Appendix 8: Spatial thinking and visualisation

contributed by Keith Jones

1 Spatial Thinking

Among the many modalities of human thought, two are particularly common: verbal reasoning and spatial reasoning. Verbal reasoning is the process of forming ideas by assembling symbols into meaningful sequences. Spatial reasoning is the process of forming ideas through the spatial relationships between objects. It is the form of mental activity which makes it possible to create spatial images and manipulate them in the course of solving practical and theoretical problems. Because space is a fundamental feature of the human environment, spatial thinking plays a crucial role in even the most ordinary human problem solving. People process spatial information when they navigate, when they manipulate objects, and when they design them. Geometry is an example of spatial reasoning at work.

The mathematician Jacques Hadamard argued that much of the thinking that is required in higher mathematics is spatial in nature. Einstein's comments on thinking in images are well known. Numerous mathematicians report using spatial skills when they visualise mathematical relations. Physical scientists also report using such skills when they visualise and reason about the models of the physical world.

Spatial thinking is an important component in solving many types of mathematics problems. This includes the use of diagrams and drawings, searching for patterns and structures, graphing numbers, considering how fractions can be broken down into geometrical regions, conceptualising mathematical functions, and so on. Spatial thinking has an important role in mathematics achievement, with positive correlations found between spatial ability and mathematics achievement at all levels.

Investigative tasks in geometry and measurement provide opportunities for students to analyse mathematically their spatial environment, to describe characteristics and relationships of geometric objects, and to use number concepts in a geometric context. In this way, students develop and use spatial thinking.

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2 Visualisation

Visualisation is generally taken to refer to "*the ability to represent, transform, generate, communicate, document, and reflect on visual information*" [Hershkowitz, 1989]. As such, it is a crucial component of learning geometrical concepts. Moreover, a visual image, by virtue of its concreteness, is "*an essential factor for creating the feeling of self-evidence and immediacy*" [Fischbein, 1987, p.101]. Therefore, it "*not only organizes data at hand in meaningful structures, but it is also an important factor guiding the analytical development of a solution*" [ibid].

Visualisation is essential to problem solving and spatial reasoning as it enables people to use concrete means to grapple with abstract images. In mathematics the process of visualisation entails the process of forming and manipulating images, whether with paper and pencil, technology or mentally, to investigate, discover and understand. The original meaning of the Greek word for 'to prove' (*deiknumi*) was to make visible or show.

There are serious reasons for being good at visualisation. From 2-dimensional pictures, it is often useful to determine the possible shapes of 3-dimensional objects, and vice versa. For instance, doctors and dentists and others in the health profession often need to determine from X-rays or MRI pictures the precise position and shape of an organ or bone or tooth or tumour. Geometry provides the concepts that assist in this work - concepts like cross section and contour curve.

Mathematics has a long tradition of interest in visualisation methods. Such classic works as *Anschauliche Geometrie* by Hilbert and Cohn-Vossen (translated as *Geometry and the Imagination*) demonstrate the influence of this visual approach to mathematics (indeed, it is worth noting that the English translation of the title only barely does justice to the complex nuances of the German *anschaulich*).

While visualisation process has been a cornerstone of the mathematical reasoning process since the times of the ancient geometers, the advent of high-performance interactive computer graphics systems has opened a new era that is still evolving. Mathematical visualisation is about much more than 'pretty' graphics; it has become a mathematical discipline and involves concepts in mathematics with growing implications and applications across a range of disciplines. The aim of mathematical visualisation is to offer efficient visualisation tools for many areas of mathematics, thereby creating tangible experiences of abstract mathematical objects and concepts. Typical geometric problems of interest to mathematical visualisation applications involve both static structures, such as real or complex manifolds, and changing structures requiring animation. In practice, the emphasis is on manifolds of dimension two or three embedded in three or four-dimensional spaces due to the practical limitations of holistic human spatial perception.

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