False-name-proof Combinatorial Auction Design via Single-minded Decomposition[1]

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Abstract. This paper proposes a new approach to building false-name-proof (FNP) combinatorial auctions from those that are FNP only with single-minded bidders, each of whom requires only one particular bundle. Under this approach, a general bidder is decomposed into a set of single-minded bidders, and after the decomposition the price and the allocation are determined by the FNP auctions for single-minded bidders. We first show that the auctions we get with the single-minded decomposition are FNP if those for single-minded bidders satisfy a condition called PIA. We then show that another condition, weaker than PIA, is necessary for the decomposition to build FNP auctions. To close the gap between the two conditions, we have found another sufficient condition weaker than PIA for the decomposition to produce strategy-proof mechanisms. Furthermore, we demonstrate that once we have PIA, the mechanisms created by the decomposition actually satisfy a stronger version of false-name-proofness, called false-name-proofness with withdrawal.

1 Introduction

With the fast growing application of auctions in the real-world, many theoretical and practical studies of auctions have been conducted [10]. Among various auctions, combinatorial auctions such as spectrum auctions have attracted considerable attention as they sell a variety of goods in bundles. Combinatorial auctions utilize the fact that a buyer/bidder might gain extra value for receiving a bundle of goods together, and therefore improve social welfare and possibly the revenue of the seller (see [5, 4] for extensive surveys).

One major challenge of designing a desirable auction mechanism is preventing cheating or strategic manipulations by participants. One kind of manipulation for a buyer in a combinatorial auction is misreporting her valuations for goods/bundles, given that buyers' valuations are privately observed. An auction mechanism preventing misreporting is known to be *strategy-proof*, e.g. *Vickrey-Clarke-Groves* (VCG) mechanisms. As Internet trading/auctions such as *ebay* have been growing tremendously, there exist another kind of manipulation called false-name manipulation. Namely, for an auction running through the Internet, an agent might be able to create multiple accounts/identifiers to participate in the auction, because, for example, many web applications require only a valid email address and an agent can create multiple email addresses at practically no cost. A mechanism preventing false-name manipulations is known to be *false-name-proof* [16], which is also strategy-proof.

Designing a strategy-proof mechanism is relatively easy, while to design a "good" false-name-proof mechanism is very difficult. For combinatorial auctions, existing work [8, 9, 7] has shown that if all bidders are *single-minded* (i.e. each bidder requires only one bundle), designing a desirable mechanism is relatively easy, no matter whether from a computational complexity aspect, an efficiency aspect or a false-name-proofness aspect. However, when we face *general bidders*, obtaining a mechanism with desirable properties becomes very challenging.

In this paper, we propose a different approach to designing false-name-proof (or strategy-proof) combinatorial auctions for general bidders from combinatorial auctions that are false-name-proof (or strategy-proof) only with single-minded bidders. The main idea is decomposing a general bidder into a set of single-minded bidders and then adapting the mechanism for single-minded bidders to get the allocation and payments. Using this decomposition approach, we analyse under what conditions desirable mechanisms for general bidders are achievable. Especially, we show that a condition, called *Prices Increase with Agents* (PIA), is sufficient to achieve a false-name-proof auction for general bidders from the auction that is false-name-proof only with single-minded bidders. We also demonstrate another condition weaker than PIA, but sufficient for the decomposition to build strategy-proof mechanisms from the mechanisms that are strategy-proof only with single-minded bidders. As well as these sufficient conditions, we further provide some necessary conditions for designing false-name-proof/strategy-proof mechanisms via the decomposition, and finally demonstrate the applicability of these conditions.

Most existing research on false-name-proof mechanism design has focused on combinatorial auctions. Yokoo [16] showed that no false-name-proof mechanism satisfies *Pareto* efficiency, i.e. maximizing social welfare. Therefore, to get false-name-proof auction mechanisms we need to sacrifice efficiency. Several false-name-proof mechanisms have been proposed for general combinatorial auction settings, such as the Set mechanism [14], the Minimal Bundle mechanism [14], and the Leveled Division Set mechanism [15]. Iwasaki *et al.* [7] showed that the worst-case efficiency ratio of any false-name-proof combinatorial auction is at most $2/(m+1)$ for selling m items. They also observed that the worst-case efficiency ratio of those existing false-name-proof mechanisms mentioned above is generally $1/m$ or 0. Furthermore, they proposed a novel false-name-proof mechanism for single-minded bidders called Adaptive Reserve Price (ARP), which has the worst-case efficiency ratio of $2/(m+1)$, i.e. it achieves the highest worst-case efficiency ratio. For general false-name-proof combinatorial auctions, Yokoo [14] and Todo *et al.* [13] characterized the payment rules and the allocation rules respectively. Guo and Conitzer [6] provided the same characterization for a stronger version of false-name-proofness, called false-name-proofness with withdrawal.

In other settings, Conitzer [3] offered a characterization of false-name-proof voting rules, and showed the difficulty of designing

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false-name-proof mechanisms for resource allocation with transfers and public decision making without transfers. Todo *et al.* [11, 12] had some case studies of designing false-name-proof mechanisms without money. Bu [2] clarified the relationship between false-name-proofness and a consistency condition, called *population monotonicity*, in general public choice problem without transfers. Aziz *et al.* [1] extended the concept of false-name manipulation to cooperative games, namely weighted voting games.

The remainder of this paper is organized as follows. Section 2 presents the model of combinatorial auction design. Section 3 demonstrates the single-minded decomposition auction design. Then, Sections 4 and 5 provide a sufficient and a necessary condition respectively for the decomposition to work. Section 6 investigates the opportunity to close the gap between the two conditions and shows another sufficient condition for strategy-proofness, and finally Section 7 discusses the applicability of the decomposition. We conclude and discuss future work in Section 8.

