Developments of numerical methods for linear and nonlinear fluid-solid interaction dynamics with applications

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Abstract This paper presents a review on some developments of numerical methods for linear and nonlinear fluid-solid interaction (FSI) problems with their applications in engineering. The discussion covers the four types of numerical methods: (1) mixed finite element (FE)-substructure-subdomain model to deal with linear FSI in a finite domain, such as sloshing, acoustic-structure interactions, pressure waves in fluids, earthquake responses of chemical vessels, dam-water couplings, etc.; (2) mixed FE-boundary element (BE) model to solve linear FSI with infinite domains, for example, very large floating structure (VLFS) subject to airplane landing impacts, ship dynamic response caused by cannon/missile fire impacts, etc.; (3) mixed FE-finite difference (FD)/volume (FV) model for nonlinear FSI problems with no separations between fluids and solids and breaking waves; (4) mixed FE-smooth particle (SP) method to simulate nonlinear FSI problems with F-S separations as well as breaking waves. The partitioned iteration approach is suggested in base of available fluid and solid codes to separately solve their gov-

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erning equations in a time step, and then through reaching its convergence in coupling iteration to forward until the problem solved. The selected application examples include air-liquid-shell three phases interactions, liquefield natural gas (LNG) ship-water sloshing; acoustic analysis of air-building interaction system excited by human foot impacts; transient dynamic response of an airplane-VLFS-water interaction system excited by airplane landing impacts; turbulence flow-body interactions; structure dropping down on the water surface with breaking waves, etc. The numerical results are compared with the available experiment or numerical data to demonstrate the accuracy of the discussed approaches and their values for engineering applications. Based on FSI analysis, linear and nonlinear wave energy harvesting devices are listed to use the resonance in a linear system and the periodical solution in a nonlinear system, such as flutter, to effectively harvest wave energy. There are 231 references are given in the paper, which provides very useful resources for readers to further investigate their interesting approaches.

Keywords linear and nonlinear fluid-solid interactions, mixed FE-substructure method, mixed FE-BE method, mixed FE-FD method, mixed FE-SP method, fluid sloshing, acoustic volume-structure coupling, breaking wave simulations, pressure waves in fluids, LNG ship/VLFS-water interaction, wave energy harvesting

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1 Introduction

1.1 FSI problems and its characteristics

As clearly indicated by its name, the FSI dynamics is a branch of mechanics, which is an interdisciplinary subject to study the dynamic behaviour of a system consisting of both solids (structures) and fluids together simultaneously. Here there are two key points to identify if a problem is an FSI problem: the first is that the system must include fluids and solids together, and the second is that it is not possible to eliminate the fluid variables or the solid ones and has to be solved simultaneously. Figure 1 shows a dam-water interaction system (Xing 1984; Xing et al. 1991, 1996, 2009). An approximate method often used in engineering to solve this problem as follows: (i) assume that the solid dam is rigid to calculate the dynamic pressure added on the wet interface caused by the prescribed pressure wave \hat{p} ; (ii) considering the elastic dam is subjected to the dynamic pressure obtained in step (i) to calculate the deformation and stress. In this approximate solution process, the first step is a pure dynamic problem in fluid mechanics and the second step deals with a pure problem in solid mechanics. Therefore, the equations governing the motions of fluid and solid are solved separately, but not considering them simultaneously, so that the dynamic interaction between the dam and the water is neglected. The most important characteristic of FSI problems is "interaction" from which the fluid motions affect the force applied to the solid and at the same time the solid motion affects fluid flows. The governing equations of

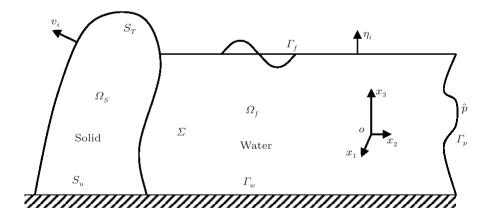


Fig. 1

A dam-water interaction system excited by a given explosion pressure wave (Xing 1984; Xing et al. 1991, 1996, 2009)

fluid and the ones for solid are coupled each other, and they cannot be solved separately, but only simultaneously.

There are two large categories of FSI problems. For the first type of problems, the fluid domain and the solid domain cannot be clearly defined and there is no a clear interface between them, such as flows through porous medium. To solve a problem of this type, the corresponding constitutive relationship specified for the problem is needed. In the second type of problems, there exists an obvious FSI interface on which the FSI happens. In this paper, we focus our attention on the second type of problems. **Figure 2** gives the relation-

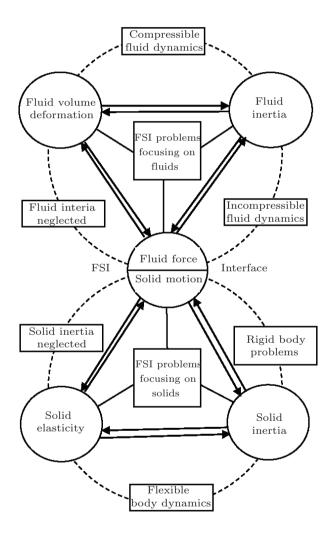


Fig. 2

The force relationships of FSI problems (Xing et al. 1997a)

ship of the forces in the second type of FSI problems, which comes from a review paper (Xing et al. 1997a) where the history, research directions, developments and published references on FSI were given. In this figure, the two large circles described by the dashed lines represent the fluid and the solid domains, respectively. A small circle, at which the two dashed circles are contacted, denotes the FSI interface through which the dynamic force of the fluid affects the motion of the solid and also the solid motion affects the fluid flows. Both the motions of the fluid and the solid on the interaction interface are unknown and they can be found only after solving the total equations governing the integrated FSI problem. For general FSI cases, denoted by the two squares at the centres of two dashed circles, both the motions of fluid and solid are also affected by their inertia forces and the elastic forces, respectively. Depending on the aim of an investigated problem, you may focus your observation point on the fluid or the solid. Normally, the researchers in fluid dynamics are mainly interested in the fluid flows while the researchers in solid mechanics are concerned in the deformation of the solid. In engineering, there are different simplifying approaches adopted to derive the simplified coupling problems. For example, when studying waterstructure interactions, the compressibility of the water could be neglected, so that we have the problem on incompressible FSI. Similarly, considering the rigid motion, we may neglect the elastic deformation of the structure to investigate the fluid-rigid body interaction. For some cases, we may neglect the inertia of fluid or the one of solid to obtain the corresponding simplified problems. For a gas, neglecting its inertia (mass) implies to consider it as a gas spring, which is often used in engineering. All of these types of simplified problems are denoted by the rectangles located on the two dashed circles in **Fig. 2**.

According to the amplitude of the relative motions between the fluid and the solid as well as their characteristics of interactions, Zienkiewicz et al. (1978) category the three types of problems: (a) FSI problems with large relative motions between the two phases; (b) Short time FSI problems with finite relative motions between the two phases; (c) Long time FSI problems with finite relative motions between the two phases. A typical problem for type (a) is the flutter problem, air-structure interactions, called the aeroelasticity problem that is a very important FSI problem in aircraft designs. Problem (b) undergoes a short time period process in which the amplitude of motion is finite. The FSI systems involving impacts and explosions have this characteristic, for which the compressibility of the fluid is more important. Problem (c) shows a long time period FSI process with finite amplitude of motion, for which designers are concerned by the dynamic responses of the system. Figure 3 lists some selected FSI problems in engineering.

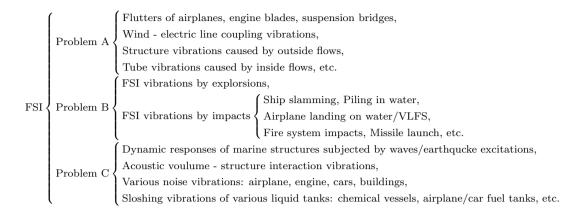


Fig. 3

The selected FSI problems in engineering

The dynamic analysis of FSI systems subject to various dynamic loads necessitates inter-disciplinary studies on fluids, rigid or flexible structures and their interactions, which belongs to the problems in the classical mechanics, therefore, the theory and methods developed in the classical mechanics can be directly used to deal with them. Due to the complexity of the problems, generally, for almost all these problems, analytical solutions are not available and recourse to numerical solutions or experimental studies is the only way forward. To carry on a full scale product test costs too high, so that the numerical solution of the problem is more important, especially for the design stage when the product has not been produced.

