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Modal Rayleigh-like streaming in layered acoustofluidic devices

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Classical Rayleigh streaming is well known and can be modelled using Nyborg's limiting velocity method as driven by fluid velocities adjacent to the walls parallel to the axis of the main acoustic resonance. We have demonstrated previously the existence and the mechanism of four-quadrant transducer plane streaming patterns in thin-layered acoustofluidic devices which are driven by the limiting velocities on the walls perpendicular to the axis of the main acoustic propagation. We have recently found experimentally that there is a third case which resembles Rayleigh streaming but is a more complex pattern related to three-dimensional cavity modes of an enclosure. This streaming has vortex sizes related to the effective wavelength in each cavity axis of the modes which can be much larger than those found in the one-dimensional case with Rayleigh streaming. We will call this here *modal Rayleigh-like streaming* and show that it can be important in layered acoustofluidic manipulation devices. This paper seeks to establish the conditions under which each of these is dominant and shows how the limiting velocity field for each relates to different parts of the complex acoustic intensity patterns at the driving boundaries. © 2016 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4939590>]

I. INTRODUCTION

In resonant acoustofluidic particle manipulation devices, acoustic streaming flows are typically found in addition to the acoustic radiation forces. These are generally considered to be a disturbance as they place a practical lower limit on the particle size that can be manipulated by the primary acoustic radiation force.^{1,2} However, acoustic streaming can also play an active role in the functioning of such systems, such as particle trapping,^{3–7} two-dimensional particle focusing,⁸ and particle separation.⁹ Most acoustofluidic particle manipulation devices utilise standing wave fields, and the acoustic streaming field is generally dominated by boundary-driven streaming which arises from the acoustic attenuation within the acoustic boundary layer due to the non-slip condition on the walls of the fluid channel. Another significant streaming pattern, Eckart streaming,¹⁰ requires acoustic absorption over longer distances (as is the case in surface acoustic wave devices^{11,12}) than those found in these bulk acoustic wave resonators. Experimental observation and modelling of surface acoustic wave devices by Tan *et al.*¹³ reveal a rich pattern of streaming vortices in such devices.

Classical boundary-driven streaming includes streaming vortices both inside and outside of the acoustic boundary layer, which are respectively referred to as Schlichting streaming¹⁴ (or inner streaming) and Rayleigh streaming^{15,16} (or outer streaming) in recognition of their contributions to analysing these streaming patterns. Following Rayleigh's early work, a series of modifications to his solution have been proposed.^{17–19} The boundary-driven streaming vortices both inside and outside of the acoustic boundary layer can be modelled by considering Reynolds stresses, the volume forces representing the time-averaged acoustic momentum flux due to the acoustic dissipation near the no-slip walls.²⁰ Alternatively, the streaming field in the bulk of the fluid can be effectively modelled using the

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limiting velocity method,^{21,22} in which the acoustic boundary layer with its non-slip boundary condition on the walls parallel to the axis of acoustic resonances is replaced by a slip boundary condition with a limiting velocity derived from the acoustic field. The former method is computationally expensive, especially for three-dimensional (3D) models, as tiny mesh elements near the no-slip walls are required to resolve the acoustic and streaming fields within the acoustic boundary layer. In contrast, the limiting velocity method is more computationally efficient and is valid so long as the boundaries have a radius of curvature that is much larger than the viscous boundary layer thickness. In practical acoustofluidic particle manipulation devices working in the MHz region this approximation is usually valid, and since the fluid channel dimensions are typically several orders larger than the acoustic boundary layer thicknesses only the acoustic streaming field in the bulk of the fluid is usually of interest.

