Investigation of ground vibration from circular tunnels using a 2.5D FE/BE model of tunnel and ground

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ABSTRACT

A numerical model of a circular tunnel using the 2.5D finite element and boundary element method is described. This makes use of the constant geometry in the axial direction so that a two-dimensional model is solved for a series of wavenumbers. The full 3D solution can be recovered by a Fourier transformation. The response of the tunnel structure is analysed first. Then, the vibration at certain points on the ground surface is predicted using the tunnel-soil model. Finally, a parametric study is carried out to show the influence of different aspects of tunnel design to the vibration in the ground.

1. INTRODUCTION

Train-induced vibration and noise from underground railways has gradually attracted public concern, as the subways are rapidly expanding under densely populated cities in many countries. Therefore, recently, many prediction models have been developed in response to the requirement of reducing the effect from ground vibration and ground-borne noise. Two types of prediction models are popularly used: analytical models and numerical models. The analytical method provides a rapid solution and is useful for engineering practice. However, to cater for more detailed design studies a numerical approach is required.

An example of an analytical model is the PiP model, developed by Hussein et al. to calculate the far-field displacement due to a train running in a tunnel (1). The tunnel is represented by a circular cylinder embedded in a whole space ground and the track is represented by a separate analytical model. The model has recently been extended to include the free ground surface and ground layering. However, the efficiency of this analytical approach is achieved at the cost of model simplification and application limitations. In order to study the tunnel and track design in more detail, to meet the requirements for vibration and noise reduction, numerical models which can provide an authentic modelling of tunnel and track structure are required.

One such numerical approach is a coupled finite element and boundary element model in which the tunnel structure is modelled using finite elements and the ground using boundary elements which avoid problems of reflections at artificial boundaries (2). In (3), the FE/BE models in two and three dimensions are compared; 2D models are found to be lacking in precision whereas 3D models are computationally expensive. The so-called 2.5D approach is a compromise between these two models. A series of 2D models are used with the third dimension represented by wavenumber. This approach has been employed for example by Sheng et al. (4) and Francois et al. (5) to predict the ground vibration from trains with good results .

In this paper the 2.5D FE/BE approach is used to model a circular tunnel. The response of the tunnel structure (without soil) to a harmonic excitation is analysed first to illustrate its dynamic behaviour. Then, the vibration at certain points on the ground surface is predicted using the tunnel-soil model. Finally, a parametric study is carried out to show the influence of different aspects of tunnel design, such as the tunnel lining and invert, on the vibration of the ground.

2. MODEL DESCRIPTION

The research object is a straight circular tunnel embedded in a homogeneous half space. In the present work the track is omitted but this can readily be included in the model.

The tunnel and ground are modelled using the 2.5D coupled finite element / boundary element software WANDS (Wave-Number-Domain FE-BE Software) (6). Based on the assumption that the soil and tunnel structure properties are invariant in the longitudinal direction, the 2.5D model is formed from a sequence of 2D FE/BE models of the cross-section of tunnel and soil, in terms of the wavenumber in the axial direction (x- direction). The 3D solution can be recovered by using an inverse Fourier transform over wavenumber. As shown in Figure 1, the tunnel structure is modelled using 8-noded solid elements; the ground is modelled using 3-noded boundary elements around the tunnel and at the ground surface (not shown). A harmonic force is applied directly on the tunnel lining or invert. Ground layers can also be included as necessary. The parameters chosen in the wavenumber domain depend mainly on the soil properties. Details of the model will be given in the following section.

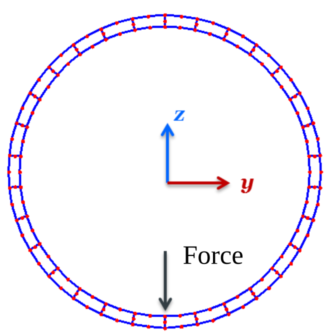


Figure 1 Model of tunnel lining in WANDS

3. MODE ANALYSIS OF FE TUNNEL MODEL

3.1 Model description

Before considering the coupled tunnel-ground problem, the circular tunnel is first modelled alone, without the consideration of the surrounding soil. The tunnel lining is modelled with a single layer of 8-noded solid elements, as shown in Figure 1. The parameters used in the model are listed in Table 1. The frequency range used in the calculation is from 1~100 Hz with a logarithmic spacing. A discussion of the parameters used in the wavenumber domain can be found in (7). Compared to the model with soil in Section 4, a relatively fine wavenumber resolution ($\Delta\beta$) is chosen here to give precise results in the axial (x) direction. The total number of wavenumbers is given by $\beta_{\max} = 0.5 \bullet \Delta\beta \bullet N_\beta$. A vertical harmonic force is applied at the bottom of the tunnel. The parameters of the tunnel are based on a typical concrete tunnel. In addition two types of tunnel invert are considered, described in Section 3.3 below. They are also modelled with the 8-noded solid elements and the same material properties.

