UNIVERSITY of SOUTHAMPTON

FACULTY OF BUSINESS AND LAW

SOUTHAMPTON BUSINESS SCHOOL

Heterogeneous Location- and Pollution-Routing Problems

by

Çağrı Koç

Thesis for the degree of Doctor of Philosophy

September 2015

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Abstract

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This thesis introduces and studies new classes of heterogeneous vehicle routing problems with or without location and pollution considerations. It develops powerful evolutionary and adaptive large neighborhood search based metaheuristics capable of solving a wide variety of such problems with suitable enhancements, and provides several important managerial insights. It is structured into five main chapters. After the introduction presented in Chapter 1, Chapter 2 classifies and reviews the relevant literature on heterogeneous vehicle routing problems, and presents a comparative analysis of the available metaheuristic algorithms for these problems. Chapter 3 describes a hybrid evolutionary algorithm for four variants of heterogeneous fleet vehicle routing problems with time windows. The algorithm successfully combines several metaheuristics and introduces a number of new advanced efficient procedures. Extensive computational experiments on benchmark instances show that the algorithm is highly competitive with state-of-the art methods for the three variants. New benchmark results on the fourth problem are also presented. In Chapter 4, the thesis introduces the fleet size and mix location-routing problem with time windows (FSMLRPTW) which extends the classical location-routing problem by considering a heterogeneous fleet and time windows. The main objective of the FSMLRPTW is to minimize the sum of depot cost, vehicle fixed cost and routing cost. The thesis presents integer programming formulations for the FSMLRPTW, along with a family of valid inequalities and an algorithm based on adaptation of the hybrid evolutionary metaheuristic. The strengths of the formulations are evaluated with respect to their ability to yield optimal solutions. Extensive computational experiments on new benchmark instances show that the algorithm is highly effective. Chapter 5 introduces

the fleet size and mix pollution-routing problem (FSMPRP) which extends the previously studied pollution-routing problem (PRP) by considering a heterogeneous vehicle fleet. The main objective is to minimize the sum of vehicle fixed costs and routing cost, where the latter can be defined with respect to the cost of fuel and CO_2 emissions, and driver cost. An adaptation of the hybrid evolutionary algorithm is successfully applied to a large pool of realistic PRP and FSMPRP benchmark instances, where new best solutions are obtained for the former. Several analyses are conducted to shed light on the trade-offs between various performance indicators. The benefit of using a heterogeneous fleet over a homogeneous one is demonstrated. In Chapter 6, the thesis investigates the combined impact of depot location, fleet composition and routing decisions on vehicle emissions in urban freight distribution characterized by several speed limits, where goods need to be delivered from a depot to customers located in different speed zones. To solve the problem, an adaptive large neighborhood search algorithm is successfully applied to a large pool of new benchmark instances. Extensive analyses are conducted to quantify the effect of various problem parameters, such as depot cost and location, customer distribution and fleet composition on key performance indicators, including fuel consumption, emissions and operational costs. The results illustrate the benefits of locating depots located in suburban areas rather than in the city centre and of using a heterogeneous fleet over a homogeneous one. The conclusions, presented in Chapter 7, summarize the results of the thesis, provide limitations of this work, as well as future research directions.

Keywords. Operational research; combinatorial optimisation; logistics; city logistics; transportation; vehicle routing; location-routing; heterogeneous fleet; fleet size and mix; fuel consumption; CO_2 emissions; sustainability; evolutionary metaheuristic; adaptive large neighborhood search.

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Özet

FACULTY OF BUSINESS AND LAW
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Doktora, Yöneylem Araştırması

Heterojen Yer Seçimi- ve Çevre Kirliliği-Rotalama Problemleri

by Çağrı Koç

Bu çalışmada, heterojen araç rotalama problemlerinin yer seçimi ve çevre kirliliği özelliklerinin olduğu ve olmadığı yeni çeşitleri tanımlanmış, oldukça güçlü, etkili ve birçok problem çeşidini başarıyla çözebilen evrime dayalı ve uyarlanabilir büyük komşuluk arama metasezgiselleri geliştirilmiş, ve çeşitli idari bakış açıları sağlanmıştır. Çalışma beş ana bölümden oluşmaktadır. Birinci bölümde sunulan giriş kısmının ardından, ikinci bölümde heterojen araç rotalama problemleri ile ilgili literatür taranmış ve sınıflandırılmış, devamında bu problemler için literatürde önerilmiş metasezgisel algoritmalar karşılaştırılmıştır. Üçüncü bölümde, heterojen filolu ve zaman pencereli araç rotalama problemlerinin dört farklı çeşidi için evrime dayalı karma bir algoritma geliştirilmiştir. Önerilen algoritma çeşitli metasezgiselleri biraraya getirmekle birlikte yeni ve etkili yöntemler içermektedir. Test problemleri üzerinde gerçekleştirilen geniş kapsamlı deneysel çalışmalar, geliştirilen algoritmanin literatürde bu tür problemler için geliştirilen en etkili yöntemlerle oldukça sıkı bir şekilde rekabet edebildiğini göstermiştir. Dördüncü bölümde, bileşik yer seçimi ve rotalama problemleminin genelleştirilmiş bir çeşidi olan heterojen filolu ve zaman pencereli yer seçimi-rotalama problemi tanımlanmış ve incelenmiştir. Bu problemin temel amacı depo, araç ve rotalama maliyetleri toplamını enküçüklemektir. Problemin çözümü için, geçerli eşitsizliklerle kuvvetlendirilmiş tamsayılı programlama formülasyonları önerilmiş, ayrıca karma evrime dayalı algoritmanın bir başka çeşidi geliştirilmiştir. Önerilen formülasyonların etkinlikleri, eniyi çözüme ulaşma yetenekleri açısından deneyler ile değerlendirilmiştir. Yeni üretilen test problemleri üzerinde gerçekleştirilen geniş kapsamlı deneyler, geliştirilen metasezgisel algoritmanın oldukça başarılı olduğunu gostermiştir. Beşinci bölümde, bileşik çevre kirliliği ve rotalama probleminin genelleştirilmiş bir çeşidi olan heterojen filolu çevre kirliliği rotalama problemi tanımlanmıştır. Bu problemin temel amacı, araç, rotalama, yakıt, CO_2 salınımı ile sürücü maliyetleri toplamını enküçüklemektir. Problemin çözümü için evrime dayalı algoritmanin başka bir çeşidi geliştirilmiştir. Hem gözönüne alınan problem, hem de problemin homojen filolu çeşidi için, geniş kapsamlı gerçekçi test problemleri üzerinde deneyler gerçekleştirilmiştir. Deneyler sonucunda problemin homojen filolu çeşidi için literatürde varolan test problemleri üzerinde yeni en iyi çözüm değerleri elde edilmiştir. Ayrıca problem parametrelerinin çeşitli performans göstergeleri üzerindeki etkilerine ışık tutmak için ek analizler gerçekleştirilmiş, bu analizler sonucunda homojen araç filosu yerine heterojen araç filosu kullanımının faydaları açıkça ortaya konulmuştur. Altıncı bölümde, çeşitli hız bölgelerine ayrılmış olan şehiriçi yük taşımacılığındaki depo yerinin, araç filosunun ve rotalama kararlarının, araç CO_2 salınımı üzerindeki bütünleşik etkisi analiz edilmiştir. Problemde, ürünlerin şehir içinde bulunan depolardan, yine şehir içinde yer alan müşterilere ulaştırılması amaçlanmaktadır. Problemi çözmek için, uyarlanabilir bir büyük komşuluk arama metasezgiseli geliştirilmiş ve çeşitli yeni test problemleri üzerinde etkinliği incelenmiştir. Depo yeri ve maliyeti, müşteri dağılımı ve heterojen araç filosu gibi problem parametrelerinin, yakıt tüketimi, CO_2 salınımı ve operasyonel maliyetler gibi performans göstergeleri üzerindeki değişimlerinin etkisini analiz etmek için, geniş kapsamlı deneyler gerçekleştirilmiştir. Elde edilen sonuçlarda, depoların şehir içi yerine banliyölere yerleştirilmesinin ve homojen araç filosu yerine heterojen araç filosu kullanımının faydaları nümerik sonuçlarla gösterilmiştir. Yedinci bölümde, tez çalışmasında elde edilen sonuçlar kısaca özetlenmiş, çalışmanın sınırları ortaya konulmuş ve gelecek çalışmalar için çeşitli öneriler sunulmuştur.

Anahtar kelimeler. Yöneylem araştırması; kesikli eniyileme; lojistik; şehir lojistiği; ulaştırma; araç rotalama; yer seçimi-rotalama; heterojen filo; yakıt tüketimi; CO_2 salınımı; sürdürebilirlik; evrime dayalı metasezgisel; uyarlanabilir büyük komşuluk arama metasezgiseli.

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Declaration of Authorship

I, Çağrı Koç, declare that this thesis titled, 'Heterogeneous Location- and Pollution-Routing Problems' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed: **Çağrı Koç**

Date: **September 2015**

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Bu tez aileme, ülkeme ve Türk Milleti'ne ithaf edilmiştir.

This thesis is dedicated to my family, my country and the Turkish people.

List of Abbreviations

ACUTR	Average Cost Per Unit Removal Operator
ALNS	Adaptive Large Neighborhood Search
AWAP	Adaptive Weight Adjustment Procedure
CO_2	Carbon Dioxide
DBR	Demand-Based Removal Operator
DCR	Depot Closing Removal Operator
DDR	Depot Distance Removal Operator
DOR	Depot Opening Removal Operator
DR	Depot Removal Operator
FLP	Facility Location Problem
FD	Fleet Size and Mix Vehicle Routing Problem with Time Windows with Distance Objective
FT	Fleet Size and Mix Vehicle Routing Problem with Time Windows with En-route time Objective
FSMLRPTW	Fleet Size and Mix Location-Routing Problem with Time Windows
FSMPRP	Fleet Size and Mix Pollution-Routing Problem
FSMVRP	Fleet Size and Mix Vehicle Routing Problem
FSMVRPTW	Fleet Size and Mix Vehicle Routing Problem with Time Windows
GHG	Green House Gas
GIET	Greedy Insertion with En-route Time Operator
GINF	Greedy Insertion with Noise Function Operator
GR	Greedy Insertion Operator
GVWR	Gross Vehicle Weight Rating
HALNS	Heterogeneous Adaptive Large Neighborhood Search
HD	Heterogeneous Fixed Fleet Vehicle Routing Problem with Time Windows

	with Distance Objective
HEA	Hybrid Evolutionary Algorithm
HESA	Hybrid Evolutionary Search Algorithm
HFFVRP	Heterogeneous Fixed Fleet Vehicle Routing Problem
HFFVRPTW	Heterogeneous Fixed Fleet Vehicle Routing Problem with Time Windows
HT	Heterogeneous Fixed Fleet Vehicle Routing Problem with Time Windows with En-route time Objective
HVRP	Heterogeneous Vehicle Routing Problem
L-HALNS	Location-Heterogeneous Adaptive Large Neighborhood Search
LRP	Location-Routing Problem
NP-hard	Nod-deterministic Polynomial-time hard
NR	Neighborhood Removal Operator
PBR	Proximity-Based Removal Operator
PRP	Pollution-Routing Problem
P-L-HALNS	Pollution-and-Location-Heterogeneous Adaptive Large Neighborhood Search
RR	Random Removal Operator
SOA	Speed Optimization Algorithm
SR	Shaw Removal Operator
SSOA	Split Algorithm with Speed Optimization Algorithm
TBR	Time-Based Removal Operator
TSP	Traveling Salesman Routing Problem
VRP	Vehicle Routing Problem
VRPTW	Vehicle Routing Problem with Time Windows
WDR	Worst Distance Removal Operator
WTR	Worst Time Removal Operator

Chapter 1

Introduction

1.1 Context of the Research Problems

Logistics encompasses the flow of goods, information and funds between sources and end users, as well as the planning and execution of their movements and of the related support activities. The main objective of logistics is to coordinate these tasks so as to that meet customer requirements at minimum cost (Ghiani et al. 2013, UPS 2015). The design of distribution networks plays an important role for companies that vie to reduce costs and improve service quality.

In the past, the traditional logistics costs were defined in purely monetary terms. In recent years, however, concerns for the environment have emerged, as a result of which companies have been challenged to increasingly consider the external costs of logistics associated mainly with climate change, greenhouse gas (GHG) emissions and air pollution. GHG and in particular CO_2 emissions are the most concerning because they have direct consequences on human health, as well as indirect effects on the environment (Green 2014). Recent works such as those of McKinnon (2007) and Sbihi and Eglese (2007) suggest that there exist many opportunities for reducing CO_2 emissions in logistics and transportation, for example by extending the traditional objectives to account for the pollution cost.

In the context of freight transportation, city logistics poses challenges to governments, businesses, carriers, and citizens. It also requires collaboration mechanisms to build innovative partnerships and an understanding of the public sector and private businesses. Trade flows within cities exhibit a high variability, both in the size and shape of the shipments. Cities often possess a transportation infrastructure that allows traffic flows within their boundaries. However, for freight transportation this infrastructure is often inadequate, which results in congestion and pollution. For relevant references and more detailed information on city logistics, the reader is referred to the books of Taniguchi et al. (2001) and of Gonzalez-Feliu et al. (2014).

In today's business environment, public and private enterprises should optimize their planning decisions in order to manage the distribution processes more efficiently. Planning decisions are usually classified into three main levels: strategic (long term), tactical (medium term) and operational (short term) (see Crainic and Laporte 1997, Bektaş and Crainic 2008).

The design of the physical structure of the distribution networks lies at the strategic level of decision making. The relevant problems include deciding on the number and location of facilities, broadly referred to as the Facility Location Problem (FLP) (see Laporte et al. 2015), and the type and quantity of equipments to install at each facility, the capacity and type of lines, and so on (see Ghiani et al. 2013).

The operational level planning is mainly related to vehicle distribution and repositioning, crew scheduling, allocation of resources such as loads to vehicles, routing of vehicles for pickup and delivery activities. The classical Vehicle Routing Problem (VRP) is a central part of road transportation planning which aims at routing a fleet of vehicles on a given network to serve a set of customers under side constraints, where all tours start and end at a single depot. Minimizing the total distance traveled by all vehicles or minimizing the overall travel cost are some of the most commonly encountered VRP objectives. They usually are a linear function of the distance traversed. More than fifty years have passed since Dantzig and Ramser (1959) introduced the VRP. The literature on the VRP and its variants is considerably rich, see, e.g., the surveys by Cordeau et al. (2007) and Laporte (2009), as well as the books of Golden et al. (2008) and of Toth and Vigo (2014).

The integration of location and routing decisions dates back to the 1960s (see Von Boventer 1961, Webb 1961, Maranzana 1964). The classical FLP and the VRP are interrelated in several contexts. Location-Routing Problems (LRP) are combinations of these two major problems (see Laporte 1988, Min et al. 1998, Nagy and Salhi 2007, Prodhon and Prins 2014, Albareda-Sambola 2015, Drexl and Schneider 2015, for surveys).

Green issues are now receiving increasing attention in the VRP literature. Thus Bektaş and Laporte (2011) recently introduced an extension of the classical Vehicle Routing Problem with Time Windows, called the Pollution-Routing Problem (PRP). This problem consists of routing vehicles to serve a set of customers, and of determining their speed on each route segment to minimize a function comprising fuel cost, emissions and driver costs. The PRP assumes that in a vehicle trip all parameters will remain constant on a given arc, but load and speed may change from one arc to another. The PRP model approximates the total amount of energy consumed on a given road segment, which directly translates into fuel consumption and further into GHG emissions. For a further coverage of green issues at the operational level, the reader is referred to the book chapter of Eglese and Bektaş (2014) and to the surveys of Demir et al. (2014b) and Lin et al. (2014).

Lying at the interface between the strategic and operational levels, fleet dimensioning is a common tactical problem faced by industry. The trade-off between owning a fleet and subcontracting transportation activities is a key concern for most companies. Fleet dimensioning decisions are affected by several market variables such as transportation rates, transportation costs and expected demand. In most real-life distribution problems, customer demands are met with heterogeneous vehicle fleets. Companies should consider the structural characteristics of the vehicles in addition to the vehicle capacity when making decisions regarding fleet dimensioning and composition (Hoff et al. 2010). Two major heterogeneous fleet vehicle routing problems are the Fleet Size and Mix Vehicle Routing Problem introduced by Golden et al. (1984), which works with an unlimited heterogeneous fleet, and the Heterogeneous Fixed Fleet Vehicle Routing Problem introduced by Taillard (1999), which works with a known fleet. These two major problems are reviewed by Baldacci et al. (2008) and Baldacci et al. (2009). However, not much research has been carried out to address green concerns within fleet size and mix problems, apart from the work of Kopfer et al. (2014).

1.2 Illustration: The FedEx Global Distribution Network

We now provide a practical example to show the relevance of the problems studied in this thesis. FedEx is one of the world's largest express transportation companies, providing fast and reliable deliveries to more than 220 countries and territories in 2015 (see Figure 1.1). The company uses a global air-and-ground network to speed up the delivery of time-sensitive shipments, usually within two business days with a guaranteed delivery time. More than 160,000 employees work for the company worldwide. The average daily volume is approximately four million packages and 11 million pounds of freight. The company serves more than 375 airports with 650 heterogeneous aircraft, the delivery fleet includes more than 48,000 heterogeneous vehicles, the company owns 1,250 operating facilities and 12 air express hubs. In the context of green issues, FedEx has made significant gains in meeting its sustainability objectives. In 2012, the company achieved a 22% fuel efficiency improvement in the vehicle fleet since 2005, by using hybrid trucks, for example. FedEx has set several goals to reduce its carbon footprint.

These include reducing aircraft emissions by 30% by 2020 on an emissions per available-ton-mile basis, increasing vehicle efficiency by 30% by 2020, and getting 30% of its jet fuel from alternative fuels by 2030 (FedEx 2015).

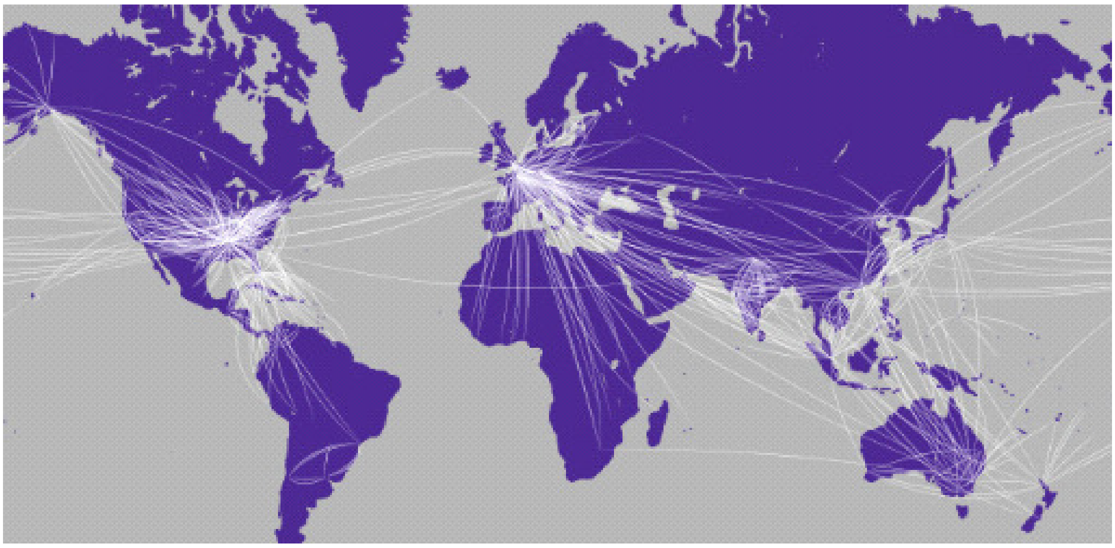


FIGURE 1.1: The FedEx global distribution network (FedEx 2015)

1.3 Context of the Methodology

Many successful and powerful metaheuristic optimization techniques have been developed for a variety of routing problems over the last decades (see Laporte et al. 2014). One of these includes evolutionary algorithms (EAs), also referred to as genetic algorithms, which are inspired from evolutionary mechanisms found in nature. EAs were introduced by Holland (1975). These algorithms incorporate natural selection mechanisms and basic genetic laws, elitist selection, crossover, education, mutation operators and diversification to evolve a population of individuals. Because of the population structure, traditional EAs have a tendency to converge slowly. To address this issue, various mechanisms, such as local search, have been devised and are generally used within EAs as education operators. EAs incorporating such additional mechanisms are sometimes called "genetic local searches" (Mühlenbein et al. 1988) or "memetic algorithms" (Moscato 1989, Moscato and Cotta 2010). Some of the enhanced evolutionary mechanisms have been shown to perform notably well on the VRP (Laporte et al. 2014) and several of its variants (see, e.g., Alba and Dorronsoro 2006, Nagata et al. 2010, Vidal et al. 2014).

Other successful optimization techniques proposed for routing problems are variations of local search algorithms, one of which is the large neighborhood search (LNS) algorithm introduced by Shaw (1998). LNS iteratively improves a solution by using both destroy and repair operators. Ropke and Pisinger (2006a) developed an extended LNS heuristic for a variant of the VRP, called adaptive large neighborhood search (ALNS), in which the LNS operators are combined within the algorithm and used with a frequency determined by their performance during the algorithm. The authors showed that such combined use of different local search operators yields a highly efficient method for the VRP. In a latter study, Ropke and Pisinger (2006b) developed an improved version of the ALNS algorithm. Finally, Pisinger and Ropke (2007) presented an unified ALNS framework to solve five different variants of the VRP. The authors indicate that this framework can be applied to many variants of routing problems.

This brief review shows that EAs and ALNS are the state-of-the-art methods for the VRP and its variants. With this motivation, our methodology is based on the combination of these two successful search paradigms.

1.4 General Research Contributions

The general research contributions of this thesis are threefold:

1. To analyze and investigate heterogeneous routing problems, to introduce new variants involving location aspects and environmental externalities, in particular pollution arising from fuel consumption.
2. To develop powerful metaheuristics capable of solving a wide variety of problems with appropriate enhancements.
3. To derive several managerial insights into the interaction of location, fleet composition and routing decisions on key performance indicators, both for long-haul transportation and city logistics.

The interactions between the three main themes of the thesis are depicted in Figure 1.2.

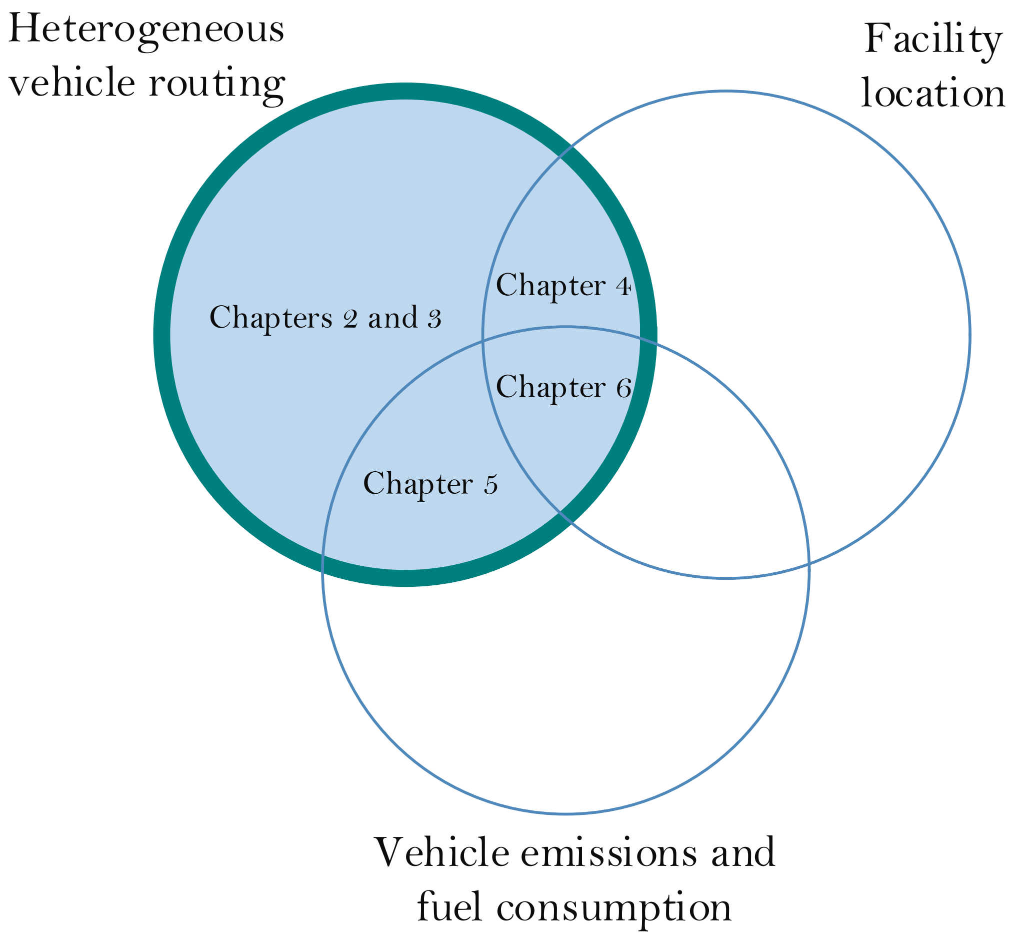


Figure 1.2: A schematic representation of the thesis structure

1.5 Specific Objectives

The remainder of this thesis is made up of five main chapters, followed by conclusions.

Here are the specific research objectives of Chapter 2 "Thirty Years of Heterogeneous Vehicle Routing":

- to classify heterogeneous vehicle routing problems,
- to present a comprehensive and up-to-date review of the existing studies by including industrial applications and case studies,
- to comparatively analyze the performance of the state-of-the-art metaheuristic algorithms.

Here are the specific research objectives of the Chapter 3 "A Hybrid Evolutionary Algorithm for Heterogeneous Fleet Vehicle Routing Problems with Time Windows":

- to review the latest developments on vehicle routing problems, and identify the state-of-the-art in solution techniques,
- to introduce several algorithmic improvements to existing techniques,
- to devise a Hybrid Evolutionary Algorithm (HEA) capable of solving the heterogeneous fleet vehicle routing problems with time windows in which various algorithmic components can be combined,
- to perform extensive computational experiments on benchmark instances.

Here are the specific research objectives of the Chapter 4 "The Fleet Size and Mix Location-Routing Problem with Time Windows: Formulations and a Heuristic Algorithm":

- to identify the latest developments on location-routing problems,
- to formulate the Fleet Size and Mix Location-Routing Problem with Time Windows (FSMLRPTW),
- to adapt the HEA for solving the FSMLRPTW,
- to perform extensive computational experiments.

Here are the specific research objectives of the Chapter 5 "The Fleet Size and Mix Pollution-Routing Problem":

- to identify functions for modelling fuel and CO_2 emissions for heterogeneous vehicle routing,
- to formulate the Fleet Size and Mix Pollution-Routing Problem (FSMPRP),
- to adapt the HEA for solving the FSMPRP,
- to perform analyses leading to managerial insights.

Here are the specific research objectives of the Chapter 6 "The impact of location, fleet composition and routing on emissions in urban freight distribution":

- to investigate the combined impact of depot location, fleet composition and routing decisions on vehicle emissions in urban freight distribution characterized by several speed limits,
- to devise a heuristic algorithm to solve the problem,
- to perform analyses in order to provide several insights.

Chapter 2

Thirty Years of Heterogeneous Vehicle Routing

Abstract

It has been around thirty years since the heterogeneous vehicle routing problem was introduced, and significant progress has since been made on this problem and its variants. The aim of this survey paper is to classify and review the literature on heterogeneous vehicle routing problems. The paper also presents a comparative analysis of the metaheuristic algorithms that have been proposed for these problems.

Keywords. vehicle routing; heterogeneous fleet; fleet size and mix; review

2.1 Introduction

In the classical Vehicle Routing Problem (VRP) introduced by Dantzig and Ramser (1959), the aim is to determine an optimal routing plan for a fleet of homogeneous vehicles to serve a set of customers, such that each vehicle route starts and ends at the depot, each customer is visited once by one vehicle, and some side constraints are satisfied. There exists a rich literature on the VRP and its variants, see, e.g., the surveys by Cordeau et al. (2007) and Laporte (2009), and the books by Golden et al. (2008) and Toth and Vigo (2014).

In most practical distribution problems, customer demands are served by means of a heterogeneous fleet of vehicles (see, e.g., Hoff et al. 2010, FedEx 2015, TNT 2015). Fleet dimensioning or composition is a common problem in industry and the trade-off between owning and keeping a fleet and subcontracting transportation is a challenging decision for companies. Fleet dimensioning decisions predominantly involve choosing the number and types of vehicles to be used, where the latter choice is often characterized by vehicle capacities. These decisions are affected by several market variables such as transportation rates, transportation costs and expected demand.

The extension of the VRP in which one must additionally decide on the fleet composition is known as the Heterogeneous Vehicle Routing Problem (HVRP). HVRPs are rooted in the seminal paper of Golden et al. (1984) published some thirty years ago and have recently evolved into a rich research area. There have also been several classifications of the associated literature from different perspectives. Baldacci et al. (2008) provided a general overview of papers with a particular focus on lower bounding techniques and heuristics. The authors also compared the performance of existing heuristics described until 2008 on benchmark instances. Baldacci et al. (2010a) presented a review of exact algorithms and a comparison of their computational performance on the capacitated VRP and HVRPs, while Hoff et al. (2010) reviewed several industrial aspects of combined fleet composition and routing in maritime and road-based transportation. More recently, Irnich et al. (2014) briefly reviewed papers on HVRPs published from 2008 to 2014.

This paper makes three main contributions. The first is to classify heterogeneous vehicle routing problems. The second is to present a comprehensive and up-to-date review of the existing studies. The third is to comparatively analyze the performance of the state-of-the-art metaheuristic algorithms. Our review differs from the previous ones by including references that have appeared since 2008, by comparing heuristic algorithms, and by including industrial applications and case studies.

The remainder of this paper is structured as follows. The HVRPs and its variants are described and classified in Section 2.2. Extended reviews of the three main problem types, namely the Fleet Size and Mix Vehicle Routing Problem, the Heterogeneous Fixed Fleet Vehicle Routing Problem and the Fleet Size and Mix Vehicle Routing Problem with Time Windows are presented in Sections 2.3, 2.4 and 2.5, respectively. Reviews of the other variants, extensions and case studies are presented in Sections 2.6 and 2.7. A tabulated summary of the literature and comparisons of the state-of-the-art heuristic algorithms are provided in Section 2.8. The paper closes with some concluding remarks and future research directions in Section 2.9.

2.2 Classification of the Heterogeneous Vehicle Routing Problem

We first define and classify the variants of HVRPs in Section 2.2.1, and then present three mathematical formulations in Section 2.2.2.

2.2.1 Problem definition and classification

HVRPs generally consider a limited or an unlimited fleet of capacitated vehicles, where each vehicle has a fixed cost, in order to serve a set of customers with known demands.

These problems consist of determining the fleet composition and vehicle routes, such that the classical VRP constraints are satisfied. Two major HVRPs are the *Fleet Size and Mix Vehicle Routing Problem* (FSM[1]) introduced by Golden et al. (1984) which works with an unlimited heterogeneous fleet, and the *Heterogeneous Fixed Fleet Vehicle Routing Problem* (HF) introduced by Taillard (1999) in which the fleet is predetermined. Other variants of the FSM and the HF also exist. In what follows, we will classify the main variants with respect to two criteria: (i) objectives and (ii) presence or absence of time window constraints. We will also mention other HVRP variants and extensions.

2.2.1.1 Objectives

The objective of both the FSM and the HF is to minimize a total cost function which includes fixed (F) and variable (V) vehicle costs. We now differentiate between five important variants: 1) the FSM with fixed and variable vehicle costs, denoted by FSM(F,V), introduced by Ferland and Michelon (1988); 2) the FSM with fixed vehicle costs only, denoted FSM(F), introduced by Golden et al. (1984); 3) the FSM with variable vehicle costs only, denoted by FSM(V), introduced by Taillard (1999); 4) the HF with fixed and variable vehicle costs, denoted by HF(F,V), introduced by Li et al. (2007); 5) the HF with variable vehicle costs only, denoted by HF(V), introduced by Taillard (1999).

2.2.1.2 Time windows

Two natural extensions of the FSM and HF arise when time window constraints are imposed on the start of service at each customer location. These problems are denoted by FSMTW and HFTW, respectively. In these extensions, two measures are used to compute the total cost to be minimized: 1) The first is based on the en-route time (T) which is the sum of the fixed vehicle cost and the trip duration but excludes the service time. In this case, service times are used only to check route feasibility and for performing adjustments to the departure time from the depot in order to minimize pre-service waiting times; 2) The second cost measure is based on distance (D) and consists

[1] Traditionally, the Fleet Size and Mix Vehicle Routing Problem has been abbreviated as FSMVRP, and its counterpart with time windows as FSMVRPTW. A similar convention has been adopted for the Heterogeneous Fixed Fleet Vehicle Routing Problem, by using HFFVRP and HFFVRPTW to denote its versions without and with time windows, respectively. In our view, some of these abbreviations are excessively long and defy the purpose of using shorthand notation. Hence we introduce shorter and simpler abbreviations in this paper.

of the fixed vehicle cost and the distance traveled by the vehicle, as is the case in the standard VRP with Time Windows (VRPTW) (Solomon 1987).

The FSM and HF, combined with the two objectives above, give rise to four problem types: 1) the FSMTW with objective T, denoted by FSMTW(T), introduced by Liu and Shen (1999b); 2) the FSMTW with objective D, denoted by FSMTW(D), introduced by Bräysy et al. (2008); 3) the HFTW with objective T, denoted by HFTW(T), introduced by Paraskevopoulos et al. (2008); 4) the HFTW with objective D, denoted by HFTW(D), recently introduced by Koç et al. (2015).

2.2.1.3 Other variants

More involved variants of the FSM or of the HF have been defined, including those with multiple depots (see Dondo and Cerda 2007, Bettinelli et al. 2011, 2014). Other extensions include stochastic demand (Teodorović et al. 1995), pickups and deliveries (Irnich 2000, Qu and Bard 2014), multi-trips (Prins 2002, Seixas and Mendes 2013), the use of external carriers (Chu 2005, Potvin and Naud 2011), backhauls (Belmecheri et al. 2013, Salhi et al. 2013), open routes (Li et al. 2012), overloads (Kritikos and Ioannou 2013), site-dependencies (Nag et al. 1988, Chao et al. 1999), multi-vehicle task assignment (Franceschelli et al. 2013), green routing (Juan et al. 2014, Koç et al. 2014), single and double container loads (Lai et al. 2013), two-dimensional loading (Leung et al. 2013, Dominguez et al. 2014), time-dependencies (Afshar-Nadjafi and Afshar-Nadjafi 2014), multi-compartments (Wang et al. 2014), multiple stacks (Iori and Riera-Ledesma 2015) and collection depot (Yao et al. 2015).

2.2.2 Mathematical formulations

We now present three formulations for the HVRP, two based on commodity flows and one based on set partitioning. The common notations of all three formulations are as follows. Each customer i has a non-negative demand q_i. Let $\mathcal{H} = \{1, \ldots, k\}$ be the set of available vehicle types. Let t^h and Q_h denote the fixed vehicle cost and the capacity of vehicle of type $h \in \mathcal{H}$, respectively. Let m_h be the available number of vehicles of type h.

2.2.2.1 Single-commodity flow formulation

The HVRP is modeled on a complete graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N} = \{0, \ldots, n\}$ is the set of nodes, node 0 corresponds to the depot, and $\mathcal{A} = \{(i,j) : 0 \leq i, j \leq n, i \neq j\}$ denote the set of arcs. The customer set is $\mathcal{N}_c = \mathcal{N} \backslash \{0\}$. Let c_{ij}^h be the travel cost on arc $(i,j) \in \mathcal{A}$ by a vehicle of type h. Furthermore, let f_{ij}^h be the amount of commodity transported on arc $(i,j) \in \mathcal{A}$ by a vehicle of type h and let the binary variable x_{ij}^h be equal to 1 if and only if a vehicle of type $h \in \mathcal{H}$ travels on arc $(i,j) \in \mathcal{A}$.

The single-commodity flow formulation of Baldacci et al. (2008) for the HVRP is as follows:

$$\text{Minimize} \quad \sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}_c} t^h x_{0j}^h + \sum_{h \in \mathcal{H}} \sum_{(i,j) \in \mathcal{A}} c_{ij}^h x_{ij}^h \tag{2.1}$$

$$\text{subject to} \quad \sum_{j \in \mathcal{N}_c} x_{0j}^h \leq m_h \qquad h \in \mathcal{H} \tag{2.2}$$

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} x_{ij}^h = 1 \qquad i \in \mathcal{N}_c \tag{2.3}$$

$$\sum_{h \in \mathcal{H}} \sum_{i \in \mathcal{N}} x_{ij}^h = 1 \qquad j \in \mathcal{N}_c \tag{2.4}$$

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} f_{ji}^h - \sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} f_{ij}^h = q_i \qquad i \in \mathcal{N}_c \tag{2.5}$$

$$q_j x_{ij}^h \leq f_{ij}^h \leq (Q_h - q_i) x_{ij}^h \qquad (i,j) \in \mathcal{A}, h \in \mathcal{H} \tag{2.6}$$

$$x_{ij}^h \in \{0,1\} \qquad (i,j) \in \mathcal{A}, h \in \mathcal{H} \tag{2.7}$$

$$f_{ij}^h \geq 0 \qquad (i,j) \in \mathcal{A}, h \in \mathcal{H}. \tag{2.8}$$

In this formulation, the objective function (2.1) minimizes the sum of vehicle fixed costs and the total travel cost. The maximum number of available vehicles of each type is imposed by constraints (2.2). In the case of the FSM, an unlimited number of vehicles for each vehicle type h ($m_h = |\mathcal{N}_c|$) are available, which effectively renders constraints (2.2) redundant. Constraints (2.3) and (2.4) ensure that each customer is visited exactly once. Constraints (2.5) and (2.6) define the commodity flows. Finally, constraints (2.7) and (2.8) enforce the integrality and non-negativity restrictions on the variables.

2.2.2.2 Two-commodity flow formulation

In the two-commodity flow formulation of Baldacci et al. (2009) for the FSM(F), the vehicle types are undominated and ordered so that $Q_1 < Q_2 < \ldots < Q_k$ and $t^1 < t^2 < \ldots < t^k$. An undirected complete graph $\hat{G} = (\hat{V}, \hat{E})$ is given, where $\hat{V} = \{0, 1, \ldots, n\}$ is the set of $n+1$ nodes and $\hat{E}$ is the set of edges. The node set $V = \hat{V} \backslash \{0\}$ includes the n customers and node 0 represents the depot. Each edge $(i, j) \in \hat{E}$ is associated with a non-negative symmetric routing cost, c_{ij}. Let $G = (V', E)$ be an undirected complete graph constructed from $\hat{G}$ as follows. The node set V' includes the set of customer nodes V and $h+1$ copies of the depot node: h origin depots, one for each vehicle type, and a common destination depot. In particular, $V' = V \cup K \cup \{n+k+1\}$, where $K = \{n+1, n+2, \ldots, n+k\}$ is the set of origin depots, and node $n' = n+k+1$ is the destination depot. Let $\pi(i) = i - n, (i \in K)$, be the vehicle type associated to node i. It is assumed that the cost matrix d_{ij} is symmetric and that $q_i = 0$ $(i \in K \cup \{n'\})$. The cost of edges d_{ij} in E is defined as follows: 1) $d_{ij} = t^{\pi(j)} + c_{0i}$, for $q_i \leq Q_{\pi(j)}, j \in K, i \in V$; 2) $d_{ij} = c_{ij}$, for $q_i + q_j \leq Q_h, i, j \in V, i < j$; 3) $d_{in'} = c_{0i}$, for $i \in V$; 4) $d_{ij} = \infty$, otherwise.

Two flow variables y_{ij} and y_{ji} are associated with each edge $(i, j) \in E$. The flow variables y_{ij} represent the vehicle load. The flow $y_{ji} = Q_k - y_{ij}$ represents the empty space on a vehicle of the largest type. The empty space on the vehicle of type h is represented by $y_{ji} - (Q_k - Q_h)$. Furthermore, for each edge $(i, j) \in E$, let x_{ij} be a binary variable, equal to 1 if and only edge (i, j) is in the solution. In addition, let $\mathcal{S} = \{S : S \subseteq V, |S| \geq 2\}$. Given a set $S \in \mathcal{S}$, let $\delta(S)$ be the cutset defined by S (i.e., $\delta(S) = \{(i, j) \in E : i \in S, j \notin S \text{ or } i \notin S, j \in S\}$). Also, let $q(S) = \sum_{i \in S} q_i$ be the total demand of customers in S.

We now formally present the two-commodity flow formulation for the FSM(F):

$$\text{Minimize} \quad \sum_{(i,j) \in E} d_{ij} x_{ij} \tag{2.9}$$

$$\text{subject to} \quad \sum_{j \in V'} (y_{ji} - y_{ij}) = 2q_i \qquad i \in V \tag{2.10}$$

$$\sum_{i \in K} \sum_{j \in V} y_{ij} = q(V) \tag{2.11}$$

$$\sum_{j \in V} y_{jn'} = 0 \tag{2.12}$$

$$\sum_{\{i,j\} \in \delta(b)} x_{ij} = 2 \qquad \forall b \in V \tag{2.13}$$

$$\sum_{i \in K} \sum_{j \in V} x_{ij} = \sum_{j \in V} x_{jn'} \tag{2.14}$$

$$y_{ij} + y_{ji} = Q_k x_{ij} \qquad (i,j) \in E \tag{2.15}$$

$$\sum_{\{i,j\} \in \delta(S)} x_{ij} \geq \sum_{i \in K} \sum_{j \in V} x_{ij} \qquad K \subset S, S \subseteq K \cup V \tag{2.16}$$

$$y_{ij} \leq Q_{\pi(i)} \qquad i \in K, j \in V \tag{2.17}$$

$$y_{ij} \geq 0, y_{ji} \geq 0 \qquad (i,j) \in E \tag{2.18}$$

$$x_{ij} \in \{0,1\} \qquad (i,j) \in E. \tag{2.19}$$

Constraints (2.10)–(2.12) and (2.18) define a feasible flow pattern. Constraints (2.12) guarantee that the inflow at node n' is equal to 0. Constraints (2.13) ensure that any feasible solution must contain two edges incident to each customer. Constraints (2.14) impose that if $p = \sum_{i \in K} \sum_{j \in V} x_{ij}$ vehicles leave node set K, then exactly p vehicles must enter node n'. Constraints (2.15) define the relation among variables in a feasible solution. Constraints (2.16) forbid the presence of simple paths starting and ending at nodes in K. The capacity requirements for each vehicle are imposed by constraints (2.17). Finally, constraints (2.18) and (2.19) enforce the integrality and non-negativity restrictions on the variables.

2.2.2.3 Set partitioning formulation

The set partitioning formulation of Baldacci and Mingozzi (2009) works with an undirected graph $G = (V', E)$, where $V' = \{0, \dots, n\}$ is the set of $n + 1$ nodes and E is the set of edges. Node 0 represents the depot and node set $V = V' \backslash \{0\}$ corresponds to n customers. A route $R = (0, i_1, \dots, i_r, 0)$ performed by a vehicle of type h, is a simple cycle in G passing through the depot and customers $\{i_1, \dots, i_r\} \subseteq V$, with $r \geq 1$. Let $\Re^h$ be the index set of all feasible routes of vehicle type $h \in \mathcal{H}$ and let $\Re = \bigcup_{h \in \mathcal{H}} \Re^h$. For each route $\ell \in \Re^h$ is associated a routing cost c^h_ℓ. Let $\Re^h_i \subset \Re^h$ be the index subset of the routes of a vehicle of type h covering customer $i \in V$. Let R^h_ℓ be the subset of customers visited by route $\ell \in \Re^h$. Furthermore, let y^h_ℓ be a binary variable that is equal to 1 if and only if route $\ell \in \Re^h$ is chosen in the solution.

We now formally present the set partitioning formulation:

$$\text{Minimize} \qquad \sum_{h\in\mathcal{H}}\sum_{\ell\in\Re^h}(t^h + c^h_\ell)y^h_\ell \tag{2.20}$$

$$\text{subject to} \qquad \sum_{h\in\mathcal{H}}\sum_{\ell\in\Re^h_i} y^h_\ell = 1 \qquad i \in V \tag{2.21}$$

$$\sum_{\ell\in\Re^h} y^h_\ell \leq m_h \qquad h \in \mathcal{H} \tag{2.22}$$

$$y^h_\ell \in \{0,1\} \qquad \ell \in \Re^h, h \in \mathcal{H}. \tag{2.23}$$

In this formulation, the objective function (2.20) minimizes the sum of all vehicle fixed costs and total routing cost. Constraints (2.21) specify that each customer $i \in \mathcal{N}_c$ must be covered once by one route. Constraints (2.22) impose the upper bound on the number of vehicles of each type that can be used ($m_h = |V|$). Finally, constraints (2.23) enforce the integrality restrictions on the variables.

2.3 The Fleet Size and Mix Vehicle Routing Problem

This section reviews the standard FSM and unifies the studies pertaining both the FSM and the HF. We first review lower bound and exact algorithms in Section 2.3.1, then continuous approximation models in Section 2.3.2, and finally heuristics in Section 2.3.3.

2.3.1 Lower bounds and exact algorithms

Several studies describe lower bounds and exact algorithms for the FSM. Yaman (2006) developed formulations and valid inequalities for this problem and proposed formulations, four of which are based on the Miller-Tucker-Zemlin (1960) subtour elimination constraints for the Travelling Salesman Problem (TSP), and two are based on commodity flows. The author compared the linear programming bounds of these formulations, derived valid inequalities and lifted several constraints to improve the lower bounds. Her results revealed that the solutions obtained from the strongest formulations were of good quality, and yielded a maximum optimality gap of 3.28%. Baldacci et al. (2009) later described a mixed integer programming formulation based on two-commodity flows and developed two new classes of valid inequalities for the FSM. These inequalities, which were new covering-type and fleet-dependent capacity inequalities, aimed to increase the lower bounds. The authors showed that their model was quite compact when compared with previous formulations, and that its linear relaxation had a reasonable quality. Fleet-dependent capacity inequalities were able to improve the lower bound by about 5% on average, and the new covering inequalities improved it by about 2.5%. Pessoa et al. (2009) presented a robust branch-cut-and-price algorithm for the FSM. Q-routes were associated with the columns, which are relaxations of capacitated elementary routes that make the pricing problem solvable in pseudo-polynomial time. These authors also proposed new families of cuts which were expressed on a large set of variables and did not increase the complexity of the pricing subproblem. The results showed that instances up to 75 nodes can be solved to optimality, a significant improvement with respect to previous exact algorithms.

Three unified exact algorithms are available to solve both the FSM and the HF. Choi and Tcha (2007) developed a column generation algorithm and solved its linear programming relaxation by column generation. They modified several dynamic programming algorithms for the classical VRP to efficiently generate feasible columns and then applied a branch-and-bound procedure to obtain an integer solution. Their results confirmed the superiority of this method over existing algorithms, both in terms of solution quality and computation time. Baldacci and Mingozzi (2009) later introduced a unified exact algorithm based on the set partitioning formulation. Three types of bounding procedures were used, based on the LP-relaxation and on Lagrangean relaxation. The new lower bounds were tighter than all previously known lower bounds. The last exact algorithm for the FSM and the HF was, to our knowledge, presented by Baldacci et al. (2010b). It combines several dual ascent procedures to generate a near-optimal dual solution of the set partitioning model and it adds valid inequalities to the set partitioning formulation within a column-and-cut generation algorithm to close the integrality gap left by the dual ascent procedures. The final dual solution is then defined to generate a reduced problem containing all optimal integer solutions. This algorithm outperformed all other available exact algorithms.

2.3.2 Continuous approximation models

Jabali et al. (2012a) developed a continuous approximation model for the FSM. Their model builds upon the work of Daganzo (1984a,b) and of Newell et al. (1986), where the latter introduced a continuous approximation model for the VRP. This model can be used at an aggregate level to analyze capacity scenarios and various cost scenarios. The authors incorporated mixed fleet considerations to the model of Newell et al. (1986) in which the vehicle routes are based on a partition of a ring-radial region into zones, each of which is serviced by a single vehicle. They presented a mixed integer non-linear formulation and developed several upper and lower bounding procedures for it. The performance of the model was tested on several instances. Computational results showed that the two proposed upper bounding procedures were more reliable than solving the original model by an off-the-shelf software. They also demonstrated the sensitivity of the models with respect to several parameters such as the vehicle variable and fixed costs, the route duration limit and customer density.

2.3.3 Heuristics

This section presents a review of heuristic methods for the FSM. We first review population search heuristics in Section 2.3.3.1, then tabu search heuristics in Section 2.3.3.2, and finally other heuristics in Section 2.3.3.3.

2.3.3.1 Population search heuristics

In contrast to the VRP, only a few population search heuristics have been developed for the FSM. Ochi et al. (1998a) described a hybrid metaheuristic which integrates genetic algorithms and scatter search with a decomposition-into-petals procedure. Ochi et al. (1998b) later used the same idea within parallel genetic algorithms. Several results of Taillard (1999) were improved with this method. However, Ochi et al. (1998a,b) did not report the exact solution values. Lima et al. (2004) proposed a memetic algorithm which is a hybrid of a genetic algorithm and of the simulated annealing heuristic of Osman (1993), and was able to find eight new best-known solutions for the Golden et al. (1984) instances. Another genetic algorithm was developed by Liu et al. (2009) who used several heuristics to generate the initial solution. Out of the 20 instances of Golden et al. (1984), 14 solutions were matched and one was improved when compared with existing algorithms such as those of Brandão et al. (2009) and Choi and Tcha (2007).

Several population heuristics for the FSM and the HF are based on variants of the split procedure of Prins (2004). Prins (2009) developed two memetic algorithms hybridized with a local search, based on chromosomes encoded as giant tours and without trip delimiters. The methods optimally splits giant tours into feasible routes and assigns a suitable vehicle type to them. The method creates new solutions from a single solution at each iteration by performing mutations and local search operations. The results revealed that the proposed method was able to efficiently handle the problems. The authors also generated a set of HF instances based on real distances from French counties and ranging from 50 to more than 250 customers. In a recent study, Vidal et al. (2014) introduced a genetic algorithm using a unified component based solution framework for different variants of the VRPs, including the FSM, the FSMTW(T) and the FSMTW(D). The authors used problem-independent local search operators such as crossover, split and a number of diversification mechanisms. A unified route-evaluation methodology was developed to increase the effectiveness of the local search. This methodology is primarily based on two procedures: move evaluations as a concatenation of known subsequences and information preprocessing on subsequences, as well as other well-known procedures. Excellent results were obtained on the FSM, the FSMTW(T) and the FSMTW(D), which will be presented in more detail in Section 2.8.2.3.

2.3.3.2 Tabu search heuristics

The first tabu search heuristic for the FSM is probably that of Osman and Salhi (1996) who modified the route perturbation procedure of Salhi and Rand (1993). The existing results for the benchmark instances were improved with the proposed method. Gendreau et al. (1999) later developed a tabu search heuristic which embedded the generalized insertion heuristic of Gendreau et al. (1992) and the adaptive memory procedure of Rochat and Taillard (1995). Their results were compared with those of Taillard (1999) and confirmed the superiority of their algorithm. Wassan and Osman (2002) presented a reactive tabu search heuristic in which several neighborhoods and special data structures were integrated and contained an intensification phase to trigger switches between simple moves. Several deterministic moves were introduced to diversify the search. The authors also proposed special data structures to explore various neighbourhoods. The method was capable of generating a number of best-known solutions. Another tabu search heuristic was developed by Lee et al. (2008) which applied a modified sweeping method with set partitioning on a giant tour was used to create initial solutions. An optimal vehicle assigment was performed for the set of routes, whenever the algorithm identified a new solution. Competitive results were obtained on the Golden et al. (1984) instances and several new best-known solutions were found. Finally, Brandão et al. (2009) proposed a tabu search algorithm in which three procedures are used to generate the initial solutions, and three moves are defined for the neighborhoods: single insertion, double insertion and swap. The algorithm also used intensification and diversification procedures during the search. The proposed method was able to obtain high quality solutions, including five new best-known solutions.

2.3.3.3 Other heuristics

Several versions of constructive heuristics and many other heuristics have been proposed for the FSM over the last thirty years. Golden et al. (1984) formally described and formulated the FSM. They also developed some heuristics based on the Clarke and Wright (1964) savings algorithm and on the partitioning of a giant tour into routes suitable for various vehicle types, using the Or-opt (1976) improvement mechanism for the TSP. They also described a procedure to calculate a lower bound. They applied the Fisher and Jaikumar (1981) heuristic to solve the generalized assignment problem for the assignment of customers to vehicles. Gheysens et al. (1984) used the Golden et al. (1984) lower bounding procedure to create a new heuristic. Their method first generated a fleet mix and then solved the resulting problem as a VRP. In a later study, Gheysens et al. (1986) showed that the proposed lower bound based heuristic of Gheysens et al. (1984), performed in general better than the heuristics of Golden et al. (1984). However, computation times were much larger and finding a feasible solution was not guaranteed. Desrochers and Verhoog (1991) developed an improved savings heuristic which is a matching based savings algorithm using successive route merging procedures. The method selects the best solution by solving a weighted matching problem at each iteration. Competitive results were obtained with respect to previous studies. Salhi et al. (1992) presented a mathematical formulation and described a perturbation based heuristic which was tested on 20 benchmark instances and yielded several best-known solutions. Salhi and Rand (1993) described a more advanced constructive heuristic which starts from a solution obtained by solving a VRP with a single vehicle capacity, selected among the available ones. Several procedures are then iteratively applied to improve it which is achieved by changing the vehicle type assigned to each route, merging or removing routes and moving customers from one route to another. On average, the proposed method performed better than the earlier algorithms. Renaud and Boctor (2002) proposed a sweep-based heuristic which extended the work of Renaud et al. (1996). The algorithm first creates a large number of routes that can be served by one or two vehicles and a set partitioning problem is then solved optimally, in polynomial time, to select the routes and vehicles to use. The method outperformed the existing algorithms and yielded competitive results with respect to tabu search. Han and Cho (2002) presented another constructive heuristic algorithm which uses generic intensification and diversification procedures. The method incorporates several mechanisms from deterministic variants of simulated annealing like threshold accepting and the great deluge algorithm (Dueck 1993). The method performed well on the Golden et al. (1984) small-size instances, but was dominated by the heuristics of Taillard (1999) and Gendreau et al. (1999) on the large-size instances.

The FSM and the HF are simultaneously considered in several papers. The earliest such work is by Taillard (1999) who developed a heuristic column generation algorithm. His method solved a homogeneous VRP by means of the Rochat and Taillard (1995) adaptive memory procedure for each of the vehicle type, where it was assumed that the number of available vehicles is unlimited. This heuristic outperformed that of Osman and Salhi (1996) on the eight largest FSM instances. The method was also tested for the HF on new benchmark instances. Imran et al. (2009) later adapted a variable neighborhood search algorithm for the FSM and the HF. Several additional features were added to the method: an adaptation of local search procedures including Dijkstra's algorithm, a diversification procedure, and the use of a dummy empty route during the search. This heuristic yielded competitive results on benchmark instances and was able to find several new best-knowns solutions. A hybrid algorithm that considered both problems was later proposed by Subramanian et al. (2012). It includes an iterated local search (ILS)-based heuristic to generate columns in a set partitioning formulation. Competitive results and new best-known solutions were obtained on benchmark instances which include large-size instances involving up to 360 customers. The same authors presented improved results by integrating the ILS with a variable neighborhood descent procedure and with a random neighborhood ordering scheme in the local search phase (Penna et al. 2013). The performance of the method was tested on 52 benchmark instances with up to 100 customers. Four new best-known solutions were obtained and 42 best-known results were matched.

2.4 The Heterogeneous Fixed Fleet Vehicle Routing Problem

To our knowledge, no exact algorithm has specifically been designed for the standard HF. However, several exact algorithms jointly consider the FSM and the HF. We therefore focus exclusively on heuristics described for the HF. We first review tabu search heuristics in Section 2.4.1, and then other heuristics in Section 2.4.2.

2.4.1 Tabu search heuristics

Euchi and Chabchoub (2010) designed a hybrid tabu search embedded within an adaptive memory heuristic for the HF. This algorithm generates three initial solutions, and at each iteration the current solution is improved by several constructive methods. The results obtained on benchmark instances were competitive in terms of solution quality and computation time. Another tabu search algorithm was proposed by Brandão et al. (2011). The algorithm is initiated with a giant tour over all customers which is then partitioned into routes that are later improved using four types of moves. Four new best-known solutions were obtained on benchmark instances.

2.4.2 Other heuristics

Tarantilis et al. (2003) proposed a list-based threshold accepting metaheuristic for the HF which explores the solution space to identify promising regions. The method was competitive on benchmark instances and could find several new best-known solutions. In a later study, the same authors developed a backtracking adaptive threshold accepting algorithm (Tarantilis et al. 2004), which generalizes that of Tarantilis and Kiranoudis (2001). The main difference between this method and the standard threshold accepting heuristic is that the value of the threshold does not always decrease but can also increase. New best solutions and better results were obtained compared with Taillard (1999). The heuristic of Gencer et al. (2006) is based on the principle of first clustering and then routing and considers the possibility of leasing vehicles when the size of the fleet is insufficient. On average, the algorithm provided lower quality solutions than that Tarantilis et al. (2004) but found better solutions in terms of vehicle capacity utilization. Li et al. (2007) adapted their previous record-to-record travel algorithm (Li et al. 2005) to solve the HF. The algorithm is a deterministic variant of a simulated annealing heuristic and produced new best-known solutions. Li et al. (2010) later developed a multistart adaptive memory programming and path relinking heuristic. This algorithm constructs multiple provisional solutions which are then improved through a modified tabu search at each iteration. New best-known solutions were found on two benchmark instances, and for the others the method found solutions of a quality comparable to that of previous algorithms. Liu (2013) developed a hybrid population heuristic with embedded local search mechanisms to diversify the population. Competitive results were obtained within short computation times. Naji-Azimi and Salari (2013) solved the HF by developing a mathematical formulation based heuristic algorithm. The method applies a mechanism in which the initial solution is destroyed and repaired by solving a mathematical model to optimality. Three new best-known solutions were obtained on bencmark instances.

2.5 The fleet size and mix vehicle routing problem with time windows

We now review the existing literature on the FSMTW. This variant of the FSM has received considerable attention, which is the reason why it is presented in a separate section. To the best of our knowledge, apart from a simple branch-and-bound scheme (Ferland and Michelon 1988), no exact algorithm has yet been proposed for the standard FSMTW. We first review tabu search heuristics in Section 2.5.1, followed by other heuristics in Section 2.5.2.

2.5.1 Tabu search heuristics

Paraskevopoulos et al. (2008) developed a two-phase heuristic based on a hybridized tabu search algorithm for the FSMTW(T) and the HFTW(T). In the first phase, initial solutions are generated by a semi-parallel construction heuristic which is followed by a sophisticated ejection chain procedure in the second phase. The quality of the solutions is further improved using variable neighborhood tabu search. To diversify the solutions, the authors describe a specialized shaking mechanism. Computational experiments conducted on the FSMTW benchmark data sets allowed the identification of better solutions than those reported by Dell'Amico et al. (2007), by about 3.4% on average. New benchmark results for the HFTW(T) were also presented for the first time.

2.5.2 Other heuristics

Ferland and Michelon (1988) showed that the VRPTW can be extended to the heterogeneous VRPTW. They presented three heuristic algorithms: discrete approximation, assignment and matching, as well as two simple branch-and-bound procedures. Liu and Shen (1999b) described a heuristic for the FSMTW(T) which starts by determining an initial solution through an adaptation of the Clarke and Wright (1964) savings algorithm previously presented by Golden et al. (1984). The second stage improves the initial solution by moving customers by means of parallel insertions. The algorithm was tested on a set of 168 FSMTW benchmark instances derived from the set of Solomon (1987) for the VRPTW and was also tested on the standard FSM. Dullaert et al. (2002) described a sequential construction algorithm for the FSMTW(T). The algorithm includes three insertion-based heuristics which are extensions of the I1 heuristic of Solomon (1987) and of the Golden et al. (1984) vehicle insertion saving method. Another paper on the FSMTW(T) is that of Dell'Amico et al. (2007) who developed a multi-start parallel regret construction heuristic embedded within a ruin-and-recreate metaheuristic. The proposed heuristic allows for the combination of routes into longer routes requiring a larger vehicle, and the splitting of routes into smaller ones. It outperformed previously published heuristics. Repoussis and Tarantilis (2010) later developed an adaptive memory programming algorithm for the FSMTW(T) which includes a probabilistic construction heuristic, a diversification mechanism, a short-term memory tabu search heuristic with edge-exchange neighbourhoods, speed-up procedures, and an iterated tabu search procedure working with a perturbation mechanism. Learning and several frequency-based long term mechanisms are also embedded into the algorithm. This method outperformed those presented in previous studies and improved upon 80 best-known solutions.

Bräysy et al. (2008) presented a three-phase multi-restart deterministic annealing metaheuristic for the FSMTW(T) and the FSMTW(D). In this algorithm, solutions are created by Clarke and Wright (1964) savings algorithm and by combining several diversification strategies. The second stage aims to reduce the number of routes in the initial solution by means of a greedy local search procedure. Four local search operators are embedded within a deterministic annealing framework to guide the improvement process in the final stage. The algorithm outperformed the earlier results on the FSMTW(T). New benchmark results on the FSMTW(D) were also presented. In a later study, Bräysy et al. (2009) described a hybrid metaheuristic algorithm for large-scale FSMTW(D) instances. They combined the well-known threshold acceptance heuristic with a guided local search metaheuristic having several search limitation strategies. Computational experiments yielded better results on the FSMTW(D) instances compared to those of Bräysy et al. (2008), and this algorithm was then used to solve a further 600 new FSMTW(D) benchmark instances with up to 1000 customers.

Prieto et al. (2011) described an asynchronous situated coevolution algorithm for a heterogeneous vehicle routing problem with time windows which uses a situated coevolution process inspired from the artificial life simulations. The authors showed that their open-ended evolutionary simulation which includes an improvement procedure yields good results. The performance was only tested on one instance with 20 nodes and 50 ships, not part of the standard benchmark FSMTW(D) instances, and the solution consists of a self-organized fleet of heterogeneous ships which satisfies the problem constraints and the market requirements.

More recently, Chapter 3 presented a unified heuristic called a hybrid evolutionary algorithm (HEA) for the FSMTW(T), the FSMTW(D), the HFTW(T) and the HFTW(D), and were the ones who introduced the last variant. The HEA combines several metaheuristics principles such as heterogeneous adaptive large scale neighborhood search and population search. The authors integrated within the HEA an innovative intensification strategy on elite solutions, a new diversification scheme based on the regeneration and the mutation procedures of solutions, and developed an advanced version of the split algorithm of Prins (2009) to determine the best fleet mix for a set of routes. Extensive computational experiments on benchmark instances showed that the HEA is highly effective on all four problems.

2.6 Variants and Extensions

Both the FSM and the HF have given rise to a multitude of extensions which have received particular attention in the last five years. In this section, we review such variants and extensions, which are classified in Figure 2.1.

2.6.1 The multi-depot HVRP

The Multi-Depot FSM was introduced by Salhi and Sari (1997) who proposed a three-level composite heuristic for it. An initial solution is first generated, a composite heuristic is executed to improve the quality of the solution, and an extension of the composite heuristic which considers all depots simultaneously is then applied. Several procedures of the algorithm are taken from Salhi and Rand (1993). Salhi et al. (2014) later considered the same problem by proposing a mixed integer linear programming formulation with new valid inequalities and several variable settings. Furthermore, the authors developed a variable neighborhood search heuristic equipped with a scheme for determining borderline customers and combined with a local search method based on a multi-level heuristic. The heuristic uses Dijkstra's algorithm to optimally partition the routes. It includes a diversification procedure and also contains a mechanism to aggregate the routes from different depots and then to disaggregate them and assign them to different depots. In total, 23 new best solutions out of the 26 benchmark instances were obtained, which makes this heuristic highly competitive.

Several metaheuristics were developed for the Multi-Depot FSMTW. Thus Dondo and Cerda (2007) proposed a mathematical model and a three-phase heuristic. The first phase determines a set of cost effective clusters and the second phase assigns clusters to vehicles and sequences them on each tour by using a cluster-based mathematical model.

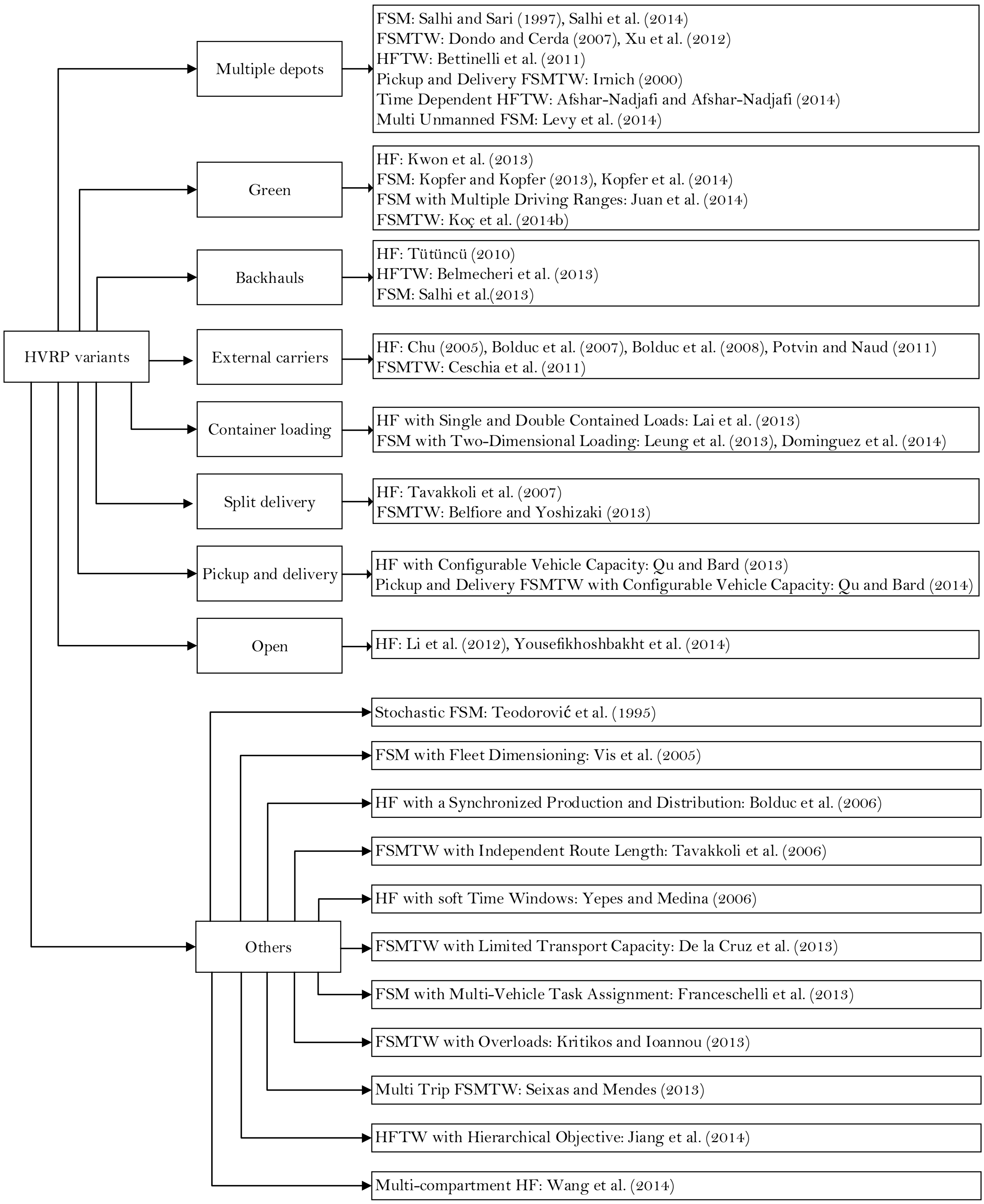


FIGURE 2.1: A classification of HVRP variants

In the final phase, customer orders and scheduling of vehicle arrival times are optimized by solving a mathematical model. Computational experiments were conducted on adapted classical Solomon (1987) VRPTW instances with up to 100 nodes. In another study, Xu et al. (2012) described a mathematical formulation and implemented a variable neighborhood search algorithm. The authors designed a hybrid insert and exchange operator for the shaking mechanism and implemented a best-improvement strategy to increase solution quality and decrease the running time. No computational experiments were conducted on the FSMTW since the authors only tested their algorithm on the classical Golden et al. (1984) FSM instances.

