UNIVERSITY OF SOUTHAMPTON

FACULTY OF BUSINESS, LAW AND ART

Southampton Business School

Multi-stage stochastic modelling for global supply chain and logistics under uncertainty

by

Lin Zhu

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ABSTRACT

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MULTI-STAGE STOCHASTIC MODELLING FOR GLOBAL SUPPLY CHAIN AND LOGISTICS UNDER UNCERTAINTY

Lin Zhu

This research focuses on the applications of multi-stage stochastic models for global supply chain and logistics, especially in global production planning problems and international air cargo forwarding problems under uncertainties. We first exam a multi-period, multi-product and multi-plant production planning problem under uncertain demand and quota limitations and develop a multi-stage stochastic model to handle this problem. Then we present three types of robust models for the same problem: the robust optimization model with solution robustness, the robust optimization model with model robustness, and the robust optimization model with the trade-off between solution robustness and model robustness. Results show that multi-stage models will bring more benefits to their decision-makers.

The second problem we look at is an international air cargo forwarding problem under uncertainty, which means the cargoes need to be transported from regions to destinations via a hub. The air forwarders not only have to make a decision about the number of containers to be booked for the regions and hub in advance before accurate customers' information becomes available, but also have to decide the number of extra containers to be required or the containers to be returned after the realisation of uncertainty. We develop stochastic models and three types of robust models for one day's flights per week and multi-days' flights per week cases for this air cargo forwarding problem. For the large scale problem which means the computer software cannot give the optimal solution, we also present a new way to design the genetic algorithm to get the better solutions.

Computational results show that the stochastic models can provide effective and cost-efficient solutions; the robust optimization models can provide a more responsive and flexible system with less risk.

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DECLARATION OF AUTHORSHIP

I, Lin Zhu

declare that this thesis and the work presented in it are my own and has been generated by me as the result of my own original research.

Multi-stage stochastic modelling for global supply chain and logistics under uncertainty

I confirm that:

1. This work was done wholly or mainly while in candidature for a research degree at this University;
2. Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated;
3. Where I have consulted the published work of others, this is always clearly attributed;
4. Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;
5. I have acknowledged all main sources of help;
6. Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;
7. None of this work has been published before submission

Attended conferences

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Signed: ....

Date: ....

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Chapter 1: Introduction

A supply chain is a network of transforming natural resources, raw materials, and components into a finished product and delivering that product to the customers (Bowersox *et al.*, 2002). Mentzer *et al.* (2001) define supply chain management as "the systemic, strategic coordination of the traditional business functions and the tactics across these business functions within a particular company and across businesses within the supply chain, for the purposes of improving the long-term performance of the individual companies and the supply chain as a whole." Supply chain management mainly contains supplier management, production planning management, inventory management, transportation management, customer service management and so on. In our research, we will focus on production planning problem and air transportation problem.

1.1 Background

In the 21st century, the outcome of globalization in the business environment has contributed to the development of supply chain management. Globalization can be characterized by the attention given to global systems of supplier relationships and the expansion of supply chains over national boundaries and into other continents. Globalization develops international operations, which require increasingly worldwide coordination and planning to achieve global optimums. This can make possible larger lot sizes, lower taxes, and better environments for the products. There are also many challenges when the supply chain is global, for example, different currencies in different countries, different tax laws and different trading policies.

Production planning management, as a fundamental part of supply chain management, has inevitably been greatly affected by the development of globalization. A very different situation from that which was common not many years previously is faced by manufacturing companies operating today. Products can be manufactured in any feasible area of the world, due to the substantial differentials in labour salary and raw material supply, continuously improving global logistics networks and dramatically decreased transportation costs. Business has been set in a global environment, where global corporations and brands dominate most markets in the world. Unless manufacturing companies develop competitive strategies, tactics, and operations for the global market, they risk being beaten by other manufacturers who have embraced more innovative approaches. Forces which are currently driving changes in the global supply chain environment include: advancement of information technology and easy access to the Internet; development of e-business, which can lead to global visibility for purchasing, production and distribution increasingly shortening products lifecycles, which leaves shorter time for manufacturers to produce; increased product variety, which makes it more difficult to accurately forecast market demand; global outsourcing of different activities; and empowered customers, who demand quick responses and speedy delivery while continuously lowering costs (Wu, 2006). Mass production, continuous production, single item manufacturing, batch production and other types of production methods have their own type of production planning. Therefore, one period plan and multi-period plan are both needed for production planning problems according to which production methods the companies choose.

Meanwhile, due to the rapid development of globalization, many domestic enterprises are facing greater competition from international firms than before. The government provides some protection policies to protect their own domestic companies, such as import quotas. Import quotas are extensively employed by various governments as a means of addressing perceived trade (import) imbalances. More specifically, import quotas are employed as a means of quantitatively restricting the importation of foreign products. For example, in order to protect their domestic companies, the importing countries will only allocate a certain amount of quotas to exporting countries. If the exporters want to export more products, they are expected to purchase quotas from importing countries first. This generally increases the exporters' production costs and, consequently, their market prices. For the designated country of import, the implication is that the unit price of such "foreign" products is expected to be higher than that of goods locally produced (thus serving as a means of protecting local production). Import quotas are therefore protectionist. In the United States, for example, although by 2009, clothing imports had totalled $63.10 billion (a 400% increase from 1990), its domestic industry continued to suffer from a steady decline not only in output, but in export as well, thus threatening the survival of domestic manufacturers (Lu and Dickerson, 2012).

Compared to a few years ago, logistics managers today encounter a totally different environment, particularly in terms of the supply chain, which has become global in outlook. Significant portions of global markets are dominated by multinational corporations and global brands; they enjoy large differentials in the costs of production, technologically advanced logistical networks and enhanced information technology. These changes enable them to purchase and/or manufacture products and materials and sell their products anywhere in the world. However, despite the ease of communication and interaction that has been perpetuated by the unprecedented growth of globalisation, the distance factor is an issue that is being taken seriously since it is necessary for shipments to move across continents and oceans before they reach their intended sites. The logistics managers need to consider not only how to make production plans, but also how to transport their products to the sales departments. The main options for long distance transportation are shipping by sea and air.

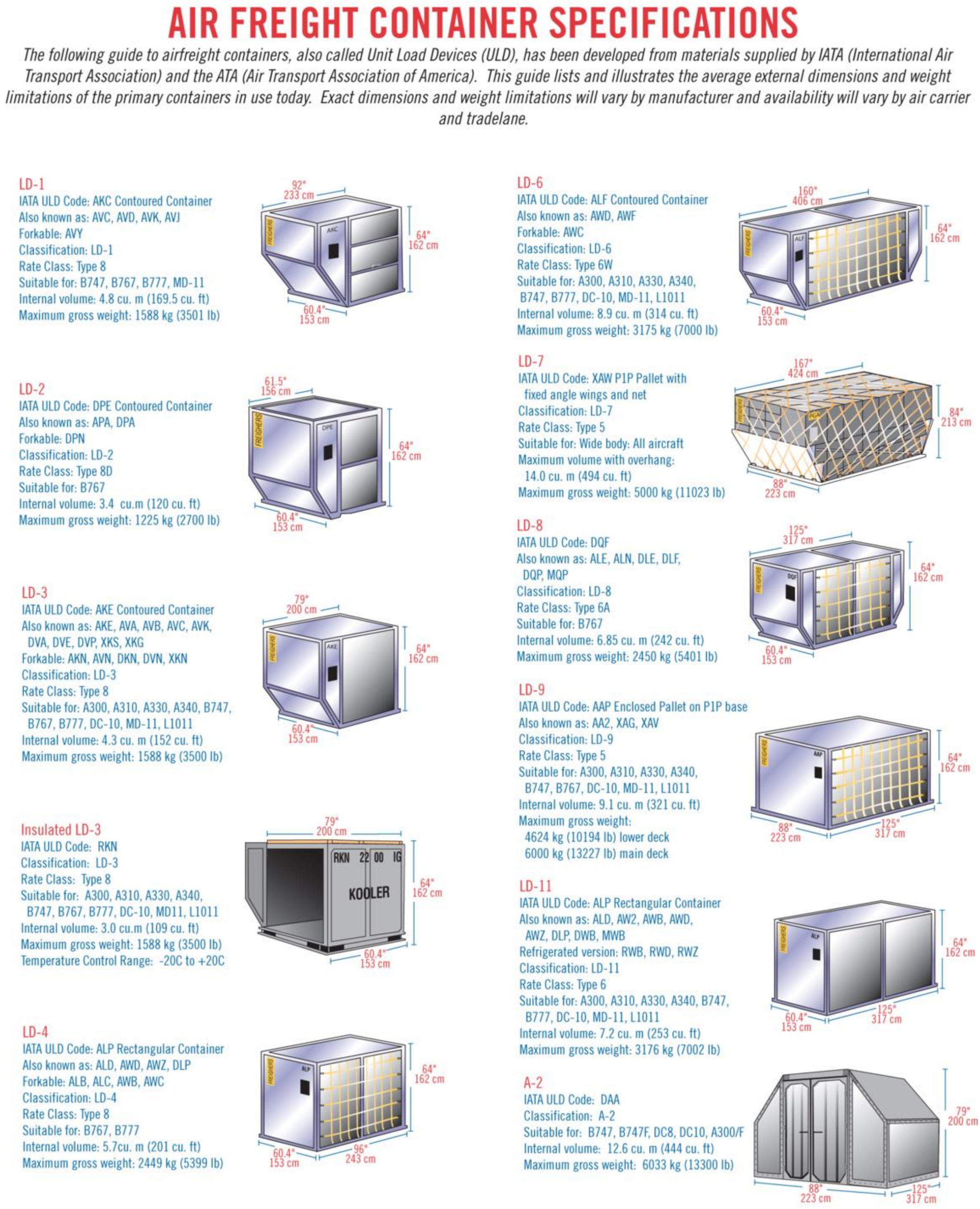

Figure 1.1 Air freight container specifications (source: http://www.maximafreight.com/UtilitesAirContainer.aspx)

Nearly five decades ago, containers were introduced as standard steel boxes designed for the sole purpose of convenient, easier and quick handling of cargo. For ship containers, they are often in cuboid format stretching 8 feet in width, and in the length either 20 feet, 40 feet or 45 feet. The length of ship containers also stand at either 8.5 feet or 9.5 feet (Christiansen *et al.*, 2007) . For air containers, the shape and size are largely dependent on the content and size of the cargo aircraft and this tends to limit its size. Figure 1.1 illustrates the air freight container specifications supplied by IATA (International Air Transport Association) and the ATA (Air Transport Association of America). There are 11 types of air container in Figure 1.1. Each type of container has its own dimensions, volume limit, weight limitation and suitable aircraft models. For examples, Containers LD-2 and LD-8 are only suitable for Aircraft B767 whereas Container LD-3 can be loaded in 10 different models of aeroplanes.

Transport by air is the quickest way to deliver items over long distances. However, time is money, and this can easily factor into the net costs of flying cargo. Transit time has been reduced from nearly 60 days for shipping, to just one day or two, because of the massive sizes of cargo planes, the high frequency of numerous airlines, and the presence of many airports in major cities around the world. Air cargo presently constitutes a marginal percentage of the world's freight in terms of weight. The nature of cargo transported via air is mainly high-value, and low density, and this converts all the value of the air freight cargo to constitute a larger proportion compared to the entire global market. Crabtree *et al.* (2014) exemplify that air cargo in the next two decades will grow, on average, 4.7% per year. They forecast that after 20 years the number of revenue tonne-kilometres (RTK) by air will reach more than twice the number reached in 2013.

Due to the rapid growth in airline business, airline hubs have been considered in recent years to facilitate more efficient use of scarce air transportation resources. When air cargo is forwarded via hubs in international commerce, goods are brought from the source region in one aircraft to the hub in which they are passed to the next aircraft, which will then convey the goods to their ultimate destination. The aircraft may or may not be of the same model. Such global air cargo forwarding via hubs usually involves two kinds of operation: either the hub may serve purely as a trans-shipment core, where products are handed over from an inbound to an outbound aircraft with not much storage involved, or the products which arrive may be kept and recorded in warehousing within the hub and sent to destinations when required. Whether trans-shipped or stored, there is no material alteration of these products. However, a restricted quantity of value-added actions such as information processing, reconsolidation, repackaging, inventory control, and break-bulking may occur at the hub. At the outset, it may appear that this procedure results in unnecessary extra handling of goods and increased transportation times but in reality it affords users improved flexibility and superior economics (Raguraman, 1997).

The problems we consider in this research are global production planning and international air container booking via a hub.

1.2 Research objectives

The main aims of this research are to provide mathematical models for production planning problems and air cargo forwarding problems via a hub, and mathematical algorithms for large-scale air cargo cases. The specific objectives are:

- To make production plans for the multi-period, multi-product and multi-plant problem under uncertain demands and import quota limits.

- To provide a more responsive and flexible system with less risk to deal with the uncertainties in the multi-period, multi-product and multi-plant problems.

- To make air containers booking plans for the air cargo forwarding problems with one day's flights per week, which means the air cargoes will be transported from different regions to different destinations via a hub in one day.

- To make air containers booking plans for the air cargo forwarding problem with multi-days' flights per week.

- To provide genetic algorithms (GA) for solving large-scale air cargoes forwarding problems.

1.3 Contribution of this research

This research makes several contributions as follows:

- In order to solve the multi-period, multi-product and multi-plant problem under uncertain demand and import quotas, we develop a multi-stage stochastic linear model. Results show that garment manufacturing firms are more likely to derive more benefits from our model than if it had adopted a two-stage model.

- For the same production planning problem, we present three types of robust optimization models to provide a more responsive and flexible system with less risk, which is particularly important in the current context of global competitiveness.

- Regarding the air cargo forwarding problems, we present a new problem that the forwarders should book air containers in advance, in order to ship the cargoes from different regions to different destinations via a hub in which the cargoes need to be repacked and consolidated before leaving.

- For the new air cargo forwarding problems we build, we develop two-stage (for one day's flights per week) and multi-stage (for multi-day's flights per week) stochastic models and also robust models to make the problems with less risk.

- For the large size air cargo forwarding problems, we design a GA with the purpose of providing solutions because the mathematical software cannot solve them.

1.4 Outline of the thesis

The rest of the thesis is organized as follows:

In Chapter 2, the literature related to the production planning problems and the air cargo forwarding problems is reviewed. Then we review the mathematical models which we use to solve the above two problems, such as two-stage stochastic models, multi-stage stochastic models and robust models. Finally, we also provide a brief review about GA, which have been used to solve large scale booking problems.

In Chapter 3, we develop a multi-stage stochastic model and three types of multi-stage robust models to solve the global production planning problems under demand uncertainty and quota limitation. The objectives are to minimise the total production costs. Numerical results and tests are then carried out to evaluate and compare these models.

In Chapter 4, we present a two-stage stochastic model and three types of two-stage robust models to solve the international air cargo forwarding problems. After that, all these models are developed to multi-stage models to solve the multi-period problems. All the two-stage and multi-stage models are formulated as mixed-integer programming models. The objectives are to minimise the air container booking costs. Due to the computational complexity of the multi-period problems, we provide a GA to solve the large-scale air cargo forwarding problem. Results and tests give the evaluation of these models and comparison between the GA results and exact solutions.

Chapter 5 summarizes the research presented in this thesis. The potential future research based on this thesis is also described in this chapter.

Chapter 2: Literature Review

In the latter years of the 20th century, the supply chains sector has witnessed considerable expansion into international locations, particularly in the computer, automobile, and apparel industries (Taylor, 1997; Dornier *et al.*, 1998). This growth in globalisation is unprecedented and rightfully so; it is accompanied by new challenges that did not exist before. Consequently, the new environment of global supply chain management has attracted both academic and practitioner interest. Bowersox *et al.* (2002) articulate a detailed introduction on the management of supply chain logistics including procurement and manufacturing, customer accommodation, inventory, transportation operations, packaging and so on. In the following sections, two parts will be the main focus of review: global production planning and international air cargo forwarding.

2.1 Production planning

The challenges associated with production planning, as a very important part of the global supply chain, have been addressed by many scholars. Such studies include that of Pyke and Cohen (1993) who develop an integrated production-distribution system for a one-product, three-location network by obtaining near-optimal solutions. On the other hand, Man *et al.* (2000) provide a multi-objective model for production scheduling planning that employed GAs. Taking into consideration production and distribution chains, Lee *et al.* (2006) develop an integrated mathematical model for the semiconductor industry supply chain system.

If all the necessary information for decision making is known before the planning time, the production planning problem will become simpler than the unclear information case. However, in the real world, there are many uncertain factors that may influence the production processes. Ho (1989) divides uncertainties into two groups: environmental uncertainty, such as demand and supply uncertainty, and system uncertainty, such as failure and maintenance time. Carino *et al.* (1994) give a review about production planning models with capacity uncertainty and demand uncertainty. This paper categorises these problems into two groups: single-period model and multi-period model. Mula *et al.* (2006) review most of the existing literature regarding production planning under uncertainty and list all the general types of uncertainty models used in manufacturing systems. They classify the uncertainty models into four areas: conceptual models, such as yield factors, safety stocks and safety lead times; analytical models, such as deterministic approximations, stochastic programming and Markov decision processes; artificial intelligence based models, such as expert systems, fuzzy set theory and multi-agent systems; and simulation models, such as Monte Carlo techniques, heuristic methods and dynamic systems. For each area, the uncertainties are divided into demand uncertainty, environmental uncertainty, system uncertainty, lead times uncertainty, operation yield uncertainty and supply lead time uncertainty.

Production planning problems under uncertainty can be classified into many topics, such as aggregate planning (Rinks, 1981; Turksen, 1988), hierarchical production planning (Hax and Meal, 1975; Gfrerer and Zäpfel, 1995), manufacturing resource planning (Billington *et al.*, 1983; Grubbström, 1999), material requirement planning (Orlicky, 1975; Murthy and Ma, 1991), supply chain planning (Petrovic, 2001; Das and Abdel-Malek, 2003), inventory management (Vujošević *et al.*, 1996; Ganeshan, 1999), capacity planning (Eppen *et al.*, 1989; Paraskevopoulos *et al.*, 1991) and so on. Thompson and Davis (1990) provide an integrated solution approach for the aggregate production planning problem demonstrated on a multi-product, fixed-workforce and multi-period example under the uncertainties in selling price, cost, demand, capacity, consumption of capacity, and retention of backorders. Meybodi and Foote (1995) develop a multi-objective hierarchical production planning and scheduling model under demand uncertainty and production failure. A mixed integer linear programming model including capacity constraints, company orders, demand forecasting and supply and subcontracting decisions for a rolling horizon planning process is built by Rota *et al.* (1997) to address the uncertainty and complex manufacturing environments. Du and Wolfe (2000) propose an active, bucketless and real-time material requirement planning system used a hybrid architecture including an object-oriented database, fuzzy logic controllers, and neural networks. Their system can particularize the exact releases and due dates for each requirement, scheduled receipt, and planned order. For the strategic supply chain planning problem, Koutsoukis *et al.* (2000) describe an integrated decision support system which has an embedded decision engine that uses two-stage stochastic programming as an example for optimisation under uncertainty. Samanta and Al-Araimi (2001) develop an inventory model using fuzzy logic by considering the periodic revision of inventory control with variable order quantity. Karabuk and Wu (2003) formulate a multi-stage stochastic programming with uncertain demand and capacity for the capacity planning problem for a major US semiconductor manufacturer.

In order to solve the production planning problems, researchers find many kinds of mathematical models and methods to deal with them. Yano (1987) uses a nonlinear programming formulation with the objective of minimising the sum of inventory holding costs and tardiness costs to address the problem of determining planned lead times in two-level assembly systems with stochastic lead times. Jolayemi and Olorunniwo (2004) formulate a deterministic model to maximise total profit over a finite planning horizon for planning production and transportation quantities in multi-plant and multi-warehouse environment with extensible capacities. Ould-Louly and Dolgui (2004) provide a mathematical formulation based on Markov chains to measure the average cost for a multi-period and multi-component supply planning problem with random lead time and fixed demand. Lütke Entrup *et al.* (2005) build mixed-integer linear programming models by considering shelf-life issues for the yoghurt production planning and scheduling problems. Wu (2011b) develops a two-stage stochastic linear recourse model for production planning problem with import quota limits and demand uncertainty. Guan and Philpott (2011) present a multi-stage stochastic programming model for the dairy industry by considering uncertain milk supply, price-demand curves and contracting. Two different robust optimization models that have differing variability measures to address the multi-product and multi-period production planning challenge of a sawmill business are proposed by Zanjani *et al.* (2010a).

There are not many literature contributions on multi-stage production planning problems especially employing robust optimization. Until now, only few researchers have addressed import quota limitations for production planning problems in the global supply chain system.

2.2 Air cargo forwarding

Nowadays, logistical services play a fundamental role in transferring or ferrying goods from their point of manufacture to where the customer requires them. Globalisation has made it possible to deliver goods within a short time, and particularly those with a short life cycle. These require an efficient and fast transportation mode. Over the last two decades or so, the air cargo industry has grown in leaps and bounds. To deal with the unprecedented demand, logistical systems are being used by all air cargo service providers in order to provide faster, more efficient and more secure transportation of goods across the globe. Despite the intervention of technological advancements in the propagation of logistical services, the loading of cargo onto aeroplanes is still largely dependent on manual labour and the decisions of the grounds crew. According to Hesse and Rodrigue (2004) air cargo loading is different from other forms of container loading in various respects, such as the fact that the containers used are not always rectangular, and this means that irregularly shaped containers are required for some of the cargo. Additionally, the mechanism for calculating the logistics costs in air cargo handling is dependent on the volume efficiency and the net loading weight of the air containers.

The first time containers were used was in the 1950s, and ever since curiosity has led to the increment of the amount of cargo that can be handled at any one time. According to Vis and De Koster (2003), containers are often explained as large boxes that are used in the transportation of goods from one point to the next, particularly to the customer’s destination. Storage of goods in an efficient manner while transporting them can be structured as a container loading challenge as purported by Bortfeldt and Gehring (2001). According to Dyckhoff and Finke (1992), the examination of container loading problems has attracted a lot of scholarly attention over the years, and as a result these problems are classified and construed in myriad ways.

The present literature contributions on the problems of container loading, unfortunately, focus mainly on sea container loading. Over the last few years, a lot of publications have been released discussing the problems faced when loading air cargo. A substantial amount of literature concentrates on the aspect of gravity encountered when loading aircraft. A comprehensive review of the computer assisted and manual approaches in loading air cargo is presented by Martin-Vega (1985), who considers the centre of gravity through pyramid loading as the most efficient way in dealing with air container loading problems. Martin-Vega’s work is extended by Mathur (1998) who provides an algorithm that had an improved worst-case performance. In addition, Amiouny *et al.* (1992) formulate a simple greedy heuristics that they apply in balancing when loading containers with the presumption that all air cargo containers have to be loaded in a particular manner; precisely in a one-dimensional hold. A military application is a very good approach from the perspective of Ng (1992), where all air cargoes are loaded and placed in a sequence of priorities. The problem of optimizing freight loading was assessed by Mongeau and Bes (2003) with the intention of reducing fuel consumption and ensuring the weight is properly balanced in a manner that is within the safety regulation provisions and stability. Through the use of a mathematical programming, a model is simulated which shows how the containers ought to be loaded into the cargo hold of an aircraft, as well as how the containers should be distributed into their respective compartments. Yan *et al.* (2008) point out that a good cargo container loading plan not only needs to minimise the airport operating costs, but should also consider the uncertain demand in real operations. They formulate a nonlinear mixed integer program to resolve daily stochastic demands in practice.

Some of the challenges encountered during the loading and shipping of air cargos include the cost of loading which is pegged on the weight of the entire container, as well as the volume of packing (Feng *et al.*, 2015). Yan *et al.* (2006) studied their model, constructed for cargo container loading plans, using the operations of the carrier FedEx. Airlines looking for a best possible baggage limit policy, whilst the goods were carried in the remaining aircraft belly space along with customers’ luggage, has been identified as a new difficulty by Wong *et al.* (2009). Most importantly, the due date of the consignment transportation has to be adhered to (Fong *et al.*, 2013). Fong *et al.* (2013) optimise the criterion followed during the loading of air cargo and conclude that it should be informed by customer needs while considering the optimum usage and the benefit to the aircraft. In their research to identify the most appropriate optimal shipment of air cargo, they applied three algorithms: the first is the GA extended using the due-date method; the second is the extended due-date method; and the third, the GA with the extended due-date method. Bing and

Bhatnagar (2013) formulate two models: a single flight and a sequential multi-flight. Both models rely on the uncertain capacity restriction from the point of view of the freight forwarders who are tasked with determining the booking amounts for spot and contract markets. The models also consider the problems of capacity booking.

A mixed integer linear programming model is formulated by Wu (2008) to assist logistic managers in making the necessary decisions when they are dealing with containerization of problems encountered while loading air cargo: precisely, when leasing air containers from air carrier service providers, as well as how to optimally load air cargo into their required containers. However, the model does not address the uncertainty challenge when the accurate information cannot be obtained at the time of booking. In order to solve this problem, Wu (2010) proposes a stochastic mixed 0-1 model for a dual-response forwarding system for booking air containers and determining how cargoes are loaded in the containers simultaneously under uncertainty. Wu (2011a) goes further and extends the stochastic model to a robust model for addressing similar challenges in which cargo is permitted to be shipped at a later date. The robust model uses a quantitative approach to measuring the trade-off between the costs involved and the risks expected.

Only few researchers focus on air container booking problems, especially considering to the addition of a hub to consolidate the air freight. There is also no paper contributing on the air container booking problems for multi-flight via a hub.

2.3 Stochastic programming

Invented in the 1950s as a derivation of mathematical programming, stochastic programming is used to assist other mathematical algorithms and models whose data has encountered a substantial degree of uncertainty (Beale, 1955; Dantzig, 1955; Charnes and Cooper, 1959). Stochastic programming is a beneficial tool that has been used in numerous areas such as fiscal planning (Carino *et al.*, 1994); financial planning (Escudero *et al.*, 1993); network planning of telecommunications (Sen and Higle, 1999); transportation (Ferguson and Dantzig, 1956); generation of electric power (Murphy *et al.*, 1982; Takriti *et al.*, 1996); control of hydropower systems (Infanger, 1994); bank portfolios (Kusy and Ziemba, 1986); and the management of supply chains (Fisher *et al.*, 1997; Santoso *et al.*, 2005).

2.3.1 Two-stage stochastic programming

Dantzig (1955) first propose a two-stage linear programming model, thus it is not surprising that by the 1960s stochastic programming has undergone a period of rapid development. For example, while Kataoka (1963) develops a nonlinear model with linear inequality constraints, Warner and Prawda (1972) present a mixed integer quadratic model for scheduling. Later, Birge (1985) employs decomposition and partitioning methods to develop a multi-stage stochastic linear model. Escudero *et al.* (1993), on the other hand, propose a mixed integer programming model for multi-product, multi-period, single-level production planning with demand uncertainty using a non-anticipative principle.

Over the past several decades, two-stage stochastic programming has been widely applied in a number of operations management areas, particularly in supply chain management and production planning. For example, Bakir and Byrne (1998) propose a two-stage stochastic linear programming model for a multi-product, multi-period production problem with only demand uncertainty. Subsequently, Kazaz *et al.* (2005) formulate a two-stage recourse program to describe a global production plan with uncertain exchange rate.

Meanwhile, two-stage stochastic methods were also explored widely (Birge and Louveaux, 1988; Huang and Loucks, 2000; Ahmed *et al.*, 2004; Barbaroso and Gcaron, 2004). Darby-Dowman *et al.* (2000) formulate two-stage stochastic programming with recourse model for planting problems in horticulture under uncertain weather. Alonso-Ayuso *et al.* (2003) present two-stage stochastic 0-1 modelling considering the production topology, plant sizing, product selection, product allocation among plants and vendor selection for raw materials. A branch-and-fix coordination heuristic is proposed for solving this model. Zanjani *et al.* (2013) propose a two-stage stochastic linear programming approach to deal with a sawmill production planning problem where the non-homogeneous characteristics of logs incur random process yields.

2.3.2 Multi-stage stochastic programming

It can be observed that, since the year 2000, the utilisation of multi-stage stochastic models has become widespread. Examples of studies that have employed such models include Ahmed and Sahinidis (2003), who build a multi-stage stochastic mixed-integer program model for a stochastic capacity expansion problem characterised by fixed-charge cost functions, and forecast uncertainties. They are able to demonstrate that the distance between heuristic and accurate optimisation solutions could almost disappear with an increase in problem size. Shortly afterwards, Alfieri and Brandimarte (2005) provide a general review of multi-stage stochastic models applicable to manufacturing. This study was taken forward the following year by Brandimarte (2006) who, utilising simulation, develop a multi-stage model for multi-item capacitated lot-sizing problems with uncertain demand. Other studies in this area include those of Goh *et al.* (2007) who design a multi-stage global supply chain network model with profit maximisation and risk minimisation objectives, by using Moreau-Yosida regularisation; Rappold and Yoho (2008), who put forward a multi-item integrated production-inventory model with highly uncertain demand and Huang and Ahmed (2009) who formulate a multi-stage stochastic programming model (for production and capacity planning) under uncertainty, although this is constrained by multiple resources, tasks and products. More recently, Zanjani *et al.* (2010b) develop a multi-stage stochastic model with uncertainty in the quality of raw materials and demand. Körpeoğlu *et al.* (2011) use multi-stage stochastic programming to handle the master production scheduling problem with finite capacity, controllable processing times, and uncertain demand values. This paper also gives an effective formulation for large instances to save computation time. Sen and Zhou (2014) provide a multi-stage stochastic decomposition for multi-stage stochastic programming models to find approximations which were very close to optimal solutions, because some of the multi-stage stochastic models are hard to solve when considering more uncertainties.

2.3.3 Robust optimisation

In the recent global supply chain management environment, accurate information about uncertainty becomes harder and harder to obtain. Comparing with stochastic optimisation, robust optimisation is provided to solve the optimisation problems with a certain measure of robustness to mitigate against uncertainty. Therefore the robust optimisation model can deal with risk and uncertainty. The robust optimisation idea was postulated by Mulvey *et al.* (1995), who use a nonlinear regularization function to come up with a robust counterpart approach. The regularization function operates as a monitor that automatically reprimands all constraint contraventions and uncertainties, which are dealt with using a discrete set of circumstances. This model has been applied in numerous areas that emanate from global supply chain challenges, particularly from uncertainty challenges. According to Vassiadou-Zeniou and Zenios (1996), when using the conventional simulation models that are used in bond pricing, when combined with elements from the robust optimisation techniques, beneficial tools for the management of callable bonds portfolios are generated. Consequently, it is possible to assert that, through robust optimisation, two beneficial models are developed for one period which are capable of addressing numerous challenges.

In the event of dealing with an incapacitated network in which a design problem occurs when deciding whether to use an assortment of probable future scenarios or just a single fixed future scenario, the robustness approach can be used in choosing the most appropriate selection (Gutiérrez *et al.*, 1996). The concept of restricted resource was introduced by Vladimirou and Zenios (1997). It integrates parameterized restrictions in stochastic models so as to execute sturdiness in alternative decisions. Vladimirou and Zenios (1997) come up with three optional models of stochastic programs with a limited recourse, carried out similar tests on each of them, and then compared their performance. They examined the exchange between the firmness of recourse decisions and the predicted expense of implementing a resolution in a robust optimisation model.

A robust optimisation model was formulated by Yu (1997) to deal with problems of stochastic logistics. He exemplified this using two logistical examples from an airline and wine company and exhibited the calculation effectiveness of the robust model. Robust formulations are preferred by Darlington *et al.* (1999) when dealing with the restricted control of systems in uncertainty challenges. In the development of stochastic and nonlinear models, a mean-variance robustness model is used. They assess the flexibility of the development using a penalty model, and to test the robust strategies an engineering and chemical optimisation challenge was used. Additionally, Yu and Li (2000) come up with a robust optimisation model to deal with stochastic logistic quandaries. They set out a seamless method which, when adapted, would minimise the computational burden in practice.

Furthermore, a robust optimisation model is proposed to address stochastic aggregate production planning by Leung and Wu (2004). In their proposal, they formulate model solutions that relied on multi-period and multi-period data and then compare their performances before considering the exchange between the robustness of the model and the solution. Leung *et al.* (2007) when encountering an uncertain environment deal with the challenge of multi-site production planning using the robust optimisation model. In trying to diminish the impacts of vacillation of the unclear boundaries involving all the likely scenarios that can occur in the future, Rahmani *et al.* (2013) formulate a robust optimisation model to realise their objectives.

2.4 Genetic algorithm

An extensive analysis of the present heuristic approaches revealed that since 1975 when Dr. John Holland formulated GAs, they have increasingly been used in various fields and are preferred by many scholars in suggesting the best ways of dealing with challenging optimisation and combinatorial problems (De Jong, 1975; Davis, 1989; Rudolph, 1994; Thierens and Goldberg, 1994; Palmer and Kershenbaum, 1995; Reeves, 1997; Cheng *et al.*, 2000). The last decade has witnessed a growing interest in the application of GAs to provide solutions for a myriad of single and multi-objective challenges encountered in global supply chains management (Dimopoulos and Zalzala, 2000; Vidal *et al.*, 2012).

GA is a familiar heuristic method, inspired by the biological development of living life forms that functions on a population of the resolutions concurrently, which has been extensively employed in solving sequencing and scheduling problems. It begins with a set of arbitrary solutions called a population. Golberg (1989) coalesces structured, yet randomized, information exchange with the notion of survival of the fittest to undertake vigorous examination and utilization of the solution space. In the natural world, each individually named chromosome is allocated a fitness value. A genetic operator, namely crossover, executes the examination process and the utilization process is executed by another operator – mutation. The Darwinian theory of evolution is the basis for the selection of the new generation, as in it individuals with better performances will have more probability of being chosen. It is managed by the parent selection and offspring acceptance strategies. Our choosing GA in this thesis has two rationales: Firstly, GA is a well-known heuristic method and its efficiency is confirmed in literature (Bazzazi *et al.*, 2009; Tavakkoli-Moghaddam *et al.*, 2009; Lee and Kim, 2010; Lu and Xi, 2010); secondly, a population-based approach such as GA is required to enhance our exploration of the solution space. It has been demonstrated that GA-based methodologies can solve large-sized problems with almost optimal solutions.

Studies by Altiparmak *et al.* (2006) establish that GAs operate on an assortment of probable solutions, selecting the most resilient of them all, which will generate better approximations to the suggested solution. Each generation witnesses the selection of new sets of possibly better solutions that are derived from the selection process in which problems are simulated by genetic operators. This process, according to Ko and Evans (2007), breeds a fresh and dynamic evolution of the population of individuals who have a higher chance of surviving the environment. Some of the GA operations are specific to real problems and the efficiency of the solution technique is dependent on how effectively the quandary is demarcated for the various genetic processes.

Torabi *et al.* (2006) assert that in GA the search parameters of all likely solutions to the problems are beaconed onto a set of fixed strings. This process is known as encoding. The GA does not directly work on the solution; rather it focuses on how the solutions will be represented. Essentially, one of the most crucial stages in GA is the encoding process of the solution space. Improper encoding of the various solutions can lead to misguiding the algorithm which will select weak or erroneous solutions (Ko and Evans, 2007). The encoding process identified should be selected carefully and sequentially so that at each point, the search space is adequately represented by a chromosome, or a string. GA commences with supplying various probable solutions to the identified problems, which are then all placed in a pool where the initial generation is selected randomly. At this stage, the chromosomes selected inherit the best qualities from both parents. In other words, the best position of packing and arranging items/boxes into the containers is identified. The positioning and packing of bins in air cargo may at times not be most favourable in the preliminary generation, and maximum fitness appears to be focused on the optimal point. At this stage, the mutation operator aids in attaining the best optimal fitness. The final generated chromosome has both feasible and optimal solutions for the packing of the boxes into air containers. Compared to other techniques for optimisation, GA conducts a parallel search over a fixed point set in the solution space, and in so doing it avoids being stuck in a local optimum.

The fitness function of a solution chromosome is what determines its survival capabilities. The fitness function is informed by the objective function of the predicament (Vidal *et al.*, 2012). The higher the fitness value of the solution chromosome, the higher the chances of its selection. After selection, the solution chromosomes are placed in a mating pool where they are expected to crossover and mutate so as to increase the chances of generating better solution chromosomes (Man *et al.*, 2000; Ko and Evans, 2007; Tang, 2011). Aytug *et al.* (2003) assert that crossover offers fundamental alterations in the genes, but the mutation provides only slight alterations to the genes that are randomly selected. The mutation probability is much lower compared to the crossover probability. Thus, only a slight part of the population solution selects mutation. Once the crossover and mutation operations are completed, the better child chromosomes are selected and sent to the preceding generation (Rudolph, 1994; Palmer and Kershenbaum, 1995; Man *et al.*, 2000). Davis (1989) and Altiparmak *et al.* (2006) stress that in some GA applications; a randomly chosen portion of chromosomes is subjected to inversion; other generic operators involve the position changing process of the genes in the chromosome while keeping their meaning. These genetic operations offer a chance to replace the bad genes with those that are good.

Chapter 3: Global production planning Problem

With today's increasingly competitive environment, many international companies, whose markets are in the EU and North America, for example, decide to build their factories in developing countries, such as Vietnam, mainland China and Thailand, to reduce costs. The sales departments collect the market demand information; the headquarters make production plans according to the information; and production plans are then distributed to the several manufacturing plants (see Figure 3.1). The headquarters usually make these plans multi-period, such as weekly plans for one month or monthly plans for one season, under uncertainty. Uncertainties in production planning areas primarily consist of four factors: demand, processing, failure and maintenance times (Bakir and Byrne, 1998).

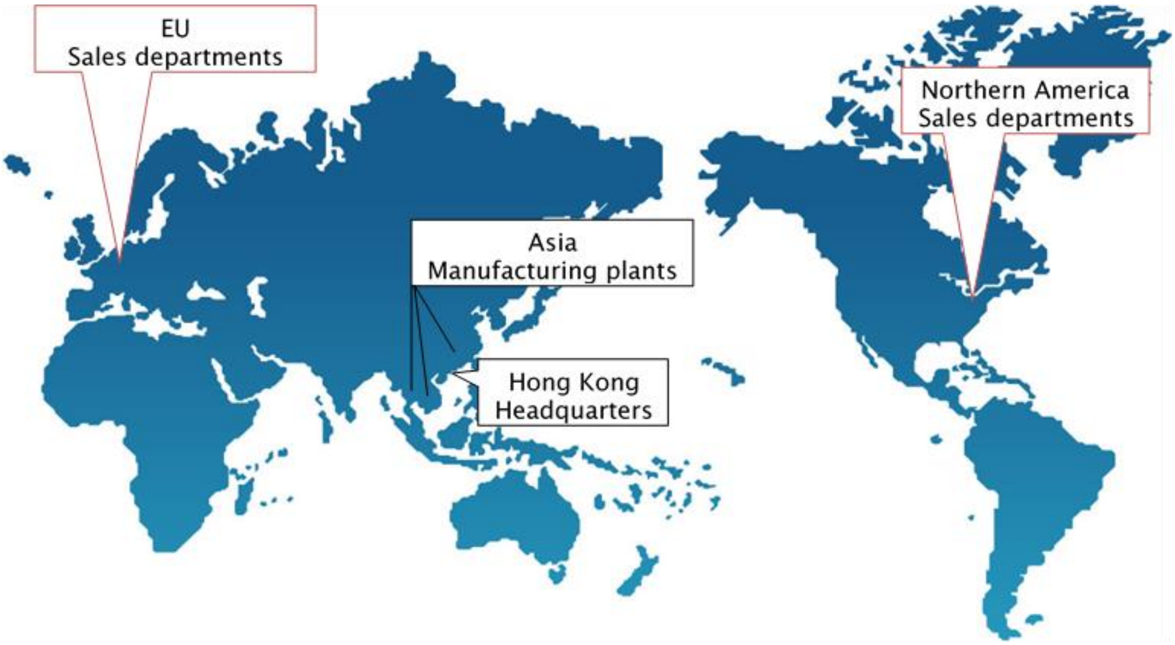

Figure 3.1 An example of international company

3.1 Problem description

In this research, it is assumed that company managers have to make production plans for the next selling season. Their manufacturing plants are allocated in several developing countries. Products can be produced by skilled or non-skilled workers in all plants. Each plant has its own machine working time and labour working time limitations. Accurate market demand information cannot be obtained before making the plans. Therefore we introduce a stochastic model to handle the uncertain demand.

The focus of this study is the optimisation of production planning within the global manufacturing industry. The purpose is using multi-stage stochastic linear model to help managers to make multi-period production plans under demand uncertainty and quota limitation. Decisions in the first stage tell plants how to produce products in the first period. Decisions in the second stage have two parts: one is how to satisfy the uncertainty when different uncertain scenarios are realised in the first period; the other one is how to produce products in the second period due to different uncertain scenarios occurring. Decisions in the following stages are similar. In the final stage, decisions consist only of how to meet the uncertainty in the final period. In this study, we assume that uncertainty satisfies a discrete stochastic process. Consequently, scenario trees are employed to describe uncertainty with stages in the scenario tree sequentially connected. A simple example illustrates that: if the demand in one stage is high, then the probability of high demand in the next stage is likely to be greater than if there had been low demand previously.

Various studies (Dupačová, 2002; Brandimarte, 2006; Huang and Ahmed, 2009), have explored the application of multi-stage stochastic models in production planning and management scenarios, the application of such models is very pertinent in today's global manufacturing planning sector for three reasons. In the first place, uncertainty is clearly identified as a major complicating characteristic of production planning and control (Guide, 2000), thus requiring research attention. Secondly, although there is substantial research articulating production planning models under uncertainty (Mula *et al.*, 2006; Wu, 2011b), there is generally only limited research that has focused on multi-period, multi-product and multi-plant production planning, particularly under conditions of import quota limits and demand uncertainty. Thirdly, studies (Bakir and Byrne, 1998; Khor *et al.*, 2008; Zanjani *et al.*, 2010b), suggest that multi-period production problems associated with uncertainty are unlikely to be robustly addressed utilising two-stage stochastic programming approaches. This is due to the fact that the approach requires the entire multi-period schedule to be designed prior to the realisation of uncertainty. However, with the multi-stage approach, planning decision modification can occur based on the incorporation of previously realised uncertainty.