Table 1 Parameters and properties used in tunnel model

Parameters in Wavenumber Domain	**Tunnel properties**
Frequency: 1~100 Hz Number of wavenumber [N_β]: 2048 Wavenumber resolution [$\Delta\beta$]: 6.28×10^{-4} rad/m Maximum wavenumber [β_{max}]: 0.64 rad/m	Inner diameter: 3.81 m Outer diameter: 4.11 m Young's Modulus: 50 MPa Density: 2500 kg/m^3 Poisson's ratio: 0.3 Loss factor: 0.03

3.2 Mode analysis

The 2.5D FE model of the tunnel structure provides a comprehensive insight into the deformation modes of the tunnel. The magnitude of the response at each frequency and wavenumber, when the tunnel is excited by a unit force at the tunnel bottom, is plotted in Figure 2. The responses at the forcing point in each direction are shown separately, although it should be noted that the force is in the vertical (z) direction in each case. Also plotted are the dispersion curves (free wavenumbers) of a beam which has the same geometry and properties as the tunnel structure.

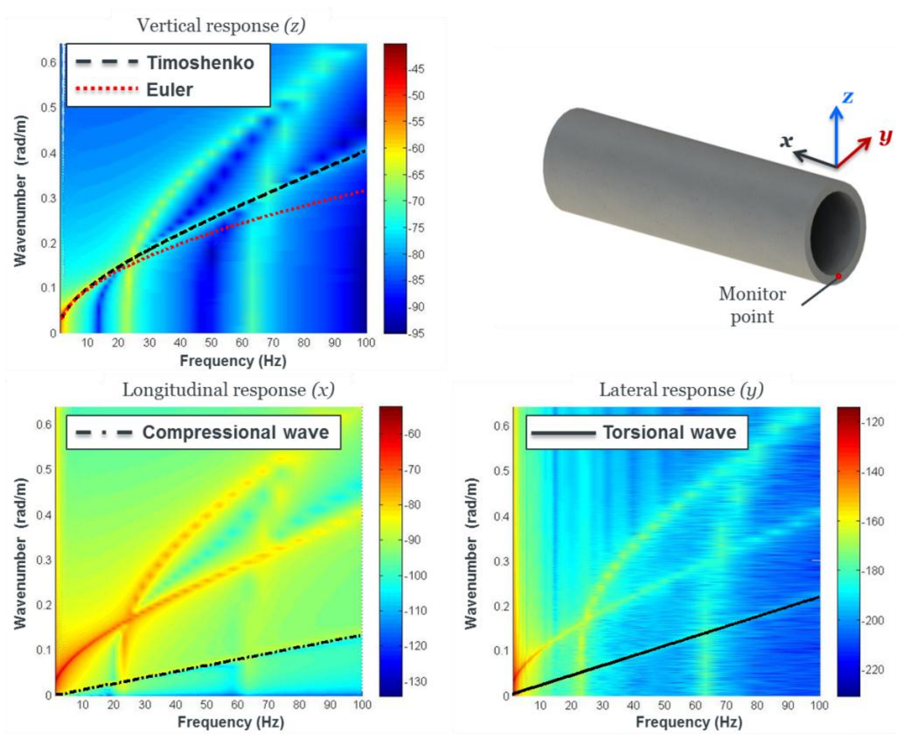
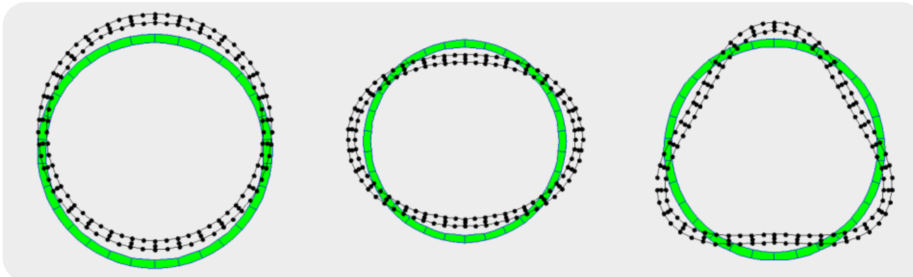