While Rayleigh streaming has been recently extensively studied within the field of acoustic particle trapping and manipulation,^{23–30} there are acoustic streaming patterns observed experimentally in acoustofluidic particle manipulation devices that cannot be explained by Rayleigh's classical theory.^{3,31–34} Recently, we have explained the mechanism behind the four-quadrant transducer-plane streaming, which has a vortex pattern parallel to the transducer face and is driven by the limiting velocity on the walls perpendicular to the axis of main acoustic propagation.³⁵ In this case, the limiting velocity field was shown to be closely related to the sound intensity field at the driving boundaries. The expression for the limiting velocity has terms corresponding to acoustic velocity gradients in different directions. Depending on which of these is dominant, different acoustic streaming patterns arise, e.g., transducer-plane streaming³⁵ (streaming vortex parallel to the driving surface, Fig. 1(b)) and Rayleigh streaming²⁸ (streaming vortex perpendicular to the driving surface, Fig. 1(d)) corresponding to the rotational and irrotational features of, respectively, the active and reactive intensity patterns in acoustic fields.³⁶

In this paper, we address the conditions under which each of these streaming patterns discussed above is dominant in layered acoustofluidic particle manipulation devices with different channel aspect ratios and show how the limiting velocity for each case relates to different parts of the complex acoustic intensity fields at the driving boundaries. We also investigate a case, here called modal Rayleigh-like streaming, which has not been discussed or shown experimentally before. In this regime, the streaming is driven by boundaries perpendicular to those driving conventional Rayleigh patterns

FIG. 1. Schematic illustrations of: (a) a layered acoustofluidic particle manipulation device; (b) four-quadrant transducer-plane streaming in a fluid channel with low aspect ratio in its yz cross section ($h/w \leq 1/20$); (c) Modal Rayleigh-like streaming (as introduced in this paper) in a fluid channel with medium aspect ratio ($1/3 \leq h/w \leq 1/2$); and (d) Rayleigh streaming in a fluid channel with high aspect ratio ($h/w > 1$), where the waveforms on the surfaces of the 3D volumes show respectively the standing wave fields established in these fluid channels, the planes through the half-heights of the fluid volumes are the pressure nodal planes, and the rolls of cylinders show the paralleled streaming vortices with the arrows representing their orientations.

and has a roll size greater than the quarter wavelength of the main acoustic resonance: the scale is instead related to a cavity mode, Fig. 1(c).

II. MODELLING

The numerical process was conducted in COMSOL 4.4.³⁷ Fig. 1(a) shows the schematic presentation of a classical layered acoustofluidic particle manipulation device, which is, typically, composed of four layers: the transducer, the carrier layer, the fluid channel, and the reflector layer.^{38,39} In this paper, only the fluid layer was considered for the numerical efficiency of 3D acoustic and streaming simulations, which is appropriate as it has been shown previously³⁵ that this simplified model is sufficient to demonstrate the fundamental behaviour of streaming fields. For a given application, however, a full model may be required to capture more complex combinations of boundary movement to determine which resonance is excited in the fluid layer.²⁸ In this case, we excite a particular cavity mode (see below) through applying a normal acceleration boundary condition on the bottom surfaces of the fluid channels.

In order to investigate the sensitivity of the acoustic streaming patterns to the channel dimensions, a series of models with various channel dimensions were considered, which are summarised in Table I, where l , w , and h are the dimensions of the fluid channel along the coordinates x , y , and z , respectively. Various channel heights ranging from 0.2 to 2 mm for four different channel widths were considered. We restrict our models to the thin-layered acoustofluidic devices ($h/w < 1$ and $h/l < 1$) for the reason that the purpose of this paper is to demonstrate the transition from streaming fields induced by limiting velocities dominated by active sound intensity to fields dominated by the reactive intensity in 3D resonant cavity modes. However, in high aspect-ratio device ($h/w > 0.5$), the acoustic field is usually more closely approximated by a one-dimensional (1D) acoustic resonance,^{27,28} leading to classical Rayleigh streaming,¹⁵ which is a different pattern to the others discussed here. Furthermore, it was found that in the thin-layered regime the contribution of the acoustic streaming field generated by the side boundaries ($y = \pm w/2$) to the overall streaming field in the bulk of the fluid channel is small. Hence, in this paper, only the driving terms on the top and bottom boundaries ($z = \pm h/2$) were taken into consideration for the acoustic streaming simulations unless otherwise stated.