Bettinelli et al. (2011) developed a branch-and-cut-and-price algorithm for the Multi-Depot HFTW. They investigated several mixed strategies, such as initializing the restricted master problem and repairing the columns in the restricted master problem by removing cycles. Their results suggest that the number of different vehicle types, as opposed to the presence of multiple depots, makes the problem more difficult to solve. The tightness of the time windows was also found to influence the difficulty of the problem.

Irnich (2000) introduced the Multi-Depot Pickup and Delivery FSMTW with a single hub. In this problem, all routes have to visit the hub once, and are cycles that start from and end at the same depot. The author developed a network model which computes lower bounds and solves a set partitioning problem.

Afshar-Nadjafi and Afshar-Nadjafi (2014) studied a Time-Dependent Multi-Depot HFTW and proposed a mathematical formulation along with a constructive heuristic with three local search operators. Their results were compared on 180 test instances and indicate that the proposed heuristic was able to identify solutions within 0.3% of optimality for small size instances

Levy et al. (2014) introduced the Multi-Depot Multi-Unmanned FSM with Fuel Constraints where the different types of vehicles, between two and nine, are expected to refuel at fuel stations or at depots as they run out of fuel. They used a variable neighborhood descent (VND) and a variable neighborhood search (VNS). The authors presented simulation results to test the efficiency of the method on a set of 23 instances on which VND produced better solutions than VNS.

Dayarian et al. (2015) introduced the Multi-Depot FSMTW with deliveries to plants. The problem is inspired by collection-redistribution activities in the raw-milk industry of Quebec, where there is a need to satisfy the plant demands by delivering the supplies collected earlier. The authors defined a new set covering model, a specialized cutting-edge column generation procedure to solve its linear relaxation, and a new branching strategy based on the special structure of the problem. Promising results were obtained on instances with up to 50 producers.

2.6.2 The green HVRP

In recent years, green issues have received increased attention in the context of the VRP. Thus, Kwon et al. (2013) studied a green extension of the HF in which the objective is to minimize carbon emissions. The authors presented a mathematical formulation and developed several tabu search algorithms. The authors performed a cost-benefit assessment of the value of selling or purchasing of carbon emission rights. Their results suggest that carbon emissions can be reduced without increasing costs due to the benefits of carbon trading.

Juan et al. (2014) introduced the FSM with Multiple Driving Ranges in which the total distance that each vehicle type can travel is limited. This problem arises in the routing of electric and hybrid-electric vehicles which can only travel limited distances due to the limited capacity of their batteries. The authors described a mathematical formulation and developed a multi-round heuristic. The method is based on a biased randomized algorithm which can be used alone to create alternative fleet choices whenever the feasibility of the prespecified fleet configuration is not guaranteed. A set of benchmark instances were created to analyze how distance-based costs increase when considering "greener" fleet configurations. The method performed well on all benchmark instances and many different alternative solutions offer competitive distance-based costs while using fewer long- or medium-range vehicles than normally required.

Kopfer and Kopfer (2013) studied the emission minimizing variant of the FSM. These authors described a mathematical formulation for the problem and computed the CO_2 emissions based on the vehicle load and distance traveled. They presented computational experiments on small size instances with up to 10 customers. Kopfer et al. (2014) later studied an extension of the problem in which emission and fuel consumption are jointly minimized. They presented a mathematical model and solved it by CPLEX on instances with up to 14 nodes. The model was used to analyze the potential of reducing CO_2 emissions by using a heterogeneous fleet. The tests confirmed that the quantity of fuel needed to serve a given customer demand can indeed be reduced.

Koç et al. (2014) studied the Fleet Size and Mix Pollution-Routing Problem where the objective is a linear combination of vehicle, fixed cost, fuel cost and CO_2 emissions, and driver cost. The authors formally defined the problem, presented a mathematical model and developed a hybrid evolutionary metaheuristic. Several algorithmic features were introduced, namely a heterogeneous adaptive large neighborhood search procedure, a split algorithm with speed optimization algorithm and a new solution education procedure. Computational experiments were conducted to shed light on the potential trade-offs between various performance indicators, such as fuel and CO_2 emissions, vehicle fixed cost, distance, driver cost and total cost. The authors quantified the benefit of using a heterogeneous fleet over a homogeneous one. An interesting finding was that a heterogeneous fleet with fixed speeds achieved greater benefits in cost as opposed to a homogeneous fleet using speed optimization.

2.6.3 The HVRP with backhauls

Tütüncü (2010) introduced the HF with Backhauls and proposed a new greedy randomised adaptive memory programming search based on a visual interactive algorithm. The method was embedded within a visual decision support system where users are allowed to generate and assess alternative decisions by using their experience and knowledge of the problem. The proposed algorithm was initially tested on classical HF instances and competitive results were obtained within a reasonable computation time. Several new benchmark instances with up to 100 nodes were also generated for the HF with Backhauls.

Belmecheri et al. (2013) described the HFTW with Mixed Backhauls where linehaul customer demands are delivered from the depot, while backhaul customers have goods to be picked up and brought back to the same depot. The authors presented a mathematical model and proposed a particle swarm optimization heuristic which was applied to randomly generated instances with 25 to 100 nodes.

A new variant of the FSM with Backhauls was introduced by Salhi et al. (2013). In this problem, there are two types of customers: delivery (linehaul) customers and pick-up (backhaul) customers. All deliveries must be made to the linehaul customers before any of the backhaul customers are serviced. Routes containing only backhaul customers are not allowed. The authors formulated the problem, presented several valid inequalities and developed a heuristic algorithm based on a set partitioning formulation. A total of 36 instances were generated, ranging from 20 to 100 nodes. Optimal solutions were obtained on small size instances, and upper and lower bounds could be computed on larger ones. The method performed reasonably well on the FSM with Backhauls as well as on the standard FSM.

2.6.4 The HVRP with external carriers

Several studies have considered the special case of the HF with the use of external carriers. In this problem, customer demands are delivered by means of a heterogeneous internal fleet or by an external carrier. The objective is minimizing a total cost function of external carriers, transportation and fixed cost of the internal fleet.

One of the first papers on this problem is by Chu (2005) who proposed a three-level heuristic. The customers served by the external carrier are first selected. Routes are then constructed to serve the remaining customers by applying a modified version of Clarke and Wright (1964) savings algorithm. Finally, a steepest descent heuristic is applied to improve the quality of the solution. The author applied the method to five generated instances with up to 29 customers. Another study on this problem was presented by Bolduc et al. (2007) who described a heuristic that first selects the customers to be served by external carriers, and then generates an initial solution subsequently improved by 4-opt moves. This heuristic yielded better results than that of Chu (2005). In a later study, Bolduc et al. (2008) proposed a perturbation metaheuristic for the same problem. The algorithm integrates a local descent on different neighbourhood structures with a randomized construction procedure, a perturbation mechanism where pairs of customers are swapped, and an improvement procedure. It also makes use of a streamlined family of edge exchanges. The method provided better results than those of Chu (2005) and Bolduc et al. (2007). Finally, Potvin and Naud (2011) developed a tabu search heuristic for the same problem with a neighbourhood structure based on ejection chains. This heuristic outperformed all previous ones and was particularly effective on large-size instances due to the use of the ejection chain mechanism which allows multiple displacements of customers served by heterogeneous vehicles.

Ceschia et al. (2011) studied another extension of the FSMTW with Carrier-Dependent Costs. The problem works with a heterogeneous fleet, a multi-day planning horizon, complex carrier-dependent vehicle costs, and the possibility of not serving some orders. The authors developed a tabu search with a combination of three different neighborhood relations. The effects of these neighborhoods were investigated on a set of real-world instances. The method was also tested on the benchmarks instances of Bolduc et al. (2007).The proposed method was able to obtain one new best-known solution.

2.6.5 The HVRP with container loading

A few studies have considered the joint HVRP and container loading problem, where the latter feature adds a significant layer of complexity. One of the earlier papers by Lai et al. (2013) considered the HF with Single and Double Container Loads where container loads must be shipped from exporters to a port and from the port to importers by trucks carrying either one or two containers. The problem was formulated as a mixed integer linear program. The authors developed an algorithm in which an initial solution is obtained through a modified version of Clarke and Wright (1964) savings heuristic, and is then improved by a sequence of local search stages.

Leung et al. (2013) have studied the FSM with Two-Dimensional Loading. In this problem, vehicles have different capacities, fixed and variable operating costs, a length and a width, and two-dimensional loading constraints. Customers demand rectangular items with a given width, length and weight. The authors developed a simulated annealing algorithm combined with a local search heuristic to improve the solution. Furthermore, six packing algorithms, five of which were proposed by Zachariadis et al. (2009) and one by Leung et al. (2010) were also used to solve the loading subproblem. The method was tested on benchmark instances derived from the VRP with Two-Dimensional Loading (Iori et al. 2007, Gendreau et al. 2008). Dominguez et al. (2014) later studied an undirected version of the FSM with Two-Dimensional Loading which differs from that of Leung et al. (2013) by allowing the items to be rotated by 90° during the truck-loading process. The work was motivated by a real-world case in which a company distributes industrial building construction equipment to customers. These items are assumed to be rectangular and must be packed so as to efficiently use the vehicle capacity. Thus, the equipments must be distributed considering not only their weight, but also their specific dimensions. The authors developed a multi-start heuristic based on biased randomization of routing and packing algorithm. Routing and packing costs are considered simultaneously to better support the decision making process. The authors used the benchmark instances of Leung et al. (2013) and were able to obtain some new best-known solutions.

2.6.6 The HVRP with split deliveries

Split deliveries in vehicle routing occur when the demand of a customer may be fulfilled by multiple vehicles. Tavakkoli et al. (2007) were among the first to allow split deliveries in the context of the HF. The cost of the fleet depends on the total unused capacity and on the number of vehicles used. The authors formulated the problem as mixed-integer linear program and then developed a hybrid simulated annealing algorithm. They generated new benchmark instances with six to 100 nodes. On the small size instances, the heuristic was compared with a branch-and-bound method which yields comptetive results. On the larger size instances the comparison was made with respect to lower bounds obtained by solving a giant tour visiting all the customers.

Belfiore and Yoshizaki (2013) developed a scatter-search algorithm for the FSMTW with Split Deliveries. Initial solutions were created by two constructive heuristics. Scatter search was then used to diversify and intensify the solutions. The authors applied their algorithm on the standard FSMTW instances of Liu and Shen (1999a) and on generated instances from the VRP with Split Deliveries of Ho and Haugland (2004) with 100 nodes for which several best-known solutions obtained.

2.6.7 The HVRP with pickup and delivery

Qu and Bard (2013) introduced the Pickup and Delivery HF with Configurable Vehicle Capacity in which the vehicle capacity can be modified by reconfiguring its interior to satisfy different types of customer demands. The authors presented a mixed-integer formulation, and developed a two-phase heuristic based on greedy randomized adaptive search procedures with multiple starts. In the first phase, several randomized procedures are used to obtain a set of good feasible solutions and in the second phase an adaptive large neighborhood search heuristic is applied to improve the solutions. Eight real instances with up to 100 nodes and four vehicle types were solved. The solutions yielded cost savings from 30% to 40%. The same authors (Qu and Bard 2014) later introduced the Pickup and Delivery FSMTW with Configurable Vehicle Capacity. They presented a mixed-integer programming model and a branch-and-price-and-cut algorithm. The authors proposed a labeling algorithm for the pricing subproblem, which is an elementary shortest path problem, developed efficient dominance conditions to speed up the method, and used subset-row inequalities to strengthen the lower bound obtained by column generation. Benchmark data sets with up to 50 nodes were generated to test the efficiency of the method. Optimal solutions were obtained in the majority of cases.

2.6.8 The open HVRP

Li et al. (2012) seem to have been the first to study the Open HF, where a vehicle starts its route at the depot but is not required to return back to it after servicing the last customer. The authors proposed a multistart adaptive memory programming algorithm with a modified tabu search heuristic which was applied to randomly generated instances with between 50 and 200 nodes and with six vehicle types. The second paper on this problem is by Yousefikhoshbakht et al. (2014), in which an adaptive memory algorithm combined with tabu search is proposed. The algorithm generates initial diversified solutions which are later intensified. The tests revealed that the algorithm is effective and can find better solutions than those of Li et al. (2012).

2.6.9 Other HVRP variants and extensions

A number of the HVRP extensions have been studied, ranging from cases in which customer demands are stochastic to cases containing realistic constraints relative to synchronization, multiple products, loading, etc. We now provide a brief overview of these studies.

Teodorović et al. (1995) considered a stochastic FSM where customer demands are drawn from a uniform distribution. The authors proposed a heuristic to construct a giant tour, which is then split into smaller routes, each of which is assigned to a suitable vehicle type. The probability of failure on a route is then computed as the probability that the total demand served on the route exceeds the vehicle capacity.

Vis et al. (2005) considered a variant of the FSMTW with fleet dimensioning between buffer areas and storage areas at a container terminal. This problem arises in maritime transportation where products can be transported in containers between ports. The containers are transferred from one transportation mode to another, and cranes remove containers from a ship to put them in a capacitated buffer area. A vehicle lifts a container from the buffer area before it is full and transports it to another buffer area before it is eventually moved to a storage area. The problem minimizes the container fleet size between buffer and storage areas within a prespecified time window. The authors described a mathematical model for the problem and used simulation to validate the results on instances with 50, 80, or 100 containers. Additional experiments were also conducted to test the performance of the model under various conditions. The objective was to minimize the vehicle fleet size.

Bolduc et al. (2006) consider the HF with Synchronized Production and Distribution for a large-scale supply chain network. The problem involves the determination of a production schedule, inventory levels and a schedule for delivering demand at the retailers. The authors presented a mathematical model, proposed four heuristics for direct deliveries and described several extensions to tackle with the multiple-retailer-routes.

Tavakkoli et al. (2006) introduced another variant of the FSMTW in which only the depot has a time window, and the cost is independent of the route length but dependent on the type and capacity of available vehicles. The authors developed a mathematical model and a hybrid simulated annealing algorithm based on the nearest neighborhood heuristic. Computational results show that 18 small scale instances of up to 10 customers were solved to optimality with the proposed mathematical model and the heuristic could find good solutions within reasonable computation time on 10 large instances with up to 300 customers.

The HF with soft Time Windows was studied by Yepes and Medina (2006) who proposed a three-step local search algorithm based on a probabilistic variable neighborhood search. The method includes a generation procedure that makes use of a greedy randomized adaptive search, a diversification procedure uses an extinctive selection evolution mechanism, and a postoptimization mechanism based on a threshold heuristic with restarts. The authors note that practical VRPs need an economic objective function to compute the solution cost under various economic scenarios whose specific conditions may change every day. Three instances with 100 nodes and three vehicle types were solved.

De la Cruz et al. (2013) studied the FSMTW with multiple products and limited transport capacity. The authors developed a hybrid ant colony heuristic with a two-pheromone trail strategy to accelerate the ants, combined with a simple tabu search heuristic. A colony of cooperative agents is used to obtain feasible solutions for the problem where the implementation is two-level iterative process (see Barbarosoglu and Ozgur 1999, Homberger and Gehring 1999). After the ant colony search, a tabu search algorithm is applied to obtain better solutions without significantly affecting the computation time. The heuristic uses recent event and frequent event memories, as well as diversification procedures. The authors generated benchmark instances with up to 200 nodes based on those of Solomon (1987) and Homberger and Gehring (1999) on which competitive results were obtained.

Franceschelli et al. (2013) developed two heuristic to solve several variants of the FSM with multi-vehicle task assignment. To improve the local task assignments, the first algorithm builds on local, asynchronous and pairwise optimizations, while the second one is linear with respect to the number of tasks. The authors proposed upper and lower bounds which consider vehicles with different movement and task execution speeds, and also tasks with several service costs. The algorithms was validated through simulations.

Another extension of the FSMTW, which considers overloads on vehicles, was investigated by Kritikos and Ioannou (2013). Overloads are allowed up to a prespecified bound, the penalty function of Gheysens et al. (1984) is embedded within the objective function to effectively control overloaded solutions. The authors developed a sequential insertion heuristic which extends the traditional insertion criteria of Solomon (1987), and adapts several algorithmic procedures introduced by Golden et al. (1984), Dullaert et al. (2002) and Liu and Shen (1999a). Competitive results were obtained on the adapted FSMTW instances of Liu and Shen (1999b).

Seixas and Mendes (2013) studied the Multi-Trip FSMTW with accessibility restrictions on customers, in which the work hours of the drivers are limited. They developed a column generation algorithm, a constructive heuristic and a tabu search heuristic. Valid inequalities were also introduced to strengthen the formulation. Instances with up to 50 customers and 25 vehicles were solved to optimality.

Jiang et al. (2014) considered the HFTW with a hierarchical objective function that minimizes the total number of vehicles and total travelled distance. Each component is multiplied by a hierarchical weight in the objective function. The authors developed a two-phase tabu-search algorithm. In the first phase the algorithm of Lau et al. (2003) is used to handle the heterogeneous fleet dimension and a post-processing procedure is applied in the second phase. The method was tested on randomly generated instances with up to 100 nodes.

Wang et al. (2014) studied the Multi-compartment HF. The problem arises in many practical application, such as the transportation of apparel products with different vehicle types. These products have different styles and packages, and are usually jointly delivered in one vehicle. Some products are hung on flexible swing rods while others are packed in boxes. In this case, the vehicle is reorganized to form multiple separated compartments, one for each product type. The authors proposed a reactive guided tabu search algorithm in which the search history is used to guide the process, and they solved instances with two different vehicle types and up to 100 nodes.

Iori and Riera-Ledesma (2015) introduced a variant of the FSM, called the Double VRP with Multiple Stacks, which is the one-to-one pickup-and-delivery VRP with backhaul deliveries. Heterogeneous vehicles carry containers divided into stacks of fixed height, the operation of the vehicles follows a last-in-first-out policy. The aim is to minimize the total routing cost by performing pickups and deliveries while ensuring feasible loading and unloading of the vehicles. The authors have developed three models: a three-index formulation, and two set partitioning formulations using different families of columns. These models were solved by branch-and-cut, branch-and-price and branch-and-price-and-cut, respectively. The branch-and-price and the branch-and-price-and-cut algorithms performed well when the number of vehicles increased. On the other hand, the branch-and-cut algorithm yielded better quality solutions on instances with a small number of vehicles. Instance with up to 50 nodes were solved to optimality by the branch-and-price-and-cut algorithm.

2.7 Case Studies

Because heterogeneous fleets are common in practice, several studies were conducted to investigate and solve real-life distribution problems, which we now review.

Tarantilis and Kiranoudis (2001) described a real-life application concerning the scheduling of distribution of fresh milk for a dairy company in the Athens area. The authors aimed to minimize the total cost of delivering fresh milk from a dairy company to supermarkets and small stores by means of a heterogeneous fleet of vehicles. There are 299 delivery points in that study, and three vehicle types are available. The authors proposed a heuristic that was able to yield considerable improvements in the operational performance of the company. In another paper, the same authors studied two real-life HF problems arising in the dairy sector and in the construction industry (Tarantilis and Kiranoudis 2007). The first case study considers the central warehouse of a dairy company that hosts 27 vehicles of 12 different types and stores bottles of fresh milk that must be delivered daily to a set of customers, the number of which varies from 240 to 320. In the second case study, a construction company has a distribution center where ready-made concrete is loaded onto a heterogeneous fixed fleet of concrete-mixer trucks. The concrete is then delivered to 100 construction sites, and each load can be blended by a specific type of concrete-mixer truck of specific capacity which can carry different blends of concrete. In total, 13 trucks of six different types are available. The authors developed a flexible adaptive memory-based algorithm which is a two-phase construction heuristic, incorporating various operational constraints. The method outperformed that of Tarantilis et al. (2003) and significantly reduced the fleet size and distribution costs when compared with the current practice.

Prins (2002) studied a multi-trip variant of the HF in which each vehicle can perform several trips. Several heuristics, namely sequential heuristics, a new merge heuristic, steepest descent local search and tabu search. Both the single trip and multi-trip versions of the HF were solved with the proposed methods. Furthermore, the merge heuristic is applied to the case of a furniture manufacturer located near Nantes on the Atlantic coast of France, with 775 destination stores and 71 trucks. In the problem, the orders must be received at the latest on Friday for a delivery the week after. The author indicated that this situation creates sufficient time on Friday night to run the algorithm. The method achieved significant savings on the average route duration and on the total cost of the weekly scheduling.

Calvete et al. (2007) described a two-phase goal programming model for the FSM with multiple objectives and hard or soft time windows to solve a real-life problem arising in the medium-size delivery company. In a first phase feasible routes are enumerated and the total penalty incurred by each route regarding to deviations from targets is computed. The second phase solves a set partitioning problem to select the best set of feasible routes. Medium-sized real-life problems containing 60 instances were grouped into six different configurations with 30, 50 and 70 nodes. Customers are clustered into four groups with respect to time windows: soft time windows reflecting town council regulations or customer requirements for delivery early in the morning, early in the afternoon or in the evening. Hard time windows allow a maximum deviation from the soft time windows of one hour on each side. Competitive results were obtained on medium-sized problems.

Belfiore and Yoshizaki (2009) proposed a scatter search algorithm for a real-life FSMTW with Split Deliveries arising in the retail industry in São Paulo. The sequential insertion heuristics of Dullaert et al. (2002) and of Ho and Haugland (2004) were adapted to generate initial solutions. Several intensification and diversification procedures were also combined. The algorithm was applied to a major Brazilian retail group which serves 519 delivery points in 11 states. Customers are served from one depot located in São Paulo with four different types of trucks. The results showed that proposed method is capable of finding better solutions than current practice and decreased the total distribution cost by 7.5% on average.

Bettinelli et al. (2014) considered the Pickup and Delivery Multi-Depot HF with Soft Time Windows to find efficient solutions for small transportation companies operating in the urban area of Milan. To this end, they proposed an exact branch-and-price algorithm based on advanced dynamic programming techniques. The method could optimally solve the instances with up to 75 customers. Furthermore, the authors analyzed the effect of managing customer preferences by soft time windows which increases routing costs from 5 to 15%.

Xu and Jiang (2014) studied the Multi-Depot FSM and proposed a variable neighborhood search algorithm based on hybrid operators to solve a problem arising in a large-scale water project in China. Simulated annealing was embedded within the algorithm to manage the acceptance process of the solutions. The project contained two distribution centers, 16 customers and two vehicle types. It aimed to achieve a material flow equilibrium including excavation sites, filling sites, transfer yards, excavation waste dump sites, material yards, a distribution center, and equipment parking. Overall, the algorithm was able to decrease the average traveled distance by 3.49% and to reduce the total costs by 7.35%.

Moutaoukil et al. (2014) performed a case study arising in a "green" context and aimed at minimizing CO_2 emissions in the FSM. The authors defined a mathematical formulation to investigate the effect of homogeneous and heterogeneous fleets on CO_2 emissions. They presented a small illustrative example, and solved a real-life problem. In this example, a collection center located in Saint-Étienne, France, serves 10 nodes with three vehicle types, compromising one light-duty and two heavy-duty vehicle, to collect parcels every day. The results show that the use of a heterogeneous fleet yields better results than a homogeneous fleet. However, the model was able to solve only small-scale instances.

Yao et al. (2015) studied a variant of the HF in which the vehicles start and end their tours at a third-party logistics company, pick up cartons from factories and then deliver them to customers. The authors proposed a particle swarm optimization heuristic integrated within a self-adaptive scheme and a local search improvement strategy. They applied the method to a case study in Dalian City where there are eight carton factories and 85 customers in the region. The total cost was reduced by 28% when compared with the current situation.

2.8 Summary and Computational Comparisons

In this section, we first provide a tabulated summary of the existing literature on the HVRPs, and then present a comparative analysis of the computational results reported in the literature for the FSM, HF and FSMTW.

2.8.1 Summary

Tables 2.1, 2.2 and 2.3 contain a summary of all publications reviewed in this paper. These tables contain, for each reference, the side problem to the main problem (if any), whether a mathematical programming formulation was described ("•" for yes), the solution method and whether a case study was included. The abbreviations of side problems used in Tables 2.1, 2.2 and 2.3 are as follows: backhauls (BC), carton with a collection depot (CCD), deliveries to plants (DTP), green (GR), multi-depot (MD), multi-compartment (MC), multi-vehicle task assignment (MV), multi-trip (MT), multiple stacks (MS), open (OP), overloads (OV), pickup and delivery (PD), single and double container loads (SDC), stochasticity (ST), time-dependencies (TD), two-dimensional loading (TDL), time windows (TW), use of external carriers (UEC). The abbreviations of solution methods are as follows: continuous approximation models (CA), branch-and-bound algorithm (BB), branch-and-cut algorithm (BCA), branch-cut-and-price algorithm (BCP), branch-and-price algorithm (BP), column generation (CG), decomposition (DE), heuristic column generation (HCG), integer programming (IP), mixed integer programming (MIP), lower bound formulations (LB), set partitioning (SP), valid inequalities (VI), adaptive memory programming (AMP), ant colony optimization (ACO), constructive heuristics (CH), iterated local search (ILS), particle swarm optimization (PSO), population search (PS), scatter search (SS), simulated annealing (SA), simulation (SIM), tabu search (TS), threshold accepting (TA), variable neighborhood search (VNS).

The following conclusions can be drawn from the tables: 1) For the HVRP, the most widely studied version is the FSM and its variants, with 60 references, comprising 58.25% of all references reviewed in this paper. This is followed by the HF and its variants studied in 32 references (31.07% of all references). In 11 references both the FSM and the HF (10.68% of all references) were studied. 2) For the FSM (Table 2.1), the most widely studied versions are the standard one and the FSMTW, each with 22 references and each comprising 36.67% of the list. 3) For the HF (Table 2.2), the most widely studied version is the standard one, with 14 references, comprising 43.75% of the list. This is followed by the HFTW, studied in five references (15.63% of the list), by the Multi-depot HF studied in three references (9% of the list), and the Pickup and Delivery HF studied in three references (9% of the list). 4) Two papers considered both the

FSMTW and the HFTW, comprising 18% of the list given in Table 2.3. 5) Of the FSM solution methods (Table 2.1), the most common are constructive heuristics with 26 references (43.34% of the list), followed by tabu search with seven references (11.67% of the list), and population search with six references (10% of the list). 6) Of the HF solution methods (Table 2.2), the most common occurence was constructive heuristics with 14 references (43.75% of the list), followed by tabu search heuristics with nine references (28.13% of the list), and threshold accepting heuristics with four references (12.5% of the list). 7) Concerning both the FSM and the HF solution methods (Table 2.3), the most common are population search with three references (27% of the list), followed by iterated local search with two references (18% of the list).

2.8.2 Metaheuristics computational comparisons

Baldacci et al. (2010a) provided a comparison of the exact algorithms produced until 2010. To the best of our knowledge, no exact algorithm has been developed for the standard FSM or the HF since 2010. For this reason, we provide a comparative analysis of the heuristic results for the HVRP in this section.

Most studies describing new algorithms for the HVRP were tested on the benchmark instances: 1) Golden et al. (1984) proposed a set of 20 instances for the FSM with 12 to 100 nodes. 2) Liu and Shen (1999b) described several data sets for the FSMTW, derived from the classical Solomon (1987) VRPTW instances with 100 nodes. These sets include 56 instances split into a random data set R, a clustered data set C and a semi-clustered data set RC. Sets denoted by R1, C1 and RC1 have a short scheduling horizon and small vehicle capacities, in contrast to the sets denoted R2, C2 and RC2 which have a long scheduling horizon and large vehicle capacities. Liu and Shen (1999b) also introduced three cost structures and several vehicle types with different capacities and fixed vehicle costs for each of the 56 instances. This results in a total of 168 benchmark instances for the FSMTW. 3) Taillard (1999) developed a benchmark data set for the HF by adapting eight of the Golden et al. (1984) benchmark instances. This set includes eight instances containing 50, 70 and 100 nodes. 4) The benchmark of Paraskevopoulos et al. (2008) for the HFTW is a subset of the FSMTW instances, in which the fleet size is set equal to that found in the best known solutions of Liu and Shen (1999a). In total, there are 24 benchmark instances derived from Liu and Shen (1999a) for the HFTW.

TABLE 2.1: Literature on the FSM

Reference		Side problem	Mathematical model	Solution method	Case study
1	Golden et al. (1984)		•	IP, CT	
2	Gheysens et al. (1984)		•	IP, CT	
3	Gheysens et al. (1986)		•	LB, CT	
4	Ferland and Michelon (1988)	TW	•	BB, CG, CT	
5	Desrochers and Verhoog (1991)			CT	
6	Salhi et al. (1992)		•	IP, CT	
7	Salhi and Rand (1993)		•	MIP, CT	
8	Teodorović et al. (1995)	ST		CT	
9	Osman and Salhi (1996)		•	MIP, CT, TS	
10	Salhi and Sari (1997)	MD		CT	
11	Ochi et al. (1998a)			DE, PS	
12	Ochi et al. (1998b)			DE, PS	
13	Gendreau et al. (1999)			TS	
14	Liu and Shen (1999a)	TW		CT	
15	Liu and Shen (1999b)	TW		CT	
16	Irnich (2000)	PD	•	LB, SP	
17	Dullaert et al. (2002)	TW		CT	
18	Han and Cho (2002)			CT	
19	Renaud and Boctor (2002)		•	SP, CT	
20	Wassan and Osman (2002)			TS	
21	Lima et al. (2004)			PS	
22	Vis et al. (2005)	TW	•	IP, SIM	
23	Tavakkoli et al. (2006)	TW	•	SA	
24	Yaman (2006)		•	LB	
25	Calvete et al. (2007)	TW	•	MIP, SP	•
26	Dell'Amico et al. (2007)	TW	•	CT	
27	Dondo and Cerda (2007)	TW, MD	•	CT	
28	Bräysy et al. (2008)	TW	•	CT	
29	Lee et al. (2008)		•	SP, TS	
30	Baldacci et al. (2009)		•	MIP, VI	
31	Belfiore and Yoshizaki (2009)	TW, PD	•	SS	•
32	Brandão et al. (2009)			TS	
33	Bräysy et al. (2009)	TW		CT	
34	Liu et al. (2009)			PS	
35	Pessoa et al. (2009)		•	BCP	
36	Repoussis and Tarantilis (2010)	TW	•	AMP	
37	Ceschia et al. (2011)	TW	•	TS	
38	Prieto et al. (2011)	TW		CT, SIM	
39	Jabali et al. (2012a)		•	AP	
40	Xu et al. (2012)	TW, MD	•	VNS	
41	Belfiore and Yoshizaki (2013)	TW, PD	•	SS	
42	De la Cruz et al. (2013)	TW, OV	•	ACO	
43	Franceschelli et al. (2013)	MV	•	CT	
44	Kopfer and Kopfer (2013)		•	MIP	
45	Kritikos and Ioannou (2013)	TW, OV	•	CT	
46	Leung et al. (2013)	TDL		SA	
47	Salhi et al. (2013)	BC	•	SP, CT	
48	Seixas and Mendes (2013)	TW, MT	•	CG, CT, TS	
49	Dominguez et al. (2014)	TDL		CT	
50	Juan et al. (2014)	GR	•	CT	
51	Koç et al. (2014)	GR	•	PS	
52	Kopfer et al. (2014)	GR	•	MIP	
53	Levy et al. (2014)	MD		VNS, SIM	
54	Moutaoukil et al. (2014)	GR		MIP	•
55	Qu and Bard (2014)	TW, PD	•	BCP	
56	Salhi et al. (2014)	MD	•	MIP, VNS	
57	Vidal et al. (2014)	TW		PS	
58	Xu and Jiang (2014)	MD	•	SA	•
59	Iori and Riera-Ledesma (2015)	MS, PD, BC	•	BCA, BP, BCP, SP	
60	Dayarian et al. (2015)	TW, MD, DTP	•	CG, BP	

Table 2.2: Literature on the HF

	Reference	Side problem	Mathematical model	Solution method	Case study
1	Tarantilis and Kiranoudis (2001)			TA	•
2	Prins (2002)	MT		CH	•
3	Tarantilis et al. (2003)			TA	
4	Tarantilis et al. (2004)			TA	
5	Chu (2005)	UEC	•	IP, CH	
6	Bolduc et al. (2006)		•	MIP, CH	
7	Gencer et al. (2006)			CH	
8	Yepes and Medina (2006)	TW	•	CH, TA	
9	Bolduc et al. (2007)			CH	
10	Li et al. (2007)			CH	
11	Tarantilis and Kiranoudis (2007)			CH, TS	•
12	Tavakkoli et al. (2007)	PD	•	SA	
13	Bolduc et al. (2008)	UEC	•	CH	
14	Euchi and Chabchoub (2010)		•	TS	
15	Li et al. (2010)			AMP	
16	Tütüncü (2010)	BC		CH	
17	Bettinelli et al. (2011)	TW, MD	•	BCP	
18	Brandão et al. (2011)			TS	
19	Potvin and Naud (2011)		•	TS	
20	Li et al. (2012)	OP		AMP, TS	
21	Naji-Azimi and Salari (2013)		•	CH	
22	Belmecheri et al. (2013)	TW, BC	•	PSO	
23	Kwon et al. (2013)	GR	•	MIP, TS	
24	Lai et al. (2013)	SDC	•	CH	
25	Liu (2013)		•	PS	
26	Qu and Bard (2013)	PD	•	CH	
27	Afshar-Nadjafi and Afshar-Nadjafi (2014)	TW, MD, TD	•	MIP, CH	
28	Bettinelli et al. (2014)	TW, MD, PD	•	BP	•
29	Jiang et al. (2014)	TW	•	TS	
30	Wang et al. (2014)	MC	•	TS	
31	Yousefikhoshbakht et al. (2014)	OP		AMP, TS	
32	Yao et al. (2015)	CCD, PD	bullet	PSO	•

TABLE 2.3: Literature on the FSM and the HF

	Reference	Side problem	Mathematical model	Solution method	Case study
1	Taillard (1999)		•	HCG	
2	Choi and Tcha (2007)		•	IP, CG	
3	Paraskevopoulos et al. (2008)	TW		TS	
4	Baldacci and Mingozzi (2009)		•	LB, SP	
5	Imran et al. (2009)			VNS	
6	Prins (2009)			PS	
7	Baldacci et al. (2010b)		•	SP	
8	Duhamel et al. (2012)			PS	
9	Subramanian et al. (2012)		•	SP, ILS	
10	Penna et al. (2013)			ILS	
11	Koç et al. (2015)	TW		PS	

We now present a comparison of the recent results for the FSM, HF and FSMTW in Sections 2.8.2.1, 2.8.2.2 and 2.8.2.3, respectively.

2.8.2.1 Comparison of recent metaheuristics on the FSM

Table 2.4 presents a summary of the comparison results of recent metaheuristics for the FSM with three costs variants, i.e., FSM(F,V), FSM(F) and FSM(V). For detailed comparison results, the reader is referred to Tables A.1, A.2 and A.3 in the Appendix. In these tables, the first column provides the references, and for each reference two performance indicators are shown: the average percentage deviation (Dev) from the value of the best-known solution (BKS) for each instance retrieved from the articles surveyed, and the average computation time in seconds (Time). The reported running times are the best solution running time over all runs. The computers and programming languages used are not comparable, hence scaled times for one reference computer would not be valid. We simply indicate the features of each computer such as processor and CPU speed in GHz in the last column of the table.

Table 2.4 indicates an effective progress since 2007. All metaheuristics for the three cost versions of the FSM achieve average deviations 0.2% or less. In terms solution quality, the top performers are Penna et al. (2013) and Vidal et al. (2014) for the FSM(F,V), Vidal et al. (2014) for the FSM(F), and Choi and Tcha (2007), Penna et al. (2013) and Vidal et al. (2014) for the FSM(V).

2.8.2.2 Comparison of recent metaheuristics on the HF

Table 2.5 provides a summary of the average results of recent metaheuristics applied to two costs variants of HF: HF(F,V) and HF(V). For detailed comparison results, the reader is referred to Tables A.4 and A.5 in the Appendix.

Table 2.5 shows a continuous progress since 1999. Recent metaheuristics for all two cost versions of the HF achieve average deviations 0.5% or less. In terms of solution quality, the top performers are Subramanian et al. (2012) for the HF(F,V), and Li et al. (2007), Subramanian et al. (2012), Liu (2013) and Penna et al. (2013) for the HF(V).

Table 2.4: Average comparison of recent metaheuristics on the FSM

Reference	FSM(F,V)		FSM(F)		FSM(V)		Processor	CPU
	Dev (%)	Time (s)	Dev (%)	Time (s)	Dev (%)	Time (s)		
Choi and Tcha (2007)	0.08	92.70	0.08	150.33	0.00	36.06	Pentium IV	2.6GHz
Brandão et al. (2009)	–	–	0.10	191.83	0.02	294.63	Pentium M	1.4GHz
Imran et al. (2009)	0.04	466.17	0.07	500.50	0.02	531.00	Pentium M	1.7GHz
Liu et al. (2009)	–	–	0.03	198.17	0.04	260.75	Pentium IV	3GHz
Prins (2009)	0.02	23.06	0.12	42.52	–	–	Pentium IV M	1.8GHz
Penna et al. (2013)	0.00	24.50	0.03	30.35	0.00	29.77	Intel i7	2.93GHz
Vidal et al. (2014)	0.00	59.30	0.01	68.10	0.00	51.15	Opt	2.4GHz

TABLE 2.5: Average comparison of recent metaheuristics on the HF

Reference	HF(F,V)		HF(V)		Processor	CPU
	Dev (%)	Time (s)	Dev (%)	Time (s)		
Taillard (1999)	–	–	1.25	2011.13	Sun Sparc	50MHz
Tarantilis et al. (2004)	–	–	0.51	607.13	Pentium II	400MHz
Gencer et al. (2006)	4.63	–	–	–	–	–
Li et al. (2007)	–	–	0.03	285.75	AMD Athlon	1.0GHz
Prins (2009)	–	–	0.06	94.76	Pentium IV M	1.8GHz
Li et al. (2010)	0.21	131.75	–	–	Intel	2.2GHz
Subramanian et al. (2012)	0.00	7.73	0.03	4.03	Intel i7	2.93GHz
Liu (2013)	0.19	127.05	0.03	71.17	Intel Pentium IV	3GHz
Penna et al. (2013)	0.07	31.97	0.03	31.89	Intel i7	2.93GHz

2.8.2.3 Comparison of recent metaheuristics on the FSMTW

Tables 2.6 and 2.7 present summaries of the results of recent metaheuristics for the FSM on two costs variants: FSMTW(T) and FSMTW(D). The first column shows the instance sets containing several instances. The second column shows the average BKS. The remaining columns show, for each reference, the average percentage deviations of each set, and the average solution time. The computer specifications are provided in the last two rows.

In the case of the FSMTW(T), a continuous progress can be observed since 2007. Recent metaheuristics, Vidal et al. (2014) and Koç et al. (2015), achieve average deviations 0.2% or less. In the case of the FSMTW(D), there has also been a significant progress in terms of average deviations. Recent metaheuristics, Vidal et al. (2014) and Koç et al. (2015), achieve average deviations of at most 0.1%. These two papers developed powerful population search based metaheuristics, and obtained very effective results on both cost variants of the FSMTW.

2.9 Conclusions and Future Research Directions

The Heterogeneous Vehicle Routing Problem (HVRP) was introduced some 30 years ago by Golden et al. (1984) and has since evolved into a rich research area. Several versions of the problem have been studied, and applications are encountered in many settings. Our survey provides a classification of the HVRP literature under two main dimensions: unlimited fleet and limited fleet. We have identified the following conclusions and future research directions: 1) All five standard versions of the HVRP (FSM(F,V), FSM(F), FSM(V), HF(F,V), HF(V)) have now been solved to near optimality by heuristics, and it is our belief that this algorithmic research on the standard problems has now reached maturity. 2) Over the years, most of the research effort has shifted towards the study of rich extensions of the standard HVRP, such as time windows, multiple depots, external carriers, pickup and delivery operations, container loading and bachauls. There still exist numerous research opportunities on these rich extensions. 3) The "green" extension of the problem has also received increasing attention in recent years. It would seem interesting to study some extensions of the standard HVRP in a green context. 4) Almost all studies, except one, have so far focused on time-independent versions of

TABLE 2.6: Comparison of recent metaheuristics on the FSMTW(T)

Instance set	BKS	Dell'Amico et al. (2007)	Paraskevopoulos et al. (2008)	Bräysy et al. (2008)	Repoussis and Tarantilis (2010)	Vidal et al. (2014)	Koç et al. (2015)
		Dev (%)	Dev (%)	Dev (%)	Dev (%)	Dev (%)	Dev (%)
R1A (12)	4103.16	1.86	0.61	0.68	0.26	0.00	0.38
R1B (12)	1891.63	1.86	0.56	0.38	0.27	0.00	0.25
R1C (12)	1574.32	2.55	0.50	0.31	0.24	0.00	0.05
C1A (9)	7138.93	1.25	0.06	0.03	0.01	0.00	0.06
C1B (9)	2359.63	1.05	0.09	0.25	0.01	0.00	0.07
C1C (9)	1618.91	0.66	0.13	0.18	0.00	0.02	0.04
RC1A (8)	4915.10	3.96	0.94	0.68	0.67	0.00	0.65
RC1B (8)	2129.04	1.59	0.64	0.03	0.36	0.00	0.27
RC1C (8)	1752.19	1.81	1.00	0.35	0.58	0.00	0.48
R2A (11)	3267.31	8.45	1.13	1.31	0.62	0.00	0.16
R2B (11)	1471.33	14.81	1.84	1.61	1.06	0.61	0.00
R2C (11)	1237.79	13.82	3.40	1.58	1.84	0.00	0.66
C2A (8)	5746.53	8.32	0.22	0.88	0.06	0.24	0.00
C2B (8)	1748.52	7.86	0.32	0.43	0.03	0.11	0.00
C2C (8)	1218.12	4.56	1.21	0.47	0.49	0.25	0.00
RC2A (8)	4381.73	7.81	0.56	0.40	0.16	0.00	0.21
RC2B (8)	1867.80	13.37	1.11	1.65	0.38	0.53	0.00
RC2C (8)	1530.08	16.34	2.37	2.06	0.72	0.98	0.00
Avg Dev (%)		6.22	0.93	0.74	0.43	0.15	0.18
Avg Time (s)		849	1200	658.2	1000.2	304.8	289.8
Runs		1	1	3	1	10	10
Processor		Pentium	Pentium IV	Ath	Pentium IV	Opt	Xeon
CPU		600MHz	1.5GHz	2.6GHz	3.4GHz	2.2.GHz	2.6.GHz

Table 2.7: Comparison of recent metaheuristics on the FSMTW(D)

Instance set	BKS	Bräysy et al. (2008)	Bräysy et al. (2009)	Vidal et al. (2014)	Koç et al. (2015)
		Dev (%)	Dev (%)	Dev (%)	Dev (%)
R1a (12)	4031.28	0.92	0.73	0.00	0.25
R1b (12)	1839.39	0.82	–	0.11	0.00
R1c (12)	1525.56	0.90	0.93	0.31	0.00
C1a (9)	7082.98	0.04	0.04	0.00	0.00
C1b (9)	2332.90	0.09	–	0.00	0.00
C1c (9)	1615.38	0.02	0.00	0.01	0.00
RC1a (8)	4891.25	1.08	0.90	0.00	0.51
RC1b (8)	2103.21	0.87	–	0.18	0.00
RC1c (8)	1725.44	0.94	1.38	0.51	0.00
R2a (11)	3150.29	1.35	0.95	0.05	0.00
R2b (11)	1351.52	2.97	–	0.03	0.00
R2c (11)	1126.42	2.02	1.97	0.20	0.00
C2a (8)	5686.75	0.07	0.05	0.00	0.00
C2b (8)	1686.75	0.69	–	0.00	0.00
C2c (8)	1185.19	0.07	0.04	0.00	0.00
RC2a (8)	4210.10	0.74	0.50	0.00	0.00
RC2b (8)	1686.47	1.04	–	0.01	0.00
RC2c (8)	1358.24	1.19	1.95	0.00	0.08
Avg Dev (%)		0.88	0.79	0.08	0.05
Avg Time (s)		213.60	–	283.20	265.80
Runs		3	1	10	10
Processor		Ath	Duo	Opt	Xeon
CPU		2.6GHz	2.4GHz	2.2GHz	2.6GHz

the HVRP. A more realistic version of the standard HVRP and its variants would be to consider time-dependencies, particularly in urban settings and in city logistics. 5) To our knowledge, no exact algorithm has yet been proposed for the FSMTW (except for a simple branch-and-bound scheme) or the HFTW. Further studies should focus on developing effective exact methods for those different problems. 6) HVRPs tend to be very hard to solve, which explains why most algorithms are heuristics. These have gradually evolved from simple interchange schemes to more sophisticated metaheuristics, sometimes combining exact methods. In general, constructive heuristics complemented with local search heuristics are the main methods of these studies, but researchers should consider hybrid schemes combining population search and local search (like ALNS or iterated local search), such as those applied to the standard VRP (Laporte et al. 2014). Such metaheuristics should also be effective on special cases of FSM/HF and on problems that contain FSM/HF as special cases. The recent tendency in the field of the VRP heuristics has been to develop algorithms that are highly accurate but often require large computing times and lack simplicity in the sense that they contain too many parameters and are difficult to reproduce. This observation applies to most of the metaheuristics we have just surveyed. While computing times have become more modest in some of the best recent implementations, the lack of simplicity of these methods remains problematic in many cases (Laporte et al. 2014). On a more positive note, we have witnessed in recent years the emergence of flexible metaheuristics, based on genetic algorithm framework, capable of solving a host of problem variants with the same parameter settings (Vidal et al. 2014, Koç et al. 2015). 7) To our knowledge, only one study has developed a continuous approximation model which is highly effective on the standard FSM. This type of modelling should be applied to the HF and to rich extensions of the HVRP.

We believe this paper has helped unify the rapidly expanding body of knowledge on the HVRP and will encourage other researchers to pursue the study of this fascinating field of research.

Chapter 3

A Hybrid Evolutionary Algorithm for Heterogeneous Fleet Vehicle Routing Problems with Time Windows

Abstract

This paper presents a hybrid evolutionary algorithm (HEA) to solve heterogeneous fleet vehicle routing problems with time windows. There are two main types of such problems, namely the Fleet Size and Mix Vehicle Routing Problem with Time Windows (F) and the Heterogeneous Fixed Fleet Vehicle Routing Problem with Time Windows (H), where the latter, in contrast to the former, assumes a limited availability of vehicles. The main objective is to minimize the fixed vehicle cost and the distribution cost, where the latter can be defined with respect to en-route time (T) or distance (D). The proposed unified algorithm is able to solve the four variants of heterogeneous fleet routing problem, called FT, FD, HT and HD, where the last variant is new. The HEA successfully combines several metaheuristics and offers a number of new advanced efficient procedures tailored to handle the heterogeneous fleet dimension. Extensive computational experiments on benchmark instances have shown that the HEA is highly effective on FT, FD and HT. In particular, out of the 360 instances we obtained 75 new best solutions and matched 102 within reasonable computational times. New benchmark results on HD are also presented.

Keywords. vehicle routing; time windows; heterogeneous fleet; genetic algorithm; neighborhood search

3.1 Introduction

In heterogeneous fleet vehicle routing problems with time windows, one considers a fleet of vehicles with various capacities and vehicle-related costs, as well as a set of customers with known demands and time windows. These problems consist of determining a set of vehicle routes such that each customer is visited exactly once by a vehicle within a prespecified time window, all vehicles start and end their routes at a depot, and the load of each vehicle does not exceed its capacity. As is normally the case in vehicle routing problem with time windows (VRPTW), customer service must start within the time window, but the vehicle may wait at a customer location if it arrives before the beginning of the time window. There are two main categories of such problems, namely the Fleet Size and Mix Vehicle Routing Problem with Time Windows (F) and the Heterogeneous Fixed Fleet Vehicle Routing Problem with Time Windows (H). In category F, there is no limit in the number of available vehicles of each type, whereas such a limit exists in category H. Note that it is easy to find feasible solutions to the instances of category F since there always exists a feasible assignment of vehicles to routes. However, this is not always the case for the instances of category H.

Two measures are used to compute the total cost to be minimized. The first is the sum of the fixed vehicle cost and of the *en-route time* (T), which includes traveling time and possible waiting time at the customer locations before the opening of their time windows (we assume that travel time and cost are equivalent). In this case, service times are only used to check feasibility and for performing adjustments to the departure time from the depot in order to minimize pre-service waiting times. The second cost measure is based on *distance* (D) and consists of the fixed vehicle cost and the distance traveled by the vehicle, as is the case in the standard VRPTW (Solomon 1987).

We differentiate between four variants defined with respect to the problem category and to the way in which the objective function is defined, namely FT, FD, HT and HD. The first variant is FT, described by Liu and Shen (1999b) and the second is FD, introduced by Bräysy et al. (2008). The third variant HT was defined and solved by Paraskevopoulos et al. (2008). Finally, HD is a new variant which we introduce in this paper. HD differs from HT by considering the objective function D instead of T. This variant has never been studied before.

Hoff et al. (2010) and Belfiore and Yoshizaki (2009) describe several industrial aspects and practical applications of heterogeneous vehicle routing problems. The most studied versions are the fleet size and mix vehicle routing problem, described by Golden et al. (1984), which considers an unlimited heterogeneous fleet, and the heterogeneous fixed fleet vehicle routing problem, proposed by Taillard (1999). For further details, the reader is referred to the surveys of Baldacci et al. (2008) and of Baldacci et al. (2009).

The FT variant has several extensions, e.g., multiple depots (Dondo and Cerda 2007, Bettinelli et al. 2011), overloads (Kritikos and Ioannou 2013), and split deliveries (Belfiore and Yoshizaki 2009, 2013). There exist several exact algorithms for the capacitated vehicle routing problem (VRP) (Toth and Vigo 2002, Baldacci et al. 2010a), and for the heterogeneous VRP (Baldacci et al. 2009). However, to the best of our knowledge, no exact algorithm has been proposed for the heterogeneous VRP with time windows, i.e., FT, FD and HT. The existing heuristic algorithms for these three variants are briefly described below.

Liu and Shen (1999b) proposed a heuristic for FT which starts by determining an initial solution through an adaptation of the Clarke and Wright (1964) savings algorithm, previously presented by Golden et al. (1984). The second stage improves the initial solution by moving customers by means of parallel insertions. The algorithm was tested on a set of 168 benchmark instances derived from the set of Solomon (1987) for the VRPTW. Dullaert et al. (2002) described a sequential construction algorithm for FT, which is an extension of the insertion heuristic of Golden et al. (1984). Dell'Amico et al. (2007) described a multi-start parallel regret construction heuristic for FT, which is embedded into a ruin and recreate metaheuristic. Bräysy et al. (2008) presented a deterministic annealing metaheuristic for FT and FD. In a later study, Bräysy et al. (2009) described a hybrid metaheuristic algorithm for large scale FD instances. Their algorithm combines the well-known threshold acceptance heuristic with a guided local search metaheuristic having several search limitation strategies. An adaptive memory programming algorithm was proposed by Repoussis and Tarantilis (2010) for FT, which combines a probabilistic semi-parallel construction heuristic, a reconstruction mechanism and a tabu search algorithm. Computational results indicate that their method is highly successful and improves many best known solutions. In a recent study, Vidal et al. (2014) introduced a genetic algorithm based on a unified solution framework for different variants of the VRPs, including FT and FD. To our knowledge, Paraskevopoulos et al. (2008) are the only authors who have studied HT. Their two-phase solution methodology is based on a hybridized tabu search algorithm capable of solving both FT and HT.

This brief review shows that the two problem categories F and H have already been solved independently through different methodologies. We believe there exists merit for the development of a unified algorithm capable of efficiently solving the two problem categories. This is the main motivation behind this paper.

This paper makes three main scientific contributions. First, we develop a unified hybrid evolutionary algorithm (HEA) capable of handling the four variants of the problem. The HEA combines two state-of-the-art metaheuristic concepts which have proved highly successful on a variety of VRPs: Adaptive Large Neighborhood Search (ALNS) (see Ropke and Pisinger 2006a, Pisinger and Ropke 2007, Demir et al. 2012) and population based search (see Prins 2004, Vidal et al. 2014). The second contribution is the introduction of several algorithmic improvements to the procedures developed by Prins (2009) and Vidal et al. (2012). We use a ALNS equipped with a range of operators as the main EDUCATION procedure within the search. We also propose an advanced version of the SPLIT algorithm of Prins (2009) capable of handling infeasibilities. Finally, we introduce an innovative aggressive INTENSIFICATION procedure on elite solutions, as well as a new diversification scheme through the REGENERATION and the MUTATION procedures of solutions. The third contribution is to introduce HD as a new problem variant.

The remainder of this paper is structured as follows. Section 3.2 presents a detailed description of the HEA. Computational experiments are presented in Section 3.3, and conclusions are provided in Section 3.4.

3.2 Description of the Hybrid Evolutionary Algorithm

We start by introducing the notation related to FT, FD, HT and HD. All problems are defined on a complete graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N} = \{0, \dots, n\}$ is the set of nodes, and node 0 corresponds to the depot. Let $\mathcal{A} = \{(i, j) : 0 \leq i, j\} \leq n, i \neq j\}$ denote the set of arcs. The distance from i to j is denoted by d_{ij}. The customer set is $\mathcal{N}_c$ in which each customer i has a demand q_i and a service time s_i, which must start within time window $[a_i,\ b_i]$. If a vehicle arrives at customer i before a_i, it then waits until a_i. Let $K = \{1, \dots, k\}$ be the set of available vehicle types. Let e_k and Q_k denote the fixed vehicle cost and the capacity of vehicle type k, respectively. The travel time from i to j is denoted by t_{ij} and is independent of the vehicle type. The distribution cost from i to j associated with a vehicle of type k is c^k_{ij} for all problem types. In HT and HD, the available number of vehicles of type $k \in K$ is n_k, whereas the constant can be set to an arbitrary large value for problems FT and FD. The objectives are as discussed in the Introduction.

The remainder of this section introduces the main components of the HEA. A general overview of the HEA is given in Section 3.2.1. More specifically, Section 3.2.2 presents the offspring EDUCATION procedure. Section 3.2.3 presents the initialization of the population. The selection of parent solutions, the ordered crossover operator and the advanced algorithm SPLIT are described in Sections 3.2.4, 3.2.5 and 3.2.6, respectively. Section 3.2.7 presents the INTENSIFICATION procedure. The survivor selection mechanism is detailed in Section 3.2.8. Finally, the diversification stage, including the REGENERATION and MUTATION procedures, is described in Section 3.2.9.

3.2.1 Overview of the hybrid evolutionary algorithm

The HEA brings together the well-known components of evolutionary algorithms, such as parent selection and crossover, some advanced operators to educate, intensify and diversify the population. The SPLIT procedure is applied on a giant tour obtained from combining two parents in the population, the quality which is further improved by a quality of ALNS. Further details on the components are explained below.

The general structure of the HEA is presented in Algorithm 1. The modified version of the classical Clarke and Wright savings algorithm and the ALNS operators are combined to generate the initial population (Line 1). Two parents are selected (Line 3) through a binary tournament, following which the crossover operation (Line 4) generates a new offspring C.

The advanced SPLIT algorithm is applied to the offspring C (Line 5), which optimally segments the giant tour by choosing the vehicle type for each route. This algorithm is a tour splitting procedure which optimally partitions a solution into feasible routes. Each solution is a permutation of customers without trip delimiters and can therefore be viewed as a giant TSP tour for a vehicle with a large enough capacity.

The EDUCATION procedure (Line 6) uses the ALNS operators to educate offspring C and inserts it back into the population. If C is infeasible, the EDUCATION procedure is iteratively applied until a modified version of C is feasible, which is then inserted into the population. The probabilities associated with the operators used in the EDUCATION procedure and the penalty parameters are updated by means of an adaptive weight adjustment procedure (AWAP) (Line 7).

At each iteration of the algorithm, all individuals in the population are ranked in decreasing order solution cost. The first n_e individuals of this ranking are called elite solutions which are carried over to the next iteration. Elite solutions are put through an aggressive INTENSIFICATION procedure, based on the ALNS algorithm (Line 8) in order to improve their quality.

If, at any iteration, the population size n_a reaches $n_p + n_o$, then a survivor selection mechanism is applied (Line 9). The population size, shown by n_a, changes during the algorithm as new offsprings are added, but is limited by $n_p + n_o$, where n_p is a constant denoting the size of the population initialized at the beginning of the algorithm and n_o is a constant showing the maximum allowable number of offsprings that can be inserted into the population. At each iteration of the algorithm, MUTATION is applied to a randomly selected individual from the population with probability p_m. If there are no improvements in the best known solution for a number of consecutive iterations it_r, the entire population undergoes a REGENERATION (Line 10). The HEA terminates when the number of iterations without improvement it_t is reached (Line 11).

Algorithm 1 The general framework of the HEA

1: *Initialization*: initialize a population with size n_p
2: **while** number of iterations without improvement $< it_t$ **do**
3: *Parent selection*: select parent solutions P_1 and P_2
4: *Crossover*: generate offspring C from P_1 and P_2
5: SPLIT: partition C into routes
6: EDUCATION: educate C with ALNS and insert into population
7: *AWAP*: update probabilities of the ALNS operators
8: INTENSIFICATION: intensify elite solution with ALNS
9: *Survivor selection*: if the population size n_a reaches $n_p + n_o$, then select survivors
10: *Diversification*: diversify the population with MUTATION or REGENERATION procedures
11: **end while**
12: Return best feasible solution

3.2.2 EDUCATION

The EDUCATION procedure is systematically applied to each offspring in order to improve its quality. The ALNS algorithm is used as a way of educating the solutions in the HEA. This is achieved by applying both the destroy and repair operators, and a number of removable nodes are modified in each iteration. An example of the removal and insertion phases is illustrated in Figure 3.1. The operators used within the HEA are either adapted or inspired from those employed by various authors (Ropke and Pisinger 2006a,b, Pisinger and Ropke 2007, Demir et al. 2012, Paraskevopoulos et al. 2008).

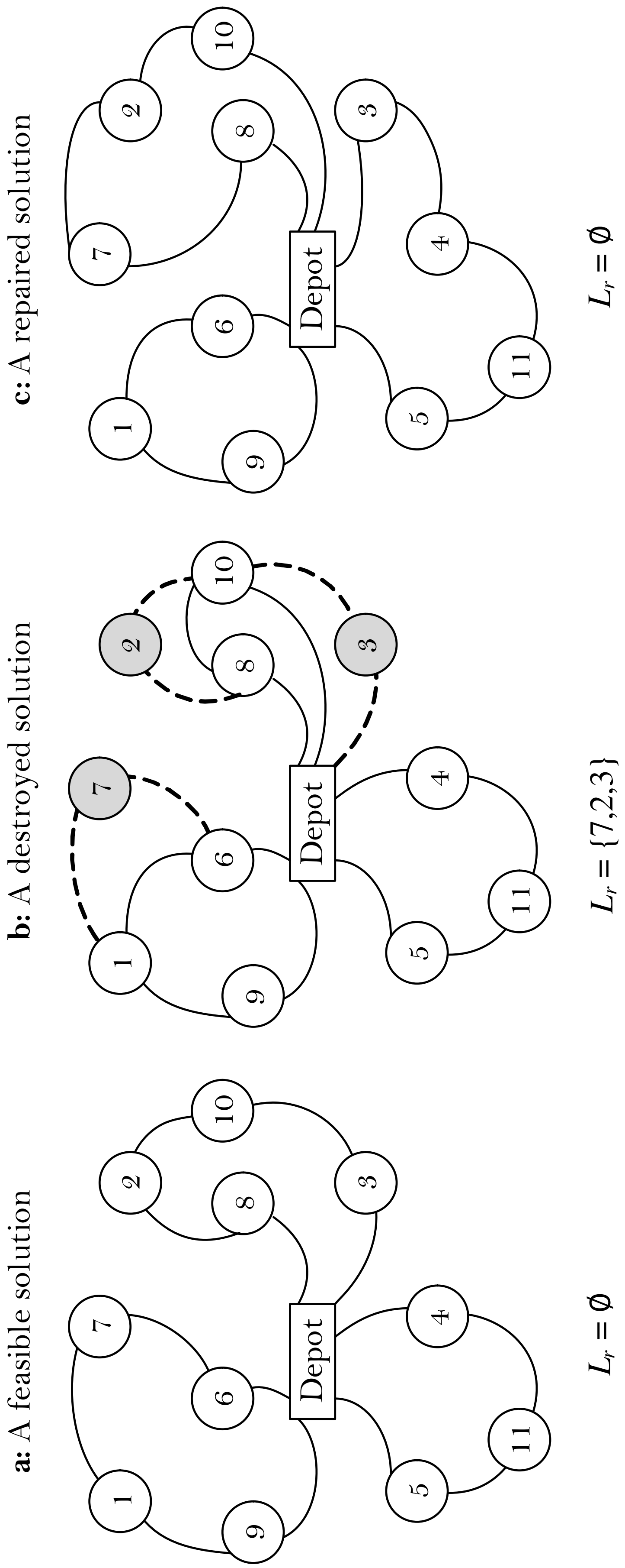


FIGURE 3.1: Illustration of the EDUCATION procedure

The EDUCATION procedure is detailed in Algorithm 2. All operators are repeated $O(n)$ times and the complexity given are the overall repeats. The removal procedure (line 4 of Algorithm 2) runs for n' iterations, removes n' customers from the solution and add to the removal list L_r, where n' is in the interval of removable nodes $[b_l^e, b_u^e]$. An insertion operator is then selected to iteratively insert the nodes, starting from the first customer of L_r, into the partially destroyed solution until L_r is empty (line 5). The feasibility conditions in terms of capacity and time windows for FT, FD, HT and HD, and in terms of fleet size for HT and HD, are always respected during the insertion process. We do not allow overcapacity of the vehicle and service start outside the time windows for all problem types, and we also do not allow the use of additional vehicles beyond the fixed fleet size for HT and HD. The removal and insertion operators are randomly selected according to their past performance and a certain probability as explained further in Section 3.2.2.3. The cost of an individual C before the removal is denoted by $\omega(C)$, and its cost after the insertion is denoted by $\omega(C^*)$.

Algorithm 2 EDUCATION

1: Initialize: An individual C, $\omega(C) \leftarrow$ cost of C, $\omega(C^*) \leftarrow 0$, iteration $\leftarrow 0$
2: **while** there is no improvement and C is feasible **do**
3: $\quad L_r = \varnothing$ and select a removal operator
4: $\quad$ Apply a removal operator to the individual C to remove a set of nodes and add them to L_r
5: $\quad$ Select an insertion operator and apply it to the partially destroyed individual C to insert the nodes of L_r
6: $\quad$ Let C^* be the new solution obtained by applying insertion operator
7: $\quad$ **if** $\omega(C^*) < \omega(C)$ and C^* is feasible **then**
8: $\quad\quad \omega(C) \leftarrow \omega(C^*)$
9: $\quad$ iteration $\leftarrow$ iteration + 1
10: **end while**
11: Return educated feasible solution

The heterogeneous fleet version of the ALNS that we use here was recently introduced in Chapter 5. It educates solutions by considering the heterogeneous fleet aspect. The ALNS integrates fleet sizing within the destroy and repair operators. In particular, if a node is removed, we check whether the resulting route can be served by a smaller vehicle. We then update the solution accordingly. If inserting a node requires additional vehicle capacity we then consider the option of using larger vehicles. For each node $i \in N_c \backslash L_r$, let $f^h(i)$ be the current vehicle fixed cost associated with the vehicle serving i. Let $\Delta(i)$ be the saving obtained as a result of using a removal operator on node i without considering the vehicle fixed cost. Let $f_1^{h*}(i)$ be the vehicle fixed cost after removal of node i. Consequently, $f_1^{h*}(i) < f^h(i)$ only if the route containing node i can be served by a smaller vehicle when removing node i. The savings in vehicle fixed cost can be expressed as $f^h(i) - f_1^{h*}(i)$, respectively. Thus, for each removal operator, the total savings of removing node $i \in N_c \backslash L_r$, denoted $RC(i)$, is calculated as follows:

$$RC(i) = \Delta(i) + (f^h(i) - f_1^{h*}(i)). \tag{3.1}$$

In a destroyed solution, the insertion cost of node $j \in L_r$ after node i is defined as $\Omega(i, j)$ for a given node $i \in N_c \backslash L_r$. Let $f_2^{h*}(i)$ be the vehicle fixed cost after the insertion of node i, i.e., $f_2^{h*} > f^h$ only if the route containing node i necessitates the use of a larger capacity vehicle after inserting node i. The cost differences in vehicle fixed cost can be expressed as $f_2^{h*}(i) - f^h(i)$. Thus, the total insertion cost of node $i \in N_c \backslash L_r$, for each insertion operator is

$$IC(i) = \Omega(i, j) + (f_2^{h*}(i) - f^h(i)). \tag{3.2}$$

3.2.2.1 Removal operators

Nine removal operators are used in the destroy phase of the EDUCATION procedure and are described in detail below. The first eight were used in several papers (Ropke and Pisinger 2006a,b, Demir et al. 2012, Koç et al. 2015), whereas the last one is new. We introduce the ninth one as a new removal operator specific to FT, FD HT and HD.

1.*Random removal* (RR): The RR operator randomly selects a node $j \in N \backslash \{0\} \backslash L_r$, removes it from the solution.

2.*Worst distance removal* (WDR): The purpose of the WDR operator is to choose a number of expensive nodes according to their distance based cost. The cost of a node $j \in N \backslash \{0\} \backslash L_r$ is the distance from its predecessor i and its distance to its successor k. The WDR operator iteratively removes nodes j^* from the solution where $j^* = \arg\max_{j \in N \backslash \{0\} \backslash L_r} \{d_{ij} + d_{jk} + f^h(i) - f_1^{h*}(i)\}$.

3.*Worst time removal* (WTR): The WTR operator is a variant of the WDR operator. For each node $j \in N \backslash \{0\} \backslash L_r$ costs are calculated, depending on the deviation between the arrival time z_j and the beginning of the time window a_j. The WTR operator iteratively removes customers from the solution, where $j^* = \arg\max_{j \in N \backslash \{0\} \backslash L_r} \{|z_j - a_j| + f^h(i) - f_1^{h*}(i)\}$. The ALNS iteratively applies this process to the solution after each removal.

4.*Neighborhood removal* (NR): In a given solution with a set $\Re$ of routes, the NR operator calculates an average distance $\bar{d}(R) = \sum_{(i,j)\in R} d_{ij}/|R|$ for each route $R \in \Re$, and selects a node $j^* = \arg\max_{(R\in\Re;j\in R)}\{\bar{d}(R) - d_{R\setminus\{j\}} + f^h(i) - f_1^{h*}(i)\}$, where $d_{R\setminus\{j\}}$ denotes the average distance of route R excluding node j.