Figure 2 Dispersion diagrams at tunnel bottom for a vertical force applied at the same location



(a) At 1 Hz (b) At 25 Hz (c) At 63 Hz

Figure 3 In-plane deformation of the tunnel lining under a vertical force at tunnel bottom

The maximum response at the tunnel bottom is in the vertical (z) direction, the results for which are shown at the top left. Three bright lines can be seen, each representing a deformation mode of the tunnel ring. The cut-on frequencies for each mode (the frequencies at which the mode starts to propagate) can also be found in the dispersion diagram as 0, 23 and 63 Hz. Figure 3 shows examples of the cross-section deformation close to the cut-on frequency of each of these modes. The first wave to occur corresponds to a rigid body movement of the cross-section, which shows a very good agreement with the bending wave of the corresponding Timoshenko beam (see Figure 2). The other two modes are higher order shell modes.

The response in the other two directions is much smaller than in the vertical direction; theoretically it should be zero but due to numerical issues a small non-zero response is found. Apart from the three modes found in the vertical response, two other curves can be observed in the dispersion diagrams for these two directions. They reflect the compressional wave in longitudinal direction and the torsional wave in the lateral direction.

3.3 Influence of lining thickness and invert

In this section, the response to a harmonic excitation at the tunnel bottom is analysed. The influence is studied of changing the thickness of the tunnel lining, as well as introducing an invert with various shapes and height.

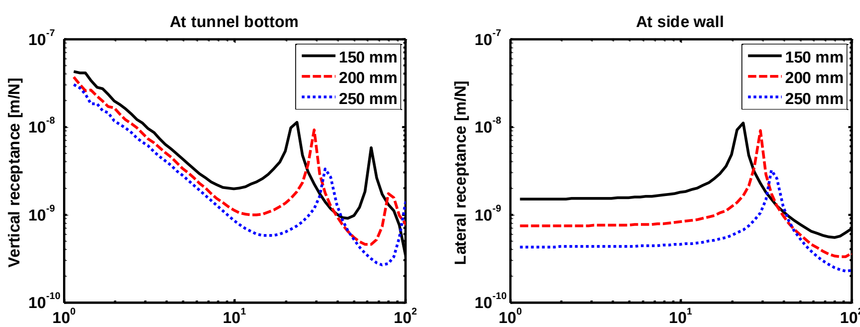


Figure 4 Responses to the excitation at tunnel bottom with different lining thickness

To study the influence of the thickness of the tunnel lining, the responses at the tunnel bottom and side wall are compared in Figure 4 from FE tunnel models with different lining thicknesses. These are shown as receptance (displacement for a unit force). All the tunnels have the same inner radius of 3.81 m. It can be seen from the figure that the cut-on frequencies (visible as peaks in the response) increase with the thickness of the tunnel lining, while the average amplitude of the response decreases. The responses at the crown of the tunnel lining (not shown) are almost identical to those at the bottom when there is no soil. The lateral response at the side wall is smaller at low frequency as the first mode has no response here. At the cut-on frequency of the second mode the response is similar to that at the tunnel bottom. The third mode also does not appear in the lateral deformation of the side wall.