1.2 Solution strategy

1.2.1 Approximate solution with no coupling

In early period when numerical computational techniques had not well developed, designers can only solve FSI problems with no coupling method as discussed for the dam-water interaction system in **Fig. 1**. This approximate solution approach approximately deals with an FSI problem by solving the two corresponding non-coupling problems: a pure fluid problem and a pure solid problem. In the initial stage to design a product, this method is now still adopted to construct a draft figure of the intended design.

1.2.2 Quasi-coupling method

For some of FSI problems, a quasi-relationship between the fluid dynamic force and the structure motion may be found, so that the original FSI problem can be solved by a quasi-coupling approach using the following steps.

- (i) Investigating the fluid dynamic force produced by a set of given motions u of the solid structure to obtain the fluid dynamic force f(u) as a function of solid motions. This force function may be an analytical function or discrete data obtained from the experiments.
 - (ii) Solve solid equation subjected by the dynamic force obtained above, i.e.,

$$Lu = f(u) \tag{1}$$

The typical examples of this method are as follows. In the history, an aerofoil oscillating with a given harmonic motion in air flows was investigated to derive the unsteady aerodynamic lift force as a function of the motion of the aerofoil (Fung 1955; Bisplinghoff et al. 1957, 1962; Bisplinghoff 1958). Also, for marine ship-water interactions (Bishop et al. 1979, 1986), giving a set of natural mode motions of the dry ship to solve the corresponding radiation problem in order to obtain the water pressure relating to each mode motion, which can be used to construct the water force as the function of mode coordinate, applied to the structure equation in the form of Eq. (1) based on the mode summation approach.

Compared with the non-coupling method described in Sect.1.2.1, here the solid structure is not rigid fixed in the space but with a set of given motions. Therefore, the first step solves a set of pure fluid problems, of which each one with a given solid motion, so that the function f(u) can be obtained. Since for real FSI problems, both fluid and solid motions are unknown and cannot be solved separately. The obtained force function f(u) in the condition of a given solid motion might not be valid for the real coupling cases. From this point of view, we call this method as a quasi-coupling approach. However, if this obtained force function is also valid for the real coupling cases, even in some conditional range, this method will provide real coupling solution.

1.2.3 Integrated-coupling field method

This method follows the principles in continuum mechanics to establish the governing equations describing both solid and fluid motions as well as their boundary and interaction conditions and then numerically solve this set of coupling equations. This paper discusses the numerical methods developed by author and his colleagues to solve integrated FSI problems, which include the following four numerical models.

- (i) Mixed finite element (FE)-substructure-subdomain approach to deal with linear incompressible/compressible FSI involving sloshing/acoustic/explosion waves in fluids (Xing 1984; Xing et al. 1991, 1996, 2009);
- (ii) Mixed FE-boundary element (BE) model to solve linear water-structure interactions with infinite fluid domain (Xing et al. 2004, 2005a, 2005b; Jin et al. 2007, 2009; Jin 2007);
- (iii) Mixed FE-finite difference (FD) model for nonlinear FSI problems but no separations between fluids and solids (Xing et al. 2002, 2003, 2005);
- (iv) Mixed FE-smooth particle (SP) method to simulate nonlinear FSI problems with F-S separations as well as breaking waves (Sun 2013; Sun et al. 2011, 2012, 2013; Javed et al. 2013a, 2013b, 2014a, 2014b; Javed 2015; Sun et al. 2014, 2015a, 2015b).

1.3 Numerical integration process

1.3.1 Direct or simultaneous integration

One used numerical integration process to solve FSI equations is a direct or simultaneous solution procedure which sets up the integrated FSI equations and solve them simultaneously (Xing et al. 2002, 2003, 2005). This procedure is more suitable to solve linear FSI problems using available numerical integration schemes with no interactive iteration process. The non-symmetrical FSI equations for mixed FE model can be symmetrised (Xing 1984; Xing et al. 1991, 1996) in order to directly use the available computer codes for symmetrical FE equations.

1.3.2 Partitioned iteration

Another solution process is called *partitioned iteration* procedure (Xing et al. 2002, 2003, 2005), which is more suitable to integrate nonlinear FSI equations. In this method, the dynamics of the fluid, structure and even electric unit if electric-mechanical coupling involved are solved for separately, and the data are exchanged at every time-step or iteration. The solution steps are shown in **Fig. 4**.

2 Mixed FE-substructure/subdomain model

In the earlier stage for solutions of linear FSI problems, based on the displacement FE model (Bathe 1996; Zienkiewicz et al. 1989, 1991), people naturally adopted the fluid displacement to model fluid domains (Xing 1986a, 1986b). Numerical practices have demonstrated that un-avoided zero energy modes affect its wide applications. For the well-developed FE models, free surface waves are often neglected (Liu et al. 1982, 1988; Kock et

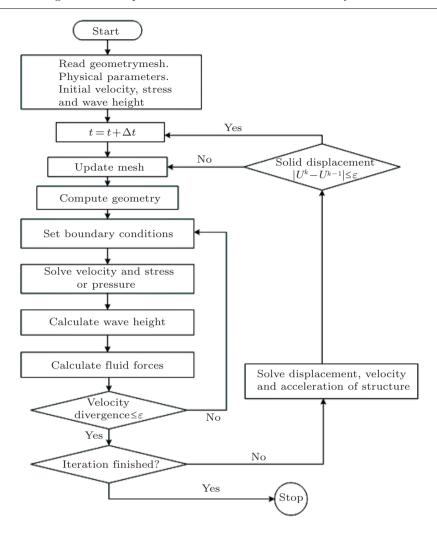


Fig. 4

The flow chart of partitioned solution procedure (Xing et al. 2002, 2003, 2005)

al. 1991; Morand et al. 1995; Dervieux 2003), which was also omitted in the fluid pressure model to avoid the related numerical difficulties (Zienkiewicz et al. 1978). Xing (1988) proposed the two variational formulations for linear FSI problems including free surface waves considered, based on which the mixed FE method was formulated (Xing et al. 1991). In this model, the fluids are non-viscous compressible and their motions are irrotational and governed by a wave equation, while the structures are exactly governed by the linear elasticity theory with small displacements. This method was further developed to include substructure-subdomain (SS) techniques (Hunn 1955; Hurty 1960, 1965; Hou 1969; Craig

et al. 1968, 1977; MacNeal 1977; Unruh 1979; Xing 1981, 1984; Xing et al. 1983), which was theoretically investigated on its convergence and mode reduction rule based on the variational principles of elastodynamics (Xing 1981, 1984; Xing et al. 1983), to reduce the computation time (Xing 1986a, 1986b; Xing et al. 1996).

The substructure-subdomain (SS) method is a development combining the Rayleigh-Ritz method and an FE analysis. It takes advantages of both methods but avoids their shortcomings. It divides a large vibration problem into smaller ones of a size more easily handled, and a solution to the larger problem is obtained by suitably synthesizing the solutions of the smaller problems. The fundamental solution process of SS methods includes the following three steps as shown in **Fig. 5**: (i) the whole FSI system under examination is divided into a number of smaller subsystems called substructures for the solid and subdomains for the fluid. Naturally, the number of subsystems can be determined according to the problem and the computer capacity used; (ii) SS analysis is carried out to derive data describing their dynamic characteristics using a theoretical, experimental, or numerical methods. More

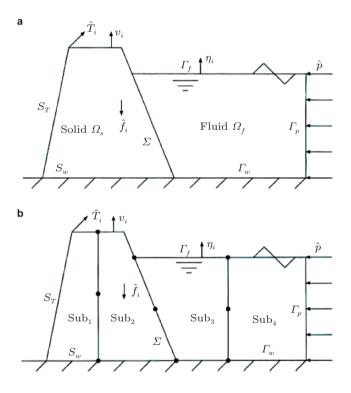


Fig. 5

A dam-water interaction system (a) is divided into SS system (b) (Xing et al. 1996, 2009)

often, FE is used in the analysis of SS. These data form the basis for the construction of the solution to the whole FSI problem. (iii) An approximate solution describing the dynamics of the whole system is obtained by the synthesis of SS data based on the coupling conditions on the SS boundaries.