To address challenges associated with multi-period-product and plant production planning, uncertainty consists of two elements: demand uncertainty and import quota limitation. If the quantity of the products produced is greater than the demand, this will incur an inventory cost. On the other hand, if the production quantity is lower than demand, it is necessary to purchase additional products at a high price to satisfy the demand. About the quota limitation, if the quotas bought cannot satisfy the demand, the managers have to buy extra quotas on the open market at the prevailing market price. Conversely, the penalty cost for used quotas will be charged.

In this study, the models aim to minimise the total costs incurred in meeting production demand. Here costs take two parts: (1) “certain costs” which include cost for labour (including regular and over working time cost for skilled and non-skilled works and hiring or firing works cost), machine (including regular and additional machine cost), raw materials and initial quota purchasing and (2) “uncertain costs” which include cost associated with import quota limitations and surplus or shortage cost to satisfy the uncertain demand. The models are tested using data from a garment manufacturing company. Results from the modelling (a 5-stage model with three possible outcome levels), suggest that using multi-stage planning affords varying levels of savings according to the profitability environment in which the company operates.

3.2 Multi-stage stochastic model

3.2.1 Scenario tree and fixed mix approach scenario tree

Within managerial thought, the notion of uncertainty generally applies to the prediction of events that may occur in (Kahneman and Tversky, 1979). Crucially, however, it implies that due to a number of factors including limited knowledge, describing the exact state of the future event is difficult (Bell, 1995). Generally, events which represent likely realisations are known as scenarios.

For multi-stage problems, uncertainty may be represented in the form of a scenario tree, as shown in Figure 3.2 which depicts a simple 4-stage scenario tree with given probabilities. The scenario tree consists of nodes which are linked by arrows. Each node represents a possible situation at the corresponding stage. The arrows denote relationship which includes probabilities. In the scenario tree shown in Figure 3.2, it is observed that scenarios “(11)” and “(21)” have the same outcomes at stage 3, but their previous stages are different. Therefore, “(11)” and “(21)” mean different scenarios in stage 3. Here we use descriptions, “(121)”, to denote different scenarios. Through this method, we can easily find the position and the route from the beginning for each scenario. In each stage, the sum of probabilities of the scenarios should be 100%. The multi-stage stochastic model principle is easy to understand from the scenario tree. The first stage is the decision plan for the first period. The second stage has two parts: urgent plans for the first period to satisfy the uncertainty when each scenario is realised and decision plans for the second period. The following stage is similarly to the second stage. The final T-stage only has one part, urgent plans for the T-1 period, because there is no period to make a plan for. Therefore, the number of stages at which decisions are made is one greater than the number of periods of uncertainty.

In order to compare the t-stage, (t-1)-stage, ⋯, 2-stage models, we introduce the fixed mix approach (Fleten *et al.*, 2002). The fixed mix and stochastic versions both need discrete probability distributions for the uncertain decision variables. The probabilities in the stochastic model can be described as a scenario tree (see Figure 3.2). Fixed mix means that the probabilities are rebalanced to fixed proportions at future decision nodes. For example in Figure 3.3, the probabilities in stage 4 are readjusted to 100%. That means the uncertainties in stage 4 exhibit similar conditions to those in the previous stage. In our research, we use this kind of fixed mix model to reduce the number of stages. Figure 3.3 shows that we can combine Stage 3 and 4 together because the uncertainties in Stage 3 and 4 are the same. Then the 4-stage model becomes 3-stage fixed mix model.

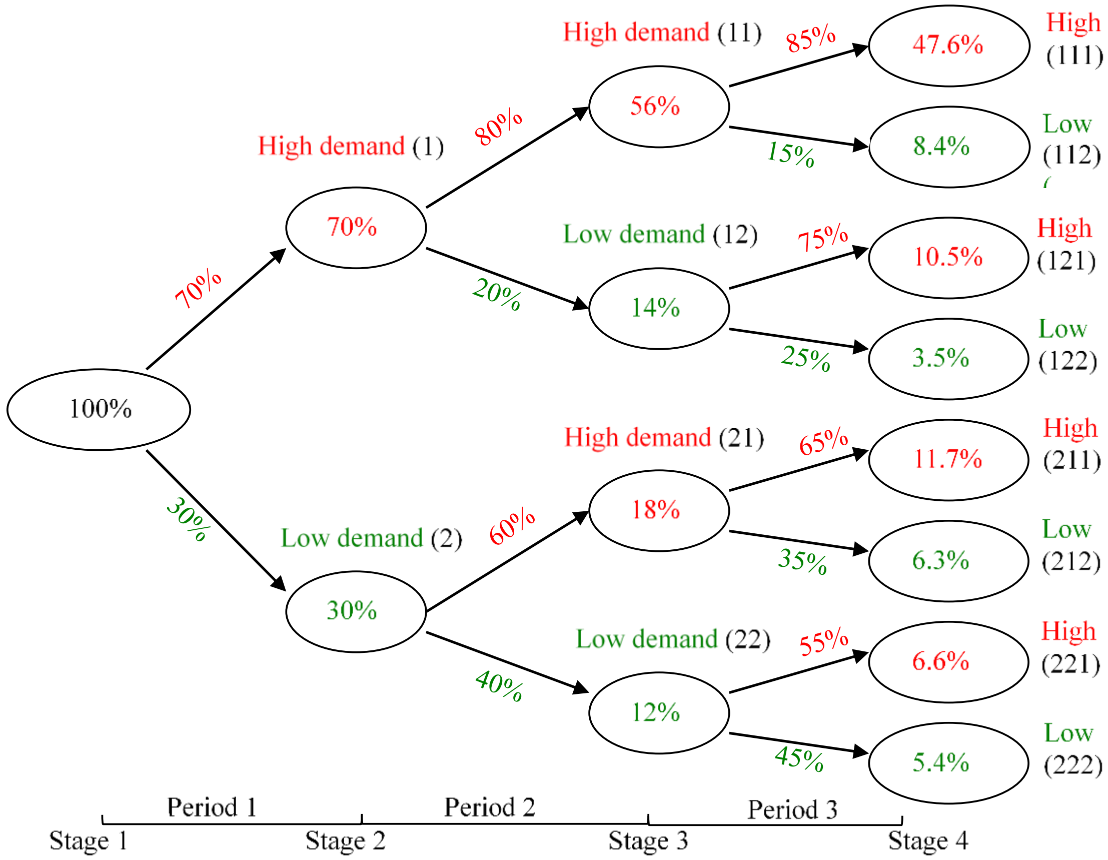

Figure 3.2 4-stage scenario tree

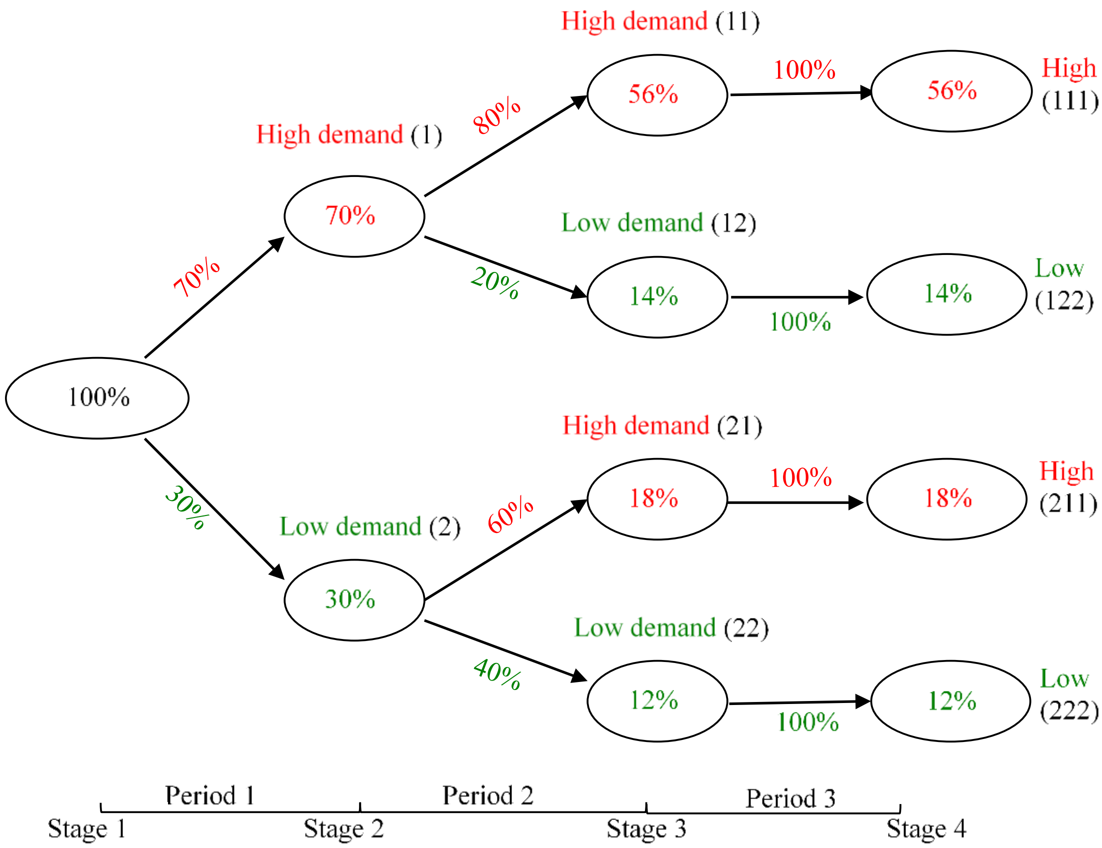

Figure 3.3 An example of fixed mix approach scenario tree

3.2.2 The general multi-stage stochastic model

The general linear deterministic model can be represented: $\min c^T x$, subject to $Ax = b$, where vector x, $x \geq 0$, is the decision variable (Birge and Louveaux, 1997). A is a fixed matrix; b is a fixed vector; and c^T is the related parameter with x.

For the multi-stage model, S_i represents the set of all scenarios in stage $i+1$ and P_{s_i} is the probability of scenario s_i, $s_i \in S_i$. x_1 denotes the decision variable for production plan in the first stage, and x_{s_i} denotes the decision variable for the production plan when scenario s_i happens. y_{s_i} denotes the decision variable for the uncertain purchasing/inventory plan when scenario s_i happens. c_i^T and $d_{s_i}^T$ are related parameters with x_{s_i} and y_{s_i} respectively. A and B are fixed matrices; and a_i and b_{is_i} are fixed vectors. Therefore, the general t-stage stochastic model (Birge and Louveaux, 1997) is:

$$\min c_1^T x_1 + \sum_{s_1 \in S_1} p_{s_1}(d_{s_1}^T y_{s_1} + c_2^T x_{s_1}) + \cdots + \sum_{s_{t-2} \in S_{t-2}} p_{s_{t-2}}(d_{s_{t-2}}^T y_{s_{t-2}} + c_{t-1}^T x_{s_{t-2}}) + \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}} \quad (3.1)$$

subject to

$$A_1x_1 = a_1, A_2x_{s_1} = a_2, \cdots, A_{t-1}x_{s_{t-2}} = a_{t-1} \tag{3.2}$$

$$B_{11}x_1 + B_{12}y_{s_1} = b_{1s_1}, B_{21}x_{s_1} + B_{22}y_{s_2} = b_{2s_2}, \cdots, B_{(t-1)1}x_{s_{t-2}} + B_{(t-1)2}y_{s_{t-1}} = b_{(t-1)s_{t-1}} \tag{3.3}$$

$$x_1, x_{s_1}, \cdots, x_{s_{t-2}}, y_{s_1}, \cdots, y_{s_{t-1}} \geq 0 \tag{3.4}$$

In the objective function, $c_1^T x_1$ is the first stage cost. $\sum_{s_i \in S_i} p_{s_i}(d_{s_i}^T y_{s_i} + c_{i+1}^T x_{s_i})$ is the cost in stage $i+1$. $\sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}}$ is the final , t-stage cost. Thus, minimizing the sum of all costs is the objective function.

3.2.3 Model formulation

Before proceeding to the modelling description, we list the assumptions used for the production planning problem, as follows:

- Uncertainty satisfies a discrete stochastic process;
- Raw material cost per unit is certain, just related to what kind of product and which plant to produce;
- Regular and additional machine costs per hour are certain, just related to the plant in question;
- The costs of skilled and non-skilled workers making a unit product are certain, just related to what kind of product is produced and at which plant;
- The costs of hiring or firing skilled and non-skilled workers are certain, just related to the plant in question and to the time period;
- The method used to ensure the quality of products is to control the ratio between skilled and non-skilled working time in each plant in the whole periods.

Notation

Indices

i products ($i = 1, \cdots, m$);

j plants ($j = 1, \cdots, n$);

t periods ($t = 1, \cdots, T$);

$(s_1 s_2 \cdots s_t)$ scenarios in period t (with outcomes $s_1, s_2, \cdots, s_t = 1, \cdots, S$).

Deterministic parameters

k_{ij}^1/k_{ij}^2 cost of skilled/non-skilled workers making a unit of product i in plant j;

o_j^1/o_j^2 overtime cost of skilled/non-skilled workers per hour in plant j;

h_{jt}^1/h_{jt}^2 cost of hiring skilled/non-skilled workers per hour in plant j at the beginning of period t;

f_{jt}^1/f_{jt}^2 cost of reduction of skilled/non-skilled working time per hour in plant j at the beginning of period t;

v_{j0}^1/v_{j0}^2 initial labour time of skilled/non-skilled workers in plant j;

α_j limit for the ratio between skilled and non-skilled workers for production in plant j;

l_{ij}^1/l_{ij}^2 labour time for production of a unit of product i in plant j by skilled/non-skilled workers;

r_{ij} raw material cost of production per unit of product i in plant j;

a_j^1/a_j^2 regular/additional machine cost of production per hour in plant j;

g_{ij}^1/g_{ij}^2 machine time for production of a unit of product i by skilled/non-skilled workers in plant j;

d_{i0}^+ initial inventory of product i at the beginning of the planning horizon;

c_i initial quota purchasing cost per unit of product i;

Q_i initial quota quantity of product i at the beginning of the planning horizon;

$p_{(s_1 s_2 \cdots s_t)}$ probability of scenario $(s_1 s_2 \cdots s_t)$ occurrence;

L_j^1/L_j^2 maximum capacity of hiring skilled/non-skilled workers in plant j;

W_j^1/W_j^2 maximum overtime for skilled/non-skilled workers in plant j;

C_j/A_j maximum regular/additional machine capacity of plant j;

V_j minimum work time in plant j;

I_i maximum inventory capacity for product i;

B_i maximum purchasing capacity for product i;

Random parameters

$D_{i(s_1 s_2 \cdots s_t)}$ demand for product i in scenario $(s_1 s_2 \cdots s_t)$;

$b^{-}_{i(s_1s_2\cdots s_t)}/b^{+}_{i(s_1s_2\cdots s_t)}$ under-/over-production cost of a unit of product i in scenario $(s_1s_2\cdots s_t)$;

$c^{-}_{i(s_1s_2\cdots s_t)}/c^{+}_{i(s_1s_2\cdots s_t)}$ under-/over-quota cost per unit of product i in scenario $(s_1s_2\cdots s_t)$.

<u>*Decision variables*</u>

x^1_{ij1}/x^2_{ij1} planned production quantities of product i by skilled/non-skilled workers in plant j for first period;

$x^1_{ij(s_1s_2\cdots s_t)}/x^2_{ij(s_1s_2\cdots s_t)}$ planned production quantities of product i by skilled/non-skilled workers in plant j for period $t+1$ when scenario $(s_1s_2\cdots s_t)$ is realised;

y^1_{j1}/y^2_{j1} planned labour time of hiring skilled/non-skilled workers in plant j for first period;

$y^1_{j(s_1s_2\cdots s_t)}/y^2_{j(s_1s_2\cdots s_t)}$ planned labour time of hiring skilled/non-skilled workers in plant j for period $t+1$ when scenario $(s_1s_2\cdots s_t)$ is realised;

z^1_{j1}/z^2_{j1} planned reduction in labour time of skilled/non-skilled workers in plant j for first period;

$z^1_{j(s_1s_2\cdots s_t)}/z^2_{j(s_1s_2\cdots s_t)}$ planned reduction in labour time of skilled/non-skilled working time in plant j for period $t+1$ when scenario $(s_1s_2\cdots s_t)$ is realised;

u^1_{j1}/u^2_{j1} planned regular/additional machine capacities in plant j for first period;

$u^1_{j(s_1s_2\cdots s_t)}/u^2_{j(s_1s_2\cdots s_t)}$ planned regular/additional machine capacities in plant j for period $t+1$ when scenario $(s_1s_2\cdots s_t)$ is realised;

v^1_{j1}/v^2_{j1} planned labour time of skilled/non-skilled workers in plant j for first period;

$v^1_{j(s_1s_2\cdots s_t)}/v^2_{j(s_1s_2\cdots s_t)}$ planned labour time of skilled/non-skilled workers in plant j for period $t+1$ when scenario $(s_1s_2\cdots s_t)$ is realised;

w^1_{j1}/w^2_{j1} planned overtime of skilled/non-skilled workers in plant j for first period;

$w^1_{j(s_1s_2\cdots s_t)}/w^2_{j(s_1s_2\cdots s_t)}$ planned overtime of skilled/non-skilled workers in plant j for period $t+1$ when scenario $(s_1s_2\cdots s_t)$ is realised;

q_{i1} initially allocated quota quantity of product i in first period;

$q_{i(s_1s_2\cdots s_t)}$ allocated quota quantity of product i for period $t+1$ when scenario $(s_1s_2\cdots s_t)$ is realised;

$d^{-}_{i(s_1s_2\cdots s_t)}/d^{+}_{i(s_1s_2\cdots s_t)}$ shortage/surplus of product i when scenario $(s_1s_2\cdots s_t)$ is realised;

$q^{-}_{i(s_1s_2\cdots s_t)}/q^{+}_{i(s_1s_2\cdots s_t)}$ under-/over-quota quantities of product i when scenario $(s_1s_2\cdots s_t)$ is realised.

3.2.3.1 A multi-stage stochastic linear recourse programming model

The objective of this model is to minimize the total costs including those costs that are known and those uncertain. Some decisions about this problem will be taken after the realization of uncertainty; these are referred to as recourse decisions (Birge and Louveaux, 1997). We therefore refer to this model as a recourse programming model.

$$\min Z = \sum_{t=1}^{T} M_t \tag{3.5}$$

subject to

$$M_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}(x^1_{ij1} + x^2_{ij1}) + \sum_{j=1}^{n}(a^1_j u^1_{j1} + a^2_j u^2_{j1}) + \sum_{i=1}^{m}\sum_{j=1}^{n}(k^1_{ij}x^1_{ij1} + k^2_{ij}x^2_{ij1}) + \sum_{j=1}^{n}(o^1_j w^1_{j1} + o^2_j w^2_{j1}) + \sum_{j=1}^{n}(h^1_{j1}y^1_{j1} + h^2_{j1}y^2_{j1} + f^1_{j1}z^1_{j1} + f^2_{j1}z^2_{j1}) + \sum_{i=1}^{m} c_i q_{i1} \tag{3.6}$$

$$\begin{aligned} M_t = &\sum_{s_1=1}^{S}\sum_{s_2=1}^{S}\cdots\sum_{s_{t-1}=1}^{S} p_{(s_1s_2\cdots s_{t-1})}\Big(\sum_{i=1}^{m}(b^{-}_{i(s_1s_2\cdots s_{t-1})}d^{-}_{i(s_1s_2\cdots s_{t-1})} + b^{+}_{i(s_1s_2\cdots s_{t-1})}d^{+}_{i(s_1s_2\cdots s_{t-1})} \\ &+ c^{-}_{i(s_1s_2\cdots s_{t-1})}q^{-}_{i(s_1s_2\cdots s_{t-1})} + c^{+}_{i(s_1s_2\cdots s_{t-1})}q^{+}_{i(s_1s_2\cdots s_{t-1})}) \\ &+ \sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}\big(x^1_{ij(s_1s_2\cdots s_{t-1})} + x^2_{ij(s_1s_2\cdots s_{t-1})}\big) + \sum_{j=1}^{n}\big(a^1_j u^1_{j(s_1s_2\cdots s_{t-1})} + a^2_j u^2_{j(s_1s_2\cdots s_{t-1})}\big) \\ &+ \sum_{i=1}^{m}\sum_{j=1}^{n}\big(k^1_{ij}x^1_{ij(s_1s_2\cdots s_{t-1})} + k^2_{ij}x^2_{ij(s_1s_2\cdots s_{t-1})}\big) + \sum_{j=1}^{n}\big(o^1_j w^1_{j(s_1s_2\cdots s_{t-1})} + o^2_j w^2_{j(s_1s_2\cdots s_{t-1})}\big) \\ &+ \sum_{j=1}^{n}\big(h^1_{j(t-1)}y^1_{j(s_1s_2\cdots s_{t-1})} + h^2_{j(t-1)}y^2_{j(s_1s_2\cdots s_{t-1})} + f^1_{j(t-1)}z^1_{j(s_1s_2\cdots s_{t-1})} + f^2_{j(t-1)}z^2_{j(s_1s_2\cdots s_{t-1})}\big) \\ &+ \sum_{i=1}^{m} c_i q_{i(s_1s_2\cdots s_{t-1})}\Big), \quad t = 2,3,\cdots,T-1 \end{aligned} \tag{3.7}$$

$$M_T = \sum_{s_1=1}^{S}\sum_{s_2=1}^{S}\cdots\sum_{s_{T-1}=1}^{S} p_{(s_1s_2\cdots s_{T-1})}\Big(\sum_{i=1}^{m}\big(b^-_{i(s_1s_2\cdots s_{T-1})}d^-_{i(s_1s_2\cdots s_{T-1})} + b^+_{i(s_1s_2\cdots s_{T-1})}d^+_{i(s_1s_2\cdots s_{T-1})} + c^-_{i(s_1s_2\cdots s_{T-1})}q^-_{i(s_1s_2\cdots s_{T-1})} + c^+_{i(s_1s_2\cdots s_{T-1})}q^+_{i(s_1s_2\cdots s_{T-1})}\big)\Big) \quad (3.8)$$

$$\sum_{i=1}^{m}(g^1_{ij}x^1_{ij1} + g^2_{ij}x^2_{ij1}) = u^1_{j1} + u^2_{j1}, \quad j = 1,\cdots,n \quad (3.9)$$

$$\sum_{i=1}^{m}(g^1_{ij}x^1_{ij(s_1s_2\cdots s_t)} + g^2_{ij}x^2_{ij(s_1s_2\cdots s_t)}) = u^1_{j(s_1s_2\cdots s_t)} + u^2_{j(s_1s_2\cdots s_t)}, \quad j = 1,\cdots,n;\ t = 1,\cdots,T-2;\ s_1,\cdots,s_t = 1,\cdots,S \quad (3.10)$$

$$\sum_{i=1}^{m} l^1_{ij}x^1_{ij1} = v^1_{j1}, \quad j = 1,\cdots,n \quad (3.11)$$

$$\sum_{i=1}^{m} l^1_{ij}x^1_{ij(s_1s_2\cdots s_t)} = v^1_{j(s_1s_2\cdots s_t)}, \quad j = 1,\cdots,n; t = 1,\cdots,T-2;\ s_1,\cdots,s_t = 1,\cdots,S \quad (3.12)$$

$$\sum_{i=1}^{m} l^2_{ij}x^2_{ij1} = v^2_{j1}, \quad j = 1,\cdots,n \quad (3.13)$$

$$\sum_{i=1}^{m} l^2_{ij}x^2_{ij(s_1s_2\cdots s_t)} = v^2_{j(s_1s_2\cdots s_t)}, \quad j = 1,\cdots,n; t = 1,\cdots,T-2;\ s_1,\cdots,s_t = 1,\cdots,S \quad (3.14)$$

$$v^1_{j1} = v^1_{j0} + y^1_{j1} - z^1_{j1} + w^1_{j1}, \quad j = 1,\cdots,n \quad (3.15)$$

$$v^1_{j(s_1s_2\cdots s_t)} = v^1_{j(s_1s_2\cdots s_{t-1})} + y^1_{j(s_1s_2\cdots s_t)} - z^1_{j(s_1s_2\cdots s_t)} + w^1_{j(s_1s_2\cdots s_t)}, \quad j = 1,\cdots,n; t = 1,\cdots,T-2;\ s_1,\cdots,s_t = 1,\cdots,S \quad (3.16)$$

$$v^2_{j1} = v^2_{j0} + y^2_{j1} - z^2_{j1} + w^2_{j1}, \quad j = 1,\cdots,n \quad (3.17)$$

$$v^2_{j(s_1s_2\cdots s_t)} = v^2_{j(s_1s_2\cdots s_{t-1})} + y^2_{j(s_1s_2\cdots s_t)} - z^2_{j(s_1s_2\cdots s_t)} + w^2_{j(s_1s_2\cdots s_t)}, \quad j = 1,\cdots,n; t = 1,\cdots,T-2;\ s_1,\cdots,s_t = 1,\cdots,S \quad (3.18)$$

$$v^1_{j1} + v^2_{j1} \geq V_j, \quad j = 1,\cdots,n \quad (3.19)$$

$$v^1_{j(s_1s_2\cdots s_t)} + v^2_{j(s_1s_2\cdots s_t)} \geq V_j, \quad j = 1,\cdots,n; t = 1,\cdots,T-2;\ s_1,\cdots,s_t = 1,\cdots,S \quad (3.20)$$

$$u^1_{j1} \leq C_j, \quad j = 1,\cdots,n \quad (3.21)$$

$$u^1_{j(s_1s_2\cdots s_t)} \le C_j\,,\quad j=1,\cdots,n; t=1,\cdots,T-2;\ s_1,\cdots,s_t=1,\cdots,S \tag{3.22}$$

$$u^2_{j1} \le A_j\,,\qquad j=1,\cdots,n \tag{3.23}$$

$$u^2_{j(s_1s_2\cdots s_t)} \le A_j\,,\quad j=1,\cdots,n; t=1,\cdots,T-2;\ s_1,\cdots,s_t=1,\cdots,S \tag{3.24}$$

$$y^1_{j1}-z^1_{j1} \le L^1_j\,,\quad j=1,\cdots,n \tag{3.25}$$

$$y^1_{j(s_1s_2\cdots s_t)}-z^1_{j(s_1s_2\cdots s_t)} \le L^1_j\,,\quad j=1,\cdots,n; t=1,\cdots,T-2;\ s_1,\cdots,s_t=1,\cdots,S \tag{3.26}$$

$$y^2_{j1}-z^2_{j1} \le L^2_j\,,\quad j=1,\cdots,n \tag{3.27}$$

$$y^2_{j(s_1s_2\cdots s_t)}-z^2_{j(s_1s_2\cdots s_t)} \le L^2_j\,,\quad j=1,\cdots,n; t=1,\cdots,T-2;\ s_1,\cdots,s_t=1,\cdots,S \tag{3.28}$$

$$w^1_{j1} \le W^1_j\,,\qquad j=1,\cdots,n \tag{3.29}$$

$$w^1_{j(s_1s_2\cdots s_t)} \le W^1_j\,,\quad j=1,\cdots,n; t=1,\cdots,T-2;\ s_1,\cdots,s_t=1,\cdots,S \tag{3.30}$$

$$w^2_{j1} \le W^2_j\,,\qquad j=1,\cdots,n \tag{3.31}$$

$$w^2_{j(s_1s_2\cdots s_t)} \le W^2_j\,,\quad j=1,\cdots,n; t=1,\cdots,T-2;\ s_1,\cdots,s_t=1,\cdots,S \tag{3.32}$$

$$\sum_{j=1}^{n}(x^1_{ij1}+x^2_{ij1})+d^+_{i0}+d^-_{i(s_1)}-d^+_{i(s_1)}=D_{i(s_1)}\,,\quad i=1,\cdots,m;\ s_1=1,\cdots,S \tag{3.33}$$

$$\sum_{j=1}^{n}(x^1_{ij(s_1s_2\cdots s_{t-1})}+x^2_{ij(s_1s_2\cdots s_{t-1})})+d^+_{i(s_1s_2\cdots s_{t-1})}+d^-_{i(s_1s_2\cdots s_t)}-d^+_{i(s_1s_2\cdots s_t)}=D_{i(s_1s_2\cdots s_t)}\,,\ i=1,\cdots,m;\ t=2,\cdots,T-1;\ s_1,\cdots,s_t=1,\cdots,S \tag{3.34}$$

$$q_{i1}+q^-_{i(s_1)}-q^+_{i(s_1)}=D_{i(s_1)}\,,\qquad i=1,\cdots,m;\ s_1=1,\cdots,S \tag{3.35}$$

$$q_{i(s_1s_2\cdots s_{t-1})}+q^+_{i(s_1s_2\cdots s_{t-1})}+q^-_{i(s_1s_2\cdots s_t)}-q^+_{i(s_1s_2\cdots s_t)}=D_{i(s_1s_2\cdots s_t)}\,,\ i=1,\cdots,m;\ t=2,\cdots,T-1;\ s_1,\cdots,s_t=1,\cdots,S \tag{3.36}$$

$$d^-_{i(s_1s_2\cdots s_t)} \le B_i\,,\qquad i=1,\cdots,m;\ t=1,\cdots,T-1;\ s_1,\cdots,s_t=1,\cdots,S \tag{3.37}$$

$$d^+_{i(s_1s_2\cdots s_t)} \le I_i\,,\qquad i=1,\cdots,m;\ t=1,\cdots,T-1;\ s_1,\cdots,s_t=1,\cdots,S \tag{3.38}$$

$$v^1_{j1}+v^1_{j(s_1)}+v^1_{j(s_1s_2)}+\cdots+v^1_{j(s_1s_2\cdots s_{T-2})} \ge \alpha_j\big(v^2_{j1}+v^2_{j(s_1)}+v^2_{j(s_1s_2)}+\cdots+v^2_{j(s_1s_2\cdots s_{T-2})}\big)\,,\quad j=1,\cdots,n;\ s_1,\cdots,s_{T-2}=1,\cdots,S \tag{3.39}$$

$$q_{i1} + q_{i(s_1)} + q_{i(s_1 s_2)} + \cdots + q_{i(s_1 s_2 \cdots s_{T-2})} = Q_i\,, \qquad i = 1, \cdots, m;\;\; s_1, \cdots, s_{T-2} = 1, \cdots, S \qquad (3.40)$$

$$x_{ij1}^{1}, x_{ij1}^{2}, y_{j1}^{1}, y_{j1}^{2}, z_{j1}^{1}, z_{j1}^{2}, u_{j1}^{1}, u_{j1}^{2}, v_{j1}^{1}, v_{j1}^{2}, w_{j1}^{1}, w_{j1}^{2}, q_{i1}, d_{i(s_1)}^{-}, d_{i(s_1)}^{+}, q_{i(s_1)}^{-}, q_{i(s_1)}^{+},$$

$$x_{ij(s_1 s_2 \cdots s_{t-1})}^{1}, x_{ij(s_1 s_2 \cdots s_{t-1})}^{2}, y_{j(s_1 s_2 \cdots s_{t-1})}^{1}, y_{j(s_1 s_2 \cdots s_{t-1})}^{2}, z_{j(s_1 s_2 \cdots s_{t-1})}^{1}, z_{j(s_1 s_2 \cdots s_{t-1})}^{2}, u_{j(s_1 s_2 \cdots s_{t-1})}^{1},$$

$$u_{j(s_1 s_2 \cdots s_{t-1})}^{2}, v_{j(s_1 s_2 \cdots s_{t-1})}^{1}, v_{j(s_1 s_2 \cdots s_{t-1})}^{2}, w_{j(s_1 s_2 \cdots s_{t-1})}^{1}, w_{j(s_1 s_2 \cdots s_{t-1})}^{2}, q_{i(s_1 s_2 \cdots s_{t-1})}, d_{i(s_1 s_2 \cdots s_t)}^{-},$$

$$d_{i(s_1 s_2 \cdots s_t)}^{+}, q_{i(s_1 s_2 \cdots s_t)}^{-}, q_{i(s_1 s_2 \cdots s_t)}^{+} \geq 0\,, \quad i = 1, \cdots, m;\; j = 1, \cdots, n;\; t = 2, \cdots, T-1;\; s_1, \cdots, s_t = 1, \cdots, S \qquad (3.41)$$

The objective function (3.5) is the total cost. Costs are divided into “certain” and “uncertain” costs. The cost for the first stage is represented by (3.6). In the first stage, there is only “certain” costs which are comprised of cost associated with raw materials, machines, labour, overtime, hiring (and firing) of workers, and initial quota purchasing. The t-stage cost is listed in (3.7). The first two lines of (3.7) are the “uncertain” costs in t stage. $b_{i(s_1 s_2 \cdots s_{t-1})}^{-} d_{i(s_1 s_2 \cdots s_{t-1})}^{-}$ is the shortage cost when scenario $(s_1 s_2 \cdots s_{t-1})$ is realised; $b_{i(s_1 s_2 \cdots s_{t-1})}^{+} d_{i(s_1 s_2 \cdots s_{t-1})}^{+}$ is the inventory cost; $c_{i(s_1 s_2 \cdots s_{t-1})}^{-} q_{i(s_1 s_2 \cdots s_{t-1})}^{-}$ is under-quota cost and $c_{i(s_1 s_2 \cdots s_{t-1})}^{+} q_{i(s_1 s_2 \cdots s_{t-1})}^{+}$ is the over-quota cost. The rest of (3.7) is the “certain” cost after scenario $(s_1 s_2 \cdots s_{t-1})$ happened. The final T stage cost is displayed in (3.8). In the final stage, there are only “uncertain” costs.

Constraints (3.9) and (3.10) imply that the total production time should be equal to the sum of regular and additional machine capacities. Constraints (3.11), (3.12), (3.13) and (3.14) are the total labour time of skilled/non-skilled workers. Constraints (3.15), (3.16), (3.17) and (3.18) denote that the labour time of skilled/non-skilled workers in this stage is equal to the time in previous stage plus the changes in this stage. Constraints (3.19) and (3.20) imply that each plant has its minimum working time. Constraints (3.33) and (3.34) indicate that in this stage of each scenario, a summation of the quantity of the same product produced, the inventory in the previous stage and the purchasing in this stage, minus the inventory in this stage will equal to the demand. Similarly, constraints (3.35) and (3.36) denote that in each scenario, a summation of the initial quota quantity for the same product, the over quota in the previous stage and the under quota in this stage, minus the over quota in this stage will equal to the demand. In order to ensure the qualities of the products, to be greater than a given constant for each route in the scenario tree, constraint (3.39) limits the total working time ratio between skilled and non-skilled workers. Constraint (3.40) ensures that for each route in the scenario tree, the sum of the initial quota quantities is equal to the value of quota quantity at the beginning of the planning horizon. Constraints (3.21)-(3.32), (3.37), (3.38) and (3.41) are the boundary conditions.

3.2.4 Results and analysis

3.2.4.1 A practical problem

The case organisation described in greater detail in Wu (2011b), has its head offices in Hong Kong. Manufacturing operations are undertaken across factories located across Asia including Vietnam, mainland China and Thailand. The biggest of its factories is located in China. The organisation produces three main types of garments. Initial deterministic data including for example material, labour and machine cost which is utilised in this study are shown in Appendix A.

Due to readily available labour, the organisation does not generally expend additional cost hiring employees (workers). This implies that h_{jt}^1/h_{jt}^2, f_{jt}^1/f_{jt}^2 and v_{j0}^1/v_{j0}^2 are all equal to zero. Generally, as the working hours for non-skilled workers cannot exceed that of the skilled workers: $\alpha_j = 1$, for $j = 1, \cdots, n$. The initial inventory d_{i0}^+ is taken to be zero.

It is assumed that not only will the company satisfy production demand, but it does have a warehouse large enough to store surplus products if demand wanes. This implies that I_i and B_i, for $i = 1, \cdots, m$, are both infinite. Considering the uncertain demand, it is assumed that the three events (outcomes) that may happen in each of the four periods are high demand s_1, medium demand s_2 and low demand s_3. In the first period, the probabilities of high, medium and low demand are 10%, 10% and 80% respectively. For the other three periods, the probabilities will depend on what happened in the previous period. Figure 3.4 shows the relationship between two periods. If the high demand event occurs in this period, then the related probabilities in the next period become 20%, 20% and 60%. If the medium demand event happens, then the related probabilities in next period changes to 10%, 30% and 60%. On the other hand, if the low demand event takes place; the related probabilities in next period will be 5%, 5% and 90%. The total probability of each scenario for each period is given by the product of the probability of the previous period and the related probability. For example, the total probability of node $(s_1 s_3)$ is given by the probability of node (s_1) (10%) times the related probability (60%), thus 6%. Table 3.1 gives the shortage/surplus cost per unit, under/over-quota cost per unit and demand in different scenarios.

Table 3.1 Shortage/surplus cost per unit, under/over-quota cost per unit and demand in different outcomes.

Outcome	Product	Period	Shortage cost ($)	Surplus cost ($)	Under-quota cost ($)	Over-quota cost ($)	Demand (units)
s_1	1	1	120	2.5	26	4	1900

		2	120	2.5	26	4	2000
		3	120	2.5	26	4	2100
		4	120	2.5	26	4	2200
	2	1	72	1.5	17	3	1500
		2	72	1.5	17	3	1700
		3	72	1.5	17	3	1900
		4	72	1.5	17	3	2100
	3	1	48	1	10	2	1200
		2	48	1	10	2	1300
		3	48	1	10	2	1400
		4	48	1	10	2	1500
s_2	1	1	100	2	24	3	1800
		2	100	2	24	3	1900
		3	100	2	24	3	2000
		4	100	2	24	3	2100
	2	1	60	1	15	2	1400
		2	60	1	15	2	1600
		3	60	1	15	2	1800
		4	60	1	15	2	2000
	3	1	40	0.5	8	1	1100
		2	40	0.5	8	1	1200
		3	40	0.5	8	1	1300
		4	40	0.5	8	1	1400
s_3	1	1	80	1.8	22	2.5	1700
		2	80	1.8	22	2.5	1800
		3	80	1.8	22	2.5	1900
		4	80	1.8	22	2.5	2000
	2	1	48	0.8	14	1.5	1300
		2	48	0.8	14	1.5	1500
		3	48	0.8	14	1.5	1700
		4	48	0.8	14	1.5	1900
	3	1	32	0.3	7	0.5	1000
		2	32	0.3	7	0.5	1100
		3	32	0.3	7	0.5	1200
		4	32	0.3	7	0.5	1300

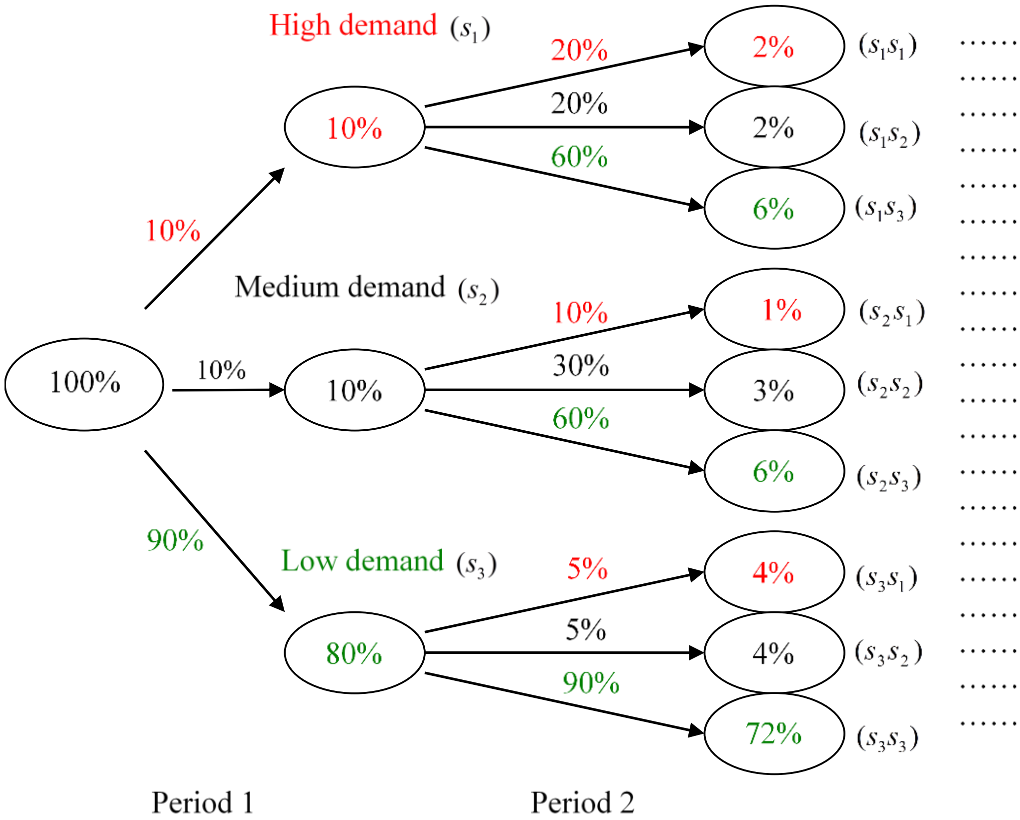

Figure 3.4 The probabilities for each scenario each stage

3.2.4.2 Staged decisions

Due to the four periods, we choose a five-stage model to make the plan. For modelling, we employ the use of AIMMS 3.11, see Bisschop and Roelofs (1999), which is known for its ability to solve large-scale optimisation and scheduling problems. In the case of this study, modelling will involve approximately 2300 constraints and 3000 variables.