First, the COMSOL “Pressure Acoustics, Frequency Domain” interface was used to simulate the first-order acoustic fields, by solving the harmonic, linearized acoustic problem, taking the form

$$\nabla^2 p = -\frac{\omega^2}{c^2} p, \quad (1)$$

where p is the complex pressure, ω is the angular frequency, and c is the sound speed in the fluid. While there are various resonant acoustofluidic systems, we investigated here the half-wave resonance in the z -direction, which is a widely used system for particle and cell manipulation.^{38–47} The origin of the coordinates in these models was set at the centre of the fluid channels such that the fluid channels are located within coordinates: $-l/2 \leq x \leq l/2$, $-w/2 \leq y \leq w/2$, $-h/2 \leq z \leq h/2$. We excited in each case the standing wave field through a “normal acceleration” boundary condition on the bottom wall. Energy gradients are created by the spatially varying source of excitation in combination with radiation boundary conditions at the two ends of the flow channels ($x = \pm l/2$). The remaining walls were set as sound reflecting boundary conditions.

We showed previously that for the four-quadrant transducer-plane streaming in a resonant cavity the streaming pattern was largely insensitive to the pattern of wall accelerations used to create the

TABLE I. Channel dimensions modelled.

Case	Channel dimensions
a	$l = 10$ mm, $w = 6$ mm, h : 0.2 – 2 mm
b	$l = 12$ mm, $w = 8$ mm, h : 0.2 – 2 mm
c	$l = 14$ mm, $w = 10$ mm, h : 0.2 – 2 mm
d	$l = 16$ mm, $w = 12$ mm, h : 0.2 – 2 mm

resonance. We thus selected the acceleration distribution used previously,³⁵ i.e., $a_n = a_0 e^{ax^2 + by^2}$ with $a_0 = 5.9 \times 10^4 \text{ m} \cdot \text{s}^{-2}$ and $a = b = -2.2 \times 10^5 \text{ m}^{-2}$. These values match those used in our previous modelling where we demonstrated there that the results are not sensitive to these values. A frequency sweep study was first used to find the half-wave resonant frequency in the z -direction of these 3D fluid channels, looking for the frequency with maximum energy density in the fluid. The excitation used above created the required resonance in all the cases examined, shown in Fig. 2, where the acoustic pressure distributions on the surfaces of two models are presented. It can be seen that a similar standing wave field was established in both models. In addition to the main resonance in the z -direction, the acoustic pressure field in the y -direction has a one-wavelength standing wave variation. There is a smaller gradient in the x -direction resulting from a resonance formed by reflections from the plane-wave radiation boundary conditions at $x = \pm l/2$; Although these boundaries are formulated with the intention of transmitting acoustic energy, in a closed duct significant reflections do occur, and mimic the conditions we expect in the experimental device. Thus, the overall cavity mode in the device is a (1, 2, 1) mode. In each channel dimension shown in Table I, the (1, 2, 1) mode is found to show a similar acoustic field pattern (Fig. 2). It will be shown later that the four-quadrant transducer-plane streaming pattern observed in thin-layered acoustofluidic particle manipulation devices^{3,35} and the modal Rayleigh-like streaming discussed below are a result of the z - and y -axes acoustic gradients of this particular cavity mode.

In the second step, the *limiting velocity method*, introduced by Nyborg,²¹ modified by Lee and Wang²² and applied by Lei *et al.*^{28,35} for 3D simulations, was used to solve the acoustic streaming fields in these layered acoustofluidic devices. Driven by the limiting velocities, the COMSOL “*Creeping Flow*” interface was used to simulate the acoustic streaming fields. While the first order part of the perturbation expansion accounts for effects relating inertia and compressibility to generate the second order Reynolds stresses, at second order the time averaged streaming field can often (for the low velocities considered here) be considered non-inertial and incompressible flow.²⁰ The governing equations for the streaming velocities, $\mathbf{u}_2$, and the associated acoustic pressure fields, p_2 , are