Theoretically, this model has further developed in the following aspects.

- (i) The interaction condition on the interface of two different fluids with different mass densities is formulated, which provides a basis to simulate air-water-structure interaction systems (Xing et al. 2009).
- (ii) The boundary condition on the FSI wet interface with floating structure motions is modified, such as a ship base, to include the gravity potential changes.
 - (iii) The surface tension effect is considered for the case when it is essential.
- (iv) The frequency shift technique, which was not demonstrated for FSI system in the original publication (Xing et al. 1996), is now mathematically proven (Xing et al. 2009).
- (v) Furthermore, in dealing with water-structure dynamic interactions involving both free surface wave and compressible wave caused by under-water explosions, it has been realised that the radiation condition at infinite water boundary, proposed by Sommerfeld and generalised by others (Sommerfeld 1912, 1949; Rellich 1943; Magnus et al. 1949; Courant et al. 1962), is difficult to be defined, since the speed of free surface wave and the speed of compressible wave are different. Xing (2007, 2008) studied it through the example and theoretical analysis and proposed a new radiation condition to model this practical case.

The new developments (i)–(v) have been included in the following two modified variational formulations.

$$\Pi_{sf}[p, W_{i}] = \Pi_{s}[W_{i}] + \Pi_{f}[p] + \Pi_{int}[p, W_{i}]$$

$$\Pi_{\chi}[p] = \int_{t_{1}}^{t_{2}} dt \int_{\Gamma} \chi p_{,It} p_{,It} / (2g^{2}\rho_{f}^{2}) d\Gamma$$

$$\Pi_{s}[W_{i}] = \int_{t_{1}}^{t_{2}} \left\{ \int_{\Omega_{s}} (\rho_{s} W_{i} W_{i} / 2 - C_{ijkl} V_{ij} V_{kl} / 2 - \hat{F}_{i} W_{i}) d\Omega_{s} - \int_{S_{T}} \hat{T}_{i} W_{i} dS \right\} dt$$

$$\Pi_{f}[p] = \sum_{\beta} \int_{t_{1}}^{t_{2}} dt \int_{\Omega_{f}^{(\beta)}} \left\{ p_{,t}^{(\beta)} p_{,t}^{(\beta)} / [\rho_{f}^{(\beta)}(c^{2})^{(\beta)}] - p_{,i}^{(\beta)} p_{,i}^{(\beta)} / \rho_{f}^{(\beta)} \right\} / 2d\Omega_{f}$$

$$\Pi_{int}[p, W_{i}] = \sum_{\beta} \int_{t_{1}}^{t_{2}} \left\{ \int_{\Gamma_{f}^{(LA)}} p_{,t}^{(\beta)} p_{,t}^{(\beta)} \nu_{3}^{(\beta)} / (2\rho_{f}^{(\beta)}g) d\Gamma - \int_{\Gamma_{w}^{(\beta)}} p^{(\beta)} \hat{w}_{i}^{(\beta)} \nu_{i}^{(\beta)} d\Gamma + \int_{\Sigma^{(\alpha)}} [pW_{i}\eta_{i} - \rho_{f}g^{(\alpha)}U_{i,t}\nu_{i}U_{j,t}\nu_{j}/2]^{(\beta)} d\Gamma \right\} dt + \sum_{\Gamma_{f}^{(LA)}} \Pi_{\chi}[p]^{(\beta)} / \lambda$$

$$\mathbf{H}_{sf}[\phi, U_{i}] = \mathbf{H}_{s}[U_{i}] - \mathbf{H}_{f}[\phi] - \mathbf{H}_{int}[\phi, U_{i}]$$

$$\mathbf{H}_{\chi}[\phi] = \int_{t_{1}}^{t_{2}} dt \int_{\Gamma} \chi \phi_{,It} \phi_{,It} / (2g^{2}) d\Gamma$$

$$\mathbf{H}_{s}[U_{i}] = \int_{t_{1}}^{t_{2}} \left\{ \int_{\Omega_{s}} (C_{ijkl} E_{ij} E_{kl} / 2 - \rho_{s} U_{i,t} U_{i,t} / 2 - \hat{F}_{i} U_{i}) d\Omega_{s} - \int_{S_{T}} \hat{T}_{i} U_{i} dS \right\} dt$$

$$\mathbf{H}_{f}[\phi] = \sum_{\beta} \int_{t_{1}}^{t_{2}} dt \int_{\Omega_{f}^{(\beta)}} \rho_{f}^{(\beta)} [\phi_{,t}^{(\beta)} \phi_{,t}^{(\beta)} / (c^{2})^{(\beta)} - \phi_{,i}^{(\beta)} \phi_{,i}^{(\beta)}] / 2d\Omega_{f}$$

$$\mathbf{H}_{int}[\phi, U_{i}] = \sum_{\beta} \int_{t_{1}}^{t_{2}} \rho_{f}^{(\beta)} \left\{ \int_{\Gamma_{f}^{(\mathrm{LA})}} \phi_{,t}^{(\beta)} \phi_{,t}^{(\beta)} \nu_{3}^{(\beta)} / (2g) d\Gamma + \int_{\Gamma_{v}^{(\beta)}} \phi^{(\beta)} \hat{v}_{i}^{(\beta)} \nu_{i}^{(\beta)} d\Gamma + \int_{\Sigma_{v}^{(\alpha)}} [U_{i} \eta_{i} (g^{(\alpha)} U_{j} \nu_{j} / 2 + \phi_{,t})]^{(\beta)} d\Gamma \right\} dt + \sum_{\Gamma_{v}^{(\mathrm{LA})}} \mathbf{H}_{\chi}[\phi]^{(\beta)} / \lambda$$

In association with the theoretically proposed mixed FE-SS model for linear FSI problems, the computer codes FSIAP was also developed (Xing 1992a, 1992b, 1995a, 1995b), which has been used to simulate many engineering problems (Tan et al. 2006; Xing et al. 1997b, 2007a, 2007b, 2008a, 2008b, 2009; Xiong et al. 2006a, 2006b, 2007, 2008a, 2008b). For examples, as shown in **Fig. 6**: (a) dynamic response of LNG tank-sea water interaction system to explosion waves in the water; (b) huge LNG storage tank impacted by airplane; (c) air-liquid-elastic spherical tank vibrations; (d) building structure-acoustic volume interaction system excited by human footfall impacts. FSI analysis has been used to design linear and nonlinear devices for effective wave energy harvesting (Xing et al. 2009a, 2011; Yang et al. 2011).

3 Mixed FE-BE model

For marine structure-water interaction dynamics, the water is normally assumed incompressible and its motions are irrotational which can be modeled by a potential of velocity (Bishop et al. 1979, 1986; Newman 1977, 1978, 1994). For this type of FSI problems, the equation governing the water motion is a Laplacian equation defined in an infinite water domain. The BE method (Brebbia 1980) is very effective to deal with problems defined in an infinite domain by a boundary integration equation on the boundary. The developed mixed FE-BE method (Xing et al. 2004, 2005a, 2005b; Jin et al. 2007, 2009; Jin 2007) for FSI dynamics combines the powerful FE approach for solid structures in a finite domain and BE for the water in an infinite domain to improve the computation efficiency. The basic idea of this mixed method is as follows.

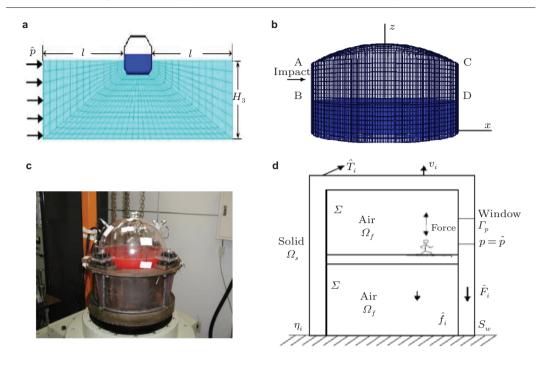


Fig. 6

Examples of linear FSI problems simulated by FSIAP code (Xing 1992a, 1992b, 1995a, 1995b)

- (i) For the solid structure, using FE method to obtain its natural modes and frequencies;
- (ii) Using the selected modes of the solid structure to construct a mode space to transfer the dynamic equations of the solid into the mode equations;
 - (iii) For the fluid domain, based on Green third identity to establish its BE equation;
 - (iv) Using the FSI conditions to generate the integrated coupling equations;
- (v) Numerically solve the integrated equations to obtain the numerical solution of the problem.