5. *Shaw removal* (SR): The general idea behind the SR operator, which was introduced by Shaw (1998), is to remove a set of customers that are related in a predefined way and are therefore easy to change. The SR operator removes a set of n' similar customers. The similarity between two customers i and j is defined by the relatedness measure $\delta(i,j)$. This includes four terms: a distance term d_{ij}, a time term $|a_i - a_j|$, a relation term l_{ij}, which is equal to -1 if i and j are in the same route, and 1 otherwise, and a demand term $|q_i - q_j|$. The relatedness measure is given by

$$\delta(i,j) = \varphi_1 d_{ij} + \varphi_2|a_i - a_j| + \varphi_3 l_{ij} + \varphi_4|q_i - q_j|, \tag{3.3}$$

where φ_1 to φ_4 are weights that are normalized to find the best candidate solution. The operator starts by randomly selecting a node $i \in N\setminus\{0\}\setminus L_r$, and selects the node j^* to remove where $j^* = \arg\min_{j\in N\setminus\{0\}\setminus L_r}\{\delta(i,j) + f^h(i) - f_1^{h*}(i)\}$. The operator is iteratively applied to select a node which is most similar to the one last added to L_r.

6. *Proximity-based removal* (PBR): This operator is a second variant of the classical Shaw removal operator. The selection criterion of a set of routes is solely based on the distance. Therefore, the weights are $\varphi_1 = 1$ and $\varphi_2 = \varphi_3 = \varphi_4 = 0$.

7. *Time-based removal* (TBR): The TBR operator removes a set of nodes that are related in terms of time. It is a special case of the Shaw removal operator where $\varphi_2 = 1$ and $\varphi_1 = \varphi_3 = \varphi_4 = 0$.

8. *Demand-based removal* (DBR): The DBR operator is yet another variant of the Shaw removal operator with $\varphi_4 = 1$ and $\varphi_1 = \varphi_2 = \varphi_3 = 0$.

9. *Average cost per unit removal* (ACUTR): The average cost per unit (ACUT) is described by Paraskevopoulos et al. (2008) to measure the utilization efficiency of a vehicle $\Pi(R)$ on a given route R. $\Pi(R)$ is expressed as the ratio of the total travel cost and fixed vehicle cost over the total demand carried by a vehicle k traversing route R:

$$\Pi(R) = \frac{\sum_{(i,j)\in A} c_{ij}x_{ij}^k + e^k}{\sum_{i\in N\setminus\{0\}} q_i x_{ij}^k}. \tag{3.4}$$

The aim of the ACUTR operator is to calculate the cost of each route and remove the one with the least $\Pi(R)$ value from the solution.

3.2.2.2 Insertion operators

Three insertion operators are used in the repair phase of the EDUCATION procedure. The first two were used in several papers (Ropke and Pisinger 2006a,b, Demir et al. 2012, Koç et al. 2015), whereas the last one is new. We introduce the third one as a new insertion operator specific to FT and HT.

1. *Greedy insertion* (GI): The aim of this operator is to find the best possible insertion position for all nodes in L_r. For node $i \in N \backslash L_r$ succeeded in the destroyed solution by $k \in N \backslash \{0\} \backslash L_r$, and node $j \in L_r$ we define $\gamma(i,j) = d_{ij} + d_{jk} - d_{ik}$. We find the least-cost insertion position for $j \in L_r$ by $i^* = \arg\min_{i \in N \backslash L_r} \{\gamma(i,j) + f_2^{h*}(i) - f^h(i)\}$. This process is iteratively applied to all nodes in L_r.

2. *Greedy insertion with noise function* (GINF): The GINF operator is based on the GI operator but extends it by allowing a degree of freedom in selecting the best place for a node. This is done by calculating the noise cost $\upsilon(i,j) = \gamma(i,j) + f_2^{h*}(i) - f^h(i) + d_{\max} p_n \epsilon_{ij}$ where $d_{\max}$ is the maximum distance between all nodes, p_n is a noise parameter used for diversification and is set equal to 0.1, and ϵ_{ij} is a random number in $[-1, 1]$ for $i \in N \backslash L_r, j \in L_r$.

3. *Greedy insertion with en-route time* (GIET): This operator calculates the *en-route* time difference $\eta(i,j)$ between before and after inserting the customer $j \in L_r$. For node $i \in N \backslash L_r$ succeeded in the destroyed solution by $k \in N \backslash \{0\} \backslash L_r$, and node $j \in L_r$, we define $\eta(i,j) = \tau_{ij} + \tau_{jk} - \tau_{ik}$ where τ_{ij} is the *en-route* time from node i to node j. We find the least-cost insertion position for $j \in L_r$ by $i^* = \arg\min_{i \in N \backslash L_r} \{\eta(i,j) + f_2^{h*}(i) - f^h(i)\}$.

3.2.2.3 Adaptive weight adjustment procedure

Each removal and insertion operator has a certain probability of being chosen in every iteration. The selection criterion is based on the historical performance of every operator and is calibrated by a roulette-wheel mechanism (Ropke and Pisinger 2006a, Demir et al. 2012). After it_w iterations of the roulette wheel segmentation, the probability of each operator is recalculated according to its total score. Initially, the probabilities of each removal and insertion operator are equal. Let p_i^t be the probability of operator i in the last it_w iterations, $p_i^{t+1} = p_i^t(1 - r_p) + r_p\pi_i/\tau_i$, where r_p is the roulette wheel probability, for operator i; π_i is its score and τ_i is the number of times it was used during the last segment. At the start of each segment, the scores of all operators are set to zero. The scores are changed by σ_1 if a new best solution is found, by σ_2 if the new solution is better than the current solution and by σ_3 if the new solution is worse than the current solution.

3.2.3 Initialization

The procedure used to generate the initial population is based on a modified version of the Clarke and Wright and ALNS algorithms. An initial individual solution is obtained by applying Clarke and Wright algorithm and by selecting the largest vehicle type for each route. Then, until the initial population size reaches n_p, new individuals are created by applying to the initial solution operators based on random removals and greedy insertions with a noise function (see Section 3.2.2). We have selected these two operators in order to diversify the initial population. The number of nodes removed is randomly chosen from the initialization interval $[b_l^i, b_u^i]$, which is defined by a lower and an upper bound calculated as a percentage of the total number of nodes in an instance.

3.2.4 Parent selection

In evolutionary algorithms, the evaluation function of individuals is often based on the solution cost. However, this kind of evaluation, does not take into account other important factors such as the diversity of the population which plays a critical role. Vidal et al. (2012) proposed a new method, named *biased fitness* $bf(C)$, to tackle this issue. This method considers the cost of an individual C, as well as its *diversity contribution* $dc(C)$ to the population. This function is continuously updated and is used to measure the quality of individuals during selection phases. The $dc(C)$ is defined as

$$dc(C) = \frac{1}{n_c} \sum_{C' \in N_c} \beta(C, C'), \tag{3.5}$$

where N_c is the set of the n_c closest neighbours of C in the population. Thus, $dc(C)$ calculates the average distance between C and its neighbours in N_c. The distance between two parents $\beta(C, C')$ is the number of pairs of adjacent requests in C which are no longer adjacent, (called broken), in C'. For example, let $C = \{4, 5, 6, 7, 8, 9, 10\}$ and $C' = \{10, 7, 8, 9, 5, 6, 4\}$, in C' the pairs $\{4, 5\}$, $\{6, 7\}$ and $\{9, 10\}$ are broken and $\beta(C, C') = 3$. The algorithm selects the broken pairs distance (see Prins 2009) to compute the distance β. The main idea behind $dc(C)$ is to assess the differences between individuals.

The evaluation function of an individual C in a population is

$$bf(C) = r_c(C) + (1 - \frac{n_e}{n_a})r_{dc}(C), \tag{3.6}$$

which is based on the rank $r_c(C)$ of solution cost, and on the rank $r_{dc}(C)$ of the *diversity contribution*. The rank $r_{dc}(C)$ is based on the diversity contribution calculated in equation (3.5), according to which the solutions are ranked in decreasing order of their $dc(C)$ value. In (3.6), n_e is the number of elite individuals and n_a is the current number of individuals.

The HEA selects two parents through a binary tournament to yield an offspring. The selection process randomly chooses two individuals from the population and keeps the one having the best biased fitness.

3.2.5 Crossover

Two parents undergo the classical Ordered Crossover or OX without trip delimiters, following the parent selection phase. The OX operator is well suited for cyclic permutations, and the giant tour encoding allows recycling crossovers designed for the traveling salesman problem (TSP) (see Prins 2004, 2009). Initially, two positions i and j are randomly selected in the first parent P_1. Subsequently, the substring $(i, ..., j)$ is copied into the first offspring O_1, at the same positions. The second parent P_2 is then swept cyclically from position $j + 1$ until the last node of P_2, and continues from the first node of P_2 and continue until position $i - 1$, such that empty positions in O_1 are filled. The second offspring O_2 is generated likewise by exchanging the roles of P_1 and P_2. In the original version of OX, two offsprings are obtained. However in the HEA, we only randomly select one offspring. Figure 3.2 shows an illustration of ORDERED CROSSOVER.

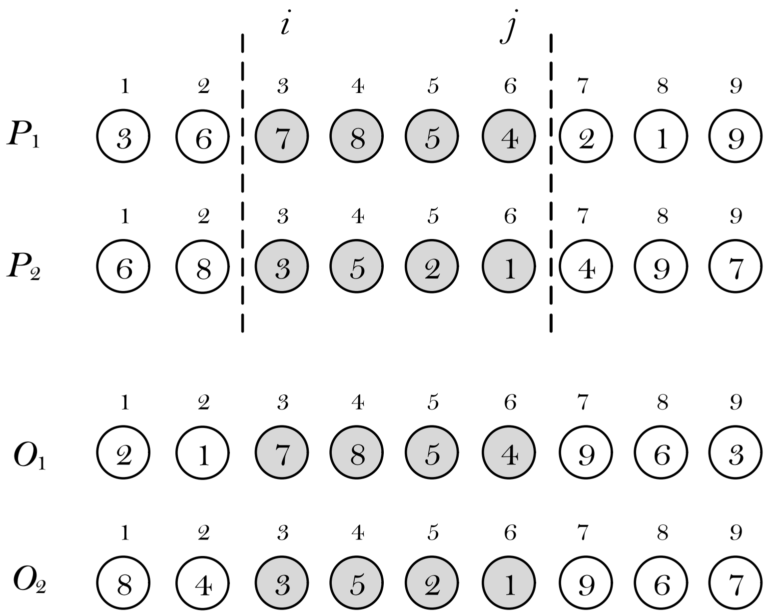


FIGURE 3.2: Illustration of ORDERED CROSSOVER

3.2.6 SPLIT algorithm

This algorithm was successfully applied in evolutionary based algorithms for several routing problems (Prins 2004, 2009, Vidal et al. 2012, 2013). We propose an advanced tour splitting procedure, denoted by SPLIT, which is embedded in the HEA to segment a giant tour and to determine the fleet mix composition. This is achieved through a controlled exploration of infeasible solutions (see Cordeau et al. 2001 and Nagata et al. 2010), by relaxing the limits on time windows and vehicle capacities. Violations of these limits are penalized through an objective function containing extra terms to acccount for infeasibilities. This is in contrast to Prins (2009) who does not allow infeasibilities, and in turn solves a resource-constrained shortest path problem using dynamic programming to determine the best fleet mix on a given solution. Our implementation also differs from those of Vidal et al. (2013) since it allows for infeasibilities that are not just related to time windows or load, but also to the fleet size in the case of HT and HD.

We now describe the SPLIT algorithm. Let $\Re$ be the set of all routes in individual C, and let R be a route. While formally R is a vector, for convenience we denote the number of its components by $|R|$. Therefore, $R = (i_0 = 0, i_1, i_2, \ldots, i_{|R|-1}, i_{|R|} = 0)$, we also write

$i \in R$ if i is a component of R, and $(i, j) \in R$ if i and j appear in succession in R. Let z_t be the arrival time at the t^{th} customer in R, thus the time window violation of route R is $\sum_{t=1}^{|R|-1} \max\{z_t - b_{i_t}, 0\}$. The total load for route R is $\sum_{t=1}^{|R|-1} q_{i_t}$, and we consider solutions with a total load not exceeding twice the capacity of the largest vehicle given by Q_{max} (Vidal et al. 2013). Furthermore, for route R and for each vehicle type k we compute $y(k)$, which is the number of vehicles of type k used in the solution.

Let λ_t, λ_l and λ_f represent the penalty values for any violations of the time windows, the vehicle capacity and the fleet size, respectively. The variable x_{ij}^k is equal to 1 if customer i immediately precedes customer j visited by vehicle of type k. The fixed cost associated with using a vehicle of type $k \in K$ is denoted by e_k. For each route $R \in \Re$ traversed by vehicle $k \in K$, the cost including penalties is

$$\nu(R, k) = \sum_{(i,j)\in R} c_{ij}^k x_{ij}^k + e_k + \lambda_t \sum_{t=1}^{|R|-1} \max\{z_t - b_{i_t}, 0\} + \lambda_l \max\{\sum_{t=1}^{|R|-1} q_{i_t} - Q_{max}, 0\}, \quad (3.7)$$

which brings various objectives together to be able to guide to the search towards infeasible solutions. Thus, the total cost of individual C is

$$\Delta(C) = \sum_{R\in\Re} \sum_{k\in K} \nu(R, k) + \lambda_f \sum_{k\in K} \max\{0, y(k) - n_k\}, \quad (3.8)$$

where n_k is set equal to a sufficiently large number (e.g., n) for FT and FD, in order for the last term in Equation (3.8) to be zero.

The Split operator works on a graph $\mathcal{G}' = (\mathcal{N}, \mathcal{A}')$ where $\mathcal{N} = \{v_0, v_1, v_2, \ldots, v_n\}$ is an ordered set of nodes in which $v_0 = 0$ and $\{v_1, v_2, \ldots, v_n\}$ is the order of customer nodes visited in the giant tour. The set of arcs in $\mathcal{G}'$ is defined by $\mathcal{A}' = \{(v_i, v_j) | v_i \in \mathcal{N}, v_j \in \mathcal{N}, v_j > v_i\}$, where each arc $(v_i, v_j) \in \mathcal{A}'$ corresponds to a route $R = \{0, v_{i+1}, v_{i+2}, \ldots, v_j, 0\}$ in $\Re$. The cost of route R is calculated as explained above. The Split procedure solves a shortest path problem on graph G' from source node 0 to destination node v_n, where the arcs that appear on the shortest path correspond to the partitioning of the new routes in individual C.

Figure 3.3 shows the steps of this advanced procedure applied to an FD instance. The arc costs, demands and time windows are given in Figure 3.3.a. In particular, the number in bold within the parentheses associated with each node is the demand for that customer;

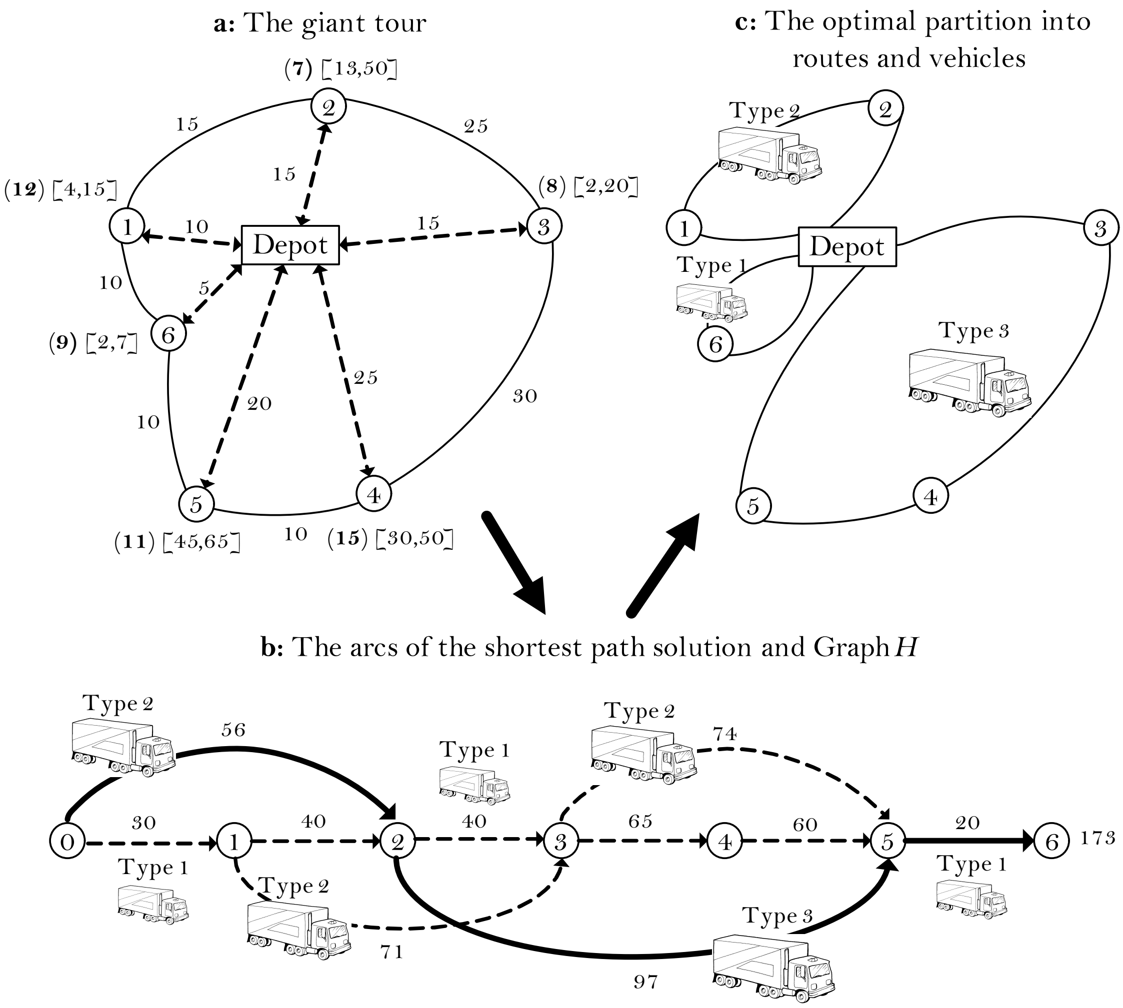


FIGURE 3.3: Illustration of procedure SPLIT

the two numbers within brackets define the time window. Service times are identical and equal to 4 for each customer, and three different types of vehicles are available. The capacity q_k and fixed cost e_k of vehicles of type {1,2,3} are $q_1 = 10$, $q_2 = 20$, $q_3 = 30$ and $e_1 = 6$, $e_2 = 8$, $e_3 = 10$, respectively. The algorithm starts with a giant TSP tour which includes six customers and uses one vehicle with unlimited capacity. The SPLIT algorithm computes an optimal compound segmentation in three routes corresponding to three sequences of customers {1,2}, {3,4,5} and {6} with three vehicle choices, Type 2, Type 3 and Type 1, respectively, as shown in Figure 3.3.b. The resulting solution is shown in Figure 3.3.c. An optimal partitioning of the giant tour into routes for offspring C corresponds to a minimum-cost path.

The penalty parameters of the SPLIT algorithm are initially set to an initial value and are dynamically adjusted during the algorithm. If an individual is still infeasible after the first EDUCATION procedure, then the penalty parameters are multiplied by λ_m and the EDUCATION procedure restarts. When this solution becomes feasible, the parameters are reset to their initial values. These values are $\lambda_t = \lambda_l = \lambda_f = 3, \lambda_m = 10$.

3.2.7 INTENSIFICATION

We introduce a two-phase aggressive INTENSIFICATION procedure to improve the quality of elite individuals. This procedure intensifies the search within promising regions of the solution space. The detailed pseude-code of this method is shown in Algorithm 3. The algorithm starts with an elite list of solutions L_e, which takes the best n_e individuals from the main population as measured by equation (3.6). Step 1 is similar to the main EDUCATION procedure (Section 3.2.2). Step 2 attempts to explore different regions of the search space with the RR operator, intensifies this area by applying the GI operator for FD and HD, and GIET for FT and HT, to a partially destroyed solution. Steps 1 and 2 terminate when there is no improvement to the solution and the main loop terminates when n_e successive iterations have been performed.

Due to the difficulty of the problems considered in this paper, we have developed a two-phase aggressive INTENSIFICATION procedure after having tried several variants such as one-phase with only Step 1 or Step 2, three-phase with Step 1, Step 2 and Step 1 and various other combinations.

3.2.8 Survivor selection

In population-based metaheuristics, avoiding premature convergence is a key challenge. Ensuring the diversity of the population, in other words to search a different location in the solution space during the algorithm, in the hope of being closer to the best known or optimal solutions constitutes a major trade-off between solutions in a population. The method of Vidal et al. (2012), aims to ensure the diversity of the population and preserve the elite solutions. The second part of this method is the survivor selection process (the first part was discussed in Section 3.2.4). In this way, elite individuals are protected.

Algorithm 3 INTENSIFICATION

Initialize $L_e = \{\chi_1, \ldots, \chi_n\}$, $i \leftarrow 1$
while all elite solutions are intensified **do**
  $\chi \leftarrow \chi_i$
  Step 1
  while there is improvement and elite solution χ is feasible **do**
    $L_r = \varnothing$ and select a removal operator
    Apply to the elite solution χ to remove nodes and add them to L_r
    Select an insertion operator and apply it to the destroyed elite solution χ by inserting the node of L_r
    Let χ^* be the new solution obtained by applying insertion operator
    if $\omega(\chi^*) < \omega(\chi)$ **then**
      $\omega(\chi) \leftarrow \omega(\chi^*)$
  end while
  Step 2
  while there is improvement and χ^* is feasible **do**
    $L_r = \varnothing$ and apply RR operator to the elite solution χ to remove nodes and add them to L_r
    Apply GI or GIET operator to the partially destroyed elite solution χ by inserting the node of L_r
    Let χ^* be the new elite solution obtained by applying insertion operator
    if $\omega(\chi^*) < \omega(\chi)$ **then**
      $\omega(\chi) \leftarrow \omega(\chi^*)$
  end while
  $i \leftarrow i + 1$
end while
Return elite solutions

3.2.9 Diversification

The efficient management of feasible solutions plays a significant role in population diversity. The performance of the HEA is improved by applying a MUTATION after the EDUCATION procedure. Over the iterations, individuals tend to become more similar, making it difficult to avoid premature convergence. To overcome this difficulty, we introduce a new scheme in order to increase the population diversity. The diversification stage includes two procedures, namely REGENERATION and MUTATION, representations of which are shown in Figure 3.4.

A REGENERATION procedure (Figure 3.4.a) takes place when the maximum allowable iterations for REGENERATION it_r is reached without an improvement in the best solution value. In this procedure, the n_e elite individuals are preserved and are transferred to the next generation. The remaining $n_p - n_e$ individuals, which are ranked according to their

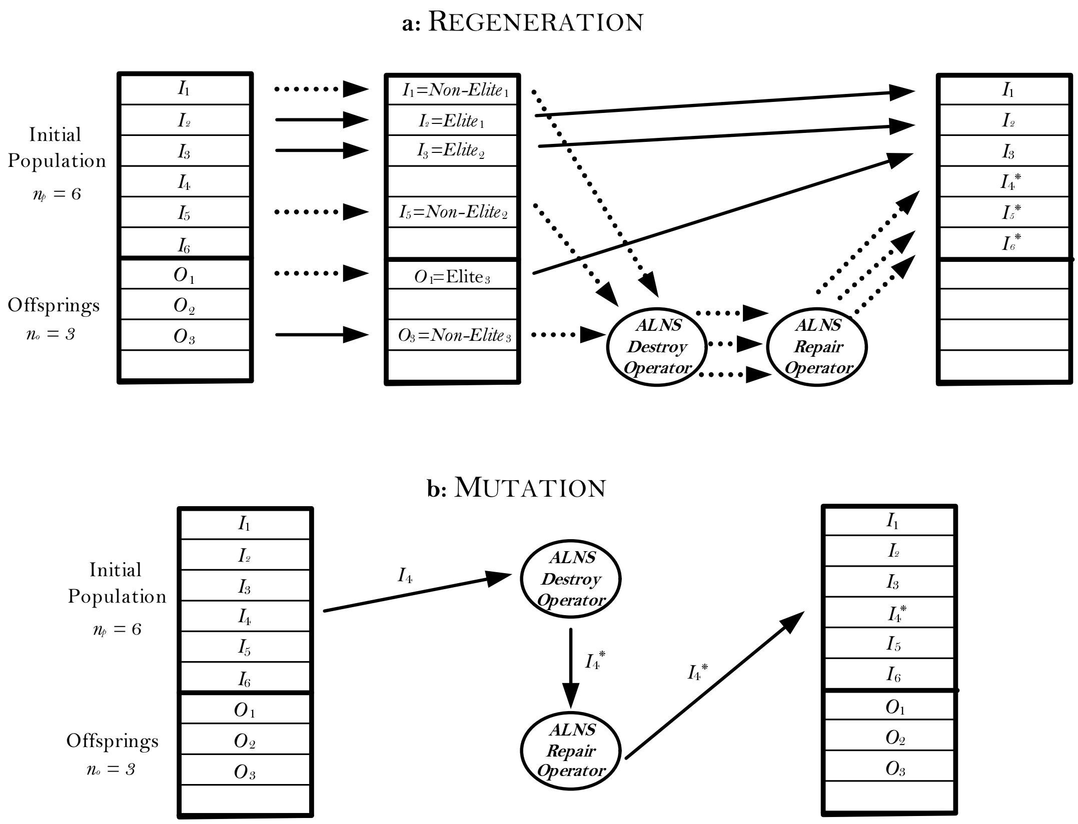


FIGURE 3.4: Illustration of the diversification stage

biased fitness, are subjected to the RR and GINF operators, to create new individuals. At the end of this procedure, only n_p new individuals are kept in the population.

The MUTATION procedure is applied with probability p_m. Figure 3.4.b illustrates the MUTATION procedure. In this procedure, an individual C different from the best solution is randomly selected. Two randomized structure based ALNS operators, the RR and the GINF, are then used to change the positions of a specific number of nodes, which are chosen from the interval $[b_l^m, b_u^m]$ of removable nodes in the MUTATION procedure.

3.3 Computational Experiments

This section presents the results of computational experiments performed in order to assess the performance of the HEA. The HEA was implemented in C++ and run on a computer with one gigabyte RAM and Intel Xeon 2.6 GHz processor. We first describe the benchmark instances and the parameters used within the algorithm. This is followed by a presentation of the results.

3.3.1 Data sets and experimental settings

The benchmark data sets of Liu and Shen (1999b), derived from the classical Solomon (1987) VRPTW instances with 100 nodes, are used as the test-bed. These sets include 56 instances, split into a random data set R, a clustered data set C and a semi-clustered data set RC. Sets shown by R1, C1 and RC1 have a short scheduling horizon and small vehicle capacities, in contrast to sets denoted R2, C2 and RC2 with a long scheduling horizon and large vehicle capacities. Liu and Shen (1999b) introduced three types of cost structures, namely large, medium and small, and have denoted them by A, B and C, respectively. The authors also introduced several vehicle types with different capacities and fixed vehicle costs for each of the 56 instances. This results in a total of 168 benchmark instances for FT or FD.

The benchmark set used by Paraskevopoulos et al. (2008) for HT is a subset of the FT instances, in which the fleet size is set equal to that found in the best known solutions of Liu and Shen (1999a). In total, there are 24 benchmark instances derived from Liu and Shen (1999a) for HT. We use the same set for HD, with the new objective.

Evolutionary algorithms use a set of correlated parameters and configuration decisions. In our implementation, we initially used the parameters suggested by Vidal et al. (2012, 2013) for the genetic algorithm, but we have conducted several experiments to further fine-tune these parameters on instances C101A, C203A, R101A, R211A, RC105A and RC207A. Following these tests, the following parameter values were used in our experiments: $it_t = 5000, it_r = 2000, it_w = 500, n_p = 25, n_o = 25, n_e = 10, n_c = 3, p_m \in [0.4, 0.6], [b_l^i, b_u^i] = [0.3, 0.8], [b_l^e, b_u^e] = [0.1, 0.16], [b_l^m, b_u^m] = [0.1, 0.16], \sigma_1 = 3, \sigma_2 = 1, \sigma_3 = 0$. For the Adaptive Large Neighborhood Search (ALNS), we have used the same parameter values as in Demir et al. (2012), namely $r_p = 0.1, \varphi_1 = 0.5, \varphi_2 = 0.25, \varphi_3 = 0.15, \varphi_4 = 0.25$. All of these settings are identical for all four considered problems.

Table 3.1 presents the results of a fine-tuning experiment on parameters n_p and n_o, and to test the effect of these parameters on the solution quality.

The table shows the percent gap between the solution value obtained by the HEA and the best-known solution (BKS) value, averaged over the six chosen instances. The maximum

TABLE 3.1: Average percentage deviations of the solution values found by the HEA from best-known solution values with varying n_p and n_o

	n_o				
n_p	10	25	50	75	100
10	0.42	0.26	0.38	0.56	0.69
25	0.19	**0.11**	0.26	0.37	0.49
50	0.39	0.29	0.30	0.45	0.57
75	0.56	0.42	0.51	0.61	0.68
100	0.67	0.53	0.61	0.72	0.78

population size is dependent on n_p and n_o, both of which have a significant impact on the solution quality, where the best setting is obtained with $n_p = n_o = 25$.

Table B.3 in Appendix B presents the number of iterations as a percentage operators by education operator. Table B.3 shows that over nine removal operators ACUTR operator is used by 15.48% on average, which is the highest one. This results indicated that ACUTR operator is able to improve solution quality, this is probably due to its heterogeneous based structure.

3.3.2 Comparative analysis

We now present a comparative analysis of the results of the HEA with those reported in the literature. In particular, we compare ourselves against LSa (Liu and Shen 1999a), LSb (Liu and Shen 1999b), T-RR-TW (Dell'Amico et al. 2007), ReVNTS (Paraskevopoulos et al. 2008), MDA (Bräysy et al. 2008), BPDRT (Bräysy et al. 2009), AMP (Repoussis and Tarantilis 2010) and UHGS (Vidal et al. 2014). The comparisons are presented in tables, where the columns show the total cost (TC), and percent deviations (Dev) of the values of solutions found by each method with respect to the HEA. The first column displays the instance sets and the number of instances in each set in parentheses. The rows named Avg (%), Min (%) and Max (%) show the average, minimum and maximum deviations across all benchmark instances, respectively. A negative deviation shows that the solution found by the HEA is of better quality. In the column labeled BKS, "=" shows the total number of matches and "<" shows the number of new BKS found for each instance set.

Ten separate runs are performed for each instance, the best one of which is reported. For each instance, a boldface refers to match with current BKS, where as a boldface with a

"*" indicates new BKS. For detailed results, the reader is referred to Appendix B. Tables B.1–B.9 present the fixed vehicle cost (VC), the distribution cost (transportation cost) (DC), the computational time in minutes (Time) and the actual number of vehicles used (Mix), where the letters A–E correspond to the vehicle types and the upper numbers denote the number of each type of vehicle used. For example, (A^2B^1) indicates that two vehicles of type A and one vehicle of type B are used in the solution.

Tables 3.2 and 3.3 summarize the average comparison results of the current state-of-the-art solution methods for FT and FD, compared with the HEA. According to Tables 3.2 and 3.3, the HEA is highly competitive, with average deviations ranging from −6.78% to 0.03% and a worst-case performance of 0.66% for FT. The average performance of our HEA is better than that of all the competitors for FT, except for the algorithm of Vidal et al. (2014) which is slightly better on average. However, the HEA found 17 new best solution and outperforms this algorithm on to the second type of FT instances, which are less tight in terms of vehicle capacity. As for FD, average cost reductions range from −0.90% to −0.02% and the worst- case performance is 0.94%. The HEA outperforms all other algorithms in the literature for FD, including the UHGS of Vidal et al. (2014).

Table 3.4 presents the comparison results for each HT instance against LSa and ReVNTS. We note that LSa only solved FT and not HT, which was the basis for setting the number of available vehicles in ReVNTS. The results show that the HEA outperforms both methods and yields higher quality solutions within short computation times. On average, the total cost reductions obtained were −12.68% and −0.34% compared to LSa and ReVNTS, with minimum deviations of −29.47% and −2.01% and maximum deviations of −1.26% and 0.35%, respectively. Finally, Table 3.5 shows the results obtained on the newly introduced HD.

Looking at the results obtained on the HT instances, on average the HEA yields 1.23% and 0.13% lower vehicle fixed costs than the LSa and ReVNTS, respectively. The HEA decreases the distribution cost (en-route time based cost) by 42.19% and 1.03%, compared with LSa and ReVNTS, respectively. These results indicate that the HEA is able find better fleet mix composition and lower distribution costs than the other methods. The results also show that when ReVNTS and HEA find the same fleet mix composition on a given instance, the distribution costs are lower in the solution found by the HEA, indicating that our algorithm is more effective in finding better routes.

TABLE 3.2: Average results for FT

Instance set	T-RR-TW		ReVNTS		MDA		AMP		UHGS		HEA	BKS	
	TC	Dev	TC	Dev	TC	Dev	TC	Dev	TC	Dev	TC	=	<
R1A (12)	4180.83	−1.51	4128.48	−0.24	4131.31	−0.31	4113.89	0.12	**4103.16**	0.38	4118.70	0	0
R1B (12)	1927.57	−1.65	1902.19	−0.31	1898.88	−0.13	1896.83	−0.03	**1891.63**	0.25	1896.35	0	**1***
R1C (12)	1615.44	−2.56	1582.18	−0.45	1579.17	−0.26	1578.12	−0.19	**1574.32**	0.05	1575.09	**1**	0
C1A (9)	7229.02	−1.20	7143.35	0.00	7141.15	0.03	7139.96	0.05	**7138.93**	0.06	7143.35	**2**	0
C1B (9)	2384.77	−0.99	2361.78	−0.02	2365.49	−0.18	2359.82	0.06	**2359.63**	0.07	2361.29	**2**	**1***
C1C (9)	1629.70	−0.62	1621.09	−0.09	1621.83	−0.14	**1618.91**	0.04	1619.18	0.00	1619.18	**6**	0
RC1A (8)	5117.96	−3.49	4961.69	−0.33	4948.53	−0.07	4948.02	−0.06	**4915.10**	0.61	4945.14	0	0
RC1B (8)	2163.51	−1.35	2142.65	−0.37	2129.60	0.24	2136.73	−0.09	**2129.04**	0.27	2134.74	0	**2***
RC1C (8)	1784.51	−1.36	1769.93	−0.53	1758.29	0.13	1762.34	−0.10	**1752.19**	0.48	1760.59	0	0
R2A (11)	3568.97	−9.06	3304.57	−0.98	3310.70	−1.17	3287.80	−0.47	**3267.31**	0.16	3272.48	**2**	**1***
R2B (11)	1727.04	−17.40	1498.97	−1.88	1495.37	−1.64	1487.09	−1.08	1480.30	−0.61	**1471.27***	**1**	**7***
R2C (11)	1436.22	−15.30	1281.31	−2.84	1257.65	−0.94	1260.97	−1.20	**1237.79**	0.66	1245.97	0	0
C2A (8)	6267.75	−9.07	5759.02	−0.22	5797.38	−0.89	5749.98	−0.06	5760.29	−0.24	**5746.44***	**4**	0
C2B (8)	1897.62	−8.53	1754.07	−0.32	1756.08	−0.43	1748.99	−0.03	1750.37	−0.11	**1748.52***	**2**	**1***
C2C (8)	1276.29	−4.78	1232.98	−1.22	1223.86	−0.47	1224.08	−0.49	1221.17	−0.25	**1218.12***	**4**	**2***
RC2A (8)	4752.95	−8.24	4406.28	−0.34	4399.12	−0.18	4388.88	0.05	**4381.73**	0.21	4391.16	0	0
RC2B (8)	2156.11	−15.40	1888.83	−1.13	1899.20	−1.68	1874.86	−0.38	1877.84	−0.54	**1867.80***	0	**2***
RC2C (8)	1828.95	−19.50	1567.22	−2.43	1562.19	−2.10	1541.13	−0.72	1545.29	−0.99	**1530.08***	0	0
Min (%)		−19.50		−2.84		−2.10		−1.20		−0.99			
Avg (%)		−6.78		−0.76		−0.57		−0.25		0.03			
Max (%)		−0.62		0.00		0.24		0.12		0.66			
All												**24**	**17***
Runs	1		1		3		1		10		10		
Processor	P 600M		PIV 1.5GHz		Ath 2.6GHz		PIV 3.4GHz		Opt 2.2GHz		Xe 2.6GHz		
Avg Time	14.15		20.00		10.97		16.67		5.08		4.83		

TABLE 3.3: Average results for FD

Instance set	MDA		BPDRT		UHGS		HEA	BKS	
	TC	Dev	TC	Dev	TC	Dev	TC	=	<
R1A (12)	4068.59	−0.67	4060.96	−0.48	**4031.28**	0.25	4041.46	0	0
R1B (12)	1854.60	−0.82	–	–	1841.43	−0.11	**1839.39***	0	**4***
R1C (12)	1539.48	−0.91	1539.90	−0.93	1530.25	−0.30	**1525.56***	0	**8***
C1A (9)	7085.56	−0.03	7085.91	−0.04	**7082.98**	0.00	**7082.98**	**9**	0
C1B (9)	2335.11	−0.09	–	–	**2332.89**	0.00	2332.90	**9**	0
C1C (9)	1615.75	−0.02	1615.40	−0.01	1615.49	−0.01	**1615.38***	**9**	**0**
RC1A (8)	4944.48	−0.57	4935.52	−0.38	**4891.25**	0.51	4916.41	0	0
RC1B (8)	2121.62	−0.87	–	–	2107.08	−0.18	**2103.21***	**0**	**7***
RC1C (8)	1741.78	−0.94	1749.66	−1.40	1734.36	−0.51	**1725.44***	**2**	**6***
R2A (11)	3193.41	−1.36	3180.59	−0.96	3151.96	−0.05	**3150.29***	**7**	**4***
R2B(11)	1392.92	−3.06	–	–	1351.905	−0.02	**1351.52***	**4**	**2***
R2C (11)	1149.65	−2.06	1149.11	−2.01	1128.708	−0.20	**1126.42***	**5**	**4***
C2A (8)	5690.87	−0.07	5689.40	−0.04	**5686.75**	0.00	**5686.75**	**8**	0
C2B (8)	1698.51	−0.69	–	–	**1686.75**	0.00	**1686.75**	**8**	0
C2C (8)	1186.03	−0.07	1185.70	−0.04	**1185.19**	0.00	**1185.19**	**8**	0
RC2A (8)	4241.33	−0.73	4231.25	−0.49	**4210.10**	0.00	**4210.10**	**5**	**1***
RC2B (8)	1704.13	−1.04	–	–	1686.63	−0.01	**1686.47***	0	**5***
RC2C (8)	1374.55	−1.11	1385.32	−1.91	1358.24	0.08	1359.33	**1**	**3***
Min (%)		−4.30		−7.74		−1.49			
Avg (%)		−0.90		−0.74		−0.02			
Max (%)		0.07		0.10		0.94			
All								**75**	**44***
Runs	3		1		10		10		
Processor	Ath 2.6G		Duo 2.4G		Opt 2.2G		Xe 2.6G		
Avg Time	3.56		–		4.72		4.56		

TABLE 3.4: Results for HT

Instance set	LSa			ReVNTS			HEA					BKS	
	Mix	TC	Dev	Mix	TC	Dev	DC	VC	Mix	TC	Time	=	<
R101A	$A^1B^{11}C^{11}D^1$	5061	−10.29	$B^{10}C^{11}D^1$	**4583.99**	0.10	1998.76	2590	$B^{10}C^{11}D^1$	4588.76	5.49	0	0
R102A	$A^1B^4C^{14}D^2$	5013	−13.25	$B^3C^{14}D^2$	4420.68	0.13	1736.54	2640	$A^1B^4C^{13}D^2$	**4376.54***	6.78	0	**1***
R103A	B^7C^{15}	4772	−13.57	B^6C^{15}	**4195.05**	0.16	1621.71	2580	B^6C^{15}	4201.71	7.45	0	0
R104A	B^9C^{14}	4455	−10.61	B^8C^{14}	4065.52	−0.94	1487.69	2540	B^9C^{13}	**4027.69***	6.14	0	**1***
C101A	A^1B^{10}	9272	−5.02	B^{10}	**8828.93**	0.00	828.93	8000	B^{10}	**8828.93**	3.67	**1**	0
C102A	A^{19}	8433	−17.89	A^{19}	**7137.79**	0.21	1453.13	5700	A^{19}	7153.13	4.12	0	0
C103A	A^{19}	8033	−12.78	A^{19}	7143.88	−0.30	1422.57	5700	A^{19}	**7122.57***	3.45	0	**1***
C104A	A^{19}	7384	−4.25	A^{19}	7104.96	−0.30	1383.74	5700	A^{19}	**7083.74***	3.13	0	**1***
RC101A	$A^7B^7C^7$	5687	−7.99	$A^4B^7C^7$	5279.92	−0.26	1876.36	3390	$A^4B^7C^7$	**5266.36***	5.73	0	**1***
RC102A	$A^5B^6C^8$	5649	−10.77	$A^4B^5C^8$	5149.95	−0.99	1709.55	3390	$A^4B^5C^8$	**5099.55***	5.14	0	**1***
RC103A	$A^{11}B^2C^8$	5419	−8.58	$A^{10}B^2C^8$	5002.41	−0.22	1691.29	3300	$A^{10}B^2C^8$	**4991.29***	4.90	0	**1***
RC104A	$A^2B^{13}C^3D^1$	5189	−3.43	$A^2B^{13}C^3D^1$	5024.25	−0.15	1596.97	3420	$A^2B^{13}C^3D^1$	**5016.97***	5.21	0	**1***
R201A	A^5	4593	−21.43	A^5	**3779.12**	0.09	1532.49	2250	A^5	3782.49	7.45	0	0
R202A	A^5	4331	−20.85	A^5	**3578.91**	0.14	1333.92	2250	A^5	3583.92	8.45	0	0
R203A	A^4B^1	4220	−18.74	A^4B^1	3582.54	−0.81	1053.92	2500	A^4B^1	**3553.92***	7.12	0	**1***
R204A	A^5	3849	−24.89	A^5	3143.68	−2.01	831.80	2250	A^5	**3081.80***	6.99	0	**1***
C201A	A^4B^1	6711	−9.29	A^4B^1	**6140.64**	0.00	740.64	5400	A^4B^1	**6140.64**	4.89	**1**	0
C202A	A^1C^3	7720	−1.26	A^1C^3	7752.88	−1.69	623.96	7000	A^1C^3	**7623.96***	4.26	0	**1***
C203A	C^2D^1	7466	−2.23	C^2D^1	**7303.37**	0.00	603.37	6700	C^2D^1	**7303.37**	4.37	**1**	0
C204A	A^5	6744	−18.72	A^5	5721.09	−0.72	680.46	5000	A^5	**5680.46***	5.29	0	**1***
RC201A	C^1E^3	5871	−6.08	C^1E^3	**5523.15**	0.21	1684.59	3850	C^1E^3	5534.59	6.47	0	0
RC202A	$A^1C^1D^1E^2$	5945	−15.43	$A^1C^1D^1E^2$	**5132.08**	0.35	1450.23	3700	$A^1C^1D^1E^2$	5150.23	6.35	0	0
RC203A	$A^1B^1C^5$	5790	−29.47	$A^1B^1C^5$	4508.27	−0.81	1221.92	3250	$A^1B^1C^5$	**4471.92***	6.01	0	**1***
RC204A	$A^{14}B^2$	4983	−17.47	$A^{14}B^2$	4252.87	−0.26	1441.83	2800	$A^{14}B^2$	**4241.83***	5.87	0	**1***
Min (%)			−29.47			−2.01							
Avg (%)			−12.68			−0.34							
Max (%)			−1.26			0.35							
Total												**3**	**14***
Runs	3			1			10						
Processor	P 233M			PIV 1.5GHz			Xe 2.6GHz						
Avg Time	–			20.00			5.61						

TABLE 3.5: Results for HD

Instance set	HEA				
	DC	VC	Mix	TC	Time
R101A	1765.41	2590	$B^{10}C^{11}D^{1}$	4355.41	5.19
R102A	1716.44	2640	$B^{4}C^{13}D^{2}$	4356.44	6.24
R103A	1500.16	2580	$B^{6}C^{15}$	4080.16	6.57
R104A	1434.72	2520	$B^{7}C14$	3954.72	5.89
C101A	828.94	8000	B^{10}	8828.94	4.25
C102A	1380.17	5700	A^{19}	7080.17	3.97
C103A	1379.21	5700	A^{19}	7079.21	3.99
C104A	1375.06	5700	A^{19}	7075.06	2.98
RC101A	1772.28	3390	$A^{4}B^{7}C^{7}$	5162.28	6.41
RC102A	1598.05	3420	$A^{2}B^{6}C^{8}$	5018.05	5.24
RC103A	1626.55	3300	$A^{10}B^{2}C^{8}$	4926.55	4.39
RC104A	1575.91	3420	$A^{2}B^{13}C^{3}D^{1}$	4995.91	4.88
R201A	1198.76	2250	A^{5}	3448.76	6.74
R202A	1058.16	2250	A^{5}	3308.16	8.13
R203A	882.39	2500	$A^{4}B^{1}$	3382.39	7.49
R204A	768.14	2250	A^{5}	3018.14	5.47
C201A	682.38	5400	$A^{4}B^{1}$	6082.38	4.21
C202A	618.62	7000	$A^{1}C^{3}$	7618.62	3.69
C203A	603.37	6700	$C^{2}D^{1}$	7303.37	3.67
C204A	677.66	5000	A^{5}	5677.66	5.11
RC201A	1494.47	3850	$C^{1}E^{3}$	5344.47	6.72
RC202A	1156.02	3700	$A^{1}C^{1}D^{1}E^{2}$	4856.02	6.48
RC203A	996.25	3250	$A^{1}B^{1}C^{5}$	4246.25	6.93
RC204A	1395.32	2800	$A^{14}B^{2}$	4195.32	6.17
Average				5224.77	5.45
Runs	10				
Processor	Xe 2.6GHz				
Avg Time	5.45				

In summary, the HEA was able to find 41 BKS for 168 FT instances, where 17 are strictly better than those obtained by competing heuristics. As for FD, the algorithm has identified 119 BKS out of the 168 instances, 44 of which are strictly better than those obtained by previous heuristics. The results are even more striking for HT, with 17 BKS on the 24 instances, 14 of which are strictly better than those reported earlier. Overall, the HEA improves 75 BKS and matches 102 BKS out of 360 benchmark instances.

3.4 Conclusions

We have proposed a unified heuristic for four types of heterogeneous fleet vehicle routing problems with time windows. The first two are the Fleet Size and Mix Vehicle Routing Problem with Time Windows (F) and the Heterogeneous Fixed Fleet Vehicle Routing Problem with Time Windows (H). Each of these two problems was solved under a time and a distance objective, yielding the four variants FT, FD, HT and HD. We have developed a unified hybrid evolutionary algorithm (HEA) capable of solving all variants without any modification. This heuristic combines state-of-the-art metaheuristic principles such as heterogeneous adaptive large scale neighborhood search and population search. We have integrated within our HEA an innovative INTENSIFICATION strategy on elite solutions and we have developed a new diversification scheme based on the REGENERATION and the MUTATION of solutions. We have also developed an advanced version of the SPLIT algorithm of Prins (2009) to determine the best fleet mix for a set of routes. Finally, we have introduced the new variant HD. Extensive computational experiments were carried out on benchmark instances. In the case of FT, our HEA clearly outperforms all previous algorithms except that of Vidal et al. (2014). It performs slightly worse on average, but is superior on instances which are less tight in terms of vehicle capacity. On the FD instances, our HEA outperforms the three existing algorithms. Overall, the HEA has identified 160 new best solutions out of 336 on the F instances, 61 of which are strictly better than previously known solutions. On the HT instances, our HEA outperforms the two existing algorithms and has identified 17 best known solutions out of 24, 14 of which are strictly better than previously found solutions. The HD instances are solved here for the first time. Overall, we have improved 75 solutions out of 360 instances, and we have matched 102 others. All instances were solved within a modest computational effort. Our algorithm is not only highly competitive, but it is also flexible in that it can solve four problem classes with the same parameter settings.

Chapter 4

The Fleet Size and Mix Location-Routing Problem with Time Windows: Formulations and a Heuristic Algorithm

Abstract

This paper introduces the fleet size and mix location-routing problem with time windows (FSMLRPTW) which extends the location-routing problem by considering a heterogeneous fleet and time windows. The main objective is to minimize the sum of vehicle fixed cost, depot cost and routing cost. We present mixed integer programming formulations, a family of valid inequalities and we develop a powerful hybrid evolutionary search algorithm (HESA) to solve the problem. The HESA successfully combines several metaheuristics and offers a number of new advanced efficient procedures tailored to handle heterogeneous fleet dimensioning and location decisions. We evaluate the strengths of the proposed formulations with respect to their ability to find optimal solutions. We also investigate the performance of the HESA. Extensive computational experiments on new benchmark instances have shown that the HESA is highly effective on the FSMLRPTW.

Keywords. location-routing; heterogeneous fleet; time windows; mixed integer programming; genetic algorithm

4.1 Introduction

The design of distribution networks is critical for most companies because it usually requires a major capital outlay. Two major types of decisions intervene in this process, namely determining the locations of depots, and designing vehicle routes supplying customers from these depots. In the classical Facility Location Problem (FLP) (Balinski 1965), it is assumed that each customer is served individually through a direct shipment, which makes sense when customer demands are close to the vehicle capacity. However, there exist several situations where customer demands can be consolidated. In such contexts, the FLP and Vehicle Routing Problem (VRP) should be solved jointly. The idea of combining location and routing decisions was put forward more than fifty years ago (Von Boventer 1961) and has given rise to a rich research known as the Location-Routing Problem (LRP) (see Laporte 1988, Min et al. 1998, Nagy and Salhi 2007, Prodhon and Prins 2014, Albareda-Sambola 2015, Drexl and Schneider 2015, for reviews). Applications of the LRP arise in areas as diverse as food and drink distribution, parcel delivery and telecommunication network design. Many algorithms, mostly heuristics, have been developed for the LRP and its variations over the past fifty years, including some population metaheuristics (Prins et al. 2006b, Duhamel et al. 2010), neighborhood-based metaheuristics (Prins et al. 2006a, Duhamel et al. 2011), simulated annealing (Karaoglan et al. 2012) and adaptive large neighborhood search (ALNS) algorithms (Hemmelmayr et al. 2012).

According to Hoff et al. (2010), heterogeneous fleets are more common in real-world distribution problems than homogeneous ones. The Fleet Size and Mix VRP with Time Windows (FSMVRPTW), introduced by Liu and Shen (1999b), is an LRP with a single depot, a heterogeneous fleet and time windows. Many heuristics have also been developed for the FSMVRPTW (see Irnich et al. 2014), including a two-stage construction heuristic (Liu and Shen 1999b), a sequential construction heuristic (Dullaert et al. 2002), a multi-start parallel regret construction heuristic (Dell'Amico et al. 2007), a three-phase multi-restart deterministic annealing metaheuristic (Bräysy et al. 2008), a hybrid metaheuristic combining a threshold acceptance heuristic with a guided local search (Bräysy et al. 2009), an adaptive memory programming algorithm (Repoussis and Tarantilis 2010), and a genetic algorithm using a unified component based solution framework (Vidal et al. 2014). This problem and a number of its variations are reviewed by Baldacci et al. (2008) and Baldacci et al. (2009).

To our knowledge, a number of studies indirectly consider heterogeneous fleets in an LRP context but without taking time windows into account (Ambrosino et al. 2009, Wu et al. 2010). Berger et al. (2007) only consider a distance constraint. Therefore, combining heterogeneous fleets and time windows in the LRP is done here for the first time. We believe there is methodological interest in solving the Fleet Size and Mix Location-Routing Problem with Time Windows (FSMLRPTW).

In the FSMLRPTW, one considers a fleet of vehicles with various capacities and vehicle-related costs, as well as a set of potential depots with opening costs and capacities, and a set of customers with known demands and time windows. The FSMLRPTW consists of opening a subset of depots, assigning customers to them and determining a set of routes for heterogeneous vehicles such that all vehicles start and end their routes at their depot, each customer is visited exactly once by a vehicle within a prespecified time window, and the load of each vehicle does not exceed its capacity. The objective is to minimize the total cost which is made up of three components: the depot operating cost, the fixed cost of the vehicles, and the total travel costs of the vehicles. It is assumed that these costs are scaled over the same time horizon.

The contributions of this paper are as follows. We introduce the FSMLPRTW as a new LRP variant. We develop a hybrid evolutionary search algorithm (HESA) with the introduction of several algorithmic procedures specific to the FSMLRPTW. Namely, we introduce the location-heterogeneous adaptive large neighborhood search (L-HALNS) procedure equipped with a range of several new operators as the main EDUCATION procedure within the search. We also propose an INITIALIZATION procedure to create initial solutions, and a PARTITION procedure for offspring solutions. Finally, we develop a new diversification scheme through the MUTATION procedure of solutions.

The remainder of this paper is structured as follows. Section 4.2 formally defines the problem and provides integer programming formulations together with valid inequalities. Section 4.3 presents a detailed description of the HESA. Computational experiments are provided in Section 4.4, and conclusions follow in Section 4.5.

4.2 Formulations for the Fleet Size and Mix Location-Routing Problem with Time Windows

In this section, we first define the FSMLRPTW, and then present several integer programming formulations and valid inequalities to strengthen them.

4.2.1 Notation and problem definition

The FSMLRPTW is defined on a complete directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N} = \mathcal{N}_0 \cup \mathcal{N}_c$ is a set of nodes in which $\mathcal{N}_0$ and $\mathcal{N}_c$ represent the potential depot and customer nodes, respectively, and $\mathcal{A} = \{(i,j) : i \in \mathcal{N}, j \in \mathcal{N}\} \setminus \{(i,j) : i \in \mathcal{N}_0, j \in \mathcal{N}_0, i \neq j\}$ is the set of arcs. Each arc $(i,j) \in \mathcal{A}$ has a nonnegative distance c_{ij}. Here, the terms distance, travel time and travel cost are used in interchangeably. Each customer $i \in \mathcal{N}_c$ has a positive demand q_i. A storage capacity D^k and a fixed opening cost g^k are associated with each potential depot $k \in \mathcal{N}_0$. The index set of vehicle types is denoted by $\mathcal{H}$. Let Q^h and f^h denote the capacity and fixed dispatch cost of a vehicle of type $h \in \mathcal{H}$. Furthermore, s_i corresponds to the service time of node $i \in \mathcal{N}_c$, which must start within the hard time window $[a_i, b_i]$. If a vehicle arrives at customer $i \in \mathcal{N}_c$ before time a_i, it waits until a_i to start servicing the customer.

To formulate the FSMLRPTW, we define the following decision variables. Let x_{ij}^h be equal to 1 if vehicle of type h travels directly from node i to node j and to 0 otherwise. Let y_k be equal to 1 if depot k is opened and to 0 otherwise. Let z_{ik} be equal to 1 if customer i is assigned to depot k and to 0 otherwise. Let u_{ij}^h be the total load of vehicle of type h immediately after visiting node i and traveling directly to node j. Let f_{ij} be the total load of the vehicle while traversing arc $(i, j) \in \mathcal{A}$, which is the aggregation of the u_{ij}^h variables over $\mathcal{H}$. Let t_i^h be the time at which a vehicle of type h starts serving at node i. Let t_i be the time at which service starts at node $i \in \mathcal{N}$, which is the aggregation of the t_i^h variables.

4.2.2 Integer programming formulations

The integer linear programming formulation of the problem, denoted by E_1, is then:

$$(E_1) \text{ Minimize} \sum_{k \in \mathcal{N}_0} g^k y_k + \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{N}_0} \sum_{i \in \mathcal{N}_c} f^h x_{ki}^h + \sum_{h \in \mathcal{H}} \sum_{(i,j) \in \mathcal{A}} c_{ij} x_{ij}^h \tag{4.1}$$

subject to

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} x_{ij}^h = 1 \qquad i \in \mathcal{N}_c \tag{4.2}$$

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} x_{ji}^h = \sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} x_{ij}^h \qquad i \in \mathcal{N} \tag{4.3}$$

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} u_{ji}^h - \sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} u_{ij}^h = q_i \qquad i \in \mathcal{N}_c \tag{4.4}$$

$$u_{ij}^h \leq Q^h x_{ij}^h \qquad i \in \mathcal{N}_0, j \in \mathcal{N}, i \neq j, h \in \mathcal{H} \tag{4.5}$$

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}_c} u_{kj}^h = \sum_{j \in \mathcal{N}_c} z_{jk} q_j \qquad k \in \mathcal{N}_0 \tag{4.6}$$

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}_c} u_{jk}^h = 0 \qquad k \in \mathcal{N}_0 \tag{4.7}$$

$$u_{ij}^h \leq (Q^h - q_i) x_{ij}^h \qquad i \in \mathcal{N}_c, j \in \mathcal{N}, h \in \mathcal{H} \tag{4.8}$$

$$u_{ij}^h \geq q_j x_{ij}^h \qquad i \in \mathcal{N}, j \in \mathcal{N}_c, h \in \mathcal{H} \tag{4.9}$$

$$\sum_{i \in \mathcal{N}_c} q_i z_{ik} \leq D^k y_k \qquad k \in \mathcal{N}_0 \tag{4.10}$$

$$\sum_{k \in \mathcal{N}_0} z_{ik} = 1 \qquad i \in \mathcal{N}_c \tag{4.11}$$

$$x_{ij}^h + \sum_{q\in\mathcal{H},q\neq h}\sum_{l\in\mathcal{N},j\neq l} x_{jl}^q \leq 1 \qquad i\in\mathcal{N}, j\in\mathcal{N}_c, i\neq j, h\in\mathcal{H} \tag{4.12}$$

$$\sum_{h\in\mathcal{H}} x_{ik}^h \leq z_{ik} \qquad k\in\mathcal{N}_0, i\in\mathcal{N}_c \tag{4.13}$$

$$\sum_{h\in\mathcal{H}} x_{ki}^h \leq z_{ik} \qquad k\in\mathcal{N}_0, i\in\mathcal{N}_c \tag{4.14}$$

$$\sum_{h\in\mathcal{H}} x_{ij}^h + z_{ik} + \sum_{m\in\mathcal{N}_0,m\neq k} z_{jm} \leq 2 \qquad k\in\mathcal{N}_0, (i,j)\in\mathcal{N}_c, i\neq j \tag{4.15}$$

$$t_i^h - t_j^h + s_i + c_{ij} \leq M(1-x_{ij}^h) \qquad i\in\mathcal{N}, j\in\mathcal{N}_c, i\neq j, h\in\mathcal{H} \tag{4.16}$$

$$a_i \leq t_i^h \leq b_i \qquad i\in\mathcal{N}, h\in\mathcal{H} \tag{4.17}$$

$$x_{ij}^h \in \{0,1\} \qquad (i,j)\in\mathcal{N}, h\in\mathcal{H} \tag{4.18}$$

$$z_{ik} \in \{0,1\} \qquad k\in\mathcal{N}_0, i\in\mathcal{N}_c \tag{4.19}$$

$$y_k \in \{0,1\} \qquad k\in\mathcal{N}_0 \tag{4.20}$$

$$u_{ij}^h \geq 0 \qquad (i,j)\in\mathcal{N}, h\in\mathcal{H} \tag{4.21}$$

$$t_i^h \geq 0 \qquad i\in\mathcal{N}_c, h\in\mathcal{H}. \tag{4.22}$$

The objective function (4.1) minimizes the total cost including depot fixed cost, vehicle fixed cost and variable travel cost. Constraints (4.2) and (4.3) are degree constraints; in particular constraints (4.2) guarantee that each customer must be visited exactly once, and constraints (4.3) ensure that entering and leaving arcs to each node are equal. Constraints (4.4) imply that the demand of each customer is satisfied. Constraints (4.5) mean that the total load on any arc cannot exceed the capacity of the vehicle traversing it. Constraints (4.6) ensure that the total load of each depot is equal to the total demand of customers assigned to it. Note that constraints (4.6) strengthen the formulation, yet are not necessary. Constraints (4.7) state that the load on a vehicle returning to each depot must be equal to zero. Contraints (4.8) and (4.9) are bounding constraints for load variables. Constraints (4.10) guarantee that total demand supplied by a depot cannot exceed its capacity. Constraints (4.11) and (4.12) ensure that each customer is assigned to only one depot and to only one vehicle type, respectively. Constraints (4.13)–(4.15) forbid illegal routes, i.e., routes that do not start and end at the same depot. Constraints (4.16) and (4.17), where M is a large number, enforce the time window restrictions. Constraints (4.18)–(4.22) define the domains of the decision variables.

The validity of constraints (4.13)–(4.15) was proven by Karaoglan et al. (2011). They prevent illegal routes starting at one depot and ending at another, the prevents the creation of infeasible routes $P = (v_{k_1}, v_1, v_2, \ldots, v_{s-1}, v_s, v_{l_2})$ with s customers $(k, l \in \mathcal{N}_0, k \neq l)$. Constraints (4.13) imply that $z_{v_1,k_1} = 1$. Since $x_{v_1,v_2} = 1$ and $z_{v_1,k_1} = 1$, constraints (4.15) ensure that $z_{v_2,k_1} = 1$ and z_{v_2,k_2} cannot be equal to one.

Formulation E_1 shown by (4.1)–(4.22) is valid for the FSMLRPTW. Other valid formulations can be derived from E_1. Before defining these formulations, we provide several variations of E_1 by aggregating some of the variables or disaggregating some of the constraints. The reason is to either reduce the size of the formulation through aggregation of variables or to tighten the linear relaxation bound by disaggregating some of the constraints. First, f_{ij} which is the aggregation of the u_{ij}^h variables, is obtained as follows:

$$\sum_{j \in \mathcal{N}} f_{ji} - \sum_{j \in \mathcal{N}} f_{ij} = q_i \qquad i \in \mathcal{N}_c \tag{4.23}$$

$$f_{ij} \leq \sum_{h \in \mathcal{H}} Q^h x_{ij}^h \qquad i \in \mathcal{N}_0, j \in \mathcal{N}, i \neq j \tag{4.24}$$

$$\sum_{j \in \mathcal{N}_c} f_{kj} = \sum_{j \in \mathcal{N}_c} z_{jk} q_j \qquad k \in \mathcal{N}_0 \tag{4.25}$$

$$\sum_{j \in \mathcal{N}_c} f_{jk} = 0 \qquad k \in \mathcal{N}_0 \tag{4.26}$$

$$f_{ij} \leq \sum_{h \in \mathcal{H}} (Q^h - q_i) x_{ij}^h \qquad i \in \mathcal{N}_c, j \in \mathcal{N} \tag{4.27}$$

$$f_{ij} \geq q_j \sum_{h \in \mathcal{H}} x_{ij}^h \qquad i \in \mathcal{N}, j \in \mathcal{N}_c. \tag{4.28}$$

Let (f, x) be a solution satisfying constraints (4.23)–(4.28) and let (u, x) be a solution satisfying constraints (4.2) and (4.4)–(4.9), where f, u and x are the respective vectors for variables f_{ij}, u_{ij}^h and x_{ij}^h. Then, for each feasible (u, x), there exists a feasible (f, x) and vice versa. Due to (4.2), there exists an $h^* \in \mathcal{H}$ for which $x_{ij}^{h^*} = 1$ and due to (4.8), there exists a $u_{ij}^{h^*} \geq 0$, implying $x_{ij}^h = u_{ij}^h = 0$ for all other $h \in \mathcal{H} \setminus \{h^*\}$. Using these arguments, we have linked f_{ij} to the original variables in the following way:

$$f_{ij} = \sum_{h \in \mathcal{H}} u_{ij}^h \qquad i, j \in \mathcal{N}. \tag{4.29}$$

We can now derive constraints (4.23)–(4.28) from (4.4)–(4.9) by using definition (4.29). Constraints (4.23)–(4.28) are satisfied, since they are the same as (4.4)–(4.9), respectively.

Second, the aggregation of the t_i^h variable is

$$t_i - t_j + s_i + c_{ij} \leq M(1 - \sum_{h \in \mathcal{H}} x_{ij}^h) \qquad i \in \mathcal{N}, j \in \mathcal{N}_c, i \neq j \tag{4.30}$$

$$a_i \leq t_i \leq b_i \qquad i \in \mathcal{N}. \tag{4.31}$$

Constraints (4.30)–(4.31) and (4.16)–(4.17) are equivalent. Due to (4.16) there exists $h^* \in \mathcal{H}$ for which $x_{ij}^{h^*} = 1$, i.e., $x_{ij}^h = 0$ for all $h \neq h^*$, $h \in \mathcal{H}$. An equivalent definition is

$$t_j = \sum_{h \in \mathcal{H}} t_j^h \qquad j \in \mathcal{N}. \tag{4.32}$$

We can now derive constraints (4.30)–(4.31) from (4.16)–(4.17) by using definition (4.32). Constraints (4.30)–(4.31) are satisfied, since they are the same as (4.16)–(4.17), respectively.

Third, the disaggregation of the balance constraints (4.3) is

$$\sum_{j \in \mathcal{N}} x_{ji}^h = \sum_{j \in \mathcal{N}} x_{ij}^h \qquad h \in \mathcal{H}, i \in \mathcal{N}. \tag{4.33}$$

Constraints (4.33) are equivalent to constraints (4.3) and (4.12). Given that i, j, k are on the same route, assume $h^1, h^2 \in \mathcal{H}$, $x_{ij}^{h^1} = 1$ and $x_{jk}^{h^2} = 1$, which is valid for (4.3). But constraints (4.12) along with (4.3) only allow the use of the same vehicle on a route, i.e, $x_{ij}^{h^1} = 1$ and $x_{jk}^{h^1} = 1$. We can now derive constraints (4.33) from (4.3). Using (4.33) also makes (4.12) redundant. Constraints (4.33) ensure that $x_{ij}^{h^1} = 1$ and $x_{jk}^{h^1} = 1$ without using (4.12). Constraints (4.33) are satisfied, since they are the same as (4.3), respectively.

We now define the following four variations of E_1 as follows:

1) Formulation E_2 minimizes (4.1), subject to (4.2), (4.3), (4.23)–(4.28) and (4.10)–(4.22).

2) Formulation E_3 minimizes (4.1), subject to (4.2)–(4.5) and (4.7)–(4.22).

3) Formulation E_4 minimizes (4.1), subject to (4.2)–(4.3), (4.23)–(4.28), (4.10)–(4.15), (4.30)–(4.31) and (4.18)–(4.22).

4) Formulation E_5 minimizes (4.1), subject to constraints (4.2), (4.33), (4.23)–(4.28), (4.10), (4.11), (4.13)–(4.15), (4.30)–(4.31) and (4.18)–(4.22).

4.2.3 Valid inequalities

We make use of four polynomial size valid inequalities which are proposed in the literature by several authors (Dantzig et al. 1954, Achuthan 2003, Labbé et al. 2004). These were used by several authors for variants of LRPs (see Karaoglan et al. 2011, 2012). It can be shown that these inequalities are also valid for the FSMLRPTW.

The first inequalities are special case of classical subtour elimination constraints for the Traveling Salesman Problem (Dantzig et al. 1954) for two nodes as follows:

$$x_{ij}^h + x_{ji}^h \leq 1 \qquad i, j \in \mathcal{N}_c, h \in \mathcal{H}. \tag{4.34}$$

Constraints (4.34) break subtours involving two customers only. The second valid inequality, which was used by Labbé et al. (2004) for plant-cycle location problem is as follows:

$$z_{ik} \leq y_k \qquad i \in \mathcal{N}_c, k \in \mathcal{N}_0. \tag{4.35}$$

Constraints (4.35) impose that customer i cannot be assigned to depot k if the latter is not open. We now adapt the next valid inequality described by Achuthan (2003) for the FSMLRPTW as follows:

$$\sum_{h\in\mathcal{H}} \sum_{k\in\mathcal{N}_0} \sum_{i\in\mathcal{N}_c} x_{ki}^h \geq r_\alpha(\mathcal{N}_c). \tag{4.36}$$

Constraints (4.36) provide a lower bound on the number of routes originating from depots where $r_\alpha(\mathcal{N}_c) = \lceil \sum_{i\in\mathcal{N}_c} q_i / \max_{h\in\mathcal{H}} Q^h \rceil$ and $\lceil \bullet \rceil$ is the smallest integer greater than or equal to $\bullet$. The fourth and final valid inequality is

$$\sum_{k\in\mathcal{N}_0} y_k \geq y_{min}. \tag{4.37}$$

Constraints (4.37) bound the number of opened depots from below, where y_{min} is taken as the minimum number of k opened depots satisfying $\sum_{k \in \mathcal{N}_0} D^k \geq \sum_{i \in \mathcal{N}_c} q_i$ after the depots have been sorted in non-decreasing order of their capacity.