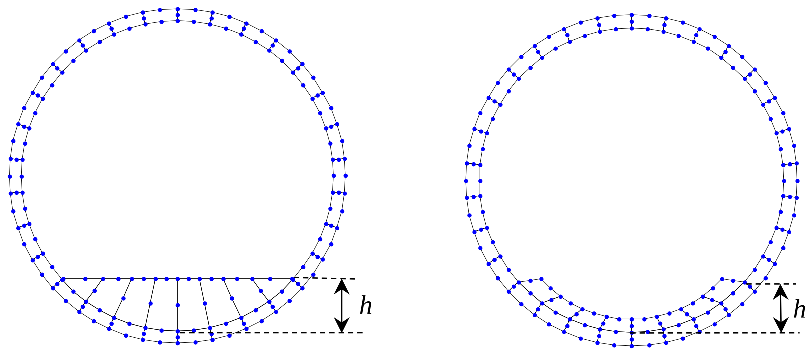


Figure 5 FE tunnel model with two types of invert

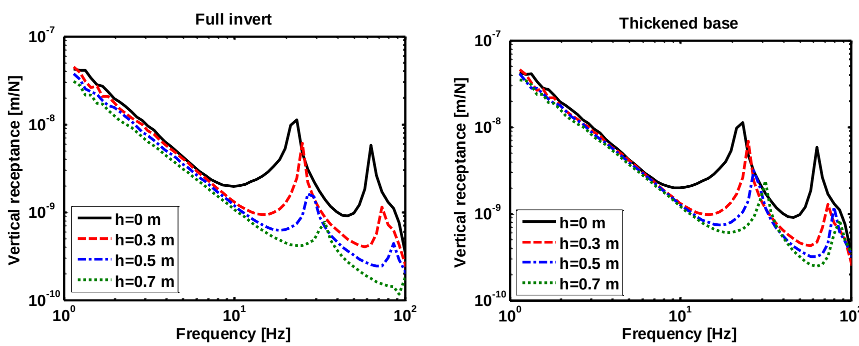


Figure 6 Comparison of responses at the tunnel bottom when invert with different shape and height is used

The tunnel invert is an important element connecting the track with the tunnel lining. Although the tunnel invert may have various shapes and geometry, it is often ignored or simplified in the modelling. Here, it will be studied how the invert influences the dynamic behaviour of the tunnel structure.

Two different shapes of tunnel invert are considered based on those which are commonly used in practice, as shown in Figure 5. The one on the left can be used with ballasted track, booted-sleeper track and so on, while the one on the right is used particularly with floating slab track.

Figure 6 compares the responses at the tunnel bottom when different shapes and heights of invert are used. For the full invert, h is the height of the invert, as shown in Figure 5(a). For the thickened base, the thickness is kept constant and h defines the extent of the invert, as shown in Figure 5(b). In each case h=0 means the tunnel without the invert. Compared with the tunnel model without invert, the responses of those with an invert are lower. The cut-on frequencies of the deformation modes also increase with the height of the invert.

4. RESULTS OF TUNNEL-GROUND MODEL

4.1 Model description

The tunnel is now considered embedded in soil. Some characteristics of the modal behaviour of tunnel structure discussed above will assist in interpreting the results obtained here.

The tunnel modelled here is the same as the one in the previous section, apart from the tunnel thickness, which is varied from 150~250 mm; unless otherwise stated, it is 150 mm as before. The tunnel is buried at a depth of 15 m or 25 m below the ground surface, the distance in each case being to the tunnel centre. The ground is formed by 3-noded boundary elements. The free ground surface is modelled to a distance of 30 m on one side and 40 m on the other side of the tunnel. At the edge of the ground surface mesh, special edge elements are used to minimise reflections (3). The element size on the ground surface is 0.25 m. The tunnel lining is modelled using 30 elements, which have a length of about 0.3 m. Two types of tunnel invert are considered as previously. The soil properties are based on London clay, as listed in Table 2. The parameters used in the wavenumber domain for the tunnel-ground model are different from those used in tunnel model, and are mainly determined by the soil properties. In order to get a precise prediction of vibration, a large maximum wavenumber is used, while the wavenumber resolution is less fine compared with that used in the tunnel model.

Table 2 Parameters and properties used in tunnel-soil model

Parameters in Wavenumber Domain	**Soil properties** (8)
Frequency: 1~100 Hz	S-wave speed: 220 m/s
Number of wavenumber [N_β]: 1024	P-wave speed: 1751 m/s
Wavenumber resolution [$\Delta\beta$]: 6.28×10^{-3} rad/m	Density: 1980 kg/m^3
Maximum wavenumber [β_{max}]: 3.2 rad/m	Loss factor: 0.078

4.2 Results overview and verification

Using the tunnel-ground model built in WANDS, the vibration produced by the vertical force acting on the tunnel bottom can be predicted.

Figure 7 shows the response on the tunnel at different positions. Compared with Figure 4, the responses are much flatter due to the radiation damping of the soil. The cut-on frequencies are not evident at all at the tunnel bottom, although the response is clearly affected by the lining thickness. The second and third cut-on frequencies appear at the side wall and tunnel crown respectively, but their amplitudes are much smaller than in Figure 4.

Figure 8 shows the waves propagating on the ground surface for different excitation frequencies. The *x*-axis is the longitudinal direction of tunnel. The force is applied on the tunnel bottom at $x=y=0$. It can be seen in Figure 8 that, as the frequency increases, the strength of the waves in the longitudinal direction is gradually moderated due to the existence of the tunnel, and the wavelength is also lengthened relative to that in the transverse direction.