2 The Model

Consider a set of bidders $N = \{1, \ldots, n\}$, and a set of goods/items $G = \{g_1, \ldots, g_m\}$. Each bidder $i \in N$ has a valuation for each bundle $B \subseteq G$, which is determined by i's privately observed **type** θ_i, denoted by $v(B, \theta_i)$. We assume $v(\emptyset, \theta_i) = 0$, and $v(B', \theta_i) \geq v(B, \theta_i)$ for all $B \subseteq B' \subseteq G$ (i.e. free disposal). Let θ be the type profile of all bidders and θ_{-i} be the type profile of all bidders except i. In a **combinatorial auction** $\mathcal{M} = (\pi, p)$, each bidder i is required to report her type θ_i (note that, she does not have to report her true type), and given bidders' type reports $\hat{\theta}$, $\mathcal{M}$ determines a **feasible allocation**[3] $\pi_i(\hat{\theta}) \subseteq G$ for each bidder i and a **payment** $p_i(\pi_i(\hat{\theta}), \hat{\theta})$ that i will pay. Given i's allocation $B \subseteq G$ and her payment $p_{B,i}$, the **utility** of bidder i is defined as $v(B, \theta_i) - p_{B,i}$.

One key criterion of combinatorial auction design (or mechanism design in general) is preventing strategic manipulations by bidders/participants, which is known as *strategy-proofness* or *incentive compatibility*.

Definition 1. *We say mechanism $\mathcal{M} = (\pi, p)$ is **strategy-proof (SP)**, if for all θ, for all $\hat{\theta}_i$, $v(\pi_i(\theta), \theta_i) - p_i(\pi_i(\theta), \theta) \geq v(\pi_i(\hat{\theta}_i, \theta_{-i}), \theta_i) - p_i(\pi_i(\hat{\theta}_i, \theta_{-i}), (\hat{\theta}_i, \theta_{-i}))$.*

That is, declaring the true type is a dominant strategy for each bidder. However, strategy-proofness cannot prevent manipulations via creating multiple identities for a single bidder, which is called *false-name manipulation*. In a false-name manipulation, a bidder uses more than one identity to report multiple types, which can be beneficial even if the mechanism is strategy-proof. We say a mechanism is *false-name-proof* if declaring the true type via a single identifier is a dominant strategy for each bidder.

Definition 2. *Mechanism $\mathcal{M} = (\pi, p)$ is **false-name-proof (FNP)**, if for all θ, for all $\theta_{i_1}, \theta_{i_2}, \ldots, \theta_{i_k}$, we have $v(\pi_i(\theta), \theta_i) - p_i(\pi_i(\theta), \theta) \geq v(\bigcup_{j=1}^{k} \pi_{i_j}(\theta_{-i} \cup \bigcup_{j=1}^{k} \theta_{i_j}), \theta_i) - \sum_{j=1}^{k} p_{i_j}(\pi_{i_j}(\theta_{-i} \cup \bigcup_{j=1}^{k} \theta_{i_j}), \theta_{-i} \cup \bigcup_{j=1}^{k} \theta_{i_j}))$.*

Intuitively, in a false-name-proof mechanism, a bidder cannot gain more utility via using any number of identifiers with any kind of misreports than reporting her type truthfully with one identifier. It is evident that false-name-proofness implies strategy-proofness.

We restrict our attention to **individually rational** mechanisms in this paper, where no bidder suffers any loss for participating in these mechanisms, i.e. bidders are not forced to participate in the mechanisms. Moreover, we restrict our attention to deterministic mechanisms, which always give the same outcome for the same input.

There are a few ways to characterize strategy-proof mechanisms such as Proposition 9.27 of [10], we will use the one given by [14], called Price-Oriented, Rationing-Free (PORF) mechanism, which characterizes a class of strategy-proof combinatorial auction mechanisms for our model. Namely, any strategy-proof combinatorial auction can be described as a PORF mechanism.

Theorem 1 ([14])**.** *Mechanism $\mathcal{M} = (\pi, p)$ is strategy-proof if and only if $\mathcal{M}$ can be described as a PORF mechanism.*

A PORF mechanism is defined as follows:

Definition 3 (**PORF Mechanism** [14])**.**

- *Each bidder i declares her type $\hat{\theta}_i$, which is not necessarily her true type θ_i.*
- *For each bidder i, for each bundle $B \subseteq G$, the price $p_{B,i}$ is defined. $p_{B,i}$ is determined independently of i's declared type $\hat{\theta}_i$, while it can depends on declared types of other bidders.*
- *We assume $p_{\emptyset,i} = 0$. Also, if $B \subseteq B'$, $p_{B,i} \leq p_{B',i}$.*
- *For bidder i, bundle B_i^* is allocated, where $B_i^* = \arg\max_{B \subseteq G}(v(B, \hat{\theta}_i) - p_{B,i})$, and i pays $p_{B_i^*,i}$. If there exist multiple bundles that maximize i's utility, one of these bundles is allocated.*
- *The result of the allocation satisfies* allocation-feasibility*, i.e. $B_i^* \cap B_j^* = \emptyset$ for any $i \neq j$.*

In a PORF mechanism, i's price is determined independently of i's declared type, and i receives a bundle maximizing her utility (i.e. a PORF mechanism is strategy-proof). Therefore, the prices must be determined appropriately to satisfy allocation-feasibility, i.e. allocation feasibility is essentially controlled by the prices. Without loss of generality, we assume that PORF always allocates one of the minimal bundles that maximizing a bidder's utility to the bidder. Bundle B is **minimal** for i if $v(B', \theta_i) < v(B, \theta_i)$ for all $B' \subset B$.

