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Modelling rough interfaces on seismic reflection profiles - the application of fractal concepts

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Abstract. The distortion of reflection continuity and amplitude by overburden structure in seismic reflection images of the sub-surface is easily recognised and modelled when the wavelength of the shallower structure is relatively large. The effects of shorter wavelength structure although giving rise to little reflective response itself, cause significant distortion of the propagating wavefield, particularly when a moderate or strong acoustic impedance contrast is present in the shallow sub-surface. Here we show how short as well as long spatial wavelengths of horizon roughness affect deeper reflection continuity, and develop a new method using fractal interpolation techniques to predict the *total roughness* of sub-surface horizons from information contained in seismic reflection sections. Fractally complete depth-velocity models are used in forward models, using the finite difference technique, to produce synthetic seismic profiles. The technique is illustrated with data from the Edoras Bank area of the Rockall Plateau, NE Atlantic, where apparently discontinuous reflectors underlying basalt flows are shown to be from continuous sedimentary horizons distorted by overlying rough horizons.

Fractal Interpolation

Several earlier studies used data acquired from exposed rock surfaces to show that natural topographies are self-affine fractals (Mandelbrot, 1977; Brown & Scholz, 1985; Farr, 1992; Huang and Turcotte; 1989). Natural topographies have roughness on all scale ranges from centimetres to kilometres and features on one scale appear statistically identical under different magnifications. The power spectrum P(k) of such a profile, which quantifies the overall distribution of roughness in terms of spatial wavelength, follows a power-law,

$$P(k)=Ak^{-\beta}, where P(k)=\frac{1}{L}|X(k)|^2, x(I)\Leftrightarrow X(k) \qquad (1)$$

where x(I) is the input topographic data at regular sampling interval I, k is the spatial frequency (the inverse of the spatial wavelength of features present), A is a constant and β is the slope of the logarithmic plot of P(k) against k see Figure 1. The relationship between β and the fractal dimension D_f is (Berry and Lewis, 1980),

$$D_f=\frac{5-\beta}{2} \qquad (2)$$

where $1 < \beta < 3$ and D_f varies between 1 and 2 for fractal profiles.

Barnsley and Demko (1985) called the process of predicting small-scale roughness from larger scale features, fractal interpolation (see also Barnsley, 1988), and we use this term to describe the concept, illustrated in Figure 1, of adding the smaller spatial scale features to horizons digitised from seismic reflection profiles. Surface roughness estimation has been used to interpolate bathymetric and seafloor datasets (Fox and Hayes, 1985; Goff and Jordan, 1988; Mareschal, 1989; Turcotte, 1992) but has not been