$$\nabla p_2 = \mu \nabla^2 \mathbf{u}_2, \quad (2a)$$

$$\nabla \cdot \mathbf{u}_2 = 0, \quad (2b)$$

FIG. 2. Simulated acoustic pressure distributions (normalised) on the surfaces of the fluid channels from two models. (a) and (b) colour plots of the pressure distributions on the surfaces of the fluid channels with dimensions of $w = 6 \text{ mm}$, $h = 0.3 \text{ mm}$ and $w = 6 \text{ mm}$, $h = 2 \text{ mm}$, respectively; (c) and (d) pressure distributions along lines B-B and A-A. The origin of the coordinates in these models was set at the centre of the fluid channels such that the fluid channels are located within coordinates: $-l/2 \leq x \leq l/2$, $-w/2 \leq y \leq w/2$, $-h/2 \leq z \leq h/2$.

where μ is the dynamic viscosity of the fluid. For the models shown in this paper, the limiting velocity equations on the driving boundaries ($z = \pm h/2$) take the form

$$u_L = -\frac{1}{4\omega} \text{Re} \left\{ u_1 \frac{du_1^*}{dx} + v_1 \frac{du_1^*}{dy} + u_1^* \left[(2+i) \left(\frac{du_1}{dx} + \frac{dv_1}{dy} + \frac{dw_1}{dz} \right) - (2+3i) \frac{dw_1}{dz} \right] \right\}, \quad (3a)$$

$$v_L = -\frac{1}{4\omega} \text{Re} \left\{ u_1 \frac{dv_1^*}{dx} + v_1 \frac{dv_1^*}{dy} + v_1^* \left[(2+i) \left(\frac{du_1}{dx} + \frac{dv_1}{dy} + \frac{dw_1}{dz} \right) - (2+3i) \frac{dw_1}{dz} \right] \right\}, \quad (3b)$$

where u_L and v_L are the two components of limiting velocities on a driving boundary, and u_1, v_1 , and w_1 are components of the complex acoustic velocity vector, $\mathbf{u}_1$ (the subscript helps distinguish this first order term from the second order streaming velocity, $\mathbf{u}_2$), along the coordinates x , y , and z , respectively. The superscript, $*$, represents the complex conjugate. A more detailed description of this method can be found in Lei *et al.*³⁵ The bottom and top walls ($z = \pm h/2$) were considered as limiting velocity boundary conditions while the remaining walls were no-slip boundary conditions unless otherwise stated. All the results presented in this paper are for fluid channels with $w = 6$ mm and $l = 10$ mm.

In layered acoustofluidic particle manipulation devices excited in the mode shown in Fig. 2, on the top and bottom surfaces ($z = \pm h/2$), du_1/dx can be neglected³⁵ as it has low contribution, compared to the other two acoustic gradients in the bulk of the fluid channel, to the divergence of the acoustic velocity vector. Therefore, the relative importance of the remaining terms in the limiting velocity equations will depend on the ratio of velocity gradients,

$$r = \frac{dw_1/dz}{dv_1/dy}. \quad (4)$$

We previously showed how the individual terms of the limiting velocity equation were proportional to the active and reactive sound intensity fields.^{28,35} Thus for $r \gg 1$, the limiting velocity field typically has a similar pattern to the active intensity field, while for $r \ll 1$, the limiting velocity field is dominated by the reactive intensity fields. More detailed explanations for this can be found in the supplementary material.⁴⁸ We show in the supplementary material⁴⁸ that, in the resonant cavity shown in Fig. 2, r can be approximated as

$$r \approx \left(\frac{w}{2h} \right)^2. \quad (5)$$

This means that the ratio of the significant acoustic velocity gradients, which determines which part of the complex acoustic intensity field dominates the limiting velocity field, is dependent on the aspect ratio of the fluid channel cross section. Our numerical models show that the approximation of Equation (5) is accurate to within an error of 2% in all the cases shown in Table I. This relationship implies that a change in the aspect ratio of the fluid channel cross sections can alter the acoustic streaming field from one pattern to another due to the rotational and irrotational properties of the active intensity and reactive intensity fields, respectively, and this is investigated further in the modelled 3D acoustic streaming fields shown below. The modelled results shown below are in devices where $w = 6$ mm unless otherwise stated.

