frac{f_t}{s_t + f_t} = 1 - \frac{s_t}{s_t + f_t} = 1 - u_t^{ss} \qquad (4.E.5)
$$

Then substituting equations (4.E.1), (4.E.2) and (4.E.5) in equation (4.E.4), we have that:

$$
\triangle u_t^{ss} = (1 - u_t^{ss}) \cdot u_{t-1}^{ss} \cdot \frac{\triangle s_t}{s_{t-1}} - u_t^{ss} \cdot (1 - u_{t-1}^{ss}) \cdot \frac{\triangle f_t}{f_{t-1}}. \qquad (4.E.6)
$$

Dividing equation (4.E.6) by u_{t-1}^{ss}:

$$
\frac{\triangle u_t^{ss}}{u_{t-1}^{ss}} = (1 - u_t^{ss}) \cdot \frac{\triangle s_t}{s_{t-1}} - (1 - u_{t-1}^{ss}) \cdot \frac{u_t^{ss}}{u_{t-1}^{ss}} \cdot \frac{\triangle f_t}{f_{t-1}}. \qquad (4.E.7)
$$

And assuming that $u_t^{ss} \approx u_{t-1}^{ss}$ we can simplify the above equation:

$$
\frac{\triangle u_t^{ss}}{u_{t-1}^{ss}} \approx (1 - u_t^{ss}) \cdot \frac{\triangle s_t}{s_{t-1}} - \cdot (1 - u_{t-1}^{ss}) \cdot \frac{\triangle f_t}{f_{t-1}}. \qquad (4.E.8)
$$

By add an error term we can make both sides of the equation (4.E.8) become equal:

$$\frac{\triangle u_t^{ss}}{u_{t-1}^{ss}} = (1-u_t^{ss})\cdot\frac{\triangle s_t}{s_{t-1}} - (1-u_{t-1}^{ss})\cdot\frac{\triangle f_t}{f_{t-1}} + \varepsilon. \tag{4.E.9}$$

4.E.2 Contribution to Unemployment Volatility

In Appendix E we find the following expression for the percentage change in unemployment in the steady state (see equation (4.E.8))is:

$$\frac{\triangle u_t^{ss}}{u_{t-1}^{ss}} = (1-u_t^{ss})\cdot\frac{\triangle s_t}{s_{t-1}} - (1-u_{t-1}^{ss})\cdot\frac{\triangle f_t}{f_{t-1}} + \varepsilon. \tag{4.E.10}$$

Then we define:

- $du_t^{ss} = \frac{\triangle u_t^{ss}}{u_{t-1}^{ss}}$,
- $du_t^{s} = (1-u_t^{ss})\cdot\frac{\triangle s_t}{s_{t-1}}$,
- $du_t^{f} = -(1-u_{t-1}^{ss})\cdot\frac{\triangle f_t}{f_{t-1}}$,
- $du_t^{\varepsilon} = \varepsilon$,

where:

- du_t^{ss} : Percentage change of unemployment rate in steady state.
- du_t^{s} : Measure of contribution of changes in instantaneous unemployment inflow (s_t) to changes in steady state unemployment.
- du_t^{f} : Measure of contribution of changes in instantaneous unemployment outflow (f_t) to changes in steady state unemployment.
- du_t^{ε} : Measure of contribution of changes in error component (ε) to changes in steady state unemployment.

Then the equation (4.E.10) can be expressed as follows:

$$du_t^{ss} = du_t^{s} + du_t^{f} + du_t^{\varepsilon}. \tag{4.E.11}$$

Taking variance to the equation (4.E.11) we have:

$$\begin{aligned} V(du_t^{ss}) &= V(du_t^{s}) + V(du_t^{f}) + V(du_t^{\varepsilon}) + 2Cov(du_t^{s}, du_t^{f}) \\ &\quad + 2Cov(du_t^{s}, du_t^{\varepsilon}) + 2Cov(du_t^{f}, du_t^{\varepsilon}), \end{aligned} \tag{4.E.12}$$

and we can show that the equation (4.E.12) can be written reduced as follows:

$$V(du_t^{ss}) = Cov(du_t^s, du_t^{ss}) + Cov(du_t^s, du_t^{ss}) + Cov(du_t^\varepsilon, du_t^{ss}). \quad (4.E.13)$$

Now, we proceed to show the relationship between equation (4.E.12) and (4.E.13). Then, from equation (4.E.11) we can isolate du_t^s, du_t^f and du_t^ε, as follows:

$$du_t^s = du_t^{ss} - du_t^f - du_t^\varepsilon. \quad (4.E.14)$$

$$du_t^f = du_t^{ss} - du_t^s - du_t^\varepsilon. \quad (4.E.15)$$

$$du_t^\varepsilon = du_t^{ss} - du_t^s - du_t^f. \quad (4.E.16)$$

Taking variance to the equation (4.E.14):

$$\begin{aligned} V(du_t^s) &= V(du_t^{ss}) + V(-du_t^f) + V(-du_t^\varepsilon) + 2Cov(du_t^{ss}, -du_t^f) \\ &\quad +2Cov(du_t^{ss}, -du_t^\varepsilon) + 2Cov(-du_t^f, -du_t^\varepsilon) \end{aligned} \quad (4.E.17)$$