First stage decisions

In this stage, we provide the production plan for the first period. Table 3.2 informs various plants on their production targets. Table 3.3 indicates the quantities of purchasing quotas for each kind of product. Notice that, although product 1 will be produced in a total of 1900 units (1200+672+28), allocated quotas are only 1800 because of the uncertainty involved in future quota.

Table 3.2 Production quantity for first period (units)

Product	1			2			3		
Plant	1	2	3	1	2	3	1	2	3
By skilled workers	1200	672	28			1166			

By non-skilled workers						334		944	256

Table 3.3 Quotas allocated for first period (units)

Product	1	2	3
Quota (units)	1800	1500	1200

<u>Second stage decisions</u>

After the first stage, the uncertainty of the first period is realised. The company not only needs to satisfy the demand that might be high, medium or low in the first period but also needs to plan the second period of production. Table 3.4 shows for each scenario that happened in the first period and how many products and quotas the company should purchase or have in storage.

Table 3.4 Shortage/surplus and under-/Over-quota for each scenario in first period (units)

		Purchased products			Inventory			Purchased quotas			Stored quotas		
Product		1	2	3	1	2	3	1	2	3	1	2	3
Scenario	(s_1)							100					
	(s_2)				100	100	100					100	100
	(s_3)				200	200	200				100	200	200

However, even though the probability of high demand event is only 10%, the company may need to produce enough products to satisfy high demand due to the inventory cost being much cheaper than the purchasing cost. Corresponding to high, medium or low demand events in the first period, Table 3.5 shows different production plans for skilled and non-skilled workers in the second period. Quota quantities allocated in the second period also have three different designs according to what has occurred over first period (see Table 3.6).

Table 3.5 Production quantity for second period (units)

		By skilled workers									By non-skilled workers								
Product		1			2			3			1			2			3		
Plant		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Scenario	(s_1)	1200	171	629			1163									537		991	309
	(s_2)		40	793			580				1067					1020		1140	60
	(s_3)		67	667			433				1067					1067		1100	

Table 3.6 Quotas allocated for second period (units)

Product		1	2	3
Scenario	(s_1)	2000	1700	1300
	(s_2)	1900	1500	1200
	(s_3)	1800	1400	1100

Third stage decisions

In the second period, the uncertain events also correspond to high, medium and low demand. However, the probabilities of these events are dependent on what had occurred during the first period. The implication is that over the second period, there are 9 scenarios (3 times 3). For each scenario, Table 3.7 shows the quantities of shortage/surplus of products and under-/over-quotas.

Table 3.7 Shortage/surplus and under-/Over-quota for each scenario in second period (units)

		Purchased products			Inventory			Purchased quotas			Stored quotas		
Product		1	2	3	1	2	3	1	2	3	1	2	3
Scenario	(s_1s_1)												
	(s_1s_2)				100	100	100				100	100	100
	(s_1s_3)				200	200	200				200	200	200
	(s_2s_1)							100	100				
	(s_2s_2)				100	100	100						100
	(s_2s_3)				200	200	200				100	100	200
	(s_3s_1)							100	100				
	(s_3s_2)				100	100	100						100
	(s_3s_3)				200	200	200				100	100	200

In the second period, the company still produces enough to satisfy high demand due to no purchasing plans. Table 3.8 and Table 3.9 present detailed plans for the third period for each scenario that occurred over the second period. This includes products and quota assignments. We can see that Plant 3 will produce the most, due to its cheaper labour cost.

Table 3.8 Production quantity for third period (units)

		By skilled workers									By non-skilled workers								
Product		1			2			3			1			2			3		
Plant		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Scena	(s_1s_1)		978	55			1215				1067					685		595	805
	(s_1s_2)		853	80			1184				1067					616		407	893

	(s_1s_3)		729	105			1152				1067					548		220	980
	(s_2s_1)	63	1026				1472				1011					428		451	949
	(s_2s_2)	40	929				1472				1031					328		222	1078
	(s_2s_3)	1200	700				914									786		153	1047
	(s_3s_1)	1200	900				1554									346		427	973
	(s_3s_2)	144	917				1554				939					246		199	1101
	(s_3s_3)	1200	700				925									775		150	1050

Table 3.9 Quotas allocated for the third period (units)

Product		1	2	3
Scenario	(s_1s_1)	1900	1700	1400
	(s_1s_2)	1900	1700	1300
	(s_1s_3)	1800	1600	1200
	(s_2s_1)	2000	1900	1400
	(s_2s_2)	2000	1800	1300
	(s_2s_3)	1900	1700	1200
	(s_3s_1)	2100	1900	1400
	(s_3s_2)	2000	1800	1300
	(s_3s_3)	1900	1700	1200

Fourth stage decisions

Similarly, the uncertain events in the third period are related to the second period. The implication being that there are 27 scenarios over this (third) period. All the plans are shown in Appendix B. From the results, it is clear that irrespective of what occurs, the company will not have a need to purchase products from the market to satisfy the demand.

Fifth stage decisions

In the final period, there are 81 scenarios. Over this period, we only need to make plans about purchase or storage of products and quotas. Due to a substantial number of scenarios, we do not list the entire plan for this period. However, examining the data shows that irrespective of an event occurring, units of purchase or inventory for every kind of product in each scenario are no more than 200. For detailed results, see Appendix B.

3.2.4.3 Comparing the 5-stage stochastic model and deterministic model

Due to the demand uncertainty and import quota limitations, accurate information cannot be obtained before the production. The decision makers are unable to construct a perfect production plan for all scenarios. Therefore, we provided a multi-stage stochastic model to help them to evaluate the benefits and losses of the plan. The total cost of the multi-stage stochastic model is called the expected objective value of the stochastic solution, denoted as ESS, and the result of the corresponding deterministic model is called expected value problem or mean value problem, denoted as EV. That means we use the expected values of stochastic parameters to replace all the stochastic parameters first; then we calculate the solution. Using the EV solution to obtain the expected result of the stochastic model is denoted as EEV. The difference between EEV and ESS is called the value of the stochastic solution (VSS). The VSS means how many bonuses we can get by comparing the stochastic solution and corresponding expected solution value model. We also run more tests with different probabilities to see how the VSS changes. The original probability in Figure 3.4 shows that the company does not expect that products will sell well over the four periods, primarily because of a high probability of low demand (80%). We term this condition as a "bad economy environment". Now we consider two other situations, "fair" and "good" economy environments. Figure 3.5 and Figure 3.6 give the probabilities of these two tests.

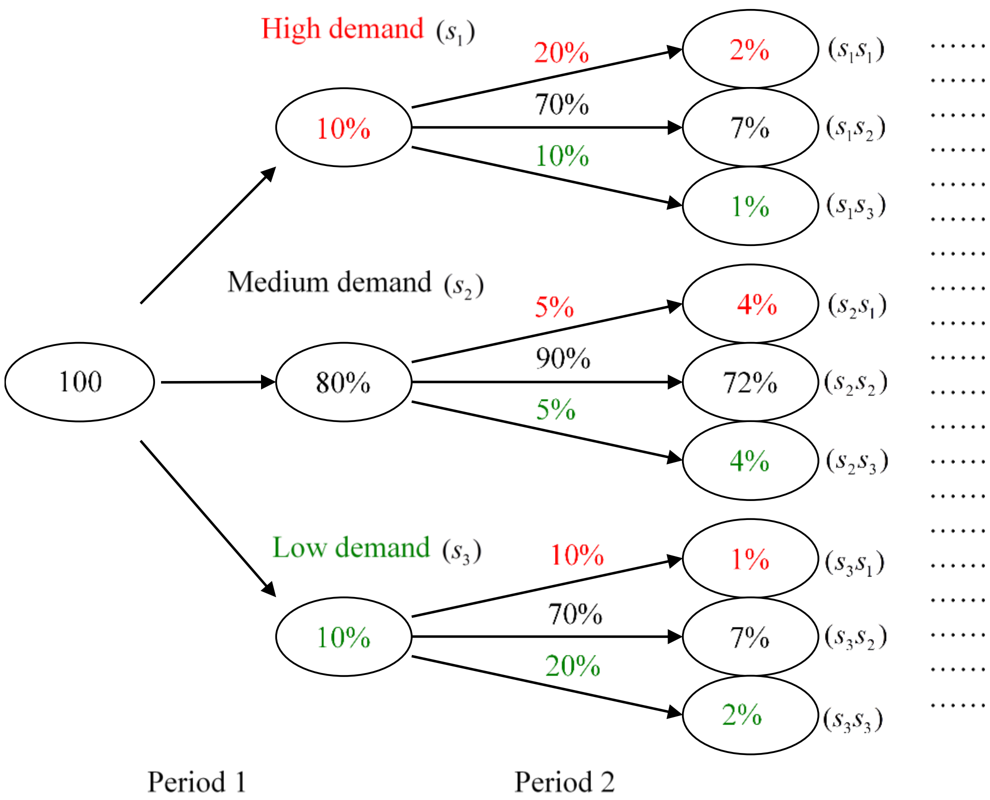

Figure 3.5 The probabilities of fair economy environment

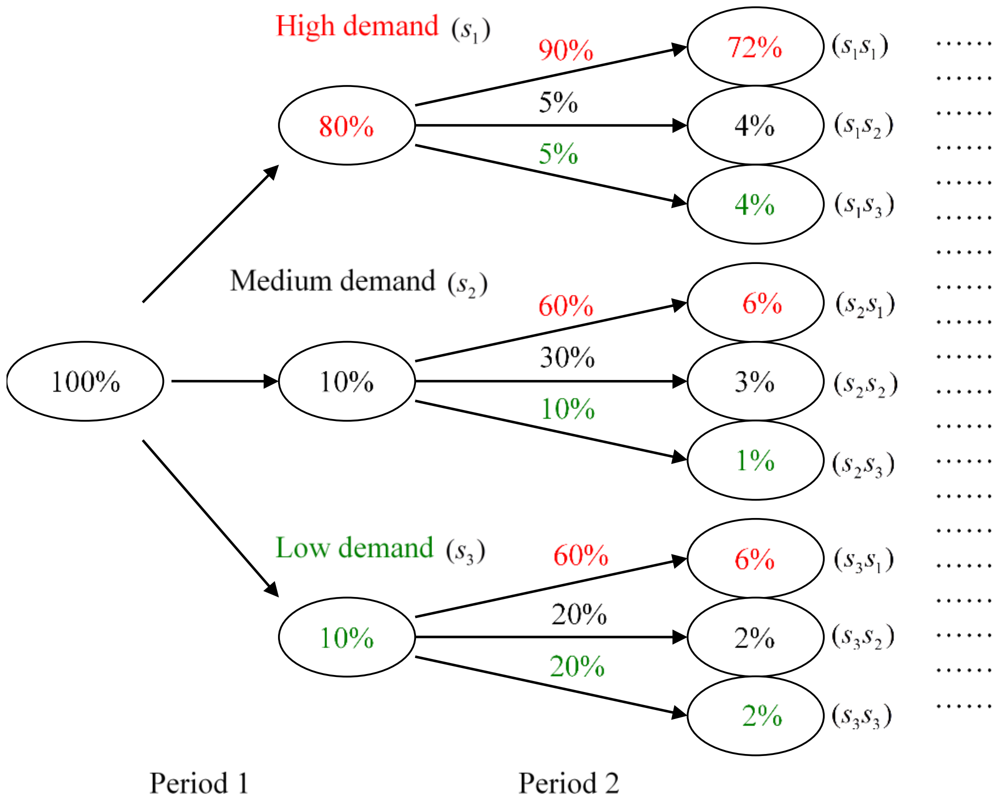

Figure 3.6 The probabilities of good economy environment

Table 3.10 Comparing the expected value model and stochastic model ($)

Test	ESS	EV	EEV	VSS(EEV-ESS)
Bad economy environment	408962	402073	421481	12519
Fair economy environment	412392	408970	416780	4388
Good economy environment	430113	428387	448481	18368

Table 3.10 lists the results of the comparison of the stochastic model and the deterministic model. It shows that all the values of EEV are larger than the values of ESS which means using stochastic solution can generate more benefits than using the deterministic solution. Particularly in the good economy environment, the total cost reduces by $18,368, from $448,481 to $430113.

3.2.4.4 Comparing the results of fixed mix approaches

Comparing with the fixed mix approaches, e.g. Fleten et al. (2002), multi-stage stochastic model can describe the dynamic information much better. In order to show the benefits of multi-stage stochastic models, we conduct additional tests. First, we calculate the results of 3-stage and 4-stage fixed mix models with the same initial data as the 5-stage model. For the 4-stage model, we use the multi-stage model to solve the first three periods and use fixed mix approach assumption to deal with the last period. Similarly, for the 3-stage model, the last two periods will be solved using a fixed mix model. Please note, as shown in Figure 3.4, that the related probabilities for the 3-stage and 4-stage models are the same as for the 5-stage model. Then we consider two other situations, "fair" and "good" economy environments. We also compute the 2-stage, 3-stage, 4-stage and 5-stage model for these different tests. We list all results of these tests in Table 3.11.

Table 3.11 The expected costs of all tests ($)

Test	Stochastic model	Expected total cost	Expected production cost	Expected shortage/ surplus cost	Expected under-/over-quota cost
Bad economy environment	2-Stage	423010	409045	8915	5050
	3-Stage	415055	402242	8685	4128
	4-Stage	411806	402450	5588	3768
	5-Stage	408962	401571	3704	3687
Fair economy environment	2-Stage	420705	409045	6955	4705
	3-Stage	415201	407277	3930	3994
	4-Stage	413576	406822	2987	3767
	5-Stage	412392	406612	2123	3657
Good economy environment	2-Stage	432865	413860	930	18075
	3-Stage	431819	413306	526	17987
	4-Stage	430800	412876	365	17559
	5-Stage	430113	412476	297	17340

Figure 3.7, Figure 3.8 and Figure 3.9 illustrate changes in total cost, shortage/surplus and under-/over- quota cost. We observe that during difficult economic conditions, it appears that compared to the 2-stage model, the 5-stage model is likely to deliver savings of more than $14,048. We also observe that under more positive economic conditions, the difference in total cost between the 2-stage and 5-stage model is $2,752.

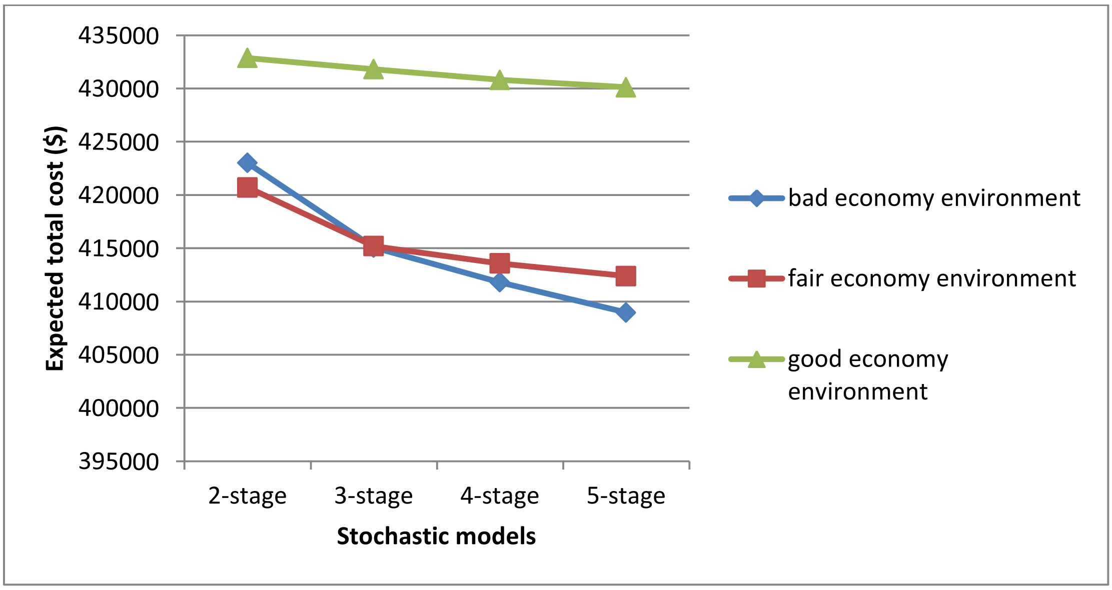

Figure 3.7 The comparison of expected total cost with different stochastic models

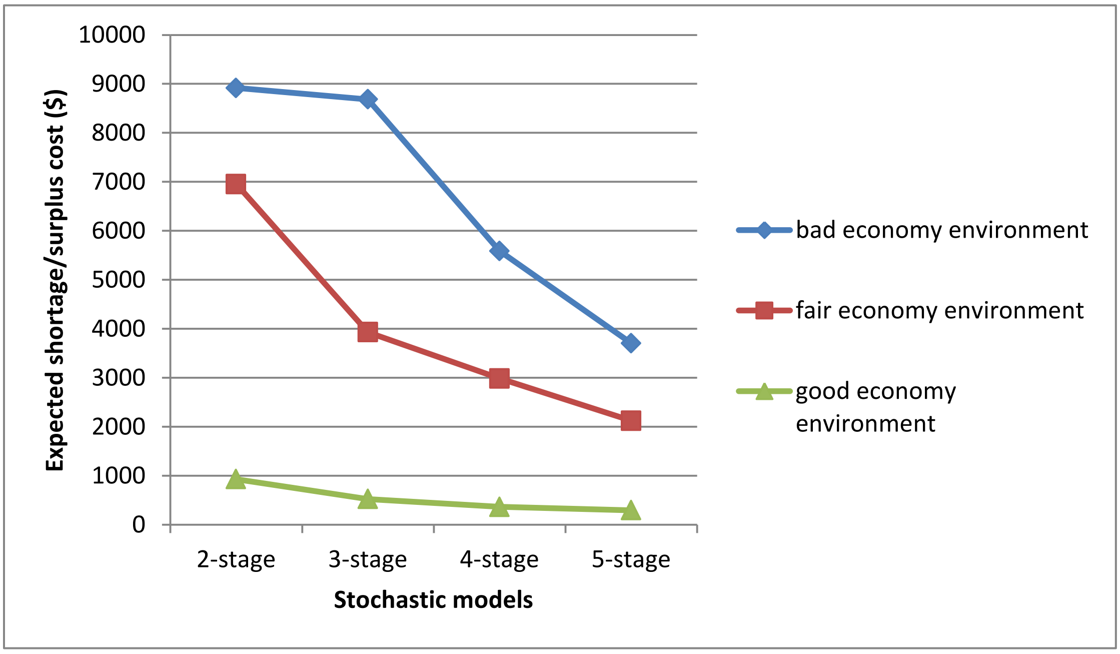

Figure 3.8 The comparison of expected shortage/surplus cost with different stochastic models

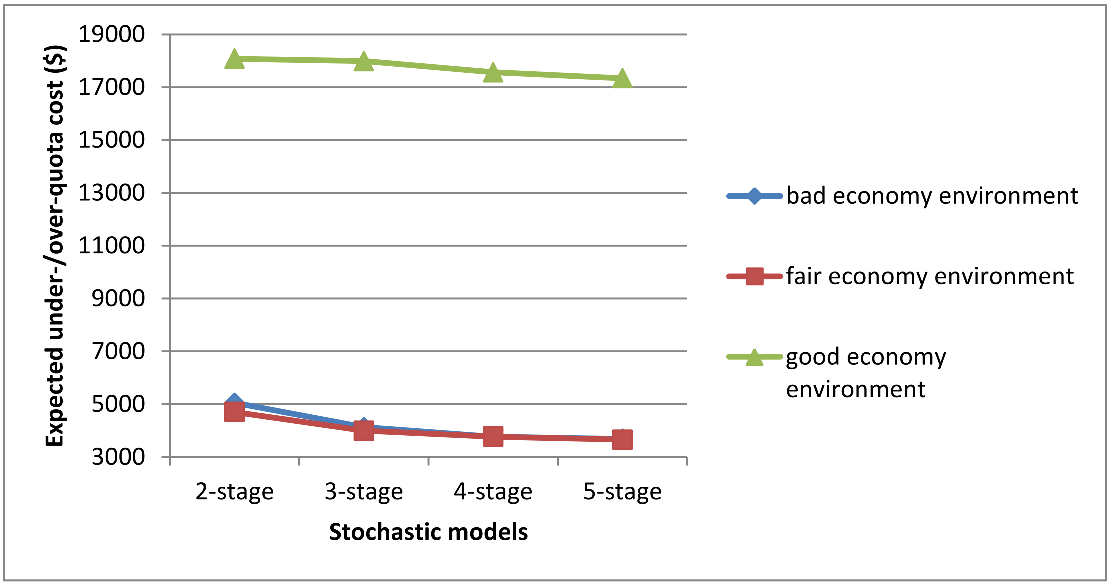

Figure 3.9 The comparison of expected under-/over- quota cost with different stochastic models

It will appear from the results shown in Figure 3.7, Figure 3.8 and Figure 3.9, that employing multi-stage decision models will deliver more benefits under poor and fair economic conditions. Under good economic conditions which are characterised by low expected shortage/surplus cost, the multi-stage production plan matches uncertain demand quite well. In the 5-stage model, the expected shortage/surplus cost is only $297. However, because the expected event is highly demand under good economic conditions, the initial quota quantity may not be enough to satisfy demand. Thus it may become necessary for the company to increase the amount it spends on purchasing quotas. As however articulated earlier, such a production strategy may ultimately increase production costs. In sum, the test do demonstrate that when in possession of accurate market prediction data, the multi-stage stochastic model appears more suitable to solve the production problems with uncertainty than the two-stage model.

Table 3.12 New test of demands in different outcomes in all periods (units)

Outcome	Product	Period			
		1	2	3	4
s_1	1	1700	1900	2100	2300
	2	1500	1600	1700	1800
	3	1200	1300	1400	1500
s_2	1	1600	1700	1800	1900
	2	1300	1500	1400	1600
	3	1100	1200	1300	1400
s_3	1	1500	1450	1400	1350
	2	1200	1300	1100	1200
	3	1000	1050	1100	1150

Table 3.13 The total costs of new tests for different models ($)

Test	Stochastic model	Total cost
Bad economy environment	2-Stage	431334
	3-Stage	419037
	4-Stage	413272
	5-Stage	405365
Fair economy environment	2-Stage	420871
	3-Stage	411327
	4-Stage	408918
	5-Stage	406111
Good economy environment	2-Stage	420946
	3-Stage	418786
	4-Stage	416959
	5-Stage	415164

We also changed the demand to do the new test for these model. Table 3.12 represents the demands of this new test in different outcomes in all periods. Based on these demands, Table 3.13 lists all the total costs for different models. We can see that 5-stage model is always the best model. In bad economy environment, using 5-stage model, the company can save $25969 than using 2-stgae model. Even in the smallest fluctuation environment, good economy environment, the company still can save $5782.

In order to show how the uncertainties influence the total cost, we introduce three types of multi-stage robust optimisation model in the following section.

3.3 Three types of multi-stage robust models

Demand uncertainty is an additional significant issue which has an effect on production loading, as production is used to meet market demands. In international supply chain administration, accurate information regarding the market turns out to be harder and harder to acquire. Market demand typically comes from various dealers situated mainly in the European and North American markets and they are likely to delay their obligations in favour of their actual needs, which then allows producers even less time in which to manufacture their merchandise (Wu, 2006).

Compared to those in the past, sellers today possess much more power. Because of the vast plethora of data available on the Internet and obtainable from other resources, many more opportunities exist in which they are able to compare quality, delivery speed, price and service. In a lot of industries, the minimum standard is now a product of high quality, as opposed to a position of demarcation. Consequently, manufacturing companies can now achieve a competitive advantage in response to fluctuating market demand by giving customers flexible, responsive and quick production while simultaneously maintaining low costs.

The goods relating to this study are fashion clothes with short lead times. Until accurate market demand is observed, for a manufacturing company it is necessary to commence production in its plants. Then, when the sales season is approaching, the demand for products will be clear and the company has to take appropriate action in its manufacturing strategy.

There is a large risk of both surplus and shortage for manufacturing goods and purchasing quotas in loading production. The use of a stochastic production loading strategy would enable a company to keep costs low when responding quickly to fluctuating market demand while simultaneously reducing any risk. In dealing with the risk and uncertainty in the stochastic production loading process, the adoption of robust optimisation is sufficient.

From the multi-stage stochastic model in Section 3.2, managers should be able to satisfy all the demand exactly. In reality, it is very difficult for the company to satisfy all the market demand due to the demand uncertainty. Sometimes, more than several times the product costs may be spent to satisfy only a small amount of demand. Therefore, we introduce a robust model with a penalty measure to allow violation of the uncertainty constraints. This model is called the robust optimisation model with model robustness. Also from the multi-stage stochastic model, although only one scenario occurs in any one stage, managers still need to make production plans for every scenario. The plans for different scenarios in the same stage may have huge differences. For example, in Table 3.5, the production plan of product 1 in plant 1 is 1200 units by skilled workers in scenario (s_1), but 1067 units by non-skilled workers in scenario (s_2). Therefore, we can use a robust model by adding a measure function to control the variabilities in each stage. This model is called the robust optimisation model with solution robustness. We also provide a trade-off robust optimisation model to combine these two measures together. The frameworks for the three robust models are provided in the following section.

3.3.1 General robust optimisation framework

The general multi-stage stochastic model is shown in Section 3.2.2. All the objective functions for these three kinds of robust models are based on (3.1).

3.3.1.1 The robust optimisation model with model robustness.

A robust optimisation model with model robustness means the violation of the uncertainty constraints is permitted, but this is done by the least amount by introducing a penalty function. A robust optimisation model with model robustness can be formulated as:

$$\min c_1^T x_1 + \sum_{s_1 \in S_1} p_{s_1}(d_{s_1}^T y_{s_1} + c_2^T x_{s_1}) + \omega_1 \sum_{s_1 \in S_1} p_{s_1} |e_{1s_1}| + \cdots$$
$$+ \sum_{s_{t-2} \in S_{t-2}} p_{s_{t-2}}\left(d_{s_{t-2}}^T y_{s_{t-2}} + c_{t-1}^T x_{s_{t-2}}\right) + \omega_{t-2} \sum_{s_{t-2} \in S_{t-2}} p_{s_{t-2}} |e_{s_{t-3}s_{t-2}}|$$
$$+ \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}} + \omega_{t-1} \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}} |e_{s_{t-2}s_{t-1}}| \tag{3.42}$$

subject to (3.2) and

$$B_{11}x_1 + B_{12}y_{s_1} + e_{1s_1} = b_{1s_1}, B_{21}x_{s_1} + B_{22}y_{s_2} + e_{s_2 s_1} = b_{2s_2}, \cdots,$$
$$B_{(t-1)1}x_{s_{t-2}} + B_{(t-1)2}y_{s_{t-1}} + e_{s_{t-2}s_{t-1}} = b_{(t-1)s_{t-1}} \tag{3.43}$$
$$x_1, x_{s_1}, \cdots, x_{s_{t-2}}, y_{s_1}, \cdots, y_{s_{t-1}}, \omega_1, \cdots, \omega_{t-1} \geq 0 \tag{3.44}$$

In the objective function, a series of $\sum_{S_{t-1}} p_{s_{t-1}} |e_{s_{t-2}s_{t-1}}|$ is defined as the expected infeasibility, which is used to measure the violation of the multiple stage constraints. And the series of $\omega_{t-1} \sum_{S_{t-1}} p_{s_{t-1}} |e_{s_{t-2}s_{t-1}}|$ is defined as the expected infeasibility cost, where ω is a parameter as a measurement of the infeasibility of the constraints of uncertainty. If $\omega = 0$, there is no penalty for not satisfying the uncertainty constraints. In this case, the quantity of violation can be as large as possible. On the other hand, if $\omega \to +\infty$, any amount of violation is hardly accepted. That means any uncertainty constraints have to be satisfied because of the large penalty ω. Therefore, when ω is set up large enough, the robust optimisation model with model robustness is converted to a multi-stage stochastic programming model.

In order to simplify this model, we can move the absolute value sign out by adding deviation variables $\delta_{s_t} \geq 0$ and some constraints. Then, (3.42) can be formulated as the following linear programming model:

$$\min c_1^T x_1 + \sum_{s_1 \in S_1} p_{s_1}(d_{s_1}^T y_{s_1} + c_2^T x_{s_1}) + \omega_1 \sum_{s_1 \in S_1} p_{s_1}(e_{1s_1} + 2\delta_{s_1}) + \cdots$$
$$+ \sum_{s_{t-2} \in S_{t-2}} p_{s_{t-2}}\left(d_{s_{t-2}}^T y_{s_{t-2}} + c_{t-1}^T x_{s_{t-2}}\right) + \omega_{t-2} \sum_{s_{t-2} \in S_{t-2}} p_{s_{t-2}}(e_{s_{t-3}s_{t-2}} + 2\delta_{s_{t-2}})$$
$$+ \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}} + \omega_{t-1} \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}}(e_{s_{t-2}s_{t-1}} + 2\delta_{s_{t-1}}) \tag{3.45}$$

subject to (3.2), (3.43), (3.44) and

$$-e_{1s_1}-\delta_{s_1}\le 0,\cdots,-e_{s_{t-2}s_{t-1}}-\delta_{s_{t-1}}\le 0 \tag{3.46}$$

The series of constraints $-e_{s_{t-2}s_{t-1}}-\delta_{s_{t-1}}\le 0$ is to make sure the simplified model is the same as the previous model. The reason is, if $e_{s_{t-2}s_{t-1}}\ge 0$, $\delta_{s_{t-1}}$ will be zero; if $e_{s_{t-2}s_{t-1}}\le 0$, $\delta_{s_{t-1}}$ will be $-e_{s_{t-2}s_{t-1}}$.

3.3.1.2 The robust optimisation model with solution robustness.

A robust optimisation model with solution robustness means the solution will not differ substantially among different scenarios and there is less variability in the objective function across scenarios, which presumes a less aggressive management style. A robust optimisation model with solution robustness can be formulated as:

$$\begin{aligned}\min c_1^T x_1 &+ \sum_{s_1\in S_1} p_{s_1}(d_{s_1}^T y_{s_1}+c_2^T x_{s_1}) + \lambda_1 \sum_{s_1\in S_1} p_{s_1}\left|d_{s_1}^T y_{s_1} - \sum_{s_1\in S_1} p_{s_1} d_{s_1}^T y_{s_1}\right| + \cdots \\ &+ \sum_{s_{t-2}\in S_{t-2}} p_{s_{t-2}}\left(d_{s_{t-2}}^T y_{s_{t-2}} + c_{t-1}^T x_{s_{t-2}}\right) \\ &+ \lambda_{t-2}\sum_{s_{t-2}\in S_{t-2}} p_{s_{t-2}}\left|d_{s_{t-2}}^T y_{s_{t-2}} - \sum_{s_{t-2}\in S_{t-2}} p_{s_{t-2}} d_{s_{t-2}}^T y_{s_{t-2}}\right| \\ &+ \sum_{s_{t-1}\in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}} \\ &+ \lambda_{t-1}\sum_{s_{t-1}\in S_{t-1}} p_{s_{t-1}}\left|d_{s_{t-1}}^T y_{s_{t-1}} - \sum_{s_{t-1}\in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}}\right|\end{aligned} \tag{3.47}$$

subject to (3.2), (3.3) and

$$x_1, x_{s_1},\cdots,x_{s_{t-2}}, y_{s_1},\cdots,y_{s_{t-1}},\lambda_1,\cdots,\lambda_{t-1}\ge 0 \tag{3.48}$$

In the objective function, the series of $\lambda_{t-1}\sum_{s_{t-1}\in S_{t-1}} p_{s_{t-1}}\left|d_{s_{t-1}}^T y_{s_{t-1}} - \sum_{s_{t-1}\in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}}\right|$ is defined as the expected variability cost, where λ is intended as a measurement of the variability of the objective function. Clearly, if $\lambda=0$, that means the variability is not considered in decision-making process. Then the above model becomes a multi-stage stochastic model. On the other hand, if $\lambda\to+\infty$, the absolute value of $d_{s_{t-1}}^T y_{s_{t-1}} - \sum_{s_{t-1}\in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}}$ will reduce to zero (as low as possible). That means no matter which kind of uncertainty is realized, the uncertain decision variables y will be similar as possible.

We can use the same method to remove the absolute value sign by adding deviation variables $\theta_{s_t} \geq 0$ and some constraints:

$$\min c_1^T x_1 + \sum_{s_1 \in S_1} p_{s_1}(d_{s_1}^T y_{s_1} + c_2^T x_{s_1}) + \lambda_1 \sum_{s_1 \in S_1} p_{s_1}(d_{s_1}^T y_{s_1} - \sum_{s_1 \in S_1} p_{s_1} d_{s_1}^T y_{s_1} + 2\theta_{s_1}) + \cdots$$
$$+ \sum_{s_{t-2} \in S_{t-2}} p_{s_{t-2}}\left(d_{s_{t-2}}^T y_{s_{t-2}} + c_{t-1}^T x_{s_{t-2}}\right)$$
$$+ \lambda_{t-2} \sum_{s_{t-2} \in S_{t-2}} p_{s_{t-2}}(d_{s_{t-2}}^T y_{s_{t-2}} - \sum_{s_{t-2} \in S_{t-2}} p_{s_{t-2}} d_{s_{t-2}}^T y_{s_{t-2}} + 2\theta_{s_{t-2}})$$
$$+ \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}}$$
$$+ \lambda_{t-1} \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}}(d_{s_{t-1}}^T y_{s_{t-1}} - \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}} + 2\theta_{s_{t-1}}) \qquad (3.49)$$

subject to (3.2), (3.3), (3.48) and

$$d_{s_1}^T y_{s_1} - \sum_{s_1 \in S_1} p_{s_1} d_{s_1}^T y_{s_1} + \theta_{s_1} \geq 0, \cdots, d_{s_{t-1}}^T y_{s_{t-1}} - \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}} + \theta_{s_{t-1}} \geq 0 \qquad (3.50)$$

The series of constraints $d_{s_{t-1}}^T y_{s_{t-1}} - \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}} + \theta_{s_{t-1}} \geq 0$ is to make sure the model has the same meaning as the previous one. This can be proved as follows:

If $d_{s_{t-1}}^T y_{s_{t-1}} \geq \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}}$, then $\theta_{s_{t-1}} = 0$.

If $d_{s_{t-1}}^T y_{s_{t-1}} \leq \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}}$, then $\theta_{s_{t-1}} = -d_{s_{t-1}}^T y_{s_{t-1}} + \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}}$.

3.3.1.3 The robust optimisation model with the trade-off between model robustness and solution robustness.

When we consider the variability and infeasibility simultaneously, a robust optimisation model featuring a trade-off between model and solution robustness can be formulated as:

$$\begin{aligned}
\min c_1^T x_1 &+ \sum_{s_1 \in S_1} p_{s_1}(d_{s_1}^T y_{s_1} + c_2^T x_{s_1}) + \omega_1 \sum_{s_1 \in S_1} p_{s_1}(e_{1s_1} + 2\delta_{s_1}) \\
&+ \lambda_1 \sum_{s_1 \in S_1} p_{s_1}(d_{s_1}^T y_{s_1} - \sum_{s_1 \in S_1} p_{s_1} d_{s_1}^T y_{s_1} + 2\theta_{s_1}) + \cdots \\
&+ \sum_{s_{t-2} \in S_{t-2}} p_{s_{t-2}}\left(d_{s_{t-2}}^T y_{s_{t-2}} + c_{t-1}^T x_{s_{t-2}}\right) + \omega_{t-2} \sum_{s_{t-2} \in S_{t-2}} p_{s_{t-2}}(e_{s_{t-3}s_{t-2}} + 2\delta_{s_{t-2}}) \\
&+ \lambda_{t-2} \sum_{s_{t-2} \in S_{t-2}} p_{s_{t-2}}(d_{s_{t-2}}^T y_{s_{t-2}} - \sum_{s_{t-2} \in S_{t-2}} p_{s_{t-2}} d_{s_{t-2}}^T y_{s_{t-2}} + 2\theta_{s_{t-2}}) \\
&+ \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}} + \omega_{t-1} \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}}(e_{s_{t-2}s_{t-1}} + 2\delta_{s_{t-1}}) \\
&+ \lambda_{t-1} \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}}(d_{s_{t-1}}^T y_{s_{t-1}} - \sum_{s_{t-1} \in S_{t-1}} p_{s_{t-1}} d_{s_{t-1}}^T y_{s_{t-1}} + 2\theta_{s_{t-1}}) \qquad (3.51)
\end{aligned}$$

subject to (3.2), (3.43), (3.44), (3.46), (3.48) and (3.50)

3.3.2 Multi-stage robust optimisation models

The multi-stage stochastic model is shown in Section 3.2.3. All these three kinds of robust models are based on that stochastic model.

3.3.2.1 New notation

New deterministic parameters

ω_t^1 unit weighting penalty for the infeasibility of the random demand constraints in period t;

ω_t^2 unit weighting penalty for the infeasibility of the random quota constraints in period t;

λ_t measurement of the variability of the objective function in period t.

New decision variables

$e_{i(s_1 s_2 \cdots s_t)}^1 / e_{i(s_1 s_2 \cdots s_t)}^2$ the amount of violation in the demand/quota constraint for product i when scenario $(s_1 s_2 \cdots s_t)$ is realised;

$\delta_{i(s_1 s_2 \cdots s_t)} / \gamma_{i(s_1 s_2 \cdots s_t)}$ deviational variables in demand/quota constraints for product i when scenario $(s_1 s_2 \cdots s_t)$ is realised;

$\theta_{(s_1 s_2 \cdots s_t)}$ deviational variables for the robust model with solution robustness when scenario $(s_1 s_2 \cdots s_t)$ is realised.

3.3.2.2 A robust optimisation model with model robustness

Based on Section 3.3.1.1, the robust optimisation model with model robustness for global production planning problem will be built as:

$$\min Z = \sum_{t=1}^{T} M_t + \sum_{t=2}^{T}(\sum_{s_1=1}^{S}\sum_{s_2=1}^{S}\cdots\sum_{s_{t-1}=1}^{S} p_{(s_1 s_2 \cdots s_{t-1})}\sum_{i=1}^{m}(\omega_{t-1}^{1}\big(e_{i(s_1 s_2 \cdots s_{t-1})}^{1} + 2\delta_{i(s_1 s_2 \cdots s_{t-1})}\big) + \omega_{t-1}^{2}\big(e_{i(s_1 s_2 \cdots s_{t-1})}^{2} + 2\gamma_{i(s_1 s_2 \cdots s_{t-1})}\big))) \quad (3.52)$$

subject to (3.6)-(3.32), (3.37)-(3.41) and

$$e_{i(s_1)}^{1} = D_{i(s_1)} - \sum_{j=1}^{n}\big(x_{ij1}^{1} + x_{ij1}^{2}\big) - d_{i0}^{+} - d_{i(s_1)}^{-} + d_{i(s_1)}^{+}\,, \quad i = 1,\cdots,m;\ s_1 = 1,\cdots,S \quad (3.53)$$

$$e_{i(s_1 s_2 \cdots s_t)}^{1} = D_{i(s_1 s_2 \cdots s_t)} - \sum_{j=1}^{n}\big(x_{ij(s_1 s_2 \cdots s_{t-1})}^{1} + x_{ij(s_1 s_2 \cdots s_{t-1})}^{2}\big) - d_{i(s_1 s_2 \cdots s_{t-1})}^{+} - d_{i(s_1 s_2 \cdots s_t)}^{-} + d_{i(s_1 s_2 \cdots s_t)}^{+}\,, \quad i = 1,\cdots,m;\ t = 2,\cdots,T-1;\ s_1,\cdots,s_t = 1,\cdots,S \quad (3.54)$$

$$e_{i(s_1)}^{2} = D_{i(s_1)} - q_{i1} - q_{i(s_1)}^{-} + q_{i(s_1)}^{+}\,, \qquad i = 1,\cdots,m;\ s_1 = 1,\cdots,S \quad (3.55)$$

$$e_{i(s_1 s_2 \cdots s_t)}^{2} = D_{i(s_1 s_2 \cdots s_t)} - q_{i(s_1 s_2 \cdots s_{t-1})} - q_{i(s_1 s_2 \cdots s_{t-1})}^{+} - q_{i(s_1 s_2 \cdots s_t)}^{-} + q_{i(s_1 s_2 \cdots s_t)}^{+}\,, \quad i = 1,\cdots,m;\ t = 2,\cdots,T-1;\ s_1,\cdots,s_t = 1,\cdots,S \quad (3.56)$$

$$-e_{i(s_1 s_2 \cdots s_t)}^{1} - \delta_{i(s_1 s_2 \cdots s_t)} \le 0\,, \qquad i = 1,\cdots,m;\ t = 1,\cdots,T-1;\ s_1,\cdots,s_t = 1,\cdots,S \quad (3.57)$$

$$-e_{i(s_1 s_2 \cdots s_t)}^{2} - \gamma_{i(s_1 s_2 \cdots s_t)} \le 0\,, \qquad i = 1,\cdots,m;\ t = 1,\cdots,T-1;\ s_1,\cdots,s_t = 1,\cdots,S \quad (3.58)$$

$$\omega_t^1, \omega_t^2, \delta_{i(s_1 s_2 \cdots s_t)}, \gamma_{i(s_1 s_2 \cdots s_t)} \ge 0\,, i = 1,\cdots,m;\ t = 1,\cdots,T-1;\ s_1,\cdots,s_t = 1,\cdots,S \quad (3.59)$$

Constraints (3.53) and (3.54) are the difference between demand, production and shortage/surplus. Similarly, Constraints (3.55) and (3.56) mean the difference between demand, initial quota allocated and under/over-quota. Constraints (3.57) and (3.58) are to make sure to take off the absolute value sign. In the objective function (3.52), when the unit weighting parameters ω_t^1 and ω_t^2 increases, the unit penalty cost for the infeasibility of the random demand constraints increase. We have to pay more for the violation of the random demand constraint. If the value of ω_t^1 is increased by enough, the value of $e_{i(s_1 s_2 \cdots s_t)}^{1}$ will be forced to become zero, which means all random demand constraints have to be satisfied for each scenario. The same phenomenon occurs at the unit weighting penalty ω_t^2 and the corresponding random quota constraint (3.55) and (3.56). If ω_t^1 and ω_t^2, for $t = 1, \cdots, T-1$, are all large enough, the values of $e^1_{i(s_1 s_2 \cdots s_t)}$ and $e^2_{i(s_1 s_2 \cdots s_t)}$, for $t = 1, \cdots, T-1$, are all equal to zero. Then the robust optimisation model with model robustness will become to stochastic model.