Furthermore, Yokoo [14] showed that a PORF mechanism satisfying additional conditions, namely WAP and NSA defined below, is false-name-proof.

Definition 4 (**Weakly-Anonymous Pricing Rule (WAP)**)**.** *For bidder i, the price of bundle B is given as a function of types of other bidders, i.e., the price can be described as $p(B, \Theta_X)$, where X is the set of bidders except i, and Θ_X is the set of types of bidders in X.*

That is, if two bidders face the same set of types of other bidders, their prices must be identical. Note that, a mechanism always allocating items to some dictators does not satisfy the WAP rule, as the prices depend on bidder's identity but not on the others' types.

Definition 5 (**No Super-Additive Price Increase (NSA)**)**.** *For all subset of bidders $S \subseteq N$ and $X = N \setminus S$, let B_i, for each $i \in S$, denote a bundle that maximizes i's utility, then $\sum_{i \in S} p(B_i, \bigcup_{j \in S \setminus \{i\}} \{\theta_j\} \cup \Theta_X) \geq p(\bigcup_{i \in S} B_i, \Theta_X)$.*

NSA says that the price of buying a combination of bundles (the right side of the inequality) must be smaller than or equal to the sum of the prices for buying these bundles separately (the left side). The following theorem characterizes a set of false-name-proof PORF mechanisms.

Theorem 2 ([14])**.** *A PORF mechanism with the WAP is false-name-proof if and only if it also satisfies the NSA condition.*

[3] Allocation π is feasible if $\pi_i(\hat{\theta}) \cap \pi_j(\hat{\theta}) = \emptyset$, for all $i \neq j$, all profile $\hat{\theta}$.

Next section will propose a single-minded decomposition approach to designing false-name-proof combinatorial auctions for general bidders from combinatorial auctions that are false-name-proof only with single-minded bidders.

3 The Decomposition

Given a set of m heterogeneous items, there are 2^m-1 different bundles, excluding the empty bundle, and each bidder has a valuation for each bundle and therefore there are at most 2^m-1 different positive valuations for a given bidder. We call a bidder is single-minded if she has only one positive valuation, i.e. she requires only one bundle or any superset of it. We call a general bidder k-minded, where $1 \leq k \leq 2^m-1$ is the number of different positive valuations the bidder assigned to all bundles. Therefore, a single-minded bidder is also 1-minded.

Definition 6. *We say bidder i is **single-minded** if i requires only one bundle A_i, i.e. for any bundle B, if $A_i \subseteq B$, then $v(B,\theta_i) = v(A_i,\theta_i) > 0$, otherwise, $v(B,\theta_i) = 0$.*

Definition 7. *We say bidder i is k-**minded** if i requires exactly one bundle from k bundles $A_{i_1},\ldots,A_{i_j},\ldots,A_{i_k}$ with distinct positive valuations $v_{i_1},\ldots,v_{i_j},\ldots,v_{i_k}$, respectively. For notation simplicity, let us assume $A_{i_0} = \emptyset$ and $v_{i_0} = 0$. i's valuation for bundle B is defined as $v(B,\theta_i) = \max_{0\leq j\leq k} v_{i_j}$, where $A_{i_j} \subseteq B$.*

Note that, each bundle A_{i_j} of k-minded bidder i is minimal for i, i.e. $v(B,\theta_i) < v(A_{i_j},\theta_i)$ for all $B \subset A_{i_j}$.

Given the above definitions, we are ready to describe our decomposition approach which aims to build a false-name-proof PORF mechanism for k-minded bidders from the one that is FNP only with single-minded bidders. The essential part of the approach is decomposing a k-minded bidder into a set of k single-minded bidders and then apply the pricing rule of the mechanism for single-minded bidders to determine the payment and the allocation for k-minded bidders. A k-minded bidder is decomposed as follows.

Definition 8. *Given a k-minded bidder of type θ_i, who requires one bundle from k bundles $A_{i_1},\ldots,A_{i_k}$, with valuations $v_{i_1},\ldots,v_{i_k}$, respectively, $dc(\theta_i)$ is a **single-minded decomposition** of θ_i, where $dc(\theta_i) = \{\theta_{i_1},\ldots,\theta_{i_k}\}$, where each θ_{i_j} is a type of a single-minded bidder who requires A_{i_j} with valuation v_{i_j}. Let $dc(\Theta_X) = \cup_{\theta_j\in\Theta_X} dc(\theta_j)$.*

Given the above single-minded decomposition, we can apply the pricing rule of a false-name-proof mechanism for single-minded bidders to define the prices for k-minded bidders. That is, a false-name-proof mechanism for general bidders is defined by the pricing rule of the mechanism for single-minded bidders with single-minded decomposition. The following definition gives the formal definition of the new mechanism, called $\mathcal{M}_k$, and the mechanism for single-minded bidders is called $\mathcal{M}_{single}$.

Definition 9. *Given a PORF mechanism $\mathcal{M}_{single}$ with a weakly-anonymous pricing rule p that is false-name-proof only with single-minded bidders, **mechanism** $\mathcal{M}_k$ for k-minded bidders is defined by pricing rule p' where $p'(B,\Theta_X) = p(B,dc(\Theta_X))$ and an allocation maximizing each bidder's utility.*

Note that, $\mathcal{M}_k$ does not satisfy allocation feasibility, i.e. not a PORF mechanism, in general, although $\mathcal{M}_{single}$ is false-name-proof or strategy-proof. In the rest, we investigate under what conditions $\mathcal{M}_k$ satisfies false-name-proofness or strategy-proofness.