4.3 Description of the Hybrid Evolutionary Search Algorithm

This section describes the proposed Hybrid Evolutionary Search Algorithm, called HESA, that has been developed to solve the FSMLRPTW. This algorithm builds on several powerful evolutionary based metaheuristic algorithms (see Koç et al. 2014, 2015, Vidal et al. 2014). In this paper, we additionaly introduce the location-heterogeneous adaptive large neighborhood search (L-HALNS) procedure which is used as a main EDUCATION component in the HESA. Furthermore, we develope an INITIALIZATION to create initial solutions and a PARTITION procedure to split offspring solutions into routes. Finally, we propose a new MUTATION procedure to diversify the solutions.

The general structure of the HESA is sketched in Algorithm 1. The initial population is generated by using the INITIALIZATION procedure (line 1). Two parents are selected (line 3) through a binary tournament process, where a crossover operation creates a new offspring C (line 4), which then undergoes the PARTITION procedure (line 5). The EDUCATION procedure uses the L-HALNS operators to educate offspring C and inserts it back into the population (line 6). The probabilities associated with the L-HALNS operators used in the EDUCATION procedure are updated by an Adaptive Weight Adjustment Procedure (AWAP) (line 7). The INTENSIFICATION procedure which is based on the L-HALNS is run on elite solutions (line 8). As new offsprings are added, the population size n_a which is limited by $n_p + n_o$, changes over the iterations. The constant n_p denotes the size of the population initialized at the beginning of the algorithm and the constant n_o is the maximum allowable number of offspring that can be inserted into the population. If the population size n_a reaches $n_p + n_o$ at any iteration, then a survivor selection mechanism is applied (line 9). The MUTATION procedure is applied to a randomly selected individual from the population with probability p_m (line 10). When the number ϖ of iterations without improvement in the incumbent solution is reached, the

HESA terminates (line 11). For further implementation details on the ALNS based education, parent selection, crossover, AWAP, intensification and survivor selection sections the reader is referred to Chapter 3.

Algorithm 1 The general framework of the HESA

1: INITIALIZATION: initialize a population with size n_p
2: **while** number of iterations without improvement $< \varpi$ **do**
3:     *Parent selection*: select parent solutions P_1 and P_2
4:     *Crossover*: generate offspring C from P_1 and P_2
5:     PARTITION: partition offspring C into routes
6:     EDUCATION: educate C with L-HALNS and insert into population
7:     AWAP: update probabilities of the L-HALNS operators
8:     INTENSIFICATION: intensify on elite solutions with L-HALNS
9:     *Survivor selection*: if the population size n_a reaches $n_p + n_o$, then select survivors
10:     MUTATION: diversify a random solution with probability p_m
11: **end while**
12: Return best feasible solution

In what follows, we detail the algorithmic features specifically designed for the FSMLRPTW. Section 4.3.1 presents the INITIALIZATION procedure of the population. The PARTITION procedure is described in Section 4.3.2. Section 4.3.3 presents the offspring EDUCATION procedure. Finally, the MUTATION procedure is desribed in Section 4.3.4.

4.3.1 INITIALIZATION

We use a three-phase INITIALIZATION procedure to generate an initial population. The first and second phases generate the initial solution while the third phase creates other solutions. In the first phase, customers are assigned to the depots. Initially, the closest depot according to its distance is calculated for each customer, and the customers are listed in non-increasing order of distance. Each customer is then assigned to its closest depot starting from the top of the list, without violating the depot capacities. These steps are applied repeatedly to the residual customers who could not be assigned to their closest depots due to depot capacity constraints. In this case they are assigned to their closest feasible depot. In the second phase, routes are constructed for each depot by applying the Clarke and Wright (1964) algorithm and by selecting the largest vehicle type for each route. In the third phase, new individuals are created by applying the L-HALNS operators to the initial solution until the initial population size reaches n_p. A removal operator is randomly selected from a list of diversification based removal operators and a greedy insertion with a noise function operator is used as an insertion operator (see Section 4.3.3). Both of these operators are used in order to diversify the initial population. The number of nodes removed is randomly chosen from the initialization interval $[b_l^i, b_u^i]$ in the destroy phase. The interval is defined by a lower and an upper bound calculated as a percentage of the total number of nodes in a given instance.

4.3.2 Partition

We introduce a Partition procedure to be used in the L-HALNS (see Section 4.3.3). The objective of this procedure is to split a given solution into routes after the parent selection and the crossover phases during the algorithm. The given solution is represented as a permutation of customers. This procedure includes two phases. In the first phase, the customer sequence is put into a list L_r. In the second phase, we use an intensification-based insertion operator (greedy insertion operator) to insert the customers of L_r to their best possible position. The Partition procedure yields a feasible solution for the FSMLRPTW which is inserted into the population.

4.3.3 Education

At every iteration, the Education procedure is applied to the newly generated offspring to improve its quality. The ALNS used as a way of educating the solutions, as in Chapter 3, is basically an improvement heuristic. It consists of two procedures: removal, followed by insertion. In the removal procedure, n' nodes are iteratively removed by the intensification based removal operators and placed in the removal list L_r, where n' is in the interval $[b_l^e, b_u^e]$ for the destroy operators. In the insertion procedure, the nodes of L_r are iteratively inserted into a least-cost position in the incomplete solution.

Here, we introduce the Location-HALNS (L-HALNS) which integrates fleet sizing and the location decisions within the removal and insertion operators. This procedure differs from the Heterogeneous ALNS (HALNS) developed by Chapter 5 to educate the solutions in the context of a heterogeneous fleet because the latter did not account for the location decisions. When a node is removed, we check whether the resulting route can be served by a smaller vehicle, and we also check whether the related depot has any customer already assigned to it. We then update the solution accordingly. If inserting a node requires additional vehicle capacity or requires opening a new depot, we then consider the option of using larger vehicles or the option of opening the least cost depot not yet open. More formally, for each node $i \in \mathcal{N}_0 \backslash L_r$, let f^h be the vehicle fixed cost and let g^k be the depot cost associated with this node. Let $\Delta(i)$ be the distance saving obtained as a result of using a removal operator on node i. Let g_r^{k*} be the depot fixed cost and let f_r^{h*} be the vehicle fixed cost after the removal of node i, i.e., g_r^{k*} is modified only if node i is the only node of depot k and f_r^{h*} is modified only if the route containing node i can be served by a smaller vehicle when removing node i. The savings in depot fixed cost and vehicle fixed cost can be expressed as $g^k - g_r^{k*}$ and $f^h - f_r^{h*}$, respectively. Thus, for each removal operator, the total savings resulting from removing node $i \in \mathcal{N}_0 \backslash L_r$, denoted $RC(i)$, is calculated as follows:

$$RC(i) = \Delta(i) + (g^k - g_r^{k*}) + (f^h - f_r^{h*}). \tag{4.38}$$

In the destroyed solution, the insertion cost of node $j \in L_r$ after node i is defined as $\Omega(i, j)$ on a given node $i \in \mathcal{N}_0 \backslash L_r$. Let g_a^{k*} be the depot fixed cost and let f_a^{h*} be the vehicle fixed cost after the insertion of node i, i.e., g_r^{k*} is modified only if node i requires to open a new depot, or f_r^{h*} is modified only if the route containing node i necessitates the use of a larger capacity vehicle after inserting node i. The cost differences in depot fixed cost and vehicle fixed cost can be expressed as $g_r^{k*} - g^k$ and $f_a^{h*} - f^h$, respectively. Thus, the total insertion cost of node $i \in \mathcal{N}_0 \backslash L_r$ is $IC(i)$, for each insertion operator is

$$IC(i) = \Omega(i, j) + (g_r^{k*} - g^k) + (f_a^{h*} - f^h). \tag{4.39}$$

Figure 4.1 provides an example of the removal and insertion phases of the L-HALNS procedure.

4.3.3.1 Diversification based removal operators

The first and second diversification based removal operators were applied by Hemmelmayr et al. (2012), and the third one was used by Ropke and Pisinger (2006a). We introduce the fourth one as a new operator specific to the FSMLRPTW.

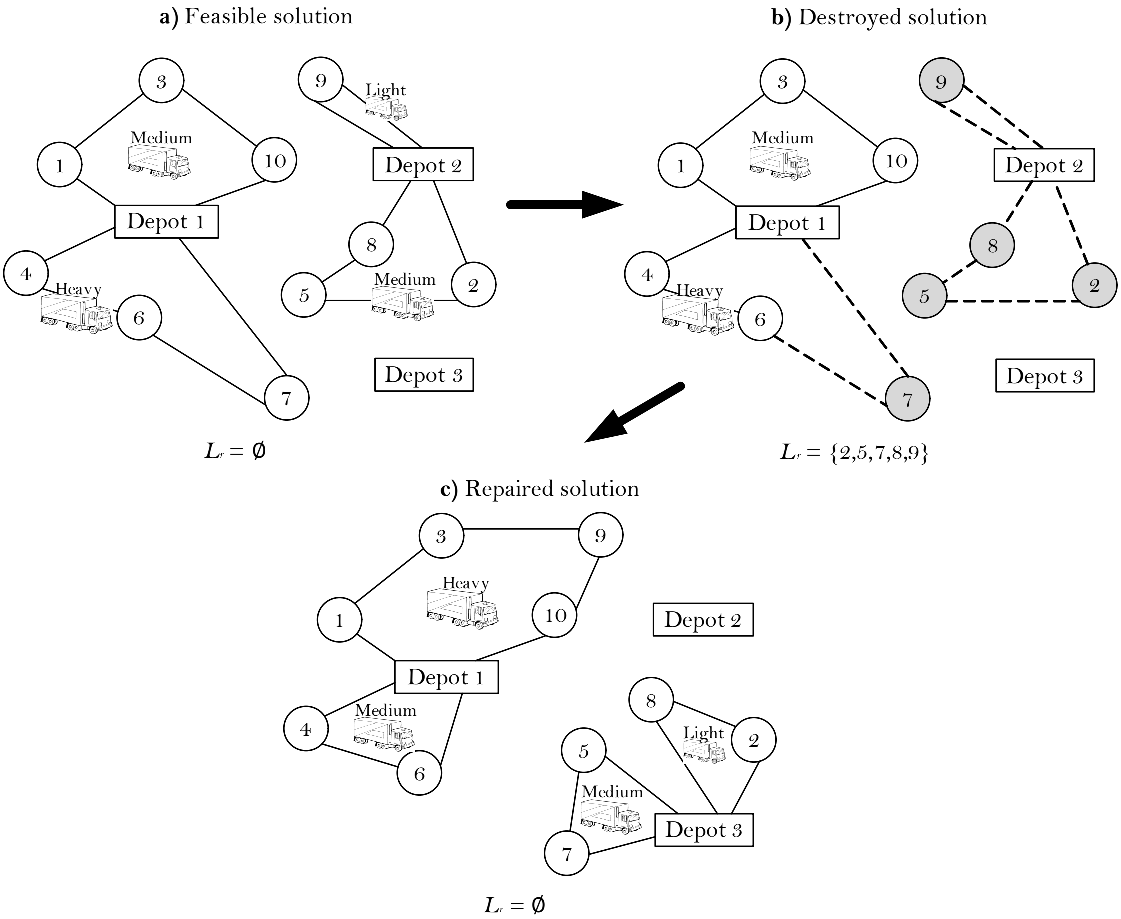


FIGURE 4.1: Illustration of the L-HALNS procedure

1. Depot closing removal (DR): This operator randomly selects an open depot and closes it. The operator removes all customers of this depot. The DR operator also randomly selects a closed depot and opens it.

2. Depot opening removal (DOR): The DOR operator randomly opens a closed depot. The n' customers removed from the solution are those which are closest to this depot according to travel cost.

3. Random removal (RR): This operator randomly selects n' customers and puts them into the removal list.

4. Depot distance removal (DDR): The DSR operator is based on the DR operator but differs from it by the criterion employed to open a closed depot. To open a depot, this operator selects a closest depot with respect to a removed one.

4.3.3.2 Intensification based removal operators

We now present the eight intensification based removal operators used in our algorithm. The first seven were used in several papers (Ropke and Pisinger 2006a,b, Demir et al. 2012, Koç et al. 2015), whereas the last one is new.

1. Neighborhood removal (NR): The general idea behind the NR operator is to remove n' customers from routes that are extreme with respect to the average distance of a route.

2. Worst distance removal (WDR): The WDR operator iteratively removes n' high cost customers, where the cost is based on the distance.

3. Worst time removal (WTR): This operator is a variant of the WDR operator. For each node, costs are calculated, depending on the deviation between the arrival time and the beginning of the time window. The WTR operator iteratively removes n' customer from the solution which has the largest deviation.

4. Shaw removal (SR): The SR operator aims to remove a set of n' similar customers. The similarity between two customers is based on distance, time, proximity and demands.

5. Proximity-based removal (PBR): This operator is a special case of SR which is solely based on distance.

6. Time-based removal (TBR): The TBR operator is another variant of SR. The selection criterion of a set of customers is solely based on time.

7. Demand based-removal (DBR): The DBR operator is yet another variant of SR which is solely based on demand.

8. Depot closing removal (DCR): The aim of the DCR operator is to calculate the utilization efficiency of each depot $\Phi(k)$ $(k \in \mathcal{N}_0)$ and remove from solution the one with the least $\Phi(k)$ value. $\Phi(k)$ is expressed as the ratio of the total demand of depot k over the capacity of depot k:

$$\Phi(k) = \frac{\sum_{i \in \mathcal{N}_c} q_i z_{ik}}{D^k}. \tag{4.40}$$

4.3.3.3 Insertion operators

Two insertion operators (Ropke and Pisinger 2006a,b, Demir et al. 2012, Koç et al. 2015) are used in the repair phase of the EDUCATION procedure.

1. Greedy insertion operator (GIO): This operator finds the best possible insertion position for all nodes in L_r while the cost calculation is based on distance. The insertion process is iteratively applied to all nodes in L_r.

2. Greedy insertion with noise function operator (GINFO): The GINFO operator is a variant of the GIO operator that extends it by allowing a degree of freedom in selecting the best possible insertion position for a node.

4.3.4 MUTATION

We introduce a MUTATION procedure to increase the population diversity over the iterations (e.g., Nagata et al. 2010). This procedure is applied with probability p_m. An individual C different from the elite individuals is randomly selected. A randomly selected diversification based removal operator and the GINFO operator are then used in order to diversify the selected individual C. These two operators are applied to change the positions of a specific number of nodes, which are chosen from the interval $[b_l^m, b_u^m]$ of removable nodes.

4.4 Computational Experiments

In this section, we present the results of computational experiments performed in order to assess the performance of the formulations and the HESA. All experiments were conducted on a server with one gigabyte RAM and Intel Xeon 2.6 GHz processor. We used CPLEX 12.5 with its default settings as the optimizer to solve the integer programming formulations. The HESA was implemented in C++.

Section 4.4.1 describes the benchmark instances and the experimental settings. The aim of the computational experiments is threefold: (i) to analyze the effect of the metaheuristic components (Section 4.4.2), (ii) to evaluate the formulation in terms of solving the FSMLRPTW to optimality on small-size instances (Section 4.4.3), and (iii) to compare and test the integer programming formulation which is integrated with valid inequalities and the HESA on small-medium-and large-size instances (Section 4.4.4).

4.4.1 Benchmark instances

Benchmark data sets for the FSMLRPTW were generated by considering the data set described by Liu and Shen (1999b) for the Fleet Size and Mix VRP with Time Windows (FSMVRPTW) and derived from the classical Solomon (1987) VRP with Time Windows instances with 100 nodes. These sets include 56 instances, split into a random data set R, a clustered data set C and a semi-clustered data set RC. The sets R1, C1 and RC1 have a short scheduling horizon and small vehicle capacities, in contrast to the sets R2, C2 and RC2 which have a longer scheduling horizon and larger vehicle capacities. Liu and Shen (1999b) also introduced three cost structures, namely A, B and C, and several vehicle types with different capacities and fixed vehicle costs for each of the 56 instances. In our data sets, we have used the cost structure A and generated small-size (10-15-20-25-30-customer) as well as medium and large-size (50-75-100-customer) instances. We have selected the first customers of each data sets to generate the instances, i.e., the first 10 customers, 15 customers, and so on, of each Liu and Shen instance.

We followed a procedure similar to that used for the LRP benchmark instances (see Barreto 2004, Albareda-Sambola et al. 2005, Prodhon 2006, Karaoglan et al. 2012) to generate our depot characteristics. To this end, new depots features were added to each instance of Liu and Shen, as shown in Appendix A.1. The depot characteristics were generated using discrete uniform distributions. The depot coordinates are in the range $[0, 100]^2$. For the sets C1, C2, R1, R2, RC1 and RC2, the depot costs are in the range [38000, 50000], [90000, 120000], [17000, 25000], [85000, 100000], [17000, 26000] and [85000, 100000], respectively. The intervals for depot capacities for each data set and for small-, medium- and large-size instances are given in Table 4.1. However, these data ensure the opening of at least two depots for each instance. The main idea behind this decision is to prevent to open only one depot in the solutions. In the original Solomon instances there is only one depot; we have used this depot's time windows for other depots as well, i.e., in each data set, time windows are same for all depots.

Evolutionary algorithms use a set of correlated parameters. All algorithmic parametric values were set as in Chapter 3, where an extensive meta-calibration procedure was used

TABLE 4.1: The intervals for depot capacities

$\|\mathcal{N}_c\|$	Data set					
	C1	C2	R1	R2	RC1	RC2
10	[90, 110]	[100, 120]	[80, 85]	[90, 105]	[150, 180]	[160, 190]
15	[150, 210]	[160, 220]	[110, 160]	[120, 180]	[210, 250]	[230, 270]
20	[190, 220]	[210, 240]	[140, 170]	[150, 190]	[220, 260]	[270, 320]
25	[220, 320]	[230, 340]	[270, 310]	[290, 305]	[290, 340]	[360, 420]
30	[360, 410]	[360, 430]	[270, 310]	[280, 340]	[370, 430]	[370, 430]
50	[540, 610]	[580, 660]	[390, 510]	[450, 530]	[630, 710]	[670, 740]
75	[810, 900]	[830, 930]	[550, 760]	[630, 790]	[730, 860]	[760, 900]
100	[780, 1000]	[800, 1030]	[720, 1000]	[700, 1100]	[790, 1180]	[780, 1300]

to generate effective parameter values for the FSMVRPTW. These authors initially used the parameters suggested by Vidal et al. (2014) for the genetic algorithm, and by Demir et al. (2012) for the ALNS. The authors then conducted several experiments to further fine tune the parameters on randomly selected instances.

4.4.2 Sensitivity analysis of method components

This section compares four versions of the HESA, the details of which can be found in Table 4.2. We present four sets of experiments on randomly selected 100-customer instances; C101, C203, R101, R211, RC105 and RC207.

TABLE 4.2: Sensitivity analysis experiment setup

Version	EDUCATION	INTENSIFICATION	MUTATION
(1)	No	No	No
(2)	Yes	No	No
(3)	Yes	Yes	No
HESA	Yes	Yes	Yes

Table 4.3 presents the best results of ten runs for each of four versions. The columns display the instance type, the total cost, percentage deterioration in solution quality (Dev) of the three versions with respect to the HESA, and the computation time in seconds (Time). The row named Avg shows the average results. These results clearly indicate the benefit of including the EDUCATION, INTENSIFICATION and MUTATION procedures within the HESA. The HESA is consistently superior to all other versions on all instances. Version (1) performs worse than all other three versions. The superiority of version (3) over version (2) confirms the usefulness of the INTENSIFICATION procedure in the algorithm. The computation times for all versions are of similar magnitude.

TABLE 4.3: Sensitivity analysis of the HESA components

Instance	Version (1)			Version (2)			Version (3)			HESA	
	Total cost	Dev	Time	Total cost	Dev	Time	Total cost	Dev	Time	Total cost	Time
C101	89091.74	4.37	274.19	87091.74	2.17	279.34	86590.81	1.61	286.38	85199.09	297.32
C203	199501.19	4.33	286.24	196801.71	3.02	294.13	192841.36	1.03	301.14	190864.00	308.09
R101	43941.23	4.91	271.39	42841.17	2.47	276.41	42640.26	2.01	282.51	41782.20	292.25
R211	185112.41	5.44	235.29	180119.73	2.81	240.13	178152.37	1.74	244.09	175051.00	247.03
RC105	41640.21	3.96	249.13	41340.21	3.27	271.31	40840.17	2.08	276.91	39990.10	281.31
RC207	183071.91	4.26	280.17	179270.13	2.23	290.43	178274.36	1.68	294.37	175280.00	300.42
Avg		4.54	266.07		2.66	275.29		1.69	280.90		287.74

4.4.3 Performance of the formulations

The formulations E_1, E_2, E_3, E_4 and E_5 are examined in terms of their ability to solve the FSMLRPTW to optimality on small-size (20-, 25-, and 30-customer) instances. To analyze the computational results, we used the following performance measures: the deviation (Dev) and computation time in seconds (Time) averaged over all instances for each instance set (over a total of 840 experiments), and the number of optimal solutions ($\#Op$) obtained within one hour of computation time. Dev is the percentage deviation between the Upper Bound (UB) and the best-known Lower Bound (LB), i.e., 100 (UB − LB)/ UB. The upper bound is the optimal or best known solution obtained by solving the formulations.

Table 4.4 presents comparative average results over the five formulations. The first column displays the instance sets, and the following two columns show the number of customers $|\mathcal{N}_c|$ and the number of depots $|\mathcal{N}_0|$, respectively. The overall results of E_1 are better than those of E_3, thus we infer that constraints (4.6) strengthen formulation E_1. The results shown in Table 4.4 indicate that formulation E_4 performs better than the other models in terms of reaching optimal solutions within one hour of computation time. These results imply that instead of constraints (4.4)–(4.9), using constraints (4.23)–(4.28) which include the aggregated variables f_{ij}, and instead of constraints (4.16)–(4.17) using constraints (4.30)–(4.31), which include the aggregated variables f_{ij}, yields better formulations for the FSMLRPTW. Formulation E_4 yields 27 optimal solutions out of 56 instances for the 20-customer instances, 13 out of 56 instances for 25-customer instances, and nine out of 56 instances for 30-customer instances within the given time limit of one hour. In terms of computation time, E_4 provides on average lower computation time than the other formulations.

We now investigate the effect of the valid inequalities on formulation E_4. To this end, we have conducted experiments on instances of the same size and we have used the same performance measures as shown in Table 4.4. We used four variations of E_4 denoted as V_1, V_2, V_3 and V_4. Variation V_1 is formulation E_4 with valid inequalities (4.34), V_2 is formulation E_4 with valid inequalities (4.34) and (4.35), V_3 is formulation E_4 with valid inequalities (4.34)–(4.36), and V_4 is formulation E_4 with valid inequalities (4.34)–(4.37). Table 4.5 summarizes the average results of the effect of the valid inequalities on formulation E_4. These results show that V_4, which includes all valid inequalities,

TABLE 4.4: Average results of the formulations

Instance set	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	E_1			E_2			E_3			E_4			E_5		
			Dev	Time	#*Op*	Dev	Time	#*Op*	Dev	Time	#*Op*	Dev	Time	#*Op*	Dev	Time	#*Op*
C1 (9)	20	5	0.07	2245.17	4	0.07	1978.42	5	0.08	2417.07	5	0.08	2505.74	4	0.08	2467.02	3
C2 (8)	20	5	0.07	2245.17	7	0.00	844.86	7	0.42	2782.58	2	0.00	573.49	8	0.00	359.59	8
R1 (12)	20	5	0.29	2947.26	3	0.28	2936.22	3	0.32	3019.03	2	0.21	3066.77	3	0.20	3019.31	2
R2 (11)	20	5	0.01	2059.15	7	0.01	1975.49	7	0.27	3291.85	1	0.00	751.98	11	0.00	894.84	10
RC1 (8)	20	5	0.17	3228.86	1	0.18	3251.18	1	0.25	3208.96	1	0.17	3222.55	1	0.17	3192.03	1
RC2 (8)	20	5	0.07	3600.00	0	0.06	3600.00	0	0.07	3600.00	0	0.07	3600.00	0	0.06	3600.00	0
Avg (Total)			0.11	2720.94	(22)	0.10	2431.03	(23)	0.23	3053.25	(11)	0.09	2286.76	(27)	0.09	2255.46	(24)
C1 (9)	25	5	0.12	2669.77	3	0.11	2480.23	3	0.16	3600.00	0	0.14	3207.57	1	0.14	3432.45	1
C2 (8)	25	5	0.02	1607.91	5	0.05	1932.03	5	0.31	3151.28	1	0.00	1215.17	6	0.00	1120.62	6
R1 (12)	25	5	0.44	3302.73	1	0.43	3302.31	1	0.32	3151.80	2	0.35	3300.77	1	0.37	3301.73	1
R2 (11)	25	5	0.17	3325.51	1	0.11	3057.42	2	0.24	3308.48	1	0.03	2597.39	4	0.06	2957.59	2
RC1 (8)	25	5	0.22	3600.00	0	0.22	3289.92	1	0.23	3305.74	1	0.20	3204.17	1	0.21	3367.11	1
RC2 (8)	25	5	0.10	3600.00	0	0.07	3600.00	0	0.10	3600.00	0	0.09	3600.00	0	0.07	3600.00	0
Avg (Total)			0.18	3017.65	(10)	0.16	2943.65	(12)	0.23	3352.88	(5)	0.14	2854.18	(13)	0.14	2963.25	(11)
C1 (9)	30	5	0.32	3600.00	0	0.26	3264.60	1	0.32	3600.00	0	0.32	3600.00	0	0.33	3600.00	0
C2 (8)	30	5	0.02	2276.50	5	0.00	1478.03	6	0.25	2975.67	2	0.10	1545.67	6	0.12	2083.98	5
R1 (12)	30	5	0.48	3305.80	1	0.50	3317.96	1	0.72	3310.17	1	0.49	3303.81	1	0.47	3303.81	1
R2 (11)	30	5	0.21	3600.00	0	0.14	3434.65	1	0.33	3600.00	0	0.08	3159.86	2	0.07	3087.20	2
RC1 (8)	30	5	0.32	3600.00	0	0.28	3600.00	0	0.34	3600.00	0	0.38	3600.00	0	0.32	3600.00	0
RC2 (8)	30	5	1.26	3600.00	0	0.75	3600.00	0	1.01	3600.00	0	0.8	3600.00	0	0.67	3600.00	0
Avg (Total)			0.43	3330.38	(6)	0.32	3115.87	(9)	0.49	3447.64	(3)	0.36	3134.89	(9)	0.33	3212.50	(8)

TABLE 4.5: Effect of the valid inequalities

Instance set	$\|\mathcal{N}_c\|$	$\|\mathcal{N}_0\|$	V_1			V_2			V_3			V_4		
			Dev	Time	$\#Op$	Dev	Time	$\#Op$	Dev	Time	$\#Op$	Dev	Time	$\#Op$
C1 (9)	20	5	0.07	1728.73	5	0.08	1694.42	5	0.06	1705.47	5	0.07	1881.50	5
C2 (8)	20	5	0.07	2245.17	7	0.01	714.34	7	0.01	628.12	7	0.00	615.45	8
R1 (12)	20	5	0.21	3007.92	2	0.24	2757.75	3	0.24	2755.51	3	0.20	2613.15	3
R2 (11)	20	5	0.01	1109.64	9	0.01	1141.27	9	0.01	1059.42	10	0.01	1029.43	10
RC1 (8)	20	5	0.17	3224.54	1	0.17	3208.28	1	0.17	3236.80	1	0.16	3199.37	1
RC2 (8)	20	5	0.07	3600.00	0	0.08	3600.00	0	0.08	3600.00	0	0.07	3600.00	0
Avg (Total)			0.10	2486.00	24	0.10	2186.01	25	0.10	2164.22	26	0.09	2156.48	27
C1 (9)	25	5	0.13	2810.01	2	0.13	2795.28	2	0.12	2795.30	2	0.10	3193.31	1
C2 (8)	25	5	0.01	1283.90	6	0.01	1369.76	6	0.01	1308.49	6	0.01	1126.88	6
R1 (12)	25	5	0.31	3058.63	2	0.20	3143.78	2	0.20	3074.96	2	0.20	3075.17	2
R2 (22)	25	5	0.06	2715.95	2	0.10	3561.73	2	0.25	3571.71	2	0.04	2839.60	3
RC1 (8)	25	5	0.22	3283.27	1	0.20	3289.40	1	0.10	3301.02	1	0.30	3258.57	1
RC2 (8)	25	5	0.10	3600.00	0	0.14	3600.00	0	0.10	3600.00	0	0.07	3600.00	0
Avg (Total)			0.14	2791.96	13	0.13	2959.99	13	0.13	2941.91	13	0.12	2848.92	13
C1 (9)	30	5	0.28	3197.45	1	0.30	3591.70	0	0.32	3591.61	1	0.28	3249.88	1
C2 (8)	30	5	0.14	2081.03	4	0.14	1822.59	5	0.17	2180.70	4	0.19	1930.52	5
R1 (12)	30	5	0.43	3296.05	1	0.50	3295.51	1	0.41	3295.57	1	0.40	3295.65	1
R2 (22)	30	5	0.10	3356.11	1	0.11	3434.18	1	0.06	3399.88	1	0.06	3321.14	2
RC1 (8)	30	5	0.28	3600.00	0	0.35	3600.00	0	0.34	3600.00	0	0.31	3600.00	0
RC2 (8)	30	5	0.89	3600.00	0	0.65	3600.00	0	0.70	3600.00	0	0.67	3600.00	0
Avg (Total)			0.35	3188.44	7	0.34	3224.00	7	0.33	3277.96	7	0.32	3066.20	9

performs better than all other variations in terms of reaching optimal solutions within one hour of computation time. For the 20-, 25-, and 30-customer instances, the solution times are 2286.76, 2854.18 and 3134.89 seconds for E_4, but these times are 2156.48, 2848.92, 3066.20 seconds for V_4. Our results indicate that V_4 yields same number of optimal solutions as E_4, but within smaller computation times.

4.4.4 Comparative analysis

We now present a comparative analysis of the results of the HESA and of the formulation E_4 integrated with valid inequalities, denoted by E_4^v. Each instance was solved once with the HESA, and once with E_4^v. For E_4^v, a common time limit of three hours was imposed on the solution time for all instances. For the HESA, ten separate runs are performed for each instance, the best one of which is reported.

Tables 4.6 and 4.7 summarize the average results of the HESA compared with E_4^v. For detailed results, the reader is referred to Appendix A. In Tables 4.6 and 4.7, the first column displays the LP relaxation value of E_4^v, obtained by relaxing the integrality constraints (4.18) on the X_{ij}^h variables only. Such a partial LP relaxation provides better quality lower bounds compared to a full relaxation where the integer restrictions (4.18)–(4.20) on all binary variables are relaxed, and in comparable solution times. The remaining columns show the total time of all 10 runs of the HESA (Total time (s.)), the time of the best solution (Time (s.)), the percent deviation values of the total cost (Dev_{TC}) and of the vehicle cost (Dev_{VC}) found by E_4^v with respect to the HESA. In the Time column, "*" denotes that the instance was not solved to optimality within three hours.

Table 4.6 shows that the HESA finds almost the same solutions as those of E_4^v but in a substantially smaller amount of time on the small-size FSMLPRTW instances. The average time required by E_4^v to solve 10-, 15-, 20-, 25- and 30-customer instances to optimality are 26.26, 3234.61, 6012.79, 8056.69 and 9359.53 seconds, respectively. For the HESA, the respective statistics are 3.18, 3.95, 6.58, 8.93 and 13.05 seconds.

As can be observed from Table 4.7, the results clearly indicate that the HESA runs quickly, also for medium and large-size instances. In particular, the algorithm requires 90.08, 167.82 and 299.98 seconds of average computation time to solve 50-, 75- and

100-customer instances, respectively. The HESA is able to produce considerably better results than E_4^v does in three hours. The improvements in solution values can be as high as 6.42%, with an average of 1.41% for the 50-customer instances. Similarly, the average improvement is 5.11% for the 75-customer instances, the highest value sitting at 23.09%. The results are even more striking for the 100-customer instances where the average total cost reduction obtained was 19.93% compared to E_4^v. In case of 50-, 75- and 100-customer instances E_4^v was not able to find optimal solutions for 166 instances out of 168 within three hours.

We now investigate the effect of the fleet mix composition on the FSMLRPTW instances using the vehicle costs and Dev_{VC} values in Table 4.7. The results show that when optimality is guaranteed by E_4^v, the HESA is able to find the optimal fleet mix composition. For the 50- and 75-customer instances, the HESA decreased the vehicle costs by 8.08% and 32.10% on average compared to E_4^v. This decrease ranges from 0.00 to 18.68% and from 4.20% to 60.60% for the 50- and 75-customer instances, respectively. As for the total costs, the results are even more striking for the 100-customer instances where the average vehicle costs reduction obtained was 53.82% compared to E_4^v. These results imply that for medium- and large-size instances, the HESA is able to produce substantially lower vehicle costs than E_4^v.

4.5 Conclusions

This paper has introduced the FSMLRPTW, a complex integrated logistics problem which, to our knowledge, was studied here for the first time, and has described a hybrid evolutionary search algorithm tailored to the problem. We have introduced several algorithmic procedures specific to the FSMLRPTW, namely, the location-heterogeneous adaptive large neighborhood search procedure equipped with a range of new operators as the main EDUCATION procedure within the search. We have also developed an INITIALIZATION procedure to create initial solutions, a PARTITION procedure for offspring solutions, and a new diversification scheme through the MUTATION procedure of solutions. Computational results on a new set of benchmark instances of up to 100 nodes and 10 potential depots were presented, which indicate that the proposed algorithm is able to identify solutions within 0.05% of optimality for small size instances and yields

Table 4.6: Average results on small-size instances

Instance set	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	E_4^v				HESA					
			LP relaxation	Vehicle cost	Total cost	Time (s.)	Vehicle cost	Total cost	Total time (s.)	Time (s.)	Dev_{TC}	Dev_{VC}
C1 (9)	10	3	82504.85	600.00	82674.64	0.38	600.00	82674.64	37.83	3.57	0.00	0.00
C2 (8)	10	3	191118.19	755.56	192201.84	2.07	755.56	192201.84	34.45	3.31	0.00	0.00
R1 (12)	10	3	39382.04	911.11	39466.34	128.64	911.11	39466.34	31.02	3.01	0.00	0.00
R2 (11)	10	3	175618.55	1066.67	176157.95	0.96	1066.67	176157.95	34.04	3.18	0.00	0.00
RC1 (8)	10	3	35614.85	1222.22	35716.90	10.77	1222.22	35716.90	34.41	3.19	0.00	0.00
RC2 (8)	10	3	173434.98	1377.78	173622.24	14.76	1377.78	173622.24	30.75	2.82	0.00	0.00
Avg						26.26			33.75	3.18	0.00	0.00
C1 (9)	15	4	81889.59	900.00	82059.92	5.51	900.00	82059.92	45.83	4.51	0.00	0.00
C2 (8)	15	4	186126.81	2000.00	187193.63	9.52	2000.00	187193.63	37.88	3.72	0.00	0.00
R1 (12)	15	4	39590.64	360.00	39702.34	6328.34	360.00	39702.41	39.27	3.86	0.00	0.00
R2 (11)	15	4	174662.67	900.00	175190.12	98.72	900.00	175190.12	39.53	3.88	0.00	0.00
RC1 (8)	15	4	35843.17	660.00	35961.05	3469.07	660.00	35961.05	36.70	3.60	0.00	0.00
RC2 (8)	15	4	173626.06	650.00	173826.21	9496.51	650.00	173826.21	42.24	4.15	0.00	0.00
Avg						3234.61			40.24	3.95	0.00	0.00
C1 (9)	20	5	82236.26	1200.00	82435.86	5263.90	1200.00	82450.45	63.52	6.25	0.02	0.00
C2 (8)	20	5	186141.23	2000.00	187224.06	601.82	2000.00	187224.06	66.28	6.46	0.00	0.00
R1 (12)	20	5	37782.90	479.17	38139.76	8220.94	479.17	38139.37	63.51	6.18	0.00	0.00
R2 (11)	20	5	174699.40	900.00	175239.55	1682.87	900.00	175238.39	64.92	6.34	0.00	0.00
RC1 (8)	20	5	36075.96	870.00	36242.45	9507.20	870.00	36246.40	71.50	7.04	0.01	0.00
RC2 (8)	20	5	173864.43	800.00	174079.45	10800.00*	800.00	174078.67	74.01	7.20	0.00	0.00
Avg						6012.79			67.29	6.58	0.00	0.00
C1 (9)	25	5	82556.95	1500.00	82767.35	8404.21	1500.00	82782.57	95.31	9.33	0.02	0.00
C2 (8)	25	5	186314.82	2000.00	187234.75	2919.09	2000.00	187234.75	87.54	8.45	0.00	0.00
R1 (12)	25	5	37930.66	613.33	38306.32	9081.98	613.33	38287.22	94.04	9.00	−0.05	0.00
R2 (11)	25	5	174795.80	900.00	175313.88	7563.09	900.00	175311.64	90.60	8.96	0.00	0.00
RC1 (8)	25	5	36252.02	990.00	36414.34	9571.79	990.00	36430.18	95.26	9.03	0.04	0.00
RC2 (8)	25	5	174075.96	1000.00	174301.53	10800.00*	1000.00	174300.67	91.18	8.82	0.00	0.00
Avg						8056.69			92.32	8.93	0.00	0.00
C1 (9)	30	5	82758.85	1800.00	83096.43	9656.17	1800.00	83115.24	143.41	13.84	0.02	0.00
C2 (8)	30	5	186488.20	2000.00	187253.95	5519.71	2000.00	187252.10	128.64	12.56	0.00	0.00
R1 (12)	30	5	38126.33	764.17	38543.11	9902.81	764.17	38495.67	119.46	11.75	−0.12	0.00
R2 (11)	30	5	174930.96	900.00	175371.60	9478.50	900.00	175356.43	147.07	14.31	−0.01	0.00
RC1 (8)	30	5	36474.96	1132.50	36715.80	10800.00*	1132.50	36717.50	129.89	12.69	0.00	0.00
RC2 (8)	30	5	36199.06	1131.25	36555.58	10800.00*	1131.25	36572.23	132.83	13.18	0.05	0.00
Avg						9359.53			133.55	13.05	−0.01	0.00

TABLE 4.7: Average results on medium- and large-size instances

Instance set	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	E_4^v				HESA					
			LP relaxation	Vehicle cost	Total cost	Time (s.)	Vehicle cost	Total cost	Total time (s.)	Time (s.)	Dev_{TC}	Dev_{VC}
C1 (9)	50	7	41386.56	2722.22	41875.93	9644.33	2722.22	41910.79	934.92	93.09	0.08	0.00
C2 (8)	50	7	92446.10	2975.00	93394.71	9450.78	2400.00	92822.45	996.13	99.21	−0.61	−18.68
R1 (12)	50	7	38870.22	1350.00	39722.87	10800.00*	1301.67	39488.63	937.20	93.32	−0.58	−3.42
R2 (11)	50	7	175518.42	1540.91	176329.57	10800.00*	1350.00	175991.78	931.02	92.70	−0.19	−9.90
RC1 (8)	50	7	19574.34	2081.25	22807.06	10800.00*	1938.75	20485.74	769.46	76.55	−6.42	−6.61
RC2 (8)	50	7	87065.42	2181.25	88183.79	10800.00*	1868.75	87536.04	860.16	85.62	−0.73	−9.87
Avg						10382.52			904.82	90.08	−1.41	−8.08
C1 (9)	75	8	82857.15	4511.11	83662.53	10800.00*	4288.89	83479.50	1739.49	173.35	−0.22	−4.20
C2 (8)	75	8	188792.53	7075.00	192980.52	10800.00*	3800.00	189374.75	1662.91	165.39	−1.84	−37.46
R1 (12)	75	8	38056.29	4041.67	64165.06	10800.00*	1969.17	39939.26	1726.14	172.11	−23.09	−41.79
R2 (11)	75	8	174160.82	5245.45	178597.02	10800.00*	1800.00	174601.27	1621.98	161.90	−2.23	−60.60
RC1 (8)	75	8	38233.86	3086.25	39523.52	10800.00*	2595.00	38768.89	1741.39	174.04	−1.90	−14.64
RC2 (8)	75	8	173677.24	4175.00	176872.38	10800.00*	2450.00	174367.88	1608.39	160.14	−1.41	−33.89
Avg						10800.00*			1683.38	167.82	−5.11	−32.10
C1 (9)	100	10	84345.45	7044.44	86863.29	10800.00*	5788.89	85234.48	3082.24	307.82	−1.83	−13.61
C2 (8)	100	10	190077.54	22887.50	330436.31	10800.00*	5200.00	190862.38	3083.79	308.28	−17.58	−56.77
R1 (12)	100	10	39151.72	9831.67	166296.26	10800.00*	2648.33	40972.43	2902.51	289.95	−65.67	−66.69
R2 (11)	100	10	174794.11	12068.18	185688.94	10800.00*	2250.00	175171.82	2934.80	292.88	−5.64	−80.01
RC1 (8)	100	10	39186.42	6123.75	49304.23	10800.00*	3405.00	39889.18	3062.54	306.05	−15.53	−40.49
RC2 (8)	100	10	174430.65	12568.75	268749.92	10800.00*	2962.50	175057.75	2956.81	294.88	−13.35	−65.35
Avg						10800.00*			3003.78	299.98	−19.93	−53.82

better solutions for larger instances as compared to an off-the-shelf solver. The running times of the algorithm are so that it can be used in practical applications.

Chapter 5

The Fleet Size and Mix Pollution-Routing Problem

Abstract

This paper introduces the fleet size and mix pollution-routing problem which extends the pollution-routing problem by considering a heterogeneous vehicle fleet. The main objective is to minimize the sum of vehicle fixed costs and routing cost, where the latter can be defined with respect to the cost of fuel and CO_2 emissions, and driver cost. Solving this problem poses several methodological challenges. To this end, we have developed a powerful metaheuristic which was successfully applied to a large pool of realistic benchmark instances. Several analyses were conducted to shed light on the trade-offs between various performance indicators, including capacity utilization, fuel and emissions and costs pertaining to vehicle acquisition, fuel consumption and drivers. The analyses also quantify the benefits of using a heterogeneous fleet over a homogeneous one.

Keywords. vehicle routing; fuel consumption; CO_2 emissions; heterogeneous fleet; evolutionary metaheuristic

5.1 Introduction

Road freight transport is a primary source of greenhouse gases (GHGs) emissions such as carbon dioxide (CO_2), the amount of which is directly proportional to fuel consumption (Kirby et al. 2000). In the United Kingdom and in the United States, around a quarter of GHGs comes from freight transportation (DfT 2012, EPA 2012). Greenhouse gases mainly result from burning fossil fuel, and over 90% of the fuel used for freight transportation is petroleum-based, which includes gasoline and diesel. These sources account for over half of the emissions from the transportation sector (Kahn Ribeiro et al. 2007).

Demir et al. (2011) have analyzed several models for fuel consumption and greenhouse gas emissions in road freight transportation. Specifically, the authors have compared six models and have assessed their respective strengths and weaknesses. These models indicate that fuel consumption depends on a number of factors that can be grouped into four categories: vehicle, driver, environment and traffic. The pollution-routing problem (PRP), introduced by Bektaş and Laporte (2011), is an extension of the classical vehicle routing problem with time windows (VRPTW). It consists of routing vehicles to serve a set of customers, and of determining their speed on each route segment to minimize a function comprising fuel cost, emissions and driver costs. To estimate pollution, the authors apply a simplified version of the emission and fuel consumption model proposed by Barth et al. (2005), Scora and Barth (2006) and Barth and Boriboonsomsin (2009). The simplified model assumes that in a vehicle trip all parameters will remain constant on a given arc, but load and speed may change from one arc to another. As such, the PRP model approximates the total amount of energy consumed on a given road segment, which directly translates into fuel consumption and further into GHG emissions. Demir et al. (2012) have developed an extended adaptive large neighbourhood search (ALNS) heuristic for the PRP. This heuristic operates in two stages: the first stage is an extension of the classical ALNS scheme to construct vehicle routes (Pisinger and Ropke 2007, Ropke and Pisinger 2006a,b) and the second stage applies a speed optimization algorithm (SOA) (Norstad et al. 2010, Hvattum et al. 2013) to compute the speed on each arc. In a later study, Demir et al. (2014a) have introduced the bi-objective PRP which jointly minimizes fuel consumption and driving time. The authors have developed a bi-objective adaptation of their ALNS-SOA heuristic and compared four *a posteriori* methods, namely the weighting method, the weighting method with normalization, the epsilon-constraint method and a new hybrid method, using a scalarization of the two objective functions.

The trade-off between minimizing CO_2 emissions and minimizing total travel times was studied by Jabali et al. (2012b) in the context of the time-dependent vehicle routing problem. The planning horizon was partitioned into two phases: free flow traffic and congestion. The authors solved the problem via a tabu search and proposed efficient bounding procedures. Franceschetti et al. (2013) have later introduced the time-dependent pollution-routing problem where a two-stage planning horizon was used, as in Jabali et al. (2012b). Such a treatment has allowed for an explicit modeling of congestion in addition to the PRP objectives. The authors developed an integer linear programming formulation in which vehicle speeds are optimally selected from a set of discrete values. Kopfer and Kopfer (2013) studied the emission minimization vehicle routing problem while considering a heterogeneous fleet. These authors described a mathematical formulation for the problem and computed the CO_2 emissions based on the payload and on the traveled distance. They presented results of computational experiments performed on small size instances with up to 10 customers. Kopfer et al. (2014) have analyzed the potential of reducing CO_2 emissions achievable by using an unlimited heterogeneous fleet of vehicles of different sizes. Kwon et al. (2013) have considered the heterogeneous fixed fleet vehicle routing problem with the objective of minimizing carbon emissions. They presented a mathematical model enabling them to perform a cost-benefit assessment of the value of purchasing or selling carbon emission rights. CO_2 emissions are calculated by fuel consumption which is based on the traveled distance of the vehicles. An upper limit for the amount of CO_2 was considered in order to introduce more flexibility into an environmentally constrained network. The authors developed tabu search algorithms and suggested that the amount of carbon emission can be reduced without sacrificing the cost because of the benefit obtained from carbon trading. For other relevant references and a state-of-the-art coverage on green road freight transportation, the reader is referred to the survey of Demir et al. (2014b).

In most real-world distribution problems, customer demands are met with heterogeneous vehicle fleets (Hoff et al. 2010). Two major problems belonging to this category are the fleet size and mix vehicle routing problem introduced by Golden et al. (1984), which works with an unlimited heterogeneous fleet, and the heterogeneous fixed fleet vehicle routing problem proposed by Taillard (1999), which works with a known fleet. These two main problems are reviewed by Baldacci et al. (2008) and Baldacci et al. (2009). To our knowledge, the fleet size and mix vehicle routing problem combining time windows with the PRP objectives, has not yet been investigated. We believe there is merit in analyzing and solving the fleet size and mix pollution-routing problem (FSMPRP), not only to quantify the benefits of using a flexible fleet with respect to fuel, emissions and the relevant costs, but also to overcome the necessary methodological challenges to solve the problem.

The contributions of this paper are threefold. First, we introduce the FSMPRP as a new PRP variant. The second contribution is to develop a new metaheuristic for the FSMPRP. Our third contribution is to perform analyses in order to provide managerial insights, using the FSMPRP model and several variants. These analyses shed light on the trade-offs between various method components and performance measures, such as distance, fuel and emissions, enroute time and vehicle types. They also highlight and quantify the benefits of using a heterogeneous fleet of vehicles over a homogeneous fleet.

The remainder of this paper is structured as follows. Section 5.2 presents a background on vehicle types and characteristics. Section 5.3 provides a formal description of the FSMPRP and the mathematical formulation. Section 5.4 contains a detailed description of the metaheuristic. Computational experiments and analyses are presented in Section 5.5, followed by conclusions in Section 5.6.

5.2 Background on Vehicle Types and Characteristics

Available studies on emission models (e.g., Demir et al. 2011, 2014b) show the significant impact that the vehicle type has on fuel consumption. In a goods distribution context, using smaller capacity vehicles is likely to increase the total distance travelled and may also increase CO_2 emissions. According to Campbell (1995a,b), if large vehicles are replaced by a larger number of small vehicles, emissions are likely to increase, even though a heavy duty vehicle which has a larger engine consumes more fuel per km than a light duty vehicle. According to Kopfer et al. (2014), replacing a large vehicle by several vehicles of different types may sometimes result in a reduction of CO_2 emissions. Vehicle type effects the engine friction factor, engine speed, engine displacement, aerodynamics drag, frontal surface area and vehicle drive train efficiency; vehicle curb-weight and payload, i.e., capacity, also play an important role in routing decisions.

In the United Kingdom, the Department of Environment, Food and Rural Affairs (DEFRA 2007) considers that higher-power engines do not necessarily result in fuel savings, and although these types of engines usually have a larger residual value, they may not be financially advantageous. The effects of curb weight and payload on fuel consumption have been studied by some authors (Bektaş and Laporte 2011, Demir et al. 2011). The payload of the vehicle has an impact on inertia force, rolling resistance and road slope force. Demir et al. (2011) point out that when compared with light and medium duty, heavy duty vehicles consume significantly more fuel, primarily due to their weight. From the perspective of payload reduction, a study by Caterpillar (2006) has shown that a 4.4% improvement in fuel savings can be reached through a 4500 kg reduction in payload and in gross weight with respect to an initial weight of 36 tonnes. The corresponding improvement is 8.8% for an initial weight of 27 tonnes. DEFRA (2012) states that a 17-tonne heavy duty vehicle emits 18% more CO_2 per km when fully loaded, and 18% less CO_2 per km when empty, relative to emissions at half-load.

The curb weight and payload constitute the Gross Vehicle Weight Rating (GVWR) of a vehicle. The United States Federal Highway Administration FHWA (2011) has categorized vehicles into three main types according to the GVWR: light duty, medium duty, and heavy duty. In practice, the prominent truck companies produce mainly three vehicle types for distribution (MAN 2014a, Mercedes-Benz 2014, Renault 2014, Volvo 2014). In our study, we consider the three main vehicle types of MAN (2014a), shown in Figure 5.1, particularly as the market share of the trucks of MAN (2014a) was around 16.3% in Western Europe in 2013 (Statista 2013). These three vehicle types, i.e., light duty, medium duty and heavy duty, are called TGL, TGM and TGX by MAN (2014a). TGL and TGM are Single-Unit Trucks and TGX is a Single-Trailer Truck (FHWA 2011).

A list of and values for the common parameters (Demir et al. 2012, 2014a, Franceschetti et al. 2013) for all vehicle types and specific parameters (MAN 2014a,b,c) for each vehicle type are given in Tables 5.1 and 5.2, respectively. For further details on TGL, TGM and TGX vehicles and their engines the reader is referred to MAN (2014a,b,c).

TABLE 5.1: Vehicle common parameters

Notation	Description	Typical values
ξ	fuel-to-air mass ratio	1
g	gravitational constant (m/s^2)	9.81
ρ	air density (kg/m^3)	1.2041
C_r	coefficient of rolling resistance	0.01
η	efficiency parameter for diesel engines	0.45
f_c	fuel and CO_2 emissions cost (£/liter)	1.4
f_d	driver wage (£/s)	0.0022
κ	heating value of a typical diesel fuel (kj/g)	44
ψ	conversion factor (g/s to L/s)	737
n_{tf}	vehicle drive train efficiency	0.45
v^l	lower speed limit (m/s)	5.5 (or 20 km/h)
v^u	upper speed limit (m/s)	27.8 (or 100 km/h)
θ	road angle	0
τ	acceleration (m/s^2)	0

Daily vehicle fixed costs f^h are determined according to the United Kingdom Department for Transport (DfT 2010). These costs combine the capital cost and the annual fixed cost, which itself includes depreciation, repairs and maintenance, tires, insurance and vehicle excise duty. In this paper, we assume that each vehicle route can be completed in one day, so that we can transform the capital and annual cost values into daily costs.

a) Light duty vehicle: TGL

b) Medium duty vehicle: TGM

c) Heavy duty vehicle: TGX

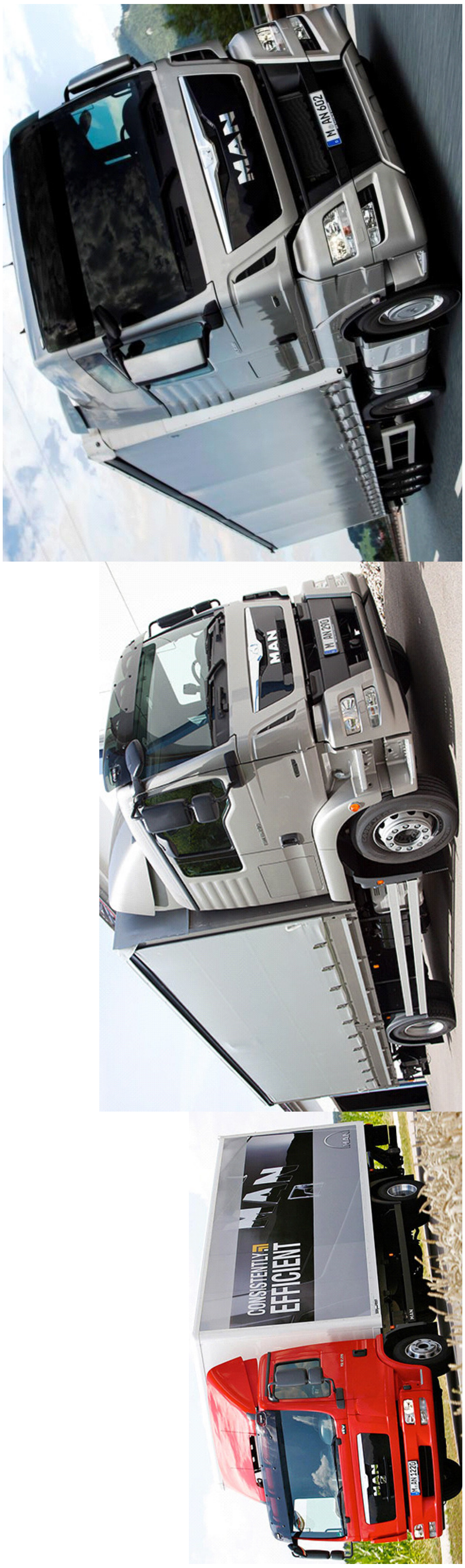


Figure 5.1: Three vehicle types (MAN 2014a)

TABLE 5.2: Vehicle specific parameters

Notation	Description	Light duty (L)	Medium duty (M)	Heavy duty (H)
w^h	curb weight (kg)	3500	5500	14000
Q^h	maximum payload (kg)	4000	12500	26000
f^h	vehicle fixed cost (£/day)	42	60	95
k^h	engine friction factor (kj/rev/liter)	0.25	0.20	0.15
N^h	engine speed (rev/s)	38.34	36.67	30.0
V^h	engine displacement (liter)	4.5	6.9	10.5
C_d^h	coefficient of aerodynamics drag	0.6	0.7	0.9
A^h	frontal surface area (m^2)	7.0	8.0	10.0

We use the comprehensive emissions model of Barth et al. (2005), Scora and Barth (2006), and Barth and Boriboonsomsin (2008) to estimate fuel consumption and emissions for a given time instant. This model has already been succesfully applied to the PRP by Bektaş and Laporte (2011), Demir et al. (2012, 2014a) and Franceschetti et al. (2013). In what follows, we adapt the comprehensive emissions model to account for the heterogeneous fleet case. The fuel consumption rate FR^h (liter/s) of a vehicle of type h is given by

$$FR^h = \xi(k^h N^h V^h + P^h/\eta)/\kappa, \tag{5.1}$$

where the variable P^h is the second-by-second engine power output (in kW) of vehicle type h. It can be calculated as

$$P^h = P^h_{tract}/n_{tf} + P_{acc}, \tag{5.2}$$

where the engine power demand P_{acc} is associated with the running losses of the engine and the operation of vehicle accessories such as air conditioning and electrical loads. We assume that $P_{acc} = 0$. The total tractive power requirement P^h_{tract} (in kW) for a vehicle of type h is

$$P^h_{tract} = (M^h\tau + M^h g\sin\theta + 0.5C^h_d\rho Av^2 + M^h gC_r\cos\theta)v/1000, \tag{5.3}$$

where M^h is the total vehicle weight (in kg) and v is the vehicle speed (m/s). The fuel consumption F^h (in liters) of vehicle type h over a distance d, is calculated as

$$F^h = k^h N^h V^h \lambda d/v + P^h\lambda\gamma^h d/v, \tag{5.4}$$

where $\lambda = \xi/\kappa\psi$, $\gamma^h = 1/1000n_{tf}\eta$ and $\alpha = \tau + g\sin\theta + gC_r\cos\theta$ are constants. Let $\beta^h = 0.5C^h_d\rho A^h$ be a vehicle-specific constant. Therefore, F^h can be rewritten as

$$F^h = \lambda(k^h N^h V^h d/v + M^h\gamma^h\alpha d + \beta^h\gamma^h dv^2). \tag{5.5}$$

In this expression the first term $k^h N^h V^h d/v$ is called the engine module, which is linear in the travel time. The second term $M^h\gamma^h\alpha_{ij}d$ is referred to as the weight module, and the third term $\beta^h\gamma^h dv^2$ is the speed module, which is quadratic in speed. These functions will be used in the objective function of our model.

5.3 Mathematical Model for the Fleet Size and Mix Pollution-Routing Problem

The FSMPRP is defined on a complete directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ where $\mathcal{N} = \{0, \ldots, n\}$ is the set of nodes, $\mathcal{A} = \{(i, j) : i, j \in \mathcal{N}, i \neq j\}$ is the set of arcs, and node 0 corresponds to the depot. The distance from i to j is denoted by d_{ij}. The customer set is $\mathcal{N}_c = \mathcal{N}\backslash\{0\}$, and each customer i has a positive demand q_i. The index set of vehicle types is denoted by $\mathcal{H}$. If a vehicle arrives at customer i before a_i, it waits until a_i before servicing the node. Furthermore, t_i corresponds to the service time of node $i \in N_0$, which must start within time window $[a_i,\ b_i]$.

The objective of the FSMPRP is to minimize a total cost function encompassing vehicle, driver, fuel and emissions costs. A feasible solution contains a set of routes for a heterogeneous fleet of vehicles that meet the demands of all customers within their respective predefined time windows. Each customer is visited once by a single vehicle, each vehicle must depart from and return to the depot, to serve a quantity of demand that does not exceed its capacity. Furthermore, the speed of each vehicle on each arc must be determined.

The binary variable x_{ij}^h is equal to 1 if and only if a vehicle of type $h \in \mathcal{H}$ travels on arc $(i, j) \in \mathcal{A}$. The formulation works with a discretized speed function, proposed by Bektaş and Laporte (2011), defined by R non-decreasing speed levels $\bar{v}^r$ $(r = 1, \ldots, R)$. The binary variable z_{ij}^{rh} is equal to 1 if and only if a vehicle of type $h \in \mathcal{H}$ travels on arc $(i, j) \in A$ at speed level $r = 1, \ldots, R$, y_j is the service start time at $j \in \mathcal{N}_c$. The total time spent on a route in which $j \in \mathcal{N}_c$ is the last visited node before returning to the depot is defined by s_j. Furthermore, let f_{ij}^h be the amount of commodity flowing on arc $(i, j) \in \mathcal{A}$ by a vehicle of type h. Therefore, the total load of vehicle of type h on arc (i, j) is $w^h + f_{ij}^h$. We now present an integer linear programming formulation for the FSMPRP:

$$\text{(FSMPRP) Minimize} \sum_{h \in \mathcal{H}} \sum_{(i,j) \in \mathcal{A}} \lambda f_c k^h N^h V^h d_{ij} \sum_{r=1}^{R} z_{ij}^{rh} / \bar{v}^r \tag{5.6}$$

$$+ \sum_{h \in \mathcal{H}} \sum_{(i,j) \in \mathcal{A}} \lambda f_c \gamma^h \alpha_{ij} d_{ij} (w^h x_{ij}^h + f_{ij}^h) \tag{5.7}$$

$$+ \sum_{h \in \mathcal{H}} \sum_{(i,j) \in \mathcal{A}} \lambda f_c \beta^h \gamma^h d_{ij} \sum_{r=1}^{R} (\bar{v}^r)^2 z_{ij}^{rh} \tag{5.8}$$

$$+ \sum_{j \in \mathcal{N}_c} f_d s_j + \sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}_c} f^h x_{0j}^h \tag{5.9}$$

subject to

$$\sum_{j \in \mathcal{N}_c} x_{0j}^h \leq m_h \qquad \forall h \in \mathcal{H} \tag{5.10}$$

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} x_{ij}^h = 1 \qquad \forall i \in \mathcal{N}_c \tag{5.11}$$

$$\sum_{h \in \mathcal{H}} \sum_{i \in \mathcal{N}} x_{ij}^h = 1 \qquad \forall j \in \mathcal{N}_c \tag{5.12}$$

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} f_{ji}^h - \sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} f_{ij}^h = q_i \qquad \forall i \in \mathcal{N}_c \tag{5.13}$$

$$q_j x_{ij}^h \leq f_{ij}^h \leq (Q^h - q_i) x_{ij}^h \qquad \forall (i,j) \in \mathcal{A}, \forall h \in \mathcal{H} \tag{5.14}$$

$$y_i - y_j + t_i + \sum_{r=1}^{R} d_{ij} z_{ij}^{rh} / \bar{v}^r \leq M_{ij} (1 - x_{ij}^h) \qquad \forall i \in \mathcal{N}, j \in \mathcal{N}_c, i \neq j, \forall h \in \mathcal{H} \tag{5.15}$$

$$a_i \leq y_i \leq b_i \qquad \forall i \in \mathcal{N}_c \tag{5.16}$$

$$y_j + t_j - s_j + \sum_{r=1}^{R} d_{j0} z_{j0}^{rh} / \bar{v}^r \leq L_{ij} (1 - x_{j0}^h) \qquad \forall j \in \mathcal{N}_c, \forall h \in \mathcal{H} \tag{5.17}$$

$$\sum_{r=1}^{R} z_{ij}^{rh} = x_{ij}^h \qquad \forall (i,j) \in \mathcal{A}, \forall h \in \mathcal{H} \tag{5.18}$$

$$x_{ij}^h \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A}, \forall h \in \mathcal{H} \tag{5.19}$$

$$z_{ij}^{rh} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A}, r = 1, \ldots, R, \forall h \in \mathcal{H} \tag{5.20}$$

$$f_{ij}^h \geq 0 \qquad \forall (i,j) \in \mathcal{A}, \forall h \in \mathcal{H} \tag{5.21}$$

$$y_i \geq 0 \qquad \forall i \in \mathcal{N}_c. \tag{5.22}$$

The first three terms of the objective function represent the cost of fuel consumption and of CO_2 emissions. In particular, term (5.6) computes the cost induced by the engine module, term (5.7) reflects the cost induced by the weight module and term (5.8) measures the cost induced by the speed module. Finally, term (5.9) computes the total driver wage and the sum of all vehicle fixed costs.

The maximum number of vehicles available for each type is imposed by constraints (5.10). We consider an unlimited number of vehicles for each vehicle type h ($m_h = |\mathcal{N}_c|$). Constraints (5.11) and (5.12) ensure that each customer is visited exactly once. Constraints (5.13) and (5.14) define the flows. Constraints (5.15)–(5.17) are time window constraints, where $M_{ij} = \max\{0, b_i + s_i + d_{ij}/\bar{v}^r - a_j\}$ and $L_{ij} = \max\{0, b_j + t_j + \max_i\{d_{ij}\}/\bar{v}^r\}$. Constraints (5.18) impose that only one speed level is selected for each arc. Finally, constraints (5.19)–(5.22) enforce the integrality and nonnegativity restrictions on the variables.

5.4 Description of the Hybrid Evolutionary Algorithm

This section describes the proposed hybrid evolutionary algorithm, called HEA++, for the FSMPRP. This algorithm builds on the HEA of Chapter 3, which is itself based on the principles put forward by Vidal et al. (2014). In this paper, we have additionaly developed the Heterogeneous Adaptive Large Neighborhood Search (HALNS) which is used as a main Higher Education component in the HEA++. An adapted version of the Speed Optimization Algorithm (SOA) (Norstad et al. 2010, Hvattum et al. 2013), which works with a continuous speed variable, is applied on a solution within the algorithm to optimize speeds between nodes. The combination of ALNS and SOA has provided good results for the PRP (Demir et al. 2012, 2014a).

The general framework of the HEA++ is sketched in Algorithm 1. We now explain the steps of the algorithm in reference to each line of Algorithm 1. The initial population is generated by using a modified version of the classical Clarke and Wright (1964) savings algorithm and the HALNS (line 1). A binary tournament process selects two parents from the population (line 3) and combines them into a new offspring C via crossover (line 4), which then undergoes an improvement step through an advanced Split algorithm with Speed Optimization Algorithm (Ssoa)(line 5). The Ssoa considers offspring C as an input, in the form of a giant tour and then optimally splits it into vehicle routes. In the Higher Education procedure, the HALNS with the SOA (line 6) are applied to offspring C. If C is infeasible, this procedure is iteratively applied until a modified version of C is feasible, which is then inserted into the population. The probabilities associated with the Higher Education procedure operators are updated by the adaptive weight adjustment procedure (AWAP) (line 7). The Intensification procedure is based on the HALNS and SOA (line 8), and is run on elite solutions. The population size n_a is limited by $n_p + n_o$, where n_p is a constant denoting the size of the initial population and n_o is a constant showing the maximum allowable number of offsprings that can be inserted into the population. A survivor selection mechanism is applied (line 9) if the populations size n_a reaches $n_p + n_o$ at any iteration. Mutation (line 10) is applied to a randomly selected individual from the population with probability p_m at each iteration of the algorithm. The entire population undergoes a Regeneration (line 11) procedure if there are no improvements in the best known solution for a given number of consecutive iterations υ. When the number ϖ of iterations without improvement in the incumbent solution is reached, the HEA++ terminates (line 13). For further implementation details on the initialization, parent selection, crossover, AWAP, survivor selection and diversification sections the reader is referred to Chapter 3.

Algorithm 1 General framework of the HEA

```
1:  Initialization: initialize a population with size n_p
2:  while number of iterations without improvement < ϖ do
3:      Parent selection: select parent solutions P_1 and P_2
4:      Crossover: generate offspring C from P_1 and P_2
5:      Ssoa: partition C into routes
6:      Higher Education: educate C with HALNS and SOA and insert into population
7:      AWAP: update probabilities of the HALNS operators
8:      Intensification: intensify elite solution with HALNS and SOA
9:      Survivor selection: if the population size n_a reaches n_p + n_o, then select survivors
10:     Mutation: diversify a random solution with probability p_m
11:     if number of iterations without improvement = υ then
12:         Regeneration: diversify the population with Regeneration procedures
13: end while
14: Return best feasible solution
```

In what follows we detail the algorithmic features specifically developed for the FSMPRP. The expanded version of the SOA is presented in Section 5.4.1, Ssoa is desribed in Section 5.4.2, and finally, the Higher Education and Intensification procedures are detailed in Section 5.4.3.

5.4.1 Speed optimization algorithm

The SOA is a heuristic procedure which optimizes the speed on each segment of a given route in order to minimize an objective function comprising fuel consumption costs and driver wages. Demir et al. (2012) adapted the arguments of Norstad et al. (2010) and Hvattum et al. (2013) to the PRP, which we describe here for the sake of completeness.