3.3.2.3 A robust optimisation model with solution robustness

Using the introduction in Section 3.3.1.2, we can list the robust optimisation model with solution robustness for global production planning problem:

$$\min Z = \sum_{t=1}^{T} M_t + \sum_{t=2}^{T} \lambda_{t-1} \sum_{s_1=1}^{S} \sum_{s_2=1}^{S} \cdots \sum_{s_{t-1}=1}^{S} p_{(s_1 s_2 \cdots s_{t-1})} \Big(\sum_{i=1}^{m} \big(b^-_{i(s_1 s_2 \cdots s_{t-1})} d^-_{i(s_1 s_2 \cdots s_{t-1})} + b^+_{i(s_1 s_2 \cdots s_{t-1})} d^+_{i(s_1 s_2 \cdots s_{t-1})} + c^-_{i(s_1 s_2 \cdots s_{t-1})} q^-_{i(s_1 s_2 \cdots s_{t-1})} + c^+_{i(s_1 s_2 \cdots s_{t-1})} q^+_{i(s_1 s_2 \cdots s_{t-1})} \big) - \sum_{s_1=1}^{S} \sum_{s_2=1}^{S} \cdots \sum_{s_{t-1}=1}^{S} p_{(s_1 s_2 \cdots s_{t-1})} \Big(\sum_{i=1}^{m} \big(b^-_{i(s_1 s_2 \cdots s_{t-1})} d^-_{i(s_1 s_2 \cdots s_{t-1})} + b^+_{i(s_1 s_2 \cdots s_{t-1})} d^+_{i(s_1 s_2 \cdots s_{t-1})} + c^-_{i(s_1 s_2 \cdots s_{t-1})} q^-_{i(s_1 s_2 \cdots s_{t-1})} + c^+_{i(s_1 s_2 \cdots s_{t-1})} q^+_{i(s_1 s_2 \cdots s_{t-1})} \big) + 2\theta_{(s_1 s_2 \cdots s_{t-1})} \Big) \qquad (3.60)$$

subject to (3.6)-(3.41) and

$$-\sum_{i=1}^{m} \big(b^-_{i(s_1 s_2 \cdots s_{t-1})} d^-_{i(s_1 s_2 \cdots s_{t-1})} + b^+_{i(s_1 s_2 \cdots s_{t-1})} d^+_{i(s_1 s_2 \cdots s_{t-1})} + c^-_{i(s_1 s_2 \cdots s_{t-1})} q^-_{i(s_1 s_2 \cdots s_{t-1})} + c^+_{i(s_1 s_2 \cdots s_{t-1})} q^+_{i(s_1 s_2 \cdots s_{t-1})} \big) + \sum_{s_1=1}^{S} \sum_{s_2=1}^{S} \cdots \sum_{s_{t-1}=1}^{S} p_{(s_1 s_2 \cdots s_{t-1})} \Big(\sum_{i=1}^{m} \big(b^-_{i(s_1 s_2 \cdots s_{t-1})} d^-_{i(s_1 s_2 \cdots s_{t-1})} + b^+_{i(s_1 s_2 \cdots s_{t-1})} d^+_{i(s_1 s_2 \cdots s_{t-1})} + c^-_{i(s_1 s_2 \cdots s_{t-1})} q^-_{i(s_1 s_2 \cdots s_{t-1})} + c^+_{i(s_1 s_2 \cdots s_{t-1})} q^+_{i(s_1 s_2 \cdots s_{t-1})} \big) - \theta_{(s_1 s_2 \cdots s_{t-1})} \le 0, \quad t = 2, \cdots, T;\ s_1, \cdots, s_t = 1, \cdots, S \qquad (3.61)$$

$$\lambda_{t-1}, \theta_{(s_1 s_2 \cdots s_{t-1})} \ge 0, \quad t = 2, \cdots, T;\ s_1, \cdots, s_t = 1, \cdots, S \qquad (3.62)$$

3.3.2.4 A robust optimisation model with the trade-off between model robustness and solution robustness

When the variability and infeasibility are considered together, a robust optimisation model with model robustness and solution robustness is developed to solve the global production planning problems with demand uncertainty and quota limits.

$$\begin{aligned}\min Z = &\sum_{t=1}^{T} M_t + \sum_{t=2}^{T}(\sum_{s_1=1}^{S}\sum_{s_2=1}^{S}\cdots\sum_{s_{t-1}=1}^{S} p_{(s_1s_2\cdots s_{t-1})}\sum_{i=1}^{m}(\omega_{t-1}^{1}\left(e_{i(s_1s_2\cdots s_{t-1})}^{1} + 2\delta_{i(s_1s_2\cdots s_{t-1})}\right)\\ &+ \omega_{t-1}^{2}\left(e_{i(s_1s_2\cdots s_{t-1})}^{2} + 2\gamma_{i(s_1s_2\cdots s_{t-1})}\right)))\\ &+\sum_{t=2}^{T}\lambda_{t-1}\sum_{s_1=1}^{S}\sum_{s_2=1}^{S}\cdots\sum_{s_{t-1}=1}^{S} p_{(s_1s_2\cdots s_{t-1})}(\sum_{i=1}^{m}(b_{i(s_1s_2\cdots s_{t-1})}^{-}d_{i(s_1s_2\cdots s_{t-1})}^{-}\\ &+ b_{i(s_1s_2\cdots s_{t-1})}^{+}d_{i(s_1s_2\cdots s_{t-1})}^{+} + c_{i(s_1s_2\cdots s_{t-1})}^{-}q_{i(s_1s_2\cdots s_{t-1})}^{-} + c_{i(s_1s_2\cdots s_{t-1})}^{+}q_{i(s_1s_2\cdots s_{t-1})}^{+})\\ &-\sum_{s_1=1}^{S}\sum_{s_2=1}^{S}\cdots\sum_{s_{t-1}=1}^{S} p_{(s_1s_2\cdots s_{t-1})}(\sum_{i=1}^{m}(b_{i(s_1s_2\cdots s_{t-1})}^{-}d_{i(s_1s_2\cdots s_{t-1})}^{-}\\ &+ b_{i(s_1s_2\cdots s_{t-1})}^{+}d_{i(s_1s_2\cdots s_{t-1})}^{+} + c_{i(s_1s_2\cdots s_{t-1})}^{-}q_{i(s_1s_2\cdots s_{t-1})}^{-} + c_{i(s_1s_2\cdots s_{t-1})}^{+}q_{i(s_1s_2\cdots s_{t-1})}^{+})\\ &+ 2\theta_{(s_1s_2\cdots s_{t-1})}) \qquad (3.63)\end{aligned}$$

subject to (3.6)-(3.32), (3.37)-(3.41), (3.53)-(3.59), (3.61) and (3.62).

3.3.3 Computational results and analysis

3.3.3.1 A practical problem

In this section, solutions will be obtained using the same initial data used in the multi-stage stochastic model case in Section 3.2.4.1. Due to the four-period case, in order to make the production plan, a five-stage robust model with the trade-off between solution robustness and model robustness has been chosen. To obtain the results, we will take $\lambda = 0.1$ and $\omega^1 = \omega^2 = 100$ as an illustration. All the five stages' decision plans are shown in Appendix C.

The robust optimisation models' computational output, using various ω and the multi-stage stochastic model, is given in Table 3.14. The full cost using the stochastic model is \$408962 and the full cost using the robust model $\lambda = 0.1$ and $\omega^1 = \omega^2 = 50$ is \$408131 (see the third row). The total cost decreases by 0.203% using the robust optimisation model, with the expected variability decreasing 79.71%. This demonstrates the fact that the robust model offers a less sensitive production loading strategy. Nevertheless, as the robust model does not satisfy all the market demand, the infeasibility cost of $1577 is involved. When ω increases to 150, there is no infeasibility cost due to too expensive penalty cost. The entire cost of the robust model increases by only 0.085% when compared with the stochastic model, which produces a decrease in expected variability of 47.79%. Therefore, the five-stage production loading plan proposed by the robust model reduces the risk and is not expensive.

Table 3.14 Comparing the robust model and the stochastic model

	Stochastic model	Robust model ($\lambda = 0.1, \omega^1 = \omega^2 = 50$)	Robust model ($\lambda = 0.1, \omega^1 = \omega^2 = 100$)	Robust model ($\lambda = 0.1, \omega^1 = \omega^2 = 150$)
Variability at second stage	488	113	113	113
Variability at third stage	492	306	306	306
Variability at fourth stage	568	362	382	374
Variability at fifth stage	3928	330	1106	2066
Expected cost	408962	406443	408510	409022
Expected variability	5476	1111	1907	2859
Expected infeasibility cost	0	1577	427	0
Total cost	408962	408131	409127	409308

3.3.3.2 More tests

In order to show how the model robustness and solution robustness influence the productions. We will provide more tests, as per the stochastic model in Section 3.2.4.4, for the three robust models separately.

Computational results for robust optimisation model with solution robustness.

The robust optimisation with solution robustness computational results for the three tests, in which λ is assigned different values, are presented in Table 3.15.

Table 3.15 Computational results for 5-stage robust optimisation model with solution robustness

Test	λ	Expected cost	Expected variability	Expected variability cost	Total cost
Bad economy environment	0	408962	4656	0	408962
	0.1	409022	2859	286	409308
	0.5	409679	1076	538	410217
	0.9	410011	515	464	410475
Fair economy environment	0	412392	4364	0	412392
	0.1	412395	4324	432	412827
	0.5	412915	2868	1434	414349
	0.9	413501	1822	1640	415141
Good economy environment	0	430113	6913	0	430113
	0.1	430140	4874	487	430627
	0.5	430154	4809	2405	432559
	0.9	433888	1	1	433889

Firstly, an analysis of the entire trend of the three tests is conducted. A multi-stage stochastic recourse model in which the variability is not considered develops from the robust optimisation model when $\lambda = 0$. The expected variability for the robust optimisation model is less than that of the multi-stage stochastic recourse model for each test, as shown in Table 3.15. Therefore, the robust optimisation model with solution robustness presents less risky than the stochastic recourse model. The overall cost of the multi-stage stochastic recourse model is less than that of the robust optimisation model. The total cost of the robust model ($\lambda = 0.9$) increases by 0.37% in a bad economy environment, 0.67% in a fair economy environment and 0.88% in a good economy environment when compared with the recourse model. However, there is a decrease in the variability of 88.94% in a bad economy environment, 58.25% in a fair economy environment and 99.99% in a good economy environment.

In order to show the benefits of multi-stage robust models, we also calculate the results of 2-stage, 3-stage and 4-stage models with the same data. In Table 3.16, we lists all results of these tests.

Table 3.16 Comparison results of different stages robust model with solution robustness

Test	λ	Robust model	Total cost	Expected cost	Expected variability cost
Bad economy environment	0.1	2-stage	424028	423429	599
		3-stage	416261	415483	778
		4-stage	412438	411925	513
		5-stage	409308	409022	286
	0.5	2-stage	425099	423926	1173
		3-stage	418886	416605	2281
		4-stage	414102	412878	1224
		5-stage	410217	409679	538
	0.9	2-stage	426038	423926	2112
		3-stage	419666	418635	1031
		4-stage	414465	413901	564
		5-stage	410475	410011	464
Fair economy environment	0.1	2-stage	421925	421059	866
		3-stage	416032	415229	803
		4-stage	414210	413586	624
		5-stage	412827	412395	432
	0.5	2-stage	423453	421945	1508
		3-stage	419018	415601	3417
		4-stage	416507	414067	2440
		5-stage	414349	412915	1434
	0.9	2-stage	424659	421945	2714
		3-stage	420346	418501	1845
		4-stage	417424	415767	1657
		5-stage	415141	413501	1640
Good economy environment	0.1	2-stage	433312	432865	447
		3-stage	432325	431851	474
		4-stage	431313	430806	507
		5-stage	430627	430140	487
	0.5	2-stage	435660	432865	2795
		3-stage	434199	431874	2325
		4-stage	433321	430825	2496

		5-stage	432559	430154	2405
	0.9	2-stage	435660	435660	0
		3-stage	434900	434900	0
		4-stage	434362	434362	0
		5-stage	433889	433888	1

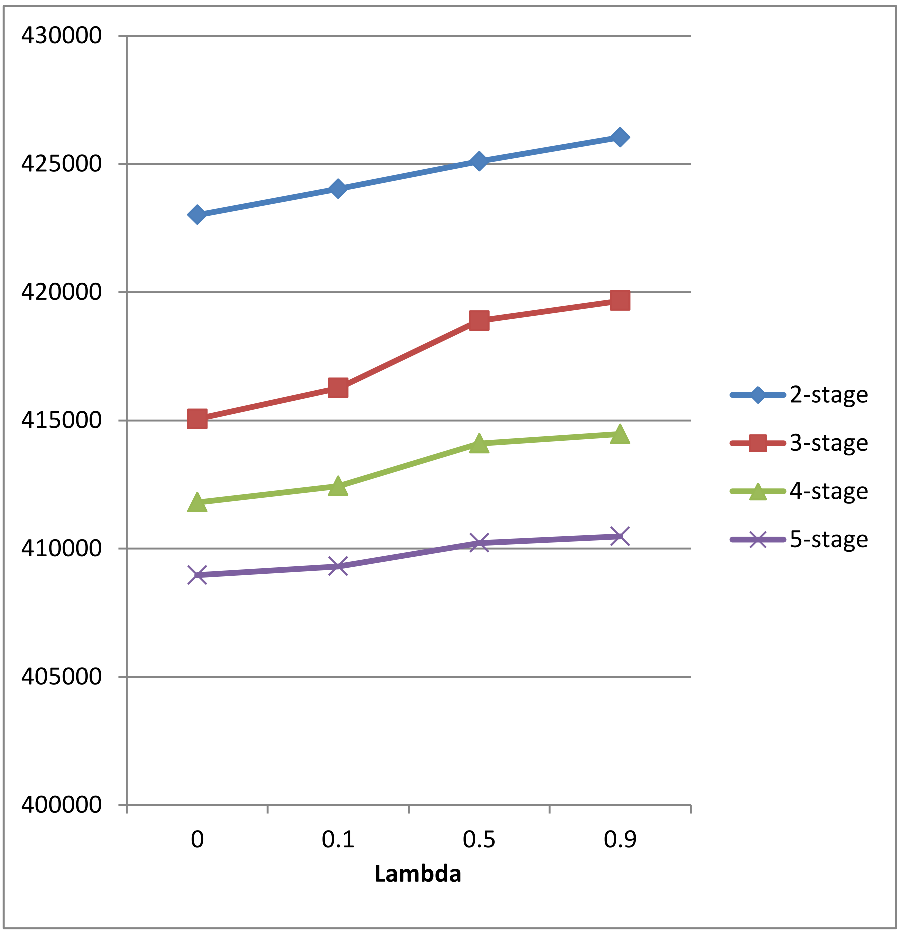

Figure 3.10 Total cost of robust models with solution robustness in bad economy environment ($)

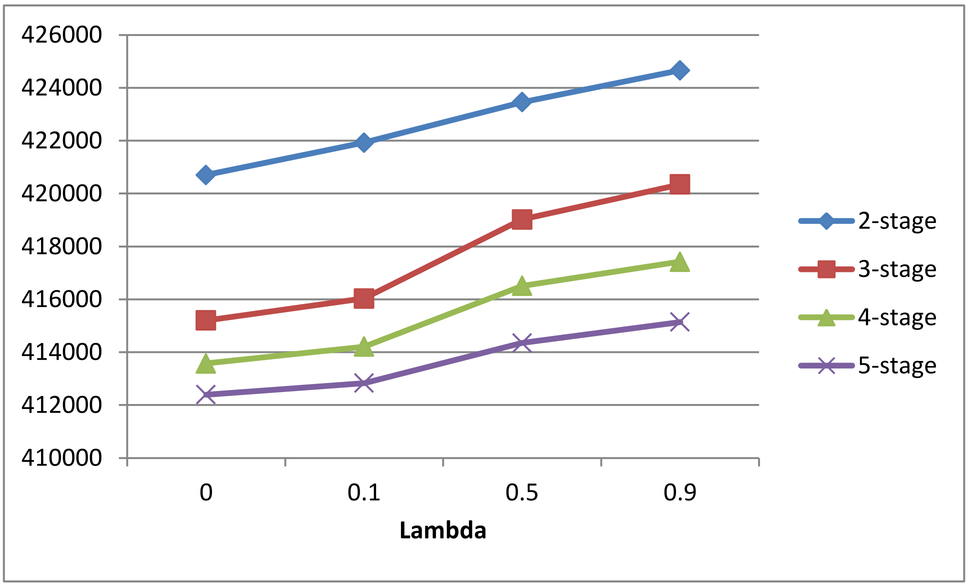

Figure 3.11 Total cost of robust models with solution robustness in fair economy environment ($)

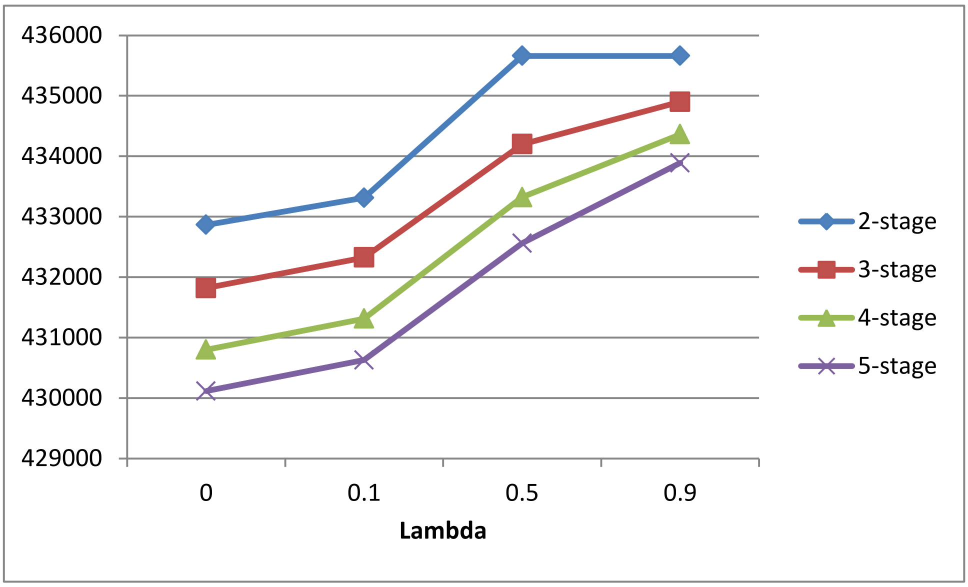

Figure 3.12 Total cost of robust models with solution robustness in good economy environment ($)

It will appear from the results shown in Figure 3.10, Figure 3.11 and Figure 3.12, that employing multi-stage decision models will deliver more positive benefits no matter under good, poor and fair economic conditions. In sum, the test do demonstrate that when in possession of accurate market prediction data, the multi-stage robust model with model robustness appears more suitable to solve the production problems with uncertainty than the two-stage model.

Computational results for robust optimisation model with model robustness.

Table 3.17 shows the computational results of the robust optimisation with model robustness for the three tests. In the tests, ω is used to represent ω^1 and ω^2. Thus we have: $\omega = \omega^1 = \omega^2$. In the three tests, when $\omega = 0$ there is no penalty for violating the random demand constraints and random quota constraints. When ω increases, the expected infeasibility decreases and the total cost increases. When ω increases by enough, the expected infeasibility becomes zero, which means that all random constraints are satisfied because of the higher penalty for the infeasibility. The robust optimisation model then becomes the stochastic recourse model (see the final row).

Table 3.17 Computational results for 5-stage robust optimisation model with model robustness

Test	ω	Expected cost	Expected infeasibility cost	Total cost
Bad economy environment	0	357094	0	357094
	20	403813	2476	406289
	50	406641	1361	408002
Fair economy environment	0	357094	0	357094
	20	407671	2978	410649
	50	411446	500	411946
Good economy environment	0	357094	0	357094
	20	419551	8145	427696
	50	430113	0	430113

In Table 3.18, we list all results regarding the total cost for 2-stage, 3-stage, 4-stage and 5-stage models. Table 3.18 shows that employing multi-stage decision models will deliver more positive benefits whether under good, poor or fair economic conditions.

Table 3.18 Comparison total cost of different stages robust model with model robustness

Text	ω	2-stage	3-stage	4-stage	5-stage
Bad economy environment	0	357094	357094	357094	357094
	20	410820	407858	406827	406289
	50	418953	411949	409723	408002
Fair economy environment	0	357094	357094	357094	357094
	20	413139	411533	410964	410649
	50	417277	413807	412668	411946

Good economy environment	0	357094	357094	357094	357094
	20	430425	429363	428366	427696
	50	432865	431816	430727	430113

Computational results for robust optimisation model with the trade-off between solution robustness and model robustness

The trade-off between model robustness and solution robustness is determined using parameters λ and ω. No penalty for the infeasibility of random constraints in the objective function exists when $\omega = 0$. The infeasibility representing un-fulfilment is a greater value. Obviously, this type of production loading plan would not be looked upon with favour by decision-makers. Yet the total objective function value is dominated by the penalty function due to the large weights ω^1 and ω^2 and would result in a greater total variability and cost. Consequently, a trade-off between the risk and the cost always exists. During the production loading process, it is necessary to check the proposed robust optimisation model with different λ in order to measure the trade-off between the risk and cost.

When λ keeps constant:

The computational results for a bad economy environment, in terms of the infeasibility, variability, and total cost when λ keeps constant, are shown in Figure 3.13, Figure 3.14 and Figure 3.15.

The variability trend when ω increases for λ =0.1, 0.5, and 0.9 respectively is given in Figure 3.13. For $\lambda = 0.1$, when ω increases, the variability sharply increases from 660 to 2859. However, the variability keeps steady at 2859 after ω increases to 150. The value of ω has a small impact on the variability when λ =0.5 and 0.9. This is due to the fact that the infeasibility cost measured by ω has less influence on the total cost and the objective function value is dominated by the variability cost when λ is given a large value.

The trend of the infeasibility when ω increases for λ =0.1, 0.5, and 0.9 respectively is shown in Figure 3.14.The fact that the value of ω has a large influence on the system's infeasibility is clear to see.

Figure 3.15 shows that when ω increases, so do the total costs. When the value of λ is small, the has the system is greater impacted by the value of ω.

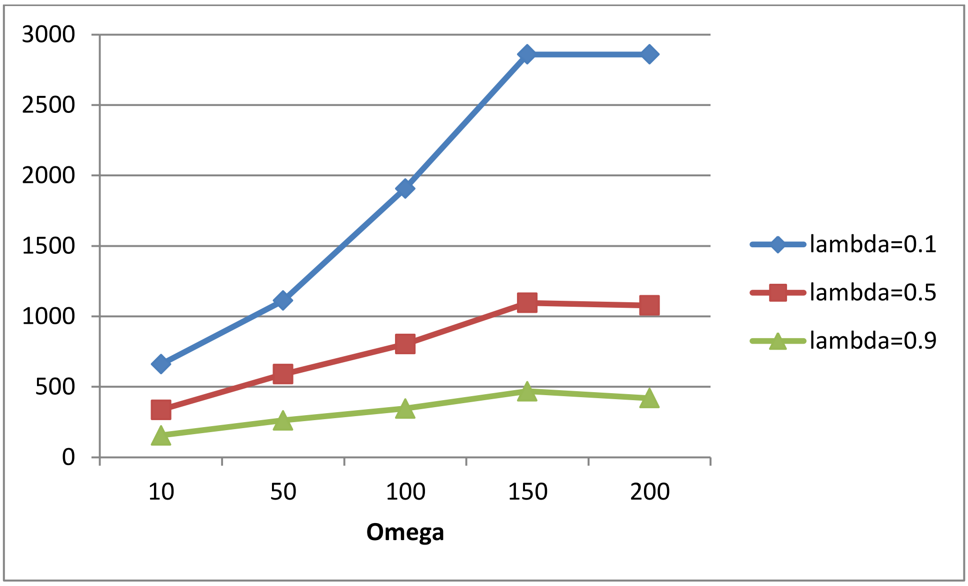

Figure 3.13 Variability when λ keeps constant

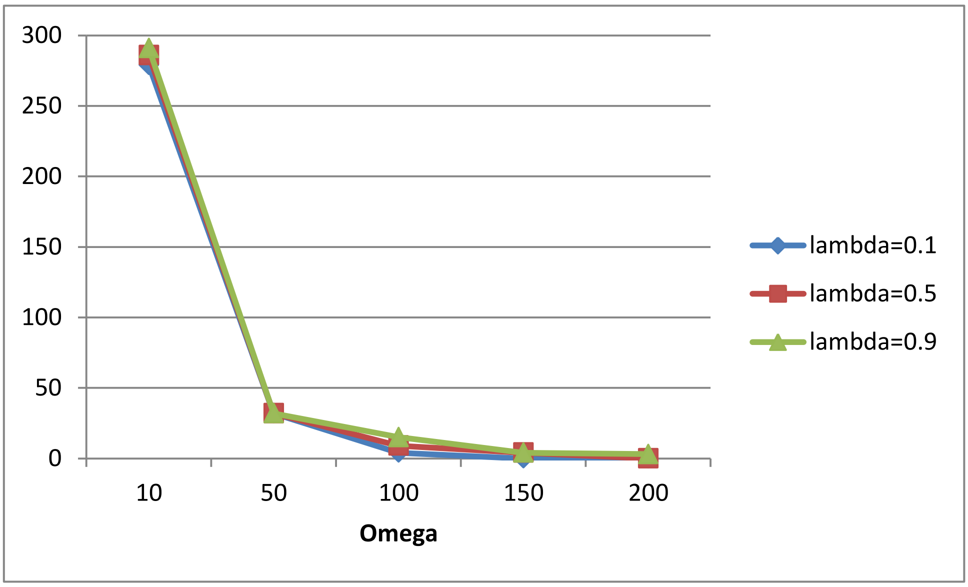

Figure 3.14 Infeasibility when λ keeps constant

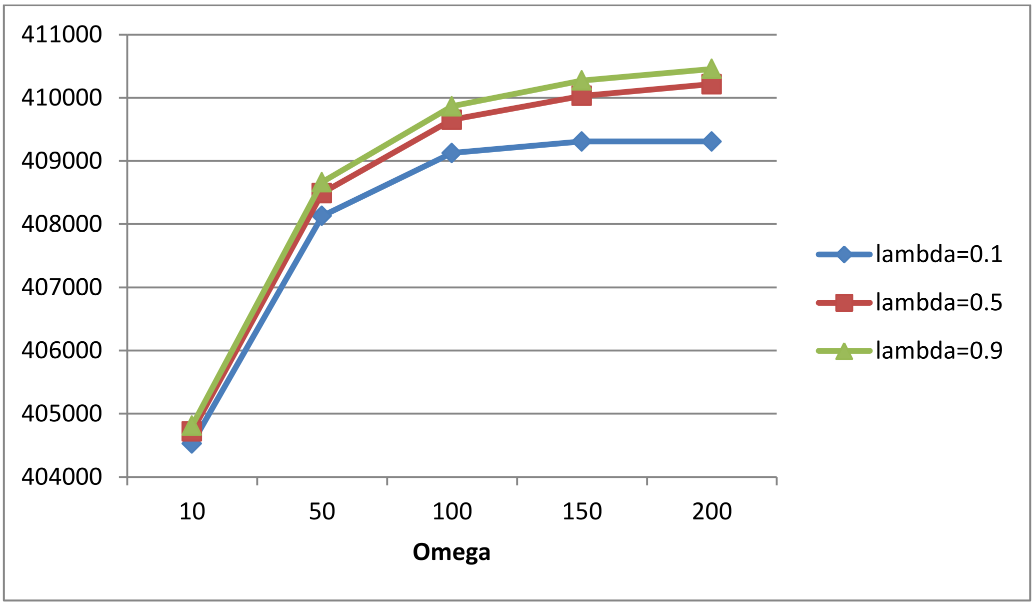

Figure 3.15 Total cost when λ keeps constant

When ω keeps constant:

The computational results of a bad economy environment, regarding the infeasibility, variability and total cost when ω keeps constant, are shown in Figure 3.16, Figure 3.17 and Figure 3.18.

The trend in the variability when λ increases for ω =10, 50, 100 and 150 respectively is shown in Figure 3.16. If λ increases from 0.1 to 0.9, for $\omega = 10$, the variability decreases by 76.52%; for $\omega = 50$, the variability decreases by 76.42%; for $\omega = 100$, the variability decreases by 81.91%; for $\omega = 150$, the variability decreases by 83.63%. The value of λ has a large effect on the variability.

The trend in the infeasibility, when λ increases for ω =10, 50, 100 and 150 respectively, can be seen in Figure 3.17. The greater the value of ω, the less the value of λ has an impact on the infeasibility. If λ increases from 0.1 to 0.9, for $\omega = 10$, the infeasibility increases by 4.3%; for ω =50, 100 and 150 the value of λ has no impact on the infeasibility. The reason for this is that when ω is given a large value, the infeasibility cost dominates the objective function value and the variability cost measured by λ has less impact on the total cost.

The trend in the total cost, when λ increases for ω =10, 50, 100 and 150 respectively, is displayed in Figure 3.18. If there is an increase in λ from 0.1 to 0.9, for $\omega = 10$, the total cost increases by 0.07%, for $\omega = 50$, the total cost increases by 0.13%; for $\omega = 100$, the total cost increases by 0.18%, for $\omega = 150$, the total costs increases by 0.24%. The total cost increases by only a small amount when λ increases, compared with the changes in infeasibility and variability in Figure 3.16 and Figure 3.17. This means that the robust model proposed in this study is not expensive for a low-risk production loading system.

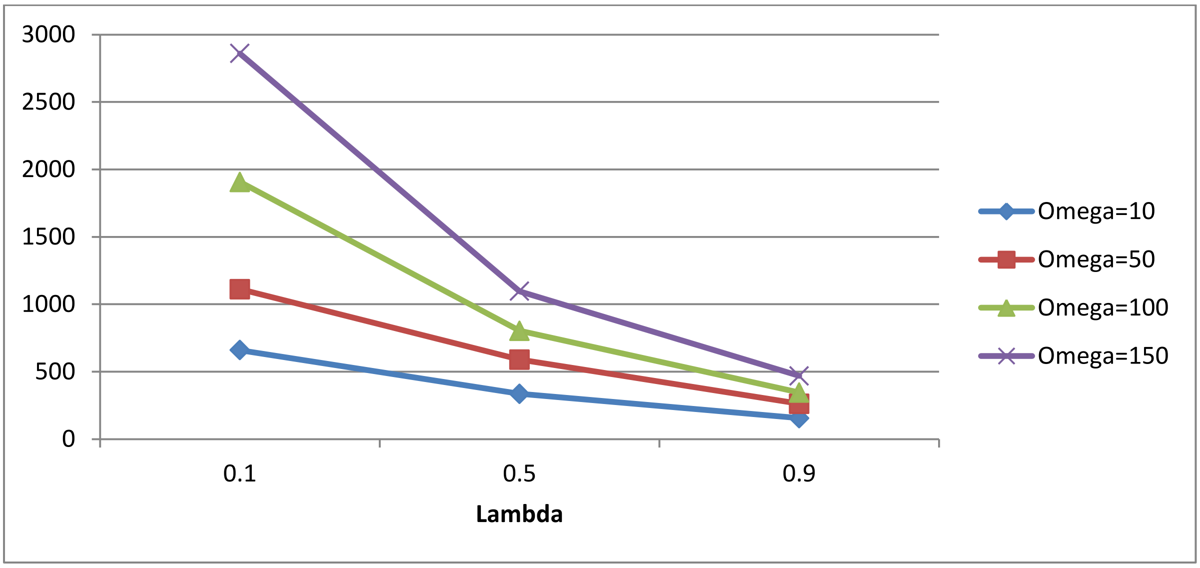

Figure 3.16 Variability when ω keeps constant

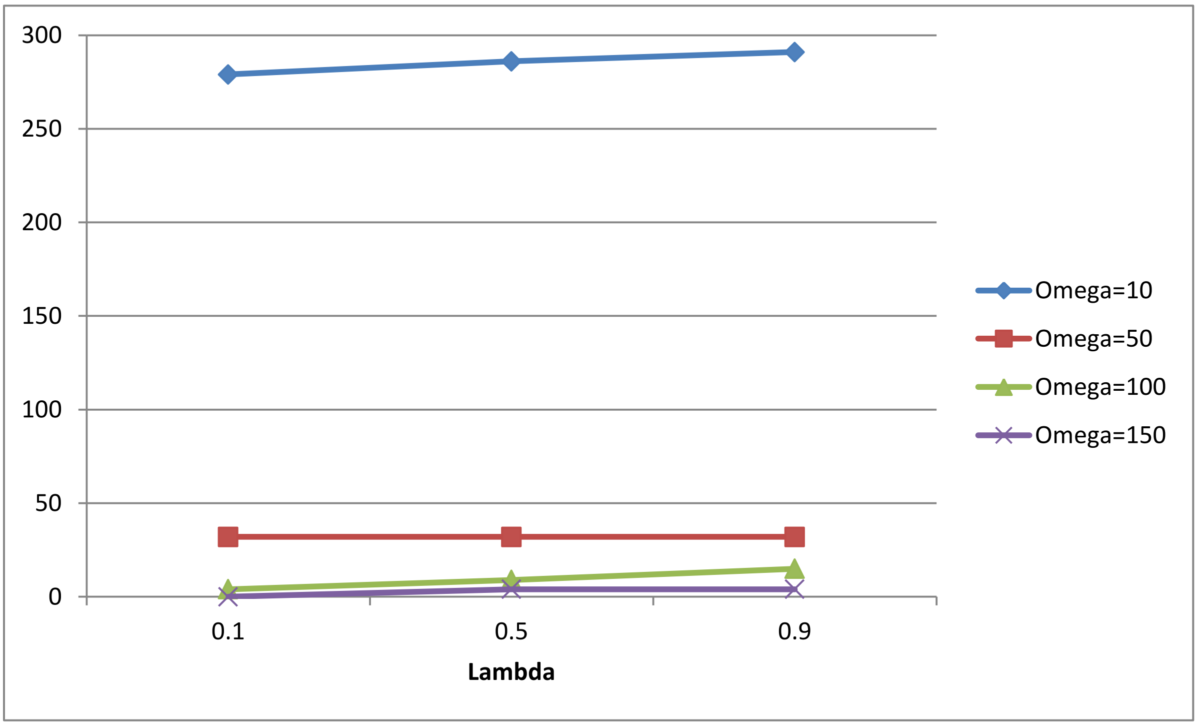

Figure 3.17 Infeasibility when ω keeps constant

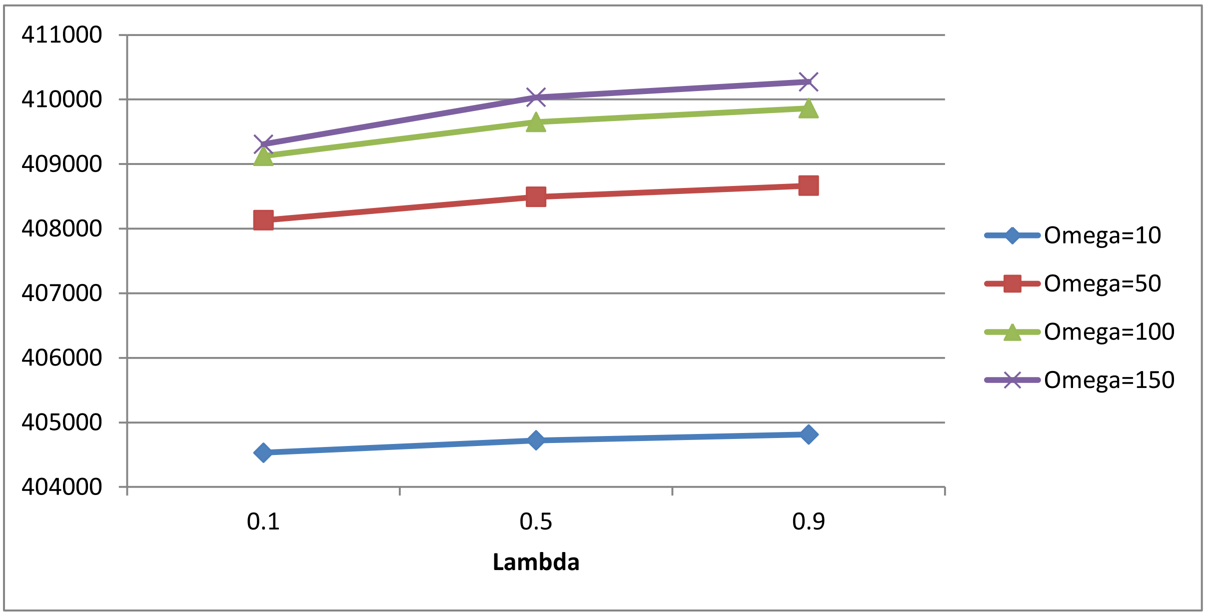

Figure 3.18 Total cost when ω keeps constant

3.4 Managerial implications and conclusions

Continued research (Dunlop and Weil, 1996; De Toni and Meneghetti, 2000; Ramcharran, 2001; Wu, 2011b) has shown on-going interest among in developing an understanding of how optimised planning and control of production may enhance the competitiveness of the garment industry. There is an urgent need for firms working within the garment industry to not only shorten their fulfilment times for orders but also to optimise their entire supply chains. Thus the effectiveness of their production and planning processes has become an essential element of the entire manufacturing business operations. For a number of reasons (including the unrealistic expectation about being able to forecast demand at a level of granularity that represents reality), there has been a robust inclination among scholars to take the view that optimised planning and control of the production process is best supported by appropriate mathematical modelling. However, as some scholars such as MacCarthy (2006), have shown, there are indeed associated limitations when production planning challenges are treated solely as mathematical problems. These limitations include for example that (1) production planning problems may not necessarily be isolated from the prevailing competitive environments, and (2) the queuing of manufacturing transactions are not static. Hence, because production planning and control has a primary interest in capacity planning and its management (Guide, 2000), and not solely scheduling (MacCarthy, 2006), there is a need for scholars to be realistic during modelling on the conditions under which one assumes the existence of certainty.

In conditions where manufacturing firms face multiple, conflicting, and fluid challenges of various configurations, stochastic programming models and robust optimisation models have been employed for the optimisation of problems associated with uncertainty.

Within this research, for the multi-period, multi-product and multi-plant production planning problem under demand uncertainty and quota limitation, we develop a multi-stage stochastic linear model to solve it. Testing shows that that garment manufacturing firms are more likely to derive more benefits from this model. However, as earlier alluded to, multi-stage stochastic linear models should be adopted cautiously as their validity is highly dependent on the accuracy of the forecasted probability of uncertainty. We find that if the probability changes by a small amount, the entire production plan may change a lot. Moreover, the addition of some other factors, such as transportation cost, exchange rate cost or uncertain raw materials fees, should make the model more applicable to real-life problem applications.

Manufacturing companies are being made to supply competitive production strategies, due to the global supply chain management environment. A quantitative approach to creating a production loading strategy when faced with increasingly shortening lead times and uncertain market information, in addition to the greater risks entailed, is offered in this study.

For the same problems in multi-stage stochastic programming, in order to make the model more responsive and flexible, three different kinds of robust optimisation model are proposed: the robust optimisation model with model robustness; the robust optimisation model with solution robustness; and the robust optimisation model with a trade-off between model robustness and solution robustness. The same example in multi-stage stochastic programming has been selected to test these three types of robust optimisation models. The production loading strategies are determined in terms of the cost and risk through the analysis of the various weights in the robust models. A series of computational tests illustrate the fact that the robust optimisation models have advantages over the stochastic recourse model in managing the uncertainty and risk. The robust model solutions can cope with the infeasibility which happens in the multi-stage recourse programming model and are progressively less sensitive to the realizations of the stochastic variables. However, there is no a priori mechanism for specifying a "correct" choice of the parameters, as is prevalent in multi-criteria programming, due to the fact that robust optimisation remains synonymous with goal programming. Additionally, a method of specifying a scenario set, which also happens when formulating a stochastic recourse programming model, is not offered by robust optimisation.

Chapter 4: International Air Cargo Forwarding Problem

In Chapter 3 we introduced the fact that many international companies decide to locate their factories in developing countries to save money, although their sales departments are in developed countries. After producing the goods in their factories, the headquarters should consider transporting their products to their markets. Air transportation is a good choice to transport low density and high value products to their sales departments. Therefore, the questions of how to make a production plan to satisfy the uncertain market demand, how to book air containers, and how to make plans to load air cargoes to these containers in order to transport the products to distant markets on time have become a challenge to decision makers. Considering the air cargoes forwarding problem, the aim of our research is to help the forwarders to book air containers in advance in order to ship the cargoes from different regions to various destinations via a hub, where the cargoes need to be repacked and consolidated before leaving (see Figure 4.1).