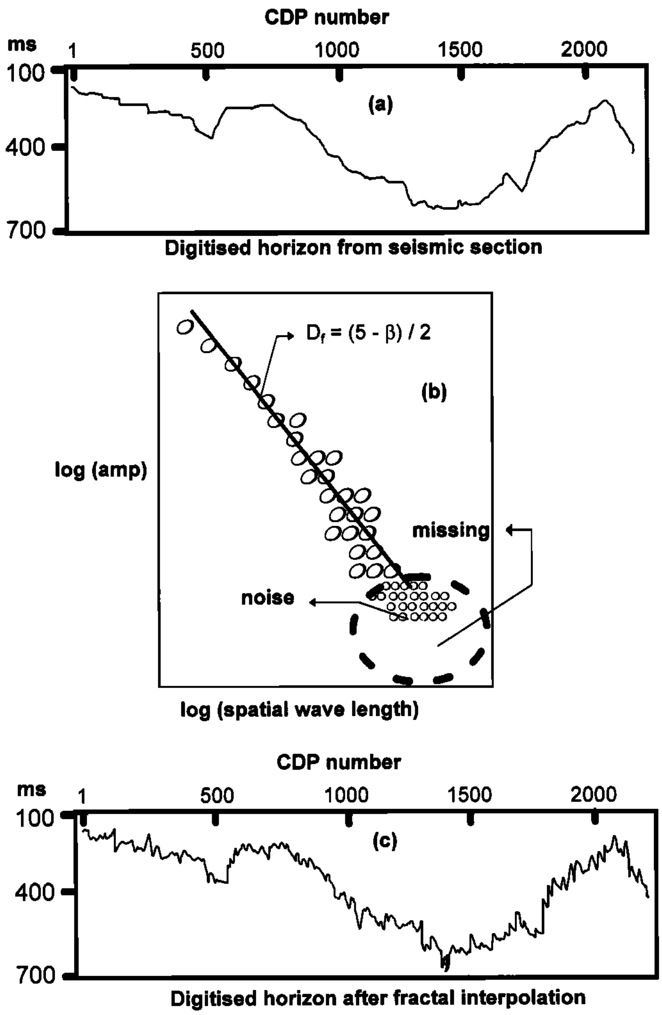

Figure 1. The concept of Fractal interpolation is illustrated. (a) A cartoon of a digitised seismic horizon and (b) its amplitude spectrum are shown. The amplitude spectrum clearly shows that information on medium to large scale features is present, however the amplitude of the small scale features are either masked by seismic noise or completely missing *(shown in circle).* Fractal interpolation can be used to restore these small scale structures. Digitised horizon after fractal interpolation is shown in (c). *(D_f is the fractal dimension and β is the slope of regression line).*

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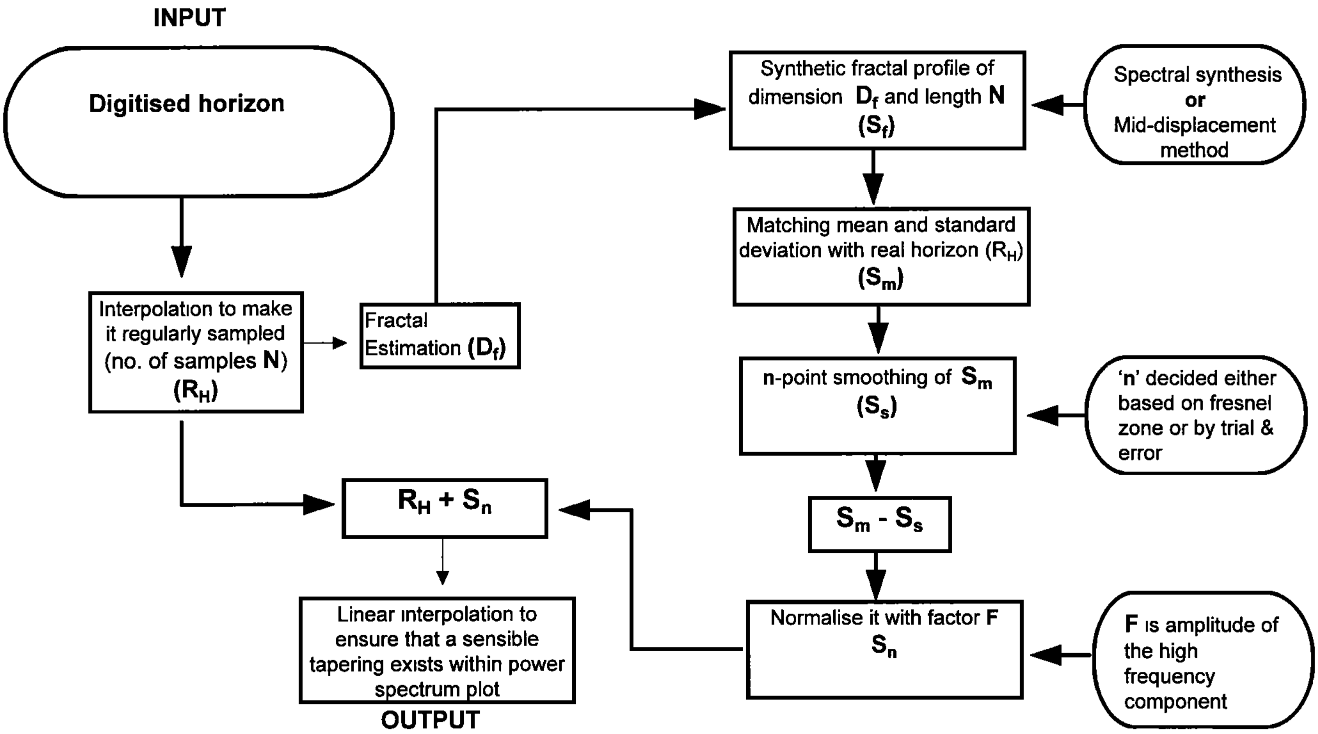

Figure 2. Flow chart showing the methodology of fractal interpolation using synthetic fractal insertion (SFI).

previously applied to seismic reflection datasets. The horizontal and vertical resolution limits of seismic reflection datasets, and errors introduced by digitising mean that higher frequency features originally present on a horizon are missing. Two techniques for fractal interpolation are described in this paper, Synthetic Fractal Insertion (SFI) and Iterative Function Systems (IFS).