The SOA is defined on a feasible path $(0, ..., n+1)$ of nodes all served by a single vehicle, where 0 and $n+1$ are two copies of the depot. The speed v_{i-1}, represents the variable speed between nodes $i-1$ and i, $\underline{e}_i$ is the arrival time at node i and $\bar{e}_i$ is the departure time from node i. The detailed pseude-code of the SOA is shown in Algorithm 2. The SOA starts with a feasible route with initial fixed speeds, it takes input parameters start node s and end node e, D and T which are respectively the total distance and total service time, and returns speed-optimized routes. Initially, the speed v_{i-1}, for each link is calculated by considering the total distance of the route and the total trip duration without the total service time (lines 4–7 of Algorithm 2). The SOA runs in two stages where the main difference between these stages is the optimal speed v^*_{i-1} calculation (line 8 of Algorithm 2). In the first stage, optimal speeds are calculated as

$$v^* = \left(\frac{k^h N^h V^h}{2\beta^h \gamma^h} + \frac{f_d}{2\beta^h \lambda \gamma^h f_c} \right)^{1/3}, \tag{5.23}$$

which minimizes fuel consumption and driver wage. The first stage fixes the arrival time to the depot and uses this value as an input to the second stage where optimal speeds are calculated using

$$v^* = \left(\frac{k^h N^h V^h}{2\beta^h \gamma^h} \right)^{1/3}, \tag{5.24}$$

which minimizes fuel consumption in the second stage. The speeds are updated (lines 9–12 of Algorithm 2) if the vehicle arrives before a_i and departs before b_i or if the vehicle arrives before b_i and departs after $b_i + t_i$. If node i is the last customer before the depot, the speeds are recalculated to arrive at node i at a_i (lines 13–14 of Algorithm 2). If v_{i-1} is lower than v^l, then it is increased to v^l, or if it is greater than v^u, then it is decreased to v^u (lines 15–18 of Algorithm 2). The optimal speed is then compared with v_{i-1}, if the optimal speed is greater, v_{i-1} is then increased to the optimal speed (lines 19–20 of Algorithm 2). The new arrival and departure times at node i are then calculated (lines 21–23 of Algorithm 2). If the departure time is less than $a_i + t_i$ or if the arrival time is greater than b_i, the violation is calculated; otherwise, it is set to zero (lines 24–27 of Algorithm 2). At each iteration, the SOA selects the arc with largest time window violation and eliminates the violation.

Algorithm 2 Speed Optimization Algorithm (s, e)

1: **Input:** $violation \leftarrow 0$, $p \leftarrow 0$, $D \leftarrow \sum_{i=s}^{e-1} d_i$, $T \leftarrow \sum_{i=s}^{e} t_i$
2: **Output:** Speed optimized routes
3: **for** $i = s + 1$ **to** e **do**
4:     **if** $\underline{e}_s \leq a_s$ **then**
5:         $v_{i-1} \leftarrow D/(\overline{e}_e - a_s - T)$
6:     **else**
7:         $v_{i-1} \leftarrow D/(\overline{e}_e - \underline{e}_s - T)$
8:     $v^*_{i-1} \leftarrow$ Optimal speed by equation (5.23) or (5.24)
9:     **if** $\overline{e}_{i-1} + d_{i-1}/v_{i-1} < a_i$ *and* $\overline{e}_i \geq a_i + t_i$ *and* $i \neq n$ **then**
10:         $v_{i-1} \leftarrow d_{i-1}/(a_i - \overline{e}_{i-1})$
11:     **else if** $\overline{e}_{i-1} + d_{i-1}/v_{i-1} < b_i$ *and* $\overline{e}_i \geq b_i + t_i$ *and* $i \neq n$ **then**
12:         $v_{i-1} \leftarrow d_{i-1}/(b_i - \overline{e}_{i-1})$
13:     **if** $i = n$ *and* $\overline{e}_i \neq \underline{e}_i$ **then**
14:         $v_{i-1} \leftarrow d_{i-1}/(a_i - \overline{e}_{i-1})$
15:     **if** $v_{i-1} < v^l$ **then**
16:         $v_{i-1} \leftarrow v^l$
17:     **else if** $v_{i-1} > v^u$ **then**
18:         $v_{i-1} \leftarrow v^u$
19:     **if** $v^*_{i-1} > v_{i-1}$ **then**
20:         $v_{i-1} \leftarrow v^*_{i-1}$
21:     $\underline{e}_i \leftarrow \overline{e}_{i-1} + d_{i-1}/v_{i-1}$
22:     **if** $i \neq n + 1$ **then**
23:         $\overline{e}_i = \underline{e}_i + t_i$
24:     $g_i \leftarrow \max\{0, \underline{e}_i - b_i, a_i + t_i - \overline{e}_i\}$
25:     **if** $g_i > violation$ **then**
26:         $violation \leftarrow g_i$
27:         $p \leftarrow i$
28: **end for**
29: **if** $violation > 0$ and $\underline{e}_p > b_p$ **then**
30:     $\overline{e}_p \leftarrow b_p + t_p$
31:     Speed Optimization Algorithm (s, p)
32:     Speed Optimization Algorithm (p, e)
33: **if** $violation > 0$ and $\overline{e}_p < a_p + t_p$ **then**
34:     $\overline{e}_p \leftarrow a_p + t_p$
35:     Speed Optimization Algorithm (s, p)
36:     Speed Optimization Algorithm (p, e)

5.4.2 The Split algorithm with the speed optimization algorithm

The Split algorithm for heterogeneous vehicle routing problems Prins (2009), takes a giant tour as an input and optimally splits it into vehicle routes. The splitting procedure is based on solving the corresponding shortest path problem. Many extensions of the Split algorithm have been successfully applied in evolutionary based heuristics for several routing problems (Prins 2009, Vidal et al. 2014, Koç et al. 2015). Chapter 3 have developed an advanced Split algorithm for a heterogeneous fleet. This algorithm was embedded in the HEA to segment a giant tour and to determine the optimal fleet mix through a controlled exploration of infeasible solutions (Cordeau et al. 2001, Nagata et al. 2010). Time windows and capacity violations are penalized through a term in the objective function. Here we introduce a new algorithmic feature, the Split algorithm with the speed optimization algorithm (Ssoa) in which we incorporate the SOA within the procedure for computing the cost of each arc in the shortest path problem.

5.4.3 Higher Education and Intensification

The classical ALNS scheme is based on the idea of gradually improving a starting solution by using both destroy and repair operators on a given fleet mix composition. The ALNS in Koç et al. (2015) uses nine removal and three insertion operators, selected from those employed by various authors (Ropke and Pisinger 2006a,b, Pisinger and Ropke 2007, Demir et al. 2012, Paraskevopoulos et al. 2008).

The ALNS is essentially a node improvement procedure and therefore does not explicitly account for the heterogenous fleet dimension. In this paper, we propose the HALNS which integrates fleet sizing within the removal and the insertion operators. For the destroy phase of the HALNS, if a node is removed, we check whether the total demand of the resulting route whether can be served by a smaller vehicle and we update the solution accordingly. For the repair phase of the HALNS, if inserting a node requires additional vehicle capacity, i.e., if the current vehicle cannot satisfy the total customer demand, then we consider the option of using larger vehicle. Figure 5.2 provides an example of the removal and insertion phases of the HALNS procedure.

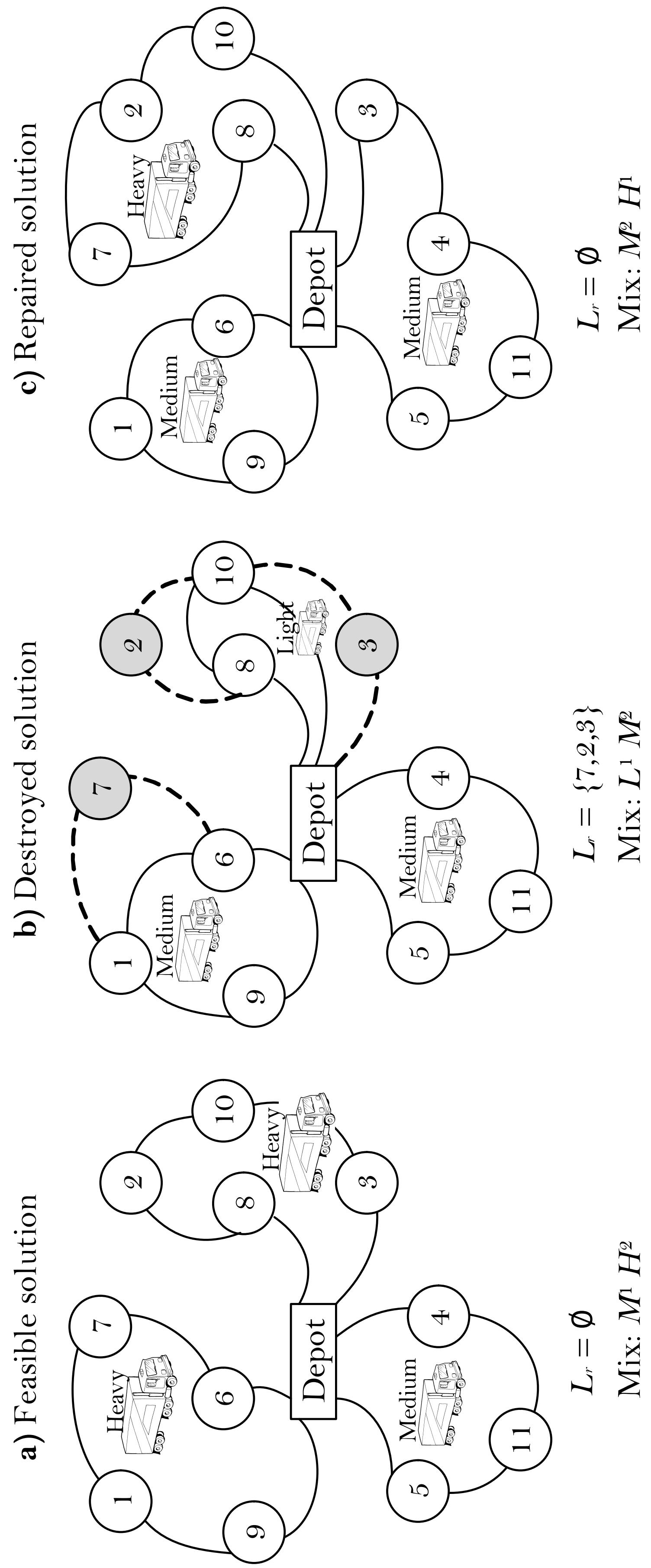


FIGURE 5.2: Illustration of the HALNS procedure

We redefine seven removal operators for the destroy phase of the HALNS procedure: worst distance, worst time, neighborhood, Shaw, proximity-based, time-based and demand-based. Furthermore, we redefine three insertion operators for the repair phase: greedy insertion, greedy insertion with noise function and greedy insertion with en-route time. Each operator has its own specific cost calculation mechanism. Aside from the distance calculations, we account for the difference in the fixed vehicle cost within each operator.

The removal operators iteratively remove nodes, add them to the removal list L_r, and update the fleet mix composition. The latter operation checks whether a vehicle with a smaller capacity can serve the route after the node removal. The insertion operators iteratively find the least-cost insertion position for node in L_r, where the cost computation includes the potential use of larger vehicles due to increasing the total demand of the route. Therefore, the insertion operators insert the nodes in their best position while updating the fleet mix composition.

For each node $i \in \mathcal{N}_c \backslash L_r$, let f^h be the current vehicle fixed cost associated with the vehicle serving i. Let $\Delta(i)$ be the saving obtained as a result of using a removal operator on node i, as defined in the ALNS. Let f_r^{h*} be the vehicle fixed cost after removal of node i, i.e., f_r^{h*} is modified only if the route containing node i can be served by a smaller vehicle when removing node i. The saving in vehicle fixed cost can be expressed as $f^h - f_r^{h*}$. Thus, the total savings of removing node $i \in \mathcal{N}_c \backslash L_r$, denoted $RC(i)$, is calculated as follows for each removal operator:

$$RC(i) = \Delta(i) + (f^h - f_r^{h*}). \tag{5.25}$$

Given a node $i \in \mathcal{N}_c \backslash L_r$ in the destroyed solution, we define the insertion cost of node $j \in L_r$ after node i as $\Omega(i, j)$. Let f_a^{h*} be the vehicle fixed cost after the insertion of node i, i.e., f_a^{h*} is modified only if the route containing node i necessitates the use of a larger capacity vehicle after inserting node i. The cost difference in vehicle fixed cost can be expressed as $f_a^{h*} - f^h$. Thus, the total insertion cost of node $i \in \mathcal{N}_c \backslash L_r$ is $IC(i)$, for each insertion operator is

$$IC(i) = \Omega(i, j) + (f_a^{h*} - f^h). \tag{5.26}$$

Chapter 3 developed a two-phase INTENSIFICATION procedure whose main idea is to improve the quality of elite individuals through intensifying the search within promising regions of the solutions space. Here we introduce an extended version of this procedure. We apply the HALNS by applying well-performing operators on the elite solutions. Furthermore, we apply the SOA on the intensified elite solutions.

5.5 Computational Experiments and Analyses

We now summarize the computational experiments performed in order to assess the performance of the HEA++. This algorithm was implemented in C++ and run on a computer with one gigabyte of RAM and an Intel Xeon 2.6 GHz processor.

We have used the PRP library of Demir et al. (2012) as the test bed. These instances were derived from real geographical distances of United Kingdom cities and are available at `http://www.apollo.management.soton.ac.uk/prplib.htm`. From this library, we have selected the four largest sets containing 75, 100, 150 and 200 nodes. Each set includes 20 instances, resulting in a total of 80 instances. These PRP instances are coupled with the parameters listed in Tables 5.1 and 5.2 for the FSMPRP. All algorithmic parametric values were set as in Chapter 3, where an extensive meta-calibration procedure was applied to generate effective parameter values for the standard heterogeneous fleet vehicle routing problem with time windows.

The aim of the computational experiments is fourfold: (i) to analyse the effect of the metaheuristic components (Section 5.5.1), (ii) to test the efficiency of the algorithm for the solution of the PRP and the FSMPRP (Section 5.5.2), (iii) to empirically calculate the savings that could be achieved by using a comprehensive objective function instead of separate objective functions (Section 5.5.3), and (iv) to quantify the benefits of using a heterogeneous fleet over a homogeneous one (Section 5.5.4).

5.5.1 Sensitivity analysis on method components

This section compares four versions of the HEA++ the details of which can be found in Table 5.3. A "No" for HALNS implies using the ALNS of Chapter 3. Similarly, a

"No" for Ssoa corresponds to using the Split algorithm without SOA. Using the four versions, we present four sets of experiments on the 100-node instances.

Table 5.3: Sensitivity analysis experiment setup

Version	HALNS	Ssoa
(1)	No	No
(2)	No	Yes
(3)	Yes	No
HEA++	Yes	Yes

Table 5.4 presents the best results of ten runs on the instances for each of the four versions. The first column displays the instances. The other columns show for each version of the algorithm, the total cost (TC) in £, percentage deterioration in solution quality (Dev) with respect to the HEA++, and the total computational time in minutes (Time). The rows named Avg, Min (%) and Max (%) show the average results, as well as minimum and maximum percentage deviations across all benchmark instances, respectively.

The results clearly indicate the benefits of including the Ssoa and HALNS within the HEA++. The HEA++ algorithm is consistently superior to all other three versions on all 20 instances. Version (1) which uses the classical ALNS and Split corresponds to the HEA of Chapter 3, performs worse than all other three versions. The superiority of version (3) over version (2) confirms the importance of the HALNS component in the algorithm. The computation times for all versions are of similar magnitude.

5.5.2 Results on the PRP and on the FSMPRP

To assess the quality of the HEA++, we have compared our algorithm with that of Demir et al. (2012), referred to as DBL12, by using a homogenous fleet of vehicles with the corresponding vehicle parameters used in the PRP. In Tables 5.5 and 5.6, we present the computational results on the PRP instances with 100 and 200 nodes, respectively. The columns show the number of vehicles used in the solution (NV) and the total distance (TD). Ten separate runs were performed for each instance as done by DBL12, the best of which is reported. For each instance, a boldface entry with a "*" indicates a new best-known solution.

Table 5.4: Sensitivity analysis of the HEA++ components

Instance	Version (1)			Version (2)			Version (3)			HEA++	
	TC	Dev	Time	TC	Dev	Time	TC	Dev	Time	TC	Time
UK100_01	1041.82	1.41	5.96	1036.13	0.87	5.28	1035.10	0.77	5.47	1027.10	5.47
UK100_02	1013.75	1.07	4.41	1012.71	0.97	4.27	1007.86	0.50	4.21	1002.87	4.38
UK100_03	948.48	2.99	4.02	942.04	2.33	4.21	925.67	0.60	4.91	920.11	4.65
UK100_04	1001.54	0.92	4.16	1002.47	1.01	4.35	994.98	0.27	4.49	992.32	4.74
UK100_05	955.12	2.47	3.47	944.74	1.40	3.64	939.83	0.88	3.95	931.56	4.21
UK100_06	1053.98	1.49	4.63	1046.18	0.76	4.55	1041.51	0.31	5.19	1038.25	5.63
UK100_07	923.60	1.10	5.39	921.22	0.85	5.41	915.88	0.27	5.47	913.39	5.19
UK100_08	982.33	3.85	3.78	967.55	2.38	4.09	953.08	0.90	4.11	944.51	4.52
UK100_09	933.29	4.53	4.59	909.09	1.99	4.19	898.45	0.83	4.49	891.02	4.69
UK100_10	1008.24	1.87	3.87	992.98	0.36	2.73	991.59	0.22	3.29	989.38	4.19
UK100_11	1054.71	1.87	4.87	1047.71	1.21	3.91	1046.18	1.07	4.19	1034.98	5.13
UK100_12	910.79	3.38	4.26	906.53	2.93	4.27	906.95	2.98	4.52	879.96	4.62
UK100_13	1023.71	2.23	4.46	1021.32	2.00	4.53	1014.79	1.37	4.79	1000.86	4.78
UK100_14	1069.87	1.63	4.69	1063.78	1.06	4.61	1058.20	0.54	4.83	1052.47	5.02
UK100_15	1106.50	2.04	4.57	1091.11	0.66	4.59	1087.32	0.31	4.81	1083.95	5.27
UK100_16	933.25	0.95	4.35	928.36	0.42	4.34	929.98	0.60	4.55	924.42	4.73
UK100_17	1076.11	1.75	3.79	1072.18	1.38	4.29	1066.71	0.88	4.29	1057.33	4.36
UK100_18	956.43	3.52	4.56	935.66	1.37	4.19	924.92	0.23	4.49	922.81	5.01
UK100_19	941.58	1.80	3.67	939.75	1.61	3.81	930.60	0.64	4.99	924.60	4.99
UK100_20	1093.54	3.27	3.43	1070.82	1.22	3.84	1066.49	0.82	4.27	1057.78	4.18
Avg	1001.43	2.21	4.38	992.62	1.34	4.26	986.81	0.75	4.57	979.48	4.79
Min (%)		0.92			0.36			0.22			
Max (%)		4.53			2.93			2.98			

The results clearly show that HEA++ outperforms DBL12 on all PRP instances in terms of solution quality. The average cost reduction is 1.60% for 100-node instances, for which the minimum and maximum improvements are 0.32% and 2.33%, respectively. For 200-node instances, the corresponding values are 1.72% (average), 0.04% (minimum) and 3.88% (maximum). On average, the Demir et al. (2012) is faster on the 100-node instances, however, this difference is less substantial on the 200-node instances.

Table 5.5: Computational results on the 100-node PRP instances

Instance	DBL12				HEA++				
	NV	TD	TC	Time	NV	TD	TC	Dev	Time
UK100_01	14	2914.40	1240.79	1.54	14	2795.08	**1212.72***	−2.31	4.37
UK100_02	13	2690.40	1168.17	1.64	13	2660.65	**1149.16***	−1.65	4.67
UK100_03	13	2531.80	1092.73	3.47	13	2487.25	**1080.87***	−1.10	5.29
UK100_04	14	2438.50	1106.48	2.49	14	2374.23	**1085.66***	−1.92	5.13
UK100_05	14	2328.50	1043.41	2.65	14	2256.48	**1033.19***	−0.99	4.93
UK100_06	14	2782.40	1213.61	2.23	14	2733.05	**1192.67***	−1.76	4.83
UK100_07	12	2463.90	1060.08	1.71	12	2412.54	**1044.58***	−1.48	4.51
UK100_08	13	2597.40	1106.78	3.49	12	2524.80	**1092.67***	−1.29	5.67
UK100_09	13	2219.20	1015.46	2.57	13	2204.89	**992.36***	−2.33	4.97
UK100_10	12	2510.10	1076.56	3.32	12	2432.26	**1063.05***	−1.27	5.64
UK100_11	15	2792.10	1210.25	1.79	14	2722.22	**1200.53***	−0.81	4.11
UK100_12	12	2427.30	1053.02	3.44	12	2336.10	**1030.17***	−2.22	5.64
UK100_13	13	2693.10	1154.83	1.47	13	2589.17	**1132.02***	−2.01	3.49
UK100_14	14	2975.30	1264.50	1.53	14	2892.45	**1241.31***	−1.87	4.29
UK100_15	15	3072.10	1315.50	1.85	15	3038.40	**1311.36***	−0.32	3.87
UK100_16	12	2219.70	1005.03	4.25	12	2203.99	**986.57***	−1.87	5.97
UK100_17	15	2960.40	1284.81	2.55	15	2860.97	**1257.44***	−2.18	4.19
UK100_18	13	2525.20	1106.00	1.54	13	2506.71	**1088.89***	−1.57	4.21
UK100_19	13	2332.60	1044.71	1.52	13	2288.50	**1024.17***	−2.01	4.19
UK100_20	14	2957.80	1263.06	3.41	14	2915.17	**1249.84***	−1.06	5.17
Avg	13.4	2621.61	1141.29	2.42	13.3	2561.75	1123.46	−1.60	4.76
Min (%)								−2.33	
Max (%)								−0.32	
Processor	Xe 3.0 GHz				Xe 2.6 GHz				
Runs	10				10				

Table 5.7 presents the average results obtained by HEA++ on the 75, 100, 150 and 200-node FSMPRP instances. For each instance set, the columns display the average fuel and CO_2 emissions cost (FEC), driver cost (DC) and vehicle cost (VC). To evaluate the environmental impact of the solutions, we also report the average amount of CO_2 emissions (in kg) based on the assumption that one liter of gasoline contains 2.32 kg of CO_2 (Coe 2005). For detailed results, the reader is referred to Tables D.1–D.4 in the Appendix C, where ten runs were performed for each instance and the best one is reported. We observe that on average, over all benchmark instances, the vehicle fixed

Table 5.6: Computational results on the 200-node PRP instances

Instance	DBL12				HEA++				
	NV	TD	TC	Time	NV	TD	TC	Dev	Time
UK200_01	28	4609.60	2111.70	12.10	28	4545.77	**2067.00***	−2.16	14.20
UK200_02	24	4444.40	1988.64	17.00	25	4332.62	**1953.35***	−1.81	15.80
UK200_03	27	4439.90	2017.63	6.74	28	4365.82	**1996.13***	−1.08	10.40
UK200_04	26	4191.90	1934.13	6.86	26	4151.74	**1905.88***	−1.48	9.47
UK200_05	27	4861.90	2182.91	15.40	27	4848.28	**2151.99***	−1.44	16.80
UK200_06	27	3980.40	1883.22	7.51	27	3980.03	**1859.40***	−1.28	11.50
UK200_07	27	4415.30	2021.95	15.70	27	4276.06	**1974.32***	−2.41	17.90
UK200_08	27	4664.40	2116.76	7.17	27	4592.54	**2088.12***	−1.37	9.17
UK200_09	25	4031.10	1894.18	9.22	25	3932.44	**1823.50***	−3.88	11.70
UK200_10	28	4921.80	2199.95	8.33	27	4847.08	**2166.59***	−1.54	9.78
UK200_11	27	4099.50	1941.19	14.10	27	4126.44	**1908.83***	−1.70	16.30
UK200_12	25	4808.50	2105.14	11.90	26	4786.39	**2104.40***	−0.04	12.80
UK200_13	25	4760.30	2141.26	7.41	25	4734.21	**2094.48***	−2.23	9.37
UK200_14	27	4369.90	2011.35	7.51	27	4369.86	**1994.49***	−0.85	10.30
UK200_15	25	4723.90	2110.86	9.04	26	4642.58	**2067.48***	−2.10	11.40
UK200_16	27	4545.90	2075.83	7.59	27	4497.75	**2023.55***	−2.58	9.71
UK200_17	26	4972.80	2218.28	6.82	26	4915.18	**2165.34***	−2.44	8.97
UK200_18	27	4370.30	2004.68	13.20	27	4406.10	**2003.75***	−0.05	14.00
UK200_19	25	3995.40	1844.90	16.20	25	3946.49	**1803.56***	−2.29	17.50
UK200_20	27	4805.40	2150.57	8.85	26	4727.98	**2114.31***	−1.71	11.30
Avg	26.35	4500.60	2047.76	10.40	26.45	4451.27	2013.32	−1.72	12.40
Min (%)								−3.88	
Max (%)								−0.04	
Processor	Xe 3.0 GHz				Xe 2.6 GHz				
Runs	10				10				

cost accounts for 38.8% of the total cost, whereas the driver cost represents 36.7% of the total, and the fuel and emissions cost accounts for 24.5%.

Table 5.7: Average results on the FSMPRP instances

Instance	TD	CO_2	FEC	DC	VC	TC	Time
75-node	1534.38	345.74	208.63	280.15	296.40	785.185	3.27
100-node	1841.48	414.17	249.93	354.56	375.00	979.484	4.79
150-node	2398.78	550.48	332.18	509.66	536.40	1378.24	7.03
200-node	2857.08	659.39	397.91	642.11	678.30	1720.42	10.4

5.5.3 The effect of cost components

This section analyzes the implications of using different cost components on the performance measures. To this end, we have conducted experiments using four different objective functions, which are presented in the rows of Table 5.8. The experiments were conducted on a 100-node FSMPRP instance, and the best results collected over ten runs are reported for each of four performance measures which we will now define. In min

TD, we consider the objective of minimizing the total distance. In min FEC, we only consider fuel and emissions cost. This setting also implies minimizing CO_2 since this is proportional to fuel consumption. In min DC, we account only for the driver cost. The min TD+VC objective corresponds to the standard heterogeneous vehicle routing problems, which consists of minimizing distance and vehicle fixed costs. Finally we present the FSMPRP objective. Aside from the objective function values, we provide the main cost components in Table 5.8. In Table 5.9, we report the deviations from the smallest cost components shown in Table 5.8. For example, the minimum value for the total distance objective (min TD) is 1921.66 km, but the FEC objective yields a solution with a total distance of 2119.87 km, corresponding to an increase of 10.31%. It is clear that considering only distance in the objective results in a poor total cost performance, yielding a 4.11% increase. This increase is more substantial when looking only at the vehicle fixed cost where min TD is 8.65% higher in terms of VC. With respect to CO_2 emissions, the closest objective value is min TD+VC. This result implies that a substantial gain in CO_2 emissions can be achieved by using the TD+VC objective. However, minimizing CO_2 emissions yields an average increase of 1.09% in TC. Similar to the TD objective, the DC objective performs poorly on all cost components, yielding an average increase of 20.67% in the CO_2 emissions.

In order to quantify the added value of changing speeds, we have experimented with three other versions of the FSMPRP in which the speed on all arcs is fixed at 70, 85 or 100 km/h. Table 5.10 presents the results of these experiments. The results suggest that while optimizing speeds with HEA++ yields the best results, using a fixed speed of 100 km/h deteriorates the solution quality by only 1.16% on average. This makes sense since high driver costs will make it economical to drive fast. On the other hand, using a fixed speed of 70 km/h deteriorates the solution value by an average value of 15.19%.

5.5.4 The effect of the heterogeneous fleet

We now analyze the benefit of using a heterogeneous fleet of vehicles as opposed to using a homogenous fleet, coupled with using fixed versus variable speeds. To do so, we have conducted three sets of experiments on the 100-node FSMPRP instances, each corresponding to using a unique vehicle type, i.e., only light duty, only medium duty and only heavy duty vehicles. This results in three sets of PRP instances which are solved

TABLE 5.8: The effect of cost components: objective function values

Objective	TD	CO_2	FEC	DC	VC	TC
Min TD (total distance)	**1921.66**	476.64	287.63	379.70	402	1069.31
Min FEC (fuel and emissions cost)	2119.87	**371.10**	**223.91**	444.40	**370**	1038.28
Min DC (driver cost)	2078.14	447.74	270.19	**368.90**	402	1041.13
Min TD and VC (total distance and vehicle fixed cost)	1994.23	407.14	245.69	410.40	384	1040.06
Min TC (total cost)	2031.46	444.12	268.00	375.10	384	**1027.10**

TABLE 5.9: The effect of cost components: percent deviation from the minimum value

Objective	TD	CO_2	FEC	DC	VC	TC
Min TD (total distance)	**0.00**	28.46	28.46	2.91	8.65	4.11
Min FEC (fuel and emissions cost)	10.31	**0.00**	**0.00**	20.40	**0.00**	1.09
Min DC (driver cost)	8.14	20.67	20.67	**0.00**	8.65	1.37
Min TD and VC (total distance and vehicle fixed cost)	3.78	9.73	9.73	11.2	3.78	1.26
Min TC (total cost)	5.71	19.69	19.69	1.67	3.78	**0.00**

TABLE 5.10: The effect of the speed

Instance	70 km/h		85 km/h		100 km/h		HEA++
	TC	Dev	TC	Dev	TC	Dev	TC
UK100_01	1218.01	18.01	1106.60	7.22	1032.10	0.49	1027.1
UK100_02	1154.54	13.31	1044.14	2.47	1018.94	1.60	1002.87
UK100_03	1066.70	13.88	982.49	4.89	936.69	1.80	920.11
UK100_04	1127.40	13.24	1028.27	3.28	995.57	0.33	992.31
UK100_05	1088.10	15.76	1007.40	7.18	939.93	0.90	931.56
UK100_06	1199.17	15.23	1099.62	5.66	1040.71	0.24	1038.25
UK100_07	1078.23	15.33	993.63	6.28	934.92	2.36	913.39
UK100_08	1128.36	14.26	1021.77	3.47	987.51	4.55	944.51
UK100_09	1059.50	17.76	963.49	7.09	899.68	0.97	891.02
UK100_10	1139.41	13.90	1055.82	5.55	1000.32	1.11	989.38
UK100_11	1197.71	14.92	1090.94	4.67	1042.25	0.70	1034.98
UK100_12	1023.51	16.19	942.27	6.97	880.87	0.10	879.96
UK100_13	1154.79	13.52	1054.11	3.63	1017.22	1.63	1000.86
UK100_14	1223.60	14.44	1112.03	4.01	1069.17	1.59	1052.47
UK100_15	1251.53	14.53	1156.86	5.86	1092.8	0.82	1083.95
UK100_16	1066.83	15.08	979.75	5.69	927.05	0.28	924.42
UK100_17	1254.68	17.99	1133.08	6.55	1063.41	0.58	1057.33
UK100_18	1091.80	15.77	991.94	5.18	943.05	2.19	922.81
UK100_19	1073.05	15.07	984.79	5.60	932.53	0.86	924.60
UK100_20	1223.29	15.56	1121.38	5.94	1058.54	0.07	1057.78
Average	1141.01	15.19	1043.52	5.36	990.66	1.16	979.48
Min (%)		13.24		2.47		0.07	
Max (%)		18.01		7.22		4.55	

with the HEA++. We have compared these results with those of the four experiments shown in Table 5.10. Table 5.11 provides a summary of this comparison. The columns Dev_{70}, Dev_{85} and Dev_{100} respectively report the percentage increase in total cost as a result of using homogeneous vehicles as in Table 5.11 over the fixed-speed results shown in Table 5.10 for 70, 85 and 100 km/h. Similarly, the colums entitled Dev_V show the deviation in total cost between the various homogeneous cases and the FSMPRP, i.e., with HEA++. Table 5.11 suggests that the total cost increases when using a heavy duty homogeneous fleet. Compared to the FSMPRP this increase ranges from 22.46% to 28.58%. For the medium duty case, the total cost increase is on average 2.49% compared to the FSMPRP. With light duty vehicles, the average increase in total cost is 19.88% compared to the FSMPRP. These results imply that for the homogeneous case, it is preferable to use medium duty vehicles. It is clear that using a heterogeneous fleet of vehicles and optimizing their speeds is superior to using a homogeneous fleet of vehicles and optimizing their speeds. Table 5.11 also indicates that using a heterogeneous fleet of vehicles with a fixed speed of 100 km/h is better than using a homogeneous fleet of vehicles and optimizing their speeds with respect to the total cost. This implies that for our experimental setting heterogeneous fleet dimension is more important than speed optimization on each arc.

The final set of experiments we now present aim at providing some insight into the capacity utilization of the vehicle fleet, for both homogenous and heterogeneous cases. In Table 5.12, we present the capacity utilizations for the three PRP settings of Table 5.11 as well as for the FSMPRP. The column CU displays the percentage of capacity utilization for the vehicle fleet, which is calculated as 100 (total demand of route/ capacity of the vehicle). In contrast to the total cost, the capacity utilization reaches its maximum level (95.44%) and worse level (55.61%) when using only light duty or medium duty vehicles, respectively. Heavy duty vehicles have approximately six and two times more capacity than light duty and medium duty vehicles, respectively. The average capacity utilization for a heavy-duty only vehicle fleet is 26.74%, but this is probably due to the limitations imposed by the time window constraints, total customer demands or vehicle capacity constraints, and interactions thereof. Using a heterogeneous fleet yields an average utilization of 62.53%, which is a compromise between light and heavy duty vehicles.

5.6 Conclusions

We have presented a hybrid evolutionary metaheuristic for the fleet size and mix pollution-routing problem (FSMPRP), which extends the pollution-routing problem (PRP) introduced by Bektaş and Laporte (2011) and further studied by Demir et al. (2012), to allow for the use of a heterogeneous vehicle fleet. The effectiveness of the algorithm was demonstrated through extensive computational experiments on realistic PRP and FSMPRP instances. These tests have enabled us to assess the effects of several algorithmic components and to measure the trade-offs between various cost indicators such as vehicle fixed cost, distance, fuel and emissions, driver cost and total cost. We have demonstrated the benefit of using a heterogeneous fleet over a homogeneous one. An interesting insight derived from this study is that using a heterogeneous fleet without speed optimization allows for a further reduction in total cost than using a homogeneous fleet with speed optimization. Furthermore, we have shown that using an adequate fixed

Table 5.11: The effect of using a heterogeneous fleet

Instance	Only light duty					Only medium duty					Only heavy duty				
	TC	Dev_{70}	Dev_{85}	Dev_{100}	Dev_V	TC	Dev_{70}	Dev_{85}	Dev_{100}	Dev_V	TC	Dev_{70}	Dev_{85}	Dev_{100}	Dev_V
UK100_01	1272.20	4.26	13.02	18.87	19.27	1066.34	−14.20	−3.78	3.21	3.68	1385.72	12.10	20.10	25.50	25.88
UK100_02	1236.88	6.67	15.58	17.62	18.92	1051.26	−9.82	0.68	3.07	4.60	1369.21	15.70	23.70	25.60	26.76
UK100_03	1208.80	11.76	18.72	22.51	23.88	928.30	−14.90	−5.84	−0.90	0.88	1209.97	11.80	18.80	22.60	23.96
UK100_04	1261.12	10.60	18.46	21.06	21.31	1013.97	−11.20	−1.41	1.81	2.14	1324.33	14.90	22.40	24.80	25.07
UK100_05	1242.92	12.46	18.95	24.38	25.05	1000.40	−8.77	−0.70	6.04	6.88	1304.31	16.60	22.80	27.90	28.58
UK100_06	1313.98	8.74	16.31	20.80	20.98	1058.53	−13.30	−3.88	1.68	1.92	1374.78	12.80	20.00	24.30	24.48
UK100_07	1130.95	4.66	12.14	17.33	19.24	930.84	−15.80	−6.75	−0.44	1.87	1204.82	10.50	17.50	22.40	24.19
UK100_08	1165.37	3.18	12.32	15.26	18.95	963.72	−17.10	−6.02	−2.47	1.99	1247.78	9.57	18.10	20.90	24.30
UK100_09	1121.13	5.49	14.06	19.75	20.52	918.53	−15.30	−4.90	2.05	2.99	1182.58	10.40	18.50	23.90	24.65
UK100_10	1137.93	−0.13	7.22	12.09	13.05	1034.73	−10.10	−2.04	3.33	4.38	1353.15	15.80	22.00	26.10	26.88
UK100_11	1305.80	8.28	16.45	20.18	20.74	1057.66	−13.20	−3.15	1.46	2.14	1376.04	13.00	20.70	24.30	24.79
UK100_12	1125.91	9.09	16.31	21.76	21.84	895.65	−14.30	−5.21	1.65	1.75	1170.16	12.50	19.50	24.70	24.80
UK100_13	1222.93	5.57	13.80	16.82	18.16	1042.52	−10.80	−1.11	2.43	4.00	1354.65	14.80	22.20	24.90	26.12
UK100_14	1323.30	7.53	15.97	19.20	20.47	1075.34	−13.80	−3.41	0.57	2.13	1400.69	12.60	20.60	23.70	24.86
UK100_15	1360.55	8.01	14.97	19.68	20.33	1087.52	−15.10	−6.38	−0.49	0.33	1417.90	11.70	18.40	22.90	23.55
UK100_16	1103.04	3.28	11.18	15.96	16.19	938.90	−13.60	−4.35	1.26	1.54	1202.76	11.30	18.50	22.90	23.14
UK100_17	1350.89	7.12	16.12	21.28	21.73	1078.64	−16.30	−5.05	1.41	1.98	1412.73	11.20	19.80	24.70	25.16
UK100_18	1141.93	4.39	13.13	17.42	19.19	939.68	−16.20	−5.56	−0.36	1.80	1216.87	10.30	18.50	22.50	24.17
UK100_19	1158.83	7.40	15.02	19.53	20.21	930.64	−15.30	−5.82	−0.20	0.65	1192.37	10.00	17.40	21.80	22.46
UK100_20	1283.81	4.71	12.65	17.55	17.61	1081.08	−13.20	−3.73	2.09	2.16	1396.63	12.40	19.70	24.20	24.26
Avg	1223.41	6.65	14.62	18.95	19.88	1004.71	−13.6	−3.92	1.36	2.49	1304.87	12.50	20.00	24.00	24.90
Min (%)		−0.13	7.22	12.09	13.05		−17.1	−6.75	−2.47	0.33		9.57	17.40	20.90	22.46
Max (%)		12.46	18.95	24.38	25.05		−8.77	0.68	6.04	6.88		16.60	23.70	27.90	28.58

Table 5.12: Capacity utilization rates

Instance	Only light duty	Only medium duty	Only heavy duty	HEA++
	CU	CU	CU	CU
UK100_01	97.81	53.66	25.80	66.59
UK100_02	91.30	50.08	24.08	62.16
UK100_03	92.10	58.95	28.34	66.48
UK100_04	94.65	56.25	27.04	62.30
UK100_05	95.53	56.77	27.30	66.24
UK100_06	94.59	56.21	27.03	62.26
UK100_07	94.69	55.55	26.71	62.65
UK100_08	96.94	56.87	27.34	64.14
UK100_09	98.29	57.66	27.72	65.03
UK100_10	94.81	47.67	22.92	59.17
UK100_11	94.65	56.25	27.04	62.30
UK100_12	96.79	56.78	27.30	64.04
UK100_13	94.39	51.78	24.90	64.27
UK100_14	91.37	54.30	26.11	60.14
UK100_15	96.92	57.60	27.69	57.60
UK100_16	95.76	56.18	27.01	56.18
UK100_17	97.09	57.70	27.74	63.91
UK100_18	97.93	57.45	27.62	64.80
UK100_19	94.07	60.20	28.94	60.20
UK100_20	99.03	54.32	26.12	60.17
Avg	95.44	55.61	26.74	62.53
Min (%)	91.30	47.67	22.92	56.18
Max (%)	99.03	60.20	28.94	66.59

speed yields results that are only slightly worse than optimizing the speed on each arc. This has a practical implication since it is easier to instruct drivers to hold a constant speed for their entire trip rather than change their speed on each segment. f

Chapter 6

The Impact of Location, Fleet Composition and Routing on Emissions in Urban Freight Distribution

Abstract

This paper investigates the combined impact of depot location, fleet composition and routing decisions on vehicle emissions in urban freight distribution. We consider a city in which goods need to be delivered from a depot to customers located in nested zones characterized by different speed limits. The objective is to minimize the total depot, vehicle and routing cost, where the latter can be defined with respect to the cost of fuel consumption and CO_2 emissions. A new powerful adaptive large neighborhood search metaheuristic is developed and successfully applied to a large pool of new benchmark instances. Extensive analyses are performed to empirically assess the effect of various problem parameters, such as depot cost and location, customer distribution and heterogeneous vehicles on key performance indicators, including fuel consumption, emissions and operational costs. Several managerial insights are presented.

Keywords. location-routing; fuel consumption; CO_2 emissions; sustainability; heterogeneous fleet; city logistics; supply chains; adaptive large neighborhood search metaheuristic.

6.1 Introduction

City logistics poses challenges to governments, businesses, carriers, and citizens, particularly in the context of freight transportation, and calls for new business operating models. It also requires an understanding of the public sector and private businesses, and collaboration mechanisms to build innovative partnerships. Trade flows within cities exhibit a high variability, both in the size and shape of the shipments. Cities often possess a transportation infrastructure that allows traffic flows within their boundaries, but this infrastructure is often inadequate for freight transportation, which translates into congestion and pollution. For relevant references and more detailed information on city logistics, the reader is referred to the books of Taniguchi et al. (2001) and of Gonzalez-Feliu et al. (2014).

Depot location, fleet composition and routing all bear on emissions in urban freight transportation. Some of their interactions are well documented. However, whereas there exists an extensive body of knowledge on the integration of location and routing, on the effect of route choice on pollution and on the impact of fleet composition on emissions, the combined effect of depot location, fleet composition and routing decisions on emissions has not yet been investigated. Yet, these decisions are clearly intertwined, especially in a city logistics context. Our purpose is to analyze these three interrelated components of urban freight distribution within a unified framework. Before we proceed with our study, we briefly review the relevant literature on some of the interactions just mentioned.

6.1.1 A brief review of the literature

Depot location and vehicle routing are two interdependent decisions. The joint study of these two problems was first suggested by Von Boventer (1961) and has since evolved into what is now commonly known as the Location-Routing Problem (LRP) (see Laporte 1988, Min et al. 1998, Nagy and Salhi 2007, Prodhon and Prins 2014, Albareda-Sambola 2015, Drexl and Schneider 2015, for reviews). Applications of the LRP arise namely in city logistics (Boudoin et al. 2014, Mancini et al. 2014).

The LRP can also be considered as a single-echelon variant of the Two-Echelon LRP, arising in city logistics where there are problems related to congestion, emissions and noise caused by heavy duty vehicles, which are generally used for long haul transportation. To circumvent such problems, the two-echelon setting assumes that freight first arrives at a central depot, from where it is transported to satellite facilities and is finally delivered to the customers by smaller vehicles. For detailed information on the Two-Echelon LRP the reader is referred to Hemmelmayr et al. (2012).

Fleet composition is yet another critical issue in city logistics. Heterogeneous vehicle fleets are commonly used in most distribution problems (Hoff et al. 2010). Heterogeneous VRPs include two major classes: the Fleet Size and Mix Vehicle Routing Problem proposed by Golden et al. (1984), which works with an unlimited fleet, and the Heterogeneous Vehicle Routing Problem (HVRP) introduced by Taillard (1999), which works with a known fleet. For further details on these problems and their variants, we refer the reader to Baldacci et al. (2008), Baldacci et al. (2009) and Jabali et al. (2012a). In recent years, green issues have received increased attention in the context of the HVRP (see Kopfer and Kopfer 2013, Kopfer et al. 2014, Kwon et al. 2013). Chapter 5 introduced the Fleet Size and Mix PRP which extends the PRP by considering a heterogeneous vehicle fleet and developed a hybrid evolutionary metaheuristic to solve it. They conducted computational experiments to shed light on the trade-offs between various performance indicators, such as fuel and CO_2 emissions, vehicle fixed cost, distance, driver cost and total cost. They demonstrated the benefit of using a heterogeneous fleet over a homogeneous one.

Greenhouse gases (GHGs) are a noxious by-product of road freight transportation (Kirby et al. 2000) which accounts for around a quarter of the total GHG emissions in the United Kingdom and the United States (DfT 2012, EPA 2012). The relationship between road freight transportation and emissions has been the object of several studies in recent years. Thus Demir et al. (2011) have surveyed several estimation models for fuel consumption and greenhouse gas emissions. More specifically, the authors have compared six models and have assessed their respective strengths and weaknesses. These models indicate that fuel consumption depends on a number of factors that can be grouped into four categories: vehicle, driver, environment and traffic. Figliozzi (2011) simultaneously considered the effects of GHG costs, new engine technologies, market conditions and fiscal policies in fleet management models. The author proposed an integer programming vehicle replacement model in order to compute some environmental and political indicators. Four factors were analysed in scenarios arising from a case study in Portland, Oregon, namely annual vehicle utilization, gasoline prices, electric vehicle tax credits, and GHG emissions costs. Bigazzi and Figliozzi (2012) examined several factors affecting GHGs emissions. The authors focused on the effects of travel demand flexibility and on the characteristics of two types of vehicles, namely light and heavy duty, across different types of pollutants. They stated that fleet composition and vehicle type are key factors driving CO_2 emissions. Furthermore, the authors indicated that several demand or vehicle based emissions strategies could have an impact on the reduction of CO_2 emissions. Jabali et al. (2012b) later studied the trade-off between the minimization of CO_2 emissions and that of total travel times in the context of the time-dependent Vehicle Routing Problem (VRP) in which the planning horizon was partitioned into two phases: free flow traffic and congestion. The authors solved the problem using tabu search and proposed efficient bounding procedures.

Van Woensel et al. (2001) noted that vehicles must often travel at traffic speed in urban areas, and changes in speed have a significant impact on CO_2 emissions. Since the label-setting algorithm was proposed by Dijkstra (1959) more than 50 years ago, several deterministic shortest path computation algorithms have been put forward by a number of researchers (see Geisberger et al. 2012). In the context of green transportation, Fagerholt et al. (2010) developed an alternative solution methodology for the minimization of fuel and emissions in ship routing and solved the problem as a shortest path problem on a directed acyclic graph. Their results showed that the shortest path method yields significant fuel and emissions savings on shipping routes. More recently, Ehmke et al. (2014) studied stochastic shortest paths with an emissions minimization objective. The authors concluded that in order to minimize emissions, vehicles may have to travel via a circuitous path rather than along a more direct shortest path.

The Pollution-Routing Problem (PRP), introduced by Bektaş and Laporte (2011), is an extension of the classical VRP with time windows. It consists of routing vehicles to serve a set of customers, and of determining their speed on each route segment to minimize a function comprising fuel cost, emissions and driver costs. To estimate fuel consumption, the authors applied a simplified version of the emission and fuel consumption model proposed by Barth et al. (2005), Scora and Barth (2006) and Barth and Boriboonsomsin (2009). This simplified model assumes that all parameters will remain constant on a given arc, but load and speed may change from one arc to another. As such, the PRP objective approximates the total amount of energy consumed on a given road segment, which directly translates into fuel consumption and further into GHG emissions. Demir et al. (2012) developed an extended adaptive large neighbourhood search (ALNS) heuristic for the PRP. This heuristic operates in two stages: the first stage is an extension of the classical ALNS scheme to construct vehicle routes (Ropke and Pisinger 2006a,b, Pisinger and Ropke 2007), and the second stage applies a speed optimization algorithm (SOA) (Norstad et al. 2010, Hvattum et al. 2013) to compute the speed on each arc. In a later study, Demir et al. (2014a) introduced the bi-objective PRP which jointly minimizes fuel consumption and driving time. The authors have developed a bi-objective adaptation of their ALNS-SOA heuristic and compared four *a posteriori* methods, namely the weighting method, the weighting method with normalization, the epsilon-constraint method and a new hybrid method, using a scalarization of the two objective functions. Franceschetti et al. (2013) studied the time-dependent PRP under a two-stage planning horizon, as in Jabali et al. (2012b), and developed an explicit congestion model in addition to the PRP objectives. The authors presented a mathematical formulation in which vehicle speeds are optimally selected from a set of discrete values. More recently, Kramer et al. (2015) proposed a matheuristic for the PRP, as well as for the the Fuel Consumption VRP and the Energy Minimizing VRP, which integrates iterated local search with a set partitioning procedure and an SOA. Their method outperformed those presented in previous studies and yielded new best-known solutions. For a state-of-the-art coverage on green road freight transportation, the reader is referred to the book chapter of Eglese and Bektaş (2014), and to the surveys of Demir et al. (2014b) and Lin et al. (2014).

6.1.2 Scientific contributions and structure of the paper

This paper studies for the first time the joint impact of location, fleet composition and routing in an urban freight distribution context. It makes three main scientific contributions. Our first contribution is to formally model this new problem and solve it by means of a powerful ALNS metaheuristic. Our second contribution is to carry out extensive computational experiments and analyses in order to gain a deep understanding into the interactions between the components of the problem. Our third contribution is to provide managerial insights.

The remainder of this paper is structured as follows. Section 6.2 presents a general framework for our analysis. Section 6.3 provides a formal description of the problem and the mathematical formulation. Section 6.4 contains a brief description of the proposed metaheuristic. Extensive computational experiments and analyses are presented in Section 6.5, followed by conclusions and managerial insights in Section 6.6.

6.2 General Description of the Problem Setting

We will first briefly provide our fuel consumption and CO_2 emissions model in Section 6.2.1. We will then describe the vehicle types and their characteristics in Section 6.2.2, followed by the specification of speed zones in Section 6.2.3, by the network structure in Section 6.2.4, and by the depot costs in Section 6.2.5.

6.2.1 Fuel consumption and CO_2 emissions

We use the comprehensive emissions model of Barth et al. (2005), Scora and Barth (2006), and Barth and Boriboonsomsin (2008) to estimate fuel consumption and emissions at a given time instant. This model has already been succesfully applied to the PRP (Bektaş and Laporte 2011, Demir et al. 2012) and to some of its extensions (Franceschetti et al. 2013, Demir et al. 2014a, Koç et al. 2014). In what follows, we briefly recall the heterogeneous fleet version of this model (Chapter 5).

The index set of vehicle types is denoted by $\mathcal{H}$. The fuel consumption rate FR^h (liter/s) of a vehicle of type $h \in \mathcal{H}$ is given by

$$FR^h = \xi(k^h N^h V^h + P^h/\eta)/\kappa, \tag{6.1}$$

where the variable P^h is the second-by-second engine power output (in kW) of vehicle type h. It can be calculated as

$$P^h = P^h_{tract}/n_{tf} + P_{acc}, \tag{6.2}$$

where the engine power demand P_{acc} is associated with the running losses of the engine and the operation of vehicle accessories such as air conditioning and electrical loads. We assume that $P_{acc} = 0$. The total tractive power requirement P^h_{tract} (in kW) for a vehicle of type h is

$$P^h_{tract} = (M^h\tau + M^h g\sin\theta + 0.5C^h_d\rho A v^2 + M^h gC_r\cos\theta)v/1000, \tag{6.3}$$

where M^h is the total vehicle weight (in kg) and v is the vehicle speed (m/s). The fuel consumption F^h (in liters) of vehicle type h over a distance d, is calculated as

$$F^h = k^h N^h V^h \lambda d/v \tag{6.4}$$

$$+ P^h\lambda\gamma^h d/v, \tag{6.5}$$

where $\lambda = \xi/\kappa\psi$, $\gamma^h = 1/1000n_{tf}\eta$ and $\alpha = \tau + g\sin\theta + gC_r\cos\theta$ are constants. Let $\beta^h = 0.5C^h_d\rho A^h$ be a vehicle-specific constant. Therefore, F^h can be rewritten as

$$F^h = \lambda(k^h N^h V^h d/v + M^h\gamma^h\alpha d + \beta^h\gamma^h dv^2). \tag{6.6}$$

In this expression the first term $k^h N^h V^h d/v$ is called the engine module, which is linear in travel time. The second term $M^h \gamma^h \alpha_{ij} d$ is referred to as the weight module, and the third term $\beta^h \gamma^h d v^2$ is the speed module, which is quadratic in speed. These functions will be used in the objective function of the mathematical formulation in Section 6.3.

6.2.2 Vehicle types and characteristics

We consider three vehicle types of MAN (2015a), a major truck manufacturer whose market share in Western Europe was around 16.3% in 2013 (Statista 2013). These three vehicle types include two light duty (TGL) vehicles and one medium duty (TGM) vehicle, classified as single-unit trucks by FHWA (2011). Table 6.1 lists the values of the parameters (Demir et al. 2012, 2014a, Franceschetti et al. 2013, Koç et al. 2014) common to all vehicle types, while Table 6.2 lists specific parameters (MAN 2015a,b,c) for each vehicle type. We refer the reader to MAN (2015a,b,c) for further details on TGL and TGM vehicles and their engines.

TABLE 6.1: Vehicle common parameters

Notation	Description	Typical values
ξ	fuel-to-air mass ratio	1
g	gravitational constant (m/s^2)	9.81
ρ	air density (kg/m^3)	1.2041
C_r	coefficient of rolling resistance	0.01
η	efficiency parameter for diesel engines	0.45
f_c	fuel and CO_2 emissions cost (£/liter)	1.4
κ	heating value of a typical diesel fuel (kj/g)	44
ψ	conversion factor (g/s to L/s)	737
n_{tf}	vehicle drive train efficiency	0.45
θ	road angle	0
τ	acceleration (m/s^2)	0

The fuel consumption function (6.6) per unit distance travelled as a function of speed is typically U-shaped (Figure 6.1) and results in an optimal speed that minimizes the fuel consumption. This function, plotted in Figure 6.1 is the sum of two components, one induced by (6.4) and the other by (6.5), for the three vehicle types considered in this paper.

TABLE 6.2: Vehicle specific parameters

Notation	Description	Light duty 1 (L1)	Light duty 2 (L2)	Medium duty (M)
w^h	curb weight (kg)	3500	4500	5500
Q^h	maximum payload (kg)	4000	7500	12500
μ^h	vehicle fixed cost (£/day)	42	49	60
k^h	engine friction factor (kj/rev/liter)	0.25	0.23	0.20
N^h	engine speed (rev/s)	38.34	37.45	36.67
V^h	engine displacement (liter)	4.5	4.5	6.9
C_d^h	coefficient of aerodynamics drag	0.6	0.64	0.7
A^h	frontal surface area (m^2)	7.0	7.4	8.0

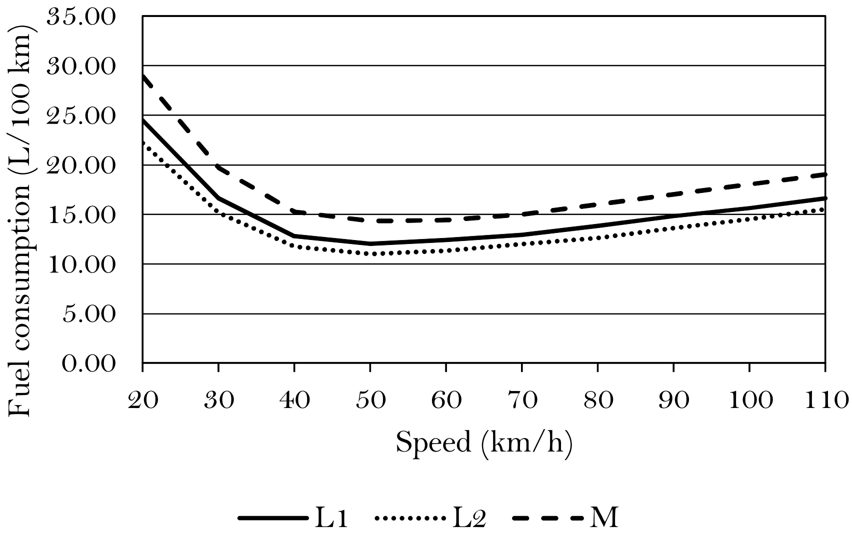


FIGURE 6.1: Fuel consumption as a function of speed

6.2.3 Speed zones

Road speed limits are commonly set by national or local governments (Wikipedia 2015). They play a key role in ensuring the safety of road users and of the public at large (UK Government 2014). In general, cities are divided into several speed zones which help traffic flow more safely and efficiently. They also provide a reasonable balance between the needs of drivers, pedestrians and cyclists who use public roads for travel, and the concerns of residents who live along these roads (Oregon 2015b). Studies have been performed in the United Kingdom by the Department for Transport (2013), in Canada by the City of Ottawa Transportation Committee (2009) and in the United States by the Oregon Department of Transportation (2015) on the best way to establish speed zones. These studies indicate that setting reasonable vehicle speeds for a variety of weather conditions results in fewer accidents. When reasonable speeds are imposed, less overtaking occurs, and one also observes smaller delays and fewer rear-end collisions. According to the above studies, speed zones in cities are generally classified under three categories:

- 15 mph (25 km/h): alleys, narrow residential roadways,
- 20 mph (32 km/h): business districts, school zones,
- 25 mph (40 km/h): residential districts, public parks, ocean shores.

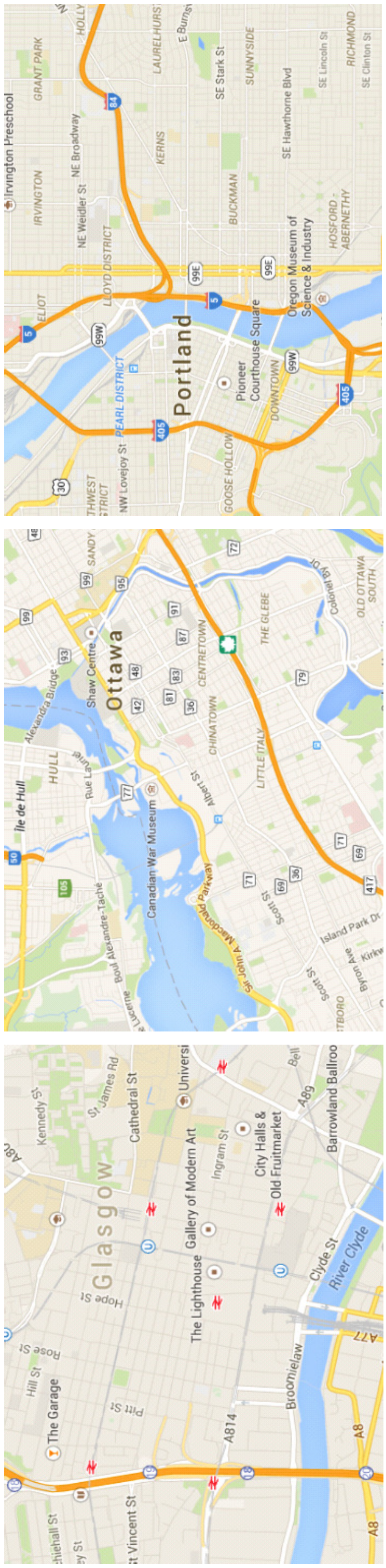
Speed zones also yield environmental benefits. For example Kirkby (2002) states that 20 mph (32 km/h) speed zones, significantly improve the quality of life of the concerned community, and encourage healthier and more sustainable transportation. This speed limit favours slower driving, saves fuel and reduces pollution, unless an unnecessarily low gear is used (DfT 2013).

6.2.4 Network structure

We consider cities in which distances are measured using the Taxicab geometry (see Krause 2012). The Taxicab geometry is also known as the rectilinear distance, the L_1 distance, the city block distance or the Manhattan distance. It implies that the shortest path between two nodes is the sum of horizontal and vertical distances between them. This metric is appropriate in several grid cities, such as Glasgow, Ottawa and Portland, shown in Figure 6.2.

In the setting considered in this study, we assume that the city centre is divided into several zones, each belonging to one of the three categories described in Section 6.2.3. We also assume that speeds are deterministic and time-independent. Zone 1 corresponds to the city centre, zone 2 is an outer urban area, and zone 3 corresponds to a suburb. The index set of speed zones is denoted by $\mathcal{Z}$. Let the zones be $z_1, z_2, z_3 \in \mathcal{Z}$ and let V_1, V_2, V_3 be the fixed speeds in zones, where $V_1 < V_2 < V_3$. Figure 6.3 illustrates a city divided into three fixed speed zones. When a vehicle travels within the same zone z_1, z_2 or z_3, its speed is equal to the speed of that zone. When it travels on the boundary of two speed zones, it uses the faster speed of the two zones.

In a city, a shortest path between i and j is not necessarily a cheapest or a least polluting path. In urban areas where a maximum speed limit of 40 km/h is imposed, a fastest path is also a least-polluting path according to Figure 6.1. However, as in Ehmke et al. (2014), this path is not always a shortest path. For example, consider the corners (A, B, C, D) of zone 2 in Figure 6.3, and nodes i and j located in zone 3. When travelling from i to j, a vehicle of type h may choose not to travel on a straight line from i to j with speed V_2 between points K and L, but may instead travel on the boundaries of zone 2 with speed $V_3 \leq 40$ km/h to avoid driving at a slower speed through congested traffic. A fastest path from customer i to j could well be (i, K, A, C, L, j) instead of (i, K, L, j), particularly in urban settings.



(a) Glasgow

(b) Ottawa

(c) Portland

FIGURE 6.2: Grid city examples (Google Maps 2015)

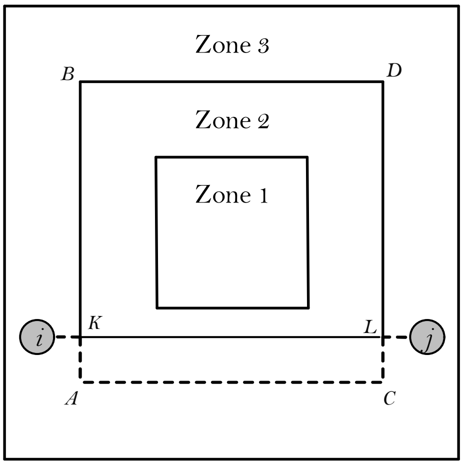


Figure 6.3: Illustration of speed zones

Using equation (6.6), we now illustrate how the load on a vehicle can affect the calculation of the cheapest path between a node pair. In Figure 6.3, let us assume that the total length of path (i, K, L, j) is 8 km, and for (i, K, A, C, L, j) is 9 km. When a vehicle of type M going from i to j carries a load equal to 1000 kg, then the cost of (i, K, L, j) is £1.85 and the cost of (i, K, A, C, L, j) is £1.95 with the former path being the cheaper one. However, when the vehicle load is equal to 12500 kg, then the cost of (i, K, L, j) is £2.20 and the cost of (i, K, A, C, L, j) is £2.05, where the cheapest path now is the latter.

6.2.5 Depot costs

There are four main categories of depot or warehouse costs: handling, storage, operations administration and general administrative expenses (see Ghiani et al. 2013). Storage expenses are the cost of occupying a facility (Speh 2009). Depot location affects the storage cost, e.g., locating a depot in the city centre (zone 1) is much more expensive than locating it in an outer zone (zone 2 or 3).

6.3 Formal Problem Description and Mathematical Formulation

Our problem is defined on a complete directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N} = \mathcal{N}_0 \cup \mathcal{N}_c$ is a set of nodes in which $\mathcal{N}_0$ and $\mathcal{N}_c$ represent the potential depots and customer nodes, respectively. A storage capacity D_k and a fixed opening cost g_k are associated with each potential depot $k \in \mathcal{N}_0$. Each customer $i \in \mathcal{N}_c$ has a positive demand q_i. The arc set $\mathcal{A}$ is defined as $\mathcal{A} = \{(i,j) : i \in \mathcal{N}, j \in \mathcal{N}\} \setminus \{(i,j) : i \in \mathcal{N}_0, j \in \mathcal{N}_0, i \neq j\}$. We assume that an unlimited heterogeneous fleet of vehicles operates with various capacities and vehicle-related costs. The index set of vehicle types is denoted by $\mathcal{H}$. Let Q^h and t^h denote the capacity and fixed dispatch cost of a vehicle of type $h \in \mathcal{H}$, respectively. Fuel and CO_2 emissions cost $c(i,j,w_i^h)$ of traveling from node i to node j with a vehicle of type h having a load equal to w_i^h upon leaving i. This cost is calculated using equation (6.6).

The problem consists of locating depots in a subset of $\mathcal{N}_0$, of assigning each customer to a depot and of determining a set of vehicle routes such that all vehicles start and end their routes at their depot, and the load of each vehicle does not exceed its capacity. The objective is to minimize the total cost which is made up of three components: the depot operating cost, the vehicle fixed cost, and the fuel and CO_2 emissions cost. Furthermore, the speed of a vehicle depends on the speeds of the zones it traverses while driving.

To formulate the problem, we define the following additional decision variables. Let x_{ij}^h be equal to 1 if a vehicle of type $h \in \mathcal{H}$ travels from node i to node j and to 0 otherwise. Let u_k be equal to 1 if depot $k \in \mathcal{N}_0$ is opened and to 0 otherwise. Let z_{ik} be equal to 1 if customer $i \in \mathcal{N}_c$ is assigned to depot $k \in \mathcal{N}_0$ and to 0 otherwise. Let f_{ij}^h be the amount of commodity carried by a vehicle of type h from node i to node j.

The integer linear programming formulation of the problem is then:

$$\text{Minimize} \sum_{k \in \mathcal{N}_0} g_k u_k + \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{N}_0} \sum_{j \in \mathcal{N}_c} t^h x_{kj}^h + \sum_{h \in \mathcal{H}} \sum_{(i,j) \in \mathcal{A}} c(i,j,w_i^h) x_{ij}^h \tag{6.7}$$

subject to

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} x_{ij}^h = 1 \qquad i \in \mathcal{N}_c \tag{6.8}$$

$$\sum_{h \in \mathcal{H}} \sum_{i \in \mathcal{N}} x_{ij}^h = 1 \qquad j \in \mathcal{N}_c \tag{6.9}$$

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} f_{ji}^h - \sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} f_{ij}^h = q_i \qquad i \in \mathcal{N}_c \tag{6.10}$$

$$f_{ij}^h \leq Q^h x_{ij}^h \qquad i \in \mathcal{N}_0, j \in \mathcal{N}, i \neq j, h \in \mathcal{H} \tag{6.11}$$

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}_c} f_{kj}^h = \sum_{j \in \mathcal{N}_c} z_{jk} q_j \qquad k \in \mathcal{N}_0 \tag{6.12}$$

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}_c} f_{jk}^h = 0 \qquad k \in \mathcal{N}_0 \tag{6.13}$$

$$f_{ij}^h \leq (Q^h - q_i) x_{ij}^h \qquad i \in \mathcal{N}_c, j \in \mathcal{N}, h \in \mathcal{H} \tag{6.14}$$

$$f_{ij}^h \geq q_j x_{ij}^h \qquad i \in \mathcal{N}, j \in \mathcal{N}_c, h \in \mathcal{H} \tag{6.15}$$

$$\sum_{i \in \mathcal{N}_c} q_i z_{ik} \leq D_k u_k \qquad k \in \mathcal{N}_0 \tag{6.16}$$

$$\sum_{k \in \mathcal{N}_0} z_{ik} = 1 \qquad i \in \mathcal{N}_c \tag{6.17}$$

$$x_{ij}^h + \sum_{q \in \mathcal{H}, q \neq h} \sum_{r \in \mathcal{N}, j \neq r} x_{jr}^h \leq 1 \qquad i \in \mathcal{N}, j \in \mathcal{N}_c, i \neq j, h \in \mathcal{H} \tag{6.18}$$

$$\sum_{h \in \mathcal{H}} x_{ik}^h \leq z_{ik} \qquad k \in \mathcal{N}_0, i \in \mathcal{N}_c \tag{6.19}$$

$$\sum_{h \in \mathcal{H}} x_{ki}^h \leq z_{ik} \qquad k \in \mathcal{N}_0, i \in \mathcal{N}_c \tag{6.20}$$

$$\sum_{h \in \mathcal{H}} x_{ij}^h + z_{ik} + \sum_{m \in \mathcal{N}_0, m \neq k} z_{jm} \leq 2 \qquad k \in \mathcal{N}_0, i, j \in \mathcal{N}_c, i \neq j \tag{6.21}$$

$$w_i^h = \sum_{j \in \mathcal{N}} f_{ij}^h \qquad i \in \mathcal{N}, h \in \mathcal{H} \tag{6.22}$$

$$x_{ij}^h \in \{0, 1\} \qquad i, j \in \mathcal{N}, h \in \mathcal{H} \tag{6.23}$$

$$u_k \in \{0, 1\} \qquad k \in \mathcal{N}_0 \tag{6.24}$$

$$z_{ik} \in \{0, 1\} \qquad k \in \mathcal{N}_0, i \in \mathcal{N}_c \tag{6.25}$$

$$f_{ij}^h \geq 0 \qquad h \in \mathcal{H}. \tag{6.26}$$

The objective function (6.7) minimizes the total cost including fixed depot and vehicle costs, as well as fuel and CO_2 emissions cost. Constraints (6.8) and (6.9) ensure that each customer is visited exactly once. Constraints (6.10) imply that the demand of each customer is satisfied. Constraints (6.11) mean that the total load on any path cannot exceed the capacity of the vehicle traversing it. Constraints (6.12) ensure that the total load of the vehicles departing from a depot is equal to the total demand of the customers assigned to it. Constraints (6.13) state that the load on all vehicles returning to each depot must be equal to zero. Contraints (6.14) and (6.15) are bounding constraints for the load variables. Constraints (6.16) guarantee that total demand associated with a depot cannot exceed its capacity. Constraints (6.17) and (6.18) ensure that each customer is assigned to only one depot and one vehicle, respectively. Constraints (6.19)–(6.21) forbid the creation of routes that do not start and end at the same depot. Constraints (6.22) define the load of a vehicle of type h upon leaving node i as the total amount of commodity on the arc (i, j) it uses to leave i. Finally, constraints (6.23)–(6.25) enforce the integrality and non-negativity restrictions on the variables.