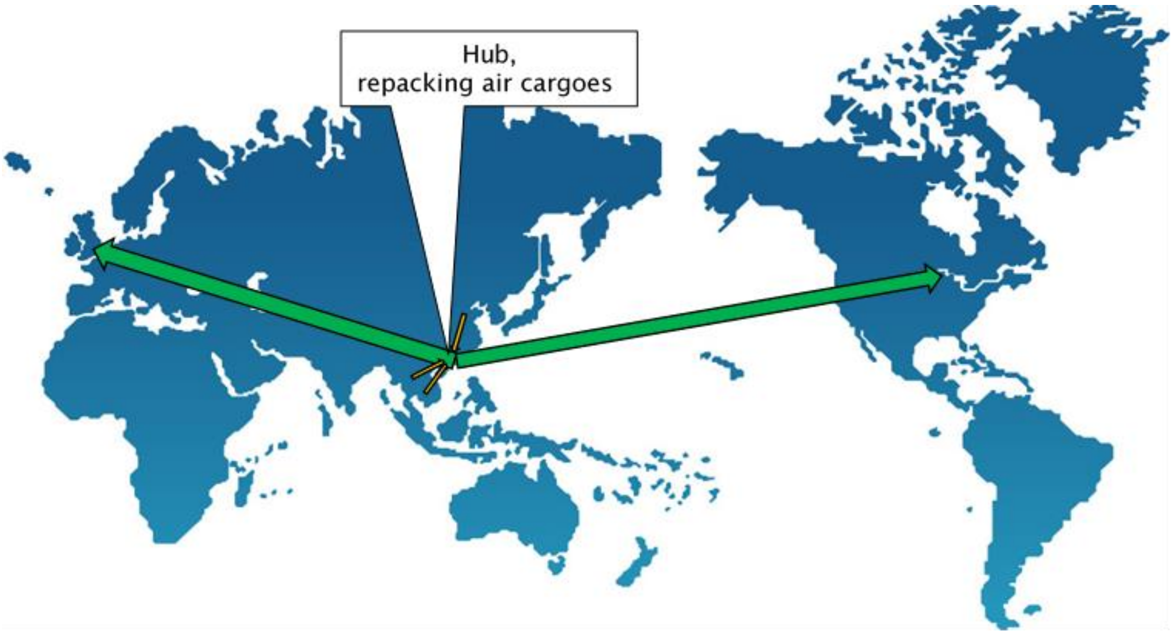

Figure 4.1 An example of air cargo forwarding problem

4.1 Problem description

With the development of high speed electronic mechanical technology in recent years, the most important factors to impact container handling have changed from labour to capital and time. Therefore, containerisation become a cost-effective and efficient method for shipment. This is also true for air cargo shipment due to the outstanding growth of airline business. The air forwarding companies handle many functions of delivering air cargoes, such as picking up items, consolidating, packing, booking air containers, preparing documents for air shipment, buying cargo insurance, warehousing and tracking. The most important one is consolidation because consolidating items can help the forwarders to save the shipment and transport costs. Nowadays, hubs have become a significant issue for the air forwarding companies due to their wide usage for saving air transportation resources.

In order to use the aircrafts' space more effectively, some types of air containers have different irregular shapes (see Figure 1.1). The air carriers cannot expect that the whole irregular space will be occupied by the air cargoes. Therefore the volume limitations provided for these containers are smaller than the exact space (Wu, 2008). In this research, we only consider the container weight and volume limitations for loading air cargoes, not the shape problems.

In this research, air cargoes need to be transported from various regions to different destinations via a hub where cargoes are combined. Every type of cargo has its own weight and volume. Cargo cannot be divided which means that each cargo must be loaded into one container. Air freight forwarders need to book air containers in all regions and hub. Notice that, in the hub, containers can come from three sources: one is from the regions, containers that can stay in use, called pre-used containers; another resource is from booking in advance, in which case they are called new containers; another resource is urgent booking on the shipping day. Using pre-used containers rather than new containers in the hub will get a little discount.

The air cargo forwarders need to make decisions not only about how many containers should be used, but also about how to load the air cargoes into these containers in order to save space and minimise the total costs. Containers are normally booked one week before the shipping date in order to get a cheap rental price from airlines. The cargo quantity that customers provide is uncertain. The forwarders do not want to wait until the actual shipment information is realized, because the price for urgent requirement or cancellation of containers on the shipping day is much higher than the booking cost one week in advance.

The price of renting a container in advance depends on container types and the cargo weight inside the container. Before the time of rental, accurate information is not available, so the forwarders have to make a decision about the quantities and types of containers in all the regions and hub, along with how to use the pre-used containers in the hub. Then, after the realization of the uncertainty, if the containers that have been booked cannot hold all cargoes, additional containers are required with a high penalty cost. On the other hand, if too many containers have been ordered, redundant containers have to be returned to the airlines with a penalty due to the breaking of a contract. Therefore, the rental cost consists of two parts: the cost of using the containers and the cost of penalties for urgent requirement or cancellation on the day of shipping. The cost of using a container is based on a fixed charge, plus a variable charge which depends on the total cargo weight that the container holds. The penalty cost includes the cost of renting the additional containers and the cost of returning the unused containers on the day of shipping.

4.2 Two-stage stochastic model and robust models

If there is one day's flights per week to deal with the forwarder's products, or there are more than one day's flights per week but each of those days can be treated as individual case, then a two-stage model will suffice. All cargoes have to be allocated to containers without delay. In this section, we provide the two-stage stochastic model and three different types of robust optimisation models similar to those in Chapter 3. In the first stage, the air freight forwarders have to make a decision, based on inaccurate information, to determine the booking quantities and types of containers in all the regions and hub, and also how to use the pre-used containers in the hub. In the second stage, in order to make sure all the air cargoes have been transported without delay, the forwarders need to make decisions not only to rent more containers or return booked containers for each scenario, but also to load all cargoes into the containers for each scenario when the uncertainty is realized on the shipping day.

All the costs will occur on the shipping day, which is quite different from those in Chapter 3. Although the air cargo forwarders should pay a deposit for booking air containers one week in advance, the deposit still can be used to pay the costs on the shipping day.

The assumptions we use for the one day's flights per week air cargo forwarding problem are as follows:

- All the cargoes should be shipped without any delay;
- Container variable costs are certain, just related to the type of containers;
- Uncertainty satisfies a discrete stochastic process;
- The container weight and volume limitations are the only two factors considered for loading air cargoes;
- The total costs of loading and repacking containers in the hub are certain, just related to the type of containers;
- The discount for using pre-used containers is a fixed proportion.

4.2.1 Container renting costs

The cost of using a container includes two parts, fixed cost and variable cost. Once a container is selected, the fixed cost needs to be paid. If the weight of cargo loaded into the container exceeds some given values, a variable cost will be incurred. These values are called break point. Let $\{1, 2, \cdots, K\}$ be K break points. The K break points divide the container weight limit into K intervals. The unit variable cost in a k interval is charged at a slope rate δ_k. In this study, the air carriers provide six break points, a_1, a_2, a_3, a_4, a_5 and a_6, for each type of container, $j = 1, 2, \cdots, n$. n represents the maximum number of air cargo types. w_j denotes weight of cargo j and y_j is the quantity of air cargo j loaded into this container. The variable cost is a piecewise function shown as follows (Wu, 2010).

$$c = \begin{cases} 0 & \sum_{j=1}^{n} w_j y_j \in (0, a_1] \\ \delta_2(\sum_{j=1}^{n} w_j y_j - a_1) & \sum_{j=1}^{n} w_j y_j \in (a_1, a_2] \\ \delta_2(a_2 - a_1) & \sum_{j=1}^{n} w_j y_j \in (a_2, a_3] \\ \delta_2(a_2 - a_1) + \delta_4(\sum_{j=1}^{n} w_j y_j - a_3) & \sum_{j=1}^{n} w_j y_j \in (a_3, a_4] \\ \delta_2(a_2 - a_1) + \delta_4(a_4 - a_3) & \sum_{j=1}^{n} w_j y_j \in (a_4, a_5] \\ \delta_2(a_2 - a_1) + \delta_4(a_4 - a_3) + \delta_6(\sum_{j=1}^{n} w_j y_j - a_5) & \sum_{j=1}^{n} w_j y_j \in (a_5, a_6] \end{cases} \tag{4.1}$$

We can simplify (4.1) by adding two new variables g_k and z_k. g_k represents the cargo weight in the range $(a_{k-1}, a_k]$ in the container. $z_k = \begin{cases} 1 & \text{if } g_k > 0 \\ 0 & \text{otherwise} \end{cases}$. Then the variable cost can change to $\sum_{k=1}^{K} \delta_k g_k$ by adding some constraints:

$$\sum_{k=1}^{K} g_k = \sum_{j=1}^{n} w_j y_j \tag{4.2}$$

$$g_k \leq z_k(a_k - a_{k-1}), \quad k = 1, 2, \cdots, K \tag{4.3}$$

$$g_k \geq z_{k+1}(a_k - a_{k-1}), \quad k = 1, 2, \cdots, K \tag{4.4}$$

Equations (4.3) and (4.4) ensure that g_k cannot be positive unless the range $(a_{k-1}, a_k]$ is fully occupied by the cargo weight.

4.2.2 Two-stage stochastic model

4.2.2.1 Notation

Indices

i types of containers ($i = 1, 2, \cdots, m$);

j types of cargoes ($j = 1, 2, \cdots, n$);

r regions ($r = 1, 2, \cdots, R$);

d destinations ($d = 1, 2, \cdots, D$);

s scenarios ($s = 1, 2, \cdots, S$);

k numbers of breaking-points for type i container ($k = 1, 2, \cdots, K_i$);

l numbers of type i container ($l = 1, 2, \cdots, L_i$).

Deterministic parameters

v_j volume of a type j cargo;

w_j weight of a type j cargo;

V_i volume limit of type i container;

W_i weight limit of type i container;

a_{ik} weight of type i container in breaking-points k;

δ_{ik} the unit charge rate of type i container in the range $(a_{i(k-1)}, a_{ik}]$;

c_{ir}^0 fixed cost by renting a type i container in region r;

c_i^{h0} fixed cost by renting a type i container in hub;

q_{jsr} type j cargo quantity in scenario s in region r;

p_s probability of scenario s;

L_{ir} type i container available quantity in region r;

L_i type i container available quantity in all regions, which means $L_i = \sum_{r=1}^{R} L_{ir}$;

L_i^h type i container available quantity in hub;

c_{ir}^{-}/c_{ir}^{+} the unit penalty cost of requiring/returning type i containers on the day of shipping in region r;

c_{i}^{h-}/c_{i}^{h+} the unit penalty cost of requiring/returning type i containers on the day of shipping in hub;

b_i the unit repacking cost of type i container in the hub (included unloading, moving the cargoes to another container);

θ the discount rate of fixed cost by using pre-used containers;

q_{jsd}^{h} quantity of type j cargo with destination d in scenario s in the hub.

Decision variables

o_{ir} number of type i container for booking in region r;

o_{i}^{h} number of type i container for booking in hub;

o_{isr}^{-}/o_{isr}^{+} number of type i container required/returned in scenario s on the day of shipping in region r;

o_{is}^{h-}/o_{is}^{h+} number of type i container required/returned in scenario s on the day of shipping in hub;

o_{i}^{hc} number of type i pre-used container for booking to continue to use in the hub;

$$x_{ilsr} = \begin{cases} 1 & \text{if the } l\text{th container of type } i \text{ is selected in scenario } s \text{ in region } r \\ 0 & \text{otherwise} \end{cases};$$

y_{iljsrd} quantity of type j cargo with destination d loaded into the lth container of type i in scenario s in region r;

y_{iljsd}^{h} quantity of type j cargo with destination d loaded into the lth container of type i in scenario s in hub;

g_{ilksr} cargo weight distributed in the range $(a_{i(k-1)}, a_{ik}]$ inside the lth container of type i in scenario s in region r;

g_{ilksd}^{h} cargo weight distributed in the range $(a_{i(k-1)}, a_{ik}]$ inside the lth container of type i in scenario s with destination d in the hub;

$$z_{ilksr} = \begin{cases} 1 & \text{if } g_{ilksr} > 0 \\ 0 & \text{otherwise} \end{cases};$$

$$z_{ilksd}^{h} = \begin{cases} 1 & \text{if } g_{ilksd}^{h} > 0 \\ 0 & \text{otherwise} \end{cases};$$

$$x_{ilsd}^{h} = \begin{cases} 1 & \text{if the } l\text{th type } i \text{ container with destination } d \text{ is selected in scenario } s \text{ in hub} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ilsd}^{hc} = \begin{cases} 1 & \text{if } l\text{th type } i \text{ pre} - \text{used container with destination } d \text{ is selected in scenario } s \text{ in hub} \\ 0 & \text{otherwise} \end{cases}$$

y_{iljsd}^{hc} quantity of type j cargo with destination d loaded into the lth pre-used container of type i in scenario s in the hub;

g_{ilksd}^{hc} cargo weight distributed in the range $(a_{i(k-1)}, a_{ik}]$ inside the lth type i pre-used container with destination d in scenario s in the hub;

$$z_{ilksd}^{hc} = \begin{cases} 1 & \text{if } g_{ilksd}^{hc} > 0 \\ 0 & \text{otherwise} \end{cases}.$$

4.2.2.2 Two-stage stochastic model

$$\min \sum_{r=1}^{R} \left(M_r + \sum_{i=1}^{m} \sum_{s=1}^{S} p_s c_{ir}^{-} o_{isr}^{-} + \sum_{i=1}^{m} \sum_{s=1}^{S} p_s c_{ir}^{+} o_{isr}^{+}\right) + \sum_{r=1}^{R} \sum_{i=1}^{m} \sum_{l=1}^{L_{ir}} \sum_{s=1}^{S} p_s b_i x_{ilsr} + \sum_{d=1}^{D} (N_d^c + N_d) + \sum_{i=1}^{m} \sum_{s=1}^{S} p_s c_i^{h-} o_{is}^{h-} + \sum_{i=1}^{m} \sum_{s=1}^{S} p_s c_i^{h+} o_{is}^{h+} \quad (4.5)$$

subject to

$$M_r = \sum_{i=1}^{m} \sum_{l=1}^{L_{ir}} \sum_{s=1}^{S} p_s c_{ir}^{0} x_{ilsr} + \sum_{i=1}^{m} \sum_{l=1}^{L_{ir}} \sum_{k=1}^{K_i} \sum_{s=1}^{S} p_s \delta_{ik} g_{ilksr} \quad r = 1, \cdots, R \quad (4.6)$$

$$N_d^c = \sum_{i=1}^{m} \sum_{l=1}^{L_i} \sum_{s=1}^{S} p_s \theta c_i^{h0} x_{ilsd}^{hc} + \sum_{i=1}^{m} \sum_{l=1}^{L_i} \sum_{k=1}^{K_i} \sum_{s=1}^{S} p_s \delta_{ik} g_{ilksd}^{hc} \quad d = 1, \cdots, D \quad (4.7)$$

$$N_d = \sum_{i=1}^{m} \sum_{l=1}^{L_i^h} \sum_{s=1}^{S} p_s c_i^{h0} x_{ilsd}^{h} + \sum_{i=1}^{m} \sum_{l=1}^{L_i^h} \sum_{k=1}^{K_i} \sum_{s=1}^{S} p_s \delta_{ik} g_{ilksd}^{h} \quad d = 1, \cdots, D \quad (4.8)$$

$$o_{ir} = \sum_{l=1}^{L_{ir}} x_{ilsr} + o_{isr}^{+} - o_{isr}^{-} \quad i = 1, \cdots, m, \quad r = 1, \cdots, R, \quad s = 1, \cdots, S \quad (4.9)$$

$$\sum_{i=1}^{m} \sum_{d=1}^{D} \sum_{l=1}^{L_{ir}} y_{iljsrd} = q_{jsr} \quad j = 1, \cdots, n, \quad r = 1, \cdots, R, \quad s = 1, \cdots, S \quad (4.10)$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i^h} y_{iljsd}^{h} + \sum_{i=1}^{m}\sum_{l=1}^{L_i} y_{iljsd}^{hc} = q_{jsd}^{h} \quad j = 1,\cdots,n, \quad d = 1,\cdots,D, \; s = 1,\cdots,S \tag{4.11}$$

$$o_i^h = \sum_{l=1}^{L_i^h}\sum_{d=1}^{D} x_{ilsd}^{h} + o_{is}^{h+} - o_{is}^{h-} \quad i = 1,\cdots,m, \quad s = 1,\cdots,S \tag{4.12}$$

$$\sum_{l=1}^{L_i}\sum_{d=1}^{D} x_{ilsd}^{hc} \le \sum_{r=1}^{R}\sum_{l=1}^{L_{ir}} x_{ilsr} \qquad i = 1,\cdots,m, \qquad s = 1,\cdots,S \tag{4.13}$$

$$\sum_{l=1}^{L_i^h}\sum_{d=1}^{D} x_{ilsd}^{h} \le L_i^h \quad i = 1,\cdots,m, \qquad s = 1,\cdots,S \tag{4.14}$$

$$\sum_{l=1}^{L_i}\sum_{d=1}^{D} x_{ilsd}^{hc} = o_i^{hc} \quad i = 1,\cdots,m, \qquad s = 1,\cdots,S \tag{4.15}$$

$$\sum_{k=1}^{K_i} g_{ilksr} = \sum_{j=1}^{n}\sum_{d=1}^{D} w_j y_{iljsrd} \quad i = 1,\cdots,m, \; l = 1,\cdots,L_{ir}, \; r = 1,\cdots,R, \; s = 1,\cdots,S \tag{4.16}$$

$$g_{ilksr} \le z_{ilksr}\left(a_{ik} - a_{i(k-1)}\right) \quad i = 1,\cdots,m, \; l = 1,\cdots,L_i, \; k = 1,\cdots,K_i, \; r = 1,\cdots,R, \; s = 1,\cdots,S \tag{4.17}$$

$$g_{ilksr} \ge z_{il(k-1)sr}\left(a_{ik} - a_{i(k-1)}\right) \quad i = 1,\cdots,m, \; l = 1,\cdots,L_i, \; k = 1,\cdots,K_i, \; r = 1,\cdots,R, \; s = 1,\cdots,S \tag{4.18}$$

$$\sum_{k=1}^{K_i} g_{ilksd}^{h} = \sum_{j=1}^{n} w_j y_{iljsd}^{h} \quad i = 1,\cdots,m, \; l = 1,\cdots,L_{ir}, \quad d = 1,\cdots,D, s = 1,\cdots,S \tag{4.19}$$

$$g_{ilksd}^{h} \le z_{ilksd}^{h}\left(a_{ik} - a_{i(k-1)}\right) \quad i = 1,\cdots,m, \; l = 1,\cdots,L_i, \quad k = 1,\cdots,K_i, \; r = 1,\cdots,R, \; s = 1,\cdots,S \tag{4.20}$$

$$g_{ilksd}^{h} \ge z_{il(k-1)sd}^{h}\left(a_{ik} - a_{i(k-1)}\right) \quad i = 1,\cdots,m, \; l = 1,\cdots,L_i, \quad k = 1,\cdots,K_i, \; r = 1,\cdots,R, \; s = 1,\cdots,S \tag{4.21}$$

$$\sum_{k=1}^{K_i} g_{ilksd}^{hc} = \sum_{j=1}^{n} w_j y_{iljsd}^{hc} \quad i = 1,\cdots,m, \; l = 1,\cdots,L_{ir}, \quad d = 1,\cdots,D, s = 1,\cdots,S \tag{4.22}$$

$$g_{ilksd}^{hc} \le z_{ilksd}^{hc}\left(a_{ik} - a_{i(k-1)}\right) \quad i = 1,\cdots,m,\ l = 1,\cdots,L_i,\ \ k = 1,\cdots,K_i,\ \ r = 1,\cdots,R,\ \ s = 1,\cdots,S \tag{4.23}$$

$$g_{ilksd}^{hc} \ge z_{il(k-1)sd}^{hc}\left(a_{ik} - a_{i(k-1)}\right) \quad i = 1,\cdots,m,\ l = 1,\cdots,L_i,\ \ k = 1,\cdots,K_i,\ \ r = 1,\cdots,R,\ \ s = 1,\cdots,S \tag{4.24}$$

$$\sum_{j=1}^{n}\sum_{d=1}^{D} v_j y_{iljsrd} \le V_i x_{ilsr} \quad i = 1,\cdots,m,\ \ l = 1,\cdots,L_{ir}, r = 1,\cdots,R,\ \ s = 1,\cdots,S \tag{4.25}$$

$$\sum_{j=1}^{n}\sum_{d=1}^{D} w_j y_{iljsrd} \le W_i x_{ilsr} \quad i = 1,\cdots,m,\ \ l = 1,\cdots,L_{ir}, r = 1,\cdots,R,\ \ s = 1,\cdots,S \tag{4.26}$$

$$\sum_{j=1}^{n} v_j y_{iljsd}^{h} \le V_i x_{ilsd}^{h} \quad i = 1,\cdots,m,\ \ l = 1,\cdots,L_{ir},\ \ d = 1,\cdots,D,\ \ s = 1,\cdots,S \tag{4.27}$$

$$\sum_{j=1}^{n} w_j y_{iljsd}^{h} \le W_i x_{ilsd}^{h} \quad i = 1,\cdots,m,\ \ l = 1,\cdots,L_{ir},\ \ d = 1,\cdots,D,\ \ s = 1,\cdots,S \tag{4.28}$$

$$\sum_{j=1}^{n} v_j y_{iljsd}^{hc} \le V_i x_{ilsd}^{hc} \quad i = 1,\cdots,m,\ \ l = 1,\cdots,L_{ir},\ \ d = 1,\cdots,D,\ \ s = 1,\cdots,S \tag{4.29}$$

$$\sum_{j=1}^{n} w_j y_{iljsd}^{hc} \le W_i x_{ilsd}^{hc} \quad i = 1,\cdots,m,\ \ l = 1,\cdots,L_{ir},\ \ d = 1,\cdots,D,\ \ s = 1,\cdots,S \tag{4.30}$$

$$o_{ir}, o_i^{h}, o_i^{hc}, o_{isr}^{-}, o_{isr}^{+}, o_{is}^{h-}, o_{is}^{h+}, y_{iljsrd}, y_{iljsd}^{h}, y_{iljsd}^{hc}, g_{ilksr}, g_{ilksd}^{h}, g_{ilksd}^{hc} \in \{0,1,2,\cdots,\inf\};$$

$$x_{ilsr}, x_{ilsd}^{h}, x_{ilsd}^{hc}, z_{ilksr}, z_{ilksd}^{h}, z_{ilksd}^{hc} \in \{0,1\} \quad i = 1,\cdots,m,\ \ l = 1,\cdots,L_{ir},\ \ j = 1,\cdots,n,\ \ r = 1,\cdots,R,\quad d = 1,\cdots,D,\ \ k = 1,\cdots,K_i,\quad s = 1,\cdots,S \tag{4.31}$$

The objective function (4.5) is the total cost including container fixed and variable costs in the regions, penalty costs for renting or returning containers on the shipping day in the regions, repacking costs in the hub, fixed and variable costs for the pre-used containers and new containers in the hub and uncertainty costs for renting or returning containers in the hub. Constraints (4.6)-(4.8) are the fixed and variable costs for containers in the regions, pre-used containers in the hub and new containers in the hub respectively. The container quantity constraints in regions and hub are (4.9) and (4.12). The cargo quantity constraints in regions and hub are (4.10) and (4.11). Constraint (4.13) means that for each type of container, the quantity of using pre-used containers in the hub cannot be greater than the sum of containers used in all regions. Constraint (4.14) ensures that the number of each type of container used in the hub cannot exceed the limit. Constraint (4.15) makes sure that the pre-used container using plan should be the same no matter which scenario is realised. Constraints (4.16)-(4.24) are container variable cost constraints according to (4.2)-(4.4). Constraints (4.25)-(4.31) are boundary conditions.

Because all the costs occur on the shipping day, the model here is quite different with previous chapter. And this kind of formulation can make the model looks simpler. The decisions in the first stage are o_{ir}, o_i^h and o_i^{hc}. o_{ir} and o_i^h are the container booking decisions in regions and hub. o_i^{hc} is the decision using pre-used containers in the hub. In the second stage, the decisions for urgent renting or returning containers are o_{isr}^-, o_{isr}^+, o_{is}^{h-} and o_{is}^{h+}; and the decisions for loading air cargoes into the containers are y_{iljsrd}, y_{iljsd}^h and y_{iljsd}^{hc}.

4.2.2.3 Computational results

A practical problem

The case organisation is described in greater detail in Wu (2010). In it, a logistics company in Hong Kong provides air transport services worldwide. It collects shipping information from its customers including the characteristics for different types of cargoes, delivery dates, destinations and uncertain demand. The air cargoes need to be transported from two regions, Mainland China (Region A) and Vietnam (Region B), to the hub in Hong Kong first. The cargoes are unloaded and consolidated in Hong Kong before they are sent to two destinations, the EU (Destination α) and Northern America (Destination β). There are three types of cargo ($n = 3$): large, medium and small, with volume 1500, 1200 and 1000 cubic decimetres and weight 750, 600 and 500 kilograms respectively.

The company contacts an airline to rent air containers in advance. There are seven types of container ($m = 7$) for rent, and currently there is only one of each type of container available in each region and hub ($L_{ir} = L_i^h = 1$ for each i and r). The airline provides the container information in Table 4.1, including the fixed cost (\$), the volume limit (dm^3), the weight limit (kg), the breaking points (kg) and the unit charge rate (\$/kg). The containers in the regions and hub have the same characteristics. If the company decides to continue using the containers in the hub, which just come from the regions, it will get 5% discount for the fixed cost, which means $\theta = 95\%$.

Table 4.1 Air container characteristics

Container type	1	2	3	4	5	6	7

Fixed cost		161617	105898	85207	74373	48713	46553	20695
Volume limit		6489	6300	5008	4882	3700	3150	1400
Weight limit		6800	5400	4200	4000	3900	3500	1200
Breaking point	a_{i1}	3968	2600	2092	1826	1196	1643	505
	a_{i2}	4722	3050	2490	2173	1423	1747	602
	a_{i3}	5290	3467	2789	2434	1594	2000	674
	a_{i4}	5976	3954	3149	2741	1825	2500	758
	a_{i5}	6273	4111	3307	2886	1917	2591	799
	a_{i6}	6800	5400	4200	4000	3900	3500	1200
Charged rate	δ_{i1}	0	0	0	0	0	0	0
	δ_{i2}	32	32	32	32	32	32	32
	δ_{i3}	0	0	0	0	0	0	0
	δ_{i4}	29	29	29	29	29	29	29
	δ_{i5}	0	0	0	0	0	0	0
	δ_{i6}	25	25	25	25	25	25	25

The uncertainty of cargo quantities of each type can be described by three scenarios: high demand s_1, medium demand s_2 and low demand s_3. Table 4.2 lists the cargo quantities under different scenarios and Table 4.3 gives the unit penalty cost for returning unused containers and renting additional containers on the day of shipping. The penalty costs in the hub will be the same in all the regions. Table 4.3 also provides the unloading cost for each type of container in the hub.

Table 4.2 Cargo quantities under different scenarios

Scenario		s_1				s_2				s_3			
Region		A		B		A		B		A		B	
Destination		α	β	α	β	α	β	α	β	α	β	α	β
Cargo type	Large	2	2	2	3	2	1	1	1	1	1	1	1
	Medium	3	3	2	2	2	3	2	2	2	2	1	1
	Small	3	2	2	2	2	2	1	2	2	1	1	2

Table 4.3 The unit penalty cost and unloading cost ($)

Container type	Unit penalty cost for returning unused containers	Unit penalty cost for renting additional containers	Unloading cost
1	100000	200000	16000
2	70000	150000	10000
3	60000	120000	8000

4	50000	100000	7000
5	40000	80000	5000
6	35000	70000	4000
7	30000	60000	2000

Results and further tests

We use the mathematical programming software AIMMS 3.14 (with CPLEX 12.6 Solver) to solve the model. The model contains 3722 constraints and 2982 variables including 1960 integer variables. Due to different probabilities of scenarios, we provide three tests, called good, fair and bad economy environments (Test I, II and III). In a good economy environment, the probabilities of high, medium and low demand scenarios are 80%, 10% and 10%. And for fair and bad economy environments, the probabilities will change to 10%, 80%, 10%, and 10%, 10%, 80% respectively. Table 4.4 shows the container booking plan for the first stage decision, and Table 4.5 provides the second stage decision about renting and returning containers on the day of shipping.

Table 4.4 Container booking plans

Container type				1	2	3	4	5	6	7
Test	I	Region	A		1		1	1	1	1
			B			1	1	1	1	
		Pre-used container				1	1	2	2	
		Hub					1	1	1	
	II	Region	A		1		1		1	1
			B				1	1	1	
		Pre-used container					2	1	2	
		Hub						1	1	
	III	Region	A				1	1	1	
			B				1	1	1	
		Pre-used container					1	2	2	
		Hub								1

Table 4.5 Renting/returning containers on the day of shipping

Container type					Scenario s_1					Scenario s_2		Scenario s_3				
					2	3	4	5	7	3	4	2	3	4	5	6
Test	I	Rent	Region	A												
				B												
			Hub													
		ne tur s	Region	A								1				

				B							1			1		
			Hub								1			1	1	1
	II	Rent	Region	A				1								
				B		1										
			Hub										1	1		
		Return	Region	A												
				B												
			Hub												1	
	III	Rent	Region	A	1				1	1						
				B		1										
			Hub			1	1	1			1					
		Return	Region	A												
				B												
			Hub													

From Table 4.4 we can see that Containers 4, 5 and 6 are the preferred choice by this model. The reason is that the costs per volume for these three containers are cheaper than the others. For example, the fixed costs per volume (using the fixed cost to divide the volume limit) for each container are \$24.91, \$16.81, \$17.01, \$15.23, \$13.17, \$14.78, \$14.78. We can see the fixed costs per volume for Container 7 is also very cheap, \$14.78. The fact is that the volume limit for container 7 is 1400 dm^3 and the volumes for large, medium and small cargo are 1500, 1200 and 1000 dm^3 respectively, which means no matter how the cargoes are loaded, it will waste at least 200 dm^3 of space. When we delete 200 dm^3 from the volume limit for Container 7, the fixed cost per volume will increase to \$17.25. That is the reason Container 7 is not preferred to Containers 4, 5 and 6. In the good economy environment, because the high demand scenario is very likely to be realized (with 80% probability), the booking plan will prefer to book enough containers to satisfy this scenario. That is the reason there is no renting plan in Test I in Table 4.5. Similarly there is no returning plan in Test III.

Now we choose the result of Test I to see how the air cargoes are loaded into the containers. Table 4.6 lists the cargoes loading plan in the regions under different scenarios. L, M and S denote the large cargo, medium cargo and small cargo needed to be transported to destination α; l, m and s represent the large, medium and small cargo need to be transported to destination β. Table 4.7 shows the cargoes loading plan in the hub under different scenarios. We can see that in the Region A plan, when medium demand scenario s_2 occurred, the solution suggests loading one large, two medium and one small cargoes into Container 2 and one small cargo into Container 7. When we calculate the volume and weight in Container 2, we find that there is still enough space to load one more small cargo. The reason we load the small cargo into Container 7 not Container 2 is that the penalty cost for returning Container 7 is relatively expensive.

Table 4.6 Cargoes loading plan in each region under different scenarios

Container type	Scenario s_1		Scenario s_2		Scenario s_3	
	Region A	Region B	Region A	Region B	Region A	Region B
1						
2	1L 2l 1s		1l 2M 1s			
3		2l 2S		1m 1S 2s		1l 1M 1s
4	2M 2m	2M 2m	3m		2M 1m	
5	1L 1m 1s	1L 2s	2S	2M 1m	1m 2S	1m 1s
6	3S	1L 1l	2L	1L 1l	1L 1l	1L 1S
7	1M		1s		1s	

Table 4.7 Cargoes loading plan in the hub under different scenarios

Container type		Scenario s_1		Scenario s_2		Scenario s_3	
		α	β	α	β	α	β
3 (From region B)		2L 2S			2l 1m		2l 1s
4	From region A	4M		1L 2M		3M	
	From hub		4m				
5	From region A	1L 2S		2M			2m
	From region B		1l 1m 1s		2m		1m 2s
	From hub	1L 1M 1S			2m 1s		
6	From region A		2l	3S		2L	
	From region B		3s		3s	3S	
	From hub		2l	2L			

Similarly with Section 3.2.4.3, we calculate VSS values to compare the two-stage stochastic model and deterministic model. Another evaluation index for the uncertainty in the stochastic model is to calculate the difference between ESS and the expected value of the corresponding wait-and-see (EWS). EWS means to find the optimal solution for each scenario after the uncertainties are realized. There are no penalty costs for return or rental containers because all the decisions are made after the realization of uncertainties. The difference between ESS and EWS is named the expected objective value of the expected value solution, denoted as EVPI. EVPI means how many benefits could be achieved if accurate information of air cargo quantity can be obtained before making decisions.

Table 4.8 Comparing the expected value model and stochastic model ($)

Test	ESS	EV	EEV	EWS	VSS(EEV-ESS)	EVPI(ESS-EWS)
I	1206444	1244798	1209838	1154860	3394	52584
II	979056	910030	987987	920522	8931	58534
III	866217	680187	875846	759632	9629	106585
Average probability	1109205	959611	1161510	948003	52305	161202

VSS and EVPI values are listed in Table 4.8. It shows that the stochastic model will save more than the deterministic model because the values of EEV for all tests are positive. We can find that VSS is quite small in Test I-III, less than 0.12% of ESS. The reason is the probability. For example, in Test I, the probability of high demand is 80%. When we calculate the expected quantity of air cargo and find the nearest integer, the value will become the same as the high demand scenario and the EV solution will become the optimal solution for a high demand scenario. Similarly, EV solutions for Test II and Test III are the optimal solutions for medium and low demand scenario respectively. Therefore, we provide one more test to see the VSS value, named “average probability”, which means the probabilities of high, medium and low demand scenarios are 34%, 33%, and 33% respectively (the last row of Table 4.8). We can see the VSS value is $52305, occupying nearly 5% of ESS. Compared with VSS, another evaluation index EVPI has more significant influence. Even the lowest value $52584 is already greater than the highest value of VSS. And the highest value of EVPI reaches $161202, occupying 14.53% of the total cost, which means the forwarders have to pay a lot of money to get the exact quantities of air cargoes before booking.

Table 4.9 lists the related costs and computing time for Test I-III. Due to too many integer variables and constraints, it takes a while to get solutions, especially in Test I. Test I-III have the same initial values, except the probabilities of scenarios. However, the first stage booking plan, second stage renting or returning plan, air cargo loading plan and the related costs are different. The total cost in Test I is 39.28% greater than in Test III. In order to examine the influence of probability on the solutions, we provide 10 more tests with different probability, with other data remaining the same. Figure 4.2 shows the total cost for each test. We can see that the total cost is highly dependent on the probability. Therefore, the forecasting probability should be carefully considered in the decision-making process.

Table 4.9 Related cost ($) and computing time

Test	I	II	III
Fixed cost in regions	525614	430550	368979
Fixed cost in hub	145238	106353	74820
Fixed cost using pre-used container	332606	276037	251660
Variable cost in regions	56921	36703	24694
Variable cost in hub	21087	12950	4673
Variable cost using pre-used container	40877	30163	21592
Penalty cost for urgent return in regions	17000	0	0
Penalty cost for urgent return in hub	17500	4000	0
Penalty cost for urgent rental in regions	0	20000	45000
Penalty cost for urgent rental in hub	0	22000	40000

Repacking cost	49600	40300	34800
Total cost	1206444	979056	866217
Computing time (seconds)	36621.38	3876.64	639.34

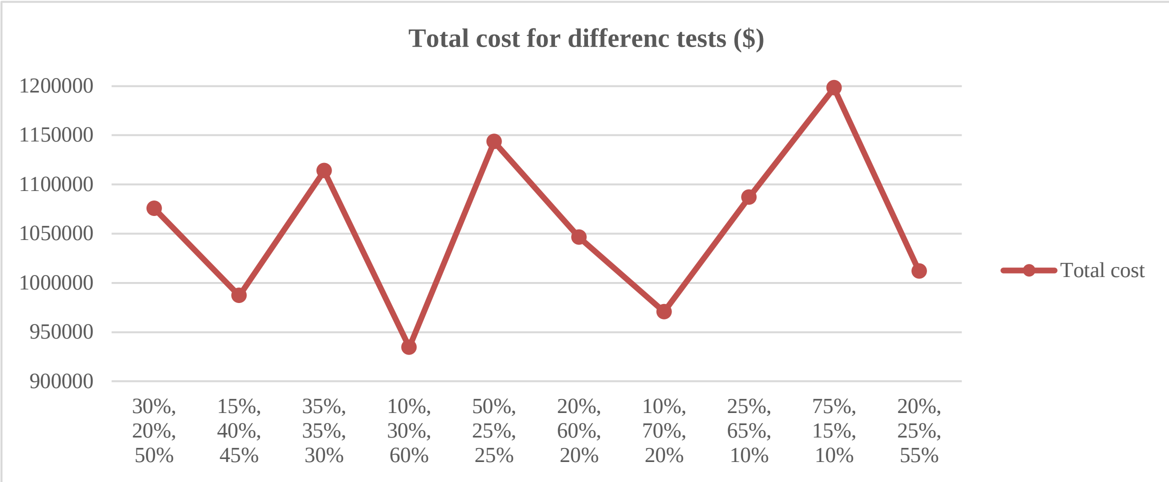

Figure 4.2 The total costs for different tests

Finally, we extend the practice problem to the three regions and three destinations case. Table 4.10 presents the cargo quantities under different scenarios for this case. Other initial data will remain the same. Table 4.11 lists the related costs and computing time in Test I-III. Similar to two regions and two destinations case, there are no penalty costs for urgent rental in Test I and no penalty costs for urgent return in Test III. The total costs still have large distance among different tests. Notice that the computing time is quite long, nearly one day, due to too many integer and binary variables. If the problem size becomes larger, the model will need much more time to return the solution even probably cannot be solved using AIMMS.

Table 4.10 Cargo quantities under different scenarios (three regions and three destinations)

Scenario		s_1									s_2									s_3								
Region		A			B			C			A			B			C			A			B			C		
Destination		α	β	γ	α	β	γ	α	β	γ	α	β	γ	α	β	γ	α	β	γ	α	β	γ	α	β	γ	α	β	γ
Cargo type	Large	2	2	3	2	3	3	2	3	2	2	1	2	1	1	2	2	3	2	1	1	2	1	1	1	2	2	2
	Medium	3	3	3	2	2	3	2	2	3	2	3	3	2	2	3	1	2	3	2	2	2	1	1	3	1	1	2
	Small	3	2	3	2	2	2	3	3	3	2	2	2	1	2	1	2	2	2	2	1	1	1	2	1	1	2	2

Table 4.11 Related cost ($) and computing time for three regions and three destinations case

Test	I	II	III
Fixed cost in regions	1478952	1107595	944606
Fixed cost in hub	237393	119228	108512
Fixed cost using pre-used container	1021164	905548	804945
Variable cost in regions	96579	116123	84831
Variable cost in hub	31586	14187	7263
Variable cost using pre-used container	110606	100689	53003
Penalty cost for urgent return in regions	50000	6000	0
Penalty cost for urgent return in hub	26500	3500	0
Penalty cost for urgent rental in regions	0	72000	132000
Penalty cost for urgent rental in hub	0	43000	89000
Repacking cost	141000	104800	89400
Total cost	3193780	2592669	2313560
Computing time (seconds)	74643.45	72342.86	69862.21

4.2.3 Three types of robust models

Similar to Section 3.3, we also provide three types of robust models with model robustness, solution robustness and the trade-off between model robustness and solution robustness according to the two-stage stochastic model in Section 4.2.2.2.

4.2.3.1 A robust optimisation model with model robustness

Based on Section 3.3.1.1, the robust optimisation model with model robustness for air cargo forwarding problems will be built as:

$$\min \sum_{r=1}^{R}\left(M_r + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_{ir}^{-} o_{isr}^{-} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_{ir}^{+} o_{isr}^{+} + \sum_{d=1}^{D}\sum_{j=1}^{n}\sum_{s=1}^{S} p_s \omega_j e_{jsrd}\right) + \sum_{r=1}^{R}\sum_{i=1}^{m}\sum_{l=1}^{L_{ir}}\sum_{s=1}^{S} b_{it} x_{ilsr} + \sum_{d=1}^{D}(N_d^c + N_d) + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^{h-} o_{is}^{h-} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^{h+} o_{is}^{h+} + \sum_{d=1}^{D}\sum_{j=1}^{n}\sum_{s=1}^{S} p_s \omega_j^h e_{jsd}^h \quad (4.32)$$

subject to (4.6)-(4.9), (4.12)-(4.31) and

$$\sum_{i=1}^{m}\sum_{d=1}^{D}\sum_{l=1}^{L_{ir}} y_{iljsrd} = q_{jsr} - e_{jsrd} \quad j = 1,\cdots,n, \quad r = 1,\cdots,R, \;\; s = 1,\cdots,S \tag{4.33}$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i^h} y_{iljsd}^{h} + \sum_{i=1}^{m}\sum_{l=1}^{L_i} y_{iljsd}^{hc} = q_{jsd}^{h} - \sum_{r=1}^{R} e_{jsrd} - e_{jsd}^{h} \quad j = 1,\cdots,n, \quad d = 1,\cdots,D, \;\; s = 1,\cdots,S \tag{4.34}$$

$$e_{jsrd}, e_{jsd}^{h} \in \{0,1,2,\cdots,\inf\} \quad j = 1,\cdots,n, \;\; d = 1,\cdots,D, \;\; r = 1,\cdots,R, \;\; s = 1,\cdots,S \tag{4.35}$$

Model robustness for air cargo forwarding problems means that the air cargoes can been transported next week by adding penalty cost ω_j. Therefore, the infeasibility variable e_{jsrd} and e_{jsd}^{h} are all positive. We do not need an absolute value sign here.