Synthetic Fractal Insertion (SFI)

In this approach lower spatial frequency (large spatial wavelength) information, reliably obtained from the seismic reflection data, is merged with higher spatial frequency components derived from a synthetic fractal profile of the same dimension. The flowchart in figure 2 summarises the procedure followed in the SFI method. Figure 3b shows horizons which have been interpolated by the SFI method and figure 4 (compare a and b with c and d) shows that the fractal dimension has not significantly changed after interpolation. It can be seen that the spatial wavelengths between 450 m and 50 m have been added after interpolation.

Iterative Function Systems (IFS)

Barnsley and Demko (1985) defined an interpolation function based on self-affine mapping which interpolated patterns from one

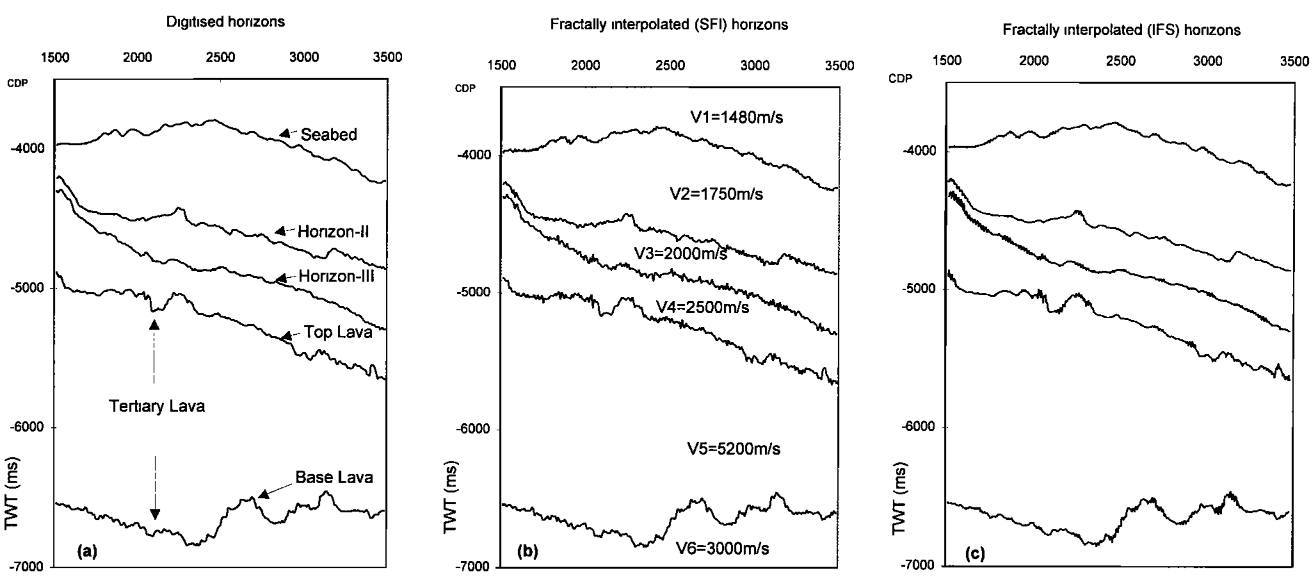

Figure 3. Horizons digitised from a time-migrated seismic reflection profile (a) (see figure 5), the fractal interpolation using (b) SFI and (c) IFS method are shown. These horizons are used to prepare the Depth-Velocity models for the Finite Difference modelling to study wave propagation through them.

scale to another, without alteration of the key fractal properties. In this work, software developed by Mareschal (1989) was adapted for fractal interpolation of seismic reflection horizons. The most critical parameter within IFS is the scaling factor which controls the roughness of the interpolated segments of the horizon. Mareschal (1989) selected the scaling factors iteratively by comparing the fractally interpolated profile with a known (bathymetric) profile. However this approach is not practical if the number of digitised points is large, as is the case of a seismic horizon. Therefore, for the IFS scheme implemented here, scaling factors for each point were calculated based on the running standard deviation of nearby digitised points (Walia, 1997). The IFS interpolation does restore features of small spatial scale (see Figure 3c), however it does tend to predict a high amplitude of roughness for those segments which are steeply dipping (as can be seen particularly for Horizon-III). This is due to the number of digitised points and the maximum