6.4 Description of the ALNS Metaheuristic

The mathematical formulation just presented is of large scale and cannot be solved for practical instances. We have therefore devised a metaheuristic algorithm, called pollution-and-location-heterogeneous adaptive large neighborhood search (P-L-HALNS), to solve the problem. This algorithm is partly based on the ALNS framework of Demir et al. (2012) which is initially put forward by Ropke and Pisinger (2006a,b) to solve several variants of the VRP (see Laporte et al. 2014). This metaheuristic has since provided very good results on several complicated variants of the VRP (see Pisinger and Ropke 2007, Koç et al. 2015), of the LRP (see Koç et al. 2016), and of the PRP (see Demir et al. 2012, 2014a, Koç et al. 2014).

The P-L-HALNS consists of two basic procedures: removal or destroy, followed by insertion or repair. In the removal procedure, n' nodes are iteratively removed by destroy operators and placed in the removal list, where n' lies in the interval $[b_l, b_u]$ for the destroy operators. In the insertion procedure, the nodes of the removal list, are iteratively inserted into a least-cost position of the incomplete solution by means of an insertion operator. The removal and insertion operators are selected dynamically according to their past performance. To this end, each operator is assigned a score which is increased whenever it improves the current solution and is periodically reset to one. Simulated annealing is used as an outer local search framework for the P-L-HALNS in order to define the acceptance rules of candidate solutions. Simulated annealing acceptance criterion has been recently shown to perform well for various routing problems (e.g., Pisinger and Ropke 2007, Demir et al. 2012), which is the motivation to use the same approach in our work.

In order to perform least-cost insertions, it is necessary to make use of cheapest path values frequently during the course of the algorithm. We explain in Section 6.4.1 how these computations are handled in the ALNS metaheuristic. This is followed in Section 6.4.2 by an overview of the metaheuristic itself.

6.4.1 Cheapest path calculation

The number of undominated paths between any two nodes is finite, but the identification of such paths is not trivial since the cost of a path depends (see equation (6.6)) on the type of vehicle traveling a path from i to j, on its load upon leaving i, and on the speed of each arc of the path. To overcome the complexity of this task, we introduce a heuristic Cheapest Path Calculation procedure which computes only three paths between i and j and selects the cheapest one. This procedure does not guarantee the calculation of the minimum cost path over all possible paths, but is suitable for iterative use within an algorithm like the P-L-HALNS described here.

Algorithm 1 presents this procedure for a node pair (i, j) which follows three steps. In step 1 (lines 2–4), we find two shortest path between (i, j). According to Taxicab geometry (see Section 6.2.4), if node i and j are not located at same horizontal or vertical coordinate, there exist two shortest paths with the same length, but not necessarily with the same cost for a vehicle with a fixed load, because of the possibility of cutting through different zones. In this case, we identify the cheapest path p_0 of the two paths and discard the other one. In steps 2 and 3, we contort p_0 to generate two alternative paths p_1 and p_2. In step 2 (lines 5–7), path p_1 follows the boundary of zone 1 on which it travels at a speed V_2 based on the assumption made in Section 6.2.3. Similarly, in step 3 (lines 8–10) path p_2 follows the boundary of zone 2. We do not consider travel on or outside the boundary of zone 3 as this is not defined. The algorithm then compares the costs of p_0, p_1 and p_2, and returns the cheapest path (line 11). In the P-L-HALNS, it should be noted that we calculate the shortest paths (Step 1) between each pair of nodes as a priori as in the VRP.

Figure 6.4 illustrates the Cheapest Path Calculation procedure for a given node pair (i, j) and a vehicle with a fixed load traveling between these nodes. Figure 6.4.a (Step 1) shows two paths, (i, A, j) and (i, B, j) that are the shortest with respect to the Taxicab geometry and distance, but the cheapest path would always be (i, B, j) since $V_2 < V_3 \leq 40$ km/h. We then calculate the cost χ_0 of $p_0 = (i, B, j)$. In Figure 6.4.b (Step 2), we first find the shortest path from node i to nearest point (A_1) of zone

Algorithm 1 Cheapest Path Calculation

Node i and node $j (i, j \in \mathcal{N})$. Let χ_0, χ_1, χ_2 be the costs of paths p_0, p_1 and p_2
Step 1
Find two shortest paths between (i, j) and choose the least cost path p_0.
Calculate the cost χ_0 of path p_0.
Step 2
Find path p_1 by contorting path p_0 part of which lies on the border of zone 1 between (i, j).
Calculate the cost χ_1 of path p_1.
Step 3
Find the path p_2 by contorting path p_0 part of which lies on the border of zone 2 between (i, j).
Calculate the cost χ_2 of path p_2.
Return Least cost path p_k where $k = \arg\min \{\chi_0, \chi_1, \chi_2\}$.

1. We then find the shortest path, on the border of zone 1, from point A_1 to nearest point (B_1) of zone 1 to node j. We finally find the shortest path from point B_1 to node j. As in Step 1, if there are two same length shortest paths between points, such as (B_1, B_{1a}, j) and (B_1, B_{1b}, j), we select the cheapest one, in this case (B_1, B_{1a}, j). We calculate the cost χ_1 of $p_1 = (i, A_1, B_1, B_{1a}, j)$. In Figure 6.4.c (Step 3), we first find the shortest path from node i to nearest point (A_2) of zone 2. We then find the shortest path, on the border of zone 2, from point A_2 to nearest point (B_2) of zone 2 to node j. We finally find the shortest path from point B_2 to node j. We calculate the cost χ_2 of $p_2 = (i, A_2, B_2, j)$.

6.4.2 Overview of the metaheuristic

The general framework of the P-L-HALNS metaheuristic is sketched in Algorithm 2. We now briefly explain its steps. Given the complexity of implementing the Cheapest Path Calculation procedure at every step of the P-L-HALNS, we work with average route demand lengths. At the beginning of the algorithm, we first define a set $\mathcal{B}$ of average route demand levels. We know the total demand of customers as a priori. For example, let $|\mathcal{B}| = 4$ and total demand of customers is 2000 kg, which results in the following intervals: level 1 ranges from zero to 500 kg, level 2 ranges from 501 to 1000 kg, level 3 ranges from 1001 to 1500 kg, and level 4 ranges from 1501 to 2000 kg. Let $v_{ij}^{\beta h}$ be the fixed cost associated with the path for each average route demand level $\beta \in \mathcal{B}$ and for each vehicle of type $h \in \mathcal{H}$. The fixed costs $v_{ij}^{\beta h}$ are calculated at the beginning of the algorithm (line 1). These fixed costs are used to compute the route

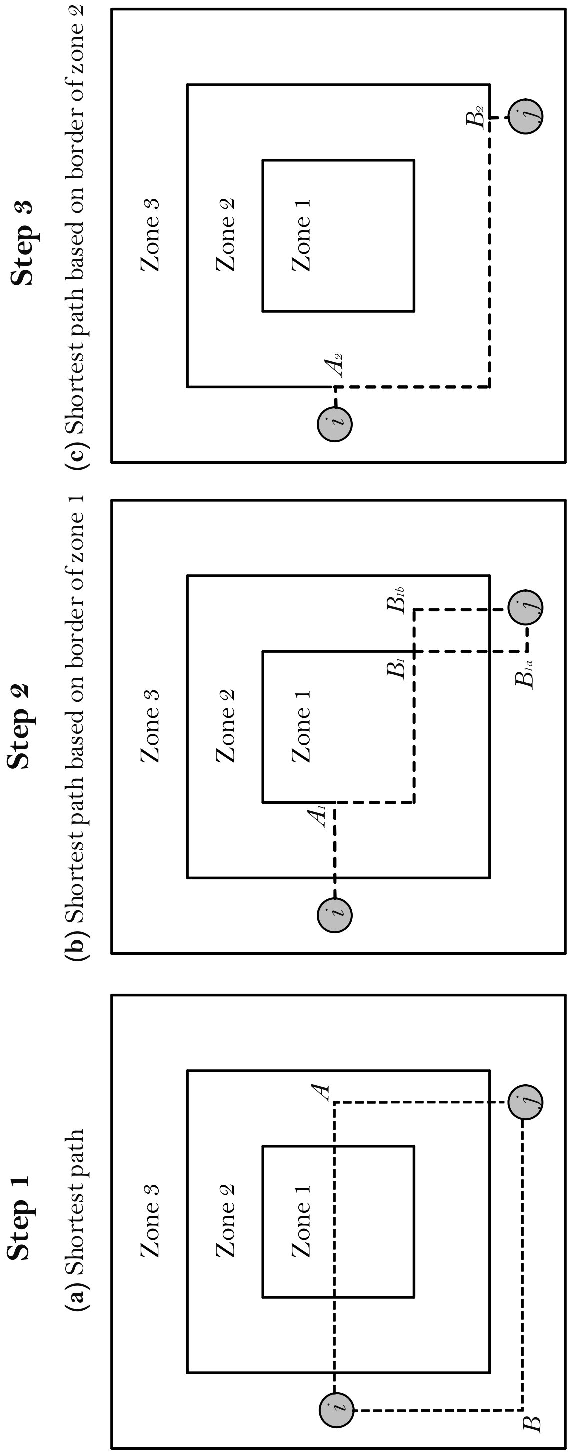


Figure 6.4: Illustration of the three speed zones

costs quickly. During the algorithm, for each solution, average route demand calculated as (total demand of customers)/(total number of vehicle routes). For example, let the total demand of customers be 2000, $|\mathcal{B}| = 4$ and the number of vehicle routes be three. The average route demand is then 2000/3 and level $\beta \in \mathcal{B}$ is equal to 2.

An initial solution ω_0 is generated by using a modified version of the classical Clarke and Wright (1964) savings algorithm for the VRP (line 2). The selection probabilities are initialized for each destroy and repair operator (line 3). In line 4, ω_b is the best solution found during the search, ω_c is the current solution obtained at the beginning of an iteration, and ω_t is a temporary solution found at the end of the iteration which can be discarded or become the current solution. The temperature is denoted by T, the iteration counter is denoted by j, and the current and the best solutions are initially set equal to the initial solution (line 4). The temperature T is initially set at $c(\omega_0)P_0$, where $c(\omega_0)$ is the cost of initial solution and P_0 is the initial temperature.

Every σ iterations, a diversification based removal operator is selected (lines 6–8) and applied to ω_c; otherwise an intensification based removal operator is selected (lines 9–11). An insertion operator is then selected and applied to the destroyed solution, and a feasible solution ω_t is obtained (line 12).

The operators are applied using the average costs up until the counter p reaches ς, following which the actual costs for ω_c, ω_t and ω_b are calculated using the Cheapest Path Calculation procedure (lines 13–15). Otherwise, the fixed costs are used to compute $c(\omega_t)$ (lines 16–18). If the cost of a repaired solution $c(\omega_t)$ is less than that of the current solution $c(\omega_c)$, then ω_c is replaced by ω_t (lines 19–20). Otherwise, the probability ϑ of accepting a non-improving solution is computed (line 21–22) as a function of the current temperature. A random number ϵ is then generated in the interval $[0, 1]$ (line 23). If ϵ is less than ϑ, ω_c is then replaced by ω_t (lines 24–25). If the cost of ω_c is less than that of ω_b, ω_b is replaced by ω_c (lines 26–27). The current temperature is gradually decreased during the algorithm as δT (line 28), where $0 < \delta < 1$ is a fixed cooling parameter. The probabilities are updated by means of an adaptive weight adjustment procedure (AWAP) (line 29). When the maximal number ϖ iterations is reached, the algorithm terminates (line 31) and returns the best found solution. For further information on the operators and on other algorithmic details the reader is referred to Demir et al. (2012) and Koç et al. (2015, 2016) and Chapters 3 and 4.

Algorithm 2 General framework of the P-L-HALNS

Fixed cost calculation: Calculate the fixed costs $\upsilon_{ij}^{\beta h}$
Initialization: Generate an initial solution
Initialize probabilities associated with the operators
$T \leftarrow temperature$, $q \leftarrow 1$, $p \leftarrow 1$, $l \leftarrow 1$, $\omega_c \leftarrow \omega_b \leftarrow \omega_0$
while the maximum number of iterations is reached $q < \varpi$ **do**
    if $l = \sigma$ **then**
        Diversification based destroy
        $l \leftarrow 1$
    else
        Intensification based destroy
        $l \leftarrow l + 1$
    Repair
    if $p = \varsigma$ **then**
        Calculate real costs
        $p \leftarrow 1$
    else
        Calculate the solution cost using fixed costs $\upsilon_{ij}^{\beta h}$
        $p \leftarrow p + 1$
    if $c(\omega_t) < c(\omega_c)$ **then**
        $\omega_c \leftarrow \omega_t$
    else
        $\vartheta \leftarrow e^{-(c(\omega_t) - c(\omega_c))/T}$
    Generate a random number ϵ
    if $\epsilon < \vartheta$ **then**
        $\omega_c \leftarrow \omega_t$
    if $c(\omega_c) < c(\omega_b)$ **then**
        $\omega_b \leftarrow \omega_c$
    $T \leftarrow \delta T$
    AWAP: update probabilities of operators
    $q \leftarrow q + 1$
end while

6.5 Computational Experiments and Analyses

We now present the results of our computational experiments. All experiments were conducted on a server with one gigabyte RAM and an Intel Xeon 2.6 GHz processor. The P-L-HALNS was implemented in C++.

We assume an area divided into three nested squares centered in the middle of the area, each corresponding to a fixed speed zone, as shown in Figure 6.3. The fixed speeds are set at 25, 32 and 40 km/h and the sizes of the nested squares are 3 km × 3 km, 6 km × 6 km and 10 km × 10 km, respectively. We generated four sets of instances where the first set contains 25 customers and four potential depots locations, the second set contains

50 customers and six potential depots locations, the third set contains 75 customers and eight potential depots locations, and the fourth set contains 100 customers and 10 potential depots depots. Each set includes three subsets: 1) customers concentrated in the city centre, denoted by CC, 2) customers concentrated in the outer city area and in the suburb, denoted by SU, and 3) customers located randomly, denoted by R. These three subsets of benchmark instances are illustrated in Figure 6.5. These configurations cover a wide variety of realistic urban settings.

Each subset includes five instances, resulting in a total of 60 instances. To generate the depot characteristics, we used a procedure similar to that used for the standard LRP benchmark instances (see Barreto 2004, Albareda-Sambola et al. 2005, Prodhon 2006). We also further explore the effect of variations in depot costs in Section 6.5.4. The customer demands and the depot capacities (in kg) were randomly generated using a uniform distribution in the range [100, 1100] and [10000, 15000], respectively. The fixed depot costs are dependent on their location, i.e., zone 1 has the highest fixed cost (per depot £5000/day), followed by zone 2 (per depot £3500/day) and finally zone 3 (per depot £2000/day). All costs relate to the same planning horizon.

The parameters used in the P-L-HALNS are provided in Table 6.3. All algorithmic parametric values, except ς and σ, are as described in Demir et al. (2012), who applied an extensive meta-calibration procedure to generate effective parameter values for their ALNS heuristic for the PRP. During the experiments, ten runs were performed for each instance and the result of the best one was retained.

The aim of the computational experiments is fivefold: 1) to solve the problem described in Section 6.3, 2) to empirically calculate the savings that could be achieved by using a comprehensive objective function instead of using individual functions for each performance indicator, 3) to analyze the effect of variations in potential depot locations and customer distribution, 4) to investigate the effect of variations in depot costs, and 5) to quantify the benefits of using a heterogeneous fleet over a homogeneous one.

6.5.1 Results obtained on the test instances

This section presents the results obtained by P-L-HALNS on the 25-, 50-, 75- and 100-customer instances. Table 6.4 presents the average results for each instance set where the

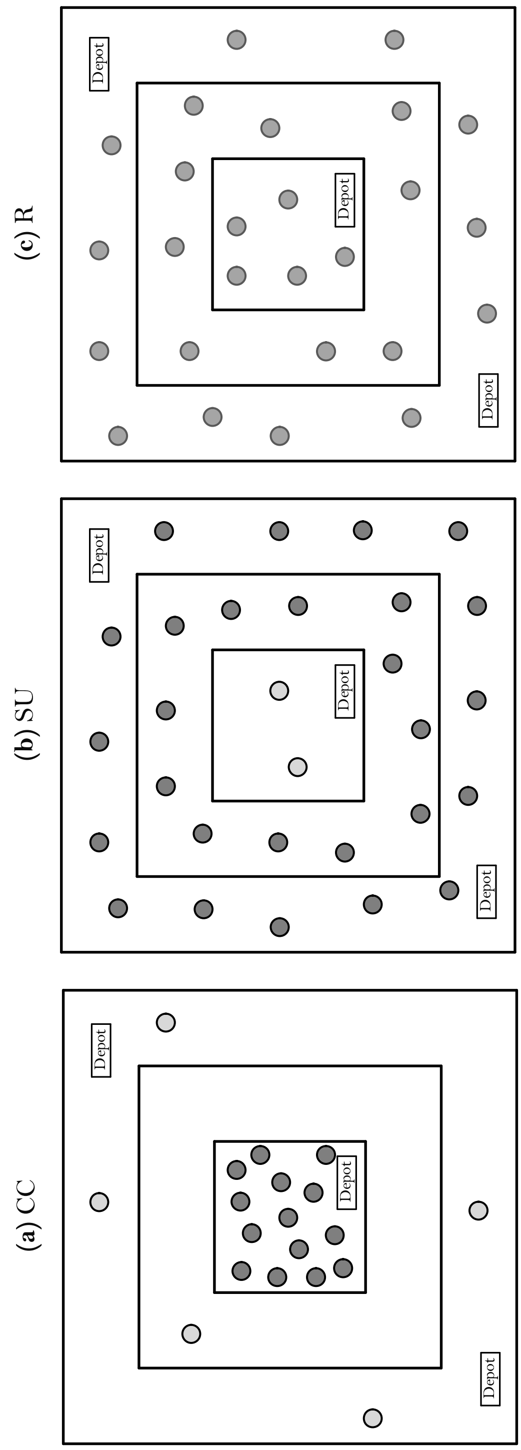


FIGURE 6.5: Geographical customer distribution in the benchmark instances

TABLE 6.3: Parameters used in the P-L-HALNS

Description	Typical values
Total number of iterations (ϖ)	25000
Number of iterations for roulette wheel	450
Roulette wheel parameter	0.1
New global solution	1
Better solution	0
Worse solution	5
Startup temperature parameter (P_0)	100
Cooling parameter (δ)	0.999
Lower limit of removable nodes	5-20% of $\lvert\mathcal{N}_c\rvert$
Upper limit of removable nodes	12-30% of $\lvert\mathcal{N}_c\rvert$
First shaw parameter	0.5
Second shaw parameter	0.15
Third shaw parameter	0.25
Noise parameter	0.1
Route cost calculation parameter (ς)	100
Diversification parameter (σ)	50

columns display the average distance (km), CO_2 emissions (kg), fuel and CO_2 emissions cost (£), depot cost (£), vehicle cost (£), total cost (£) and time (s). We also report the average number of opened depots for each subset. In this column, the first, second and third elements within the parentheses represent the number of opened depots in zone 1, 2 and 3, respectively. To evaluate the environmental impact of the solutions, we also report the average amount of CO_2 emissions (in kg) based on the assumption that one liter of gasoline contains 2.32 kg of CO_2 (Coe 2005). For detailed results, the reader is referred to Tables A.1–A.4 in the Appendix.

From Table 6.4, it is clear that the total cost is dominated by the large depot costs which force the P-L-HALNS to first minimize the number of depots, then minimize the vehicle fixed costs, and lastly fuel and CO_2 emission costs.

6.5.2 The effect of the various cost components of the objective function

In this section, we analyze the implications of using different objectives on a number of performance measures. To this end, we have conducted experiments using four special cases of the objective function, which are presented in the first column of Table 6.5. The experiments were conducted on all 100-customer R, SU and CC instances. In the first version, we only consider minimizing the fuel and CO_2 emissions costs (F). This setting

Table 6.4: Average results on the instances

Instance	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	Opened depots	Total distance (km)	CO_2 emissions (kg)	Fuel and CO_2 emissions costs (£)	Depot cost (£)	Vehicle cost (£)	Total cost (£)	Time (s)
CC25	25	4	(1.2, 0.2,0.5)	37.40	21.56	13.01	7900.00	106.20	8019.21	5.46
SU25	25	4	(0.4,0.8,0.6)	56.01	20.63	12.45	5400.00	102.60	5515.05	5.44
R25	25	4	(0.4,0.8,0.4)	64.78	19.48	11.76	5600.00	104.00	5715.76	5.41
CC50	50	6	(1.6,0.6,0.8)	80.66	23.68	14.29	11700.00	175.60	11889.88	32.11
SU50	50	6	(0.4,0.6,2.0)	125.15	21.44	12.94	8100.00	175.60	8288.53	31.00
R50	50	6	(0.2,1.0,1.8)	128.60	23.40	14.12	8100.00	169.00	8283.12	31.53
CC75	75	8	(2.2,1.0,0.8)	106.14	28.43	17.16	16100.00	256.80	16373.96	68.21
SU75	75	8	(0.0,2.0,2.0)	200.39	32.73	19.75	11300.00	288.80	11608.54	64.29
R75	75	8	(0.0,2.2,1.8)	197.89	38.22	23.06	11000.00	300.00	11323.04	64.02
CC100	100	10	(2.6,1.4,1.4)	140.24	43.61	26.31	20700.00	358.20	21084.52	169.51
SU100	100	10	(0.0,2.2,3.0)	244.44	42.02	25.35	14900.00	397.40	15322.76	151.15
R100	100	10	(0.0,3.0,2.2)	259.42	54.66	32.98	13600.00	395.80	14028.78	158.13

also implies minimizing CO_2 since emissions are proportional to fuel consumption. We then consider the objective of minimizing only the depot cost (D) and the vehicle fixed cost (V) in the second and third versions, respectively. The next objective corresponds to that of the HVRP which jointly minimizes distance and vehicle fixed costs (DV). Finally, we present the comprehensive objective of minimizing the total cost function (T) as defined by (6.7).

Table 6.6 presents the average deviations of each component from the smallest value of each column. For example, in the case of the R100 instances, the minimum average value for objective D is £13,600 across the five objective functions, but objective V yields a solution in which the average depot cost is £18,289.66, corresponding to an increase of 34.48% over the former. For the R100, SU100 and CC100 instances, it is clear that objective F results in a poor total cost performance, yielding a 20.39%, 33.66% and 6.69% average increases over the value found through objective T, respectively. In the case of the R100 instances, this increase is more substantial for objective V, which is on average 33.06% higher. For the R100, SU100 and CC100 instances, as for emissions, objective F yields an increase of 20.91%, 18.18% and 6.67% in depot cost over the value provided by objective D, respectively. For the R100, SU100 and CC100 instances, objective DV performs very poorly on all cost components, yielding average increases of 151.36%, 151.67% and 64.00%, respectively. These results indicate that traveling on a shortest path does not always result in a cheapest solution. In urban settings, due to the effect of speed zones on the objective function, longer paths outside the city centre have the potential to decrease the solution cost, a situation that was explained in Section 6.2. Maden et al. (2010) reached a similar conclusion relative to long-haul transportation.

6.5.3 The effect of variations in depot and customer locations

In this section, we investigate the effect of the variations in potential depot locations and customer distribution. To this end, we have selected five R type instances with 100 customers and 10 potential depots. We consider three variations, namely all depots are potentially located in zone 1, in zone 2, and in zone 3, respectively. Customer locations are kept the same across all variations. In the tables, the columns Dev_{CO_2} and Dev_T show the deviations in CO_2 emissions (in kg) and in total cost (£) between the various depot or customer location cases and the base case.

Table 6.5: The effect of cost components: objective function values.

Objective	Opened depots	Total distance (km)	CO_2 emissions (kg)	Fuel and CO_2 emissions costs (£)	Depot cost (£)	Vehicle cost (£)	Total cost (£)
R100 instances							
Fuel and CO_2 emissions cost (F)	(1.0,3.1,1.0)	257.66	30.50	18.40	16443.79	363.51	16889.10
Depot cost (D)	(0.0,3.1,2.2)	238.34	52.41	31.63	13600.00	399.97	14030.00
Vehicle fixed cost (V)	(1.1,3.2,2.1)	248.40	36.34	21.93	18289.66	356.22	18666.46
Distance and vehicle fixed cost (DV)	(4.0,3.1,3.2)	333.19	119.59	72.17	34234.48	962.42	35263.31
Total cost (T)	(0.0,3.0,2.2)	259.42	54.66	32.98	13600.00	395.80	14028.78
SU100 instances							
Fuel and CO_2 emissions cost (F)	(1.1,3.0,2.0)	244.44	42.02	25.35	20037.93	416.03	20480.95
Depot cost (D)	(0.0,3.2,3.0)	304.11	43.62	26.32	16955.17	498.82	17480.01
Vehicle fixed cost (V)	(1.0,3.1,2.0)	250.09	62.91	37.96	20037.93	361.18	20438.29
Distance and vehicle fixed cost (DV)	(4.0,3.2,3.1)	340.56	92.48	55.81	37506.90	999.71	38562.30
Total cost (T)	(0.0,2.2,3.0)	248.49	43.52	26.26	14900.00	397.40	15322.76
CC100 instances							
Fuel and CO_2 emissions cost (F)	(3.0,2.0,1.1)	117.68	36.15	21.82	22080.00	369.49	22494.35
Depot cost (D)	(3.1,1.2,2.1)	161.18	69.36	41.85	20700.00	408.49	21140.94
Vehicle fixed cost (V)	(3.0,2.2,1.1)	124.22	43.97	26.53	22080.00	358.20	22467.62
Distance and vehicle fixed cost (DV)	(4.3,3.1,3.0)	256.51	86.30	52.08	33580.00	991.46	34579.22
Total cost (T)	(2.6,1.4,1.4)	140.24	43.61	26.31	20700.00	358.20	21084.52

TABLE 6.6: The effect of cost components: percent deviation from the minimum value.

Objective	Total distance (km)	CO_2 emissions (kg)	Fuel and CO_2 emissions costs (£)	Depot cost (£)	Vehicle cost (£)	Total cost (£)
R100 instances						
Fuel and CO_2 emissions cost (F)	8.11	0.00	0.00	20.91	2.05	20.39
Depot cost (D)	0.00	71.86	71.86	0.00	12.28	0.01
Vehicle fixed cost (V)	4.22	19.15	19.15	34.48	0.00	33.06
Distance and vehicle fixed cost (DV)	39.79	292.13	292.13	151.72	170.18	151.36
Total cost (T)	8.84	79.22	79.21	0.00	11.11	0.00
SU100 instances						
Fuel and CO_2 emissions cost (F)	0.00	0.00	0.00	18.18	15.19	33.66
Depot cost (D)	24.41	3.82	3.82	0.00	38.11	14.08
Vehicle fixed cost (V)	2.31	49.73	49.73	18.18	0.00	33.39
Distance and vehicle fixed cost (DV)	39.32	120.12	120.12	121.21	176.79	151.67
Total cost (T)	1.66	3.57	3.57	0.00	10.03	0.00
CC100 instances						
Fuel and CO_2 emissions cost (F)	0.00	0.00	0.00	6.67	3.15	6.69
Depot cost (D)	36.97	91.85	91.85	0.00	14.04	0.27
Vehicle fixed cost (V)	5.56	21.63	21.63	6.67	0.00	6.56
Distance and vehicle fixed cost (DV)	117.97	138.73	138.73	62.22	176.79	64.00
Total cost (T)	19.18	20.62	20.62	0.00	0.00	0.00

TABLE 6.7: The effect of variations in depot location.

Instance	All depots in zone 1				All depots in zone 2				All depots in zone 3				Mix	
	CO_2 (kg)	Total cost (£)	Dev_{CO_2}	Dev_T	CO_2 (kg)	Total cost (£)	Dev_{CO_2}	Dev_T	CO_2 (kg)	Total cost (£)	Dev_{CO_2}	Dev_T	CO_2 (kg)	Total cost (£)
R100_1	110.68	25390.30	3.45	41.14	90.10	17866.20	−18.60	16.35	77.36	10367.00	−38.12	−44.15	106.86	14944.50
R100_2	52.29	25464.60	16.50	47.34	43.26	17917.10	−0.93	25.15	40.40	10461.00	−8.06	−28.19	43.66	13410.30
R100_3	50.77	25421.60	14.23	47.19	38.62	17907.30	−12.75	25.03	40.77	10422.60	−6.82	−28.80	43.55	13424.30
R100_4	54.14	25465.70	21.44	47.34	40.92	17946.70	−3.94	25.28	38.28	10488.10	−11.12	−27.86	42.53	13409.70
R100_5	77.70	25388.90	52.80	41.10	35.66	17866.50	−2.86	16.30	35.20	10366.30	−4.20	−44.27	36.68	14955.10
Avg (%)			21.68	44.82			−7.82	21.62			−13.66	−34.65		

We first report in Table 6.7 the effect of varying the depot locations. Table 6.7 shows that when all depots are located in zone 1, CO_2 emissions increase by 21.68%. When they are located in zones 2 and 3, CO_2 emissions decrease by 7.82% and 13.66%, respectively. Table 6.7 suggests that the average increase in the total cost is 44.82% and 21.62% on average when all depots are located in zone 1 and 2, respectively over the base case. When all the depots are located in zone 3, the total cost decreases by about 34.65% on average. This analysis indicates that in terms of cost, it is preferable to locate the depots in suburban areas rather than in the city centre when the customers are uniformly distributed, i.e., for R instances. This also helps reduce congestion in city centres. A similar observation was made by Dablanc (2014) who conducted an empirical study on depot location in the Los Angeles area and concluded that warehouses moved out an average of six miles from the area barycentre between 1998 and 2009. Dablanc's findings are mainly a consequence of the fact that land is cheaper in the suburbs than in inner-cities, which translates into lower depot costs. Our study goes one step further in that it shows that locating depots in peripheral zones also helps reduce pollution since more travel can be made at an optimal speed. Locating depots outside the city centre translates into larger driving distances to the inner city customers but yields overall economic and environmental benefits.

We now analyze the effect of variations in customer locations. Table 6.8 provides a comparison of three variations, namely all customers located in zone 1, all customers located in zone 2, and all customers located in zone 3. The depot locations are kept the same across all variations. Table 6.8 shows that when all customers are located in zone 3, CO_2 emissions increase by 11.42%. On the other hand, when all customers are located in zone 1 and 2, CO_2 emissions decrease by 38.97% and 50.14%, respectively. Table 6.8 suggests that the average total cost increase over the base case is 38.16%, 6.04% and 8.12% on average when all customers are located in zone 1, 2 and 3, respectively. For the case where all customers are located in zone 1, 2 and 3, the increase in the total cost ranges from 33.25% to 41.50%, from -0.33% to 10.36%, and from -0.29% to 20.57%, respectively. Our results suggest that when all customers are located only in the city centre this is always more expensive than for the other settings.

TABLE 6.8: The effect of variations in customer location.

Instance	All customers in zone 1				All customers in zone 2				All customers in zone 3				Mix	
	CO_2 (kg)	Total cost (£)	Dev_{CO_2}	Dev_T	CO_2 (kg)	Total cost (£)	Dev_{CO_2}	Dev_T	CO_2 (kg)	Total cost (£)	Dev_{CO_2}	Dev_T	CO_2 (kg)	Total cost (£)
R100_1	63.6614	22411.30	−67.86	33.32	65.15	14906.20	−64.02	−0.26	107.55	14912.70	0.64	−0.21	106.86	14944.50
R100_2	29.6308	22877.90	−47.35	41.38	27.18	14938.40	−60.61	10.23	46.55	14950.10	6.22	10.30	43.66	13410.30
R100_3	30.663	22878.50	−42.02	41.32	25.80	14948.60	−68.83	10.20	47.04	14954.40	7.42	10.23	43.55	13424.30
R100_4	33.7631	22922.40	−25.98	41.50	40.16	14959.30	−5.92	10.36	54.87	16882.10	22.48	20.57	42.53	13409.70
R100_5	32.8477	22406.20	−11.66	33.25	24.24	14905.60	−51.34	−0.33	46.05	14911.80	20.35	−0.29	36.68	14955.10
Avg (%)			−38.97	38.16			−50.14	6.04			11.42	8.12		

6.5.4 The effect of variations in depot costs

In practice, it is very difficult to estimate depot costs because these depend on factors such as land and building cost, staffing and technology. In general, these factors are highly variable and hard to quantify. In our benchmark instances, the depot costs are high with respect to other costs and dependent on their location, i.e., every zone has its own fixed depot cost. We now investigate the effect of variations in depot costs.

Our first experiments analyze the effect of same depot costs on opened depots. To this end, we have selected five R type instances with 100 customers and 10 depots. We consider five versions in which all depot costs are fixed at £5000, £3500, £2000, £1000 and £500 per day in all zones. Table 6.9 shows that when the variable depot cost (Mix) is used for each zone, 5.5 depots are opened in zones 2 and 3 on average. For the £5000, £3500, £2000, £1000 and £500 fixed costs, 3.4, 3.8, 3.8, 4.0 and 4.0 depots are opened in zones 2 and 3 on average. On the other hand, for these three fixed costs variants, 1.6, 1.2, 1.4, 1.2 and 0.0 depots are opened in zone 1 on average. The average number of opened depots in the city centre is always lower than the total of number of opened depots in the outer urban area and in the suburb. Our results clearly show that even if depot costs are the same in everywhere, it is still preferable to locate depots outside the city centre because of the pollution aspect (see Section 6.5.3).

Our next experiments investigate the effect of decreasing the variable depot costs. To this end, we have conducted four series of tests on all 100-customer CC, SU and R instances using our original variable depot costs structure. In these tests, we decrease the depot costs by 90%, 70%, 50% and 30%, respectively. For example, decreasing the depot cost by 90% means that the depot costs in zone 1, 2 and 3 are £500, £350 and £200, respectively. Looking at the results presented in Table 6.10, we observe no change in the locations of opened depots for all instances and for all variations. For example, for the CC100 instances, when we decrease depot costs by 90%, 70%, 50% and 30%, it is still preferable to open three depots in zone 1, one depot in zone 2 and two depots in zone 3. Even though customers are concentrated in the city centre, half of the depots are still located in the suburb. When we look at the SU100 instances, no depot is located in city centre, but six depots are located in outer city area and in the suburb. The R100 instances follows the same pattern with no depot located in the city centre, but five depots located in the outer city area and in the suburb. Again, these results clearly

TABLE 6.9: The effect of same depot costs on opened depots.

Instance	£5000	£3500	£2000	£1000	£500	Mix
	Opened depots	Opened depots	Opened depots	Opened depots	Opened depots	Opened depots
R100_1	(2,3,0)	(2,3,0)	(2,3,0)	(1,3,1)	(1,3,1)	(0,3,2)
R100_2	(2,3,0)	(1,3,1)	(1,3,1)	(2,2,1)	(1,3,1)	(0,3,2)
R100_3	(2,3,0)	(1,3,1)	(1,3,1)	(1,3,1)	(1,3,1)	(0,3,2)
R100_4	(1,3,1)	(1,3,1)	(2,2,2)	(1,3,2)	(2,2,2)	(0,3,3)
R100_5	(1,3,1)	(1,3,1)	(1,3,1)	(1,3,1)	(1,3,1)	(0,3,2)
Avg	(1.6,3.0,0.4)	(1.2,3.0,0.8)	(1.4,2.8,1.0)	(1.2,2.8,1.2)	(1.2,2.8,1.2)	(0.0,3.0,2.2)

show that no matter what the depot cost is, it is still preferable to locate the depots outside the city centre due to the impact of their location on CO_2 emissions.

6.5.5 The effect of fleet composition

This section analyzes the benefit of using a heterogeneous fleet of vehicles over a homogenous one. To this end, we have conducted three sets of experiments on three 100-customer instances, each using a unique vehicle type, i.e., only light duty 1 (L1), only light duty 2 (L2) and only medium duty (M). This results in three instances of the homogeneous version of the problem which are solved with the P-L-HALNS. Table 6.11 provides the results of this comparison. The columns Dev_{CO_2} and Dev_T show the deviations in CO_2 emissions (in kg) and in total cost between the various homogeneous cases and the heterogeneous case.

Table 6.11 shows that for the CC instances, CO_2 emissions increase by 29.16% when L1 vehicles are used, and decrease by 10.54% and 12.45% when L2 and M vehicles are used, respectively. The results of the SU and R instances yield similar values for CO_2 emissions, which decrease by L1 and L2 vehicles and increase by M type vehicles. Table 6.11 indicates that the average increase in total cost for the CC instances is 17.00%, 7.06% and 1.87%, for the SU instances 5.53%, 4.45% and 4.67%, for the R instances 12.09%, 10.21% and 10.44% when using L1, L2 and M homogeneous fleet over the heterogeneous case, respectively. These results imply that if one is to use a homogeneous fleet, it is preferable to use vehicles of type M in city centres (CC). For the suburban (SU) and randomly distributed customer (R) location scenarios, homogeneous vehicles of types L2 and M yield almost the same average total cost increase. This result shows that both the L2 and M vehicles are suitable for the SU and R instances. Our results also show that using a heterogeneous vehicle fleet is preferable to using a homogeneous one since the total cost decreases by about 17% at most. For urban settings or short-haul transportation, using a heterogeneous fleet does not seem to have same impact on the total cost as in long-haul transportation. Chapter 5 have indeed shown that using a heterogeneous fleet can decrease the total cost by up to 25% in inter-city travel.

Our final experiments aim at providing some insight into the capacity utilization of the vehicle fleet, both for the homogenous and the heterogeneous cases, and also into the capacity utilization of the depots. In Table 6.12, we present the capacity utilizations for

TABLE 6.10: The effect of decreasing the depot costs.

Instance	Change in depot cost (%)	Opened depots	Total distance (km)	CO_2 emissions (kg)	Fuel and CO_2 emissions costs (£)	Depot cost (£)	Vehicle cost (£)	Total cost (£)
CC100	−90%	(3,1,2)	106.74	63.93	38.58	2070.00	358.20	2424.02
SU100	−90%	(0,3,3)	255.71	24.29	14.66	1695.52	372.56	2080.63
R100	−90%	(0,3,2)	250.70	34.63	20.90	1360.00	395.80	1756.22
CC100	−70%	(3,1,2)	155.11	71.33	43.05	6210.00	358.20	6576.01
SU100	−70%	(0,3,3)	268.67	45.66	27.56	5086.55	372.56	5485.30
R100	−70%	(0,3,2)	279.48	25.98	15.68	4080.00	407.26	4479.27
CC100	−50%	(3,1,2)	129.82	66.44	40.09	10350.00	358.20	10722.59
SU100	−50%	(0,3,3)	239.59	44.95	27.12	8477.59	372.56	8876.44
R100	−50%	(0,3,2)	258.55	54.19	32.70	6800.00	395.80	7222.50
CC100	−30%	(3,1,2)	115.79	60.75	36.66	14490.00	358.20	14868.95
SU100	−30%	(0,3,3)	247.62	13.93	8.41	11868.62	372.56	12249.06
R100	−30%	(0,3,2)	253.99	52.10	31.44	9520.00	399.97	9946.26

TABLE 6.11: The effect of using a heterogeneous fleet

Instance	Only light duty 1				Only light duty 2				Only medium duty				Heterogeneous fleet	
	CO_2 (kg)	Total cost (£)	Dev_{CO_2}	Dev_T	CO_2 (kg)	Total cost (£)	Dev_{CO_2}	Dev_T	CO_2 (kg)	Total cost (£)	Dev_{CO_2}	Dev_T	CO_2 (kg)	Total cost (£)
CC100_1	50.12	26286.20	46.55	14.94	28.56	23007.20	−16.48	0.60	29.33	22877.70	−14.22	0.04	34.20	22869.60
CC100_2	51.80	22745.30	41.40	27.32	35.47	21011.40	−3.19	17.62	32.82	19379.80	−10.43	8.48	36.64	17864.10
CC100_3	71.24	26251.50	1.69	17.02	55.17	22508.80	−21.25	0.34	55.14	22441.20	−21.29	0.04	70.06	22433.30
CC100_4	46.78	24784.20	22.27	8.31	39.50	23013.80	3.24	0.57	35.77	22981.60	−6.52	0.43	38.26	22883.10
CC100_5	52.06	22745.40	33.91	17.41	33.04	22509.90	−15.02	16.20	35.08	19441.20	−9.77	0.35	38.88	19372.50
Avg (%)			29.16	17.00			−10.54	7.06			−12.45	1.87		
SU100_1	40.33	17280.30	−2.67	15.91	35.31	17011.30	−14.78	14.10	42.59	16885.70	2.77	13.26	41.44	14909.00
SU100_2	34.32	15234.70	−21.48	1.89	32.66	14960.70	−25.28	0.06	41.37	15445.00	−5.36	3.29	43.71	14952.40
SU100_3	41.72	15739.20	7.96	5.23	35.59	15511.50	−7.90	3.71	43.12	15446.00	11.60	3.27	38.64	14956.30
SU100_4	38.36	17295.10	−14.16	2.42	35.00	17511.10	−21.67	3.70	42.15	17445.40	−5.67	3.31	44.68	16887.00
SU100_5	35.40	15235.40	−14.92	2.19	29.83	15008.00	−28.31	0.66	42.67	14943.30	2.55	0.23	41.61	14909.10
Avg (%)			−9.05	5.53			−19.59	4.45			1.18	4.67		
R100_1	98.27	15237.10	−8.04	1.96	73.68	15010.30	−31.05	0.44	111.96	15145.30	4.77	1.34	106.86	14944.50
R100_2	38.89	15279.50	−10.92	13.94	34.67	15010.90	−20.59	11.94	44.55	15006.90	2.03	11.91	43.66	13410.30
R100_3	40.07	15280.20	−8.00	13.82	31.36	15008.90	−27.99	11.80	43.05	15004.20	−1.15	11.77	43.55	13424.30
R100_4	38.81	17279.40	−8.76	28.86	33.74	17010.40	−20.68	26.85	42.57	16885.70	0.09	25.92	42.53	13409.70
R100_5	36.10	15235.80	−1.57	1.88	32.49	14960.60	−11.41	0.04	40.59	15144.50	10.65	1.27	36.68	14955.10
Avg (%)			−7.46	12.09			−22.34	10.21			3.28	10.44		

the three homogeneous settings of Table 6.11 as well as for the heterogeneous version. The column VCU displays the average percentage capacity utilization of the vehicle fleet, which is calculated as 100 (total demand of route)/(capacity of the vehicle) for each vehicle, and DCU displays the average percentage of capacity utilization for depots, which is calculated as 100 (total demand of customers assigned to corresponding depot)/(capacity of the depot) for each depot.

As can be seen from Table 9, for the CC, SU and R instances the VCU reaches its maximum average level of 91.85%, 93.11% and 92.13% and its minimum average level of 80.42%, 74.94% and 72.91% when using only L1 and M duty vehicles, respectively. Using L1 vehicles yields the maximum average VCU level over all types of instances. Using a heterogeneous fleet yields an average VCU of 90.00%, 85.33% and 88.45% for the CC, SU and R instances, respectively. These results indicate that for a heterogeneous fleet, the best VCU is obtained with L1 vehicles for the CC instances, and with L2 vehicles for the SU or the R instances.

For all instance types and all homogeneous vehicle combinations, the DCU level reaches at least 92.00%, which is very similar to the heterogeneous vehicle fleet level. Our results shows that because of the very high effect of the depot costs in the objective function (see Section 6.5.1), increasing the DCU has more effect than increasing the VCU in urban settings.

6.6 Conclusions and Managerial Insights

We have studied and analyzed the combined impact of depot location, fleet composition and routing on vehicle emissions in urban freight distribution. We have formulated a new problem arising in urban settings and designed a powerful ALNS metaheuristic to solve it. We have derived managerial insights by investigating the effect of various problem components on cost and CO_2 emissions. In what follows we summarize our main conclusions.

Our first observation relates to shortest paths. Because of the effect of speed zones, a shortest path is not always a fastest, cheapest or least polluting path in city logistics since it may be advantageous to follow circuitous routes to achieve faster speeds and hence lower costs and CO_2 emissions. The explanation lies in the fact that emissions are

TABLE 6.12: Capacity utilization rates

Instance	Only light duty 1		Only light duty 2		Only medium duty		Heterogeneous fleet	
	DCU	VCU	DCU	VCU	DCU	VCU	DCU	VCU
CC100_1	91.67	90.40	89.16	86.79	92.98	86.79	89.16	92.98
CC100_2	96.39	90.72	99.50	82.26	93.47	82.26	97.92	92.77
CC100_3	87.92	93.09	97.39	84.40	97.39	72.35	97.39	85.54
CC100_4	88.76	92.46	99.36	88.76	91.19	88.76	91.19	88.76
CC100_5	96.86	92.59	98.38	83.95	96.86	71.96	93.97	89.94
Avg (%)	92.32	91.85	96.76	85.23	94.38	80.42	93.93	90.00
SU100_1	88.22	94.35	88.22	90.57	88.22	90.57	88.22	90.57
SU100_2	93.28	91.90	94.69	92.59	99.20	71.42	99.20	88.65
SU100_3	97.13	89.99	97.13	81.59	97.13	69.94	97.13	78.96
SU100_4	92.07	94.95	98.01	81.02	96.45	69.45	98.01	86.19
SU100_5	95.78	94.37	95.78	85.56	95.78	73.34	97.23	82.27
Avg (%)	93.30	93.11	94.77	86.27	95.36	74.94	95.96	85.33
R100_1	94.37	92.99	97.28	84.31	94.37	72.26	94.37	91.64
R100_2	98.18	91.36	98.18	87.71	98.18	65.78	98.18	88.30
R100_3	98.92	90.68	97.44	87.05	97.44	65.29	97.44	83.70
R100_4	86.76	92.79	86.76	89.08	86.76	89.08	86.76	89.08
R100_5	94.20	92.82	94.20	93.51	94.20	72.13	94.20	89.53
Avg (%)	94.49	92.13	94.77	88.33	94.19	72.91	94.19	88.45

a U-shaped function of speed (Figure 6.1) whose optimal value is reached at 40 km/h since this is the fastest speed used in this study. It is often the maximal allowed speed in city centres. Hence faster driving is clearly cheaper and less polluting in this context. This is consistent with what was observed by Ehmke et al. (2014) for urban areas but different from what occurs in inter-city travel where faster driving entails more pollution which must be weighted against reduced driver wages (Bektaş and Laporte 2011, Demir et al. 2014a).

We have also shown that the highest costs are attained when all customers are located only in the city centre. Our experimental results indicate that even for same depot costs or lower variable depot costs, it is preferable to locate the depots outside the city centre. This decreases the total cost by about 34.65% on average, a finding in line with that of Dablanc (2014) on the Los Angeles data. Furthermore, we have demonstrated that locating depots in the outer areas is also highly beneficial in terms of reducing pollution. Indeed, an average decrease of 13.66% can be achieved by locating depots in the suburbs. These results are remarkably stable over a wide range of fixed depot costs.

We have demonstrated that in an urban setting, using a heterogenous fleet instead of a homogeneous one can decrease average costs by up to 17%, but this is not as much as what was observed by Chapter 5 for long-haul transportation. Furthermore, we have shown that the depot capacity utilization levels tend to be higher than the vehicle capacity utilization levels. This has an important implication since in practice depot costs are often considerably larger than vehicle costs and significantly affect the total distribution cost.

Our results depend of course on the parameter values used in the experimental design but the extensive sensitivity analyses we have carried out convince us that our conclusions are highly robust. Beyond the computational comparisons we have just made, we stress the importance of the availability of a decision support tool, such as the one we have developed, capable of analyzing the trade-offs that can be established between depot location, fleet composition, routing and polluting emissions reductions in urban freight distribution networks. In particular, given the nature of the objective function encompassing different criteria, it would also be possible to treat the problem in a multi-objective optimization framework.

Chapter 7

Conclusions

7.1 Overview

This thesis has studied a number of the heterogeneous location- and pollution-routing problems. Section 7.2 summarizes the overall content and the scientific contributions of each of the five main chapters. Section 7.3 presents an overview of research outputs of the thesis. Sections 7.4 and 7.5 present the limitations of the selected techniques and methodologies, and identify future research directions. The thesis closes with some words of excitement in Section 7.6.

7.2 Summary of the Main Scientific Contributions

This thesis has introduced new classes of heterogeneous vehicle routing problems with or without location and pollution components. It has developed powerful evolutionary and adaptive large neighborhood search based metaheuristics, capable of effectively and efficiently solving a wide variety of complex logistics problems with suitable enhancements. It has also provided several managerial insights.

In Chapter 2, we have classified and reviewed the literature on heterogeneous vehicle routing problems. We have also presented a comparative analysis of the performance of the metaheuristic algorithms proposed for these problems.

In Chapter 3, we have developed a unified heuristic, namely a hybrid evolutionary algorithm (HEA), capable of solving four types of heterogeneous fleet vehicle routing problems with time windows, without any modification and with the same parameter settings. The HEA combines state-of-the-art metaheuristic principles such as population search and heterogeneous adaptive large scale neighborhood search. An innovative Intensification strategy on elite solutions was integrated within the HEA, and a new diversification scheme was developed, based on the Regeneration and the Mutation of solutions. An advanced version of the Split algorithm was also created to determine the best fleet mix for a set of routes. Overall, we have improved 75 solutions out of 360 benchmark instances, and we have matched 102 others in solution quality within modest computational effort.

In Chapter 4, we have introduced the fleet size and mix location-routing problem with time windows. We have proposed several formulations strengthened with valid inequalities, as well as a hybrid evolutionary search algorithm (HESA). This algorithm is a version of the HEA enhanced by several new algorithmic features, such as a location-heterogeneous adaptive large neighborhood search procedure equipped with a range of several new operators as the main EDUCATION procedure within the search. We have also developed an INITIALIZATION procedure to create initial solutions, a PARTITION procedure for offspring solutions, and a new diversification scheme through the MUTATION procedure of solutions. Extensive computational experiments conducted on a new set of benchmark instances of up to 100 customers and 10 potential depots, have indicated that the HESA is able to generate solutions within 0.05% of optimality for small size instances and yields better solutions than an off-the-shelf solver on larger instances, within the same amount of computation time.

In Chapter 5, we have introduced the fleet size and mix pollution-routing problem (FSMPRP), which extends the pollution-routing problem (PRP), to allow for the use of a heterogeneous vehicle fleet. The objective of the FSMPRP is to minimize the sum of vehicle fixed costs and routing cost, where the latter can be defined with respect to the cost of fuel and CO_2 emissions, and driver cost. Using a non-trivial adaptation of the HEA, extensive computational experiments were conducted on realistic PRP and FSMPRP instances. The effects of several algorithmic components and trade-offs between various cost indicators were analysed. Our results demonstrate the benefit of using a heterogeneous fleet over a homogeneous one and indicate that using a heterogeneous fleet without speed optimization allows for a further reduction in total cost than using a homogeneous fleet with speed optimization. Furthermore, applying an adequate fixed speed yields results that are only slightly worse than optimizing the speed on each arc in terms of the objective function.

In Chapter 6, we have studied and analysed the combined impact of depot location, fleet acquisition and routing decisions on vehicle emissions in an urban freight distribution setting featuring multiple nested speed zones. We have formulated a new problem arising in urban settings and we have solved it using a version of the HESA. Our results have led to several interesting managerial insights into the effect of various problem components measures, including fuel and CO_2 emissions, fixed costs of vehicles and depots, fleet composition, depot locations and customer distribution. We have shown that the highest costs are incurred when all customers are located in the city centre. Our experiments suggest that even for depot costs remaining uniform across the multiple zones, it is preferable to locate the depots in the outermost zones, which yields benefits not only in terms of operating cost, but also in terms of pollution. Our results also indicate that using a heterogenous fleet over a homogeneous one in an urban setting can decrease the average costs. Finally, we have shown that depot capacity utilization levels are higher than vehicle capacity utilization levels. This has an important implication since in practice depot costs are considerably larger than vehicle costs and affect the total cost more significantly than the latter.

7.3 Research Outputs

An overview of the research outputs of the thesis is given below.

Five publications:

- Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., 2015. "Thirty Years of Heterogeneous Vehicle Routing", *European Journal of Operational Research*, in press. [Chapter 2].
- Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., 2015. "A Hybrid Evolutionary Algorithm for Heterogeneous Fleet Vehicle Routing Problems with Time Windows", *Computers & Operations Research* 64, 11–27. [Chapter 3].
- Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., 2016. "The Fleet Size and Mix Location-Routing Problem with Time Windows: Formulations and a Heuristic Algorithm", *European Journal of Operational Research* 248, 33–51. [Chapter 4].
- Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., 2014. "The Fleet Size and Mix Pollution-Routing Problem", *Transportation Research Part B* 70, 239–254. [Chapter 5].
- Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., 2015. "The impact of location, fleet composition and routing on emissions in urban freight distribution", under evaluation after a first revision *Transportation Research Part B*. CIRRELT Technical Report 2015–33. [Chapter 6].

Nine conference presentations and seminars:

- Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., "The Fleet Size and Mix Pollution-Routing Problem", 27th European Conference on Operational Research (EURO 2015), 2015, Glasgow.
- Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., "The Fleet Size and Mix Pollution-Routing Problem", 4th INFORMS Transportation Science and Logistics Society Workshop. 2015, Berlin.
- Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., "The Fleet Size and Mix Pollution-Routing Problem", Fourth Annual Conference of the EURO Working Group on Vehicle Routing and Logistics Optimization (VeRoLog 2015), 2015, Vienna.
- Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., "A Hybrid Evolutionary Algorithm for Heterogeneous Fleet Vehicle Routing Problems with Time Windows", Third Annual Conference of the EURO Working Group on Vehicle Routing and Logistics Optimization (VeRoLog 2014), 2014, Oslo.
- Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., "A Hybrid Evolutionary Algorithm for Heterogeneous Fleet Vehicle Routing Problems with Time Windows", Optimization Days, HEC Montréal, 2014. Montréal.
- Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., "A Hybrid Evolutionary Algorithm for Heterogeneous Fleet Vehicle Routing Problems with Time Windows", CIRRELT Seminar, Invited talk, April 22, 2014, Montréal.
- Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., "An Evolutionary Algorithm for the Heterogeneous Fleet Pollution-Routing Problem", Second Annual Conference of the EURO Working Group on Vehicle Routing and Logistics Optimization (VeRoLog 2013), 2013, Southampton.
- Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., "An Evolutionary Algorithm for the Heterogeneous Fleet Pollution-Routing Problem", 26th European Conference on Operational Research (EURO 2013), 2013, Rome.
- Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., "An Evolutionary Algorithm for the Heterogeneous Fleet Pollution-Routing Problem", University of Southampton First Young CORMSIS Conference, 2013, Southampton.

7.4 Limitations of the Research Results

We acknowledge some limitations and shortcomings of this research.

- In the Location-Routing Problem, it is assumed that location and routing decisions made simultaneously and this has been the basis of the models and algorithms developed to solve the problem. However, in practical situations, managers may wish to make such decisions in a hierarchial fashion overal several periods, even though such solutions are likely to be suboptimal.
- Due to the difficulty of the problems considered here, metaheuristic techniques were performed. These powerful metaheuristics are capable of finding effective and robust feasible solutions within relatively short computation times, but optimality is not necessarily guaranteed.
- We have assumed that all parameters are deterministic. This is likely to be the case for location costs and facility capacities, but not for other parameters such as speed limits. This study has not ventured into the exploration of stochastic or dynamic environments.
- We have assumed that time windows are not flexible, and all customers must be served within their respective time windows. We have not investigated situations in which the time windows are flexible. For example, customer service could be allowed to start earlier or later than what the time windows prescribe by using penalties.
- We have assumed that the goods are ready for delivery in depot at the beginning of travel for all the routing decisions. However, in situations where the depot is also a manufacturing facility, not all goods are likely to be ready for delivery when vehicles are dispatched.
- We have assumed that all problems considered here are time-independent, as is the case for the majority of routing problems studied in the vehicle routing literature. However, particularly in city logistics, the departure time of the vehicles from the depot or from the customers may have a significant impact on travel time because of congestion.

7.5 Future Research Directions

In order to address the above limitations, we have identified the following four research of areas as promising:

- **Data:** Instead of using deterministic parameters, stochasticity and dynamism can be taken account in the problem definition, although this would require new models and solution algorithms, such as stochastic optimization (Birge and Louveaux 2011), and dynamic or real-time optimization (Bektaş et al. 2014).
- **Applications:** To validate the effectiveness of the algorithms, a larger and a more varied data set including case studies, should be considered. In particular, very large instances could be solved, at least heuristically.
- **Techniques:** Further studies are needed in order to develop new effective exact methods, such as Lagrangean relaxation to obtain lower bounds, or decomposition techniques to solve large scale instances to optimality. In addition, continuous approximation models could probably be applied in the spirit of Daganzo (1984a,b) and Jabali et al. (2012a).
- **Extensions:** A more realistic version of the considered problems and their variants would be to consider time-dependencies. There also exist numerous research opportunities on the study of rich extensions of heterogeneous vehicle routing problems, such as problems with pickups and deliveries, two-echelon, open routes, periodicities and backhauls, particularly with pollution objectives as was done in this thesis. These extensions are interesting not only because of the methodological challenge they pose, but also from a managerial perspective.

7.6 Excitement!

I am very glad to have conducted my PhD research on this topic. I believe this thesis has greatly contributed to my knowledge and understanding of Heterogeneous Location- and Pollution-Routing Problems. The field is still rich, with many open research questions waiting to be addressed and with benefits to reap. I hope my contributions will help advance knowledge on this important topic and will encourage other researchers to pursue the study of this rich and fascinating field of research.

Appendix A

Supplement to Chapter 2

Tables A.1, A.2 and A.3 provide a computational results on three costs variants of the FSM: the FSM(F,V), the FSM(F) and the FSM(V), respectively. Tables A.4 and A.5 provide statistics relative to recent metaheuristics for two cost variants of the HF: the HF(F,V) and the HF(V). The first column of each table is the instance number. The second column shows the number of customers, and the third column shows the value of the best-known solution (BKS) for each instance, where a boldface entry indicates that the value is optimal. The remaining columns show, for each reference, two performance indicators: the percentage deviation (Dev) from BKS obtained from the articles surveyed, and the computation time in seconds (Time).

TABLE A.1: Comparison of recent metaheuristics on the FSM(F,V)

Problems	$\lvert\mathcal{N}_0\rvert$	BKS	Choi and Tcha (2007)		Prins (2009)		Imran et al. (2009)		Penna et al. (2013)		Vidal et al. (2014)	
			Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)
3	20	**1144.22**	0.00	0.25	0.00	0.01	0.00	19.00	0.00	3.87	0.00	10.20
4	20	**6437.33**	0.00	0.45	0.00	0.07	0.00	17.00	0.00	2.77	0.00	13.80
5	20	**1322.26**	0.00	0.19	0.00	0.02	0.00	24.00	0.00	4.57	0.00	10.20
6	20	**6516.47**	0.00	0.41	0.00	0.07	0.00	21.00	0.00	2.80	0.00	13.80
13	50	**2964.65**	0.00	3.95	0.00	0.32	0.00	328.00	0.00	27.67	0.00	30.60
14	50	**9126.90**	0.00	51.70	0.00	8.90	0.00	250.00	0.00	11.27	0.00	47.40
15	50	**2634.96**	0.00	4.36	0.01	1.04	0.00	275.00	0.00	13.47	0.00	42.60
16	50	**3168.92**	0.00	5.98	0.01	13.05	0.00	313.00	0.00	17.55	0.00	48.00
17	75	**2004.48**	0.95	68.11	0.00	23.92	0.00	641.00	0.00	43.33	0.00	79.80
18	75	**3147.99**	0.00	18.78	0.16	24.85	0.18	835.00	0.05	47.39	0.03	76.80
19	100	**8661.81**	0.03	905.20	0.03	163.25	0.05	1411.00	0.00	60.33	0.00	234.60
20	100	4153.02	0.04	53.02	0.04	41.25	0.28	1460.00	0.00	58.97	0.00	103.80
Average			0.08	92.70	0.02	23.06	0.04	466.17	0.00	24.50	0.00	59.30
Runs			5		1		10		30		10	
Processor			Pentium IV		Pentium IV M		Pentium M		Intel i7		Opt	
CPU			2.6GHz		1.8GHz		1.7GHz		2.93GHz		2.4GHz	

TABLE A.2: Comparison of recent metaheuristics on the FSM(F)

Problems	$\|\mathcal{N}_0\|$	BKS	Choi and Tcha (2007)		Brandão et al. (2009)		Prins (2009)		Imran et al. (2009)		Liu et al. (2009)		Penna et al. (2013)		Vidal et al. (2014)	
			Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)
3	20	**961.03**	0.00	0.00	0.00	21.00	0.00	0.04	0.00	21.00	0.00	0.00	0.00	4.60	0.00	12.00
4	20	**6437.33**	0.00	1.00	0.00	22.00	0.00	0.03	0.00	18.00	0.00	0.00	0.00	3.00	0.00	13.80
5	20	**1007.05**	0.00	1.00	0.00	20.00	0.00	0.09	0.00	13.00	0.00	2.00	0.00	5.53	0.00	13.80
6	20	**6516.47**	0.00	0.00	0.00	25.00	0.00	0.08	0.00	22.00	0.00	0.00	0.00	2.91	0.00	13.80
13	50	**2406.36**	0.00	10.00	0.00	145.00	0.00	17.12	0.00	252.00	0.00	91.00	0.09	30.37	0.00	61.20
14	50	**9119.03**	0.00	51.00	0.00	220.00	0.00	19.66	0.00	274.00	0.00	42.00	0.00	11.45	0.00	52.80
15	50	**2586.37**	0.00	10.00	0.02	110.00	0.00	25.10	0.00	303.00	0.00	48.00	0.00	19.29	0.00	43.80
16	50	**2720.43**	0.00	11.00	0.28	111.00	0.32	16.37	0.00	253.00	0.14	107.00	0.00	19.98	0.00	39.60
17	75	**1734.53**	0.59	207.00	0.09	322.00	0.66	52.22	0.43	745.00	0.00	109.00	0.00	53.70	0.05	105.00
18	75	**2369.65**	0.08	70.00	0.30	267.00	0.00	36.92	0.00	897.00	0.00	197.00	0.08	54.22	0.00	103.80
19	100	**8661.81**	0.03	1179.00	0.06	438.00	0.04	169.93	0.04	1613.00	0.01	778.00	0.01	64.90	0.01	222.00
20	100	4029.74	0.24	264.00	0.45	601.00	0.37	172.73	0.37	1595.00	0.22	1004.00	0.20	94.22	0.12	135.60
Average			0.08	150.33	0.10	191.83	0.12	42.52	0.07	500.50	0.03	198.17	0.03	30.35	0.01	68.10
Runs			5		10		1		10		10		30		10	
Processor			Pentium IV		Pentium M		Pentium IV M		Pentium M		Pentium IV		Intel i7		Opt	
CPU			2.6GHz		1.4GHz		1.8GHz		1.7GHz		3GHz		2.93GHz		2.4GHz	

TABLE A.3: Comparison of recent metaheuristics on the FSM(V)

Problems	$\|\mathcal{N}_0\|$	BKS	Choi and Tcha (2007)		Brandão et al. (2009)		Prins (2009)		Imran et al. (2009)		Liu et al. (2009)		Penna et al. (2013)		Vidal et al. (2014)	
			Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time	Dev (%)	Time (sec.)
3	20	**623.22**	0.00	0.19	–	–	–	–	–	–	–	–	0.00	4.31	0.00	10.20
4	20	**387.18**	0.00	0.44	–	–	–	–	–	–	–	–	0.00	2.59	0.00	11.40
5	20	**742.87**	0.00	0.23	–	–	–	–	–	–	–	–	0.00	5.23	0.00	12.00
6	20	**415.03**	0.00	0.92	–	–	–	–	–	–	–	–	0.00	3.18	0.00	13.20
13	50	**1491.86**	0.00	4.11	0.00	101.00	0.00	3.45	0.00	310.00	0.00	117.00	0.00	30.68	0.00	43.20
14	50	**603.21**	0.00	20.41	0.00	135.00	0.00	0.86	0.00	161.00	0.00	26.00	0.00	13.92	0.00	33.60
15	50	**999.82**	0.00	4.61	0.00	137.00	0.00	9.14	0.00	218.00	0.00	37.00	0.00	14.70	0.00	36.60
16	50	**1131.00**	0.00	3.36	0.00	95.00	0.00	13.00	0.00	239.00	0.00	54.00	0.00	17.25	0.00	34.20
17	75	**1038.60**	0.00	69.38	0.00	312.00	0.00	9.53	0.00	509.00	0.00	153.00	0.00	48.15	0.00	68.40
18	75	**1800.80**	0.03	48.06	0.03	269.00	0.00	18.92	0.00	606.00	0.03	394.00	0.00	52.66	0.03	80.40
19	100	**1105.44**	0.00	182.86	0.00	839.00	0.00	52.31	0.00	1058.00	0.00	479.00	0.00	77.89	0.00	102.60
20	100	**1530.43**	0.00	98.14	0.09	469.00	0.31	104.41	0.18	1147.00	0.26	826.00	0.01	86.66	0.00	168.00
Average			0.00	36.06	0.02	294.63	0.04	26.45	0.02	531.00	0.04	260.75	0.00	29.77	0.00	51.15
Runs			5		10		1		10		10		30		10	
Processor			Pentium IV		Pentium M		Pentium IV M		Pentium M		Pentium IV		Intel i7		Opt	
CPU			2.6GHz		1.4GHz		1.8GHz		1.7GHz		3GHz		2.93GHz		2.4GHz	

TABLE A.4: Comparison of recent metaheuristics on the HF(F,V)

Problems	$\lvert\mathcal{N}_0\rvert$	BKS	Gencer et al. (2006)		Li et al. (2010)		Subramanian et al. (2012)		Liu (2013)		Penna et al. (2013)	
			Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)
13	50	**3185.09**	1.78	–	0.00	92.00	0.00	1.99	0.00	129.88	0.00	18.87
14	50	**10107.53**	0.85	–	0.00	41.00	0.00	1.29	0.00	51.78	0.00	10.58
15	50	**3065.29**	1.75	–	0.00	57.00	0.00	1.77	0.02	87.29	0.00	11.78
16	50	**3265.41**	3.74	–	0.41	83.00	0.00	1.67	0.10	73.85	0.00	11.87
17	75	**2076.96**	4.13	–	0.00	151.00	0.00	5.95	0.00	128.65	0.00	29.44
18	75	**3743.58**	4.32	–	0.00	126.00	0.00	16.47	0.00	115.31	0.00	35.75
19	100	10420.34	9.24	–	0.00	295.00	0.00	15.80	0.00	238.67	0.00	70.55
20	100	4761.26	6.48	–	1.51	209.00	0.00	16.87	1.51	190.96	0.57	66.88
Average			4.63	–	0.21	131.75	0.00	7.73	0.19	127.05	0.07	31.97
Runs			–	–	10		10		10		30	
Processor			–	–	Intel		Intel i7		Intel Pentium IV		Intel i7	
CPU			–	–	2.2GHz		2.93GHz		3GHz		2.93GHz	

Table A.5: Comparison of recent metaheuristics on the HF(V)

Problems	$\lvert\mathcal{N}_0\rvert$	BKS	Taillard (1999)		Tarantilis et al. (2004)		Li et al. (2007)		Prins (2009)		Subramanian et al. (2012)		Liu (2013)		Penna et al. (2013)	
			Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)	Dev (%)	Time (sec.)
13	50	**1517.84**	0.01	473.00	0.14	843.00	0.00	358.00	0.00	33.20	0.00	1.33	0.00	57.42	0.00	19.29
14	50	**607.53**	1.33	575.00	0.64	387.00	0.00	141.00	0.00	37.60	0.00	1.09	0.00	86.98	0.00	11.20
15	50	**1015.29**	0.15	335.00	0.00	368.00	0.00	166.00	0.00	6.60	0.00	2.13	0.00	4.85	0.00	12.56
16	50	**1144.94**	0.80	350.00	0.05	341.00	0.00	188.00	0.00	7.50	0.00	1.41	0.00	13.51	0.00	12.29
17	75	**1061.96**	0.93	2245.00	0.85	363.00	0.00	216.00	0.20	81.50	0.00	4.22	0.00	115.88	0.00	29.92
18	75	**1823.58**	2.55	2876.00	1.25	971.00	0.00	366.00	0.00	190.60	0.00	4.06	0.00	97.98	0.00	38.34
19	100	1117.51	2.55	5833.00	0.57	428.00	0.25	404.00	0.25	177.80	0.25	9.12	0.25	77.21	0.25	67.72
20	100	**1534.17**	1.67	3402.00	0.57	1156.00	0.00	447.00	0.00	223.30	0.00	8.89	0.00	115.49	0.00	63.77
Average			1.25	2011.13	0.51	607.13	0.03	285.75	0.06	94.76	0.03	4.03	0.03	71.17	0.03	31.89
Runs			5		1		1		1		10		10		30	
Processor			Sun Sparc		Pentium II		AMD Athlon		Pentium IV M		Intel i7		Intel Pentium IV		Intel i7	
CPU			50MHz		400MHz		1.0GHz		1.8GHz		2.93GHz		3GHz		2.93GHz	

Appendix B

Supplement to Chapter 3

Table B.1 shows the four versions of the HEA. Table B.2 presents the best results of ten runs for each of four versions. Table B.3 shows the number of times for each removal and insertion operators was called within the HEA. Tables B.4 to B.9 present the detailed results on all benchmark instances for FT and FD.