4.2.3.2 A robust optimisation model with solution robustness

Using the introduction in Section 3.3.1.2, we can list the robust optimisation model with solution robustness for the air cargo forwarding problem:

$$\begin{aligned}
\min \sum_{r=1}^{R} \Bigg(& M_r + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_{ir}^{-} o_{isr}^{-} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_{ir}^{+} o_{isr}^{+} + \lambda_r^1 \sum_{s=1}^{S} p_s (\sum_{i=1}^{m}\sum_{l=1}^{L_{ir}} c_{ir}^{0} x_{ilsr} \\
& + \sum_{i=1}^{m}\sum_{l=1}^{L_{ir}}\sum_{k=1}^{K_i} \delta_{ikr} g_{ilksr} + \sum_{i=1}^{m} c_{ir}^{-} o_{isr}^{-} + \sum_{i=1}^{m} c_{ir}^{+} o_{isr}^{+} - M_r - \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_{ir}^{-} o_{isr}^{-} \\
& - \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_{ir}^{+} o_{isr}^{+} + 2\theta_{rs}^{1}) \Bigg) + \sum_{r=1}^{R}\sum_{i=1}^{m}\sum_{l=1}^{L_{ir}}\sum_{s=1}^{S} b_{it} x_{ilsr} + \sum_{d=1}^{D} (N_d^c + N_d) \\
& + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^{h-} o_{is}^{h-} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^{h+} o_{is}^{h+} \\
& + \lambda_h^2 \sum_{s=1}^{S} p_s (\sum_{d=1}^{D} \left(\sum_{i=1}^{m}\sum_{l=1}^{L_i} \theta c_i^{h0} x_{ilsd}^{hc} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K_i} \delta_{ik}^{h} g_{ilksd}^{hc} \right) \\
& + \sum_{d=1}^{D} \left(\sum_{i=1}^{m}\sum_{l=1}^{L_i^h} c_i^{h0} x_{ilsd}^{h} + \sum_{i=1}^{m}\sum_{l=1}^{L_i^h}\sum_{k=1}^{K_i} \delta_{ik}^{h} g_{ilksd}^{h} \right) + \sum_{i=1}^{m} c_i^{h-} o_{is}^{h-} + \sum_{i=1}^{m} c_i^{h+} o_{is}^{h+} \\
& - \sum_{d=1}^{D} (N_d^c + N_d) - \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^{h-} o_{is}^{h-} - \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^{h+} o_{is}^{h+} + 2\theta_s^{\mathrm{h}2})
\end{aligned} \tag{4.36}$$

subject to (4.6)-(4.31) and

$$\begin{aligned}
-\theta_{rs}^{1} - \sum_{i=1}^{m}\sum_{l=1}^{L_{ir}} c_{ir}^{0} x_{ilsr} - \sum_{i=1}^{m}\sum_{l=1}^{L_{ir}}\sum_{k=1}^{K_i} \delta_{ikr} g_{ilksr} - \sum_{i=1}^{m} c_{ir}^{-} o_{isr}^{-} - \sum_{i=1}^{m} c_{ir}^{+} o_{isr}^{+} + M_r + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_{ir}^{-} o_{isr}^{-} \\
+ \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_{ir}^{+} o_{isr}^{+} \le 0, \qquad r = 1,\cdots,R; \;\; s = 1,\cdots,S
\end{aligned} \tag{4.37}$$

$$-\theta_s^{h2} - \sum_{d=1}^{D}\left(\sum_{i=1}^{m}\sum_{l=1}^{L_i}\theta c_i^{h0} x_{ilsd}^{hc} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K_i}\delta_{ik}^{h} g_{ilksd}^{hc}\right) - \sum_{d=1}^{D}(\sum_{i=1}^{m}\sum_{l=1}^{L_i^h} c_i^{h0} x_{ilsd}^{h}$$

$$+ \sum_{i=1}^{m}\sum_{l=1}^{L_{it}^h}\sum_{k=1}^{K_i}\delta_{ik}^{h} g_{ilksd}^{h}) - \sum_{i=1}^{m} c_i^{h-} o_{is}^{h-} - \sum_{i=1}^{m} c_i^{h+} o_{is}^{h+} + \sum_{d=1}^{D}(N_d^c + N_d)$$

$$+ \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^{h-} o_{is}^{h-} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^{h+} o_{is}^{h+} \le 0, \;\; s = 1, \cdots, S \qquad (4.38)$$

$$\theta_{rs}^{1}, \theta_s^{h2} \ge 0, \;\; r = 1, \cdots, R; \quad s = 1, \cdots, S \qquad (4.39)$$

4.2.3.3 A robust optimisation model with the trade-off between model robustness and solution robustness

Based on Section 3.3.1.3, we consider the variability and infeasibility together. A robust optimisation model with the trade-off between model robustness and solution robustness is developed to solve the air cargo forwarding problem with uncertainty. The objective function is combined with (4.32) and (4.36).

$$
\begin{aligned}
\min \sum_{r=1}^{R} \Bigg(& M_r + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_{ir}^{-} o_{isr}^{-} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_{ir}^{+} o_{isr}^{+} + \lambda_r^1 \sum_{s=1}^{S} p_s \Big(\sum_{i=1}^{m}\sum_{l=1}^{L_{ir}} c_{ir}^{0} x_{ilsr} \\
& + \sum_{i=1}^{m}\sum_{l=1}^{L_{ir}}\sum_{k=1}^{K_i} \delta_{ikr} g_{ilksr} + \sum_{i=1}^{m} c_{ir}^{-} o_{isr}^{-} + \sum_{i=1}^{m} c_{ir}^{+} o_{isr}^{+} - M_r - \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_{ir}^{-} o_{isr}^{-} \\
& - \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_{ir}^{+} o_{isr}^{+} + 2\theta_{rs}^{1}\Big) + \sum_{d=1}^{D}\sum_{j=1}^{n}\sum_{s=1}^{S} p_s \omega_j e_{jsrd} \Bigg) + \sum_{r=1}^{R}\sum_{i=1}^{m}\sum_{l=1}^{L_{irt}}\sum_{s=1}^{S} b_{it} x_{ilsr} \\
& + \sum_{d=1}^{D} (N_d^c + N_d) + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^{h-} o_{is}^{h-} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^{h+} o_{is}^{h+} \\
& + \lambda_h^2 \sum_{s=1}^{S} p_s \Bigg(\sum_{d=1}^{D} \left(\sum_{i=1}^{m}\sum_{l=1}^{L_i} \theta c_i^{h0} x_{ilsd}^{hc} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K_i} \delta_{ik}^{h} g_{ilksd}^{hc} \right) \\
& + \sum_{d=1}^{D} \left(\sum_{i=1}^{m}\sum_{l=1}^{L_i^h} c_i^{h0} x_{ilsd}^{h} + \sum_{i=1}^{m}\sum_{l=1}^{L_i^h}\sum_{k=1}^{K_i} \delta_{ik}^{h} g_{ilksd}^{h} \right) + \sum_{i=1}^{m} c_i^{h-} o_{is}^{h-} + \sum_{i=1}^{m} c_i^{h+} o_{is}^{h+} \\
& - \sum_{d=1}^{D} (N_d^c + N_d) - \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^{h-} o_{is}^{h-} - \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^{h+} o_{is}^{h+} + 2\theta_s^{h2}\Bigg) \\
& + \sum_{d=1}^{D}\sum_{j=1}^{n}\sum_{s=1}^{S} p_s \omega_j^h e_{jsd}^h
\end{aligned}
\tag{4.40}
$$

subject to (4.6)-(4.9), (4.12)-(4.31), (4.33)-(4.35) and (4.37)-(4.39).

4.2.3.4 Computational results

<u>*A practical problem result*</u>

Here we will use the same initial data from the two-stage stochastic model case in Section 4.2.2.3 to get the solutions. We will take the two-stage robust model with the trade-off between model robustness and solution robustness ($\lambda_r^1, \lambda_h^2 = 0.1$ for $r = 1, \cdots, R$ and $\omega_1 = \omega_1^h = 32000$, $\omega_2 = \omega_2^h = 28000$, $\omega_3 = \omega_3^h = 24000$) as an example for the good economy environment to see the results. $\omega_1, \omega_1^h, \omega_2, \omega_2^h, \omega_3, \omega_3^h$ are the penalty costs per unit for large, medium and small cargo in the regions and hub. Table 4.12 presents the booking plan for this model. Container 4, 5, 6 and 7 are booked in both in regions A and B because these containers are more economical than other types of container. One Container 4 and two Container 7 will be returned to regions when they arrive in the hub. There are no urgent returns or rentals occurring, due to the reduction of the variability cost. Table 4.13 gives the details of unshipped cargoes which means they will be considered in the following week.

Table 4.12 Container booking plan for good economy environment

Container type		1	2	3	4	5	6	7
Region	A				1	1	1	1
	B				1	1	1	1
Pre-used container					1	2	2	
Hub						1	1	

Table 4.13 Unshipped cargoes

Cargo		Scenario s_1			Scenario s_2			Scenario s_3		
		Large	Medium	Small	Large	Medium	Small	Large	Medium	Small
Region	A	2	1				2			
	B	2		1						
Hub										

Table 4.14 lists the computational results of the robust models using different penalty costs for unshipped cargoes and the multi-stage stochastic model. The total cost under the stochastic model is \$1206444 and the total cost under the robust model with λ_r^1, λ_h^2 =0.1 and $\omega = \omega^h$ =28000, 24000, 2000 is \$1149952. Using the robust optimisation model, the total cost decreases by 4.68%, and the expected variability of the robust model decreases 84%, which means the robust model presents a less sensitive air cargoes transportation strategy. However, the robust model involves the huge infeasibility cost of \$382400 for unshipped cargoes. If we increase the penalty to 34000, 30000 and 26000, no random constraint is violated. Compare this with the stochastic model, in which the expected variability decreases 2.51%, and the total cost of the robust model only increases by 0.55%. It means that the robust model is trying to find the balance between variability and infeasibility.

Table 4.14 Comparing the robust model and the stochastic model for good economy environment

	Stochastic model	Robust model ($\lambda_r^1, \lambda_h^2 = 0.1$, $\omega_1 = \omega_1^h =$ 28000,	Robust model ($\lambda_r^1, \lambda_h^2 = 0.1$, $\omega_1 = \omega_1^h =$ 30000,	Robust model ($\lambda_r^1, \lambda_h^2 = 0.1$, $\omega_1 = \omega_1^h =$ 32000,	Robust model ($\lambda_r^1, \lambda_h^2 = 0.1$, $\omega_1 = \omega_1^h =$ 34000,

		$\omega_2 = \omega_2^h =$ 24000, $\omega_3 = \omega_3^h =$ 20000)	$\omega_2 = \omega_2^h =$ 26000, $\omega_3 = \omega_3^h =$ 22000)	$\omega_2 = \omega_2^h =$ 28000, $\omega_3 = \omega_3^h =$ 24000)	$\omega_2 = \omega_2^h =$ 30000, $\omega_3 = \omega_3^h =$ 26000)
Expected cost	1206444	766475	851396	851396	1206537
Expected variability	67317	10768	20233	20233	65624
Expected infeasibility cost	0	382400	325600	348800	0
Total cost	1206444	1149952	1179019	1202219	1213099

Computational results for robust model with solution robustness

Table 4.15 gives the computational results of the robust optimisation with solution robustness for the three tests, in which λ is assigned different values.

Table 4.15 Computational results for robust optimisation model with solution robustness

Test	λ_r^1, λ_h^2	Expected cost in regions	Expected cost in hub	Variability in regions	Variability in hub	Expected variability cost	Total cost
I	0	599535	606909	32126	35191	0	1206444
	0.1	599856	606681	31612	34012	6562	1213099
	0.5	603060	606777	26485	29451	27969	1237806
	0.9	617517	625185	3361	1	3025	1245727
II	0	487253	491803	66474	73735	0	979056
	0.1	487253	491803	66474	73735	14020	993076
	0.5	487253	495775	66474	64564	65518	1048546
	0.9	508945	508060	42134	44169	77672	1094677
III	0	438673	427544	131152	127289	0	866217
	0.1	438673	427544	131152	127289	25844	892061
	0.5	466029	450145	68549	78649	73599	989773
	0.9	472314	451103	57355	77178	121080	1044497

When $\lambda = 0$, the robust optimisation model becomes a two-stage stochastic model in which the variability is not considered. In Table 4.15, for each test the expected variability for the stochastic model is greater than or equal to that of the robust optimisation model. This means that the stochastic model is riskier than the robust optimisation model with solution robustness. The total cost of the robust optimisation model is greater than that of the two-stage stochastic model. Compared with the recourse model, the total cost of robust model ($\lambda = 0.9$) increases by 3.26% in Test I, 11.81% in Test II and 20.58% in Test III. However, the variability decreases by 95.01% in Test I, 38.47% in Test II and 47.94% in Test III. The variabilities in Test II and III are more than twice the value of variability in Test I. That means that it is more important to use the robust model with solution robustness in Tests II and III than in Test I, as the risk is higher.

<u>Computational results for robust model with model robustness</u>

Table 4.16, Table 4.17 and Table 4.18 show the computational results of the robust optimisation with model robustness for the three tests. In the tests, "L", "M" and "S" means large, medium and small cargo respectively. When the penalty index $\omega = 0$, there is no penalty for unshipped cargoes which means all the cargoes will be considered in the next week. When ω increases, the trend of expected infeasibility decreases, and the total cost increases. When ω increases by a large amount, the expected infeasibility becomes zero, which means that all the cargoes should be loaded without delay. The robust optimisation model then becomes the stochastic model (see the final column in each table).

Table 4.16 Computational results of robust optimisation model with model robustness for Test I

$\omega_1 = \omega_1^h$ $\omega_2 = \omega_2^h$ $\omega_3 = \omega_3^h$			24000 20000 16000	26000 22000 18000	28000 24000 20000	30000 26000 22000	32000 28000 24000	34000 30000 26000
Expected cost in regions			0	216718	378606	423029	496214	599535
Expected cost in hub			0	225191	387870	428366	501181	606909
Unshipped cargoes in region A	scenario	s_1	4L 6M 5S	4L 3M 2S	4L 1M	3L 1M	3L 1S	0
		s_2	3L 5M 4S	3L 2M 1S	2L	2S	2S	0
		s_3	2L 4M 3S	1M 3S	0	0	0	0
Unshipped cargoes in region B	scenario	s_1	5L 4M 4S	5L 1M 1S	2L 2S	2L 1S	0	0
		s_2	2L 4M 3S	2L 1M	0	0	0	0
		s_3	2L 2M 3S	2S	0	0	0	0
Unshipped cargoes in hub	scenario	s_1	0	0	0	0	0	0
		s_2	0	0	0	0	0	0
		s_3	0	0	0	0	0	0

infeasibility cost ($)	1040800	666800	382400	325600	201600	0
Total cost ($)	1040800	1108709	1148876	1176995	1198995	1206444

Table 4.17 Computational results of robust optimisation model with model robustness for Test II

$\omega_1 = \omega_1^h$ $\omega_2 = \omega_2^h$ $\omega_3 = \omega_3^h$			24000 20000 16000	26000 22000 18000	30000 26000 22000	34000 30000 26000	38000 34000 30000	58000 54000 50000	62000 58000 54000	74000 70000 66000	76000 72000 68000
Expected cost in regions			0	216718	380006	380006	452278	452278	475178	475178	487253
Expected cost in hub			0	225191	387870	387870	449512	449512	468410	468410	491803
Unshipped cargoes in region A	scenario	s_1	4L 6M 5S	4L 3M 2S	4L 1M	4L 1M	2L	2L	1L 1S	1L 1S	0
		s_2	3L 5M 4S	3L 2M 1S	2L	2L	0	0	0	0	0
		s_3	2L 4M 3S	1M 3S	0	0	0	0	0	0	0
Unshipped cargoes in region B	scenario	s_1	5L 4M 4S	5L 1M 1S	2L 2S	2L 2S	3L 1S	3L 1S	0	0	0
		s_2	2L 4M 3S	2L 1M	0	0	0	0	0	0	0
		s_3	2L 2M 3S	2S	0	0	0	0	0	0	0
Unshipped cargoes in hub	scenario	s_1	0	0	0	0	0	0	1L	1L	0
		s_2	0	0	0	0	0	0	0	0	0
		s_3	0	0	0	0	0	0	0	0	0
infeasibility cost ($)			833600	440000	146000	166000	44000	68000	29400	35400	0
Total cost ($)			833600	881909	913876	933876	945790	969790	972988	978988	979056

Table 4.18 Computational results of robust optimisation model with model robustness for Test III

$\omega_1 = \omega_1^h$ $\omega_2 = \omega_2^h$ $\omega_3 = \omega_3^h$			22000 18000 14000	26000 22000 18000	30000 26000 22000	34000 30000 26000	46000 42000 38000	50000 46000 42000	62000 58000 54000	70000 66000 62000	78000 74000 70000
Expected cost in regions			0	214688	316834	335960	335960	347238	384544	419349	438672
Expected cost in hub			0	223161	320437	344027	344027	351805	371514	408560	427545
a r g o	nari	s_1	4L 6M	4L 3M	3L 1M	4L 1M	4L 1M	3L 1M	4L 1S	0	0

			5S	2S	1S			1S			
		s_2	3L 5M 4S	3L 2M 1S	2L	2L	2L	2L	2L	2L	0
		s_3	2L 4M 3S	1M 3S	0	0	0	0	0	0	0
Unshipped cargoes in region B	scenario	s_1	5L 4M 4S	5L 1M 1S	5L 1S	4L 2S	4L 2S	4L 1S	0	0	0
		s_2	2L 4M 3S	2L 1M	2L	2L	2L	1L	0	0	0
		s_3	2L 2M 3S	2S	1S	0	0	0	0	0	0
Unshipped cargoes in hub	scenario	s_1	0	0	0	0	0	0	1L	0	0
		s_2	0	0	1M	0	0	0	1M	1M	0
		s_3	0	0	0	0	0	0	0	0	0
infeasibility cost ($)			622800	297200	126400	98000	134000	126000	97200	34600	0
Total cost ($)			622800	735049	763671	777987	813987	825043	853258	862509	866217

Computational results for robust optimisation model with the trade-off between solution robustness and model robustness

Parameters λ and ω are used to measure the trade-off between solution robustness and model robustness. When $\omega = 0$, there is no penalty for the infeasibility of random constraints in the objective function. The infeasibility representing un-fulfilment is a higher value. Clearly, decision makers would not like this kind of production loading plan. However, a large weight of ω means the penalty function dominates the total objective function value and would result in a higher variability and a higher total cost. Therefore, there is always a trade-off between the risk and the cost. Figure 4.3, Figure 4.4 and Figure 4.5 show the computational results for Test I in terms of the variability, infeasibility, and total cost, when λ keeps constant.

Figure 4.3 gives the trend of the variability when ω increases for λ =0.1, 0.5, and 0.9, respectively. For λ =0.1 and 0.5, when ω increases, the variabilities sharply increase from 1044 to 65624 and from 1044 to 34494. When $\lambda =$ 0.9, the value of ω has a small impact on the variability. The reason for this is that when λ is given a large value, the variability cost dominates the objective function value, and the infeasibility cost measured by ω has less impact on the total cost. Figure 4.4 gives the trend of the infeasibility when ω increases for λ =0.1, 0.5, and 0.9, respectively. Clearly, the value of ω has a big influence on the system's infeasibility. In Figure 4.5, when ω increases, the total cost increases accordingly. The value of ω has more impact on the system when the value of λ is small.

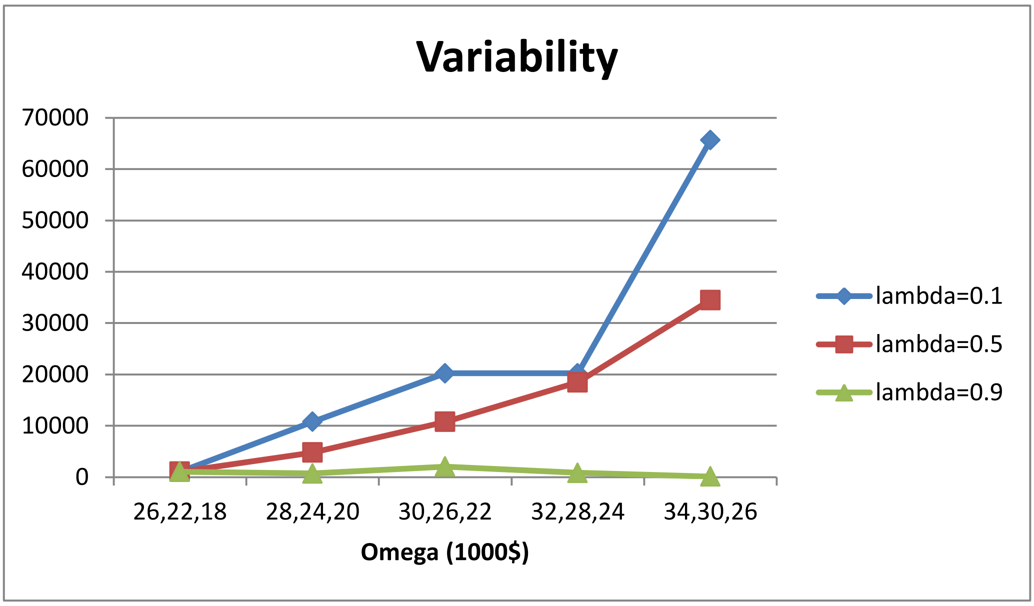

Figure 4.3 Variability when λ keeps constant

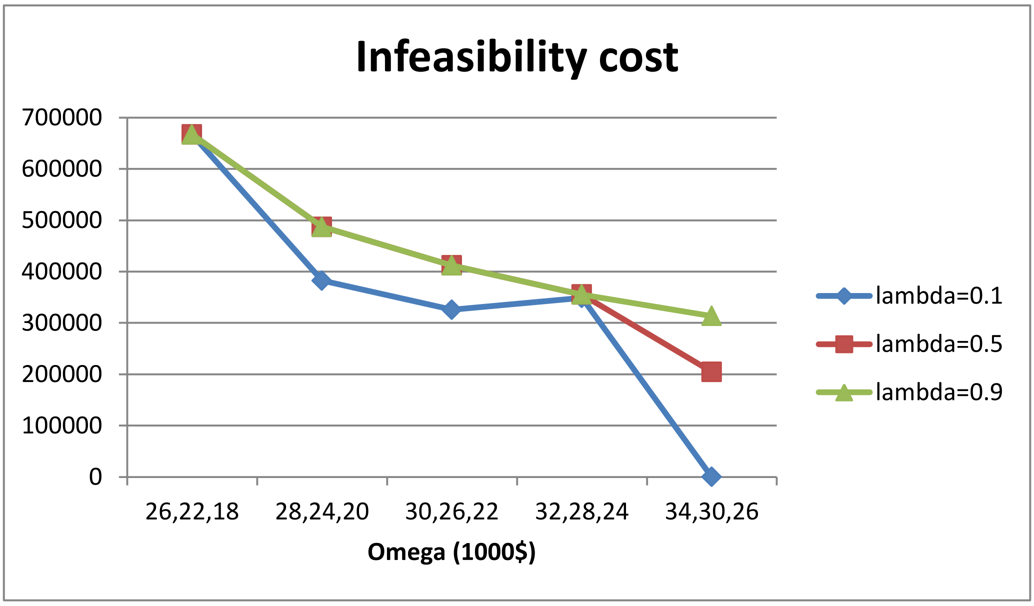

Figure 4.4 Infeasibility when λ keeps constant

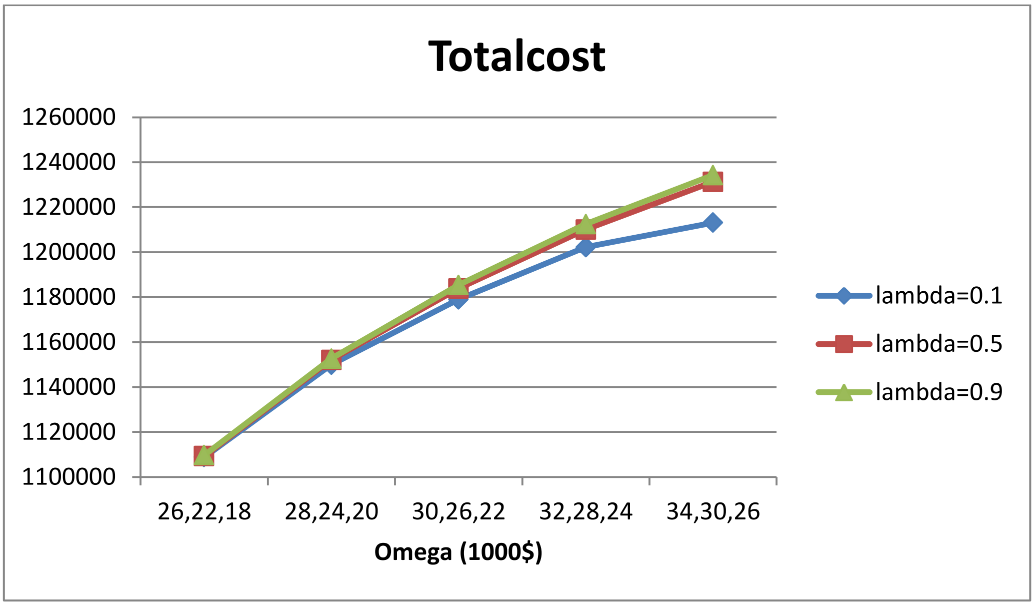

Figure 4.5 Total cost when λ keeps constant

Figure 4.6, Figure 4.7 and Figure 4.8 show the computational results of Test I in terms of the variability, infeasibility, and total cost, when ω keeps constant.

Figure 4.6 shows the trend in the variability when λ increases for different infeasibility penalty ω values. When ω =26000, 22000 and 18000, the variability is the same and very small for different λ. The reason for this is that the decision plan for each scenario is nearly the same. The different cargo quantities among the scenarios will be considered in the next week due to the relatively cheap penalty ω values. When λ increases from 0.1 to 0.9, for ω =28000, 24000 and 20000, the variability decreases by 93.40%; for ω =30000, 26000 and 22000, the variability decreases by 89.90%; for ω =32000, 28000 and 24000, the variability decreases by 95.79%; and for ω =34000, 30000 and 26000, the variability decreases by 99.80%. The value of λ has a great impact on the variability.

Figure 4.7 shows the trend of the infeasibility when λ increases for different infeasibility penalty ω values. When ω =26000, 22000 and 18000, the infeasibility costs are the same for different λ which means the variability cost measured by λ has no impact on the infeasibility owing to the huge infeasibility cost. When λ increases from 0.1 to 0.9, for ω =28000, 24000 and 20000, the infeasibility cost increases by 27.41%; for ω =30000, 26000 and 22000, the infeasibility cost increases by 26.54%; for ω =32000, 28000 and 24000, the infeasibility cost increases by 1.83%; and for ω =34000, 30000 and 26000, the infeasibility cost increases very dramatically, from \$0 to \$313600. The value of λ has a great impact on the infeasibility cost in the last test.

Figure 4.8 shows the trend of the total cost when λ increases for different infeasibility penalty ω values. If λ increases from 0.1 to 0.9, for ω =26000, 22000 and 18000, the total cost increases by

0.08%, for ω =28000, 24000 and 20000, the total cost increases by 0.23%; for ω =30000, 26000 and 22000, the total cost increases by 0.54%, for ω =32000, 28000 and 24000, the total cost increases by 0.85%; and for ω =34000, 30000 and 26000, the total costs increases by 1.75%. Compared with the changes in variability and infeasibility in Figure 4.6 and Figure 4.7, the total cost only increases by a small amount when λ increases.

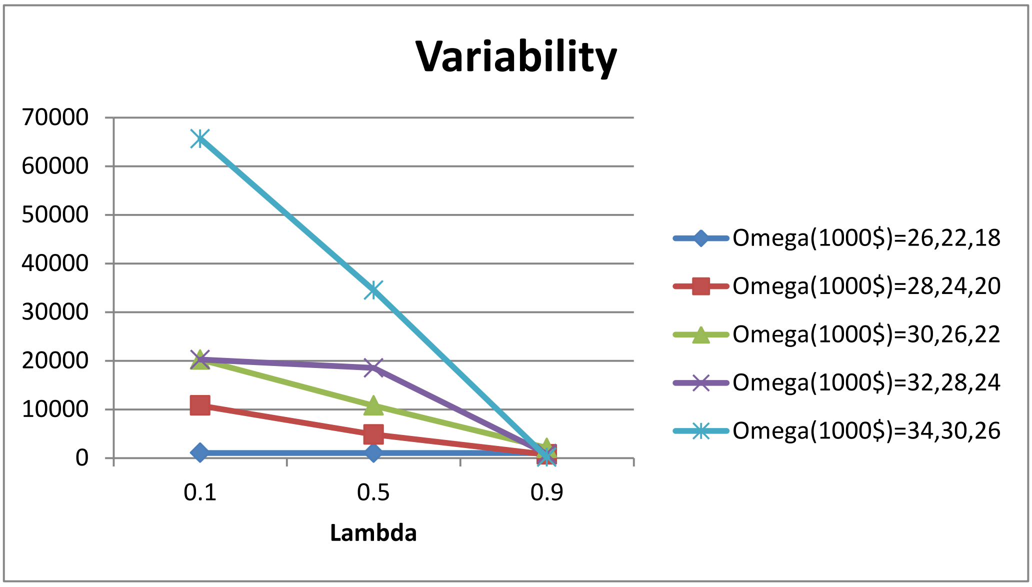

Figure 4.6 Variability when ω keeps constant

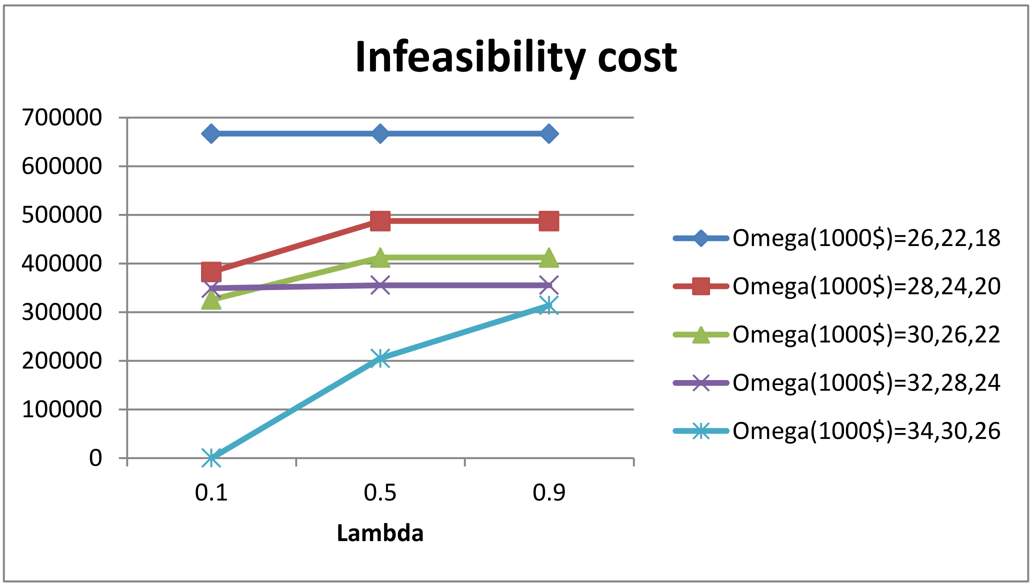

Figure 4.7 Infeasibility when ω keeps constant

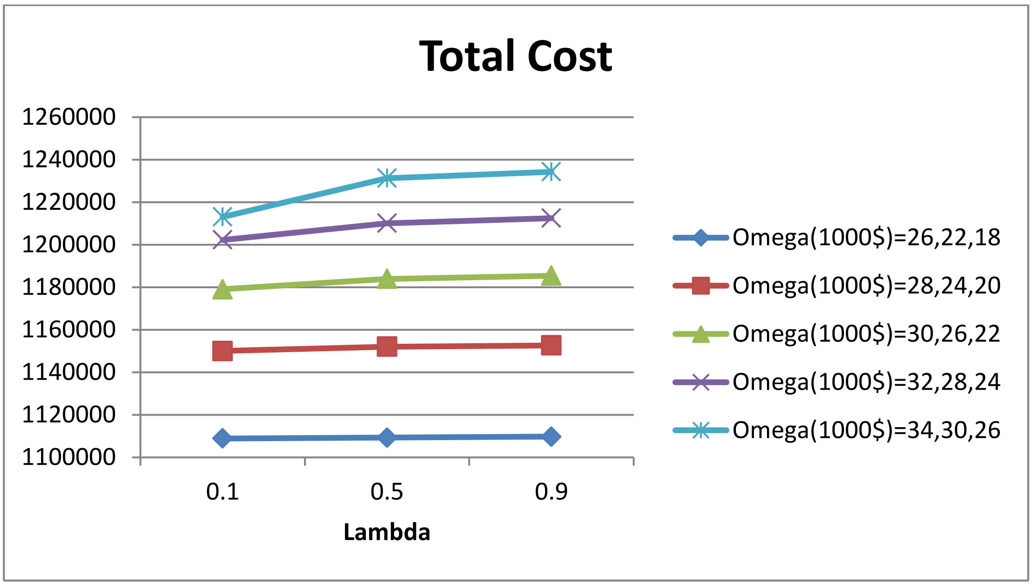

Figure 4.8 Total cost when ω keeps constant

The problem assumes there is one day's flights per week; the decisions should be made one week earlier and unshipped cargo is not allowed. In the next section, we introduce multi-stage model for a multi-day's flights case.

4.3 Multi-stage stochastic model and robust model with model robustness

If there are multiple days' flights per week to deal with the forwarders' products, a multi-stage model will be very suitable to solve the air cargo forwarding problem. In the first stage, the air freight forwarders have to make booking decisions for all days in the next week, including the quantities and types of booked containers in all the regions and hub, and also the pre-used containers in the hub. In the second stage, the forwarders need to make decisions not only about renting more containers or returning booked containers for each scenario in the first day, but also about loading cargoes into the containers for each scenario. The decisions in the following stage will be similar with the second stage.

The assumptions for this problem are the same as with the two-stage case in Section 4.2 except the first one. In the two-stage model we assume all the cargoes should be shipped without any delay. In the multi-stage model, we assume the air cargoes can be transported with one day's delay but all cargoes must be transported before the final day.

4.3.1 Multi-stage stochastic model

4.3.1.1 Notation

Indices

i types of containers ($i = 1, 2, \cdots, m$);

j types of cargoes ($j = 1, 2, \cdots, n$);

r regions ($r = 1, 2, \cdots, R$);

d destinations ($d = 1, 2, \cdots, D$);

t periods ($t = 1, 2, \cdots, T$);

$(s_1 s_2 \cdots s_t)$ scenarios in period t (with outcomes $s_1, s_2, \cdots, s_t = 1, \cdots, S$);

k numbers of breaking-points for type i container ($k = 1, 2, \cdots, K_i$);

l numbers of type i container ($l = 1, 2, \cdots, L_i$).

Deterministic parameters

v_j volume of a type j cargo;

w_j weight of a type j cargo;

V_i volume limit of type i container;

W_i weight limit of type i container;

a_{ikr} weight of type i container in breaking-points k in region r;

δ_{ikr} the unit charge rate of type i container in the range $(a_{i(k-1)r}, a_{ikr}]$ in region r;

c_{irt}^0 fixed cost by renting a type i container in region r in period t;

L_{irt} type i container available quantity in region r in period t;

L_{it} type i container available quantity in all regions in period t, which means $L_{it} = \sum_{r=1}^{R} L_{irt}$;

c_{irt}^- / c_{irt}^+ the unit penalty cost of requiring/returning type i containers on the day of shipping in region r in period t;

b_{jrt}^+ the storage cost of a type j cargo in region r in period t;

A_r volume limit of storage room in region r;

B_r weight limit of storage room in region r;

$p_{(s_1 s_2 \cdots s_t)}$ probability of scenario $(s_1 s_2 \cdots s_t)$;

a_{ik}^h weight of type i container in breaking-points k in region r;

δ_{ik}^h the unit charge rate of type i container in the range $(a_{i(k-1)}^h, a_{ik}^h]$ in the hub;

c_{it}^{h0} fixed cost by renting a type i container in the hub in period t;

L_{it}^h type i container available quantity in the hub in period t;

c_{it}^{h-}/c_{it}^{h+} the unit penalty cost of requiring/returning type i containers on the day of shipping in the hub in period t;

b_{it} the unit repacking cost of type i container in the hub in period t (included unloading, moving the cargoes to another container);

μ the discount rate of fixed cost by using pre-used containers;

b_{jt}^{h+} the storage cost of a type j cargo in the hub in period t;

A^h volume limit of storage room in the hub;

B^h weight limit of storage room in the hub.

<u>*Random parameters*</u>

$q_{jr(s_1 s_2 \cdots s_t)}$ quantity of type j cargo in scenario $(s_1 s_2 \cdots s_t)$ in region r;

$q_{jd(s_1 s_2 \cdots s_t)}^h$ quantity of type j cargo with destination d in scenario $(s_1 s_2 \cdots s_t)$ in the hub.