4 A Sufficient Condition for $\mathcal{M}_k$ to be FNP

In this section, we show a condition, called Prices Increase with Agents, on the price function p of the FNP mechanism $\mathcal{M}_{single}$, that is sufficient for $\mathcal{M}_k$ to be false-name-proof. For notation simplicity, we assume in the rest of the paper that p of $\mathcal{M}_{single}$ satisfies WAP without further mention.

Definition 10 (Prices Increase with Agents (PIA) [6]**).** *Given a set of single-minded bidders S, for each $i \in S$, each bundle $B \subseteq G$, $p(B,\Theta_S) \geq p(B,\Theta_{S\setminus\{i\}})$.*

Intuitively, price p satisfies PIA if for each bidder the price for each bundle is non-decreasing after adding more bidders.

Theorem 3. *Given FNP mechanism $\mathcal{M}_{single}$ with price p, if p satisfies PIA, then $\mathcal{M}_k$ is also FNP.*

To prove above theorem, according to Theorem 1 and 2, we need to prove that the price p' of $\mathcal{M}_k$ satisfies both allocation feasibility and NSA.

Lemma 1. *Given FNP mechanism $\mathcal{M}_{single}$ with price p, if p satisfies PIA, then p' of $\mathcal{M}_k$ satisfies allocation feasibility.*

Proof. Given any set of single-minded bidders X and any two k-minded bidders k_1 and k_2 who can be decomposed into to a set of single-minded bidders K_1, K_2 respectively. To guarantee that p' satisfies allocation feasibility, we need to show:

- Firstly, for k_1, the bundle B_{k_1} maximizing k_1's utility under price $p(B,\Theta_{X\cup K_2})$ is not conflicting with the allocation of $X \cup K_2$ when there are only single-minded bidders $X \cup K_1 \cup K_2$ (note that the allocation for $X\cup K_2$ is the same no matter the rest bidders are k_1 or K_1 because the price p' for $X \cup K_2$ is the same). The same must hold for k_2.
- Secondly, the allocations for k_1 and k_2 are not conflicting, i.e. $B_{k_1} \cap B_{k_2} = \emptyset$.

Assume the bundle B_{k_1} maximizing k_1's utility is the bundle interested by $b_1^* \in K_1$ (there is no need to check for $B_{k_1} = \emptyset$), and let B_{K_2} and B_X be the items allocated to K_2 and X respectively. If we substitute b_1^* for k_1, i.e. consider the situation with bidders $\{b_1^*, K_2, X\}$, the price for b_1^* is the same as for k_1, and the price for $j \in X\cup K_2$ is $p(B,\Theta_{(X\cup\{b_1^*\}\cup K_2)\setminus\{j\}}) \leq p(B,\Theta_{(X\cup K_1\cup K_2)\setminus\{j\}})$ (according to PIA). Therefore, the allocation for b_1^* is still B_{k_1}. The allocation for each $j \in X \cup K_2$ might change and assume that the new allocation for K_2 and X are $B^1_{K_2}$ and B^1_X respectively. From the situation with bidders $\{b_1^*, K_2, X\}$ to the situation with bidders $\{k_1, K_2, X\}$, the price for each $j \in X \cup K_2$ is non-decreasing, so j's allocation is either the same or empty, i.e. $B^1_{K_2} \supseteq B_{K_2}$ and $B^1_X \supseteq B_X$. We already know that p satisfies allocation feasibility in the situation with bidders $\{b_1^*, K_2, X\}$, i.e. $B_{k_1} \cap (B^1_{K_2} \cup B^1_X) = \emptyset$, thus $B_{k_1} \cap (B_{K_2} \cup B_X) = \emptyset$, i.e. the allocation is feasible under the situation with bidders $\{k_1, K_2, X\}$. Similarly, we can show for k_2.

We summarize the above proof by Table 1, where Case I shows the situation with (single-minded) bidders $\{K_1, K_2, X\}$, Case II shows the situation with bidders $\{b_1^*, K_2, X\}$, and Case III shows the situation with bidders $\{K_1, b_2^*, X\}$. Case IV shows the situation with bidders $\{b_1^*, b_2^*, X\}$, which is used in the rest of the proof.

In the rest, we show that $B_{k_1} \cap B_{k_2} = \emptyset$. Under the situation with bidders $\{b_1^*, b_2^*, X\}$, we can show that the allocation for b_i^* is B_{k_i} given PIA. Take b_1^* as example, we know from PIA that $p(B,\Theta_{X\cup\{b_2^*\}}) \leq p(B,\Theta_{X\cup K_2})$, therefore, the bundle maximizing b_1^*'s utility is still B_{k_1}. From the allocation feasibility of $\mathcal{M}_{single}$, we get that $B_{k_1} \cap B_{k_2} = \emptyset$. □

I	Bidders:	K_1	K_2	X
	Allocation:	B_{K_1}	B_{K_2}	B_X
II	Bidders:	b_1^*	K_2	X
	Allocation:	B_{k_1}	$B^1_{K_2} \supseteq B_{K_2}$	$B^1_X \supseteq B_X$
III	Bidders:	K_1	b_2^*	X
	Allocation:	$B^2_{K_1} \supseteq B_{K_1}$	B_{k_2}	$B^2_X \supseteq B_X$
IV	Bidders:	b_1^*	b_2^*	X
	Allocation:	B_{k_1}	B_{k_2}	$B^{1,2}_X \supseteq B^1_X \cup B^2_X$

Table 1. Summary of the Proof of Lemma 1

Lemma 2. *Given FNP mechanism $\mathcal{M}_{single}$ with price p, if p satisfies PIA, then p' of $\mathcal{M}_k$ satisfies NSA.*

Proof. By contradiction, assume p' does not satisfy NSA, by suing PIA, we show that p also violates NSA.