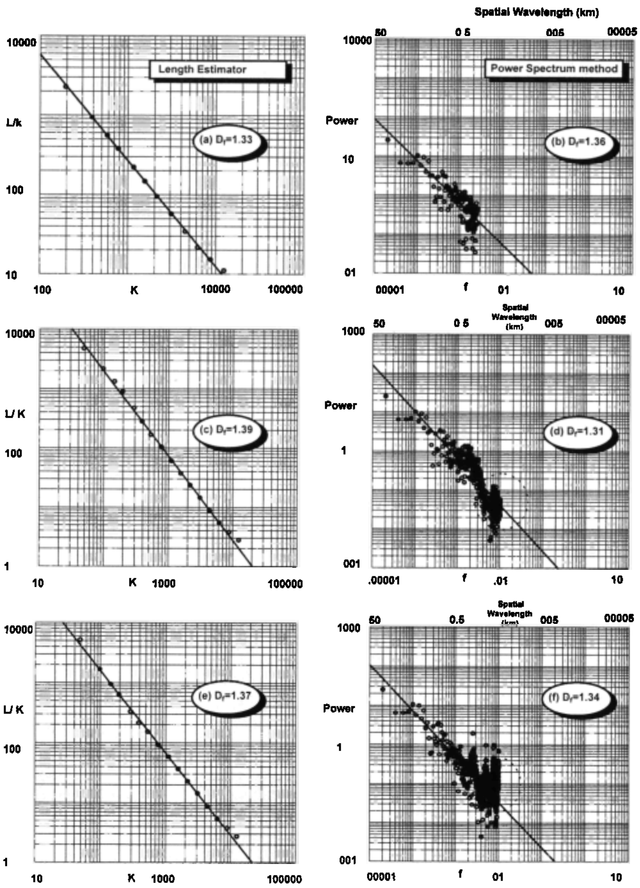

Figure 4. Estimating the fractal dimension of the digitised Top lava (a & b), the fractally interpolated Top lava using the SFI algorithm (c & d) and using the IFS algorithm (e & f). Length estimator method (shown in a, c and e) calculates precise D_f values based on a technique proposed by *Higuchi (1988)*. Power spectrum method (shown in b, d and f) considers digitised horizon data as a regular time series and calculates its power-spectrum, P(k), using Fourier transformation. P(k) quantifies the overall distribution of the roughness in terms of the spatial wavelength. The slope of the curve P(k) against k in log-log space is related to D_f as discussed in the main text.