References (Xing et al. 2004, 2005a, 2005b; Jin et al. 2007, 2009; Jin 2007) presented the details of this approach to solve a transient dynamic response problem of an airplane landing on a very large floating structure (VLFS) on the water. As shown in Fig. 7(a), in this model, the aircraft and the floating body are two solid substructures, modelled by the mode function derived from FEA, with their interactions by the landing gear; while the water domain is modelled by BE method, which is interacting with the floating body. Due to the aircraft landing and travelling on the floating body, the problem is a transient dynamic problem involving moving loads. To validate the proposed numerical approach, a

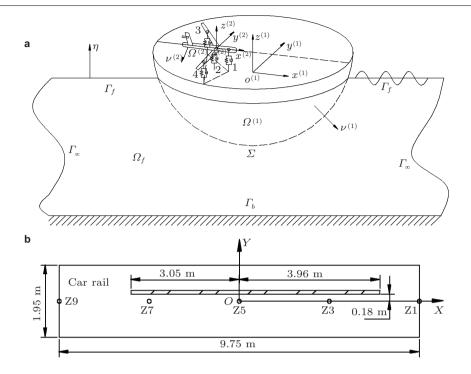


Fig. 7

(a) An integrated aircraft-VLFS-water interaction system subject to aircraft landing impact (Xing et al. 2004, 2005a, 2005b; Jin 2007); (b) The arrangement of a car running test completed by the reference (Endo et al. 1998)

car running test reported by Endo et al. (1998) shown in Fig. 7(b) was simulated and the very good agreement between the test and the numerical results has been observed as shown in Fig. 8.

The developed computer code of this method (Jin 2007) was used to simulate an airplane model landing on a VLFS shown in **Fig. 9**. The dynamic characteristics of the simplified airplane model were defined based on the published data of Boeing 747-400 (Boeing webpage) shown in **Fig. 9**. The total weight and geometrical size of the model are similar to the real airplane and the first natural frequency of symmetrical bending mode of the wing was set as 1 Hz based on the ground vibration test experience. The floating airport is a pontoon type/mat like VLFS, which has a length $5\,000\,\mathrm{m}$, breadth $1\,000\,\mathrm{m}$, depth $10\,\mathrm{m}$, draught $5\,\mathrm{m}$ and bending rigidity/unit breadth $1.764\times10^{11}\,\mathrm{Nm}$. The airplane landing speed is in the range of the real aircraft landing speed.

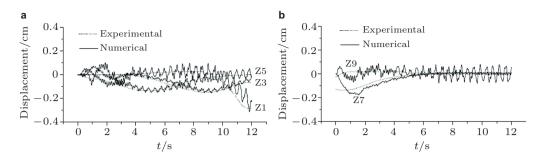


Fig. 8

The comparison of the experimental results (Endo et al. 1998) and the numerical results (Jin et al. 2007, Jin 2007)

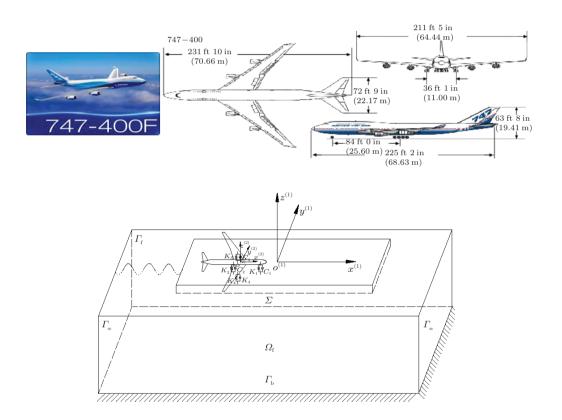


Fig. 9

The simulation of a simplified Boeing 747 model landing on a VLFS (Jin 2007)

4 Variational principles of nonlinear FSI problems

Some generalised variational principles to model nonlinear FSI problems involving var-

ious potential flows including heat effects were developed, of which the theory and mathematical works have been given in the paper (Xing et al. 1997). This provides a theoretical background to construct an approximate solution of nonlinear FSI systems (Xing et al. 1998) and to analyse nonlinear water-ship dynamics (Xing et al. 2000). Recently, one of variational principle was used to construct the numerical equations in order to investigate the dynamic behaviour of a water-filled cylindrical vessel subjected to high-frequency excitations, from which an important nonlinear phenomenon on low-frequency gravity waves produced by high-frequency excitations was revealed (Zhuo et al. 2013).

It is most important to notice the main differences in deriving variations of this functional for nonlinear FSI if compared with linear cases. For linear theory, we assume that the motions of fluid and solid are small so that its original configuration is taken as our reference state, and there is no need to distinguish Lagrange and Euler coordinates as well as involved variations. For example, when we take the variation of a quantity defined in a fluid domain, we consider its boundary fixed at the original position and neglect the effect caused by the boundary motion. Furthermore, we freely exchange the order of time and space integrations without any considerations in linear variation process. For nonlinear cases, these operations are no longer valid, since the large motions cause the boundaries changes which have to be considered in the variation process. To address these differences, the related theories concerning the material derivative of a moving continuum domain, local and material variations, transmission and translation velocities of moving curves in the space are presented. These provide convenience to model nonlinear FSI dynamics using arbitrary-Lagrangian-Eulerian (ALE) system to coordinate the solid Lagrange meshes with the fluid spatial meshes.

As examples, here lists the functional $\Pi[\phi, h, U_i]$ for incompressible fluid-solid interactions and a one $\Pi[\rho_f, \phi, h, U_i]$ for compressible fluid cases as follows, which can be used to model the water-ship interactions as shown in **Fig. 10**.

$$\Pi[\phi, h, U_i] = \int_{t_1}^{t_2} \left\{ \int_{\Omega_f} \tilde{\rho}_f \left[-\frac{1}{2} \phi_{,i} \phi_{,i} - \phi_{,t} - g x_j \delta_{3j} \right] d\Omega + \int_{\Gamma_v} \tilde{\rho}_f \phi \hat{v}_i \nu_i d\Gamma \right\} dt - \int_{t_1}^{t_2} \left\{ \int_{\Omega_S} \left[A(E_{ij}) - B(V_i) - U_i \hat{F}_i \right] d\Omega - \int_{S_T} U_i \hat{T}_i dS \right\} dt, \tag{4}$$

$$\Pi[\rho_f, \phi, h, U_i] = \int_{t_1}^{t_2} \left\{ \int_{\Omega_f} \rho_f \left[-\frac{1}{2} \phi_{,i} \phi_{,i} - \phi_{,t} - e - g x_j \delta_{3j} \right] d\Omega + \int_{\Gamma_v} \rho_f \phi \hat{v}_i \nu_i d\Gamma \right\} dt - \int_{t_1}^{t_2} \left\{ \int_{\Omega_S} \left[A(E_{ij}) - B(V_i) - U_i \hat{F}_i \right] d\Omega - \int_{S_T} U_i \hat{T}_i dS \right\} dt. \tag{5}$$

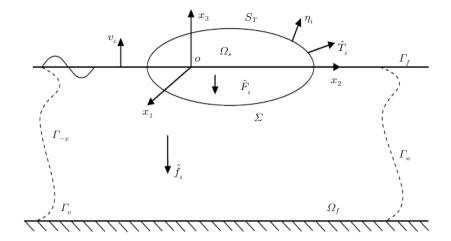


Fig. 10

Nonlinear water-structure interactions

Here, $\tilde{\rho}_f$ represents a constant mass density of incompressible fluid while ρ_f is the one for compressible fluid with its internal energy e involved; the notations $h = x_j \delta_{3j}$, ϕ and U_i represent the wave height, the fluid potential of velocity and the solid displacement, respectively.