Then, using variance and covariance properties over equation (4.E.17) it can be expressed as:

$$\begin{aligned} V(du_t^s) &= V(du_t^{ss}) + V(du_t^f) + V(du_t^\varepsilon) - 2Cov(du_t^{ss}, du_t^f) \\ &\quad -2Cov(du_t^{ss}, du_t^\varepsilon) + 2Cov(du_t^f, du_t^\varepsilon) \end{aligned} \quad (4.E.18)$$

Performing the same procedure above, this is, taking variance to the equation (4.E.15):

$$\begin{aligned} V(du_t^f) &= V(du_t^{ss}) + V(-du_t^s) + V(-du_t^\varepsilon) + 2Cov(du_t^{ss}, -du_s^s) \\ &\quad +2Cov(du_t^{ss}, -du_t^\varepsilon) + 2Cov(-du_t^s, -du_t^\varepsilon). \end{aligned} \quad (4.E.19)$$

And using properties of variance y covariance over equation (4.E.19):

$$\begin{aligned} V(du_t^f) &= V(du_t^{ss}) + V(du_t^s) + V(du_t^\varepsilon) - 2Cov(du_t^{ss}, du_t^s) \\ &\quad -2Cov(du_t^{ss}, du_t^\varepsilon) + 2Cov(du_t^s, du_t^\varepsilon). \end{aligned} \quad (4.E.20)$$

Finally taking variance to the equation (4.E.16):

$$\begin{aligned} V(du_t^{\varepsilon}) &= V(du_t^{ss}) + V(-du_t^{s}) + V(-du_t^{f}) + 2Cov(du_t^{ss}, -du_t^{s}) \\ &\quad +2Cov(du_t^{ss}, -du_t^{f}) + 2Cov(-du_t^{s}, -du_t^{f}). \end{aligned} \tag{4.E.21}$$

And using properties of variance y covariance over equation (4.E.21):

$$\begin{aligned} V(du_t^{\varepsilon}) &= V(du_t^{ss}) + V(du_t^{s}) + V(du_t^{f}) - 2Cov(du_t^{ss}, du_t^{s}) \\ &\quad -2Cov(du_t^{ss}, du_t^{f}) + 2Cov(du_t^{s}, du_t^{f}). \end{aligned} \tag{4.E.22}$$

Adding equation (4.E.18), (4.E.20) and (4.E.22) we have:

$$\begin{aligned} V(du_t^{s}) + V(du_t^{f}) + V(du_t^{\varepsilon}) &= V(du_t^{ss}) + V(du_t^{f}) + V(du_t^{\varepsilon}) - 2Cov(du_t^{ss}, du_t^{f}) \\ &\quad -2Cov(du_t^{ss}, du_t^{\varepsilon}) + 2Cov(du_t^{f}, du_t^{\varepsilon}) \\ &\quad V(du_t^{ss}) + V(du_t^{s}) + V(du_t^{\varepsilon}) - 2Cov(du_t^{ss}, du_t^{s}) \\ &\quad -2Cov(du_t^{ss}, du_t^{\varepsilon}) + 2Cov(du_t^{s}, du_t^{\varepsilon}) \\ &\quad V(du_t^{ss}) + V(du_t^{s}) + V(du_t^{f}) - 2Cov(du_t^{ss}, du_t^{s}) \\ &\quad -2Cov(du_t^{ss}, du_t^{f}) + 2Cov(du_t^{s}, du_t^{f}) \end{aligned} \tag{4.E.23}$$

Grouping and canceling terms in equation (4.E.23):

$$\begin{aligned} 0 &= 3V(du_t^{ss}) + [V(du_t^{s}) + V(du_t^{f}) + V(du_t^{\varepsilon}) \\ &\quad +2Cov(du_t^{s}, du_t^{f}) + 2Cov(du_t^{s}, du_t^{\varepsilon}) + 2Cov(du_t^{f}, du_t^{\varepsilon})] \\ &\quad -4Cov(du_t^{ss}, du_t^{s}) - 4Cov(du_t^{ss}, du_t^{f}) - 4Cov(du_t^{ss}, du_t^{\varepsilon}) \end{aligned} \tag{4.E.24}$$