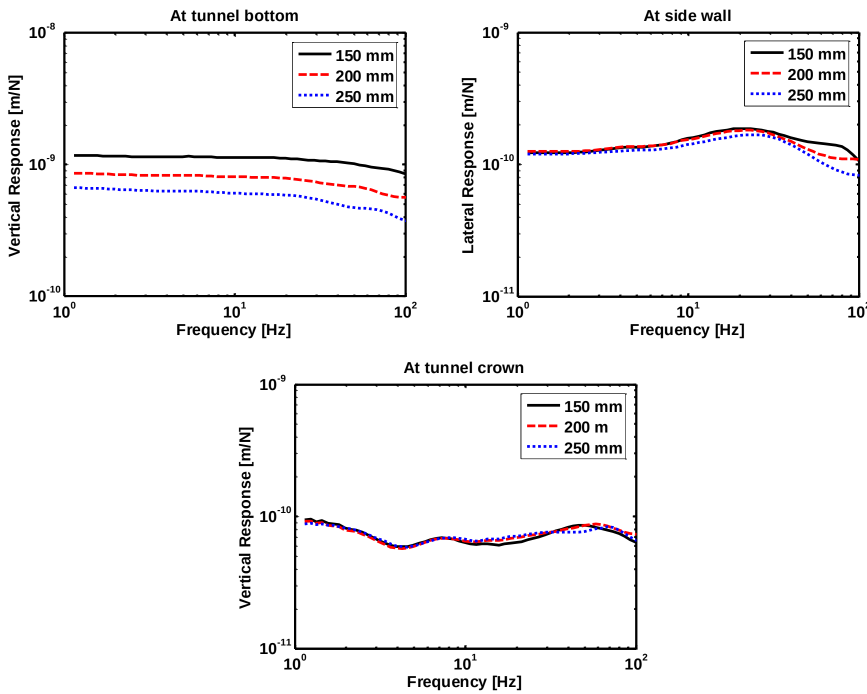


Figure 7 Responses to the excitation at tunnel bottom with different lining thickness for tunnel in soil at 25 m depth

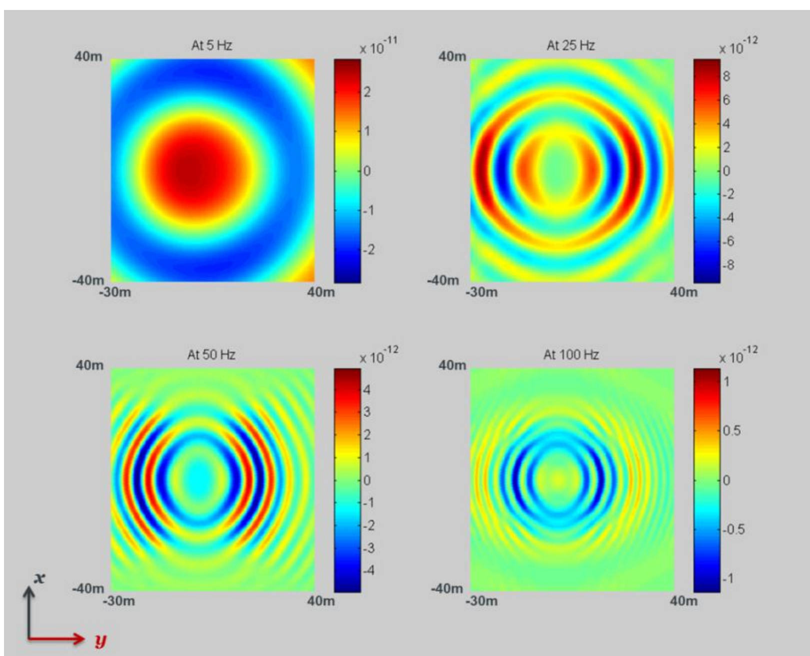


Figure 8 Surface plot of instantaneous vertical displacement (m/N) on the ground surface on the ground surface excited at different frequencies