5 Mixed FE-FD model for nonlinear FSI problems

Based on an arbitrary-Lagrangian-Eulerian (ALE) mesh system, the mixed FE-FD numerical model to solve nonlinear FSI problems was proposed in the papers by Xing et al. (2002, 2003, 2005). In this numerical strategy, the structure modelled by the powerful FE scheme (Bathe 1996; Zienkiewicz et al. 1989, 1991) is assumed to undergo large motions, while the fluid flow modelled by the effective FD/FV schemes in computational fluid dynamics (Hirsch 1988, 1990; Anderson 1995) is required to satisfy nonlinear, viscous or non-viscous, field equations and nonlinear boundary conditions on the free surface and FSI interfaces. As it is well known, applications of FE methods in structural analyses and CFD in fluid flows have achieved increased sophistication and success in recent years. However, when these methods are used to solve nonlinear FSI problems, problems arise. For example, in solid mechanics, a Lagrangian description usually adopts a material point formulation of the solid structure whereas in fluid dynamics, an Eulerian description describes the behaviour of the fluid at each point fixed in space. Thus variables describing a structural

motion are functions of the material coordinates fixed to each particle in the solid and time whereas variables describing a fluid flow are functions of the coordinates fixed in space and time. When the structure moves, the material coordinates also move from their original positions to new positions in space but the Eulerian coordinates remain unchanged. On FSI interfaces involving large disturbances, this difference of descriptions produces a separation of the mesh points on the solid from those in the fluid which initially coincided before the system was disturbed. For linear small disturbance problems, this separation is neglected and the original statically equilibrium configuration of the FSI system forms the basis of the mathematical model on which a numerical analysis is developed. However, for nonlinear problems, the difference between Lagrangian and Eulerian descriptions of a motion has to be fully described to keep the validity of compatibility conditions on the coupling interfaces.

For the numerical solution of Navier-Stokes fluid equations, references (Noh 1964, Trulio 1966) proposed an ALE coordinate system using an FD formulation. Its further development by Hirt et al. (1974) is particularly noteworthy in its application to arbitrary FD meshes and permits flows at all speeds to be treated. Further refined numerical schemes of study to calculate 3-D fluid flows around sharp interfaces were reported by Pracht (1975) and Chan (1975) and discussions of the analysis of free-surface fluid flows and moving boundaries were presented by Ramaswamy et al. (1987) and Floryan (1989). These contributions to the development of ALE techniques adopting FD techniques for the description of the fluid dynamics have produced powerful numerical tools. A detailed review on ALE system was given by Zhang et al. (1997). To retain the validity of compatibility conditions on the coupling interfaces in nonlinear FSI problems, references (Belytschko et al. 1978, 1982; Donea 1980, 1983; Donea et al. 1977, 1982; Hughes et al. 1981; Ward et al. 1988; Nitikitpaiboon et al. 1993; Bath et al. 1995) developed an ALE numerical model using FE both in the solid and fluid domains, in which the developed powerful techniques of FE are used to describe the dynamics of the solid, but the benefit of adopting an FD/FV formulation to describe the dynamics of the fluid is reduced. Therefore references (Xing et al. 2002, 2003, 2005) combine FE for solid with the FD/FV for fluid to develop a mixed FE-FD numerical scheme of study to calculate nonlinear FSI problems. This mathematical model provides a numerical scheme in association with an overset grid (Freitas et al. 1999) to solve nonlinear FSI problems based on powerful FE and CFD commercial codes. The developed approach was validated by the benchmark problems and experimental results (Durao et al. 1988) as well as other publications (Caughey 2001, Franke et al. 1991, Koobus et al. 2000).

6 Mixed FE-SP method for nonlinear FSI dynamics with breaking waves

The proposed FE-FD method for nonlinear FSI problems is one of grid-based numerical methods which present difficulties in simulating nonlinear FSI problems with breaking waves, where the mesh is distorted or broken resulting in computation going down. Therefore, it demands to develop an alternative numerical method without grids to avoid these problems caused by mesh generation. Since 1970s efforts have been put into developing meshless methods, most of which are inherently Lagrangian methods which are suitable to simulate violent nonlinear FSI problems involving breaking waves. The history, characteristics, development and applications of this type of methods are summarised as follows.

6.1 History, characteristics and development of SPM methods

SP method, such as smoothed particle hydrodynamics (SPH), has been largely developed in the last three decades since its introduction in astrophysics (Gingold et al. 1977, Lucy 1977). It is based on the theory of integral interplant to approximately express partial differential equations by integral formulism through different kernel functions. The interpolation method used in particle method is closely related to the standard interpolation methods used in other more traditional numerical methods (Monaghan 1982). Apart from interpolation methods, SP formulations for governing equations can be derived based on Lagrangian formalisms (Bonet et al. 2004). In SP methods, the system is discretised into many particles which carry material properties such like density, velocity, stress and so on. These particles move following the physical laws without directly connections between each other. Unlike traditional mesh required methods, the particles are free of moving in the space hence this method is convenient to simulate fragmental sprays such like breaking waves or explorations, fluid flow with free surface and other type of large deformation problems. Elaboration of SP methods including general issues such as the particle forms of conservation laws, construction of artificial viscosity, kernel functions, nearest particle searching algorithm, accuracy analysis etc. were presented in the early works (Monagham 1982, 1988, 1992; Monagham et al. 1985a, 1985b; Libersky et al. 1991; Swegle et al. 1995; Randles et al. 1996) and some publications (Capuzzo-Dolcetta 2000; Liu 2003; Liu et al. 2003a, 2003b; Wroblewski et al. 2007; Dominguez et al. 2010).

6.2 Applications

6.2.1 Incompressible flows

SPH was first applied to incompressible fluids with free surface problems (Monagham 1994) and then for incompressible flows of low Reynolds number (Morris et al. 1997). In those early works, fluid such as water was assumed to be slightly compressible. Therefore, quasi-incompressible equation of state was applied to calculate pressure. Time stepping size depended on sound speed adjusted to confine the fluid density variation. This weakly compressible SPH method (WCSPH) was proved to successfully simulate Poiseuille flow through comparing the results obtained from finite volume method (Lobovsky et al. 2007). However, it requires a very small time step and a small density error may cause significant non-physical pressure fluctuation (Lee et al. 2008). An approximated pressure projection method was developed (Cummins et al. 1999) to enforce fluid incompressibility by solving a pressure Poisson's equation. Afterwards, a truly incompressible SPH method was proposed (Shao et al. 2003) using prediction-correction fractional steps to upgrade the related physical properties. The temporal velocity field was integrated forward in time without considering the pressure effect in the first step. The temporal density obtained from the first step was then implicitly projected onto a velocity divergence-free space to satisfy incompressibility in the second step. The pressure values were calculated through a Poisson's equation matrix. This incompressible SPH (ICSPH) was widely used since then (Hosseini et al. 2007). However, a more straightforward way to ensure the incompressibility is to use constant density (Lee et al. 2010). More publications on incompressible flows especially waves simulated using SPH are given in references (Schussler et al. 1981, Morris et al. 1997, Pozorski et al. 2002, Oger et al. 2006, Ellero et al. 2007, Ataie-Ashtiani et al. 2008, Colagrossi et al. 2009, Shao 2009, Dalrymple et al. 2010).

6.2.2 Multi-phase flows

For more realistic cases, multi-phase flows need to be considered. The early SPH application on multi-phase flows is about compressible fluid like dusty gas. The mixed fluid can be treated as a new type of fluid and void fraction is used (Monagham et al. 1995c; Johnson et al. 1996a, 1996b). The mass density of this new fluid can be updated based on the continuity equation. Since the density of dust is known so the gas density can be obtained through the void fraction. It is more difficult to consider the situation when air mixed into water as large density ratio may cause instability for the algorithm. It needs to be careful

to choose an appropriate particle evolution form to ensure the density to be continuous for each phase. If WCSPH method is used, the densities may need to be re-initialized during the computation process (Colagrossi et al. 2003). In the latest work about compressible multiphase flows, the pressure of gas is assumed to be constant and the density was calculated from the equation of state (Ritchie et al. 2001). Different forms of particle evolution are developed to get rid of the influence of large density ratio (Grenier et al. 2008, 2009; Hu et al. 2006). However, most of these works are based on WCSPH method, an improvement of ICSPH is still necessary for multi-phase incompressible flows. SMH have been used to simulate more multi-phase flows, porous flows, shock/impact problems, solitary waves etc. as given by Monagham (1987, 1996), Monagham et al. (1983, 1995c, 1999), Stellingwerf (1994), Hu et al. (2007) and Jiang et al. (2007).