Given a set of k-minded bidders X, and two k-minded bidders $1, 2 \notin X$ with type θ_1, θ_2 who can be decomposed into single-minded bidders of types $\{\theta_{1_1}, ..., \theta_{1_i}\}$ and $\{\theta_{2_1}, ..., \theta_{2_j}\}$ respectively. Without loss of generality, assume NSA of p' does not hold for $S = \{1, 2\}$, i.e.

$$p(B_1, \{\theta_{2_1}, ..., \theta_{2_j}\} \cup dc(\Theta_X)) + p(B_2, \{\theta_{1_1}, ..., \theta_{1_i}\} \cup dc(\Theta_X)) < p(B_1 \cup B_2, dc(\Theta_X)),$$

where $B_i \neq \emptyset$ is a bundle maximizing i's utility.

Assume that B_1 (B_2) is the bundle interested by θ_{1_1} (θ_{2_1}). If we simply substitute θ_{1_1} for θ_1 and θ_{2_1} for θ_2, from PIA, we know that the prices for bidder $1, 2$ are non-increasing, so the allocation keeps the same for bidder $1, 2$ before and after the substitution. Therefore, we conclude that p also violates NSA:

$$p(B_1, \{\theta_{2_1}\} \cup dc(\Theta_X)) + p(B_2, \{\theta_{1_1}\} \cup dc(\Theta_X)) < p(B_1 \cup B_2, dc(\Theta_X)).$$

□

5 What is Necessary for $\mathcal{M}_k$ to be FNP

We have shown that PIA on p is sufficient to guarantee that $\mathcal{M}_k$ is FNP. In this section, we discuss what are necessary in order to achieve false-name-proofness for $\mathcal{M}_k$.

In the following, we show that p of $\mathcal{M}_{single}$ has to satisfy some kind of weaker PIA in order to achieve false-name-proofness for $\mathcal{M}_{single}$ (even without considering the false-name-proofness of $\mathcal{M}_k$). Namely, by adding a new bidder j who requires a bundle A_j, then for each old bidder the price of any bundle B s.t. $B \cap A_j \neq \emptyset$ is non-decreasing. Moreover, if new bidder j's allocation is empty, or j's allocation is non-empty but with zero payment, then for each old bidder the prices of all bundles should be non-decreasing.

Theorem 4. *Given FNP mechanism $\mathcal{M}_{single}$ with price p and a set of single-minded bidders N, for any $i \neq j \in N$, we have for all $B \subseteq G$, $p(B, \Theta_{N\setminus\{i\}}) \geq p(B, \Theta_{N\setminus\{i,j\}})$ if one of the following conditions hold:*

- *$B \cap A_j \neq \emptyset$, where A_j is the bundle asked by j.*
- *the bundle B_j maximizing $v(B_j, \theta_j) - p(B_j, \Theta_{N\setminus\{j\}}) > 0$ is empty.*
- *the bundle B_j maximizing $v(B_j, \theta_j) - p(B_j, \Theta_{N\setminus\{j\}}) > 0$ is not empty, but $p(B_j, \Theta_{N\setminus\{j\}}) = 0$.*

Proof. By contradiction, for the first condition in the theorem, assume there exists a bundle B^* such that $p(B^*, \Theta_{N\setminus\{i\}}) < p(B^*, \Theta_{N\setminus\{i,j\}})$ and $B^* \cap A_j \neq \emptyset$. We can always find a single-minded bidder i who requires B^* and her valuation satisfies $p(B^*, \Theta_{N\setminus\{i\}}) < v(B^*, \theta_i) < p(B^*, \Theta_{N\setminus\{i,j\}})$, then i can manipulate via adding false-name bidder j so that her price for B^* is decreased and her allocation should be B^* according to the allocation feasibility of p. Similarly, we can get the same manipulation for the other two conditions. □

Note that, Theorem 4 does not say how much the price should increase after adding j, because allocation feasibility requires more, which is something not explicitly specified for a general PORF mechanism [14]. It is clear that the price must also satisfy the following additional condition to achieve allocation feasibility: for any θ_i, θ_j requiring A_i, A_j respectively, where $A_i \cap A_j \neq \emptyset$, if $v(A_i, \theta_i) > p(A_i, \Theta_{N\setminus\{i,j\}})$, i.e. i is allocated the bundle A_i without j, then one of the following conditions must hold after adding j:

1. $v(A_j, \theta_j) \leq p(A_j, \Theta_{N\setminus\{j\}})$.
2. $v(A_i, \theta_i) \leq p(A_i, \Theta_{N\setminus\{i\}})$.

That is, either i or j cannot receive the bundle she requires.

Furthermore, Theorem 4 induces that for any $i \neq j \in N$, $p(B, \Theta_{N\setminus\{i\}})$ can be less than $p(B, \Theta_{N\setminus\{i,j\}})$ if and only if $B \cap A_j = \emptyset$ and j receives A_j with payment $p(A_j, \Theta_{N\setminus\{j\}}) > 0$, given that NSA and allocation feasibility are not violated. Therefore, the necessary condition given in Theorem 4 is part of the PIA condition.

Now we have a sufficient condition, PIA, to guarantee false-name-proofness of $\mathcal{M}_k$ and also a necessary condition, a weaker PIA, for $\mathcal{M}_{single}/\mathcal{M}_k$ to be false-name-proof. We will discuss in next section the possibilities of closing the gap to obtain a both sufficient and necessary condition on p for $\mathcal{M}_k$ to be FNP.