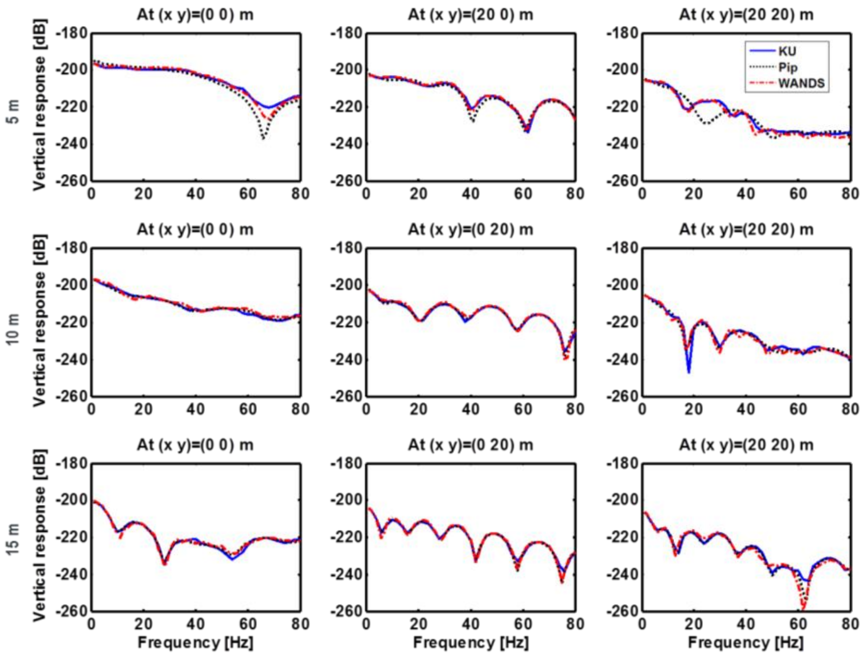


Figure 9 Comparison of vertical responses on the the ground surface between WANDS, PiP and KU Leuven model (9)

To verify the correctness of the 2.5D tunnel-ground model, a comparison is made between the results from WANDS, PiP (1) and a model from KU Leuven (5). PiP (1) is an analytical model while the KU Leuven model is a numerical model similar to WANDS. The parameters used in the comparison model are different from those introduced previously. For the tunnel, the inner radius is 2.75 m and the lining thickness is 0.25 m. For the soil, the shear and compressional wave speeds are 200 m/s and 400 m/s, with density of 1800 kg/m^3 and loss factor of 0.04. The vibration predicted by the three models at three different points on the ground surface and for three different tunnel depths is compared in Figure 9. The comparison shows a good agreement, especially for the tunnels with buried depths of 10 m and 15 m.

4.3 Parametric analysis

Results are presented for a series of models with different tunnel size and invert depth to study the impact on the responses at the ground surface. They are all converted to one-third octave bands for ease of comparison.

Figure 10 and Figure 11 show the responses in different directions at various positions on the ground surface from the models with the same buried depth of 25 m. Above the tunnel axis, in Figure 10, there is no response in the lateral direction, while in the cross-section of the forcing point, Figure 11, the longitudinal response is zero. At 10 m from the tunnel, the lateral response exceeds the vertical response above 6 Hz, but further away the vertical response increases. This is due to the shadow effect (2) of the tunnel structure, see Figure 8.

These results illustrate that the vibration on the ground surface is not only dominated by the vertical component; the longitudinal and lateral components are also important at the points away from $(x, y)=(0, 0)$. Therefore, a way of evaluating the total vibration is adopted from (2) known as the pseudo-resultant amplitude:

$$u = \sqrt{|u_z|^2 + |u_y|^2 + |u_x|^2}$$

where u_z, u_y and u_x are the complex amplitudes of vertical, lateral and longitudinal responses, respectively.

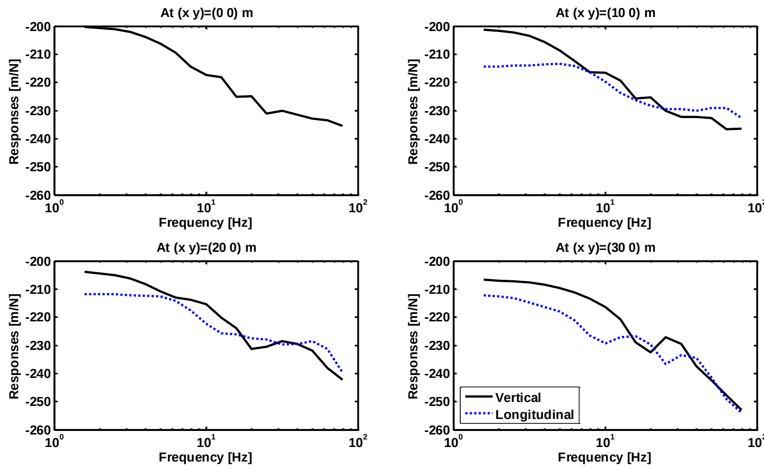


Figure 10 Responses at various locations along x direction on the ground surface predicted for a tunnel at a depth of 25 m