6.2.3 Solids and correction methods

When a solid continuum is in a state of isotropic tension, it is stretched and the SPH particles attract each other to resist the stretching, which causes particle clumping. The use of the standard SPH equation in conjunction with an explicit time stepping scheme leads to unstable time integration for any time step size and this instability cannot be eliminated by introducing artificial viscosity (Bonet et al. 1999). This phenomenon is called "tensile instability" that is a major deficiency of traditional SPH. Besides, SPH cannot produce correct estimation on the boundaries as not enough particles near around. Various methods were developed to improve the accuracy, stability and consistency of SPH. Reproducing kernel particle methods (RKPM) uses a correction function multiplying the kernel function to provide correction on boundaries and reproduce kernel stability (Liu et al. 1995b). The correction function can be expressed in a linear combination of polynomial basis function with unknown coefficients determined by ensuring the approximated function or its derivative to be reproduced exactly. The number of these coefficients involved in the definition of the correction function is determined by the order of the highest derivative term in the governing equations (Aluru 1999). The increased stability of the reconstruction equation enabled the ability of using very few particles and eliminated the tensile instability associated with SPH methods (Liu et al. 1995a, Jun et al. 1998). Normalized smoothing function adjusted the standard smoothing functions for every node to normalize the kernels to handle boundary effects and reduce the tensile stability (Johnson et al. 1996c). This algorithm was applied to simulate impact problems (Johnson et al. 1996a, 1996b). Corrective smoothed particle method (CSPM) was developed for solving the general boundary deficient problems with

heat conduction (Chen et al. 1999). It resolved the general problem of particle deficiency at boundaries by applying the kernel estimate to the Taylor series expansions. The number of the terms involved in the Taylor expression depends on the order of the approximated function. A similar method called generalized smoothed particle hydrodynamics method (Chen et al. 2000) focus on reproducing the first and second derivatives in 2-D case and finally extends to any order of derivatives. By modifying CSPM, a modified smoothed particle hydrodynamics (MSPH) method was developed (Zhang et al. 2004) to improve the accuracy of the approximation near the boundary but it will take more CPU time since that big matrix is required to be solved. The truncation error of the gradient approximation analysis (Quinlan et al. 2006) for studying the robustness and accuracy of SPH formulations indicated that the accuracy of SPH discretization could be improved if the absolute values of pressure and velocity can be reduced before calculating the gradients for non-uniformly distributed particles. But a uniform distribution of particles is better and smaller smoothing length gives more accurate results. For Poiseuille flow, by subtracting hydrostatic pressure from the absolute pressure in the momentum equation, the absolute value of pressure can be reduced so that the truncation error can be reduced as well (Basa et al. 2009). In some cases, particle oscillation may happen and results in unphysical approximation, the original SPH is not able to provide accurate estimations therefore correction is necessary. Usually, the velocity may be corrected by using XSPH variant, where X is an unknown factor, to avoid two or more particles with different velocities occupying the same position (Monaghan 1989, 1992, 2002). The XSPH variant makes a particle move with a velocity that is closer to the average velocity in its neighbourhood. This velocity corrective term was to smooth out oscillations of particle velocities calculated by integration of the momentum equation (Antoci et al. 2007, Crespo et al. 2007, Lobovsky et al. 2007, Sun 2013).

6.2.4 FSI problems

SP method is good for simulating large deformation fluid motions with breaking waves and it has been applied to FSI problems by combing FE for structure simulation. This can be done through a master-slave algorithm to couple FSI (Attawy et al. 1994; Johnson 1994; Johnson et al. 1996a, 1996b). The contact constraint was satisfied by applying a contact force to both the slave node and the master surface, which is to remove any penetration in the next time step. The force is always normal to the corresponding element surface. Sliding between particles and elements in tangential direction is allowed since that in the algorithm if make move of the slave nodes, the master nodes move in a manner consistent with the

velocity changes. When a slave node overlaps master segment, the normal velocities of these involved three nodes are adjusted to conserve linear momentum, angular momentum and a velocity match of the slave node on master segment. Hybrid approximation coupling method has advantage since the particle sums remain symmetric so that the deformation gradient is computed correctly (Rabczuk et al. 2006). Besides, the approximation is independent of the relation between the particle distance and FE length. SPH coupled with FE method has a strong coupled form, the pressure of fluid and the stress of structure can be calculated at the same time step. The deformation of FE model becomes the actual boundaries of fluid domain in the dynamic process, so that the FE coupled with SP are a good solution for boundaries of SP (Monaghan 1994). Another possibility to simplify the coupling is to apply SPH method on solid as well. The shear stress, deviator stress and pressure formalism can be derived by applying SPH directly to strain rate tensor and the rotation rate tensor. This makes the transfer of information between the fluid and structure domains easier as similar algorithm is used on both parts. It also makes the simulation more efficient in the case of large deformation happened on solid. There are two coupling models of using SPH for both solid and fluid. One is to treat all the particles in the same way regardless of their nature and a XSPH correction is applied to avoid the particle penetration (Rabczuk et al. 2006; Rafiee et al. 2008, 2009); another is to treat the fluid and solid separately and find out the right position of interface surface and its normal direction (Antoci et al. 2007). The force acting on the solid from the fluid is computed through the evaluation of an approximated surface integral of fluid pressure. This force on the interface follows action-reaction principle. In some cases when the tensile instability becoming serious artificial viscosity and artificial stress term can be used to deal with the problem (Antoci et al. 2007, Bui et al. 2007).

6.3 Mixed FE-SP approaches

References (Sun 2013; Sun et al. 2011, 2012, 2013; Javed et al. 2013a, 2013b, 2014a, 2014b; Javed 2015; Sun et al. 2014, 2015a, 2015b) investigated a mixed FE-SP approach to simulate nonlinear FSI problems involving breaking waves and FS separations. In this approach, the solid motion includes large rigid motion with small elastic deformation represented by mode summation, while the fluid is formulated by SP and on the FSI interface the velocity consistence and force balance conditions are required. The new developments are: (i) to coordinate with the projection method for incompressible fluids, the force balance condition is expressed in a pressure gradient form using the acceleration at the FE nodes of the solid, which can avoid the calculation of the solid stress; (ii) A mixed source term in the

pressure Poisson equation with no artificial term in the formulation; and (iii) A new version of "cell-link" neighbour particle searching strategy, which reduces about 6.5/9 (~72%) of the searching area compared with traditional "cell-linked" algorithm (Sun et al. 2014, 2015a, 2015b); (iv) Hybrid SP-Eulerian grid is used to deal with FSI problems involving viscous flows in large domain, in which the active domain around the FSI interface adopts SP scheme while the outside domain use Eulerian grid to improve the calculation efficiency (Javed 2015; Javed et al. 2013a, 2013b, 2014a, 2014b). The developed method has been validated by experiments or other publications. **Figure 11** shows the results for a 2-DOF vibrating cylinder in the fluid presented in the thesis (Javed 2015). **Figures 12–14** show the results for wedge dropping and water-floating box interactions (Sun 2013; Sun et al. 2014, 2015a, 2015b). In the PhD theses (Sun 2013, Javed 2015), more examples involving two phases flows, aircraft landing on the water surface, aerofoil-air interactions etc. are given.