6 The Possibility of Closing the Gap

PIA condition given in Section 4 is intuitive and sufficient for $\mathcal{M}_k$ to be FNP, but not all FNP mechanisms for single-minded bidders can satisfy PIA (e.g. the ARP mechanism discussed in Proposition 3), while the condition given in Theorem 4 is necessary but not enough for $\mathcal{M}_k$ to be FNP. This section investigates the possibility of closing the gap between these two conditions.

As we demonstrated in the proof of Theorem 3, to prove that $\mathcal{M}_k$ is FNP, we need to show that the price p' of $\mathcal{M}_k$ satisfies both allocation feasibility and NSA. From the the first part of the proof of Lemma 1, we get another necessary condition presented in the following proposition, which is necessary to achieve the allocation feasibility of p'.

Proposition 1. *Given single-minded bidders N, if $\mathcal{M}_k$ is FNP, then for all $i \in N$ and non-empty $S \subseteq N \setminus \{i\}$, if $v(A_i, \theta_i) - p(A_i, \Theta_{N\setminus(S\cup\{i\})}) > \max_{j\in S}(v(A_j, \theta_j) - p(A_j, \Theta_{N\setminus(S\cup\{i\})}))$, then for all $j \in N \setminus (S \cup \{i\})$, $p(A_j, \Theta_{N\setminus\{j\}}) \geq v(A_j, \theta_j)$ if $A_j \cap A_i \neq \emptyset$, where A_i, A_j are the minimal bundles asked by i, j respectively.*

Proposition 1 says that if we extend a single-minded bidder i to a k-minded bidder i' who can be decomposed into a set of single-minded bidders $\{i\} \cup S$ and the allocation is A_i for both i and i', then other bidders $N \setminus (S \cup \{i\})$ cannot get any allocation with positive utility that is conflicting with A_i after substituting i' for i. Otherwise, allocation feasibility is violated.

Note that, Proposition 1 does not depend on the allocation feasibility of p for $\mathcal{M}_{single}$, although it provides a necessary condition on p to achieve the allocation feasibility of p'. By utilizing the allocation feasibility of p, we get the following stronger version of the condition given in Proposition 1, but weaker than PIA.

Condition 1. *Given single-minded bidders N, for all $i \in N$ and non-empty $S \subseteq N \setminus \{i\}$, if $v(A_i, \theta_i) - p(A_i, \Theta_{N\setminus(S\cup\{i\})}) > \max_{j\in S}(v(A_j, \theta_j) - p(A_j, \Theta_{N\setminus(S\cup\{i\})}))$, then for all $j \in N \setminus (S \cup \{i\})$, $p(B, \Theta_{N\setminus\{j\}}) \geq p(B, \Theta_{N\setminus(S\cup\{j\})})$ if $B \cap A_i \neq \emptyset$.*

Condition 1 guarantees the first part of the allocation feasibility of p' (i.e. the allocation for a k-minded bidder will not conflict with the allocation of the other bidders given that the others are all single-minded), because from allocation feasibility of p, we know $p(A_j, \Theta_{N\setminus(S\cup\{j\})}) \geq v(A_j, \theta_j)$ if $A_j \cap A_i \neq \emptyset$, and therefore $p(A_j, \Theta_{N\setminus\{j\}}) \geq v(A_j, \theta_j)$ which guarantees that j's allocation is still not conflicting with A_i after adding bidders S. For the second part of the allocation feasibility of p' (i.e. the allocation for any two k-minded bidders is not conflicting), we propose Condition 2.

Condition 2. *Given two k-minded bidders k_1, k_2 and a set of single-minded bidders X, if k_i receives bundle $B_i \neq \emptyset$ with positive utility under price p', then the allocations for single-minded bidders b_1^*, b_2^* are not both empty under the situation of bidders $\{b_1^*, b_2^*, X\}$ with price p, where b_i^* is a single-minded bidder decomposed from k_i and the bundle required by b_i^* is B_i.*

Condition 2 says that if two k-minded bidders should receive a bundle each with a positive utility under price p', then if we substitute b_1^* and b_2^* for k_1 and k_2 respectively, the allocation for either b_1^* or b_2^* should be non-empty under price p. We will prove in the following that to guarantee allocation feasibility of p', Conditions 1 and 2 together, which is weaker than PIA[4], is sufficient. Note that, Condition 2 is a high level constraint on price p and the only situation it excludes is when the allocations for b_1^*, b_2^* are both empty. To include this situation to guarantee allocation feasibility of p', we need more less-intuitive constraints. Nonetheless, similar to PIA, Condition 1 and 2 are also verifiable for a given PORF mechanism.

Theorem 5. *If p of FNP mechanism $\mathcal{M}_{single}$ satisfies Conditions 1 and 2, then p' of $\mathcal{M}_k$ satisfies allocation feasibility.*

Proof. Recall the two steps to prove the allocation feasibility of $\mathcal{M}_k$ in Lemma 1:

1. For any k-minded bidder k_1, any set of single-minded bidders X, the allocation for k_1 is not conflicting with the allocation for X.
2. For any two k-minded bidders k_1, k_2, any set of single-minded bidders X, the allocation for k_1, k_2 is not conflicting.

For the first step, assume the allocation for k_1 is $B_1 \neq \emptyset$ under price p', and B_1 is interested by single-minded bidder b_1^* decomposed from k_1. It is evident that the allocation for b_1^* is still B_1 when we substitute b_1^* for k_1, because the price for k_1, b_1^* is the same. From the allocation feasibility of p, we know that B_1 is not conflicting with the allocation of X after the substitution. From Condition 1, we know that if we substitute k_1 for b_1^* back, the price of bidders in X for any bundle that intersects with B_1 is non-decreasing, and therefore the allocation for X is also not conflicting with k_1's allocation B_1.