<u>*Decision variables*</u>

o_{irt} number of type i container for booking in region r in period t;

$o_{ir(s_1 s_2 \cdots s_t)}^-/o_{ir(s_1 s_2 \cdots s_t)}^+$ number of type i container required/returned in scenario $(s_1 s_2 \cdots s_t)$ on the day of shipping in region r;

$$x_{ilr(s_1 s_2 \cdots s_t)} = \begin{cases} 1 & \text{if the } l\text{th container of type } i \text{ is selected in scenario } (s_1 s_2 \cdots s_t) \text{ in region } r \\ 0 & \text{otherwise} \end{cases};$$

$y_{iljrd(s_1 s_2 \cdots s_t)}$ quantity of type j cargo with destination d loaded into the lth container of type i in scenario $(s_1 s_2 \cdots s_t)$ in region r;

$y^{+}_{jrd(s_1s_2\cdots s_t)}$ quantity of type j cargo with destination d stored for next period in scenario $(s_1s_2\cdots s_t)$ in region r;

$g_{ilkr(s_1s_2\cdots s_t)}$ cargo weight distributed in the range $(a_{i(k-1)r}, a_{ikr}]$ inside the lth container of type i in scenario $(s_1s_2\cdots s_t)$ in region r;

$$z_{ilkr(s_1s_2\cdots s_t)} = \begin{cases} 1 & \text{if } g_{ilkr(s_1s_2\cdots s_t)} > 0 \\ 0 & otherwise \end{cases};$$

o^{h}_{it} number of type i container for booking in the hub in period t;

$o^{h-}_{i(s_1s_2\cdots s_t)}/o^{h+}_{i(s_1s_2\cdots s_t)}$ number of type i container required/returned in scenario $(s_1s_2\cdots s_t)$ on the day of shipping in the hub;

o^{hc}_{it} number of type i pre-used container for booking to continue to use in the hub in period t;

$$x^{h}_{ild(s_1s_2\cdots s_t)} = \begin{cases} 1 & \text{if the } l\text{th type } i \text{ container with destination } d \text{ is selected in scenario } (s_1s_2\cdots s_t) \text{ in the hub} \\ 0 & \text{otherwise} \end{cases};$$

$$x^{hc}_{ild(s_1s_2\cdots s_t)} = \begin{cases} 1 & \text{if the } l\text{th type } i \text{ pre} - \text{used container with destination } d \text{ is selected in scenario } (s_1s_2\cdots s_t) \text{ in the hub} \\ 0 & \text{otherwise} \end{cases};$$

$y^{h}_{iljd(s_1s_2\cdots s_t)}$ quantity of type j cargo with destination d loaded into the lth container of type i in scenario $(s_1s_2\cdots s_t)$ in the hub;

$y^{hc}_{iljd(s_1s_2\cdots s_t)}$ quantity of type j cargo with destination d loaded into the lth pre-used container of type i in scenario $(s_1s_2\cdots s_t)$ in the hub;

$y^{h+}_{jd(s_1s_2\cdots s_t)}$ quantity of type j cargo with destination d stored for next period in scenario $(s_1s_2\cdots s_t)$ in the hub;

$g^{h}_{ilkd(s_1s_2\cdots s_t)}$ cargo weight distributed in the range $(a_{i(k-1)}, a_{ik}]$ inside the lth container of type i in scenario $(s_1s_2\cdots s_t)$ with destination d in the hub;

$g^{hc}_{ilkd(s_1s_2\cdots s_t)}$ cargo weight distributed in the range $(a^{h}_{i(k-1)}, a^{h}_{ik}]$ inside the lth type i pre-used container with destination d in scenario $(s_1s_2\cdots s_t)$ in the hub;

$$z^{h}_{ilkd(s_1s_2\cdots s_t)} = \begin{cases} 1 & \text{if } g^{h}_{ilkd(s_1s_2\cdots s_t)} > 0 \\ 0 & \text{otherwise} \end{cases};$$

$$z^{hc}_{ilkd(s_1 s_2 \cdots s_t)} = \begin{cases} 1 & \text{if } g^{hc}_{ilkd(s_1 s_2 \cdots s_t)} > 0 \\ 0 & \text{otherwise} \end{cases}.$$

4.3.1.2 Multi-stage stochastic model

$$\min \sum_{t=1}^{T} (\sum_{s_1}^{S} \sum_{s_2}^{S} \cdots \sum_{s_t}^{S} p_{(s_1 s_2 \cdots s_t)} (\sum_{r=1}^{R} M_{rt} + N_t)) \tag{4.41}$$

subject to

$$\begin{aligned} M_{rt} = & \sum_{i=1}^{m} \sum_{l=1}^{L_{irt}} c^{0}_{irt} x_{ilr(s_1 s_2 \cdots s_t)} + \sum_{i=1}^{m} \sum_{l=1}^{L_{irt}} \sum_{k=1}^{K_i} \delta_{ikr} g_{ilkr(s_1 s_2 \cdots s_t)} + \sum_{i=1}^{m} c^{-}_{irt} o^{-}_{ir(s_1 s_2 \cdots s_t)} \\ & + \sum_{i=1}^{m} c^{+}_{irt} o^{+}_{ir(s_1 s_2 \cdots s_t)} + \sum_{d=1}^{D} \sum_{j=1}^{n} b^{+}_{jrt} y^{+}_{jrd(s_1 s_2 \cdots s_t)} \\ & + \sum_{r=1}^{R} \sum_{i=1}^{m} \sum_{l=1}^{L_{irt}} b_{it} x_{ilr(s_1 s_2 \cdots s_t)} \end{aligned} \tag{4.42}$$

$$\begin{aligned} N_t = & \sum_{d=1}^{D} \Bigg(\sum_{i=1}^{m} \sum_{l=1}^{L_{it}} \mu c^{h0}_{it} x^{hc}_{ild(s_1 s_2 \cdots s_t)} + \sum_{i=1}^{m} \sum_{l=1}^{L_{it}} \sum_{k=1}^{K_i} \delta^{h}_{ik} g^{hc}_{ilkd(s_1 s_2 \cdots s_t)} + \sum_{i=1}^{m} \sum_{l=1}^{L^{h}_{it}} c^{h0}_{it} x^{h}_{ild(s_1 s_2 \cdots s_t)} \\ & + \sum_{i=1}^{m} \sum_{l=1}^{L^{h}_{it}} \sum_{k=1}^{K_i} \delta^{h}_{ik} g^{h}_{ilkd(s_1 s_2 \cdots s_t)} + \sum_{j=1}^{n} b^{h+}_{jt} y^{h+}_{jd(s_1 s_2 \cdots s_t)} \Bigg) + \sum_{i=1}^{m} c^{h-}_{it} o^{h-}_{i(s_1 s_2 \cdots s_t)} \\ & + \sum_{i=1}^{m} c^{h+}_{it} o^{h+}_{i(s_1 s_2 \cdots s_t)} \end{aligned} \tag{4.43}$$

$$\sum_{j=1}^{n} \sum_{d=1}^{D} v_j y_{iljrd(s_1 s_2 \cdots s_t)} \le V_i x_{ilr(s_1 s_2 \cdots s_t)} \quad i = 1, \cdots, m;\ l = 1, \cdots, L_{irt};\ t = 1, \cdots, T; s_1, \cdots, s_t = 1, \cdots, S;\ r = 1, \cdots, R; \tag{4.44}$$

$$\sum_{j=1}^{n} \sum_{d=1}^{D} w_j y_{iljrd(s_1 s_2 \cdots s_t)} \le W_i x_{ilr(s_1 s_2 \cdots s_t)} \quad i = 1, \cdots, m;\ l = 1, \cdots, L_{irt};\ t = 1, \cdots, T;\ s_1, \cdots, s_t = 1, \cdots, S;\ r = 1, \cdots, R; \tag{4.45}$$

$$\sum_{j=1}^{n} v_j y^{h}_{iljd(s_1 s_2 \cdots s_t)} \le V_i x^{h}_{ild(s_1 s_2 \cdots s_t)} \qquad i = 1,\cdots,m;\ l = 1,\cdots,L^{h}_{it};\ t = 1,\cdots,T;\ s_1,\cdots,s_t = 1,\cdots,S;\ d = 1,\cdots,D; \tag{4.46}$$

$$\sum_{j=1}^{n} w_j y^{h}_{iljd(s_1 s_2 \cdots s_t)} \le W_i x^{h}_{ild(s_1 s_2 \cdots s_t)} \qquad i = 1,\cdots,m;\ l = 1,\cdots,L^{h}_{it};\ t = 1,\cdots,T;\ s_1,\cdots,s_t = 1,\cdots,S;\ d = 1,\cdots,D; \tag{4.47}$$

$$\sum_{j=1}^{n} v_j y^{hc}_{iljd(s_1 s_2 \cdots s_t)} \le V_i x^{hc}_{ild(s_1 s_2 \cdots s_t)} \qquad i = 1,\cdots,m;\ l = 1,\cdots,L_{it};\ t = 1,\cdots,T;\ s_1,\cdots,s_t = 1,\cdots,S;\ d = 1,\cdots,D; \tag{4.48}$$

$$\sum_{j=1}^{n} w_j y^{hc}_{iljd(s_1 s_2 \cdots s_t)} \le W_i x^{hc}_{ild(s_1 s_2 \cdots s_t)} \qquad i = 1,\cdots,m;\ l = 1,\cdots,L_{it};\ t = 1,\cdots,T;\ s_1,\cdots,s_t = 1,\cdots,S;\ d = 1,\cdots,D; \tag{4.49}$$

$$y^{+}_{jrds_0} = 0,\ \ y^{+}_{jrd(s_1 s_2 \cdots s_T)} = 0 \qquad j = 1,\ \cdots,\ n;\ r = 1,\ \cdots,\ R;\ d = 1,\ \cdots,\ D \tag{4.50}$$

$$\sum_{i=1}^{m}\sum_{d=1}^{D}\sum_{l=1}^{L_{irt}} y_{iljrd(s_1 s_2 \cdots s_t)} + \sum_{d=1}^{D} y^{+}_{jrd(s_1 s_2 \cdots s_t)} = q_{jr(s_1 s_2 \cdots s_t)} + \sum_{d=1}^{D} y^{+}_{jrd(s_1 s_2 \cdots s_{t-1})} \qquad j = 1,\ \cdots,\ n;\ r = 1,\ \cdots,\ R;\ t = 1,\cdots,T;\ s_1,\cdots,s_t = 1,\cdots,S \tag{4.51}$$

$$y^{+}_{jrd(s_1 s_2 \cdots s_t)} \le \sum_{i=1}^{m}\sum_{l=1}^{L_{irt}} y_{iljrd(s_1 s_2 \cdots s_{t+1})} \qquad j = 1,\ \cdots,\ n; r = 1,\ \cdots,\ R;\ d = 1,\ \cdots,\ D;\ t = 1,\cdots,T;\ s_1,\cdots,s_t = 1,\cdots,S \tag{4.52}$$

$$y^{h+}_{jds_0} = 0,\ \ y^{h+}_{jd(s_1 s_2 \cdots s_T)} = 0 \qquad j = 1,\ \cdots,\ n;\ d = 1,\ \cdots,\ D \tag{4.53}$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L^{h}_{it}} y^{h}_{iljd(s_1 s_2 \cdots s_t)} + \sum_{i=1}^{m}\sum_{l=1}^{L_{it}} y^{hc}_{iljd(s_1 s_2 \cdots s_t)} + y^{h+}_{jd(s_1 s_2 \cdots s_t)} = q^{h}_{jd(s_1 s_2 \cdots s_t)} + y^{h+}_{jd(s_1 s_2 \cdots s_{t-1})} - \sum_{r=1}^{R} y^{+}_{jrd(s_1 s_2 \cdots s_t)} \qquad j = 1,\ \cdots,\ n;\ d = 1,\ \cdots,\ D;\ t = 1,\cdots,T; s_1,\cdots,s_t = 1,\cdots,S \tag{4.54}$$

$$y^{h+}_{jd(s_1 s_2 \cdots s_t)} \le \sum_{i=1}^{m}\sum_{l=1}^{L^{h}_{it}} y^{h}_{ilj(s_1 s_2 \cdots s_{t+1})} + \sum_{i=1}^{m}\sum_{l=1}^{L_{it}} y^{hc}_{iljd(s_1 s_2 \cdots s_{t+1})} \qquad j = 1,\ \cdots,\ n;\ d = 1,\ \cdots,\ D;\ t = 1,\cdots,T-1;\ s_1,\cdots,s_t = 1,\cdots,S \tag{4.55}$$

$$o_{irt} = \sum_{l=1}^{L_{irt}} x_{ilr(s_1 s_2 \cdots s_t)} + o^{+}_{ir(s_1 s_2 \cdots s_t)} - o^{-}_{ir(s_1 s_2 \cdots s_t)} \qquad i = 1,\cdots,m;\ r = 1,\cdots,R, t = 1,\cdots,T;\ s_1,\cdots,s_t = 1,\cdots,S \tag{4.56}$$

$$o_{it}^{h} = \sum_{l=1}^{L_{it}^{h}} \sum_{d=1}^{D} x_{ild(s_1 s_2 \cdots s_t)}^{h} + o_{i(s_1 s_2 \cdots s_t)}^{h+} - o_{i(s_1 s_2 \cdots s_t)}^{h-} \quad i = 1, \cdots, m;\ t = 1, \cdots, T;\ s_1, \cdots, s_t = 1, \cdots, S \tag{4.57}$$

$$\sum_{r=1}^{R} \sum_{l=1}^{L_{irt}} x_{ild(s_1 s_2 \cdots s_t)}^{hc} \leq \sum_{r=1}^{R} \sum_{l=1}^{L_{irt}} x_{ilr(s_1 s_2 \cdots s_t)} \quad i = 1, \cdots, m;\ t = 1, \cdots, T;\ s_1, \cdots, s_t = 1, \cdots, S \tag{4.58}$$

$$\sum_{l=1}^{L_{it}^{h}} \sum_{d=1}^{D} x_{ild(s_1 s_2 \cdots s_t)}^{h} \leq L_{it}^{h} \quad i = 1, \cdots, m;\ t = 1, \cdots, T;\ s_1, \cdots, s_t = 1, \cdots, S \tag{4.59}$$

$$\sum_{r=1}^{R} \sum_{l=1}^{L_{irt}} \sum_{d=1}^{D} x_{ild(s_1 s_2 \cdots s_t)}^{hc} = o_{it}^{hc} \quad i = 1, \cdots, m;\ t = 1, \cdots, T;\ s_1, \cdots, s_t = 1, \cdots, S \tag{4.60}$$

$$\sum_{k=1}^{K_i} g_{ilkr(s_1 s_2 \cdots s_t)} = \sum_{j=1}^{n} \sum_{d=1}^{D} w_j y_{iljrd(s_1 s_2 \cdots s_t)} \quad i = 1, \cdots, m;\ l = 1, \cdots, L_{irt};\ t = 1, \cdots, T;\ s_1, \cdots, s_t = 1, \cdots, S;\ r = 1, \cdots, R \tag{4.61}$$

$$g_{ilkr(s_1 s_2 \cdots s_t)} \leq z_{ilkr(s_1 s_2 \cdots s_t)} \left(a_{ikr} - a_{i(k-1)r} \right) \quad i = 1, \cdots, m;\ l = 1, \cdots, L_{irt};\ k = 1, \cdots, K_i;\ t = 1, \cdots, T;\ r = 1, \cdots, R;\ s_1, \cdots, s_t = 1, \cdots, S \tag{4.62}$$

$$g_{ilkr(s_1 s_2 \cdots s_t)} \geq z_{il(k-1)r(s_1 s_2 \cdots s_t)} \left(a_{ikr} - a_{i(k-1)r} \right) \quad i = 1, \cdots, m;\ l = 1, \cdots, L_{irt};\ k = 1, \cdots, K_i;\ t = 1, \cdots, T;\ r = 1, \cdots, R;\ s_1, \cdots, s_t = 1, \cdots, S \tag{4.63}$$

$$\sum_{k=1}^{K_i} g_{ilkd(s_1 s_2 \cdots s_t)}^{h} = \sum_{j=1}^{n} y_{iljd(s_1 s_2 \cdots s_t)}^{h} \quad i = 1, \cdots, m;\ l = 1, \cdots, L_{it}^{h};\ t = 1, \cdots, T;\ d = 1, \cdots, D;\ s_1, \cdots, s_t = 1, \cdots, S \tag{4.64}$$

$$g_{ilkd(s_1 s_2 \cdots s_t)}^{h} \leq z_{ilkd(s_1 s_2 \cdots s_t)}^{h} \left(a_{ik}^{h} - a_{i(k-1)}^{h} \right) \quad i = 1, \cdots, m;\ l = 1, \cdots, L_{it}^{h};\ t = 1, \cdots, T;\ k = 1, \cdots, K_i;\ s_1, \cdots, s_t = 1, \cdots, S \tag{4.65}$$

$$g_{ilkd(s_1 s_2 \cdots s_t)}^{h} \geq z_{il(k-1)d(s_1 s_2 \cdots s_t)}^{h} \left(a_{ik}^{h} - a_{i(k-1)}^{h} \right) \quad i = 1, \cdots, m;\ l = 1, \cdots, L_{it}^{h};\ t = 1, \cdots, T;\ k = 1, \cdots, K_i;\ s_1, \cdots, s_t = 1, \cdots, S \tag{4.66}$$

$$\sum_{k=1}^{K_i} g^{hc}_{ilkd(s_1s_2\cdots s_t)} = \sum_{j=1}^{n} y^{hc}_{iljd(s_1s_2\cdots s_t)} \quad i = 1,\cdots,m;\ l = 1,\cdots,L^h_{it};\ t = 1,\cdots,T;\ d = 1,\cdots,D;\ s_1,\cdots,s_t = 1,\cdots,S \tag{4.67}$$

$$g^{hc}_{ilkd(s_1s_2\cdots s_t)} \le z^{hc}_{ilkd(s_1s_2\cdots s_t)}\left(a^h_{ik} - a^h_{i(k-1)}\right) \quad i = 1,\cdots,m;\ l = 1,\cdots,L_{irt};\ t = 1,\cdots,T;\ k = 1,\cdots,K_i;\ r = 1,\cdots,R;\ s_1,\cdots,s_t = 1,\cdots,S \tag{4.68}$$

$$g^{hc}_{ilkd(s_1s_2\cdots s_t)} \ge z^{hc}_{il(k-1)d(s_1s_2\cdots s_t)}\left(a^h_{ik} - a^h_{i(k-1)}\right) \quad i = 1,\cdots,m;\ l = 1,\cdots,L_{irt};\ t = 1,\cdots,T;\ k = 1,\cdots,K_i;\ r = 1,\cdots,R;\ s_1,\cdots,s_t = 1,\cdots,S \tag{4.69}$$

$$o_{irt}, o^-_{ir(s_1s_2\cdots s_t)}, o^+_{ir(s_1s_2\cdots s_t)}, o^h_{it}, o^{h-}_{i(s_1s_2\cdots s_t)}, o^{h+}_{i(s_1s_2\cdots s_t)}, o^{hc}_{it}, y_{iljrd(s_1s_2\cdots s_t)}, y^+_{jrd(s_1s_2\cdots s_t)}, y^h_{iljd(s_1s_2\cdots s_t)},$$
$$y^{hc}_{iljd(s_1s_2\cdots s_t)}, y^{h+}_{jd(s_1s_2\cdots s_t)}, g_{ilkr(s_1s_2\cdots s_t)}, g^h_{ilkd(s_1s_2\cdots s_t)}, g^{hc}_{ilkd(s_1s_2\cdots s_t)} \in \{0,1,\cdots,\inf\};\ x_{ilr(s_1s_2\cdots s_t)},$$
$$x^h_{ild(s_1s_2\cdots s_t)}, x^{hc}_{ild(s_1s_2\cdots s_t)}, z_{ilkr(s_1s_2\cdots s_t)}, z^h_{ilkd(s_1s_2\cdots s_t)}, z^{hc}_{ilkd(s_1s_2\cdots s_t)} \in \{0,1\} \quad i = 1,\cdots,m,$$
$$l = 1,\cdots,L_{irt};\quad j = 1,\cdots,n;\quad r = 1,\cdots,R;\ d = 1,\cdots,D, t = 1,\cdots,T;\ k = 1,\cdots,K_i;\ s_1,\cdots,s_t = 1,\cdots,S \tag{4.70}$$

The objective function (4.41) is the total cost including container fixed and variable cost in the regions, penalty cost for renting or returning containers on the shipping day in the regions, inventory costs in regions, repacking cost in the hub, fixed and variable cost for the pre-used containers and new containers in the hub, inventory costs in the hub and uncertainty cost for renting or returning containers to the hub. The details of the objective function can be found in the constraints (4.42) and (4.43). Constraints (4.44)-(4.49) ensure that the cargoes loaded into the containers cannot exceed the volume and weight limitations. The cargo quantity constraints in the regions and hub are (4.50)-(4.55). (4.50) and (4.53) mean the quantities of storage cargo before the first period and in the final period must be 0. (4.51) and (4.54) ensure that the quantity of cargo should be same. (4.52) and (4.55) mean the quantity of storage cargo in this period is less than or equal to the quantity of transported cargo in next. These two constraints make sure the assumption that the air cargoes can be transported with one day's delay. If the assumption should be changed to two days or more, we just need to change these two constraints. The container quantity constraints in regions and hub are (4.56) and (4.57). Constraint (4.58) means that for each type of container, the quantity of using pre-used containers in the hub cannot be greater than the sum of containers used in all regions. Constraint (4.59) ensures that the number of each type of container used in the hub cannot exceed the limit. Constraint (4.60) makes sure that the pre-used container using plan should be the same no matter which scenario is realised. Constraints (4.61)-(4.69) are container variable cost constraints according to (4.2)-(4.4). Constraints (4.70) is the variable range.

4.3.1.3 Computational results

Here we take a two days' flight case with two regions and two destinations via a hub as an example to see how multi-stage model works. We use the same initial data from the two-stage stochastic model case in Section 4.2.2.3. The probabilities for scenarios are the same as in the production planning problem in Section 3.2.4. The quantities of air cargo under different scenarios in the first day is the same as in Section 4.2.2.3. The quantities of air cargo under different outcomes in the second day is in Table 4.19. The one day storage costs for large, medium and small air cargo in regions and hub are same, \$4000, \$3600 and \$3200 respectively. Table 4.20 lists the container booking plan for the three-stage stochastic model for Test III. We can see that fewer containers are booked in the first period than the second period. The reason is that a lot of air cargoes are stored for the second period due to cheap inventory costs. Table 4.21 and Table 4.22 give the urgent rental or return plan on the day of shipping in the first and second periods.

Table 4.19 Cargo quantities under different outcomes in the second period

Outcomes		s_1				s_2				s_3			
Region		A		B		A		B		A		B	
Destination		α	β	α	β	α	β	α	β	α	β	α	β
Cargo type	Large	3	2	3	3	2	1	3	3	2	1	2	2
	Medium	3	2	2	3	3	2	1	2	2	1	1	1
	Small	3	3	3	2	2	2	2	1	2	2	2	1

Table 4.20 Container booking plan for three-stage model (made one week before day of shipping)

Container type	First period				Second period			
	Region		Pre-used container	Hub	Region		Pre-used container	Hub
	A	B			A	B		
1					1			
2					1	1	2	
3						1	1	
4					1		1	
5	1	1	2			1	1	
6	1	1	2	1	1	1	2	1
7	1	1				1	1	

Table 4.21 Renting/Returning containers on the day of shipping (for first period)

Container type		Renting			Returning		
		Region		Hub	Region		Hub
		A	B		A	B	
Scenario s1	4	1	1	1			
	5			1			
Scenario s2	4	1					
	5						
Scenario s3	4						
	5						

Table 4.22 Renting/Returning containers on the day of shipping (for second period)

Container type			Rent			Return		
			Region		Hub	Region		Hub
			A	B		A	B	
Scenario	(s1,ss1)	1		1				
		2			1			
		3	1		1			
		4			1			
	(s1,ss2)	4		1	1			
		7	1					
	(s1,ss3)							
	(s2,ss1)	2			1			
		3	1					
		4		1	1			
		5			1			
	(s2,ss2)	4		1	1			
		5			1			
	(s2,ss3)							
	(s3,ss1)	3	1		1			
		4		1	1			
	(s3,ss2)	5			1			
	(s3,ss3)							

Table 4.23 shows the tests in different probabilities for the three-stage stochastic model calculated by computer software AIMMS. We can find that AIMMS cannot provide the optimal solutions, there are clear distances between the solution and lower bound. The lower bounds are provided by AIMMS using Branch and Bound Algorithm. And the computing time takes too long especially in Test III, more than three days.

Table 4.23 AIMMS results for three-stage stochastic model

Test	I	II	III
Total cost by AIMMS	2836720	2266843	2022581
Lower bound	2606252	2073880	1805892
Distance between AIMMS and Lower bound	8.12%	8.51%	10.71%
Computing time by AIMMS (seconds)	70860.69	46873.65	140961.85

4.3.2 Three types of robust models

Similarly with Section 4.2.3, we also provide three types of robust models with model robustness, solution robustness, and the trade-off between model robustness and solution robustness according to the multi-stage stochastic model in Section 4.3.1.2.

4.3.2.1 New notation

New deterministic parameters

ω_{jrt} unit penalty for type j cargo which cannot be transported any more in region r in period t;

ω_{jt}^{h} unit penalty for type j cargo which cannot be transported any more in the hub in period t;

λ_{rt} measurement of the variability of the objective costs in region r in period t;

λ_{t}^{h} measurement of the variability of the objective costs in the hub in period t;

New decision variables

$e_{jrd(s_1 s_2 \cdots s_t)}$ the quantity of type j cargo which cannot be transported in this week when scenario $(s_1 s_2 \cdots s_t)$ is realised in region r;

$e_{jd(s_1 s_2 \cdots s_t)}^{h}$ the quantity of type j cargo which cannot be transported in this week when scenario $(s_1 s_2 \cdots s_t)$ is realised in the hub;

$\theta_{r(s_1 s_2 \cdots s_t)}$ deviational variables for the robust model with solution robustness when scenario $(s_1 s_2 \cdots s_t)$ is realised in region r.

$\theta_{(s_1 s_2 \cdots s_t)}^{h}$ deviational variables for the robust model with solution robustness when scenario $(s_1 s_2 \cdots s_t)$ is realised in the hub.

4.3.2.2 Multi-stage robust model with model robustness

Base on Section 4.2.3.1, the multi-stage robust optimisation model with model robustness for air cargo forwarding problem will be built as:

$$\min \sum_{t=1}^{T}\left(\sum_{s_1}^{S}\sum_{s_2}^{S}\cdots\sum_{s_t}^{S} p_{(s_1 s_2\cdots s_t)}\left(\sum_{r=1}^{R}(M_{rt}+\sum_{d=1}^{D}\sum_{j=1}^{n}\omega_{jrt}e_{jrd(s_1 s_2\cdots s_t)})+N_t + \sum_{d=1}^{D}\sum_{j=1}^{n}\omega_{jt}^{h}e_{jd(s_1 s_2\cdots s_t)}^{h}\right)\right) \quad (4.71)$$

Subject to (4.42)-(4.50), (4.52), (4.53) and (4.55)-(4.70)

$$\sum_{i=1}^{m}\sum_{d=1}^{D}\sum_{l=1}^{L_{irt}} y_{iljrd(s_1 s_2\cdots s_t)} + \sum_{d=1}^{D} y_{jrd(s_1 s_2\cdots s_t)}^{+} + \sum_{d=1}^{D} e_{jrd(s_1 s_2\cdots s_t)} = q_{jr(s_1 s_2\cdots s_t)} + \sum_{d=1}^{D} y_{jrd(s_1 s_2\cdots s_{t-1})}^{+}, \quad j=1,\ \cdots,\ n;\ r=1,\ \cdots,\ R; s_1,\cdots,s_t=1,\cdots,S \quad (4.72)$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_{it}^{h}} y_{iljd(s_1 s_2\cdots s_t)}^{h} + \sum_{r=1}^{R}\sum_{i=1}^{m}\sum_{l=1}^{L_{irt}} y_{iljd(s_1 s_2\cdots s_t)}^{hc} + y_{jd(s_1 s_2\cdots s_t)}^{h+} + e_{jd(s_1 s_2\cdots s_t)}^{h} = q_{jd(s_1 s_2\cdots s_t)}^{h} + y_{jd(s_1 s_2\cdots s_{t-1})}^{h+} - \sum_{r=1}^{R} y_{jrd(s_1 s_2\cdots s_t)}^{+} \quad j=1,\ \cdots,\ n;\ d=1,\ \cdots,\ D;\ s_1,\cdots,s_t=1,\cdots,S \quad (4.73)$$

$$e_{jrd(s_1 s_2\cdots s_t)}, e_{jd(s_1 s_2\cdots s_t)}^{h} \in \{0,1,2,\cdots,\inf\} \quad j=1,\ \cdots,\ n;\ \ d=1,\ \cdots,\ D;\ \ r=1,\cdots,R;\ \ s_1,\cdots,s_t = 1,\cdots,S \quad (4.74)$$

4.3.2.3 Multi-stage robust model with solution robustness

Similarly with Section 4.2.3.2, we can list the multi-stage robust optimisation model with solution robustness for air cargo forwarding problem:

$$\min \sum_{t=1}^{T}(\sum_{s_1}^{S}\sum_{s_2}^{S}\cdots\sum_{s_t}^{S}p_{(s_1s_2\cdots s_t)}(\sum_{r=1}^{R}M_{rt}+N_t))$$
$$+\sum_{t=1}^{T}\sum_{r=1}^{R}\lambda_{rt}(\sum_{s_1}^{S}\sum_{s_2}^{S}\cdots\sum_{s_t}^{S}p_{(s_1s_2\cdots s_t)}(M_{rt}-\sum_{s_1}^{S}\sum_{s_2}^{S}\cdots\sum_{s_t}^{S}p_{(s_1s_2\cdots s_t)}M_{rt}$$
$$+2\theta_{r(s_1s_2\cdots s_t)}))$$
$$+\sum_{t=1}^{T}\lambda_t^h(\sum_{s_1}^{S}\sum_{s_2}^{S}\cdots\sum_{s_t}^{S}p_{(s_1s_2\cdots s_t)}(N_t-\sum_{s_1}^{S}\sum_{s_2}^{S}\cdots\sum_{s_t}^{S}p_{(s_1s_2\cdots s_t)}N_t$$
$$+2\theta_{(s_1s_2\cdots s_t)}^h) \tag{4.75}$$

Subject to (4.42)-(4.70),

$$-\theta_{r(s_1s_2\cdots s_t)}-M_{rt}+\sum_{s_1}^{S}\sum_{s_2}^{S}\cdots\sum_{s_t}^{S}p_{(s_1s_2\cdots s_t)}M_{rt}\le 0, r=1,\ \cdots,\ R; s_1,\cdots,s_t=1,\cdots,S \tag{4.76}$$

$$-\theta_{(s_1s_2\cdots s_t)}^h-N_t+\sum_{s_1}^{S}\sum_{s_2}^{S}\cdots\sum_{s_t}^{S}p_{(s_1s_2\cdots s_t)}N_t\le 0 \qquad s_1,\cdots,s_t=1,\cdots,S \tag{4.77}$$

$$\theta_{r(s_1s_2\cdots s_t)},\theta_{(s_1s_2\cdots s_t)}^h\ge 0,\ \ r=1,\cdots,R;\quad s_1,\cdots,s_t=1,\cdots,S \tag{4.78}$$

4.3.2.4 Multi-stage robust model with the trade-off between model robustness and solution robustness

According to Section 4.2.3.3, we consider the variability and infeasibility together. A robust optimisation model with the trade-off between model robustness and solution robustness is developed to solve the air cargo forwarding problem with uncertainty. The objective function is combined with combined (4.71) and (4.75).

$$\min \sum_{t=1}^{T} (\sum_{s_1}^{S}\sum_{s_2}^{S}\cdots\sum_{s_t}^{S} p_{(s_1 s_2 \cdots s_t)} (\sum_{r=1}^{R} (M_{rt} + \sum_{d=1}^{D}\sum_{j=1}^{n} \omega_{jrt} e_{jrd(s_1 s_2 \cdots s_t)}) + N_t$$
$$+ \sum_{d=1}^{D}\sum_{j=1}^{n} \omega_{jt}^{h} e_{jd(s_1 s_2 \cdots s_t)}^{h}))$$
$$+ \sum_{t=1}^{T}\sum_{r=1}^{R} \lambda_{rt} (\sum_{s_1}^{S}\sum_{s_2}^{S}\cdots\sum_{s_t}^{S} p_{(s_1 s_2 \cdots s_t)} (M_{rt} - \sum_{s_1}^{S}\sum_{s_2}^{S}\cdots\sum_{s_t}^{S} p_{(s_1 s_2 \cdots s_t)} M_{rt}$$
$$+ 2\theta_{r(s_1 s_2 \cdots s_t)}))$$
$$+ \sum_{t=1}^{T} \lambda_t^{h} (\sum_{s_1}^{S}\sum_{s_2}^{S}\cdots\sum_{s_t}^{S} p_{(s_1 s_2 \cdots s_t)} (N_t - \sum_{s_1}^{S}\sum_{s_2}^{S}\cdots\sum_{s_t}^{S} p_{(s_1 s_2 \cdots s_t)} N_t$$
$$+ 2\theta_{(s_1 s_2 \cdots s_t)}^{h}) \quad (4.79)$$

subject to (4.42)-(4.50), (4.52), (4.53), (4.55)-(4.70), (4.72)-(4.74) and (4.76)- (4.78).

4.3.2.5 Computational results

Here we use the same initial data from the two-stage stochastic model case in Section 4.3.1.3. We will take the three-stage robust model with the trade-off between model robustness and solution robustness ($\lambda = 0.1$ and ω =30000, 26000, 22000) as an example for the bad economy environment to see the results. Table 4.24 and Table 4.25 present the results of three-stage robust optimisation models for Test III. There is no urgent rental or return plan in the first period. The reason is the cargoes left in the first period are either stored for the second period or do not transported this week. The computing time is also too long, more than 20 hours (see Table 4.26). Therefore, a GA for quickly finding a better solution is introduced in the next section.

Table 4.24 Container booking plan for three-stage model (made one week before shipping day)

Container type	First period				Second period			
	Region		Pre-used container	Hub	Region		Pre-used container	Hub
	A	B			A	B		
1								
2		1						1
3					1			
4					1	1	2	
5	1		1	1	1	1	2	
6	1	1	2	1	1	1	2	1

7						1	1	

Table 4.25 Renting/Returning containers on the day of shipping (for second period)

Container type			Rent			Return		
			Region		Hub	Region		Hub
			A	B		A	B	
Scenario	$(s_1 s_1)$	1		1				
		2	1					
		3		1	1			
		4			1			
		5			1			
		7			1			
	$(s_1 s_2)$	3		1	1			
		4			1			
	$(s_1 s_3)$							
	$(s_2 s_1)$	1	1	1				
		2		1				
		3			1			
		4			1			
	$(s_2 s_2)$	2		1				
		5			1			
	$(s_2 s_3)$							
	$(s_3 s_1)$	1		1				
		2	1	1				
		3			1			
		4			1			
		7	1					
	$(s_3 s_2)$	2	1	1				
		4			1			
		5			1			
	$(s_3 s_3)$							

Table 4.26 AIMMS results for three-stage robust optimisation model

Test	I	II	III
Total cost by AIMMS	2727075	2286350	1950941
Lower bound	2572298	2032928	1720935
Distance between AIMMS and Lower bound	5.68%	11.08%	11.79%
Computing time by AIMMS (seconds)	52308.69	84363.07	72315.36

4.4 Genetic Algorithm

The reasons we choose GA to solve the air cargo forwarding problems are the following. Firstly, GA is a well-known heuristic approach, inspired by the natural evolution of the living organisms that works on a population of the solutions simultaneously. Secondly, GAs have been used extensively in solving scheduling and loading problems. The air container booking problem in this research also includes the air cargo loading problem. If we know which cargo will be loaded into which container, we will obtain the container booking plan automatically. Therefore GAs are suitable for solving this problem. Thirdly, GAs can solve large-sized problems, and thus are suited to this research. Fourthly, use of GA can increase the chance of finding better solutions because the population-based GA needs a large solution space to be explored.

4.4.1 Design of GA

Chromosome representation and initialisation

The initial step of the GA is to design the initial chromosome, which is the most important part. Because we transform the air container booking problem into the air cargo loading problem, the air cargo loading plan in regions and the hub will be the chromosomes.

By considering the cargo loading variables, we use a matrix structure to represent the solution of the proposed problem. Here we use a very simple case to show how to get the initial solution, one day's cargo loading plan with two regions, two destinations and two scenarios. Cargo loading variables in the hub are quite different from those with-in regions because there are more options, such as pre-used containers, in the hub. Table 4.27 gives the air cargo loading solutions in the two regions. In the column named "cargo type", "1", "2" and "3" mean large, medium and small cargo respectively. Column "container type" means the corresponding cargo should be loaded into which type of container. Column "container number" means which number of the corresponding type of container is used. For the same type of container, the container number should start from 1. When the container is fully occupied by cargoes, if the next cargo still needs to be loaded into this type of container, the container number will become 2. The container number cannot exceed the available quantity for this type of container. Therefore the data in column "container number" can be provided by the data in column "container type". In this simple case, the available quantity is one for each type of container. The data in columns "scenario" and "cargo type" come from the initial data. Hence, the column "container type" will be the chromosome for regions (Table 4.28).

Table 4.27 The air cargo loading solutions in regions

Cargo loading plan in Region 1			
Scenario	Cargo type	Container number	Container type
1	1	1	2
1	1	1	2
1	2	1	4
1	2	1	7
1	3	1	5
1	3	1	6
2	1	1	3
2	1	1	5
2	2	1	3
2	3	1	4
2	3	1	6
2	3	1	5

Cargo loading plan in Region 2			
Scenario	Cargo type	Container number	Container type
1	1	1	6
1	1	1	1
1	2	1	4
1	2	1	1
1	2	1	3
1	3	1	7
2	1	1	2
2	2	1	4
2	2	1	3
2	3	1	5
2	3	1	3

Table 4.28 Chromosomes for regions

Chromosome in Region 1
2
2
4
7
5
6
3
5
3
4
6
5

Chromosome in Region 2
6
1
4
1
3
7
2
4
3
5
3

Table 4.29 lists the containers used with the plan by regions. We want to explain how to transfer the cargo loading problem back to a container booking problem by introducing the container use plan. Generally, and not only in this example, if the use plans of all scenarios equal 1 for the same container, we should book this container in advance to save money. If the use plans of all scenarios equal 0, we do not need to book it. If some are 1 and others are 0, we need a comparison to make the decision. If we book it in advance, the “0” cases, will cause urgent return costs; if we do not book it, for the “1” cases we need to pay the urgent rental costs. We have to

compare these two costs and choose the cheaper to decide whether to book this container. Thus the problem can become a container booking problem.

Table 4.29 Corresponding container use plans in regions

Container type	Container use in Region 1	
	Scenario 1	Scenario 2
1	0	0
2	1	0
3	0	1
4	1	1
5	1	1
6	1	1
7	1	0

Container type	Container use in Region 2	
	Scenario 1	Scenario 2
1	1	0
2	0	1
3	1	1
4	1	1
5	0	1
6	1	0
7	1	0

Similarly, Table 4.30 provides the cargo loading plan in the hub. There is a little difference in "Container number": "1" represents a pre-used container from Region 1; "2" represents a pre-used container from Region 2; "3" represents a container from hub. If we do not get the booking plan in the regions, we will not know how many containers are available in hub. Therefore, the region plans should be considered first. The data in "container type" is the chromosome for the hub (see Table 4.31). Table 4.32 gives the corresponding container using plan for the hub.

Table 4.30 The cargo loading plan in hub

Scenario	1	1	1	1	1	1	2	2	2	2	2	2
Cargo type	1	1	1	2	3	3	1	2	3	3	3	3
Container number	2	2	3	2	2	1	1	3	2	1	2	1
Container destination	1	1	1	1	1	1	1	1	1	1	1	1
Container type	2	3	1	2	5	6	3	1	2	4	2	3

Scenario	1	1	1	1	1	1	2	2	2	2	2	
Cargo type	1	2	2	2	2	3	1	1	2	2	3	
Container number	1	1	1	1	3	1	2	1	3	1	1	
Container destination	2	2	2	2	2	2	2	2	2	2	2	
Container type	3	4	5	4	7	5	3	5	2	6	5	

Table 4.31 The chromosome for hub

2	3	1	2	5	6	3	1	2	4	2	3	3	4	5	4	7	5	3	5	2	6	5

Table 4.32 Corresponding container using plan in hub

Container type	Container number	Scenario 1		Scenario 2	
		Container destination	Container using	Container destination	Container using
1	3	1	1	1	1
2	2	1	1	1	1
	3	0	0	2	1
3	1	2	1	1	1
	2	1	1	2	1
	3	0	0	0	0
4	1	2	1	1	1
	2	0	0	0	0
	3	0	0	0	0
5	1	2	1	2	1
	2	1	1	0	0
	3	0	0	0	0
6	1	1	1	2	1
	3	0	0	0	0
7	3	2	1	0	0

Once we have the cargo loading plans, we can calculate the values of the container variable according to the container quantity constraints (4.9) and (4.12)-(4.15). Container variable cost constraints (4.16)-(4.24) do not need to be considered because they are only used to calculate the total cost. We just need to make sure that all the cargoes have been loaded in the containers and that the total loaded volume and weight for each container do not exceed the limitations. The initial population is constructed by the following steps:

(1) Sort the cargoes in Region 1 firstly by scenario and secondly by cargo type (decreasing volume of cargo, which means large cargoes are at the front). Constraint (4.10) is thus satisfied.

(2) Start from the first cargo, randomly choose a type of container from the container set (1 to 7). If the total loaded volume and weight for this container do not exceed its limitations, load this cargo to this container. If the limitations are exceeded, delete this type of container from the container set and randomly choose another element. Continue doing this in a loop until a suitable container is found. Record the type of container in the last column of the cargo loading matrix. Constraints (4.25) and (4.26) are thus satisfied.

(3) Continue to do step (2) until all the cargoes in Region 1 have suitable containers.

(4) Calculate the container booking plan and the urgent rental/return plan for Region 1.

(5) Schedule the cargoes for all regions according to steps (1)-(4).

(6) Divide the cargoes in the hub into different groups regarding the destinations and sorting them by scenario first and cargo type second (decreasing volume of cargo), as in Table 4.30. Constraint (4.11) is thus satisfied.

(7) Starting from the first cargo with Destination 1, randomly choose a type of container from the container set (1 to 7). If this type of container is booked in Region 1 and has not been chosen before, load this cargo into this container; if this type of container is booked in Region 1 and has been chosen for the same destination, if the total loaded volume and weight for this container do not exceed its limitations, load this cargo into this container; otherwise, check for other regions. If a suitable container still has not been found, consider the same type of container in the hub. If this fails, delete this type of container from the container set and randomly choose another element to do the same loop until a suitable one is found. Record the container number, destination and container type. Constraints (4.27)-(4.30) are thus satisfied.

(8) Schedule the rest of the cargoes in the hub according to step (7). The sequence is ordering the first cargo in different groups, then the second cargo in different groups and so on.

(9) Calculate the container and pre-used container booking plan, the urgent rental/return plan in the hub.

(10) Calculate the total cost.

<u>*Genetic operators design*</u>

In order to efficiently explore the solution space, the crossover and mutation operations are needed. We use a five-point crossover for all chromosomes in regions and hub. We will do the crossover for regions one by one first, then for the hub. The crossover should follow the rule that, after crossover, the total loaded volume and weight for a changed container should not exceed its limitations. Otherwise, we will not do a crossover for this cargo. Table 4.33 gives an example of two parents which are selected to do a five-point crossover. The blue rows should be kept; the red rows are for a crossover. After the third crossover, we find that the third and fourth cargo in Parent 1 are both loaded into container 7: this will exceed the volume limitation of container 7. Therefore we do not do the third crossover. Other crossovers have no problems. Table 4.34 lists the two children after crossover. Hence, the five-point crossover guarantees that the generated children will remain feasible if parents are feasible.

Table 4.33 An illustration of two parents which are selected to do a five-point crossover

Parent 1 **Parent 2**

2	6
2	1
4	7
7	1
5	3
6	4
1	2
5	4
3	3
4	5
6	3
5	5

Table 4.34 The two children after crossover

Child 1	Child 2
2	6
1	2
4	7
7	1
5	3
4	6
2	1
4	5
3	3
4	5
3	6
5	5

It should be noticed that after crossover in regions, the container booking plan may change. For example, after crossover, one container may be cancelled, but it is still used in the pre-used container plan in the hub. Therefore, there should be a correction for the cargo loading plan in the hub. The steps for this correction are as follow: find the cargoes loaded into that container first and move each item one by one into the same type of pre-used container from other regions. Otherwise, try to move them one-by-one into the same container type in the hub. If some of them are still left, randomly find other container types for them just like the steps in choosing the initial population. We will record all the total costs, including doing crossover, for one region or the hub and doing crossover for all cargo loading variables and then choose the first two cheaper choices one as the output crossover solutions. The mutation operator has a similar process to crossover.

Each operator is performed with a certain probability that is known in advance by the GA parameter settings. The crossover rate and mutation rate determine the performance of GA; therefore, proper value settings are needed in order to ensure the convergence of GA to the global optimal neighbourhood in a reasonable time. The population size is kept unchanged during the crossover and mutation operations.

Offspring acceptance strategy

We use a semi-greedy strategy to accept the offspring created by the GA operators. In this strategy, an offspring is accepted as the new generation only if its total cost is less than the average of its parents. This could make sure the best function value in any generation is no worse than that of previous generations. This approach enables GA to reduce the computation time and results in a fast convergence toward an optimal solution.

Parents selection strategy

The fitness of each solution is obtained by calculating its objective function value, the total cost. We use the “roulette wheel” method to select parents. It is preferable that the individuals with smaller total costs are chosen as parents for the next generation.

Stopping criterion

In order to balance the searching computation time, as well as evolving an approximate optimal solution, we use two criteria as stopping rules: (1) the maximum number of evolving generations allowed for GA, and (2) the standard deviation of the fitness values of chromosomes in the current generation is below a small value.

4.4.2 Computational results by GA

We use the same initial values in the two-stage stochastic case in Section 4.2.2.3 to test the GA results. The corresponding parameters we choose are: maximum generation 50, population size 200, crossover rate 0.7 and mutation rate 0.1. Table 4.35 lists the tests for GA and the comparisons for the two-stage stochastic model. We run the GA programme 30 times and choose the average of the results. From Table 4.35, we know that AIMMS can get the optimal solution for the two-stage stochastic model. However, the computing time is quite different between different tests from 10 minutes to 10 hours. GA only takes no more than 10 minutes, but does not find the optimal solution. The distance between GA and AIMMS in Test I is smaller than others, occupying 6.82% of the optimal solution. That means in the good economy environment, GA could provide better solutions. We also do more tests by changing the corresponding parameters. If the population size decreases, the results become worse. And for other tests the average of the total costs will be similar, do not have significant change. Therefore we do not list the tests results here.

Table 4.35 Comparison between GA and AIMMS for Two-stage stochastic model

Test	I	II	III
Total cost by GA	1288726	1067032	955656
Total cost by AIMMS	1206444	979056	866217
Distance between GA and AIMMS	6.82%	8.99%	10.33%
Computing time for AIMMS to get the optimal result (seconds)	36621.38	3876.64	639.34

Figure 4.9 gives one example of the typical convergence process for GA. We can observe that our proposed GA reached convergence quickly after 25 generations and stopped at generation 43 due to the very low deviation of the population.