Where the expression in parentheses represents the variance of du_t^{ss} (see equation (4.E.12)), then:

$$\begin{aligned} 0 &= 4V(du_t^{ss}) - 4Cov(du_t^{ss}, du_t^{s}) - 4Cov(du_t^{ss}, du_t^{f}) - 4Cov(du_t^{ss}, du_t^{\varepsilon}) \\ 4V(du_t^{ss}) &= 4Cov(du_t^{ss}, du_t^{s}) + 4Cov(du_t^{ss}, du_t^{f}) + 4Cov(du_t^{ss}, du_t^{\varepsilon}) \\ V(du_t^{ss}) &= Cov(du_t^{ss}, du_t^{s}) + Cov(du_t^{ss}, du_t^{f}) + Cov(du_t^{ss}, du_t^{\varepsilon}) \end{aligned} \tag{4.E.25}$$

Then, by taking variance on both sides of equation (4.E.25) and dividing by $Var(du_t^{ss})$ it is obtained that:

$$1 = \frac{Cov(du_t^{ss}, du_t^{s})}{Var(du_t^{ss})} + \frac{Cov(du_t^{ss}, du_t^{f})}{Var(du_t^{ss})} + \frac{Cov(du_t^{ss}, du_t^{\varepsilon})}{Var(du_t^{ss})} \tag{4.E.26}$$

Following Fujita and Ramey (2006, 2007), the finance beta concept is used to define the unemployment variation total portion attributed to hazard rates and to an error term at business cycle frequency, as follows:

$$\beta^s = \frac{Cov(du_t^{ss}, du_t^s)}{Var(du_t^{ss})} \tag{4.E.27}$$

$$\beta^f = \frac{Cov(du_t^{ss}, du_t^f)}{Var(du_t^{ss})} \tag{4.E.28}$$

$$\beta^\varepsilon = \frac{Cov(du_t^{ss}, du_t^\varepsilon)}{Var(du_t^{ss})} \tag{4.E.29}$$

So by replacing the values of equation (4.E.27), (4.E.28) and (4.E.29) in equation (4.E.26) we have:

$$1 = \beta^s + \beta^f + \beta^\varepsilon \tag{4.E.30}$$

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5. Conclusion

This thesis contributes to the study of youth unemployment by applying direct search models and tries to answer if youth unemployment is a real economic problem and if the active employment policies in the labor market for young people tend to improve their position competitive. Dynamic work flows of juvenile people in the labor market is also studied. The models and empirical analysis are applied to the labor market in the United States.

In the chapter 2, through a competitive direct search model, this research has been able to model the patterns of unemployment by age in the United States for individuals aged 18 to 28 years old. Also the theoretical model developed sheds light on the, causes for high youth unemployment. High youth unemployment is jointly explained by the fact that they enter the labour market as unemployed and because they have to compete for a job position with more productive workers. The model is also successful in explaining the fact that as young people get older their employability situation improves, reducing their unemployment levels. Additionally, the model is able to predict roughly - as the stylized facts show - that by the 28 years of age unemployment tends to converge. The result of this research is interesting from the economic policy point of view. The model shows that the pattern observed in youth unemployment is the most efficient market allocation for the young people entering the labour market. Given the above, this research theoretically discards that active policies in the labour market for the young people, make the economy better, as instead generate a welfare loss. Ultimately, this research concludes that the high youth unemployment compared to adult unemployment does not represent a real economic problem, as it is in essence, a temporary problems following the natural transition between education and work.

In Chapter 3, this research uses a static model of direct search with heterogeneous agents to respond from, a theoretical perspective, whether the juvenile Active Labour Market

Policies (ALMPs) actually benefit those workers to whom the program is addressed to (the youths, in this research). At the same time, the model allows us to analyse if there are any labour substitution effect between young workers benefited by a subsidy and those low-skilled adult workers, who are not part of the programme. Presence of an eventual labour substitution effect has been neglected in the literature despite its relevance from a social-optimum macroeconomic perspective. To analyse the effectiveness of ALMPs this model will be calibrated to the US economy, so that to perform later numerical experiments that allow to assess the effects of the AMLPs; particularly the effect of training and hiring subsidies on employment levels for different types of workers and to find out if there are displacement effects. This research confirms that the Active Labour Market Policies (ALMP) which intend to improve the position of young people in the labour market do not necessarily increase employment overall as these policies produce a substitution effect between subsidized workers and those who are not benefited from it, as it happens to low-skilled adult workers. This research will also confirm that those young workers who may have access to the subsidy experience a decrease in their unemployment levels.

Finally, in the Chapter 4, after analysing the instantaneous flow separation and job creation in the U.S. in terms of correlation, volatility, comovement and their contribution to the unemployment volatility, a high heterogeneity of results arise when controlling by age and gender. This high heterogeneity found in the microdata compels us to avoid generalizations when trying to provide quantitative answers at the macroeconomic level to labour policy issues by using general equilibrium dynamic models that incorporate labour-type friction. As a result, we can conclude that regarding our target group (young people aged between 18 and 24) findings rates tends to be the main factor that would explain its unemployment dynamics, becoming a crucial factor to keep in mind when designing policies to reduce unemployment in this group. As mentioned earlier, this is not the case for prime-age population, where both factors (inflow and outflow) play an important role in explaining the unemployment dynamics.

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