The modelled active and reactive sound intensity fields at the bottom surface of the fluid channels ($z = -h/2$) for the pressure distributions shown in Fig. 2 are plotted in Figs. 3(a) and 3(b), respectively. It can be seen that the active (mean) intensity fields on the limiting velocity boundaries have a regular four-quadrant vortex pattern while the reactive intensity field is irrotational, diverging from the pressure maxima and concentrating at the pressure minima. Figs. 3(c)-3(g) plots the limiting velocity vector fields for five models with different fluid channel heights. It is clear that, for the first model where $r \gg 1$ ($h = 0.2$ mm), shown in Fig. 3(c), the limiting velocity field has a similar vortex pattern to that shown in Fig. 3(a) demonstrating that, as predicted, it is dominated by the active sound intensity field. In Figs. 3(d)-3(f), the aspect ratio, and hence r , becomes steadily smaller and the predicted transition towards a pattern dominated by the vortex pattern of the reactive intensity field is seen.

For the different channel aspect ratios, qualitatively different vortex patterns are also seen in the streaming velocity fields due to the varying contribution of the active and reactive intensity fields discussed above. Fig. 4 plots the simulated in-plane acoustic streaming fields on the central xy pressure

FIG. 3. Vector plots showing the relation between the active and reactive components of sound intensity and the limiting velocity fields on the bottom surface of the 3D fluid channels ($z = -h/2$) for various aspect ratios of their yz cross sections ($w = 6$ mm, $l = 10$ mm): (a) active (mean) intensity field; (b) reactive intensity field; (c)–(g) limiting velocity fields for, respectively, $h = 0.2$ mm, $h = 0.5$ mm, $h = 0.8$ mm, $h = 1$ mm, and $h = 2$ mm, where the length of arrows shows normalised magnitudes.

nodal plane ($z = 0$) of the fluid channels for the five models illustrated above. It can be seen that a well-defined four-quadrant transducer-plane streaming pattern^{3,35} is only observed in Fig. 4(a), where $h = 0.2$ mm. With the increase of h and the related decrease in r , the in-plane acoustic streaming vortices transition towards modal Rayleigh-like streaming (see discussion below), as seen in Fig. 4(e). In this case, the in-plane acoustic streaming field was found to be nearly irrotational, as the limiting velocities for this case are dominated by the irrotational reactive intensity field.

In addition, the acoustic streaming fields on the central yz cross section of the fluid channel ($x = 0$) for all these five models are plotted in Fig. 5. It can be seen that for the model $h = 0.2$ mm, where the transducer-plane streaming vortex pattern is observed, the streaming field is similar at all heights of the fluid channel (z -direction) as all the velocities are parallel to the bottom wall (limiting velocity boundary), Fig. 5(a). With the increase of h to 0.5 mm, Fig. 5(b), the streaming velocities are non-uniform in the z -direction near the channel centre ($y = 0$) and the side walls ($y = \pm w/2$), where the pressure antinodes for the one-wave mode in the y -direction are positioned. Further, small vortices appear and increase in size in these areas with the further increase of h , forming well developed modal Rayleigh-like streaming when the fluid height $h = 2$ mm, Fig. 5(e).

FIG. 4. Vector plots of the modelled acoustic streaming fields in the central xy plane ($z = 0$) of the fluid channels ($w = 6$ mm, $l = 10$ mm) for: (a) $h = 0.2$ mm, (b) $h = 0.5$ mm, (c) $h = 0.8$ mm, (d) $h = 1$ mm, and (e) $h = 2$ mm, where the maximum streaming velocities shown in models (a)–(e) are, respectively, 0.14, 1.08, 2.03, 2.83, and 12.38 $\mu\text{m/s}$, which are obtained from acoustic pressure amplitude of approximately 0.60 MPa. A transition from transducer plane streaming to modal Rayleigh-like streaming is seen with deeper channels. Limiting velocities on the side walls are set to zero for simplicity. The two-way arrows in (a) and (e) show the sizes of, respectively, the transducer plane streaming and modal Rayleigh-like streaming, where λ_y represents the modal wavelength in the y -direction of the fluid channel.