Table B.1: Sensitivity analyis experiment setup

Version	Education	Intensification	Diversification
1	No	No	No
2	Yes	No	No
3	Yes	Yes	No
HEA	Yes	Yes	Yes

Table B.2: Sensitivity analyis of the HEA components

Instance	Version (1)			Version (2)			Version (3)			HEA	
	TC	Dev	Time	TC	Dev	Time	TC	Dev	Time	TC	Time
C101A	7271.60	0.62	2.79	7259.60	0.46	2.85	7246.51	0.28	2.91	7226.51	2.97
C203A	5795.12	0.93	4.53	5781.12	0.69	4.61	5761.12	0.35	4.69	5741.12	4.76
R101A	4591.80	1.09	4.49	4580.80	0.85	4.58	4571.70	0.66	5.09	4541.70	5.26
R211A	3061.11	1.32	7.72	3050.11	0.97	7.81	3040.56	0.66	7.87	3020.56	7.99
RC105A	5146.19	0.55	5.14	5136.19	0.35	5.21	5128.19	0.20	5.27	5118.10	5.32
RC207A	4362.19	1.08	5.59	4351.19	0.83	6.11	4335.19	0.46	6.21	4315.19	6.27
Avg		0.93	5.04		0.69	5.20		0.43	5.34		5.43

Table B.3: Number of iterations as a percentage by education operators

Instance	Removal operators									Insertion operators	
	RR	WDR	WTR	NR	SR	PBR	TBR	DR	ACUTR	GI	GINF
C101A	6.30	13.70	12.80	14.30	14.20	7.20	8.60	7.20	15.70	65.20	34.80
C203A	6.10	13.90	12.40	14.50	14.00	7.90	9.00	7.00	15.20	60.80	39.20
R101A	6.90	12.50	13.60	14.80	13.90	7.90	8.00	7.00	15.40	71.20	28.80
R211A	6.30	13.50	12.80	14.50	14.20	6.70	8.60	7.90	15.50	67.80	32.20
RC105A	5.30	14.70	11.80	13.50	15.20	8.90	7.70	7.60	15.30	69.10	30.90
RC207A	7.40	12.60	14.50	13.60	12.90	8.20	7.10	7.90	15.80	66.40	33.60
Avg	6.38	13.48	12.98	14.20	14.07	7.80	8.17	7.43	15.48	66.75	33.25

TABLE B.4: Results for FT for cost structure A

Instance set	ReVNTS		MDA		AMP		UHGS		HEA				
	TC	Dev	TC	Dev	TC	Dev	TC	Dev	DC	VC	Mix	TC	Time
R101A	4539.99	0.04	4631.31	−1.97	**4536.4**	0.12	4608.62	−1.50	1951.70	2590	$A^1B^2C^{17}$	4541.70	5.26
R102A	4375.70	−0.47	4401.31	−1.06	**4348.92**	0.14	4369.74	−0.30	1775.10	2580	B^6C^{15}	4355.10	5.87
R103A	4120.63	0.26	4182.16	−1.23	**4119.04**	0.30	4145.68	−0.30	1551.23	2580	B^6C^{15}	4131.23	4.19
R104A	3992.65	−0.01	3981.28	0.27	**3986.35**	0.14	3961.39	0.77	1302.10	2690	$B^5C^{11}D^3$	3992.10	5.02
R105A	4229.69	0.07	4236.84	−0.10	4229.67	0.07	**4209.84**	0.54	1672.54	2560	B^4C^{16}	4232.54	4.73
R106A	4137.96	0.01	4118.48	0.48	4130.82	0.18	**4109.08**	0.71	1538.30	2600	B^1C^{18}	4138.30	5.13
R107A	4061.10	−0.66	4035.96	−0.04	4031.16	0.08	**4007.87**	0.66	1474.32	2560	B^4C^{16}	4034.32	5.4
R108A	3986.07	−0.50	3970.26	−0.10	3962.2	0.10	**3934.48**	0.80	1406.10	2560	B^4C^{16}	3966.10	4.78
R109A	4086.72	−0.68	4060.17	−0.03	4052.21	0.17	**4020.75**	0.94	1429.02	2630	$C^{17}D^1$	4059.02	4.6
R110A	4030.85	−0.86	3995.18	0.03	3999.09	−0.07	**3965.88**	0.76	1436.31	2560	B^4C^16	3996.31	4.17
R111A	4018.80	0.03	4017.81	0.06	4016.19	0.10	**3985.68**	0.86	1460.10	2560	$B^4C^{13}D^2$	4020.10	4.98
R112A	3961.63	−0.10	3947.30	0.26	3954.65	0.07	**3918.88**	0.98	1397.60	2560	B^4C^{16}	3957.60	5.78
C101A	**7226.51**	0.00	**7226.51**	0.00	**7226.51**	0.00	**7226.51**	0.00	1526.51	5700	A^{19}	**7226.51**	2.97
C102A	7137.79	0.11	**7119.35**	0.37	7137.79	0.11	**7119.35**	0.37	1445.65	5700	A^{19}	7145.65	3.10
C103A	7143.88	0.00	7107.01	0.52	7141.03	0.04	**7102.86**	0.57	1443.88	5700	A^{19}	7143.88	2.70
C104A	7104.96	−0.31	**7081.50**	0.02	7086.70	−0.05	7081.51	0.02	1382.92	5700	A^{19}	7082.92	2.01
C105A	7171.48	0.05	7199.36	−0.34	**7169.08**	0.08	7196.06	−0.3	1475.00	5700	A^{19}	7175.00	2.45
C106A	**7157.13**	0.09	7180.03	−0.23	**7157.13**	0.09	7176.68	−0.20	1463.32	5700	A^{19}	7163.32	3.01
C107A	7135.43	0.07	7149.17	−0.13	**7135.38**	0.07	7144.49	−0.10	1440.20	5700	A^{19}	7140.20	2.78
C108A	7115.71	0.07	7115.81	0.07	7113.57	0.10	**7111.23**	0.14	1420.98	5700	A^{19}	7120.98	2.45
C109A	7095.55	−0.05	7094.65	−0.04	7092.49	−0.01	**7091.66**	0.00	1391.66	5700	A^{19}	**7091.66**	2.37
RC101A	5253.86	−0.35	5253.97	−0.35	5237.19	−0.03	**5217.90**	0.33	1815.42	3420	$A^2B^8C^7$	5235.42	4.97
RC102A	5053.48	−0.47	5059.58	−0.59	5053.62	−0.48	**5018.47**	0.22	1639.69	3390	$A^4B^3C^9$	5029.69	5.64
RC103A	4892.80	−0.47	4868.94	0.02	4885.58	−0.32	**4822.21**	0.98	1480.00	3390	$A^4B^3C^9$	4870.00	5.14
RC104A	4783.31	−0.29	4762.85	0.14	4761.28	0.17	**4737.00**	0.68	1289.30	3480	$A^3B^1C^9D^1$	4769.30	4.97
RC105A	5112.91	0.10	5119.80	−0.03	5110.86	0.14	**5097.35**	0.41	1788.10	3330	$A^3B^{11}C^5$	5118.10	5.32
RC106A	4997.98	−0.79	4960.78	−0.04	4966.27	−0.15	**4935.91**	0.46	1568.62	3390	$A^4B^9C^6$	4958.62	6.01
RC107A	4862.67	−0.78	4828.17	−0.06	4819.91	0.11	**4783.08**	0.87	1405.21	3420	$A4B7C7$	4825.21	5.37
RC108A	4736.50	0.38	4734.15	0.43	4749.44	0.11	**4708.85**	0.97	1244.77	3510	$A^1B^2C^9D^1$	4754.77	4.71
R201A	3779.12	−0.50	3922.00	−4.3	**3753.42**	0.19	3782.88	−0.6	1510.43	2250	A^5	3760.43	8.97
R202A	3578.91	−0.70	3610.38	−1.58	3551.12	0.09	**3540.03**	0.40	1304.20	2250	A^5	3554.20	9.98
R203A	3334.08	−0.56	3350.18	−1.05	3336.60	−0.64	**3311.35**	0.13	1065.50	2250	A^5	3315.50	8.76
R204A	3143.68	−2.20	3390.14	−10.20	3103.84	−0.91	**3075.95**	0.00	825.95	2250	A^5	**3075.95**	7.98
R205A	3371.47	−1.12	3465.81	−3.95	3367.90	−1.01	**3334.27**	0.00	1084.27	2250	A^5	**3334.27**	8.45
R206A	3272.79	−0.29	3268.36	−0.15	3264.70	−0.04	**3242.40**	0.64	1013.40	2250	A^5	3263.40	8.17
R207A	3213.60	−1.94	3231.26	−2.51	3158.69	−0.20	**3145.08**	0.23	902.29	2250	A^5	3152.29	9.29
R208A	3064.76	−1.58	3063.10	−1.52	3056.45	−1.30	3017.52	−0.01	767.12	2250	A^5	**3017.12***	8.51
R209A	3191.63	0.08	3192.95	0.04	3194.74	−0.01	**3183.36**	0.34	944.28	2250	A^5	3194.28	9.37
R210A	3338.75	−0.89	3375.38	−2.00	3325.28	−0.48	**3287.66**	0.65	1059.26	2250	A^5	3309.26	8.79
R211A	3061.47	−1.35	3042.48	−0.73	3053.08	−1.08	**3019.93**	0.02	770.56	2250	A^5	3020.56	7.99
C201A	**5820.78**	0.16	5891.45	−1.05	**5820.78**	0.16	5878.54	−0.80	830.20	5000	A^5	5830.20	5.00
C202A	5779.59	−0.05	5850.26	−1.27	5783.76	−0.12	**5776.88**	0.00	776.88	5000	A^5	**5776.88**	5.17
C203A	5750.58	−0.15	5741.90	−0.00	5736.94	0.09	**5741.12**	0.00	741.89	5000	A^5	**5741.12**	4.76
C204A	5721.09	−0.72	5691.51	−0.19	5718.49	−0.67	**5680.46**	0.00	680.46	5000	A^5	**5680.46**	4.21
C205A	5750.53	0.02	5786.71	−0.61	**5747.67**	0.06	5781.15	−0.50	751.40	5000	A^5	5751.40	6.79
C206A	5757.93	−0.29	5795.15	−0.94	**5738.09**	0.06	5767.70	−0.50	741.30	5000	A^5	5741.30	4.3
C207A	5723.91	0.02	5743.52	−0.32	**5721.16**	0.07	5731.44	−0.10	725.10	5000	A^5	5725.10	4.17
C208A	5767.78	−0.75	5884.20	−2.78	5732.95	−0.14	**5725.03**	0.00	725.03	5000	A^5	**5725.03**	5.21
RC201A	4726.22	−0.39	4740.21	−0.69	**4701.88**	0.13	4737.59	−0.60	2007.80	2700	A^{18}	4707.80	4.50
RC202A	4518.49	0.02	4522.36	−0.07	4509.11	0.23	**4487.48**	0.71	1619.40	2900	$A^{10}B^4$	4519.40	4.67
RC203A	4327.57	−0.20	4312.52	0.15	4313.42	0.13	**4305.49**	0.32	1469.10	2850	$A^{12}B^3$	4319.10	5.27
RC204A	4166.73	−0.26	4141.04	0.35	4157.32	−0.04	**4137.93**	0.43	1005.77	3150	$A^2B^5C^2$	4155.77	5.19
RC205A	4645.41	−1.08	4652.57	−1.24	4585.20	0.23	**4615.04**	−0.40	1795.67	2800	$A^{14}B^2$	4595.67	6.89
RC206A	4416.41	0.40	4431.64	0.06	4427.73	0.15	**4405.16**	0.66	1584.30	2850	$A^9B^3C^1$	4434.30	5.03
RC207A	4338.94	−0.53	4310.11	0.13	4313.07	0.07	**4290.14**	0.60	1215.90	3100	A^4B^7	4315.90	6.27
RC208A	4109.90	−0.70	4091.92	−0.26	4103.31	−0.54	**4075.04**	0.16	1031.37	3050	$A^5B^5C^1$	4081.37	5.17

Table B.5: Results for FT for cost structure B

Instance set	ReVNTS		MDA		AMP		UHGS		HEA				
	TC	Dev	TC	Dev	TC	Dev	TC	Dev	DC	VC	Mix	TC	Time
R101B	**2421.19**	0.16	2486.76	−2.54	**2421.19**	0.16	**2421.19**	0.16	1849.10	576	$A^1B^4C^9D^5$	2425.10	3.78
R102B	2219.03	−0.30	2227.48	−0.68	**2209.50**	0.13	**2209.50**	0.13	1608.37	604	$A^2B^1C^6D^8$	2212.37	3.97
R103B	1955.57	−0.18	**1938.93**	0.67	1953.50	−0.08	**1938.93**	0.67	1313.99	638	$A^1B^1C^4D^6E^2$	1951.99	4.28
R104B	1732.26	−1.01	1714.73	0.01	**1713.36**	0.09	**1713.36**	0.09	1026.86	688	$A^1C^1D^5E^4$	1714.86	4.01
R105B	2030.83	−0.29	2027.98	−0.15	2030.83	−0.29	2027.98	−0.15	1436.91	588	$B^3C^5D^8$	**2024.91***	3.68
R106B	1924.03	−0.1	1919.03	0.16	**1919.02**	0.16	**1919.02**	0.16	1338.10	584	$B^1C^6D^8$	1922.10	4.19
R107B	1781.01	0.12	1789.58	−0.36	**1780.52**	0.15	**1780.52**	0.15	1127.20	656	$C^2D^8E^2$	1783.20	5.30
R108B	1667.51	−0.36	1649.24	0.74	1665.78	−0.25	**1649.24**	0.74	983.58	678	$C^1D^5E^4$	1661.58	4.78
R109B	1844.99	−0.87	**1828.63**	0.03	1840.54	−0.63	**1828.63**	0.03	1185.10	644	$B^1C^1D^{10}E^1$	1829.10	4.91
R110B	1792.75	−0.78	**1774.46**	0.24	1788.18	−0.53	**1774.46**	0.24	1178.80	600	$B^1C^3D^{10}$	1778.80	5.21
R111B	1780.03	−0.27	**1769.71**	0.31	1772.51	0.15	**1769.71**	0.31	1141.24	634	$C^3D^7E^2$	1775.24	4.78
R112B	1677.13	−0.01	**1669.78**	0.43	1667.00	0.60	1667.00	0.60	1071.00	606	C^2D^{11}	1677.00	6.21
C101B	**2417.52**	0.00	**2417.52**	0.00	**2417.52**	0.00	**2417.52**	0.00	977.52	1440	A^8B^6	**2417.52**	1.99
C102B	**2350.54**	0.00	**2350.54**	0.00	**2350.54**	0.00	**2350.54**	0.00	930.54	1420	A^5B^7	**2350.54**	2.45
C103B	2349.42	−0.18	2353.64	−0.36	2347.99	−0.11	2347.99	−0.11	925.31	1420	A^5B^7	**2345.31***	3.47
C104B	2332.94	−0.10	2328.62	0.08	**2325.78**	0.21	**2325.78**	0.21	950.59	1380	A^7B^6	2330.59	3.09
C105B	2374.01	0.10	**2373.53**	0.12	2375.04	0.06	**2373.53**	0.12	956.45	1420	A^5B^7	2376.45	3.06
C106B	2381.14	0.22	2404.56	−0.76	**2381.14**	0.22	**2381.14**	0.22	966.43	1420	A^5B^7	2386.43	2.95
C107B	2357.52	0.06	2370	−0.47	2357.67	0.06	**2357.52**	0.06	939.00	1420	A^5B^7	2359.00	2.45
C108B	**2346.38**	0.08	**2346.38**	0.08	**2346.38**	0.08	**2346.38**	0.08	968.15	1380	A^7B^6	2348.15	2.79
C109B	2346.58	−0.38	2339.89	−0.10	**2336.29**	0.06	**2336.29**	0.06	957.6	1380	A^7B^6	2337.60	2.56
RC101B	2469.50	−0.22	**2462.60**	0.06	2464.66	−0.02	**2462.60**	0.06	1732.19	732	$A^1B^4C^{10}$	2464.19	4.47
RC102B	2277.79	−0.32	**2263.45**	0.31	2272.68	−0.10	**2263.45**	0.31	1538.43	732	$A^1B^3C^9D^1$	2270.43	4.12
RC103B	2057.55	−0.80	**2035.62**	0.27	2041.24	−0.00	**2035.62**	0.27	1291.20	750	$B^1C^9D^2$	2041.20	3.98
RC104B	1914.93	0.38	**1905.06**	0.90	1916.85	0.28	**1905.06**	0.90	1172.27	750	$B^1C^6D^4$	1922.27	4.21
RC105B	2337.93	−0.44	**2308.59**	0.82	2325.99	0.07	**2308.59**	0.82	1625.70	702	$A^1B^7C^8$	2327.70	4.56
RC106B	2168.44	−0.99	**2149.56**	−0.11	2160.45	−0.62	2149.56	−0.11	1415.14	732	$A^1B^2C^8D^2$	**2147.14***	4.21
RC107B	2008.39	−0.62	**2000.77**	−0.23	2003.26	−0.36	2000.77	−0.23	1264.09	732	$A^1B^2C^5D^4$	**1996.09***	4.19
RC108B	1906.69	0.12	1910.83	−0.10	1908.72	0.01	**1906.69**	0.12	1176.89	732	$A^1B^1C^7D^3$	1908.89	3.11
R201B	1965.10	−0.45	2002.53	−2.37	**1953.42**	0.14	**1953.42**	0.14	1456.21	500	A^4B^1	1956.21	6.21
R202B	1765.09	−0.72	1790.38	−2.17	**1751.12**	0.07	**1751.12**	0.07	1302.4	450	A^5	1752.40	8.00
R203B	1535.08	−1.31	1541.19	−1.72	1536.60	−1.41	1535.08	−1.31	1065.17	450	A^5	**1515.17***	5.78
R204B	1306.72	−2.12	1284.33	−0.37	1303.84	−1.90	1284.33	−0.37	829.57	450	A^5	**1279.57***	6.89
R205B	1575.75	−1.70	1563.62	−0.92	1560.07	−0.69	1560.07	−0.69	1099.39	450	A^5	**1549.39***	6.49
R206B	1477.34	−1.86	1464.53	−0.98	1464.70	−0.99	1464.53	−0.98	1000.37	450	A^5	**1450.37***	5.21
R207B	1386.84	−2.04	1380.41	−1.56	**1358.69**	0.04	**1358.69**	0.04	909.18	450	A^5	1359.18	6.31
R208B	1261.09	−3.34	1244.74	−2.00	1256.45	−2.96	1244.74	−2.00	770.36	450	A^5	**1220.36***	5.47
R209B	1418.51	−2.37	1431.37	−3.30	1394.74	−0.66	1394.74	−0.66	935.65	450	A^5	**1385.65***	7.14
R210B	1529.04	−2.23	1516.66	−1.40	1525.28	−1.97	1516.66	−1.40	1045.75	450	A^5	**1495.75***	6.93
R211B	1268.14	−3.95	1255.06	−2.88	1253.08	−2.72	**1219.93**	0.00	770.56	450	A^5	**1219.93**	7.45
C201B	**1816.14**	0.25	1820.64	0.00	**1816.14**	0.25	1820.64	0.00	740.64	1080	A^4B^1	1820.64	3.11
C202B	**1768.51**	0.09	1795.40	−1.43	**1768.51**	0.09	**1768.51**	0.09	690.10	1080	$A^2B^1C^1$	1770.10	4.58
C203B	1744.28	−0.61	1733.63	0.00	1734.82	−0.07	**1733.63**	0.00	653.63	1080	$A^2B^1C^1$	**1733.63**	3.19
C204B	1736.09	−3.31	1708.69	−1.68	1716.18	−2.13	**1680.46**	0.00	680.46	1000	A^5	**1680.46**	3.17
C205B	1747.68	0.50	1782.74	−1.49	**1747.68**	0.50	1778.30	−1.24	716.54	1040	A^1B^3	1756.54	5.21
C206B	1756.93	0.92	1772.87	0.02	**1756.01**	0.97	1767.70	0.31	733.17	1040	A^1B^3	1773.17	3.46
C207B	1732.20	−0.16	1729.49	−0.01	1729.39	−0.00	1729.49	−0.01	689.39	1040	A^1B^3	**1729.39***	2.97
C208B	1730.72	−0.38	1724.2	0.00	**1723.2**	0.06	1724.20	0.00	684.20	1040	A^1B^3	1724.20	3.13
RC201B	2231.69	0.19	2343.79	−4.83	2230.54	0.24	**2329.59**	−4.19	1615.90	620	$A^4B^4C^2$	2235.90	4.17
RC202B	2002.62	0.96	2091.53	−3.44	2022.54	−0.03	2057.66	−1.76	1392.00	630	$A^3B^3C^3$	**2022.00***	5.47
RC203B	1843.72	−0.18	1852.74	−0.67	1841.26	−0.05	**1824.54**	0.86	1190.40	650	B^3C^4	1840.40	5.12
RC204B	1611.28	−3.57	1565.31	−0.62	1575.18	−1.25	1555.75	−0.01	885.74	670	$B^1C^4D^1$	**1555.74***	4.98
RC205B	2195.62	−1.23	2195.75	−1.23	**2166.62**	0.11	2174.74	−0.26	1529.00	640	$A^2B^2C^4$	2169.00	6.47
RC206B	1887.23	0.60	1923.56	−1.31	1893.13	0.29	**1883.08**	0.82	1218.70	680	$B^5C^1D^1$	1898.70	4.14
RC207B	1780.72	−2.93	1745.85	−0.92	1743.23	−0.76	**1714.14**	0.92	1080.00	650	B^3C^4	1730.00	5.14
RC208B	1557.74	−4.50	1488.19	0.16	1526.78	−2.42	**1483.20**	0.50	830.64	660	C^6	1490.64	4.43

Table B.6: Results for FT for cost structure C

Instance set	ReVNTS		MDA		AMP		UHGS		HEA				
	TC	Dev	TC	Dev	TC	Dev	TC	Dev	DC	VC	Mix	TC	Time
R101C	**2134.90**	0.11	2199.78	−2.93	**2134.90**	0.11	2199.79	−2.93	1840.20	297	$A^1B^2C^9D^6$	2137.20	3.14
R102C	**1913.37**	0.08	1925.55	−0.56	**1913.37**	0.08	1925.56	−0.56	1599.87	315	$A^2B^3C^4D^7E^1$	1914.87	6.21
R103C	1633.62	−0.77	**1609.94**	0.69	1631.47	−0.63	1615.38	0.36	1310.20	311	$A^1C^4D^8E^1$	1621.20	3.24
R104C	1382.82	−0.52	1370.84	0.35	1377.81	−0.16	**1363.26**	0.90	1025.60	350	D^8E^3	1375.60	4.47
R105C	1729.57	−0.44	**1722.05**	0.00	1729.57	−0.44	**1722.05**	0.00	1403.05	319	$B^2C^2D^{11}$	**1722.05**	3.17
R106C	1607.96	0.15	1602.87	0.47	1607.96	0.15	**1599.04**	0.71	1285.40	325	$A^1C^5D^6E^2$	1610.40	4.08
R107C	1455.09	−0.05	1456.02	−0.12	1452.52	0.12	**1442.97**	0.78	1126.30	328	$C^2D^8E^2$	1454.30	3.51
R108C	1331.54	−0.12	1336.28	−0.48	1330.28	−0.03	**1321.68**	0.62	979.92	350	D^6E^4	1329.92	5.33
R109C	1525.65	−1.23	1507.77	−0.04	1519.37	−0.81	**1505.59**	0.10	1185.10	322	$B^1C^1D^{10}E^1$	1507.10	4.73
R110C	1463.91	−0.89	1446.41	0.32	1457.43	−0.44	**1443.92**	0.49	1109.06	342	$C^3D^4E^4$	1451.06	5.46
R111C	1451.92	−1.09	1447.88	−0.80	1443.34	−0.49	**1423.47**	0.89	1098.32	338	$B^1D^9E^2$	1436.32	6.14
R112C	1355.78	−1.09	1335.41	0.42	1339.44	0.12	**1329.07**	0.90	988.10	353	$C^2D^5E^4$	1341.10	4.17
C101C	1628.94	0.00	**1628.31**	0.04	1628.94	0.00	1628.94	0.00	828.94	800	B^{10}	1628.94	1.97
C102C	**1610.96**	0.00	**1610.96**	0.00	**1610.96**	0.00	**1610.96**	0.00	860.96	750	A^1B^9	**1610.96**	2.53
C103C	1611.14	−0.25	1619.68	−0.78	**1607.14**	0.00	**1607.14**	0.00	857.14	750	A^1B^9	**1607.14**	3.79
C104C	1610.07	−0.68	1613.96	−0.92	**1598.50**	0.04	1599.90	−0.04	869.21	730	A^3B^8	1599.21	2.89
C105C	1628.94	0.00	**1628.38**	0.03	1628.94	0.00	1628.94	0.00	828.94	800	B^{10}	1628.94	1.97
C106C	**1628.94**	0.00	**1628.94**	0.00	**1628.94**	0.00	**1628.94**	0.00	828.94	800	B^{10}	**1628.94**	2.01
C107C	1628.94	0.00	**1628.38**	0.03	1628.94	0.00	1628.94	0.00	828.94	800	B^{10}	1628.94	1.99
C108C	**1622.89**	0.13	**1622.89**	0.13	**1622.89**	0.13	**1622.89**	0.13	825	800	B^{10}	1625.00	2.45
C109C	1619.02	−0.03	**1614.99**	0.22	**1614.99**	0.22	1615.93	0.17	888.61	730	A^3B^8	1618.61	3.54
RC101C	2089.37	0.13	2084.48	0.36	2089.37	0.13	**2082.95**	0.44	1702.10	390	$B^7C^5D^3$	2092.10	4.54
RC102C	1918.96	−0.90	1895.92	0.31	1906.68	−0.25	**1895.05**	0.36	1529.89	372	$A^2B^2C^8D^2$	1901.89	4.19
RC103C	1674.50	−0.83	1660.62	0.00	1666.24	−0.33	**1650.30**	0.63	1300.7	360	C^{12}	1660.70	3.56
RC104C	1543.55	−0.19	1537.09	0.23	1540.13	0.03	**1526.04**	0.95	1159.60	381	$A^1C^5D^5$	1540.60	3.47
RC105C	1972.57	−0.84	1957.52	−0.07	**1953.99**	0.11	1957.14	−0.05	1584.09	372	$A^2B^2C^8D^2$	1956.09	4.16
RC106C	1793.12	−0.71	1776.08	0.25	1787.69	−0.41	**1774.94**	0.31	1393.45	387	$A^2B^1C^6D^4$	1780.45	3.49
RC107C	1635.65	−0.95	1614.04	0.39	1622.90	−0.16	**1607.11**	0.81	1245.30	375	$B^3C^5D^4$	1620.30	3.07
RC108C	1531.69	0.06	1535.14	−0.17	1531.69	0.06	**1523.96**	0.56	1157.60	375	$B^2C^6D^4$	1532.60	3.56
R201C	1745.39	−0.82	1729.92	0.07	1728.42	0.16	**1716.02**	0.88	1461.20	270	A^6	1731.20	6.78
R202C	1537.33	−0.50	1537.35	−0.50	1527.92	0.12	**1515.96**	0.90	1304.70	225	A^5	1529.70	8.14
R203C	1338.42	−3.22	1308.70	−0.92	1311.60	−1.15	**1286.35**	0.80	1071.72	225	A^5	1296.72	6.50
R204C	1080.66	−2.64	1062.46	−0.91	1085.71	−3.12	**1050.95**	0.19	802.90	250	A^5	1052.90	7.89
R205C	1350.12	−2.66	1311.84	0.26	1335.07	−1.51	**1309.27**	0.45	1090.20	225	A^5	1315.20	6.71
R206C	1254.67	−2.26	1251.51	−2.00	1239.70	−1.04	**1216.35**	0.86	1001.93	225	A^5	1226.93	6.59
R207C	1186.05	−5.38	1149.23	−2.11	1139.61	−1.25	**1120.08**	0.48	900.50	225	A^5	1125.50	6.98
R208C	1022.31	−2.44	1009.26	−1.13	1022.11	−2.42	**992.12**	0.59	772.97	225	A^5	997.97	5.87
R209C	1233.07	−5.91	1178.45	−1.21	1171.41	−0.61	**1155.79**	0.73	939.31	225	A^4B^1	1164.31	7.14
R210C	1284.72	−1.18	1289.35	−1.55	1281.08	−0.90	**1257.89**	0.93	1019.70	250	A^4B^1	1269.70	6.14
R211C	1061.70	−6.64	1013.84	−1.83	1028.08	−3.26	**994.93**	0.07	770.58	225	A^5	995.58	6.17
C201C	1269.41	−1.47	1269.41	−1.47	1269.41	−1.47	1269.41	−1.47	650.97	600	A^2C^2	**1250.97***	2.97
C202C	1252.24	−0.92	1242.66	−0.15	1244.54	−0.30	**1239.54**	0.11	700.86	540	$A^2B^1C^1$	1240.86	3.54
C203C	1228.13	−2.89	**1193.63**	0.00	1203.42	−0.82	**1193.63**	0.00	653.63	540	$A^2B^1C^1$	**1193.63**	3.14
C204C	1207.03	−2.59	**1176.52**	0.00	1188.18	−0.99	**1176.52**	0.00	636.52	540	$A^2B^1C^1$	**1176.52**	3.67
C205C	1245.51	−0.44	1245.62	−0.45	1239.60	0.04	**1238.30**	0.15	640.1	600	A^2B^2	1240.10	4.29
C206C	1229.63	−0.03	1245.05	−1.29	**1229.23**	0.00	1238.30	−0.74	629.23	600	A^2C^2	**1229.23**	4.38
C207C	1221.16	−0.97	1215.42	−0.49	1213.07	−0.30	1209.49	−0.01	689.48	520	$A^2B^1C^1$	**1209.48***	3.56
C208C	1210.72	−0.54	**1204.20**	0.00	1205.18	−0.08	**1204.20**	0.00	684.2	520	A^1B^3	**1204.20**	3.01
RC201C	1957.60	−2.07	2004.53	−4.52	**1915.42**	0.13	1996.79	−4.11	1577.90	340	$A^3B^3C^2D^1$	1917.90	4.65
RC202C	1699.48	−1.16	1766.52	−5.15	**1677.62**	0.14	1732.66	−3.13	1355.00	325	$A^1B^5C^1D^1$	1680.00	6.10
RC203C	1510.13	−0.66	1517.98	−1.19	1504.35	−0.28	**1496.11**	0.27	1160.20	340	$A^2B^1C^3E^1$	1500.20	6.27
RC204C	1256.91	−2.84	1238.66	−1.35	1241.45	−1.58	**1220.75**	0.12	887.16	335	$B^1C^4E^1$	1222.16	5.47
RC205C	1901.71	−4.32	1854.22	−1.71	**1822.07**	0.05	1844.74	−1.19	1453	370	$B^2C^4D^1$	1823.00	5.29
RC206C	1598.84	−2.21	1590.22	−1.66	1586.61	−1.43	**1553.65**	0.68	1224.3	340	$B^5C^1E^1$	1564.30	4.70
RC207C	1431.65	−3.61	1396.16	−1.05	1406.26	−1.78	**1377.52**	0.30	1026.71	355	$C^3D^1E^1$	1381.71	5.67
RC208C	1181.47	−2.61	1145.84	0.48	1175.23	−2.07	**1140.10**	0.98	821.40	330	C^6	1151.40	5.17

Table B.7: Results for FD for cost structure A

Instance set	MDA		BPDRT		UHGS		HEA				
	TC	Dev	TC	Dev	TC	Dev	DC	VC	Mix	TC	Time
R101A	4349.80	−0.75	4342.72	−0.58	**4314.36**	0.07	1787.52	2530	$A^1B^{10}C^{12}$	4317.52	4.14
R102A	4196.46	−0.54	4189.21	−0.37	**4166.28**	0.18	1623.84	2550	$A^1B^5C^{15}$	4173.84	5.98
R103A	4052.85	−0.53	4051.62	−0.50	**4027.36**	0.10	1401.40	2630	B^1C^{18}	4031.40	5.21
R104A	3978.48	−0.81	3972.65	−0.66	**3936.40**	0.25	1276.44	2670	$B^3C^{15}D^1$	3946.44	4.12
R105A	4161.72	−0.67	4152.50	−0.45	**4122.50**	0.28	1574.06	2560	$A^1B^5C^{15}$	4134.06	6.01
R106A	4095.20	−0.87	4085.30	−0.62	**4048.59**	0.28	1500.05	2560	B^4C^{16}	4060.05	5.12
R107A	4006.61	−0.54	3996.74	−0.29	**3970.51**	0.37	1395.12	2590	$B^3C^{15}D^1$	3985.12	4.78
R108A	3961.38	−0.73	3949.50	−0.43	**3928.12**	0.11	1342.60	2590	$B^3C^{15}D^1$	3932.60	6.54
R109A	4048.29	−0.58	4035.89	−0.27	**4015.71**	0.23	1464.83	2560	B^4C^{16}	4024.83	6.12
R110A	3997.88	−0.61	3991.63	−0.46	**3961.68**	0.30	1373.51	2600	B^1C^{18}	3973.51	5.21
R111A	4011.63	−0.59	4009.61	−0.54	**3964.99**	0.58	1368.00	2620	$B^3C^{15}D^1$	3988.00	5.12
R112A	3962.73	−0.83	3954.19	−0.61	**3918.88**	0.29	1300.19	2630	$C^{17}D^1$	3930.19	4.71
C101A	7098.04	−0.06	7097.93	−0.06	**7093.45**	0.00	1393.45	5700	A^{19}	**7093.45**	2.47
C102A	7086.11	−0.08	7085.47	−0.07	**7080.17**	0.00	1380.17	5700	A^{19}	**7080.17**	2.65
C103A	7080.35	−0.02	7080.41	−0.02	**7079.21**	0.00	1379.21	5700	A^{19}	**7079.21**	2.01
C104A	7076.90	−0.03	**7075.06**	0.00	**7075.06**	0.00	1375.06	5700	A^{19}	**7075.06**	1.97
C105A	7096.19	−0.04	7096.22	−0.04	**7093.45**	0.00	1393.45	5700	A^{19}	**7093.45**	2.65
C106A	7086.91	−0.04	7088.35	−0.06	**7083.87**	0.00	1383.87	5700	A^{19}	**7083.87**	2.17
C107A	7084.92	−0.00	7090.91	−0.09	**7084.61**	0.00	1384.61	5700	A^{19}	**7084.61**	2.39
C108A	7082.49	−0.04	7081.18	−0.02	**7079.66**	0.00	1379.66	5700	A^{19}	**7079.66**	1.97
C109A	7078.13	−0.01	7077.68	−0.01	**7077.30**	0.00	1377.30	5700	A^{19}	**7077.30**	2.19
RC101A	5180.74	−0.14	5168.23	0.10	**5150.86**	0.44	1843.47	3330	$A^3B^{13}C^4$	5173.47	5.14
RC102A	5029.59	−0.21	5025.22	−0.13	**4987.24**	0.63	1658.83	3360	$A^6B^6C^7$	5018.83	4.26
RC103A	4895.57	−0.94	4888.53	−0.79	**4804.61**	0.94	1430.20	3420	$A^2B^6C^8$	4850.20	6.47
RC104A	4760.56	−0.74	4747.38	−0.47	**4717.63**	0.16	1395.40	3330	$A^3B^2C^8D^1$	4725.40	5.29
RC105A	5060.37	−0.23	5068.54	−0.39	**5035.35**	0.27	1748.86	3300	$A^5B^8C^6$	5048.86	4.78
RC106A	4997.86	−0.68	4972.11	−0.16	**4936.74**	0.55	1514.13	3450	B^7C^8	4964.13	5.29
RC107A	4865.76	−0.83	4861.04	−0.73	**4788.69**	0.76	1435.60	3390	$A^4B^5C^8$	4825.60	4.17
RC108A	4765.37	−0.86	4753.12	−0.60	**4708.85**	0.34	1334.79	3390	$A^4B^2C^8D^1$	4724.79	4.63
R201A	3484.95	−1.11	3530.24	−2.42	**3446.78**	0.00	1196.78	2250	A^5	**3446.78**	6.13
R202A	3335.95	−1.17	3335.61	−1.16	3308.16	−0.33	1047.42	2250	A^5	**3297.42***	7.46
R203A	3173.95	−1.05	3164.03	−0.73	**3141.09**	0.00	891.09	2250	A^5	**3141.09**	6.14
R204A	3065.15	−1.56	3029.83	−0.39	**3018.14**	0.00	768.14	2250	A^5	**3018.14**	6.28
R205A	3277.69	−1.82	3261.19	−1.31	**3218.97**	0.00	968.97	2250	A^5	**3218.97**	6.38
R206A	3173.30	−0.86	3165.85	−0.62	**3146.34**	0.00	896.34	2250	A^5	**3146.34**	8.14
R207A	3136.47	−1.92	3102.79	−0.83	3077.58	−0.01	827.36	2250	A^5	**3077.36***	6.47
R208A	3050.00	−1.76	3009.13	−0.40	**2997.24**	0.00	747.25	2250	A^5	**2997.25**	6.34
R209A	3155.73	−1.16	3155.60	−1.16	3122.42	−0.09	869.56	2250	A^5	**3119.56***	4.99
R210A	3219.23	−1.54	3206.23	−1.13	3174.85	−0.14	920.41	2250	A^5	**3170.41***	5.47
R211A	3055.04	−1.16	3026.02	−0.20	**3019.93**	0.00	769.93	2250	A^5	**3019.93**	7.93
C201A	5701.45	−0.11	5700.87	−0.10	**5695.02**	0.00	695.02	5000	A^5	**5695.02**	3.46
C202A	5689.70	−0.08	5689.70	−0.08	**5685.24**	0.00	685.24	5000	A^5	**5685.24**	3.17
C203A	5685.82	−0.08	**5681.55**	0.00	**5681.55**	0.00	681.55	5000	A^5	**5681.55**	4.29
C204A	5690.30	−0.22	**5677.69**	0.00	**5677.66**	0.00	677.67	5000	A^5	**5677.66**	3.97
C205A	5691.70	−0.01	5691.70	−0.01	**5691.36**	0.00	691.36	5000	A^5	**5691.36**	3.46
C206A	5691.70	−0.04	5691.70	−0.04	**5689.32**	0.00	689.32	5000	A^5	**5689.32**	2.97
C207A	5689.82	−0.04	5692.36	−0.09	**5687.35**	0.00	687.35	5000	A^5	**5687.35**	4.10
C208A	**5686.50**	0.00	5689.59	−0.05	**5686.50**	0.00	686.50	5000	A^5	**5686.50**	3.56
RC201A	4407.68	−0.71	4404.07	−0.62	4374.09	0.06	1476.82	2900	$A^{10}B^4$	4376.82	5.14
RC202A	4277.67	−0.78	4266.96	−0.53	**4244.63**	0.00	1294.63	2950	A^8B^5	**4244.63**	4.26
RC203A	4204.85	−0.83	4189.94	−0.47	**4170.17**	0.00	1120.17	3050	$A^6B^3C^2$	**4170.17**	6.14
RC204A	4109.86	−0.56	4098.34	−0.27	**4087.11**	0.00	937.112	3150	$A^5B^2C^3$	**4087.11**	5.47
RC205A	4329.96	−0.84	4304.52	−0.25	**4291.93**	0.04	1343.73	2950	A^8B^5	4293.73	4.19
RC206A	4272.08	−0.48	4272.82	−0.49	**4251.88**	0.00	1251.88	3000	A^6B^6	**4251.88**	4.27
RC207A	4232.81	−1.20	4219.52	−0.89	4185.98	−0.08	1182.44	3000	A^6B^6	**4182.44***	5.64
RC208A	4095.71	−0.51	4093.83	−0.46	**4075.04**	0.00	975.04	3100	$A^4B^4C^2$	**4075.04**	5.31

Table B.8: Results for FD for cost structure B

Instance set	MDA		BPDRT		UHGS		HEA				
	TC	Dev	TC	Dev	TC	Dev	DC	VC	Mix	TC	Time
R101B	2226.94	−0.20	–	–	2228.67	−0.27	1664.56	558	$B^5C^{13}D^2$	**2222.56***	4.27
R102B	2071.90	−1.16	–	–	2073.63	−1.25	1476.12	572	$A^1B^2C^{10}D^5$	**2048.12***	3.28
R103B	1857.22	−0.08	–	–	**1853.66**	0.11	1249.74	606	$A^1C^7D^6E^1$	1855.74	5.27
R104B	1707.31	−1.24	–	–	**1683.33**	0.18	1026.42	660	$A^1C^1D^{10}E^1$	1686.42	5.09
R105B	1995.07	−0.71	–	–	1988.86	−0.40	1390.96	590	$C^{10}D^6$	**1980.96***	3.37
R106B	1903.95	−0.72	–	–	**1888.31**	0.10	1290.28	600	$C^9D^5E^1$	1890.28	4.19
R107B	1766.18	−0.81	–	–	1753.35	−0.08	1140.02	612	$C^4D^8E^1$	**1752.02***	5.26
R108B	1666.89	−1.06	–	–	**1647.88**	0.09	983.37	666	$B^1C^1D^8E^1$	1649.37	3.97
R109B	1833.54	−0.79	–	–	**1818.15**	0.05	1209.10	610	$B^1C^4D^8E^1$	1819.10	3.99
R110B	1781.74	−1.12	–	–	**1758.64**	0.19	1161.96	600	C^2D^{11}	1761.96	5.47
R111B	1768.74	−1.47	–	–	**1740.86**	0.13	1121.16	622	$C^4D^8E^1$	1743.16	5.69
R112B	1675.76	−0.76	–	–	**1661.85**	0.07	1029.09	634	$C^1D^{10}E^1$	1663.09	5.01
C101B	2340.98	−0.04	–	–	**2340.15**	0.00	960.15	1380	A^7B^6	**2340.15**	2.98
C102B	2326.53	−0.04	–	–	**2325.70**	0.00	945.70	1380	A^7B^6	**2325.70**	2.73
C103B	2325.61	−0.04	–	–	**2324.60**	0.00	944.60	1380	A^7B^6	**2324.60**	3.64
C104B	**2318.04**	0.00	–	–	**2318.04**	0.00	938.04	1380	A^7B^6	**2318.04**	2.98
C105B	2344.64	−0.19	–	–	**2340.15**	0.00	960.15	1380	A^7B^6	**2340.15**	2.71
C106B	2345.85	−0.24	–	–	**2340.15**	0.00	960.15	1380	A^7B^6	**2340.15**	3.19
C107B	2345.60	−0.23	–	–	**2340.15**	0.00	960.15	1380	A^7B^6	**2340.15**	2.94
C108B	2340.17	−0.07	–	–	**2338.58**	0.00	958.58	1380	A^7B^6	**2338.58**	3.88
C109B	**2328.55**	0.00	–	–	**2328.55**	0.00	948.55	1380	A^7B^6	**2328.55**	3.12
RC101B	2417.16	−0.40	–	–	2412.71	−0.22	1693.43	714	$A^2B^7C^8$	**2407.43***	3.46
RC102B	2234.47	−0.69	–	–	**2213.92**	0.24	1487.23	732	$A^2B^7C^5D2$	2219.23	5.14
RC103B	2025.74	−0.51	–	–	2016.28	−0.04	1295.55	720	$B^1C^{10}D^1$	**2015.55***	3.69
RC104B	1912.65	−0.86	–	–	1897.04	−0.03	1146.40	750	$B^1C^6D^4$	**1896.40***	4.57
RC105B	2296.16	−0.96	–	–	2287.51	−0.58	1530.28	744	$A^1B^6C^6D^2$	**2274.28***	5.69
RC106B	2157.84	−1.21	–	–	2140.86	−0.41	1400.13	732	$A^1B^2C^8D^2$	**2132.13***	3.12
RC107B	2008.02	−1.18	–	–	1989.34	−0.24	1252.67	732	$A^1B^2C^5D^1$	**1984.67***	2.45
RC108B	1920.91	−1.32	–	–	1898.96	−0.16	1133.97	762	$B^1C^6D^4$	**1895.97***	2.67
R201B	1687.44	−2.47	–	–	**1646.78**	0.00	1196.78	450	A^5	**1646.78***	6.79
R202B	1527.74	−1.73	–	–	1508.16	−0.42	1051.81	450	A^5	**1501.81***	7.23
R203B	1379.15	−2.84	–	–	**1341.09**	0.00	891.092	450	A^5	**1341.09**	4.56
R204B	1243.56	−2.09	–	–	**1218.14**	0.00	768.14	450	A^5	**1218.14**	4.11
R205B	1471.97	−3.60	–	–	**1418.97**	0.13	970.81	450	A^5	1420.81	6.47
R206B	1400.84	−3.97	–	–	**1346.34**	0.08	897.41	450	A^5	1347.41	6.99
R207B	1333.53	−4.30	–	–	**1277.58**	0.08	828.57	450	A^5	1278.57	6.78
R208B	1225.37	−2.23	–	–	**1197.24**	0.12	748.6	450	A^5	1198.70	5.47
R209B	1370.30	−3.62	–	–	**1322.42**	0.00	872.42	450	A^5	**1322.42**	5.47
R210B	1418.54	−3.51	–	–	1374.31	−0.28	920.41	450	A^5	**1370.41***	5.93
R211B	1263.72	−3.54	–	–	**1219.93**	0.05	770.57	450	A^5	1220.57	7.81
C201B	1700.87	−0.35	–	–	**1695.02**	0.00	695.02	1000	A^5	**1695.02**	2.11
C202B	1687.84	−0.15	–	–	**1685.24**	0.00	685.24	1000	A^5	**1685.24**	2.33
C203B	1696.25	−0.87	–	–	**1681.55**	0.00	681.55	1000	A^5	**1681.55**	2.57
C204B	1705.94	−1.69	–	–	**1677.66**	0.00	677.66	1000	A^5	**1677.66**	3.69
C205B	1711.00	−1.16	–	–	**1691.36**	0.00	691.36	1000	A^5	**1691.36**	3.07
C206B	1691.70	−0.14	–	–	**1689.32**	0.00	689.32	1000	A^5	**1689.32**	3.19
C207B	1704.88	−1.04	–	–	**1687.35**	0.00	687.35	1000	A^5	**1687.35**	3.76
C208B	1689.59	−0.18	–	–	**1686.50**	0.00	686.50	1000	A^5	**1686.50**	2.41
RC201B	1965.31	−1.24	–	–	**1938.36**	0.14	1321.16	620	$A^4B^1C^4$	1941.16	6.98
RC202B	1771.87	−0.22	–	–	1772.81	−0.27	1128.04	640	$A^1B^1C^5$	**1768.04***	6.47
RC203B	1619.55	−1.00	–	–	1604.04	−0.03	943.548	660	$A^1B^1C^5$	**1603.55***	6.15
RC204B	1501.10	−0.79	–	–	1490.25	−0.07	829.27	660	C^6	**1489.27***	3.47
RC205B	1853.58	−1.10	–	–	**1832.53**	0.04	1193.34	640	$A^1B^7C^1$	1833.34	3.98
RC206B	1761.49	−2.15	–	–	1725.44	−0.06	1074.41	650	$A^3B^1C^3D^1$	**1724.41***	4.54
RC207B	1666.03	−0.96	–	–	**1646.37**	0.23	1000.23	650	B^3C^4	1650.23	5.01
RC208B	1494.11	−0.83	–	–	1483.20	−0.1	821.743	660	C^6	**1481.74***	4.08

Table B.9: Results for FD for cost structure C

Instance set	MDA		BPDRT		UHGS		HEA				
	TC	Dev	TC	Dev	TC	Dev	DC	VC	Mix	TC	Time
R101C	1951.20	−0.71	1951.89	−0.75	1951.20	−0.71	1629.38	308	$A^1B^8C^5D^6$	**1937.38***	4.17
R102C	1770.40	−0.46	1778.29	−0.91	1785.35	−1.31	1465.22	297	$A^2C^{11}D^5$	**1762.22***	3.23
R103C	1558.17	−0.72	1555.26	−0.54	1552.34	−0.35	1224.98	322	$A^1C^6D^7E^1$	**1546.98***	3.69
R104C	1367.82	−1.14	1372.08	−1.46	1355.15	−0.21	1013.37	339	$A^1C^1D^5E^4$	**1352.37***	5.17
R105C	1696.67	−0.91	1698.26	−1.00	1694.56	−0.78	1381.44	300	$B^3C^4D^9$	**1681.44***	4.13
R106C	1589.25	−0.23	1590.11	−0.28	**1583.17**	0.16	1274.65	311	$B^2C^5D^7E^1$	1585.65	3.67
R107C	1435.21	−0.76	1439.81	−1.08	1428.08	−0.26	1080.37	344	$A^1C^1D^7E^3$	**1424.37***	5.98
R108C	1334.75	−1.24	1334.68	−1.23	**1314.88**	0.27	968.444	350	$A^1C^1D^5E^4$	1318.44	4.78
R109C	1515.22	−0.54	1514.13	−0.47	**1506.59**	0.03	1185.1	322	$B^1C^1D^{10}E^1$	1507.10	4.11
R110C	1457.42	−0.97	1461.85	−1.28	1443.92	−0.04	1101.37	342	$B^1C^1D^{10}E^1$	**1443.37***	4.78
R111C	1439.43	−1.41	1439.14	−1.39	1420.15	−0.05	1089.43	330	$A^1B^1D^7E^3$	**1419.43***	5.14
R112C	1358.17	−2.27	1343.26	−1.15	**1327.58**	0.03	989.01	339	$C^1D^7E^3$	1328.01	4.67
C101C	**1628.94**	0.00	**1628.94**	0.00	**1628.94**	0.00	828.94	800	B^{10}	**1628.94**	1.99
C102C	**1597.66**	0.00	**1597.66**	0.00	**1597.66**	0.00	847.66	750	A^1B^9	**1597.66**	2.14
C103C	**1596.56**	0.00	**1596.56**	0.00	**1596.56**	0.00	846.56	750	A^1B^9	**1596.56**	2.65
C104C	1594.06	−0.21	1590.86	−0.01	**1590.76**	0.00	840.76	750	A^1B^9	**1590.76**	2.11
C105C	**1628.94**	0.00	**1628.94**	0.00	**1628.94**	0.00	828.94	800	B^{10}	**1628.94**	2.41
C106C	**1628.94**	0.00	**1628.94**	0.00	**1628.94**	0.00	828.94	800	B^{10}	**1628.94**	1.74
C107C	1628.94	0.00	1628.94	0.00	**1628.94**	0.00	828.94	800	B^{10}	**1628.94**	2.03
C108C	1622.75	0.00	1622.75	0.00	**1622.75**	0.00	892.75	730	A^3B^8	**1622.75**	2.56
C109C	**1614.99**	0.00	**1614.99**	0.00	1615.93	0.06	864.99	750	A^1B^9	**1614.99**	2.97
RC101C	2048.44	−0.72	2053.55	−0.97	2043.48	−0.47	1637.89	396	$A^1B^6C^8D^1$	**2033.89***	4.16
RC102C	1860.48	−0.68	1872.49	−1.33	**1847.92**	0.00	1481.92	366	$A^1B^5C^5D^3$	**1847.92**	4.03
RC103C	1660.81	−0.88	1663.08	−1.02	**1646.35**	0.00	1271.35	375	C^8D^3	**1646.35**	4.17
RC104C	1536.24	−1.14	1540.61	−1.43	1522.04	−0.20	1143.96	375	C^4D^6	**1518.96***	5.14
RC105C	1913.09	−1.49	1929.89	−2.39	1913.06	−1.49	1497.92	387	$A^2B^3C^8D^2$	**1884.92***	4.57
RC106C	1772.05	−1.03	1776.52	−1.28	1770.95	−0.97	1372.99	381	$A^1B^2C^8D^2$	**1753.99***	3.44
RC107C	1615.74	−0.91	1633.29	−2.01	1607.11	−0.37	1211.12	390	$B^1C^6D^4$	**1601.12***	3.47
RC108C	1527.35	−0.72	1527.87	−0.76	1523.96	−0.50	1126.36	390	$A^1C^4D^6$	**1516.36***	3.64
R201C	1441.46	−0.84	1466.13	−2.56	1443.41	−0.97	1204.50	225	A^5	**1429.50***	4.54
R202C	1298.10	−1.96	1296.78	−1.86	1283.16	−0.79	1048.11	225	A^5	**1273.11***	7.12
R203C	1145.38	−2.62	1127.28	−1.00	**1116.09**	0.00	891.09	225	A^5	**1116.09**	4.58
R204C	1019.77	−2.68	1000.89	−0.78	**993.14**	0.00	768.14	225	A^5	**993.14**	6.81
R205C	1222.03	−2.19	1240.74	−3.76	**1193.97**	0.15	970.81	225	A^5	1195.81	6.21
R206C	1138.26	−1.51	1141.13	−1.76	**1121.34**	0.00	896.34	225	A^5	**1121.34**	5.14
R207C	1086.42	−3.21	1067.97	−1.46	**1052.58**	0.00	827.58	225	A^5	**1052.58**	5.23
R208C	976.11	−0.25	979.50	−0.60	**969.90**	0.39	748.70	225	A^5	973.70	5.47
R209C	1140.96	−4.20	1140.96	−4.20	1097.42	−0.22	869.97	225	A^5	**1094.97***	5.64
R210C	1161.87	−1.43	1170.29	−2.17	1149.85	−0.38	920.48	225	A^5	**1145.48***	6.17
R211C	1015.84	−2.10	1008.54	−1.37	**994.93**	0.00	769.93	225	A^5	**994.93**	6.17
C201C	**1194.33**	0.00	**1194.33**	0.00	**1194.33**	0.00	694.33	500	A^5	**1194.33**	4.50
C202C	1189.35	−0.35	**1185.24**	0.00	**1185.24**	0.00	685.24	500	A^5	**1185.24**	2.36
C203C	**1176.25**	0.00	**1176.25**	0.00	**1176.25**	0.00	656.25	520	A^1B^3	**1176.25**	3.07
C204C	1176.55	−0.10	1176.55	−0.10	**1175.37**	0.00	675.37	500	A^5	**1175.37**	3.09
C205C	**1190.36**	0.00	**1190.36**	0.00	**1190.36**	0.00	690.36	500	A^5	**1190.36**	4.50
C206C	**1188.62**	0.00	**1188.62**	0.00	**1188.62**	0.00	668.62	520	A^1B^3	**1188.62**	3.99
C207C	**1184.88**	0.00	**1187.71**	−0.24	**1184.88**	0.00	684.88	500	A^5	**1184.88**	3.17
C208C	1187.86	−0.11	1186.50	0.00	1186.50	0.00	686.50	500	A^5	**1186.50**	2.87
RC201C	1632.41	−0.41	1630.53	−0.30	**1623.36**	0.14	1285.71	340	$A^1B^7C^1$	1625.71	6.01
RC202C	1459.84	−1.02	1461.44	−1.13	1447.27	−0.15	1095.12	350	$A^1B^3C^4$	**1445.12***	4.12
RC203C	1295.07	−1.69	1292.92	−1.52	1274.04	−0.04	943.55	330	B^3C^4	**1273.55***	3.67
RC204C	1171.26	−1.15	1162.91	−0.43	1159.00	−0.09	807.94	350	C^2D^3	**1157.94***	5.14
RC205C	1525.28	−0.66	1632.67	−7.74	**1512.53**	0.19	1180.34	335	$A^1B^4C^3$	1515.34	5.01
RC206C	1425.15	−1.84	1420.89	−1.53	**1395.18**	0.30	1074.41	325	$A^1B^1C^5$	1399.41	3.27
RC207C	1332.40	−1.13	1328.29	−0.82	**1314.44**	0.23	987.50	330	C^6	1317.50	5.47
RC208C	1155.02	−1.31	1152.92	−1.12	**1140.10**	0.00	790.09	350	C^2D^3	**1140.10**	5.99

Appendix C

Supplement to Chapter 4

Table C.1 presents the characteristics of the FSMLRPTW instances. Tables C.2 to C.9 present the detailed results on all benchmark instances for the FSMLRPTW instances.