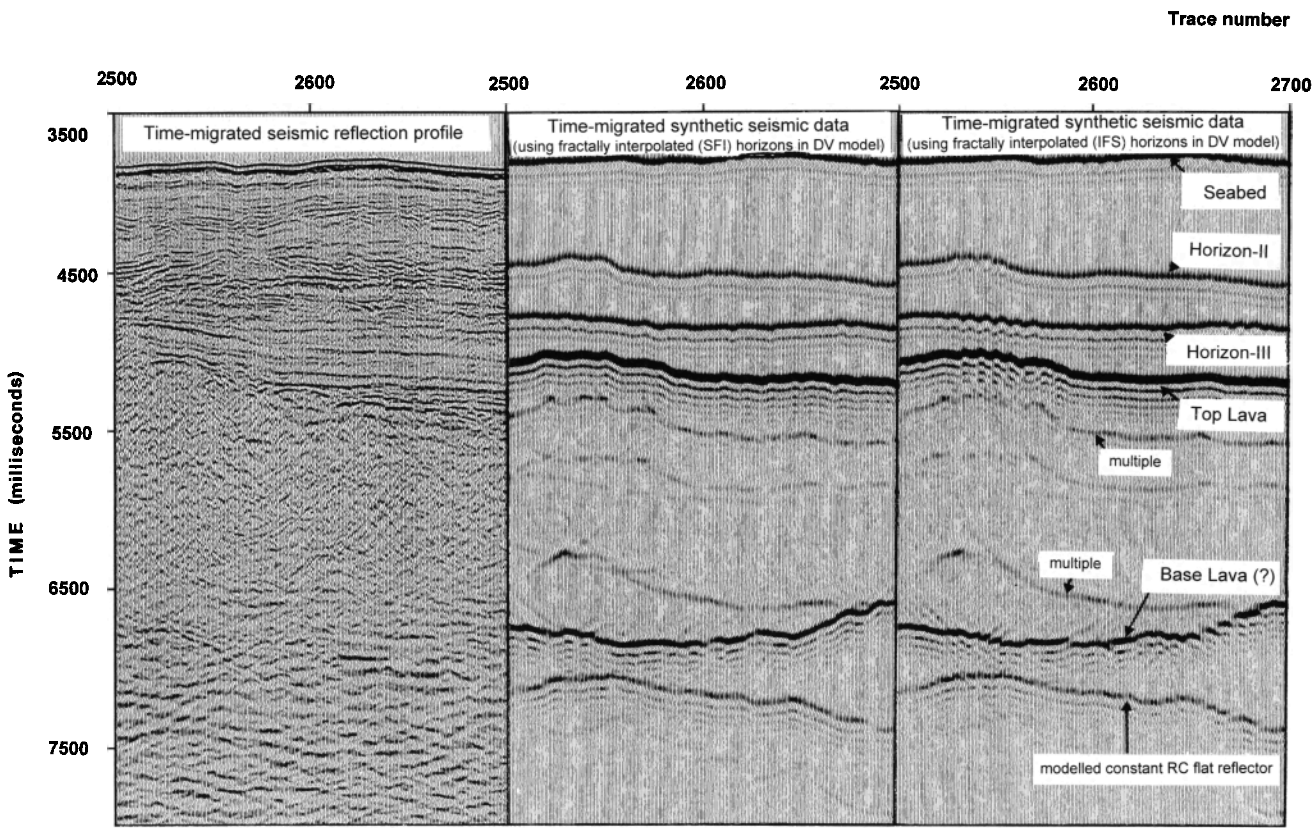

Figure 5. Comparison of seismic data with synthetics produced by finite difference models with depth-velocity models incorporating (b) SFI fractal interpolation and (c) SFI fractal interpolation. In the input models for finite difference modelling a flat constant reflection coefficient layer was included beneath the deepest rough interface (the base lava here) to enable the distortion of deeper horizons to be investigated. Note that in (b) and (c) there is substantial amplitude variation along the originally constant reflection coefficient horizon caused by the rough overburden. See Bull and Masson (1996) for a complete description of the seismic stratigraphy, but note that the base lava interface is picked with less confidence than the other horizons.

amplitude allowed for the computation of the scaling factors. The optimal values for these two parameters can be selected iteratively. Figure 4 (compare a and b with e and f) shows that the fractal dimension is unchanged within error.

Finite-Difference Modelling

One of the applications of fractal time velocity or depth velocity models is illustrated with data from south of Edoras Bank, south-west Rockall Plateau, NW British Isles (see Bull & Masson, 1996 for more detail). In this area Tertiary sediments overlie Palaeocene lavas which largely obscure the underlying stratigraphy. The base of the lava can be followed around the area described by Bull & Masson (1996) as a change in seismic attribute, from a transparent scattering zone within the lava to sub-horizontal discontinuous reflectors beneath the lavas. (Figure 5).

Five horizons which were relatively rough (as measured by their D value) and/or had a significant acoustic impedance contrast were modelled. The sea-bed (D value, D_f=1.2), Horizon II - the latest Early Miocene unconformity (D_f=1.3), Horizon III - Mid-Eocene to Early Oligocene unconformity (D_f=1.2), the top lava (D_f =1.3) and base lava (D_f=1.4). Figure 3 illustrates the fractally interpolated horizons digitised from Figure 5a. In order to assess the effect of the shallower rough interfaces on sub-lava structure, the five interfaces above, together with a sub-basalt constant reflection coefficient flat horizon, were input to a 4th order acoustic finite difference modelling code. The time migrated synthetics using SFI and IFS interpolation are shown in Figure 5b and c. The originally constant reflection coefficient sub-basalt reflector now appears with substantial amplitude variation which is similar to that observed in the real data (Figure 5a). It can be concluded that continuous flat sedimentary units present sub-basalt would appear as layers with substantial amplitude variation on a seismic reflection profile. The coherency of the modelled sub-basalt reflector would be further reduced if noise had been simulated within the synthetic.

Conclusions

We introduce the concept of fractal interpolation of horizons digitised from seismic reflection data and describe two methods for achieving the interpolation. One application of fractal seismic horizons is illustrated by the production of a fractal depth-velocity model used as input to a finite difference modelling program. This example shows that the roughness of the overburden is responsible for severe distortion of reflection amplitudes, particularly where interfaces are rough and where there are strong acoustic impedance contrasts.

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