We refer to the vortex pattern seen Figs. 4(e) and 5(e) as “modal Rayleigh-like streaming” as it has a similar pattern to the classical Rayleigh streaming pattern, four vortex pairs per wavelength along the acoustic standing wave. However, it is “Rayleigh-like” as this streaming pattern depends on the resonant cavity mode excited in the 3D rectangular cavities rather than a simple one-dimensional acoustic standing wave. To clarify, we can take the model $h = 2$ mm as an example: the one-wavelength variation of acoustic pressure field generated in the y -direction of the fluid channel was excited at the resonant cavity mode at frequency $f \approx 0.45$ MHz, where the main standing wave field is established in the z -direction; thus, the observed streaming rolls have a width related to the cavity resonance, and its characteristic wavelength rather than the wavelength of sound in an unbounded medium as seen in conventional Rayleigh streaming. We show below experimental demonstration of both transducer-plane streaming patterns and modal Rayleigh-like streaming patterns in layered acoustofluidic particle manipulation devices with different channel aspect ratios. It should be emphasised that, in this section, we have not including the driving forces resulting from limiting velocities on the $y = \pm 3$ mm sidewalls; this has simplified the streaming fields to help show the transition we wish to illustrate; however, a model of the case when limiting velocities on all boundaries are included can be seen below.

III. EXPERIMENTAL

The experiments were conducted in glass capillaries, which have been widely used to form small acoustofluidic particle manipulation systems.^{3,35,49–54} In this paper, measurements performed

FIG. 5. Vector plots of the modelled acoustic streaming fields in the central yz plane ($x = 0$) of the fluid channels ($w = 6$ mm, $l = 10$ mm): (a) $h = 0.2$ mm, (b) $h = 0.5$ mm, (c) $h = 0.8$ mm, (d) $h = 1$ mm, and (e) $h = 2$ mm, where the arrows show streaming velocity fields and the red lines show the streamlines. The arrows in (c)–(e) have been normalised in order to show clearly the velocity vectors. A transition from transducer plane streaming to modal Rayleigh-like streaming is seen with deeper channels. Limiting velocities on the side walls are set to zero for simplicity.

in two glass capillaries are presented to show, respectively, the transducer-plane streaming and the *modal Rayleigh-like streaming* in layered acoustofluidic particle manipulation devices. The two glass capillaries (Vitrocom, USA) have inner dimensions of 0.3×6 mm² and 2×6 mm² ($h \times w$) and wall thicknesses of 0.3 mm and 0.8 mm, respectively. A function generator (TTi, TG1304 Programmable) drives a RF amplifier (ENI, Model 240L) that powers the transducer, with a signal monitored by an oscilloscope (Agilent Technologies, DOS1102B Digital Storage Oscilloscope). An Olympus BXFM epi-fluorescent microscope with a *pixelfly* dual-frame CCD camera was used to image the device. The experimental measurements can be split into the following steps:

- impedance measurements were first used to identify the resonant frequencies in these two devices;
- $10\ \mu\text{m}$ particles were used to characterise the acoustic fields in the capillaries by examining acoustic radiation forces on them;
- micro-particle-image-velocimetry (μpiv) measurements of $1\ \mu\text{m}$ polystyrene tracer particles (Fluoresbrite microspheres, Polysciences, Inc.) were performed to characterise the acoustic streaming fields;
- the *voltage drop method*,⁵⁵ based on the balance of the buoyancy force and acoustic radiation force on $10\ \mu\text{m}$ particles, was used to estimate the acoustic pressure magnitudes in the fluid channels.

More detailed information regarding the device configuration, the μpiv setup and the process of measuring and characterising the acoustic streaming fields can be found in our previous work.³⁵ It was found (see Fig. 2 in supplementary material⁴⁸) that most of the $10\ \mu\text{m}$ particles used to characterise the fields were aligned to two planes in the fluid channel, $y \approx \pm 1.2$ mm, suggesting that, in addition to the primary half-wave standing wave in the z -direction of the fluid channels, acoustic pressure fields in the y -direction of the fluid channels also have one-wave variations. This confirms that the devices

TABLE II. Experimental parameters.