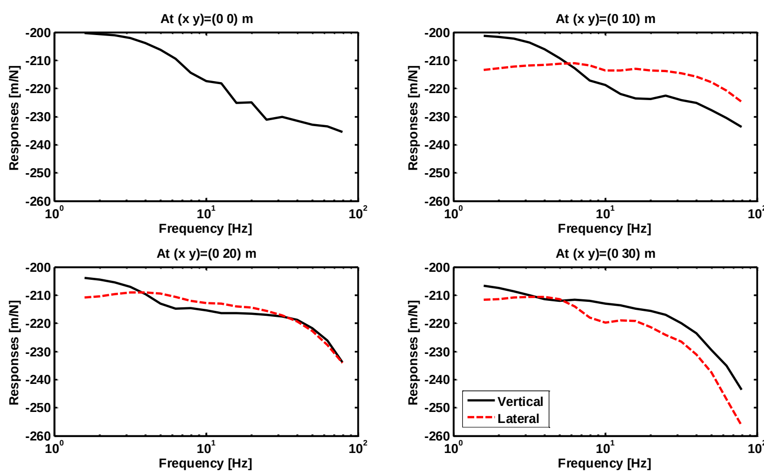


Figure 11 Responses at various locations along y direction on the ground surface predicted for a tunnel at a depth of 25 m

The pseudo-resultant vibration from models with different tunnel depth and lining thickness is compared in Figure 12. The black lines from the shallow tunnel model are higher than the red lines from the deep tunnel model. This gap increases slightly with frequency. The resultant amplitude is higher at the points further away from the tunnel (figures on the right when y=10 m) than that directly above the tunnel (figures on the left when y=0 m). The differences due to the tunnel thickness are small and mainly limited to high frequencies. Furthermore, the difference caused by the tunnel thickness is more evident in the lower figures for *x*=10 m.

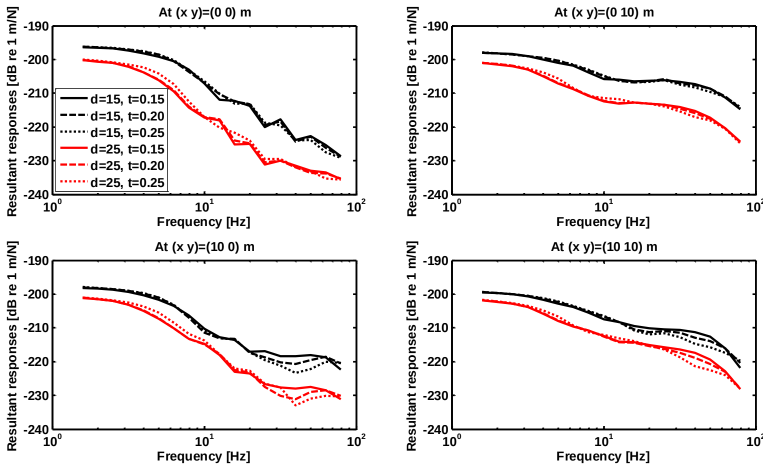


Figure 12 Pseudo-resultant response predicted for tunnels with different depth (d) and lining thickness (t) (unit: m)

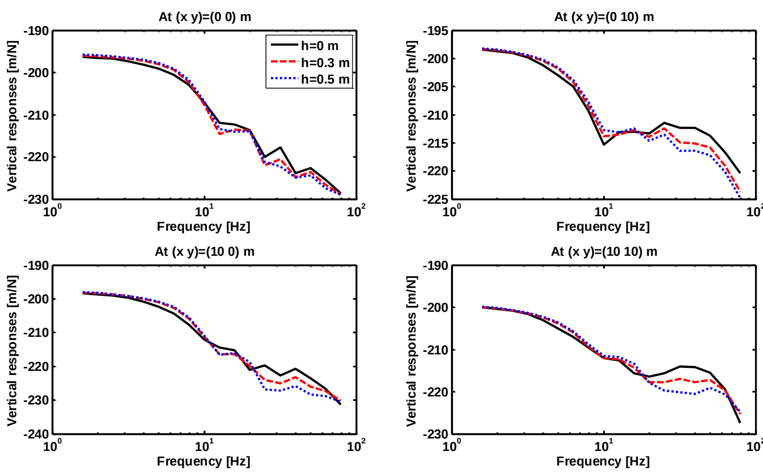


Figure 13 Vertical responses predicted with and without tunnel invert

Figure 13 compares the ground responses between models with and without invert. *h* is the height of the invert, as indicated in Figure 5. The tunnel geometry and soil properties of the model are based on those introduced previously, while the tunnel depth is 15 m. The invert adopted in the model is the full invert with the same material properties of the tunnel lining, see Figure 5(a). The black solid lines give the results from the model without invert. Generally speaking, the existence of an invert slightly increases the response on the ground surface at low frequencies, while decreasing the responses at high frequencies (above about 10 Hz).