Table C.1: The FSMLRPTW benchmark instances

Data set	Depot	Cost	Cap_{10}	Cap_{15}	Cap_{20}	Cap_{25}	Cap_{30}	Cap_{50}	Cap_{75}	Cap_{100}	X	Y
C1	1	40000	100	200	210	320	400	610	850	990	40	50
	2	45000	90	170	205	280	410	580	810	800	64	13
	3	42000	110	210	220	300	370	620	820	900	35	79
	4	41000	–	150	190	220	360	540	860	850	44	57
	5	48000	–	–	215	290	390	550	880	840	29	40
	6	50000	–	–	–	–	–	600	900	970	18	82
	7	38000	–	–	–	–	–	590	830	1000	63	93
	8	49000	–	–	–	–	–	–	870	910	85	8
	9	47000	–	–	–	–	–	–	–	930	11	63
	10	46000	–	–	–	–	–	–	–	780	37	17
C2	1	90000	110	220	240	340	430	640	880	950	40	50
	2	100000	100	180	230	290	410	610	840	800	8	95
	3	120000	120	210	240	310	380	630	830	1010	91	46
	4	95000	–	160	210	230	360	660	880	970	35	43
	5	105000	–	–	230	320	420	580	900	920	20	69
	6	97000	–	–	–	–	–	620	930	990	51	100
	7	115000	–	–	–	–	–	640	870	1030	29	28
	8	112000	–	–	–	–	–	–	890	930	60	43
	9	99000	–	–	–	–	–	–	–	870	98	97
	10	117000	–	–	–	–	–	–	–	890	96	42
R1	1	20000	80	150	160	300	330	490	750	960	35	35
	2	19000	95	110	140	280	280	440	690	750	10	54
	3	22000	85	130	140	270	300	420	610	910	52	56
	4	21000	–	160	150	290	310	510	650	820	46	60
	5	18000	–	–	170	310	270	500	760	720	81	24
	6	23000	–	–	–	–	–	390	740	790	11	59
	7	24000	–	–	–	–	–	460	590	1000	94	40
	8	17000	–	–	–	–	–	–	550	800	78	77
	9	25000	–	–	–	–	–	–	–	790	88	66
	10	20000	–	–	–	–	–	–	–	890	72	49
R2	1	85000	100	170	190	305	340	530	780	1020	35	35
	2	90000	105	120	160	290	300	480	730	810	92	77
	3	94000	90	140	150	300	310	470	650	720	82	17
	4	89000	–	180	160	315	330	530	670	790	25	82
	5	100000	–	–	190	310	280	520	790	890	64	17
	6	92000	–	–	–	–	–	450	750	1070	100	87
	7	97000	–	–	–	–	–	480	640	740	10	72
	8	87000	–	–	–	–	–	–	630	700	3	51
	9	99000	–	–	–	–	–	–	–	1100	8	99
	10	96000	–	–	–	–	–	–	–	760	64	60
RC1	1	18000	160	240	250	310	420	700	860	1050	40	50
	2	19000	180	250	260	270	370	680	790	900	15	52
	3	17000	150	230	220	320	390	710	780	1090	40	3
	4	21000	–	210	240	290	430	640	870	850	24	93
	5	26000	–	–	230	340	400	630	840	790	50	76
	6	24000	–	–	–	–	–	640	730	940	62	60
	7	23000	–	–	–	–	–	670	800	970	79	100
	8	19000	–	–	–	–	–	–	850	1180	10	95
	9	20000	–	–	–	–	–	–	–	900	80	11
	10	25000	–	–	–	–	–	–	–	1020	87	75
RC2	1	86000	180	270	320	420	420	720	900	1300	40	50
	2	91000	190	260	300	360	370	710	830	1200	86	37
	3	87000	160	240	270	380	390	740	800	900	23	94
	4	99000	–	230	290	370	430	690	890	800	55	100
	5	96000	–	–	310	410	400	680	860	1080	28	92
	6	100000	–	–	–	–	–	670	760	780	68	52
	7	85000	–	–	–	–	–	710	830	1090	72	19
	8	94000	–	–	–	–	–	–	870	1240	34	61
	9	93000	–	–	–	–	–	–	–	900	26	88
	10	97000	–	–	–	–	–	–	–	1100	61	44

Table C.2: Results on the 10-customer instances

Instance set	$\|\mathcal{N}_c\|$	$\|\mathcal{N}_0\|$	E_4^v				HESA				
			LP relaxation	Vehicle cost	Total cost	Time (s.)	Vehicle cost	Total cost	Time (s.)	Dev_{TC}	Dev_{VC}
C101	10	3	82504.85	600.00	82675.76	0.04	600.00	82675.76	4.59	0.00	0.00
C102	10	3	82504.85	600.00	82673.74	0.68	600.00	82673.74	4.91	0.00	0.00
C103	10	3	82504.85	600.00	82673.74	0.68	600.00	82673.74	2.83	0.00	0.00
C104	10	3	82504.85	600.00	82673.74	1.46	600.00	82673.74	2.60	0.00	0.00
C105	10	3	82504.85	600.00	82675.76	0.04	600.00	82675.76	3.82	0.00	0.00
C106	10	3	82504.85	600.00	82675.76	0.03	600.00	82675.76	3.96	0.00	0.00
C107	10	3	82504.85	600.00	82675.76	0.03	600.00	82675.76	4.32	0.00	0.00
C108	10	3	82504.85	600.00	82673.74	0.17	600.00	82673.74	2.03	0.00	0.00
C109	10	3	82504.85	600.00	82673.74	0.25	600.00	82673.74	3.06	0.00	0.00
C201	10	3	191118.19	2000.00	192212.53	0.06	2000.00	192212.53	4.87	0.00	0.00
C202	10	3	191118.19	2000.00	192205.36	5.04	2000.00	192205.36	1.57	0.00	0.00
C203	10	3	191118.19	2000.00	192205.36	5.03	2000.00	192205.36	1.94	0.00	0.00
C204	10	3	191118.19	2000.00	192195.93	4.97	2000.00	192195.93	3.57	0.00	0.00
C205	10	3	191118.19	2000.00	192207.74	0.19	2000.00	192207.74	4.51	0.00	0.00
C206	10	3	191118.19	2000.00	192195.93	0.07	2000.00	192195.93	3.22	0.00	0.00
C207	10	3	191118.19	2000.00	192195.93	0.16	2000.00	192195.93	3.13	0.00	0.00
C208	10	3	191118.19	2000.00	192195.93	1.00	2000.00	192195.93	3.69	0.00	0.00
R101	10	3	39382.04	260.00	39523.07	0.11	260.00	39523.07	1.18	0.00	0.00
R102	10	3	39382.04	210.00	39459.59	100.29	210.00	39459.59	4.53	0.00	0.00
R103	10	3	39382.04	210.00	39459.59	100.31	210.00	39459.59	3.80	0.00	0.00
R104	10	3	39382.04	220.00	39446.25	195.61	220.00	39446.25	4.18	0.00	0.00
R105	10	3	39382.04	210.00	39498.13	0.20	210.00	39498.13	1.99	0.00	0.00
R106	10	3	39382.04	210.00	39459.59	204.49	210.00	39459.59	4.18	0.00	0.00
R107	10	3	39382.04	210.00	39459.59	204.84	210.00	39459.59	1.99	0.00	0.00
R108	10	3	39382.04	220.00	39446.25	202.89	220.00	39446.25	2.33	0.00	0.00
R109	10	3	39382.04	210.00	39483.11	3.78	210.00	39483.11	1.97	0.00	0.00
R110	10	3	39382.04	210.00	39452.56	33.73	210.00	39452.56	4.48	0.00	0.00
R111	10	3	39382.04	210.00	39462.11	205.15	210.00	39462.11	1.40	0.00	0.00
R112	10	3	39382.04	220.00	39446.25	292.26	220.00	39446.25	4.11	0.00	0.00
R201	10	3	175618.55	900.00	176202.33	0.17	900.00	176202.33	4.04	0.00	0.00
R202	10	3	175618.55	900.00	176152.08	0.76	900.00	176152.08	2.65	0.00	0.00
R203	10	3	175618.55	900.00	176152.08	0.76	900.00	176152.08	3.54	0.00	0.00
R204	10	3	175618.55	900.00	176147.33	1.40	900.00	176147.33	4.91	0.00	0.00
R205	10	3	175618.55	900.00	176175.45	0.67	900.00	176175.45	3.17	0.00	0.00
R206	10	3	175618.55	900.00	176147.33	1.45	900.00	176147.33	2.20	0.00	0.00
R207	10	3	175618.55	900.00	176147.33	1.44	900.00	176147.33	1.12	0.00	0.00
R208	10	3	175618.55	900.00	176147.33	0.95	900.00	176147.33	2.77	0.00	0.00
R209	10	3	175618.55	900.00	176162.07	0.41	900.00	176162.07	3.81	0.00	0.00
R210	10	3	175618.55	900.00	176152.08	1.84	900.00	176152.08	4.99	0.00	0.00
R211	10	3	175618.55	900.00	176152.08	0.75	900.00	176152.08	1.79	0.00	0.00
RC101	10	3	35614.85	450.00	35766.20	1.47	450.00	35766.20	4.55	0.00	0.00
RC102	10	3	35614.85	450.00	35704.15	12.61	450.00	35704.15	1.98	0.00	0.00
RC103	10	3	35614.85	450.00	35704.15	12.60	450.00	35704.15	3.71	0.00	0.00
RC104	10	3	35614.85	450.00	35704.15	6.50	450.00	35704.15	1.83	0.00	0.00
RC105	10	3	35614.85	450.00	35731.49	36.21	450.00	35731.49	2.28	0.00	0.00
RC106	10	3	35614.85	450.00	35716.72	6.38	450.00	35716.72	4.94	0.00	0.00
RC107	10	3	35614.85	450.00	35704.15	3.39	450.00	35704.15	4.12	0.00	0.00
RC108	10	3	35614.85	450.00	35704.15	7.02	450.00	35704.15	2.08	0.00	0.00
RC201	10	3	173434.98	450.00	173626.44	2.62	450.00	173626.44	2.28	0.00	0.00
RC202	10	3	173434.98	450.00	173622.36	8.67	450.00	173622.36	2.04	0.00	0.00
RC203	10	3	173434.98	450.00	173622.36	8.67	450.00	173622.36	4.61	0.00	0.00
RC204	10	3	173434.98	450.00	173620.76	15.82	450.00	173620.76	1.62	0.00	0.00
RC205	10	3	173434.98	450.00	173623.73	8.32	450.00	173623.73	1.89	0.00	0.00
RC206	10	3	173434.98	450.00	173620.76	5.30	450.00	173620.76	3.34	0.00	0.00
RC207	10	3	173434.98	450.00	173620.76	56.11	450.00	173620.76	3.72	0.00	0.00
RC208	10	3	173434.98	450.00	173620.76	12.57	450.00	173620.76	3.07	0.00	0.00

Table C.3: Results on the 15-customer instances

Instance set	$\|\mathcal{N}_c\|$	$\|\mathcal{N}_0\|$	E_4^v				HESA				
			LP relaxation	Vehicle cost	Total cost	Time (s.)	Vehicle cost	Total cost	Time (s.)	Dev_{TC}	Dev_{VC}
C101	15	4	81889.59	900.00	82060.38	0.36	900.00	82060.38	5.56	0.00	0.00
C102	15	4	81889.59	900.00	82059.67	5.10	900.00	82059.67	4.09	0.00	0.00
C103	15	4	81889.59	900.00	82059.67	14.04	900.00	82059.67	5.66	0.00	0.00
C104	15	4	81889.59	900.00	82058.48	14.50	900.00	82058.48	3.34	0.00	0.00
C105	15	4	81889.59	900.00	82060.38	0.66	900.00	82060.38	5.35	0.00	0.00
C106	15	4	81889.59	900.00	82060.38	0.25	900.00	82060.38	3.76	0.00	0.00
C107	15	4	81889.59	900.00	82060.38	0.63	900.00	82060.38	2.15	0.00	0.00
C108	15	4	81889.59	900.00	82060.38	3.71	900.00	82060.38	5.98	0.00	0.00
C109	15	4	81889.59	900.00	82059.54	10.38	900.00	82059.54	4.73	0.00	0.00
C201	15	4	186126.81	2000.00	187199.20	0.24	2000.00	187199.20	2.64	0.00	0.00
C202	15	4	186126.81	2000.00	187190.12	11.17	2000.00	187190.12	4.56	0.00	0.00
C203	15	4	186126.81	2000.00	187190.12	12.93	2000.00	187190.12	4.26	0.00	0.00
C204	15	4	186126.81	2000.00	187190.12	26.52	2000.00	187190.12	3.58	0.00	0.00
C205	15	4	186126.81	2000.00	187194.86	2.38	2000.00	187194.86	3.31	0.00	0.00
C206	15	4	186126.81	2000.00	187194.86	3.17	2000.00	187194.86	2.84	0.00	0.00
C207	15	4	186126.81	2000.00	187194.86	9.43	2000.00	187194.86	5.18	0.00	0.00
C208	15	4	186126.81	2000.00	187194.86	10.29	2000.00	187194.86	3.37	0.00	0.00
R101	15	4	39590.64	340.00	39753.66	0.32	340.00	39753.66	3.61	0.00	0.00
R102	15	4	39590.64	400.00	39714.93	3465.17	400.00	39714.93	2.24	0.00	0.00
R103	15	4	39590.64	400.00	39714.93	9902.70	400.00	39714.93	2.17	0.00	0.00
R104	15	4	39590.64	340.00	39691.17	10800.00*	340.00	39692.00	4.28	0.00	0.00
R105	15	4	39590.64	370.00	39748.26	14.69	370.00	39748.26	3.56	0.00	0.00
R106	15	4	39590.64	350.00	39692.47	3703.37	350.00	39692.47	4.92	0.00	0.00
R107	15	4	39590.64	350.00	39692.47	8024.12	350.00	39692.47	2.40	0.00	0.00
R108	15	4	39590.64	350.00	39678.30	10800.00*	350.00	39678.30	4.91	0.00	0.00
R109	15	4	39590.64	350.00	39699.94	239.23	350.00	39699.94	4.38	0.00	0.00
R110	15	4	39590.64	360.00	39683.72	10800.00*	360.00	39683.72	5.37	0.00	0.00
R111	15	4	39590.64	350.00	39692.47	9580.68	350.00	39692.47	2.49	0.00	0.00
R112	15	4	39590.64	360.00	39665.72	8609.80	360.00	39665.72	5.95	0.00	0.00
R201	15	4	174662.67	900.00	175234.54	0.30	900.00	175234.54	4.97	0.00	0.00
R202	15	4	174662.67	900.00	175188.92	131.56	900.00	175188.92	4.12	0.00	0.00
R203	15	4	174662.67	900.00	175188.92	82.87	900.00	175188.92	3.80	0.00	0.00
R204	15	4	174662.67	900.00	175188.16	300.89	900.00	175188.16	2.03	0.00	0.00
R205	15	4	174662.67	900.00	175198.70	0.48	900.00	175198.70	2.94	0.00	0.00
R206	15	4	174662.67	900.00	175188.16	60.86	900.00	175188.16	2.43	0.00	0.00
R207	15	4	174662.67	900.00	175188.16	125.80	900.00	175188.16	4.40	0.00	0.00
R208	15	4	174662.67	900.00	175186.29	146.41	900.00	175186.29	5.54	0.00	0.00
R209	15	4	174662.67	900.00	175185.29	88.86	900.00	175185.29	3.01	0.00	0.00
R210	15	4	174662.67	900.00	175188.92	138.54	900.00	175188.92	5.39	0.00	0.00
R211	15	4	174662.67	900.00	175155.24	9.32	900.00	175155.24	4.08	0.00	0.00
RC101	15	4	35843.17	660.00	36000.10	68.82	660.00	36000.10	3.28	0.00	0.00
RC102	15	4	35843.17	660.00	35959.01	403.37	660.00	35959.01	2.44	0.00	0.00
RC103	15	4	35843.17	660.00	35954.04	1735.09	660.00	35954.04	4.44	0.00	0.00
RC104	15	4	35843.17	660.00	35951.15	7053.61	660.00	35951.15	2.61	0.00	0.00
RC105	15	4	35843.17	660.00	36003.22	6482.74	660.00	36003.22	3.35	0.00	0.00
RC106	15	4	35843.17	660.00	35950.90	842.69	660.00	35950.90	4.96	0.00	0.00
RC107	15	4	35843.17	660.00	35936.06	8415.18	660.00	35936.06	5.27	0.00	0.00
RC108	15	4	35843.17	660.00	35933.94	2751.09	660.00	35933.94	2.45	0.00	0.00
RC201	15	4	173626.06	650.00	173836.24	372.04	650.00	173836.24	3.63	0.00	0.00
RC202	15	4	173626.06	650.00	173829.47	10800.00*	650.00	173829.47	5.14	0.00	0.00
RC203	15	4	173626.06	650.00	173829.47	10800.00*	650.00	173829.47	2.48	0.00	0.00
RC204	15	4	173626.06	650.00	173823.01	10800.00*	650.00	173823.01	2.17	0.00	0.00
RC205	15	4	173626.06	650.00	173829.47	10800.00*	650.00	173829.47	5.64	0.00	0.00
RC206	15	4	173626.06	650.00	173824.84	10800.00*	650.00	173824.84	4.36	0.00	0.00
RC207	15	4	173626.06	650.00	173819.37	10800.00*	650.00	173819.37	4.90	0.00	0.00
RC208	15	4	173626.06	650.00	173817.77	10800.00*	650.00	173817.77	4.91	0.00	0.00

Table C.4: Results on the 20-customer instances

Instance set	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	E_4^v				HESA				
			LP relaxation	Vehicle cost	Total cost	Time (s.)	Vehicle cost	Total cost	Time (s.)	Dev_{TC}	Dev_{VC}
C101	20	5	82236.26	1200.00	82436.10	1.87	1200.00	82436.10	5.91	0.00	0.00
C102	20	5	82236.26	1200.00	82436.10	1382.38	1200.00	82436.10	8.04	0.00	0.00
C103	20	5	82236.26	1200.00	82436.10	10800.00*	1200.00	82436.10	4.30	0.00	0.00
C104	20	5	82236.26	1200.00	82435.04	10800.00*	1200.00	82566.30	7.41	0.16	0.00
C105	20	5	82236.26	1200.00	82436.10	2766.31	1200.00	82436.10	5.36	0.00	0.00
C106	20	5	82236.26	1200.00	82436.10	3.39	1200.00	82436.10	4.58	0.00	0.00
C107	20	5	82236.26	1200.00	82436.10	21.17	1200.00	82436.10	6.91	0.00	0.00
C108	20	5	82236.26	1200.00	82436.10	10800.00*	1200.00	82436.10	6.89	0.00	0.00
C109	20	5	82236.26	1200.00	82435.04	10800.00*	1200.00	82435.04	6.87	0.00	0.00
C201	20	5	186141.23	2000.00	187227.98	0.68	2000.00	187227.98	5.00	0.00	0.00
C202	20	5	186141.23	2000.00	187221.32	143.12	2000.00	187221.32	8.58	0.00	0.00
C203	20	5	186141.23	2000.00	187219.23	1655.04	2000.00	187219.23	5.03	0.00	0.00
C204	20	5	186141.23	2000.00	187218.28	2657.51	2000.00	187218.28	7.23	0.00	0.00
C205	20	5	186141.23	2000.00	187227.98	21.85	2000.00	187227.98	4.27	0.00	0.00
C206	20	5	186141.23	2000.00	187227.98	112.96	2000.00	187227.98	4.50	0.00	0.00
C207	20	5	186141.23	2000.00	187227.03	87.66	2000.00	187227.03	8.10	0.00	0.00
C208	20	5	186141.23	2000.00	187222.64	135.71	2000.00	187222.64	8.95	0.00	0.00
R101	20	5	37782.90	510.00	39130.26	3.58	510.00	39130.26	6.47	0.00	0.00
R102	20	5	37782.90	480.00	39055.61	10800.00*	480.00	39055.61	4.47	0.00	0.00
R103	20	5	37782.90	490.00	37973.97	10800.00*	490.00	37965.50	5.91	−0.02	0.00
R104	20	5	37782.90	460.00	37931.84	10800.00*	460.00	37918.90	6.96	−0.03	0.00
R105	20	5	37782.90	510.00	38054.09	221.14	510.00	38054.09	6.75	0.00	0.00
R106	20	5	37782.90	460.00	37965.26	10800.00*	460.00	37993.40	4.00	0.07	0.00
R107	20	5	37782.90	460.00	37940.97	10800.00*	460.00	37947.90	4.50	0.02	0.00
R108	20	5	37782.90	500.00	37927.36	10800.00*	500.00	37915.90	8.88	−0.03	0.00
R109	20	5	37782.90	460.00	37931.63	1226.56	460.00	37931.63	7.41	0.00	0.00
R110	20	5	37782.90	460.00	37931.63	10800.00*	460.00	37931.63	8.51	0.00	0.00
R111	20	5	37782.90	460.00	37920.97	10800.00*	460.00	37920.97	4.44	0.00	0.00
R112	20	5	37782.90	500.00	37913.51	10800.00*	500.00	37906.70	5.87	−0.02	0.00
R201	20	5	174699.42	900.00	175334.77	13.84	900.00	175335.00	6.83	0.00	0.00
R202	20	5	174699.40	900.00	175253.40	895.21	900.00	175253.40	6.29	0.00	0.00
R203	20	5	174699.40	900.00	175265.96	10800.00*	900.00	175253.00	6.12	−0.01	0.00
R204	20	5	174699.40	900.00	175210.76	1169.83	900.00	175210.76	5.76	0.00	0.00
R205	20	5	174699.41	900.00	175242.43	2.47	900.00	175242.43	5.50	0.00	0.00
R206	20	5	174699.40	900.00	175225.04	170.27	900.00	175225.04	8.00	0.00	0.00
R207	20	5	174699.40	900.00	175225.04	3394.52	900.00	175225.04	5.81	0.00	0.00
R208	20	5	174699.40	900.00	175207.73	228.61	900.00	175207.73	6.40	0.00	0.00
R209	20	5	174699.41	900.00	175224.09	25.81	900.00	175224.09	4.39	0.00	0.00
R210	20	5	174699.40	900.00	175237.25	1028.92	900.00	175237.25	8.88	0.00	0.00
R211	20	5	174699.40	900.00	175208.53	782.13	900.00	175208.53	5.78	0.00	0.00
RC101	20	5	36075.96	870.00	36274.22	457.60	870.00	36288.50	5.30	0.04	0.00
RC102	20	5	36075.96	870.00	36241.99	10800.00*	870.00	36241.99	8.59	0.00	0.00
RC103	20	5	36075.96	870.00	36229.55	10800.00*	870.00	36236.60	7.17	0.02	0.00
RC104	20	5	36075.96	870.00	36229.32	10800.00*	870.00	36229.32	7.02	0.00	0.00
RC105	20	5	36075.96	870.00	36271.24	10800.00*	870.00	36281.50	6.87	0.03	0.00
RC106	20	5	36075.96	870.00	36248.47	10800.00*	870.00	36248.47	8.25	0.00	0.00
RC107	20	5	36075.96	870.00	36229.15	10800.00*	870.00	36229.15	7.59	0.00	0.00
RC108	20	5	36075.96	870.00	36215.68	10800.00*	870.00	36215.68	5.53	0.00	0.00
RC201	20	5	173864.43	800.00	174093.90	10800.00*	800.00	174093.90	6.05	0.00	0.00
RC202	20	5	173864.43	800.00	174086.74	10800.00*	800.00	174086.74	8.81	0.00	0.00
RC203	20	5	173864.43	800.00	174087.25	10800.00*	800.00	174081.00	8.14	0.00	0.00
RC204	20	5	173864.43	800.00	174069.80	10800.00*	800.00	174069.80	7.86	0.00	0.00
RC205	20	5	173864.43	800.00	174086.74	10800.00*	800.00	174086.74	6.56	0.00	0.00
RC206	20	5	173864.43	800.00	174082.07	10800.00*	800.00	174082.07	5.03	0.00	0.00
RC207	20	5	173864.43	800.00	174064.56	10800.00*	800.00	174064.56	7.90	0.00	0.00
RC208	20	5	173864.43	800.00	174064.56	10800.00*	800.00	174064.56	7.26	0.00	0.00

Table C.5: Results on the 25-customer instances

Instance set	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	E_4^v				HESA				
			LP relaxation	Vehicle cost	Total cost	Time (s.)	Vehicle cost	Total cost	Time (s.)	Dev_{TC}	Dev_{VC}
C101	25	5	82556.95	1500.00	82767.06	5.94	1500.00	82767.06	9.16	0.00	0.00
C102	25	5	82556.95	1500.00	82767.06	3941.41	1500.00	82767.06	10.76	0.00	0.00
C103	25	5	82556.95	1500.00	82766.82	10800.00*	1500.00	82773.80	9.91	0.01	0.00
C104	25	5	82556.95	1500.00	82764.87	10800.00*	1500.00	82902.10	10.35	0.17	0.00
C105	25	5	82556.95	1500.00	82767.06	10800.00*	1500.00	82767.06	9.75	0.00	0.00
C106	25	5	82556.95	1500.00	82767.06	6890.55	1500.00	82767.06	11.80	0.00	0.00
C107	25	5	82556.95	1500.00	82767.06	10800.00*	1500.00	82767.06	6.61	0.00	0.00
C108	25	5	82556.95	1500.00	82767.06	10800.00*	1500.00	82767.06	8.68	0.00	0.00
C109	25	5	82556.95	1500.00	82772.10	10800.00*	1500.00	82764.90	6.96	−0.01	0.00
C201	25	5	186314.82	2000.00	187237.51	1.13	2000.00	187237.51	11.31	0.00	0.00
C202	25	5	186314.82	2000.00	187231.15	217.90	2000.00	187231.15	9.45	0.00	0.00
C203	25	5	186314.82	2000.00	187231.15	10800.00*	2000.00	187231.15	7.06	0.00	0.00
C204	25	5	186314.82	2000.00	187232.33	10800.00*	2000.00	187232.33	10.29	0.00	0.00
C205	25	5	186314.82	2000.00	187237.51	25.17	2000.00	187237.51	6.24	0.00	0.00
C206	25	5	186314.82	2000.00	187237.51	333.59	2000.00	187237.51	8.77	0.00	0.00
C207	25	5	186314.82	2000.00	187233.45	248.47	2000.00	187233.45	7.80	0.00	0.00
C208	25	5	186314.82	2000.00	187237.35	926.44	2000.00	187237.35	6.71	0.00	0.00
R101	25	5	37930.66	580.00	39242.52	8.97	580.00	39242.52	8.24	0.00	0.00
R102	25	5	37930.66	620.00	39163.49	10800.00*	620.00	39153.10	8.13	−0.03	0.00
R103	25	5	37930.66	640.00	38144.61	10800.00*	640.00	38126.50	10.35	−0.05	0.00
R104	25	5	37930.66	640.00	38111.15	10800.00*	640.00	38074.50	9.99	−0.10	0.00
R105	25	5	37930.66	600.00	38225.10	999.73	600.00	38225.10	10.69	0.00	0.00
R106	25	5	37930.66	630.00	38168.91	10800.00*	630.00	38151.70	7.54	−0.05	0.00
R107	25	5	37930.66	580.00	38075.13	10775.00	580.00	38069.40	11.55	−0.02	0.00
R108	25	5	37930.66	580.00	38040.62	10800.00*	580.00	38040.62	6.54	0.00	0.00
R109	25	5	37930.66	600.00	38150.69	10800.00*	600.00	38154.90	8.60	0.01	0.00
R110	25	5	37930.66	640.00	38114.13	10800.00*	640.00	38100.50	10.30	−0.04	0.00
R111	25	5	37930.66	610.00	38126.42	10800.00*	610.00	38058.30	6.39	−0.18	0.00
R112	25	5	37930.66	640.00	38113.11	10800.00*	640.00	38049.50	9.73	−0.17	0.00
R201	25	5	174795.80	900.00	175439.55	32.28	900.00	175439.55	9.61	0.00	0.00
R202	25	5	174795.80	900.00	175361.14	10800.00*	900.00	175361.14	7.58	0.00	0.00
R203	25	5	174795.80	900.00	175331.29	10775.00	900.00	175325.00	7.00	0.00	0.00
R204	25	5	174795.80	900.00	175267.13	10800.00*	900.00	175267.13	9.10	0.00	0.00
R205	25	5	174795.80	900.00	175319.60	567.39	900.00	175319.60	8.53	0.00	0.00
R206	25	5	174795.80	900.00	175294.33	10800.00*	900.00	175294.33	10.14	0.00	0.00
R207	25	5	174795.80	900.00	175295.41	10800.00*	900.00	175284.00	10.34	−0.01	0.00
R208	25	5	174795.80	900.00	175256.29	1588.22	900.00	175256.29	7.30	0.00	0.00
R209	25	5	174795.80	900.00	175296.89	5431.05	900.00	175296.89	8.43	0.00	0.00
R210	25	5	174795.80	900.00	175326.94	10800.00*	900.00	175320.00	11.40	0.00	0.00
R211	25	5	174795.80	900.00	175264.11	10800.00*	900.00	175264.11	9.13	0.00	0.00
RC101	25	5	36252.02	990.00	36450.48	974.29	990.00	36484.40	8.09	0.09	0.00
RC102	25	5	36252.02	990.00	36432.10	10800.00*	990.00	36447.40	8.48	0.04	0.00
RC103	25	5	36252.02	990.00	36381.15	10800.00*	990.00	36403.40	11.18	0.06	0.00
RC104	25	5	36252.02	990.00	36378.72	10800.00*	990.00	36418.00	8.54	0.11	0.00
RC105	25	5	36252.02	990.00	36468.03	10800.00*	990.00	36489.50	10.76	0.06	0.00
RC106	25	5	36252.02	990.00	36413.53	10800.00*	990.00	36415.20	8.21	0.00	0.00
RC107	25	5	36252.02	990.00	36381.34	10800.00*	990.00	36392.40	9.16	0.03	0.00
RC108	25	5	36252.02	990.00	36409.38	10800.00*	990.00	36391.10	7.79	−0.05	0.00
RC201	25	5	174075.96	1000.00	174326.81	10800.00*	1000.00	174326.81	7.98	0.00	0.00
RC202	25	5	174075.96	1000.00	174319.65	10800.00*	1000.00	174319.65	6.77	0.00	0.00
RC203	25	5	174075.96	1000.00	174309.48	10800.00*	1000.00	174309.48	11.03	0.00	0.00
RC204	25	5	174075.96	1000.00	174285.70	10800.00*	1000.00	174281.00	6.32	0.00	0.00
RC205	25	5	174075.96	1000.00	174318.88	10800.00*	1000.00	174318.88	9.16	0.00	0.00
RC206	25	5	174075.96	1000.00	174295.72	10800.00*	1000.00	174295.72	11.03	0.00	0.00
RC207	25	5	174075.96	1000.00	174280.19	10800.00*	1000.00	174278.00	11.92	0.00	0.00
RC208	25	5	174075.96	1000.00	174275.82	10800.00*	1000.00	174275.82	6.33	0.00	0.00

Table C.6: Results on the 30-customer instances

Instance set	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	E_4^v				HESA				
			LP relaxation	Vehicle cost	Total cost	Time (s.)	Vehicle cost	Total cost	Time (s.)	Dev_{TC}	Dev_{VC}
C101	30	5	82758.85	1800.00	83097.51	10800.00*	1800.00	83097.50	14.16	0.00	0.00
C102	30	5	82758.85	1800.00	83097.51	505.56	1800.00	83109.40	17.76	0.01	0.00
C103	30	5	82758.85	1800.00	83098.28	10800.00*	1800.00	83109.40	12.33	0.01	0.00
C104	30	5	82758.85	1800.00	83094.88	10800.00*	1800.00	83229.20	15.51	0.16	0.00
C105	30	5	82758.85	1800.00	83097.51	10800.00*	1800.00	83097.51	14.21	0.00	0.00
C106	30	5	82758.85	1800.00	83097.51	10800.00*	1800.00	83097.51	15.38	0.00	0.00
C107	30	5	82758.85	1800.00	83094.88	10800.00*	1800.00	83094.88	10.70	0.00	0.00
C108	30	5	82758.85	1800.00	83094.88	10800.00*	1800.00	83100.60	11.04	0.01	0.00
C109	30	5	82758.85	1800.00	83094.88	10800.00*	1800.00	83101.20	13.48	0.01	0.00
C201	30	5	186488.20	2000.00	187255.37	1.86	2000.00	187255.37	11.09	0.00	0.00
C202	30	5	186488.20	2000.00	187249.00	10800.00*	2000.00	187249.00	11.17	0.00	0.00
C203	30	5	186488.20	2000.00	187263.79	10800.00*	2000.00	187249.00	11.56	−0.01	0.00
C204	30	5	186488.20	2000.00	187245.82	10800.00*	2000.00	187245.82	10.66	0.00	0.00
C205	30	5	186488.20	2000.00	187255.37	34.37	2000.00	187255.37	12.78	0.00	0.00
C206	30	5	186488.20	2000.00	187255.37	543.48	2000.00	187255.37	16.63	0.00	0.00
C207	30	5	186488.20	2000.00	187251.31	377.96	2000.00	187251.31	13.87	0.00	0.00
C208	30	5	186488.20	2000.00	187255.56	10800.00*	2000.00	187255.56	12.75	0.00	0.00
R101	30	5	38126.33	710.00	39421.28	33.74	710.00	39421.28	10.72	0.00	0.00
R102	30	5	38126.33	730.00	39356.38	10800.00*	730.00	39341.30	10.62	−0.04	0.00
R103	30	5	38126.33	750.00	38346.59	10800.00*	750.00	38330.30	13.11	−0.04	0.00
R104	30	5	38126.33	780.00	38337.89	10800.00*	780.00	38278.40	10.11	−0.16	0.00
R105	30	5	38126.33	730.00	38451.92	10800.00*	730.00	38442.10	10.83	−0.03	0.00
R106	30	5	38126.33	790.00	38457.92	10800.00*	790.00	38353.70	10.77	−0.27	0.00
R107	30	5	38126.33	780.00	38357.13	10800.00*	780.00	38295.40	12.79	−0.16	0.00
R108	30	5	38126.33	780.00	38326.78	10800.00*	780.00	38257.70	11.83	−0.18	0.00
R109	30	5	38126.33	780.00	38396.41	10800.00*	780.00	38355.90	10.24	−0.11	0.00
R110	30	5	38126.33	780.00	38351.22	10800.00*	780.00	38321.90	13.68	−0.08	0.00
R111	30	5	38126.33	780.00	38363.48	10800.00*	780.00	38285.10	15.04	−0.20	0.00
R112	30	5	38126.33	780.00	38350.33	10800.00*	780.00	38265.00	11.21	−0.22	0.00
R201	30	5	174930.96	900.00	175537.23	628.20	900.00	175542.00	13.23	0.00	0.00
R202	30	5	174930.96	900.00	175502.53	10800.00*	900.00	175443.00	15.93	−0.03	0.00
R203	30	5	174930.96	900.00	175386.34	10800.00*	900.00	175360.00	11.43	−0.02	0.00
R204	30	5	174930.96	900.00	175283.96	10800.00*	900.00	175283.96	11.37	0.00	0.00
R205	30	5	174930.96	900.00	175382.37	6435.25	900.00	175382.37	12.45	0.00	0.00
R206	30	5	174930.96	900.00	175370.08	10800.00*	900.00	175344.00	15.26	−0.01	0.00
R207	30	5	174930.96	900.00	175327.23	10800.00*	900.00	175303.00	14.98	−0.01	0.00
R208	30	5	174930.96	900.00	175283.11	10800.00*	900.00	175273.00	14.42	−0.01	0.00
R209	30	5	174930.96	900.00	175338.33	10800.00*	900.00	175338.33	15.22	0.00	0.00
R210	30	5	174930.96	900.00	175353.04	10800.00*	900.00	175353.04	15.09	0.00	0.00
R211	30	5	174930.96	900.00	175323.38	10800.00*	900.00	175298.00	18.00	−0.01	0.00
RC101	30	5	36474.96	1080.00	36803.52	10800.00*	1080.00	36814.10	14.93	0.03	0.00
RC102	30	5	36474.96	1110.00	36766.79	10800.00*	1110.00	36786.90	13.19	0.05	0.00
RC103	30	5	36474.96	1140.00	36696.26	10800.00*	1140.00	36684.60	13.31	−0.03	0.00
RC104	30	5	36474.96	1140.00	36667.39	10800.00*	1140.00	36697.90	13.36	0.08	0.00
RC105	30	5	36474.96	1200.00	36746.25	10800.00*	1200.00	36748.20	10.20	0.01	0.00
RC106	30	5	36474.96	1140.00	36688.56	10800.00*	1140.00	36688.56	11.40	0.00	0.00
RC107	30	5	36474.96	1110.00	36699.96	10800.00*	1110.00	36664.60	13.51	−0.10	0.00
RC108	30	5	36474.96	1140.00	36657.64	10800.00*	1140.00	36655.10	11.61	−0.01	0.00
RC201	30	5	36199.06	1100.00	36585.08	10800.00*	1100.00	36578.50	13.03	−0.02	0.00
RC202	30	5	36199.06	1100.00	36543.55	10800.00*	1100.00	36539.90	13.79	−0.01	0.00
RC203	30	5	36199.06	1100.00	36540.45	10800.00*	1100.00	36606.90	10.67	0.18	0.00
RC204	30	5	36199.06	1100.00	36565.95	10800.00*	1100.00	36556.30	12.14	−0.03	0.00
RC205	30	5	36199.06	1150.00	36548.96	10800.00*	1150.00	36548.96	15.53	0.00	0.00
RC206	30	5	36199.06	1150.00	36554.74	10800.00*	1150.00	36584.10	14.00	0.08	0.00
RC207	30	5	36199.06	1150.00	36528.09	10800.00*	1150.00	36528.09	11.64	0.00	0.00
RC208	30	5	36199.06	1200.00	36577.81	10800.00*	1200.00	36635.10	14.66	0.16	0.00

Table C.7: Results on the 50-customer instances

Instance set	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	E_4^v				HESA				
			LP relaxation	Vehicle cost	Total cost	Time (s.)	Vehicle cost	Total cost	Time (s.)	Dev_{TC}	Dev_{VC}
C101	50	7	41386.56	2700.00	41865.26	398.97	2700.00	41892.30	70.96	0.06	0.00
C102	50	7	41386.56	2700.00	41859.80	10800.00*	2700.00	41859.80	103.47	0.00	0.00
C103	50	7	41386.56	2900.00	41968.98	10800.00*	2900.00	41922.80	61.00	−0.11	0.00
C104	50	7	41386.56	2700.00	41890.98	10800.00*	2700.00	42000.50	114.55	0.26	0.00
C105	50	7	41386.56	2700.00	41852.69	10800.00*	2700.00	41852.69	81.52	0.00	0.00
C106	50	7	41386.56	2700.00	41853.79	10800.00*	2700.00	41853.79	95.53	0.00	0.00
C107	50	7	41386.56	2700.00	41851.28	10800.00*	2700.00	41903.50	106.71	0.12	0.00
C108	50	7	41386.56	2700.00	41850.25	10800.00*	2700.00	41895.50	105.26	0.11	0.00
C109	50	7	41386.56	2700.00	41890.31	10800.00*	2700.00	42016.20	98.83	0.30	0.00
C201	50	7	92446.10	2400.00	92880.93	6.21	2400.00	92880.93	109.43	0.00	0.00
C202	50	7	92446.10	3000.00	93384.81	10800.00*	2400.00	92825.00	62.39	−0.60	−20.00
C203	50	7	92446.10	3400.00	93869.63	10800.00*	2400.00	92818.60	118.84	−1.12	−29.41
C204	50	7	92446.10	3000.00	93470.84	10800.00*	2400.00	92770.40	115.62	−0.75	−20.00
C205	50	7	92446.10	3000.00	93391.05	10800.00*	2400.00	92876.60	115.28	−0.55	−20.00
C206	50	7	92446.10	3000.00	93391.05	10800.00*	2400.00	92811.40	66.99	−0.62	−20.00
C207	50	7	92446.10	3000.00	93378.45	10800.00*	2400.00	92798.30	117.47	−0.62	−20.00
C208	50	7	92446.10	3000.00	93390.88	10800.00*	2400.00	92798.40	87.68	−0.63	−20.00
R101	50	7	38870.22	1280.00	40443.15	10800.00*	1250.00	40443.15	83.30	0.00	−2.34
R102	50	7	38870.22	1380.00	40440.99	10800.00*	1350.00	40328.30	95.11	−0.28	−2.17
R103	50	7	38870.22	1430.00	41424.71	10800.00*	1280.00	40195.20	83.30	−2.97	−10.49
R104	50	7	38870.22	1360.00	39368.54	10800.00*	1370.00	39155.50	112.85	−0.54	0.74
R105	50	7	38870.22	1280.00	39380.48	10800.00*	1260.00	39356.20	113.54	−0.06	−1.56
R106	50	7	38870.22	1320.00	39459.31	10800.00*	1280.00	39263.10	119.24	−0.50	−3.03
R107	50	7	38870.22	1430.00	39443.09	10800.00*	1310.00	39152.30	92.82	−0.74	−8.39
R108	50	7	38870.22	1300.00	39322.58	10800.00*	1370.00	39157.10	104.15	−0.42	5.38
R109	50	7	38870.22	1350.00	39322.03	10800.00*	1280.00	39250.70	72.21	−0.18	−5.19
R110	50	7	38870.22	1320.00	39411.26	10800.00*	1310.00	39211.80	61.42	−0.51	−0.76
R111	50	7	38870.22	1300.00	39255.20	10800.00*	1280.00	39179.70	84.72	−0.19	−1.54
R112	50	7	38870.22	1450.00	39403.04	10800.00*	1280.00	39170.50	97.18	−0.59	−11.72
R201	50	7	175518.42	1350.00	176218.56	10800.00*	1350.00	176218.56	86.87	0.00	0.00
R202	50	7	175518.42	2050.00	177013.02	10800.00*	1350.00	176088.00	91.59	−0.52	−34.15
R203	50	7	175518.42	1350.00	176249.23	10800.00*	1350.00	175995.00	93.72	−0.14	0.00
R204	50	7	175518.42	1350.00	175917.21	10800.00*	1350.00	175866.00	118.70	−0.03	0.00
R205	50	7	175518.42	1350.00	176200.21	10800.00*	1350.00	176061.00	90.21	−0.08	0.00
R206	50	7	175518.42	1350.00	176081.07	10800.00*	1350.00	176000.00	117.02	−0.05	0.00
R207	50	7	175518.42	2050.00	176815.10	10800.00*	1350.00	175932.00	119.13	−0.50	−34.15
R208	50	7	175518.42	1350.00	175962.50	10800.00*	1350.00	175855.00	63.31	−0.06	0.00
R209	50	7	175518.42	1350.00	176089.81	10800.00*	1350.00	175995.00	69.61	−0.05	0.00
R210	50	7	175518.42	1600.00	176374.18	10800.00*	1350.00	175994.00	82.15	−0.22	−15.63
R211	50	7	175518.42	1800.00	176704.38	10800.00*	1350.00	175905.00	87.41	−0.45	−25.00
RC101	50	7	19574.34	1950.00	21031.12	10800.00*	1860.00	20978.50	83.08	−0.25	−4.62
RC102	50	7	19574.34	2280.00	21334.83	10800.00*	1950.00	20892.30	63.81	−2.07	−14.47
RC103	50	7	19574.34	2130.00	38142.81	10800.00*	1950.00	20778.80	74.35	−45.52	−8.45
RC104	50	7	19574.34	2250.00	20274.62	10800.00*	2010.00	20020.00	105.47	−1.26	−10.67
RC105	50	7	19574.34	2010.00	20991.89	10800.00*	1950.00	20889.10	72.03	−0.49	−2.99
RC106	50	7	19574.34	2040.00	20325.56	10800.00*	1950.00	20223.20	76.47	−0.50	−4.41
RC107	50	7	19574.34	2040.00	20291.25	10800.00*	1860.00	20085.60	61.74	−1.01	−8.82
RC108	50	7	19574.34	1950.00	20064.39	10800.00*	1980.00	20018.40	75.42	−0.23	1.54
RC201	50	7	87065.42	1700.00	87586.41	10800.00*	1800.00	87641.90	94.32	0.06	5.88
RC202	50	7	87065.42	1900.00	87992.95	10800.00*	1800.00	87546.10	65.43	−0.51	−5.26
RC203	50	7	87065.42	2250.00	88339.08	10800.00*	1900.00	87558.60	114.14	−0.88	−15.56
RC204	50	7	87065.42	3550.00	89274.98	10800.00*	1900.00	87456.90	61.20	−2.04	−46.48
RC205	50	7	87065.42	1900.00	87784.60	10800.00*	1800.00	87530.90	76.40	−0.29	−5.26
RC206	50	7	87065.42	1750.00	87989.08	10800.00*	1900.00	87619.80	69.17	−0.42	8.57
RC207	50	7	87065.42	1850.00	88092.32	10800.00*	1900.00	87469.30	104.49	−0.71	2.70
RC208	50	7	87065.42	2550.00	88410.89	10800.00*	1950.00	87464.80	99.78	−1.07	−23.53

Table C.8: Results on the 75-customer instances

Instance set	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	E_4^v				HESA				
			LP relaxation	Vehicle cost	Total cost	Time (s.)	Vehicle cost	Total cost	Time (s.)	Dev_{TC}	Dev_{VC}
C101	75	8	82857.15	4200.00	83309.83	10800.00*	4200.00	83376.60	187.36	0.08	0.00
C102	75	8	82857.15	4200.00	83343.28	10800.00*	4200.00	83389.00	185.65	0.05	0.00
C103	75	8	82857.15	5350.00	84468.47	10800.00*	4200.00	83321.10	157.32	−1.36	−21.50
C104	75	8	82857.15	5450.00	84670.05	10800.00*	4800.00	83766.60	159.44	−1.07	−11.93
C105	75	8	82857.15	4200.00	83311.92	10800.00*	4200.00	83489.80	164.07	0.21	0.00
C106	75	8	82857.15	4200.00	83307.33	10800.00*	4200.00	83372.90	184.56	0.08	0.00
C107	75	8	82857.15	4200.00	83308.44	10800.00*	4200.00	83632.90	179.85	0.39	0.00
C108	75	8	82857.15	4200.00	83450.51	10800.00*	4200.00	83411.10	176.41	−0.05	0.00
C109	75	8	82857.15	4600.00	83792.90	10800.00*	4400.00	83555.50	165.48	−0.28	−4.35
C201	75	8	188792.53	4000.00	189585.40	10800.00*	3800.00	189401.00	163.23	−0.10	−5.00
C202	75	8	188792.53	6400.00	192152.42	10800.00*	3800.00	189371.00	161.08	−1.45	−40.63
C203	75	8	188792.53	7500.00	193453.62	10800.00*	3800.00	189360.00	174.90	−2.12	−49.33
C204	75	8	188792.53	13800.00	200531.39	10800.00*	3800.00	189328.00	141.79	−5.59	−72.46
C205	75	8	188792.53	4000.00	189582.69	10800.00*	3800.00	189400.00	144.57	−0.10	−5.00
C206	75	8	188792.53	5000.00	190731.82	10800.00*	3800.00	189376.00	180.62	−0.71	−24.00
C207	75	8	188792.53	7100.00	193275.65	10800.00*	3800.00	189390.00	170.03	−2.01	−46.48
C208	75	8	188792.53	8800.00	194531.18	10800.00*	3800.00	189372.00	186.91	−2.65	−56.82
R101	75	8	38056.29	1970.00	40600.20	10800.00*	1880.00	40609.90	189.28	0.02	−4.57
R102	75	8	38056.29	2040.00	40913.94	10800.00*	1900.00	40437.50	162.13	−1.16	−6.86
R103	75	8	38056.29	8370.00	176468.82	10800.00*	1960.00	40252.60	147.57	−77.19	−76.58
R104	75	8	38056.29	3290.00	40584.42	10800.00*	2010.00	40256.90	188.58	−0.81	−38.91
R105	75	8	38056.29	2060.00	40643.62	10800.00*	1920.00	40368.50	172.37	−0.68	−6.80
R106	75	8	38056.29	4140.00	103148.58	10800.00*	1920.00	40295.60	157.16	−60.93	−53.62
R107	75	8	38056.29	4860.00	81617.99	10800.00*	2010.00	40332.90	173.12	−50.58	−58.64
R108	75	8	38056.29	5390.00	61602.75	10800.00*	2120.00	40234.80	156.94	−34.69	−60.67
R109	75	8	38056.29	2760.00	39933.80	10800.00*	1980.00	38825.00	171.10	−2.78	−28.26
R110	75	8	38056.29	4660.00	42288.53	10800.00*	1960.00	38715.10	182.79	−8.45	−57.94
R111	75	8	38056.29	5290.00	61206.47	10800.00*	2010.00	40281.90	186.65	−34.19	−62.00
R112	75	8	38056.29	3670.00	40971.58	10800.00*	1960.00	38660.40	177.68	−5.64	−46.59
R201	75	8	174160.82	2250.00	175775.01	10800.00*	1800.00	174859.00	152.31	−0.52	−20.00
R202	75	8	174160.82	5900.00	179505.22	10800.00*	1800.00	174767.00	145.66	−2.64	−69.49
R203	75	8	174160.82	8700.00	182253.80	10800.00*	1800.00	174601.00	171.49	−4.20	−79.31
R204	75	8	174160.82	5000.00	177907.86	10800.00*	1800.00	174472.00	173.79	−1.93	−64.00
R205	75	8	174160.82	2950.00	176368.20	10800.00*	1800.00	174659.00	141.53	−0.97	−38.98
R206	75	8	174160.82	3700.00	176973.52	10800.00*	1800.00	174597.00	178.87	−1.34	−51.35
R207	75	8	174160.82	4650.00	178021.12	10800.00*	1800.00	174524.00	173.78	−1.96	−61.29
R208	75	8	174160.82	5750.00	178926.32	10800.00*	1800.00	174445.00	150.08	−2.50	−68.70
R209	75	8	174160.82	6550.00	179798.66	10800.00*	1800.00	174563.00	176.27	−2.91	−72.52
R210	75	8	174160.82	5700.00	179194.44	10800.00*	1800.00	174605.00	150.63	−2.56	−68.42
R211	75	8	174160.82	6550.00	179843.07	10800.00*	1800.00	174522.00	166.47	−2.96	−72.52
RC101	75	8	38233.86	2520.00	39061.10	10800.00*	2520.00	38940.60	172.15	−0.31	0.00
RC102	75	8	38233.86	2640.00	39221.20	10800.00*	2550.00	38832.50	174.34	−0.99	−3.41
RC103	75	8	38233.86	3510.00	39820.43	10800.00*	2580.00	38717.80	183.92	−2.77	−26.50
RC104	75	8	38233.86	3480.00	39830.82	10800.00*	2700.00	38677.30	158.68	−2.90	−22.41
RC105	75	8	38233.86	2760.00	39258.60	10800.00*	2580.00	38866.70	167.90	−1.00	−6.52
RC106	75	8	38233.86	2910.00	39318.93	10800.00*	2610.00	38795.00	183.80	−1.33	−10.31
RC107	75	8	38233.86	3390.00	39692.26	10800.00*	2670.00	38676.20	165.33	−2.56	−21.24
RC108	75	8	38233.86	3480.00	39984.80	10800.00*	2550.00	38645.00	186.19	−3.35	−26.72
RC201	75	8	173677.24	2250.00	174872.25	10800.00*	2350.00	174546.00	159.46	−0.19	4.44
RC202	75	8	173677.24	4700.00	177692.77	10800.00*	2350.00	174332.00	162.91	−1.89	−50.00
RC203	75	8	173677.24	5550.00	178208.59	10800.00*	2400.00	174373.00	173.16	−2.15	−56.76
RC204	75	8	173677.24	6250.00	179071.17	10800.00*	2550.00	174241.00	185.53	−2.70	−59.20
RC205	75	8	173677.24	2900.00	175681.10	10800.00*	2400.00	174444.00	152.35	−0.70	−17.24
RC206	75	8	173677.24	2650.00	175218.43	10800.00*	2450.00	174419.00	147.26	−0.46	−7.55
RC207	75	8	173677.24	5300.00	178097.38	10800.00*	2550.00	174346.00	144.99	−2.11	−51.89
RC208	75	8	173677.24	3800.00	176137.37	10800.00*	2550.00	174242.00	155.45	−1.08	−32.89

TABLE C.9: Results on the 100-customer instances

Instance set	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	E_4^v				HESA				
			LP relaxation	Vehicle cost	Total cost	Time (s.)	Vehicle cost	Total cost	Time (s.)	Dev_{TC}	Dev_{VC}
C101	100	10	84345.45	5700.00	85199.09	10800.00*	5700.00	85199.09	297.32	0.00	0.00
C102	100	10	84345.45	6300.00	86605.08	10800.00*	5700.00	85209.00	304.48	−1.61	−9.52
C103	100	10	84345.45	8300.00	88128.63	10800.00*	5700.00	85190.80	305.06	−3.33	−31.33
C104	100	10	84345.45	11000.00	90622.34	10800.00*	6500.00	85509.90	319.80	−5.64	−40.91
C105	100	10	84345.45	5700.00	85258.60	10800.00*	5700.00	85208.50	319.99	−0.06	0.00
C106	100	10	84345.45	5700.00	85461.36	10800.00*	5700.00	85204.90	309.38	−0.30	0.00
C107	100	10	84345.45	5700.00	85306.67	10800.00*	5700.00	85167.30	307.85	−0.16	0.00
C108	100	10	84345.45	5900.00	85570.18	10800.00*	5700.00	85222.80	294.20	−0.41	−3.39
C109	100	10	84345.45	9100.00	89617.66	10800.00*	5700.00	85198.00	312.34	−4.93	−37.36
C201	100	10	190083.07	6000.00	191687.14	10800.00*	5200.00	190876.00	271.94	−0.42	−13.33
C202	100	10	190074.71	9900.00	195858.81	10800.00*	5200.00	190865.00	259.04	−2.55	−47.47
C203	100	10	190075.95	17900.00	304096.37	10800.00*	5200.00	190864.00	308.09	−37.24	−70.95
C204	100	10	190074.71	98300.00	1154483.31	10800.00*	5200.00	190848.00	337.23	−83.47	−94.71
C205	100	10	190077.15	8400.00	194711.89	10800.00*	5200.00	190863.00	304.44	−1.98	−38.10
C206	100	10	190084.61	15600.00	202111.00	10800.00*	5200.00	190861.00	321.63	−5.57	−66.67
C207	100	10	190075.31	13200.00	199617.46	10800.00*	5200.00	190856.00	330.40	−4.39	−60.61
C208	100	10	190074.82	13800.00	200924.51	10800.00*	5200.00	190866.00	333.46	−5.01	−62.32
R101	100	10	39159.76	2770.00	42095.43	10800.00*	2560.00	41782.20	292.25	−0.74	−7.58
R102	100	10	39146.09	7090.00	105133.04	10800.00*	2600.00	41577.10	324.69	−60.45	−63.33
R103	100	10	39149.89	10860.00	225357.73	10800.00*	2620.00	41398.40	293.22	−81.63	−75.87
R104	100	10	39151.07	12740.00	227609.71	10800.00*	2680.00	41269.90	289.51	−81.87	−78.96
R105	100	10	39154.42	4790.00	81860.12	10800.00*	2670.00	41522.20	250.56	−49.28	−44.26
R106	100	10	39145.82	12070.00	226936.85	10800.00*	2650.00	41397.50	291.75	−81.76	−78.04
R107	100	10	39145.98	14680.00	230534.40	10800.00*	2680.00	41329.90	339.02	−82.07	−81.74
R108	100	10	39148.67	12390.00	227135.02	10800.00*	2680.00	40574.50	336.96	−82.14	−78.37
R109	100	10	39161.60	8300.00	106140.17	10800.00*	2710.00	40135.40	247.20	−62.19	−67.35
R110	100	10	39146.09	8800.00	68339.42	10800.00*	2600.00	40052.30	267.95	−41.39	−70.45
R111	100	10	39145.98	12590.00	228378.35	10800.00*	2600.00	40614.10	296.14	−82.22	−79.35
R112	100	10	39165.22	10900.00	226034.92	10800.00*	2730.00	40015.60	250.16	−82.30	−74.95
R201	100	10	174794.14	6400.00	180128.14	10800.00*	2250.00	175457.00	329.90	−2.59	−64.84
R202	100	10	174794.09	14450.00	188288.16	10800.00*	2250.00	175303.00	256.58	−6.90	−84.43
R203	100	10	174794.09	12850.00	186985.40	10800.00*	2250.00	175186.00	289.80	−6.31	−82.49
R204	100	10	174794.09	11250.00	184455.99	10800.00*	2250.00	175037.00	297.46	−5.11	−80.00
R205	100	10	174794.09	12450.00	186185.59	10800.00*	2250.00	175237.00	339.93	−5.88	−81.93
R206	100	10	174794.14	14150.00	187873.20	10800.00*	2250.00	175168.00	268.04	−6.76	−84.10
R207	100	10	174794.09	11950.00	185389.11	10800.00*	2250.00	175096.00	270.70	−5.55	−81.17
R208	100	10	174794.09	7500.00	180572.09	10800.00*	2250.00	175010.00	322.05	−3.08	−70.00
R209	100	10	174794.09	14700.00	188632.91	10800.00*	2250.00	175156.00	288.98	−7.14	−84.69
R210	100	10	174794.14	14850.00	188607.92	10800.00*	2250.00	175189.00	311.21	−7.11	−84.85
R211	100	10	174794.14	12200.00	185459.83	10800.00*	2250.00	175051.00	247.03	−5.61	−81.56
RC101	100	10	39180.69	3510.00	40800.29	10800.00*	3300.00	40111.60	323.52	−1.69	−5.98
RC102	100	10	39186.00	7530.00	44712.85	10800.00*	3330.00	39940.70	328.25	−10.67	−55.78
RC103	100	10	39199.13	7890.00	70169.17	10800.00*	3510.00	39828.90	316.82	−43.24	−55.51
RC104	100	10	39181.17	7530.00	67342.83	10800.00*	3420.00	39727.00	317.03	−41.01	−54.58
RC105	100	10	39203.16	4710.00	42222.87	10800.00*	3390.00	39990.10	281.31	−5.29	−28.03
RC106	100	10	39176.91	5100.00	42502.09	10800.00*	3420.00	39912.40	323.17	−6.09	−32.94
RC107	100	10	39176.91	6930.00	44066.07	10800.00*	3420.00	39827.50	275.89	−9.62	−50.65
RC108	100	10	39187.37	5790.00	42617.67	10800.00*	3450.00	39775.20	282.44	−6.67	−40.41
RC201	100	10	174428.16	4700.00	177634.48	10800.00*	3150.00	175461.00	282.26	−1.22	−32.98
RC202	100	10	174429.52	8500.00	181651.48	10800.00*	3100.00	175284.00	278.25	−3.51	−63.53
RC203	100	10	174428.48	11700.00	185169.73	10800.00*	3250.00	175271.00	313.55	−5.35	−72.22
RC204	100	10	174435.83	37550.00	873972.63	10800.00*	1350.00	173214.00	260.57	−80.18	−96.40
RC205	100	10	174432.47	9850.00	183169.93	10800.00*	3100.00	175409.00	317.51	−4.24	−68.53
RC206	100	10	174427.49	11600.00	184800.00	10800.00*	3150.00	175333.00	318.89	−5.12	−72.84
RC207	100	10	174426.83	6350.00	179681.12	10800.00*	3250.00	175280.00	300.42	−2.45	−48.82
RC208	100	10	174436.39	10300.00	183920.02	10800.00*	3350.00	175210.00	287.60	−4.74	−67.48

Appendix D

Supplement to Chapter 5

Tables D.1 to D.4 present the detailed computational results on the 75, 100, 150, and 200-node FSMPRP instances. In all tables, columns TD, CO_2, FEC, DC, VC, TC and Time are as explained in the main body of text. Column Mix shows the resulting fleet composition where L, M and H refer to light, medium and heavy vehicles and the subscripts denote the number of such vehicles used in the fleet.

TABLE D.1: Computational results on the 75-node FSMPRP instances

Instance	HEA++							
	TD	CO_2	FEC	DC	VC	Mix	TC	Time
UK75_01	1615.33	373.65	225.48	295.05	300	M^5	820.52	2.37
UK75_02	1295.66	289.32	174.59	268.08	282	L^1M^4	724.67	3.39
UK75_03	1565.67	349.94	211.17	274.58	282	L^1M^4	767.75	3.56
UK75_04	1322.99	298.26	179.99	274.17	282	L^1M^4	736.16	3.74
UK75_05	1559.89	358.89	216.57	274.98	300	M^5	791.56	3.27
UK75_06	1588.73	366.90	221.41	287.88	300	M^5	809.29	3.34
UK75_07	1586.55	367.38	221.70	294.25	300	M^5	815.94	2.81
UK75_08	1721.67	361.28	218.01	299.22	306	L^3M^3	823.23	3.35
UK75_09	1609.60	354.01	213.62	278.57	282	L^1M^4	774.19	3.49
UK75_10	1575.76	365.46	220.54	285.23	300	M^5	805.77	3.71
UK75_11	1161.03	255.72	154.31	247.39	282	L^1M^4	683.70	3.47
UK75_12	1445.46	334.10	201.61	260.38	300	M^5	761.99	3.19
UK75_13	1786.96	383.63	231.50	297.89	324	L^2M^4	853.39	2.59
UK75_14	1585.74	367.00	221.47	279.06	300	M^5	800.52	3.38
UK75_15	1707.01	366.13	220.94	296.41	324	L^2M^4	841.35	3.32
UK75_16	1536.76	356.44	215.09	277.45	300	M^5	792.54	3.93
UK75_17	1552.07	359.02	216.65	287.94	300	M^5	804.59	2.41
UK75_18	1483.53	327.60	197.69	274.30	282	L^1M^4	753.99	3.33
UK75_19	1467.55	328.35	198.14	269.84	282	L^1M^4	749.99	3.49
UK75_20	1519.60	351.64	212.20	280.38	300	M^5	792.57	3.19

TABLE D.2: Computational results on the 100-node FSMPRP instances

Instance	HEA++							
	TD	CO_2	FEC	DC	VC	Mix	TC	Time
UK100_01	2031.46	444.12	268.00	375.10	384	L^2M^5	1027.10	5.47
UK100_02	1970.65	428.68	258.69	360.19	384	L^2M^5	1002.87	4.38
UK100_03	1756.29	392.38	236.78	341.34	342	L^1M^5	920.11	4.65
UK100_04	1674.29	384.01	231.73	358.58	402	L^1M^6	992.32	4.74
UK100_05	1583.76	368.75	222.52	349.04	360	M^6	931.56	4.21
UK100_06	1898.02	430.76	259.94	376.31	402	L^1M^6	1038.25	5.63
UK100_07	1790.55	400.89	241.91	329.48	342	L^1M^5	913.39	5.19
UK100_08	1919.84	424.64	256.25	346.27	342	L^1M^5	944.51	4.52
UK100_09	1633.23	364.06	219.69	329.33	342	L^1M^5	891.02	4.69
UK100_10	1955.77	429.26	259.04	346.34	384	L^2M^5	989.38	4.19
UK100_11	1904.88	429.65	259.27	373.71	402	L^1M^6	1034.98	5.13
UK100_12	1603.76	361.60	218.21	319.75	342	L^1M^5	879.96	4.62
UK100_13	1944.83	424.91	256.41	360.45	384	L^2M^5	1000.86	4.78
UK100_14	2025.79	454.55	274.30	376.17	402	L^1M^6	1052.47	5.02
UK100_15	1983.24	459.42	277.24	386.71	420	M^7	1083.95	5.27
UK100_16	1695.44	392.00	236.55	327.87	360	M^6	924.42	4.73
UK100_17	1980.20	447.53	270.06	385.27	402	L^1M^6	1057.33	4.36
UK100_18	1788.57	401.39	242.22	338.59	342	L^1M^5	922.81	5.01
UK100_19	1645.89	382.38	230.75	333.86	360	M^6	924.60	4.99
UK100_20	2043.21	462.36	279.01	376.77	402	L^1M^6	1057.78	4.18

TABLE D.3: Computational results on the 150-node FSMPRP instances

Instance	HEA++							
	TD	CO_2	FEC	DC	VC	Mix	TC	Time
UK150_01	2142.69	486.39	293.51	490.91	522	L^1M^8	1306.42	6.28
UK150_02	2550.10	591.98	357.23	519.50	540	M^9	1416.73	6.78
UK150_03	2140.64	472.50	285.13	478.47	504	L^2M^7	1267.60	7.79
UK150_04	2385.71	555.33	335.11	520.18	540	M^9	1395.29	7.19
UK150_05	2284.05	521.09	314.45	493.46	522	L^1M^8	1329.91	6.47
UK150_06	2049.41	476.33	287.44	492.41	540	M^9	1319.85	7.13
UK150_07	2485.44	580.09	350.05	524.80	540	M^9	1414.86	7.34
UK150_08	2232.72	513.78	310.04	492.72	522	L^1M^8	1324.76	7.98
UK150_09	2587.83	581.73	351.04	529.16	564	L^2M^8	1444.20	6.37
UK150_10	2423.75	564.42	340.60	509.53	540	M^9	1390.13	6.93
UK150_11	2508.51	582.74	351.65	515.03	540	M^9	1406.69	7.96
UK150_12	2487.69	572.14	345.25	537.39	582	L^1M^9	1464.65	7.73
UK150_13	2437.12	556.33	335.72	506.81	522	L^1M^8	1364.52	6.67
UK150_14	2518.63	586.03	353.64	519.74	540	M^9	1413.38	6.67
UK150_15	2115.20	486.33	293.47	470.56	522	L^1M^8	1286.03	7.79
UK150_16	2515.94	570.23	344.11	517.33	522	L^1M^8	1383.43	7.07
UK150_17	2501.54	566.66	341.95	512.78	522	L^1M^8	1376.73	7.09
UK150_18	2389.19	554.28	334.48	502.72	540	M^9	1377.20	6.37
UK150_19	2509.16	584.23	352.55	523.01	540	M^9	1415.56	6.91
UK150_20	2710.19	606.91	366.24	536.72	564	L^2M^8	1466.96	6.09

TABLE D.4: Computational results on the 200-node FSMPRP instances

Instance	HEA++							
	TD	CO_2	FEC	DC	VC	Mix	TC	Time
UK200_01	2844.46	649.17	391.74	664.28	702	L^1M^{11}	1758.02	9.48
UK200_02	2829.05	642.55	387.75	617.43	642	L^1M^{10}	1647.17	11.27
UK200_03	2780.80	649.79	392.12	650.58	660	M^{11}	1702.70	10.43
UK200_04	2694.43	628.51	379.27	629.06	660	M^{11}	1668.33	11.48
UK200_05	3018.01	691.92	417.54	661.99	702	L^1M^{11}	1781.53	10.12
UK200_06	2590.47	603.91	364.43	631.82	660	M^{11}	1698.24	9.79
UK200_07	2798.22	648.55	391.37	648.49	702	L^1M^{11}	1741.86	13.64
UK200_08	2888.09	660.78	398.75	660.65	702	L^1M^{11}	1761.40	8.64
UK200_09	2697.92	615.58	371.47	610.83	642	L^1M^{10}	1624.30	9.19
UK200_10	3036.77	696.08	420.05	668.45	702	L^1M^{11}	1790.50	11.28
UK200_11	2677.34	620.81	374.63	641.25	702	L^1M^{11}	1717.88	9.49
UK200_12	3147.88	731.72	441.56	642.67	660	M^{11}	1744.23	10.83
UK200_13	3052.43	712.56	430.00	639.22	660	M^{11}	1729.22	9.18
UK200_14	2845.53	648.49	391.33	648.09	702	L^1M^{11}	1741.42	9.49
UK200_15	2953.85	686.59	414.32	637.81	660	M^{11}	1712.13	11.28
UK200_16	2780.94	649.33	391.84	633.79	660	M^{11}	1685.63	9.79
UK200_17	3092.09	711.40	429.29	655.59	702	L^1M^{11}	1786.88	12.27
UK200_18	2832.24	656.36	396.08	645.11	702	L^1M^{11}	1743.19	9.79
UK200_19	2636.59	604.11	364.55	597.29	642	L^1M^{10}	1603.83	9.79
UK200_20	2944.54	679.61	410.11	657.80	702	L^1M^{11}	1769.91	11.35

Appendix E

Supplement to Chapter 6

Tables E.1–E.4 present the detailed computational results on the 25-, 50-, 75- and 100-customer instances.

TABLE E.1: Computational results on the 25-customer instances.

Instance	$\|\mathcal{N}_c\|$	$\|\mathcal{N}_0\|$	P-L-HALNS							
			Opened depots	Total distance (km)	CO_2 emissions (kg)	Fuel and CO_2 emissions costs (£)	Depot cost (£)	Vehicle cost (£)	Total cost (£)	Time (s)
CC25_1	25	4	(1,1,0)	36.10	26.12	15.76	8500.00	109.00	8624.76	5.16
CC25_2	25	4	(2,0,0)	45.59	10.09	6.09	10000.00	109.00	10115.10	5.20
CC25_3	25	4	(1,0,1)	37.60	2.63	1.59	7000.00	102.00	7103.59	5.50
CC25_4	25	4	(1,0,1)	40.24	29.35	17.71	7000.00	109.00	7126.71	6.02
CC25_5	25	4	(1,0,1)	27.47	39.62	23.91	7000.00	102.00	7125.91	5.42
SU25_1	25	4	(0,1,1)	54.65	9.68	5.84	5500.00	102.00	5607.84	5.48
SU25_2	25	4	(1,0,1)	63.99	8.93	5.39	7000.00	109.00	7114.39	5.48
SU25_3	25	4	(0,1,0)	52.97	45.60	27.52	3500.00	98.00	3625.52	5.42
SU25_4	25	4	(1,1,0)	46.88	0.15	0.09	5500.00	102.00	5602.09	5.38
SU25_5	25	4	(0,1,1)	61.55	38.78	23.40	5500.00	102.00	5625.40	5.44
R25_1	25	4	(0,1,0)	73.07	28.63	17.28	3500.00	98.00	3615.28	5.36
R25_2	25	4	(0,1,1)	63.54	21.66	13.07	5500.00	102.00	5615.07	5.38
R25_3	25	4	(0,1,0)	68.13	10.17	6.14	3500.00	102.00	3608.14	5.36
R25_4	25	4	(1,1,0)	58.57	26.00	15.69	8500.00	109.00	8624.69	5.32
R25_5	25	4	(1,0,1)	60.58	10.95	6.61	7000.00	109.00	7115.61	5.62

Table E.2: Computational results on the 50-customer instances.

Instance	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	P-L-HALNS							
			Opened depots	Total distance (km)	CO_2 emissions (kg)	Fuel and CO_2 emissions costs (£)	Depot cost (£)	Vehicle cost (£)	Total cost (£)	Time (s)
CC50_1	50	6	(1,0,2)	92.86	20.74	12.52	9000.00	180.00	9192.52	33.56
CC50_2	50	6	(2,1,0)	70.76	19.83	11.97	13500.00	180.00	13692.00	33.80
CC50_3	50	6	(1,1,1)	75.47	36.37	21.94	10500.00	169.00	10690.90	31.08
CC50_4	50	6	(2,0,1)	77.13	21.10	12.73	12000.00	169.00	12181.70	31.14
CC50_5	50	6	(2,1,0)	87.07	20.34	12.27	13500.00	180.00	13692.30	30.97
SU50_1	50	6	(1,0,2)	144.13	24.37	14.71	10500.00	180.00	10694.70	30.92
SU50_2	50	6	(0,1,2)	134.87	41.57	25.08	7500.00	169.00	7694.08	30.93
SU50_3	50	6	(0,1,2)	123.49	1.76	1.06	7500.00	169.00	7670.06	31.30
SU50_4	50	6	(0,1,2)	123.17	20.70	12.49	7500.00	180.00	7692.49	30.85
SU50_5	50	6	(1,0,2)	100.09	18.79	11.34	7500.00	180.00	7691.34	31.00
R50_1	50	6	(1,1,1)	116.06	15.38	9.28	9000.00	169.00	9178.28	30.74
R50_2	50	6	(0,1,2)	133.68	21.97	13.26	7500.00	169.00	7682.26	31.62
R50_3	50	6	(0,1,2)	136.19	39.99	24.13	7500.00	169.00	7693.13	32.03
R50_4	50	6	(0,1,2)	131.86	19.17	11.57	7500.00	169.00	7680.57	31.16
R50_5	50	6	(0,1,2)	125.23	20.49	12.36	9000.00	169.00	9181.36	32.07

Table E.3: Computational results on the 75-customer instances.

Instance	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	P-L-HALNS							
			Opened depots	Total distance (km)	CO_2 emissions (kg)	Fuel and CO_2 emissions costs (£)	Depot cost (£)	Vehicle cost (£)	Total cost (£)	Time (s)
CC75_1	75	8	(2,1,1)	110.65	27.51	16.60	15500.00	240.00	15756.60	71.18
CC75_2	75	8	(2,1,1)	113.19	29.38	17.73	15500.00	282.00	15799.70	71.62
CC75_3	75	8	(2,1,1)	117.68	30.03	18.12	15500.00	240.00	15758.10	68.32
CC75_4	75	8	(3,1,0)	95.75	28.62	17.27	18500.00	282.00	18799.30	64.60
CC75_5	75	8	(2,1,1)	93.41	26.61	16.06	15500.00	240.00	15756.10	65.33
SU75_1	75	8	(0,2,2)	219.38	35.14	21.20	11000.00	320.00	11341.20	64.55
SU75_2	75	8	(0,2,2)	201.09	32.91	19.86	11000.00	324.00	11343.90	64.72
SU75_3	75	8	(0,2,2)	169.80	26.54	16.02	11000.00	229.00	11245.00	63.34
SU75_4	75	8	(0,2,2)	201.77	33.19	20.03	11000.00	240.00	11260.00	65.21
SU75_5	75	8	(0,2,2)	209.93	35.85	21.64	12500.00	331.00	12852.60	63.64
R75_1	75	8	(0,2,2)	172.62	29.32	17.70	11000.00	289.00	11306.70	63.91
R75_2	75	8	(0,2,2)	212.28	33.72	20.35	11000.00	278.00	11298.30	64.69
R75_3	75	8	(0,2,2)	207.77	61.32	37.00	11000.00	324.00	11361.00	63.58
R75_4	75	8	(0,2,2)	191.46	34.50	20.82	11000.00	289.00	11309.80	64.13
R75_5	75	8	(0,3,1)	205.30	32.22	19.44	11000.00	320.00	11339.40	63.80

Table E.4: Computational results on the 100-customer instances.

Instance	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	P-L-HALNS							
			Opened depots	Total distance (km)	CO_2 emissions (kg)	Fuel and CO_2 emissions costs (£)	Depot cost (£)	Vehicle cost (£)	Total cost (£)	Time (s)
CC100_1	100	10	(3,1,2)	145.59	34.20	20.64	22500.00	349.00	22869.60	159.53
CC100_2	100	10	(2,1,2)	125.57	36.64	22.11	17500.00	342.00	17864.10	176.07
CC100_3	100	10	(3,2,0)	143.70	70.06	42.27	22000.00	391.00	22433.30	176.89
CC100_4	100	10	(3,1,2)	143.15	38.26	23.09	22500.00	360.00	22883.10	178.28
CC100_5	100	10	(2,2,1)	143.21	38.88	23.46	19000.00	349.00	19372.50	156.76
SU100_1	100	10	(0,3,3)	236.50	41.44	25.01	14500.00	384.00	14909.00	167.88
SU100_2	100	10	(0,2,3)	242.52	43.71	26.38	14500.00	426.00	14952.40	146.06
SU100_3	100	10	(0,2,3)	246.52	38.64	23.32	14500.00	433.00	14956.30	147.51
SU100_4	100	10	(0,2,3)	255.20	44.68	26.96	16500.00	360.00	16887.00	147.15
SU100_5	100	10	(0,2,3)	241.49	41.61	25.11	14500.00	384.00	14909.10	147.17
R100_1	100	10	(0,3,2)	250.47	106.86	64.48	14500.00	380.00	14944.50	148.83
R100_2	100	10	(0,3,2)	262.33	43.66	26.35	13000.00	384.00	13410.30	147.92
R100_3	100	10	(0,3,2)	263.72	43.55	26.28	13000.00	398.00	13424.30	159.90
R100_4	100	10	(0,3,3)	257.90	42.53	25.67	13000.00	384.00	13409.70	157.72
R100_5	100	10	(0,3,2)	262.70	36.68	22.13	14500.00	433.00	14955.10	176.27

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