7 Designs of wave/wind energy harvesting devices

With increasing requirement on green energies to reduce environment pollutions, scientists and engineers have put more effort to extract energy from sea waves/wind. Investigations on wave energy harvesting devices have attracted a wide interest around the world; for example, see references (Thorpe 1999, Falnes 2002, Department of the Navy 2003, Bedard et al. 2005, Rhinefrank 2005, Wave Dragon 2005, US Department of the Interior 2006, JAM-STEC 2006, Ocean Power Technologies 2006, Ocean Power Delivery Ltd 2006, Wave Plane Production 2006). Various type of aerofoil designs to harvest wind/flow energy are discussed in the review papers (Young et al. 2014a, 2014b; Xiao et al. 2014), in which full-/semipassive one requires excitations to keep the device in a designed motion by an input energy from other source, while the full active one does not need this excitation energy that is more beneficial to harvest natural energies. Among these designs, the fundamental principle is to use waves/winds to excite mechanical motions of energy harvesting devices and then to convert mechanical energy to storable energies. Therefore, the motions of energy harvesting devices excited by fluids are required as large as possible. Two approaches may be adopted to realise this aim. One is to design a linear device with its natural frequency closing to the wave frequency so that a resonance is reached. In considering FSI and using the developed numerical method (Xing et al. 1991, 1996) with computer code FSIAP (Xing 1992a, 1992b, 1995a, 1995b), it was investigated a wave energy harvesting device-water interaction system

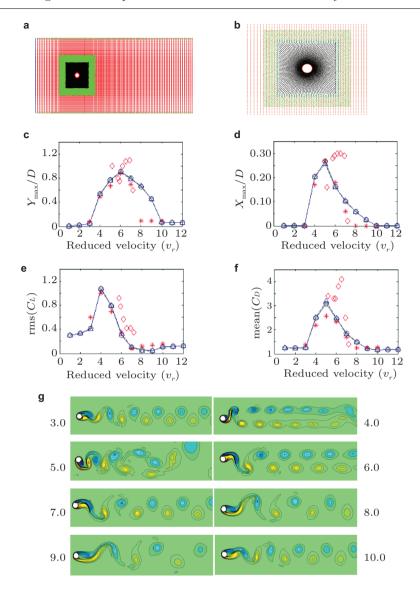
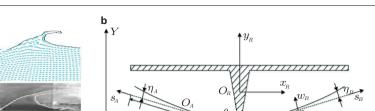


Fig. 11

The results of a 2-DOF vibrating cylinder by Javed (2015). (a) Full domain and (b) local view of hybrid grid around cylinder, Grey: active meshless; Green: inactive meshless overset on special grid; Red: special grid. (c) Crossflow amplitudes; (d) Inflow amplitudes; (e) RMS of lift coeffident; (f) Mean drag coeffident. (c)–(f) Comparisons: -o- present study; -*-reference (Zhou et al. 2012); \diamondsuit experimental (Dahl et al. 2010); (g) Vortex structure velocities v_r

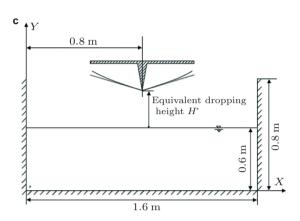
subject to the wave maker excitation in a towing tank (Xing et al. 2009a). The results demonstrated that FSI changes the natural frequency of the device designed in dry cases

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Fig. 12

Wedge dropping simulations, (a) Rigid one comparison with experiment photo (Zhao et al. 1997) by Sun (2013); (b) Elastic wedge structure; (c) FSI system of wedge dropping (Sun et al. 2014, 2015a, 2015b)

and therefore obviously affected the system efficiency. Another idea is to design a nonlinear energy harvesting system and to use its inherent large stable orbit motion to extract energy. References (Xu et al. 2005, 2007; Litaka et al. 2008, 2010; Lencia et al. 2008; Horton et al. 2011; Nandakumar et al. 2012; Pavlovskaia et al. 2012) proposed and investigated the possible rotational motions of a nonlinear pendulum subject to different base motion excitations aiming to wave energy harvest but not considering FSI. Based on the limit cycle of a nonlinear flapping foil energy harvesting system, paper (Yang et al. 2011) revealed that the efficiency of energy harvest was largely increased. Here, it might be necessary to mention that the vibration resonance in structure dynamics and the flutter in aircraft dynamics, which are usually intended to be avoided for safe operations of the designed products, are now in a reverse case to be generated to harvest large wave energy. It is more important that the investigation (Xing et al. 2009a) revealed that the collected energy plays a role like an active damping added to the system, so that it is useful to keep the stability of nonlinear

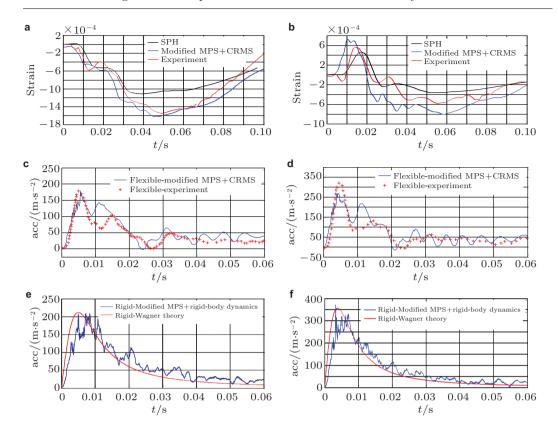


Fig. 13

Comparisons given by Sun et al. (2014, 2015a, 2015b) with experiments (Panciroli et al. 2012), (a)-(b) strains at two points from wedge tip: a) $30 \,\mathrm{mm}$, (b) $120 \,\mathrm{mm}$; (c)-(d) accelerations of elastic wedge: (c) case 1 (4.29 m/s), (d) case 2 (5.57 m/s) (Panciroli 2003); (e)-(f) accelerations of rigid wedge: (e) case 1, (f) case 2 with Wagner's theory in (Khabakh-pasheva et al. 2003, 2013)

systems as demonstrated for the flutter aerofoil system where a stable electric current was obtained (Yang et al. 2011).

Sea wave/wind energy harvesting devices are operated on or in fluids, so that the total system is a typical FSI system. The dynamic behaviours of any devices designed in dry cases are affected by fluids. A minor change of the characteristic frequency of device can cause a large difference of dynamic response (Xing et al. 2009a, Yang et al. 2011). Therefore, FSI have to be considered to design effective energy harvesting devices. Furthermore, as studied in the papers (Xing et al. 2009a, 2011), electric units in energy harvesting devices cause mechanical behaviours affected by electro-magnetic dynamics, which involves

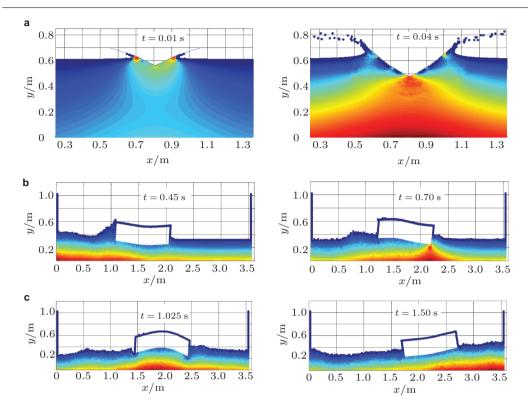


Fig. 14

FSI interaction pictures by Sun et al. (2014, 2015a, 2015b): (a) wedge dropping; (b), (c) floating box on the water surface

electric-mechanical interactions (EMI). Therefore, for efficient energy harvesting designs, an interdisciplinary research concerning nonlinear dynamics, fluids, solids and electric systems as well as their interactions is necessary. To this purpose, reference (Xing et al. 2011) proposed a mathematical model (pressure-displacement) to study the dynamics of nonlinear oscillators, pendulums and SD oscillator (Cao et al. 2006, 2008a, 2008b) coupled with water as well as an electric-magnetic energy converter aiming to wave energy harvest, as shown in Fig. 15. The paper qualitatively discussed the integrated FSI/EMI characteristics and proposed numerical solution approaches. Moreover, the velocity of potential for water can also be adopted as a variable to describe fluid motions to investigate nonlinear wave energy harvesting systems. The nonlinear energy flow theory (Xing 2015) can be used to analyse more complex system based on energy conservation laws. For practical applications, more practical problems, such as, random sea waves, structure fatigues, allowable environments, sloshing stability (Ibrahim 2005) of integrated system, etc. are required to be considered.