Quantity	Abbreviation	Value	Unit
First capillary size	$l \times h \times w$	$80 \times 6 \times 0.3$	mm^3
Second capillary size	$l \times h \times w$	$80 \times 6 \times 2$	mm^3
Dimension of transducer	$l \times h \times w$	$3 \times 3 \times 1$	mm^3
Fluid density	ρ_0	998	$\text{kg} \cdot \text{m}^{-3}$
Particle radius	r	1	μm
Dynamic viscosity of fluid	μ	0.893	$\text{mPa} \cdot \text{s}$
Speed of sound in fluid	c	1480	$\text{m} \cdot \text{s}^{-1}$

were excited in the (1, 2, 1) cavity mode matching the modelling above. All the parameters for fluid and microparticles are shown in Table II.

Fig. 6 shows the measured acoustic streaming fields in the first glass capillary ($h = 0.3 \text{ mm}$), in which the acoustic streaming fields on two xy planes, respectively, $z = 0$ and $z = 0.45h$, are presented. It can be seen that, in this low aspect ratio device, four-quadrant transducer-plane streaming pattern is seen on both planes with the same orientation on each vortex, which compares well with

FIG. 6. Experimentally measured streaming field near device centre (field of view: $1.8 \text{ mm} \leq x \leq 1.8 \text{ mm}$, $-1 \text{ mm} \leq y \leq 1 \text{ mm}$) in the first glass capillary ($h = 0.3 \text{ mm}$ and $w = 6 \text{ mm}$). (a) on plane $z = 0.4h$; and (b) on plane $z = 0$. The streaming velocities presented are obtained from acoustic pressure amplitude of approximately 0.8 MPa .

that shown in Figs. 4(a) and 5(a). However, in the second glass capillary ($h = 2$ mm), instead of uniform transducer-plane streaming pattern along the height of the fluid channel, vortices on the yz cross sections were observed.

The measured acoustic streaming fields near the centre of the second device are shown in Fig. 7 and correspond to modal Rayleigh-like streaming. The acoustic streaming fields on two xy planes, $z = 0$ and $z = 0.45h$, are presented in order to compare with those measured in the first device. It is clear that there is no in-plane streaming vortex in these xy planes and that there must be vortices in the yz cross sections (see Fig. 8(c)) to complete the flow field (compare these results to the dashed box in Fig. 8(d)). The size of these vortices is approximately one-quarter of the fluid channel width, $w/4$, which is much bigger than the size of classical Rayleigh streaming vortices, $h/2$, which would be caused from the main half-wave resonance in the z -direction. It can be seen that the measured acoustic streaming patterns in the central area compare well with those predicted in Figs. 4(e) and 5(e). However, in addition to the y -directed flows that form the pattern we are describing as modal Rayleigh-like streaming, there are some x -components to the flow in the top plane shown in Fig. 7(a). These are also seen in a smaller way in the model (Fig. 3(g)) and result from the gradient of energy density towards the central maximum. We hypothesise that the boundary conditions in the experiments cause larger gradients than those modelled and hence the larger x -component of the velocity vectors.

In order to accurately accord with experimental results, the limiting velocities at the side walls ($y = \pm w/2$) were included in the model results shown in Fig. 5(e). This creates a streaming field that has contributions from both Rayleigh streaming (driven by the sidewalls) and modal Rayleigh-like streaming (driven by the upper and lower channel boundaries), resulting in the streaming pattern shown in Fig. 8(d). This pattern has extra vortices, which accords well with the measured streaming fields shown in Figs. 8(a) and 8(b).

FIG. 7. Experimentally measured streaming field near device centre (field of view: $-1 \text{ mm} \leq x \leq 1 \text{ mm}$, $-1.5 \text{ mm} \leq y \leq 1.5 \text{ mm}$) in the second glass capillary ($h = 2$ mm and $w = 6$ mm). (a) on plane $z = 0.45h$; and (b) on plane $z = 0$, where d shows the size of the modal Rayleigh-like streaming vortices. The streaming velocities presented are obtained from acoustic pressure amplitude of approximately 0.68 MPa.