For the second step, assume the allocation for k_1, k_2 are $B_1, B_2 \neq \emptyset$ respectively, and B_1, B_2 are interested by single-minded bidders b_1^*, b_2^* decomposed from k_1, k_2 respectively. From Condition 2, we know that the allocation for either b_1^* or b_2^* is non-empty in the situation with bidders $\{b_1^*, b_2^*, X\}$. If both b_1^* and b_2^* receive a non-empty allocation, i.e. B_1 and B_2 respectively, then from allocation feasibility of p, we get that $B_1 \cap B_2 = \emptyset$. Otherwise, assume the allocation for b_1^* is empty (i.e. the allocation for b_2^* is B_2), if we substitute k_2 for b_2^*, then the allocation for b_1^* changes to B_1 and the allocation for k_2 is B_2. Thus, from Condition 1, we know that the allocation B_2 for k_2 is not conflicting with b_1^*'s allocation B_1. □

Once p' satisfies allocation feasibility, $\mathcal{M}_k$ is a PORF mechanism, i.e. $\mathcal{M}_k$ is strategy-proof, which leads to the following corollary.

Corollary 1. *For any PORF mechanism $\mathcal{M}_{single}$ with price p satisfying Conditions 1 and 2, $\mathcal{M}_k$ based on p is strategy-proof.*

We know from Theorem 2 that NSA is a necessary and sufficient condition for a PORF mechanism with the WAP to be false-name-proof. The challenge here is how to utilize NSA of p to get NSA of p'. With PIA, we can easily have NSA for p' from NSA for p, because we can simply replace each k-minded bidder by a single-minded bidder where their allocations keep the same (see the proof of Lemma 2). However, we will lose this advantage if we weaken PIA condition. It seems very hard to get another sufficient and intuitive condition, weaker than PIA, that guarantees NSA of p' via utilizing NSA of p. The key reason is that once the price p can both increase and decrease after adding new bidders, we will need to know more about the precise definition of p, which is unknown for a general PORF mechanism because of the allocation feasibility assumption.

7 The Applicability

We have shown some sufficient or necessary conditions in previous sections to achieve false-name-proofness or strategy-proofness of $\mathcal{M}_k$. This section discusses the applicability of them.

For the sufficient condition PIA given in Section 4 which guarantees false-name-proofness of $\mathcal{M}_k$, it also guarantees another stronger version of false-name-proofness proposed by Guo and Conitzer [6], called false-name-proofness with withdrawal.

Definition 11 ([6])**.** *A mechanism $\mathcal{M} = (\pi, p)$ is **false-name-proof with withdrawal (FNPW)**, if for all θ, for all $\theta_{i_1}, \theta_{i_2}, \ldots, \theta_{i_k}, \ldots, \theta_{i_m}$, we have $v(\pi_i(\theta), \theta_i) - p_i(\pi_i(\theta), \theta) \geq v(\bigcup_{j=1}^{k} \pi_{i_j}(\theta_{-i} \cup \bigcup_{j=1}^{m} \theta_{i_j}), \theta_i) - \sum_{j=1}^{k} p_{i_j}(\pi_{i_j}(\theta_{-i} \cup \bigcup_{j=1}^{m} \theta_{i_j}), \theta_{-i} \cup \bigcup_{j=1}^{m} \theta_{i_j}))$.*

That is, truthful reporting using a single identifier is always better than submitting multiple false-name bids and then withdrawing some of them, i.e. some of the false-name identifiers ($\theta_{i_{k+1}}, \ldots, \theta_{i_m}$ in Definition 11) refuse to receive the allocation and to pay the payment. According to [6], PIA is also a sufficient condition to get false-name-proofness with withdrawal for $\mathcal{M}_k$.

Proposition 2. *Given FNP mechanism $\mathcal{M}_{single}$ with price p, if p satisfies PIA, then both $\mathcal{M}_{single}$ and $\mathcal{M}_k$ are FNPW.*

Proof. It is evident that FNPW is equivalent to FNP plus that a bidder's utility for reporting truthfully does not increase if we add more bidders. With PIA, we know that all prices are non-decreasing, so a bidder's utility is non-increasing by adding more bidders for both $\mathcal{M}_{single}$ and $\mathcal{M}_k$. □

There exist very few FNP mechanisms in the literature and to our knowledge, all of them were designed for general bidders except Adaptive Reserve Price (ARP) [7]. In the rest, we show that ARP cannot be applied for the decomposition to achieve even strategy-proofness, because it does not satisfy the necessary condition given in Proposition 1.

Proposition 3. *The decomposition mechanism $\mathcal{M}_k$ using ARP does not satisfy allocation feasibility.*

[4] It is worth mentioning that PIA guarantees that the allocations for both b_1^* and b_2^* are non-empty under the same situation.

To prove the above proposition, we need to briefly review the definition of ARP, which is also a PORF mechanism. In ARP, all bidders are single-minded, and if a bidder bids on a bundle of more than one item, it is treated as a bid on the bundle of all the items. Let $v^{-i}_{\{g_1\}}, \ldots, v^{-i}_{\{g_m\}}, v^{-i}_{\{g_1,\ldots,g_m\}}$ denote the highest bids for each bundle from all bidders except i. Let $g_{(k)}$ indicate the item that has the k-th highest bid among $v^{-i}_{\{g_1\}}, \ldots, v^{-i}_{\{g_m\}}$, i.e. $v^{-i}_{\{g_{(1)}\}} \geq \ldots \geq v^{-i}_{\{g_{(m)}\}}$. For bidder i, the price for each bundle is defined as:

$$p(\{g_1, \ldots, g_m\}, \theta_{-i}) = \max(v^{-i}_{\{g_1,\ldots,g_m\}}, v^{-i}_{\{g_{(1)}\}}, 2v^{-i}_{\{g_{(2)}\}})$$

$$p(\{g_{(1)}\}, \theta_{-i}) = \begin{cases} \max(v^{-i}_{\{g_{(1)}\}}, v^{-i}_{\{g_1,\ldots,g_m\}}/2) \\ \text{if } v^{-i}_{\{g_1,\ldots,g_m\}} < 2v^{-i}_{\{g_{(2)}\}}, \\ \max(v^{-i}_{\{g_{(1)}\}}, v^{-i}_{\{g_1,\ldots,g_m\}}) \\ otherwise \end{cases}$$