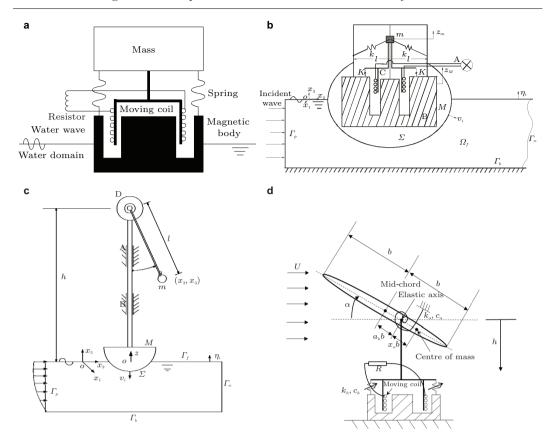


Fig. 15

Integrated FSI/EMI wave energy harvesting systems: (a) linear one based on resonance (Xing et al. 2009); (b) nonlinear SD oscillator (Xing et al. 2011); (c) nonlinear pendulum (Xu et al. 2005) extended to an integrated FSI system (Xing et al. 2011); (d) aerofoil flutter system (Yang et al. 2011)

8 Conclusion

A short review on numerical methods for linear and nonlinear FSI dynamics is given in this paper. The fundamental concepts, solution strategies, historical references and developments on FSI are summarised. This paper follows the first review paper by author and his colleagues published in the same journal (Xing et al. 1997a) to provide a further developed information on FSI dynamics. The emphasis of this paper is to present the developed four numerical methods to effectively solve various FSI dynamic problems in engineering. The mixed FE-substructure-subdomain approach with the corresponding FE code is more suitable to model linear FSI problems: sloshing, acoustic volume-structure interactions, dynamic

responses by earthquake and explosion wave excitations, etc. The mixed FE-BE model provides a convenience to simulate FSI problems involving infinite fluid domain, such as VLFS floating on the sea surface subject to aircraft landing/taking off impacts. The mixed FE-FD model gives an approach to use powerful FE and CFD commercial software in association with proposed partitioned iteration procedure to simulate nonlinear FSI problems, but which is difficult to model breaking waves. The mixed FE-SP model is more powerful to simulate violent FSI problems involving breaking waves, FS separations and three phases' flows. The review paper provides selected validation and application examples to confirm the investigated numerical methods as well as their engineering values. The discussion on wave/wind energy harvesting device designs proposes the ideas to use the resonance in linear systems and the flutter phenomenon/periodical solutions in nonlinear systems, which are usually avoided in structure designs, to collect large wave energy. The collected energy converter physically plays as an active damping which can keep the stability of nonlinear systems. Table 1 gives the comparison of the four discussed numerical approaches for readers to

Table 1 The comparison of 4 discussed methods

Contents		Methods				
		Mixed FE-	Mixed FE-BE	Mixed FE-FD	Mixed FE-SP	
		substructure method	method	method	method	
Reference coordinate systems	Solid	Linear modelling with		Updated Lagrange	Updated Lagrange	
		small disturbances without		system	system	
	Fluid	distinguishing Lagrange		Updated ALE	Total Lagrange	
		and Euler systems		system	system	
Variables	Solid	Displacement				
	Fluid	Pressure	Potential of	Velocity &	Velocity &	
			velocity	pressure	pressure	
	Solid	Linear dynamic equation		Nonlinear dynamic equation		
Governing equations	Fluid	Linear wave equation in domain and on possible free surface	Laplacian equation in domain with linear free surface wave	Nonlinear N–S equation with possible nonlinear free surface wave		
Meshes	Solid Fluid		FE meshes BE meshes	FE meshes	FE meshes Meshless particle	

Table 1 The comparison of 4 discussed methods (continued)

	Methods					
Contents	Mixed FE-sub-	Mixed FE-BE	Mixed FE-FD	Mixed FE-SP		
	structure method	method	method	method		
	Global FE					
	equation to					
	be solved by	Global FE-BE	Partitioned iteration to solve solid/fluid equations			
Solution	subspace iteration					
-	for eigenvalues	equation to be	separately in each time			
strategy	and modes;	solved by	step and through iteration			
	Mode summation	direct integration	convergence forward			
	for dynamic					
	response					
	Natural frequencies	Transient dynamic	Transient dynamic	Transient dynamic		
	modes & transient	responses of	responses for	responses for		
Academic	dynamic responses	FSI problems	nonlinear FSI	nonlinear FSI		
problems	of FSI problems	involving infinite	problems with no	problems involving		
	defined in	fluid domain	$F ext{-}S\ separations/$	$F ext{-}S\ separations/$		
	finite domain	jiaia aomain	breaking waves	breaking waves		
	LNG/Liquid tanks			Structure landing		
	sloshing		Large amplitude	on/into water with		
	Acoustic volume-	ŭ		breaking waves		
	structure interaction	water interactions	liquid tank	Green water		
	Explosion pressure	Water-VLFS-airplane interaction system Firing/landing	Flows around	of ships		
Engineering	waves in fluids		structures	Fluid flows		
examples			Structure vibrations	around structures		
	Impact and		excited by vortex	with separations		
	earthquake dynamic	impacts response of	Airplane flutter	Missiles out		
	responses Marine structure	military ships	Engine blade-air	of water		
	Marine structure—		interactions	Large amplitude		
	water interaction			sloshing		

choose a suitable method to tackle their interested FSI problems as well as further possible investigations.

The author does not claim the completeness of the cited references to include all publications of the world in this field, but to the author's knowledge and experience, the most

important publications have been mentioned. The readers may refer the published books on fluid-structure interaction dynamics, for example: the ones by Crolet et al. (1994), Morand & Ohayon (1995), Howe (1998), Dervieux (2003), Axisa et al. (2007), Wang (2008), Galdi et al. (2010), Souli et al. (2010), Khodabakhshi (2011), Paidoussis et al. (2011), Bazilevs et al. (2013), Chen (2013), Zslav (2013), Paidoussis (2013), Brebbia et al. (2013), Bodnar et al. (2014) and some FSI conferences, such as: ASME PVP series conferences (Panahi 1997), which covers theory, developments, numerical methods and engineering applications in different areas as well as more references links, to find more useful information for their particular research problems.

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线性与非线性流固耦合动力学数值方法的进展及应用

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摘 要 本文综述了线性与非线性流固耦合问题数值方法的进展及工程应用. 讨论了四种数值分析方法: (1) 混合有限元 - 子结构 - 子区域数值模型, 以求解有限域线性流固耦合问题, 如流体晃动, 声腔 - 结构耦合, 流体中的压力波, 化工容器的地震响应, 坝水耦合等; (2) 混合有限元 - 边界元数值模型, 以求解涉及无限域的线性流固耦合问题, 如大型浮体承受飞机降落冲击, 船舰的炮击回应等; (3) 混合有限元 - 有限差分(体积) 数值模型, 以求解涉及破浪和两相分离的非线性流固耦合问题. 文中推荐分区迭代求解过程, 以便应用现有的固体及流体求解器, 于每一时间步长分别求解固体及流体的方程, 通过耦合迭代收敛, 向前推进以达问题求解. 文中选用的工程应用例子包含气 - 液 - 壳三相耦合, 液化天然气船水晃动, 人体步行冲击引起的声腔 - 建筑结构耦合, 大型浮体承受飞机降落冲击的瞬态动力回应, 涉及破浪和两相分离的气 - 翼耦合及结构于水上降落的冲击. 数值分析结果与可用的实验或计算结果作了比较, 以说明所述方法的精度及工程应用价值. 文中列出了基于流固耦合的波能采积装置模型, 以应用线性系统的共振及非线性系统的周期解原理, 有效地采积波能. 本文列出了 231 篇参考文献, 以便读者进一步研讨所感兴趣方法.

关键词 线性与非线性流固耦合,混合有限元-子结构法,混合有限元-边界元法,混合有限元-有限差分法,混合有限元-光滑粒子法,流体晃动,声腔-结构耦合,破浪模拟,流体中的压力波,液化天然气船/超大浮体-水耦合,波能采积

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