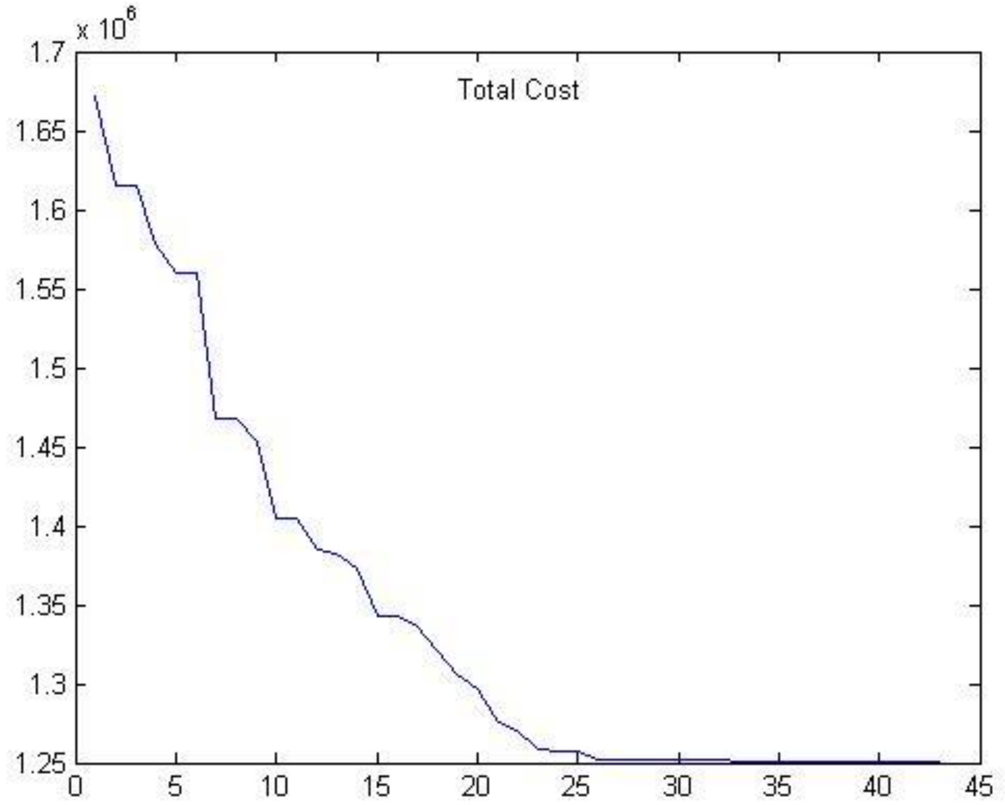

Figure 4.9 The typical convergence process of GA in Test I

GA for the multi-stage stochastic model could use a similar method of design by adding a new container, type 8. If the cargo chooses Container 8, this cargo should be stored for the next day's flight; the stored cargo should be considered first in the next day's loading plan and Container 8 cannot be chosen anymore. It could ensure the assumption that the air cargoes can be transported with one day's delay. Table 4.36 and Table 4.37 present the comparison between GA and AIMMS for the three-stage stochastic model and robust model with the trade-off between model robustness and solution robustness. We can see that for many tests, the GA can achieve better solutions than AIMMS but still has an obvious distance with the lower bound which is provided by AIMMS. Not few times AIMMS provides a little better solution than GA. However time is the biggest advantage for GA. All the tests by GA run less than 30 minutes. Figure 4.10 lists the tests for the three-stage stochastic model with different probabilities. The results of these tests are calculated by GA. We can find a similar conclusion in that the three-stage model is greatly dependent on the probabilities of scenarios.

Table 4.36 Comparison between GA and AIMMS for three-stage stochastic model

Test	I	II	III
Total cost by AIMMS	2836720	2266843	2022581
Total cost by GA	2773415	2208467	1978423
Lower bound	2606252	2073880	1805892
Distance between AIMMS and Lower bound	8.12%	8.51%	10.71%
Distance between GA and Lower bound	6.03%	6.09%	8.72%
Computing time by AIMMS (seconds)	70860.69	46873.65	140961.85

Table 4.37 Comparison between GA and AIMMS for three-stage robust optimisation model

Test	I	II	III
Total cost by AIMMS	2727075	2286350	1950941
Total cost by GA	2793648	2253259	1892472
Lower bound	2572298	2032928	1720935
Distance between AIMMS and Lower bound	5.68%	11.08%	11.79%
Distance between GA and Lower bound	7.92%	9.78%	9.06%
Computing time by AIMMS (seconds)	52308.69	84363.07	72315.36

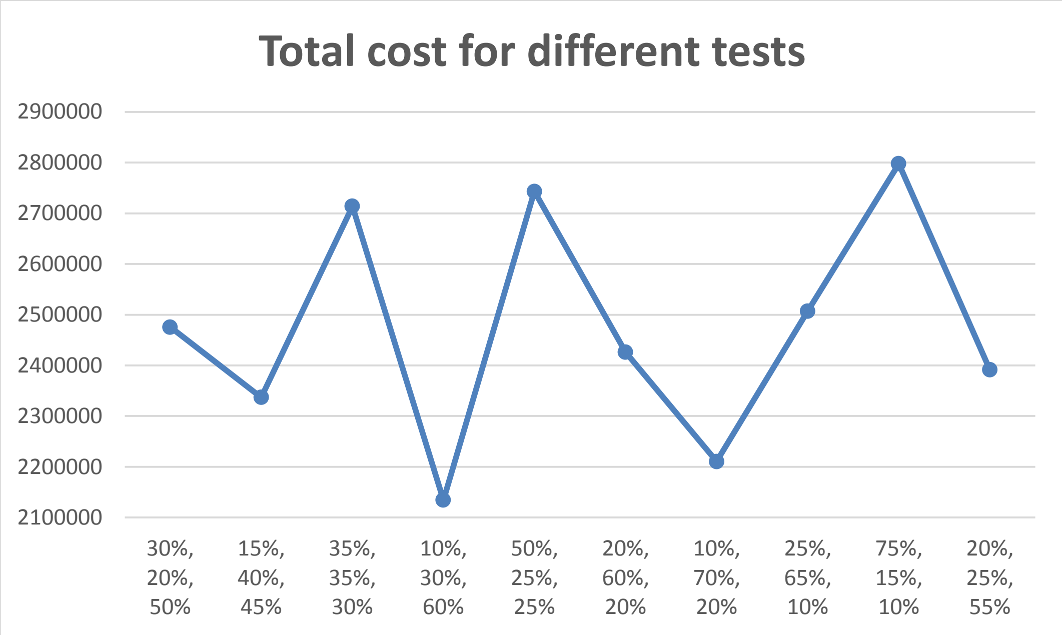

Figure 4.10 Total costs for different tests for three-stage stochastic model

4.5 Conclusion

In this chapter we build a new international air cargo forwarding problem under uncertainty, which means the cargoes need to be transported from regions to destinations via a hub. The air forwarders not only have to make a decision about the number of containers to be booked for the regions and hub in advance, before accurate customers' information becomes available, but also have to decide the number of extra containers to be required or the containers to be returned after the realisation of uncertainty.

For this air cargo forwarding problem, we develop stochastic models and three types of robust optimisation models for one period and multi-period cases. For the large scale problem, for which the computer software cannot give an optimal solution, we also present a new way to design a genetic algorithm to get better solutions.

Computational results show that the stochastic models can provide effective and cost-efficient solutions; the robust optimisation models can provide a more responsive and flexible system with less risk. Moreover, GA can provide better results than mathematical programming software for the large size problem.

Chapter 5: Conclusion

This thesis focuses on the global production planning and international air cargo forwarding problems under uncertainty. The final chapter summarizes the overall content of my thesis, highlights the specific contributions and introduces the limitations and further research.

5.1 Summary of research

Our thesis firstly helps supply chain managers to solve the multi-period production planning problems with demand uncertainty and quota limitation. Next, we introduce a new logistics problem: the air cargo forwarders should make decisions in regions and hub with uncertain cargo quantity to transport their cargoes from different regions to different destinations via a hub. We provide both stochastic and robust models to solve the one-day's flight case and multi-day's flight case.

At the beginning, we build up a multi-stage stochastic model for multi-period, multi-product and multi-plant production planning problems under uncertain demand and limited quota. The objective function is to minimise the total production cost. In this model we assume that the production plan should satisfy uncertainties exactly. Then we introduce the multi-stage robust model with model robustness to remove that assumption. Meanwhile, two more multi-stage robust models with solution robustness and the trade-off between model robustness and solution robustness are developed to test how the uncertainty impacts the total production cost.

In the logistics area, we present a new air cargo forwarding problem that means air cargoes should be transported from different regions to different destinations via a hub for consolidating. The forwarders should make the air containers booking plans for all regions and hub one week in advance. According to the general formulation of the multi-stage stochastic model, if there is one day's flight per week to do the air cargo forwarding work, a two-stage model will be enough; if there is more than one day's flights per week, a multi-stage model could be considered. Therefore, we provide all the corresponding models for this new problem, including two-stage and multi-stage stochastic models and three types of two-stage and multi-stage robust models. When the problem size increases, using computer software cannot get the optimal solution. And when the problem continues to increase, the software may not be able to run. Therefore, we produce a new way to design the GA in order to achieve a feasible and better solution in short computing time.

Many results and tests show that multi-stage model stochastic models can provide effective and cost-efficient solutions; the robust optimisation models can provide a more responsive and flexible system with less risk; and the GA we proposed can achieve better solutions than computer software for the large-scale problem.

5.2 Limitations of the thesis

It is noticed that there are some limitations in this thesis.

First, the multi-stage stochastic model for production planning problems only focuses on the uncertainties with demand and quota limitations. In the real world, a lot of things are uncertain. For example, the exchange rate changes at least every hour, which is an important factor for international companies; the cost of raw materials may change due to different scenarios being realised.

Second, the multi-stage stochastic models strongly depend on the forecasting probabilities of uncertainty. In this thesis, we assume the probabilities are known data. However, it is very difficult to obtain extremely accurate forecasting data for multi-period problems. Therefore, a perfect forecasting method would help the multi-stage stochastic model to develop much more quickly in global supply chains and logistics problems.

There are many assumptions to make our model simpler. For examples, the raw material costs are fixed for the production planning problem; the ratio between skilled and non-skilled working time is used only in whole periods to control the products quality; loading and repacking containers costs in the hub are fixed.

Finally, the multi-stage models for air cargo forwarding problems are quite complex. Although the GA method we introduced in this thesis could find a feasible and better solution than optimisation software, the solution is still the optimal one and has 10% distance with the lower bound.

5.3 Recommendations

To address the limitations introduced above, the following were identified as further research directions.

Firstly, the addition of some other factors, such as transportation cost and exchange rate cost, should make the multi-stage stochastic model for production planning problems more applicable to real-life problem application.

Secondly, cooperation with a forecasting team to get more accurate forecasting data will make the multi-stage models more practical.

Thirdly, reduction of some assumptions could make the modelling more related to reality. The raw material costs could be uncertain. The ratio between skilled and non-skilled working time can be controlled in each period for each kind of product by adding some constraints. The costs for loading and repacking containers could be considered in detail. For example, some containers may not need to be repacked if they are fully occupied by cargoes and all the cargoes are going to the same destination.

Finally, considering some other heuristic approaches such as Tabu Search, Simulated Annealing and so on might find better solutions or even optimal solutions.

Appendix A

Table A1. Raw material cost, labour cost, labour time and machine time per unit

Product	Plant	Raw material cost ($)	Labour cost of skilled workers ($)	Labour cost of non-skilled workers ($)	Labour time for skilled workers (hrs)	Labour time for non-skilled workers (hrs)	Machine time for skilled workers (hrs)	Machine time for non-skilled workers (hrs)
1	1	4	4.5	4	2	2.25	1.75	2.25
	2	4.2	4	3.5	2.25	2.5	2	2.5
	3	4.3	3.5	3	2.5	2.75	2.25	2.75
2	1	3	4	3.5	1.5	1.75	1.25	1.75
	2	3.2	3.5	3	1.75	2	1.5	2
	3	3.3	3	2.5	2	2.25	1.75	2.25
3	1	2	3	2.5	1	1.25	0.75	1.25
	2	2.2	2.5	2	1.25	1.5	1	1.5
	3	2.3	2	1.5	1.5	1.75	1.25	1.75

Table A2. Maximum machine, labour and overtime capacity and minimum work time

Plant	Period	Maximum machine regular capacity (hrs)	Maximum machine additional capacity (hrs)	Maximum capacity of skilled workers (hrs)	Maximum capacity of non-skilled workers (hrs)	Maximum overtime by skilled workers (hrs)	Maximum overtime by non-skilled workers (hrs)	Minimum labour work time (hrs)
1	1	5500	250	4800	2400	2400	1200	2400
	2	5500	250	4800	2400	2400	1200	2400
	3	5500	250	4800	2400	2400	1200	2400
	4	5500	250	4800	2400	2400	1200	2400
2	1	5000	250	3840	1920	1920	960	1800
	2	5000	250	3840	1920	1920	960	1800
	3	5000	250	3840	1920	1920	960	1800
	4	5000	250	3840	1920	1920	960	1800
3	1	5000	200	2400	1200	1200	600	1500
	2	5000	200	2400	1200	1200	600	1500
	3	5000	200	2400	1200	1200	600	1500
	4	5000	200	2400	1200	1200	600	1500

Table A3. Machine cost and overtime labour cost per hour.

Plant	Regular machine cost for production ($)	Additional machine cost for production ($)	Overtime cost for skilled worker ($)	Overtime cost for non-skilled worker ($)
1	0.05	0.055	6	5
2	0.06	0.065	5	4
3	0.07	0.075	4	3

Table A4. Initial quota cost per unit and the initial quota quantity.

Product	1	2	3
Initial quota cost per unit ($)	20.5	13	6.55
Initial quota quantity	7700	6800	5200

Appendix B

Table B1. Shortage/surplus and under-/Over-quota for each scenario in third period (units)

		Purchased products			Inventory			Purchased quotas			Stored quotas		
Product		1	2	3	1	2	3	1	2	3	1	2	3
Scenario	$(s_1 s_1 s_1)$							200	200				
	$(s_1 s_1 s_2)$				100	100	100	100	100				100
	$(s_1 s_1 s_3)$				200	200	200						200
	$(s_1 s_2 s_1)$							100	100				
	$(s_1 s_2 s_2)$				100	100	100						100
	$(s_1 s_2 s_3)$				200	200	200				100	100	200
	$(s_1 s_3 s_1)$							100	100				
	$(s_1 s_3 s_2)$				100	100	100						100
	$(s_1 s_3 s_3)$				200	200	200				100	100	200
	$(s_2 s_1 s_1)$							100					
	$(s_2 s_1 s_2)$				100	100	100					100	100
	$(s_2 s_1 s_3)$				200	200	200				100	200	200
	$(s_2 s_2 s_1)$							100	100				
	$(s_2 s_2 s_2)$				100	100	100						100
	$(s_2 s_2 s_3)$				200	200	200				100	100	200
	$(s_2 s_3 s_1)$							100	100				
	$(s_2 s_3 s_2)$				100	100	100						100
	$(s_2 s_3 s_3)$				200	200	200				100	100	200
	$(s_3 s_1 s_1)$												
	$(s_3 s_1 s_2)$				100	100	100				100	100	100
	$(s_3 s_1 s_3)$				200	200	200				200	200	200
	$(s_3 s_2 s_1)$							100	100				
	$(s_3 s_2 s_2)$				100	100	100						100
	$(s_3 s_2 s_3)$				200	200	200				100	100	200
	$(s_3 s_3 s_1)$							100	100				
	$(s_3 s_3 s_2)$				100	100	100						100
	$(s_3 s_3 s_3)$				200	200	200				100	100	200

Table B2. Production quantity by skilled workers for fourth period (units)

Product		1			2			3		
Plant		1	2	3	1	2	3	1	2	3
Scenario	$(s_1 s_1 s_1)$		865	268			256			
	$(s_1 s_1 s_2)$		740	293			224			
	$(s_1 s_1 s_3)$		418	415			69			
	$(s_1 s_2 s_1)$		865	268			256			
	$(s_1 s_2 s_2)$		740	293			224			
	$(s_1 s_2 s_3)$		418	416			69			
	$(s_1 s_3 s_1)$		865	268			256			
	$(s_1 s_3 s_2)$		740	293			225			
	$(s_1 s_3 s_3)$		418	416			69			
	$(s_2 s_1 s_1)$	1137	823	184			550			

$(s_2 s_1 s_2)$	1137	698	209			518			
$(s_2 s_1 s_3)$	1137	376	331			363			
$(s_2 s_2 s_1)$	1160	796	209			518			
$(s_2 s_2 s_2)$	1160	671	234			487			
$(s_2 s_2 s_3)$	1160	348	356			331			
$(s_2 s_3 s_1)$		900	234			1045			
$(s_2 s_3 s_2)$		775	259			1013			
$(s_2 s_3 s_3)$		452	381			858			
$(s_3 s_1 s_1)$		909	224			580			
$(s_3 s_1 s_2)$		784	249			548			
$(s_3 s_1 s_3)$		462	372			393			
$(s_3 s_2 s_1)$	1056	767	249			548			
$(s_3 s_2 s_2)$	1056	642	274			517			
$(s_3 s_2 s_3)$	1056	319	397			361			
$(s_3 s_3 s_1)$		859	274			1146			
$(s_3 s_3 s_2)$		734	299			1114			
$(s_3 s_3 s_3)$		412	422			959			

Table B3. Production quantity by non-skilled workers for fourth period (units)

	Product	1			2			3		
	Plant	1	2	3	1	2	3	1	2	3
Scenario	$(s_1 s_1 s_1)$	1067					1844		1500	
	$(s_1 s_1 s_2)$	1067					1776		1313	
	$(s_1 s_1 s_3)$	1067					1631		829	271
	$(s_1 s_2 s_1)$	1067					1844		1500	
	$(s_1 s_2 s_2)$	1067					1776		1313	87
	$(s_1 s_2 s_3)$	1067					1631		829	271
	$(s_1 s_3 s_1)$	1067					1844		1500	
	$(s_1 s_3 s_2)$	1067					1775		1313	87
	$(s_1 s_3 s_3)$	1067					1631		829	271
	$(s_2 s_1 s_1)$	56					1550		1308	192
	$(s_2 s_1 s_2)$	56					1482		1120	280
	$(s_2 s_1 s_3)$	56					1337		637	463
	$(s_2 s_2 s_1)$	36					1582		1349	151
	$(s_2 s_2 s_2)$	36					1513		1162	238
	$(s_2 s_2 s_3)$	36					1369		678	422
	$(s_2 s_3 s_1)$	1067					1055		1231	269
	$(s_2 s_3 s_2)$	1067					987		1043	357
	$(s_2 s_3 s_3)$	1067					842		559	541
	$(s_3 s_1 s_1)$	1067					1520		1351	149
	$(s_3 s_1 s_2)$	1067					1452		1163	237
	$(s_3 s_1 s_3)$	1067					1307		680	420
	$(s_3 s_2 s_1)$	128					1552		1392	108
	$(s_3 s_2 s_2)$	128					1483		1205	195
	$(s_3 s_2 s_3)$	128					1339		721	379
	$(s_3 s_3 s_1)$	1067					954		1254	246
	$(s_3 s_3 s_2)$	1067					886		1066	334
	$(s_3 s_3 s_3)$	1067					741		582	518

Table B4. Quotas allocated for the fourth period (units)

Product		1	2	3
	$(s_1s_1s_1)$	2000	1900	1300
	$(s_1s_1s_2)$	2000	1900	1300
	$(s_1s_1s_3)$	2000	1900	1300
	$(s_1s_2s_1)$	2000	1900	1400
	$(s_1s_2s_2)$	2000	1900	1400
	$(s_1s_2s_3)$	2000	1900	1400
	$(s_1s_3s_1)$	2100	2000	1500
	$(s_1s_3s_2)$	2100	2000	1500
	$(s_1s_3s_3)$	2100	2000	1500
	$(s_2s_1s_1)$	2000	1900	1400
	$(s_2s_1s_2)$	2000	1900	1400
	$(s_2s_1s_3)$	2000	1900	1400
	$(s_2s_2s_1)$	2000	2000	1500
Scenario	$(s_2s_2s_2)$	2000	2000	1500
	$(s_2s_2s_3)$	2000	2000	1500
	$(s_2s_3s_1)$	2100	2100	1600
	$(s_2s_3s_2)$	2100	2100	1600
	$(s_2s_3s_3)$	2100	2100	1600
	$(s_3s_1s_1)$	2000	2000	1500
	$(s_3s_1s_2)$	2000	2000	1500
	$(s_3s_1s_3)$	2000	2000	1500
	$(s_3s_2s_1)$	2100	2100	1600
	$(s_3s_2s_2)$	2100	2100	1600
	$(s_3s_2s_3)$	2100	2100	1600
	$(s_3s_3s_1)$	2200	2200	1700
	$(s_3s_3s_2)$	2200	2200	1700
	$(s_3s_3s_3)$	2200	2200	1700

Table B5. Shortage/surplus and under-/Over-quota for each scenario in fourth period (units)

		Purchased products			Inventory			Purchased quotas			Stored quotas		
Product		1	2	3	1	2	3	1	2	3	1	2	3
Scenario	$(s_1s_1s_1s_1)$							200	200	200			
	$(s_1s_1s_1s_2)$				100	100	100	100	100	100			
	$(s_1s_1s_1s_3)$				200	200	200						
	$(s_1s_1s_2s_1)$							200	200	100			
	$(s_1s_1s_2s_2)$				100	100	100	100	100				
	$(s_1s_1s_2s_3)$				200	200	200						100
	$(s_1s_1s_3s_1)$	100	200	200				200	200				
	$(s_1s_1s_3s_2)$		100	100				100	100				100
	$(s_1s_1s_3s_3)$				100								200
	$(s_1s_2s_1s_1)$							200	200	100			
	$(s_1s_2s_1s_2)$				100	100	100	100	100				
	$(s_1s_2s_1s_3)$				200	200	200						100
	$(s_1s_2s_2s_1)$							200	200				
	$(s_1s_2s_2s_2)$				100	100	100	100	100				100
	$(s_1s_2s_2s_3)$				200	200	200						200

$(s_1s_2s_3s_1)$	100	200	200				100	100				100
$(s_1s_2s_3s_2)$		100	100									200
$(s_1s_2s_3s_3)$				100						100	100	300
$(s_1s_3s_1s_1)$							100	100				
$(s_1s_3s_1s_2)$				100	100	100						100
$(s_1s_3s_1s_3)$				200	200	200				100	100	200
$(s_1s_3s_2s_1)$							100	100				100
$(s_1s_3s_2s_2)$				100	100	100						200
$(s_1s_3s_2s_3)$				200	200	200				100	100	300
$(s_1s_3s_3s_1)$	100	200	200									200
$(s_1s_3s_3s_2)$		100	100							100	100	300
$(s_1s_3s_3s_3)$				100						200	200	400
$(s_2s_1s_1s_1)$							200	200	100			
$(s_2s_1s_1s_2)$				100	100	100	100	100				
$(s_2s_1s_1s_3)$				200	200	200						100
$(s_2s_1s_2s_1)$							200	100				
$(s_2s_1s_2s_2)$				100	100	100	100					100
$(s_2s_1s_2s_3)$				200	200	200					100	200
$(s_2s_1s_3s_1)$	100	200	200				100					100
$(s_2s_1s_3s_2)$		100	100								100	200
$(s_2s_1s_3s_3)$				100						100	200	300
$(s_2s_2s_1s_1)$							200	100				
$(s_2s_2s_1s_2)$				100	100	100	100					100
$(s_2s_2s_1s_3)$				200	200	200					100	200
$(s_2s_2s_2s_1)$							200	100				100
$(s_2s_2s_2s_2)$				100	100	100	100					200
$(s_2s_2s_2s_3)$				200	200	200					100	300
$(s_2s_2s_3s_1)$	100	200	200				100					200
$(s_2s_2s_3s_2)$		100	100								100	300
$(s_2s_2s_3s_3)$				100						100	200	400
$(s_2s_3s_1s_1)$							100					100
$(s_2s_3s_1s_2)$				100	100	100					100	200
$(s_2s_3s_1s_3)$				200	200	200				100	200	300
$(s_2s_3s_2s_1)$							100					200
$(s_2s_3s_2s_2)$				100	100	100					100	300
$(s_2s_3s_2s_3)$				200	200	200				100	200	400
$(s_2s_3s_3s_1)$	100	200	200								100	300
$(s_2s_3s_3s_2)$		100	100							100	200	400
$(s_2s_3s_3s_3)$				100						200	300	500
$(s_3s_1s_1s_1)$							200	100				
$(s_3s_1s_1s_2)$				100	100	100	100					100
$(s_3s_1s_1s_3)$				200	200	200					100	200
$(s_3s_1s_2s_1)$							100					100
$(s_3s_1s_2s_2)$				100	100	100					100	200
$(s_3s_1s_2s_3)$				200	200	200				100	200	300
$(s_3s_1s_3s_1)$	100	200	200								100	200
$(s_3s_1s_3s_2)$		100	100							100	200	300
$(s_3s_1s_3s_3)$				100						200	300	400
$(s_3s_2s_1s_1)$							100					100
$(s_3s_2s_1s_2)$				100	100	100					100	200
$(s_3s_2s_1s_3)$				200	200	200				100	200	300
$(s_3s_2s_2s_1)$							100					200

	$(s_3s_2s_2s_2)$				100	100	100					100	300
	$(s_3s_2s_2s_3)$				200	200	200				100	200	400
	$(s_3s_2s_3s_1)$	100	200	200								100	300
	$(s_3s_2s_3s_2)$		100	100							100	200	400
	$(s_3s_2s_3s_3)$				100						200	300	500
	$(s_3s_3s_1s_1)$											100	200
	$(s_3s_3s_1s_2)$				100	100	100				100	200	300
	$(s_3s_3s_1s_3)$				200	200	200				200	300	400
	$(s_3s_3s_2s_1)$											100	300
	$(s_3s_3s_2s_2)$				100	100	100				100	200	400
	$(s_3s_3s_2s_3)$				200	200	200				200	300	500
	$(s_3s_3s_3s_1)$	100	200	200							100	200	400
	$(s_3s_3s_3s_2)$		100	100							200	300	500
	$(s_3s_3s_3s_3)$				100						300	400	600

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Table C1. Production quantity for first period (units)

Product	1			2			3		
Plant	1	2	3	1	2	3	1	2	3
By skilled workers	1200	513	187			967			
By non-skilled workers						533		1200	

Table C2. Quotas allocated for first period

Product	1	2	3
Quota (units)	1828	1500	1200

Table C3. Shortage/surplus and under-/Over-quota for each scenario in first period (units)

		Purchased products			Inventory			Purchased quotas			Stored quotas		
Product		1	2	3	1	2	3	1	2	3	1	2	3
Scenario	(s_1)							72					
	(s_2)				100	100	100				28	100	100
	(s_3)				200	200	200				128	200	200

Table C4. Production quantity for second period (units)

		By skilled workers									By non-skilled workers								
Product		1			2			3			1			2			3		
Plant		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Scenario	(s_1)		181	752			633				1067					1067		1300	
	(s_2)		40	793			580				1067					1020		1140	60
	(s_3)		67	667			433				1067					1067		1100	

Table C5. Quotas allocated for second period (units)

Product		1	2	3
Scenario	(s_1)	1972	1700	1300
	(s_2)	1872	1583	1200
	(s_3)	1772	1400	1100

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Table C6. Shortage/surplus and under-/Over-quota for each scenario in second period (units)

		Purchased products			Inventory			Purchased quotas			Stored quotas		
Product		1	2	3	1	2	3	1	2	3	1	2	3
Scenario	$(s_1 s_1)$							28					
	$(s_1 s_2)$				100	100	100				72	100	100
	$(s_1 s_3)$				200	200	200				172	200	200
	$(s_2 s_1)$							100	17				
	$(s_2 s_2)$				100	100	100					83	100
	$(s_2 s_3)$				200	200	200				100	183	200
	$(s_3 s_1)$							100	100				
	$(s_3 s_2)$				100	100	100						100
	$(s_3 s_3)$				200	200	200				100	100	200

Table C7. Production quantity for third period (units)

		By skilled workers									By non-skilled workers								
Product		1			2			3			1			2			3		
Plant		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Scenario	$(s_1 s_1)$	1200	857	43			859									1041		682	718
	$(s_1 s_2)$		865	68			1383				1067					417		336	964
	$(s_1 s_3)$	1200	607	93			794									906		307	893
	$(s_2 s_1)$		1007	27			880				1067					1020		654	746
	$(s_2 s_2)$		933				1471				1067					329		223	1077
	$(s_2 s_3)$		757	77			816				1067					884		280	920
	$(s_3 s_1)$		966	67			1466				1067					434		539	861
	$(s_3 s_2)$	1200	708	92			1434									366		351	949
	$(s_3 s_3)$		816	18			1530				1067					170			1200

Table C8. Quotas allocated for the third period (units)

Product		1	2	3
Scenario	$(s_1 s_1)$	1914	1800	1400
	$(s_1 s_2)$	1900	1700	1300
	$(s_1 s_3)$	1800	1600	1200
	$(s_2 s_1)$	2000	1817	1400
	$(s_2 s_2)$	2000	1717	1300
	$(s_2 s_3)$	1900	1617	1200
	$(s_3 s_1)$	2100	1900	1400
	$(s_3 s_2)$	2000	1900	1300
	$(s_3 s_3)$	1900	1700	1200

Table C9. Shortage/surplus and under-/Over-quota for each scenario in third period (units)

		Purchased products			Inventory			Purchased quotas			Stored quotas		
Product		1	2	3	1	2	3	1	2	3	1	2	3
Scenario	$(s_1s_1s_1)$							186	100				
	$(s_1s_1s_2)$				100	100	100	86					100
	$(s_1s_1s_3)$				200	200	200				14	100	200
	$(s_1s_2s_1)$							128	100				
	$(s_1s_2s_2)$				100	100	100	28					100
	$(s_1s_2s_3)$				200	200	200				72	100	200
	$(s_1s_3s_1)$							128	100				
	$(s_1s_3s_2)$				100	100	100	28					100
	$(s_1s_3s_3)$				200	200	200				72	100	200
	$(s_2s_1s_1)$							100	83				
	$(s_2s_1s_2)$				100	100	100					17	100
	$(s_2s_1s_3)$				200	200	200				100	117	200
	$(s_2s_2s_1)$							100	100				
	$(s_2s_2s_2)$				100	100	100						100
	$(s_2s_2s_3)$				200	200	200				100	100	200
	$(s_2s_3s_1)$							100	100				
	$(s_2s_3s_2)$				100	100	100						100
	$(s_2s_3s_3)$				200	200	200				100	100	200
	$(s_3s_1s_1)$												
	$(s_3s_1s_2)$				100	100	100				100	100	100
	$(s_3s_1s_3)$				200	200	200				200	200	200
	$(s_3s_2s_1)$							100					
	$(s_3s_2s_2)$				100	100	100					100	100
	$(s_3s_2s_3)$				200	200	200				100	200	200
	$(s_3s_3s_1)$							100	100				
	$(s_3s_3s_2)$				100	100	100						100
	$(s_3s_3s_3)$				200	200	200				100	100	200

Table C10. Production quantity by skilled works for fourth period (units)

Product		1			2			3		
Plant		1	2	3	1	2	3	1	2	3
Scenario	$(s_1s_1s_1)$		1133				1341			
	$(s_1s_1s_2)$		994	12			1326			
	$(s_1s_1s_3)$		758	75			1246			
	$(s_1s_2s_1)$	1200	1000				785			
	$(s_1s_2s_2)$	1200	875	25			753			
	$(s_1s_2s_3)$	1200	625	75			690			
	$(s_1s_3s_1)$		1133				1342			
	$(s_1s_3s_2)$		1008	25			1310			
	$(s_1s_3s_3)$		758	75			1247			
	$(s_2s_1s_1)$	1200	1000				1341			
	$(s_2s_1s_2)$	1200	875	25			1310			
	$(s_2s_1s_3)$	1200	625	75			1246			
	$(s_2s_2s_1)$	1200	948	52			719			
	$(s_2s_2s_2)$	1200	823	77			687			

$(s_2 s_2 s_3)$	1200	573	127			624			
$(s_2 s_3 s_1)$	1200	1000				1342			
$(s_2 s_3 s_2)$	1200	875	25			1311			
$(s_2 s_3 s_3)$	1200	625	75			1247			
$(s_3 s_1 s_1)$	1200	1000				867			
$(s_3 s_1 s_2)$	1200	875	25			835			
$(s_3 s_1 s_3)$	1200	625	75			772			
$(s_3 s_2 s_1)$		1133				867			
$(s_3 s_2 s_2)$		1008	25			835			
$(s_3 s_2 s_3)$		758	75			772			
$(s_3 s_3 s_1)$	1200	901	99			740			
$(s_3 s_3 s_2)$	1200	776	124			708			
$(s_3 s_3 s_3)$	1200	526	174			645			

Table C11. Production quantity by non-skilled works for fourth period (units)

	Product	1			2			3		
	Plant	1	2	3	1	2	3	1	2	3
Scenario	$(s_1 s_1 s_1)$	1067					759		845	655
	$(s_1 s_1 s_2)$	1067					674		636	764
	$(s_1 s_1 s_3)$	1067					554		283	917
	$(s_1 s_2 s_1)$						1315		1004	496
	$(s_1 s_2 s_2)$						1247		817	583
	$(s_1 s_2 s_3)$						1110		442	758
	$(s_1 s_3 s_1)$	1067					758		845	655
	$(s_1 s_3 s_2)$	1067					690		658	742
	$(s_1 s_3 s_3)$	1067					553		283	917
	$(s_2 s_1 s_1)$						759		845	655
	$(s_2 s_1 s_2)$						690		658	742
	$(s_2 s_1 s_3)$						554		283	917
	$(s_2 s_2 s_1)$						1381		1090	410
	$(s_2 s_2 s_2)$						1313		902	498
	$(s_2 s_2 s_3)$						1176		527	673
	$(s_2 s_3 s_1)$						758		845	655
	$(s_2 s_3 s_2)$						689		658	742
	$(s_2 s_3 s_3)$						553		283	917
	$(s_3 s_1 s_1)$						1233		981	519
	$(s_3 s_1 s_2)$						1165		794	606
	$(s_3 s_1 s_3)$						1028		419	781
	$(s_3 s_2 s_1)$	1067					1233		981	519
	$(s_3 s_2 s_2)$	1067					1165		793	607
	$(s_3 s_2 s_3)$	1067					1028		419	781
	$(s_3 s_3 s_1)$						1360		1145	355
	$(s_3 s_3 s_2)$						1292		957	443
	$(s_3 s_3 s_3)$						1155		582	618

Table C12. Quotas allocated for the fourth period (units)

Product		1	2	3
Scenario	$(s_1s_1s_1)$	1986	1800	1300
	$(s_1s_1s_2)$	1986	1800	1300
	$(s_1s_1s_3)$	1986	1800	1300
	$(s_1s_2s_1)$	2000	1900	1400
	$(s_1s_2s_2)$	2000	1900	1400
	$(s_1s_2s_3)$	2000	1900	1400
	$(s_1s_3s_1)$	2100	2000	1500
	$(s_1s_3s_2)$	2100	2000	1500
	$(s_1s_3s_3)$	2100	2000	1500
	$(s_2s_1s_1)$	2000	1900	1400
	$(s_2s_1s_2)$	2000	1900	1400
	$(s_2s_1s_3)$	2000	1900	1400
	$(s_2s_2s_1)$	2000	2000	1500
	$(s_2s_2s_2)$	2000	2000	1500
	$(s_2s_2s_3)$	2000	2000	1500
	$(s_2s_3s_1)$	2100	2100	1600
	$(s_2s_3s_2)$	2100	2100	1600
	$(s_2s_3s_3)$	2100	2100	1600
	$(s_3s_1s_1)$	2000	2000	1500
	$(s_3s_1s_2)$	2000	2000	1500
	$(s_3s_1s_3)$	2000	2000	1500
	$(s_3s_2s_1)$	2100	2000	1600
	$(s_3s_2s_2)$	2100	2000	1600
	$(s_3s_2s_3)$	2100	2000	1600
	$(s_3s_3s_1)$	2200	2200	1700
	$(s_3s_3s_2)$	2200	2200	1700
	$(s_3s_3s_3)$	2200	2200	1700

Table C13. Shortage/surplus and under-/Over-quota for each scenario in fourth period (units)

		Purchased products			Inventory			Purchased quotas			Stored quotas		
Product		1	2	3	1	2	3	1	2	3	1	2	3
Scenario	$(s_1s_1s_1s_1)$							214	300	200			
	$(s_1s_1s_1s_2)$				100	100	100	114	200	100			
	$(s_1s_1s_1s_3)$				200	200	200	14	100				
	$(s_1s_1s_2s_1)$							214	300	100			
	$(s_1s_1s_2s_2)$				72	100	100	114	200				
	$(s_1s_1s_2s_3)$				172	200	200	14	100				100
	$(s_1s_1s_3s_1)$		100	100				200	200				
	$(s_1s_1s_3s_2)$							100	100				100
	$(s_1s_1s_3s_3)$				100	100	100						200
	$(s_1s_2s_1s_1)$							200	200	100			
	$(s_1s_2s_1s_2)$				100	100	100	100	100				
	$(s_1s_2s_1s_3)$				200	200	200						100
	$(s_1s_2s_2s_1)$							200	200				
	$(s_1s_2s_2s_2)$				100	100	100	100	100				100
	$(s_1s_2s_2s_3)$				200	200	200						200

$(s_1s_2s_3s_1)$		100	100				128	100				100
$(s_1s_2s_3s_2)$							28					200
$(s_1s_2s_3s_3)$				100	100	100				72	100	300
$(s_1s_3s_1s_1)$							100	100				
$(s_1s_3s_1s_2)$				100	100	100						100
$(s_1s_3s_1s_3)$				200	200	200				100	100	200
$(s_1s_3s_2s_1)$							100	100				100
$(s_1s_3s_2s_2)$				100	100	100						200
$(s_1s_3s_2s_3)$				200	200	200				100	100	300
$(s_1s_3s_3s_1)$		100	100				28					200
$(s_1s_3s_3s_2)$										72	100	300
$(s_1s_3s_3s_3)$				100	100	100				172	200	400
$(s_2s_1s_1s_1)$							200	200	100			
$(s_2s_1s_1s_2)$				100	100	100	100	100				
$(s_2s_1s_1s_3)$				200	200	200						100
$(s_2s_1s_2s_1)$							200	183				
$(s_2s_1s_2s_2)$				100	100	100	100	83				100
$(s_2s_1s_2s_3)$				200	200	200					17	200
$(s_2s_1s_3s_1)$		100	100				100	83				100
$(s_2s_1s_3s_2)$											17	200
$(s_2s_1s_3s_3)$				100	100	100				100	117	300
$(s_2s_2s_1s_1)$							200	100				
$(s_2s_2s_1s_2)$				100	100	100	100					100
$(s_2s_2s_1s_3)$				200	200	200					100	200
$(s_2s_2s_2s_1)$							200	100				100
$(s_2s_2s_2s_2)$				100	100	100	100					200
$(s_2s_2s_2s_3)$				200	200	200					100	300
$(s_2s_2s_3s_1)$		100	100				100					200
$(s_2s_2s_3s_2)$											100	300
$(s_2s_2s_3s_3)$				100	100	100				100	200	400
$(s_2s_3s_1s_1)$							100					100
$(s_2s_3s_1s_2)$				100	100	100					100	200
$(s_2s_3s_1s_3)$				200	200	200				100	200	300
$(s_2s_3s_2s_1)$							100					200
$(s_2s_3s_2s_2)$				100	100	100					100	300
$(s_2s_3s_2s_3)$				200	200	200				100	200	400
$(s_2s_3s_3s_1)$		100	100								100	300
$(s_2s_3s_3s_2)$										100	200	400
$(s_2s_3s_3s_3)$				100	100	100				200	300	500
$(s_3s_1s_1s_1)$							200	100				
$(s_3s_1s_1s_2)$				100	100	100	100					100
$(s_3s_1s_1s_3)$				200	200	200					100	200
$(s_3s_1s_2s_1)$							100					100
$(s_3s_1s_2s_2)$				100	100	100					100	200
$(s_3s_1s_2s_3)$				200	200	200				100	200	300
$(s_3s_1s_3s_1)$		100	100								100	200
$(s_3s_1s_3s_2)$										100	200	300
$(s_3s_1s_3s_3)$				100	100	100				200	300	400
$(s_3s_2s_1s_1)$							100	100				100
$(s_3s_2s_1s_2)$				100	100	100						200
$(s_3s_2s_1s_3)$				200	200	200				100	100	300
$(s_3s_2s_2s_1)$							100					200

	$(s_3s_2s_2s_2)$				100	100	100					100	300
	$(s_3s_2s_2s_3)$				200	200	200				100	200	400
	$(s_3s_2s_3s_1)$		100	100								100	300
	$(s_3s_2s_3s_2)$										100	200	400
	$(s_3s_2s_3s_3)$				100	100	100				200	300	500
	$(s_3s_3s_1s_1)$											100	200
	$(s_3s_3s_1s_2)$				100	100	100				100	200	300
	$(s_3s_3s_1s_3)$				200	200	200				200	300	400
	$(s_3s_3s_2s_1)$											100	300
	$(s_3s_3s_2s_2)$				100	100	100				100	200	400
	$(s_3s_3s_2s_3)$				200	200	200				200	300	500
	$(s_3s_3s_3s_1)$		100	100							100	200	400
	$(s_3s_3s_3s_2)$										200	300	500
	$(s_3s_3s_3s_3)$				100	100	100				300	400	600

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