FIG. 8. ((a) and (b)) Measured streaming fields near the side wall (field of view: $-1 \text{ mm} \leq x \leq 1 \text{ mm}$, $1.5 \text{ mm} \leq y \leq 3 \text{ mm}$) in the second glass capillary on the plane $z=0$ and the plane $z=0.45h$ respectively; (c) schematic representation of the acoustic streaming vortices in the whole channel, where solid and dashed arrows show the measured and deduced streaming fields, respectively; (d) the modelled acoustic streaming field including Rayleigh streaming driven by limiting velocities at the side boundaries ($y = \pm w/2$). The vortices identified by the green dashed line in (d) are discussed in the text. The streaming velocities presented in (a) and (b) are obtained from an acoustic pressure amplitude of approximately 0.68 MPa. The two-way arrows in (d) show the sizes of the modal Rayleigh-like streaming and Rayleigh streaming, where λ_y and λ_z are the modal wavelengths in the y - and z -directions of the fluid channel ($w = 6 \text{ mm}$, $h = 2 \text{ mm}$, $l = 80 \text{ mm}$).

In the case of boundary-driven streaming, it is acoustic attenuation in the viscous boundary layer, forming Reynolds stress force fields, that induces streaming vortices in that thin layer. The fluid movement inside the viscous boundary layer induces outer streaming vortices in the bulk of the fluid. This means that the maximum streaming velocities will be no more than a few multiples of δ_v away from the wall. However, in some regions of the second capillary ($h = 2 \text{ mm}$), the maximum streaming velocity was found to be further from the boundaries (about 50% higher than at the boundary at a distance of $\sim 250 \mu\text{m}$), which is not consistent with our modelling. Initial modelling suggests that this is due to the energy loss in the bulk of the fluid producing Reynolds stress forces that drive non-negligible Eckart type streaming velocities in the volume of the fluid channel additional to the dominant boundary-driven streaming. This effect is also likely to be observable in acoustofluidic devices with thinner layers, in which the active intensity flow (which generates Eckart streaming) has a significant contribution to the limiting velocity fields generating the boundary-driven streaming. The effects of Eckart streaming on the overall streaming fields in planar acoustofluidic devices will be further examined in future work.

IV. CONCLUSION

The acoustic streaming patterns in 3D rectangular fluid cavities, where a $(1, 2, 1)$ resonant cavity mode was excited, have been investigated. It has been shown that the ratio of the two dominant acoustic velocity gradients, governed by the aspect ratio of the channel, determines the driving terms of the acoustic streaming fields. These driving terms can be identified as having vortex patterns matching those of the active and reactive components of the complex streaming field. This provides a basis for predicting the acoustic streaming patterns in layered acoustofluidic devices, for particle and cell manipulation.

In addition to the well-known Rayleigh streaming and the transducer-plane streaming recently described, a third type of boundary-driven streaming, *modal Rayleigh-like streaming*, has been experimentally observed and numerically verified. This streaming field is driven by limiting velocities that

have the same pattern as classical Rayleigh streaming but which depend on the resonant mode excited in a 3D cavity rather than on simple one-dimensional standing wave patterns. Since cavity modes can have much larger characteristic wavelengths than a 1D mode, the resulting streaming patterns can have much larger vortex sizes than those found in classical Rayleigh streaming.

The modelled and experimental results demonstrate that the acoustic streaming patterns in layered half-wave acoustofluidic particle manipulation devices excited in the resonant cavity modes can be split into three regions, bounded approximately by

- (1) $h/w \leq 1/20$, transducer-plane streaming;
- (2) $1/20 < h/w < 1/3$, transducer-plane streaming and modal Rayleigh-like streaming; and
- (3) $h/w \geq 1/3$, modal Rayleigh-like streaming.

The cases considered above focus on the boundary-driven streaming in layered acoustofluidic systems, without considering the Eckart type streaming generated from the energy dissipation in the bulk of the fluid, which becomes significant when the characteristic dimensions become comparable to the attenuation length (e.g., in surface acoustic wave based systems^{13,56}).

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