$\forall k \in [2, m]$,

$$p(\{g_{(k)}\}, \theta_{-i}) = \begin{cases} \max(v^{-i}_{\{g_{(2)}\}}, v^{-i}_{\{g_1,\ldots,g_m\}}/2) \\ \text{if } v^{-i}_{\{g_1,\ldots,g_m\}} < 2v^{-i}_{\{g_{(1)}\}}, \\ \max(v^{-i}_{\{g_{(1)}\}}, v^{-i}_{\{g_1,\ldots,g_m\}}) \\ otherwise \end{cases}$$

Proof of Proposition 3. The proof is by example. Consider three bidders $\{1, 2, 3\}$ with types $\{\theta_1, \theta_2, \theta_3\}$ and two items $\{g_1, g_2\}$. Bidders' valuations are the following.

	$\{g_1\}$	$\{g_2\}$	$\{g_1, g_2\}$
Bidder 1:	5	3.6	7
Bidder 2:	5.5	0	5.5
Bidder 3:	0	3	3

That is, bidder 1 is 3-minded, who can be decomposed into 3 single-minded bidders $\{\theta_{1_1}, \theta_{1_2}, \theta_{1_3}\}$ asking for $\{g_1\}, \{g_2\}, \{g_1, g_2\}$ respectively. Bidder 2 and 3 are single-minded.

By applying the price of ARP, we get the following prices, beside the valuations, for each bidder:

	$\{g_1\}$	$\{g_2\}$	$\{g_1, g_2\}$
Bidder 1:	5(5.5)	3.6(3)	7(6)
Bidder 2:	5.5(5)	0(3.6)	5.5(7.2)
Bidder 3:	0(5.5)	3(3.6)	3(7.2)

For bidder 1, we get $v^{-1}_{\{g_1\}} = 5.5$, $v^{-1}_{\{g_2\}} = 3$ and $v^{-1}_{\{g_1,g_2\}} = 0$. Therefore, $p(\{g_1, g_2\}, \{\theta_2, \theta_3\}) = 2v^{-1}_{\{g_2\}} = 6$, $p(\{g_1\}, \{\theta_2, \theta_3\}) = v^{-1}_{\{g_1\}} = 5.5$ and $p(\{g_2\}, \{\theta_2, \theta_3\}) = v^{-1}_{\{g_2\}} = 3$. For bidder 2, we have to consider bidder 1 as 3 single-minded bidders, i.e. $\{\theta_{1_1}, \theta_{1_2}, \theta_{1_3}\}$, we get $v^{-2}_{\{g_1\}} = 5$, $v^{-2}_{\{g_2\}} = 3.6$ and $v^{-2}_{\{g_1,g_2\}} = 7$. Thus, $p(\{g_1, g_2\}, \{\theta_{1_1}, \theta_{1_2}, \theta_{1_3}, \theta_3\}) = 2v^{-2}_{\{g_2\}} = 7.2$, $p(\{g_1\}, \{\theta_{1_1}, \theta_{1_2}, \theta_{1_3}, \theta_3\}) = v^{-2}_{\{g_1\}} = 5$ and $p(\{g_2\}, \{\theta_{1_1}, \theta_{1_2}, \theta_{1_3}, \theta_3\}) = v^{-2}_{\{g_2\}} = 3.6$. Similarly, we get the prices for bidder 3.

Maximizing their utility, bidder 1 receives $\{g_1, g_2\}$, bidder 2 receives $\{g_1\}$ and bidder 3 gets nothing, which violates allocation feasibility and the necessary condition given in Proposition 1. □

8 Conclusion

We have proposed a single-minded decomposition approach to designing false-name-proof (FNP) or strategy-proof (SP) combinatorial auctions based on those that are FNP or SP only with single-minded bidders. By doing that, we are able to utilize the advantage of designing FNP or SP mechanisms for single-minded bidders to build those for general bidders, although not every FNP or SP combinatorial auction for single-minded bidders can be successfully adapted by the decomposition. We have showed that if the single-minded FNP mechanism satisfies PIA, then the mechanism built by the decomposition is not only FNP but also FNP with withdrawal. If we only consider strategy-proofness, then another condition (Conditions 1 and 2 together, weaker than PIA) is sufficient for the decomposition to build SP mechanisms. We have also proved some necessary conditions (Theorem 4 and Proposition 1) for the decomposition to output FNP/SP mechanisms, and investigated the possibility of closing the gap between the sufficient conditions and the necessary conditions.

This is the very first attempt to design general FNP/SP combinatorial auctions from relatively "simple" ones. We believe that single-minded decomposition is not the only approach following this direction. Also, for the single-minded decomposition approach, we have not successfully closed the gap to obtain a both sufficient and necessary condition for the decomposition to produce FNP or SP mechanisms. The major challenge we have faced is that the PORF mechanisms used by the decomposition are very general and they assume allocation feasibility but without explicit characterization. Once we have a better understanding of the hidden constraints behind the allocation feasibility assumption, we might be able to utilize them to close the gap. Furthermore, to fully explore the power the decomposition approach, we need another line of research to designing dedicated mechanisms for single-minded bidders.

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