5. CONCLUSIONS

In this paper, a tunnel with and without ground is modelled with the 2.5D finite element and boundary element method. With these models, the modes of the tunnel structure are investigated and a parametric study is carried out to investigate the influence of the depth, tunnel lining and invert structure.

The behaviour of the tunnel structure can be interpreted in terms of the deformation modes of a cylindrical shell. An increase in the lining thickness will increase the stiffness of the tunnel structure; therefore, the overall response amplitude is reduced but the cut-on frequencies of the deformation modes increase. In the same way, the tunnel invert also plays an important role in the deformation. The influence of the invert may differ for different types, but the tendency of the changes with the increase of the height of invert is the same as that of the tunnel thickness.

The responses at different points on the ground surface to a vertical harmonic force at the tunnel bottom are compared with two existing models and show good agreement. From a parametric study, the following conclusions can be made:

1) The decay rate against distance is slightly higher in the axial direction than that in the lateral direction. Above 6 Hz, the lateral displacements at 10 m away from the tunnel exceed the vertical displacements, but the vertical displacements are higher at further distances.

2) For a deeper tunnel, the vibration on the ground surface is predicted to be smaller for all excitation frequencies.

3) The influence of lining thickness and tunnel invert on the ground vibration is limited to high frequencies. However, there is a greater difference between the decay rates against distance in the tunnel direction and the direction perpendicular to the tunnel in the presence of a tunnel invert.

It is clear that the geometry of the tunnel structure has a significant influence on the distribution and radiation of vibration in the ground. Therefore, it is important to include such details in the modelling of the tunnel-ground system.

REFERENCES

(1) Hussein, M. F. M., Hunt, H. E. M., Rikse, L., Gupta, S., Degrande, G., Talbot, J. P., François, S. & Schevenels, M. (2008) *Using the PiP model for fast calculation of vibration from a railway tunnel in a multi-layered half-space*. Noise and Vibration Mitigation for Rail transportation systems. Springer Berlin Heidelberg, 136-142.

(2) Jones, C. J. C., Thompson, D. J., & M. Petyt. (2002) *A model for ground vibration from railway tunnels*. Proceedings of the ICE-Transport 153.2: 121-129.

(3) Andersen, Lars, & Jones, C. J. C. (2006) *Coupled boundary and finite element analysis of vibration from railway tunnels—a comparison of two-and three-dimensional models*. Journal of Sound and Vibration 293.3: 611-625.

(4) Sheng, X., Jones, C. J. C. & Thompson. D. J., (2005) *Modelling ground vibration from railways using wavenumber finite-and boundary-element methods*. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science 461.2059: 2043-2070.

(5) Francois, S., Schevenels, M., Lombaert, G., Galvın, P. & Degrande, G. (2010) *A 2.5D coupled FE-BE methodology for the dynamic interaction between longitudinally invariant structures and a layered halfspace*. Computer Methods in Applied Mechanics and Engineering, 199(23-24):1536–1548

(6) Nilsson, C. M., Jones, C. J. C. (2007) *Theory manual for WANDS 2.1*, ISVR Technical Memorandum N. **975**. University of Southampton
(7) Jin, Q. & Thompson, D. J. (2014) *Predictions of ground vibration from trains using a multi-body vehicle model coupled to a 2.5D FE/BE model of the track and ground*. The 21st International Congress on Sound and Vibration, Beijing, July
(8) Clouteau, D., Othman, R., Arnst, M. Chebli, H., Degrande, G., Klein, R., Chatterjee, P. & Janssens, B. (2004) *A numerical model for ground-borne vibrations from underground railway traffic based on a periodic FE-BE formulation*. In Proceedings of the Eighth International Workshop on Railway Noise.
(9) Hussein, M. F. M., Francois, S., Schevenels, M., Hunt, H. E. M., Talbot, J. P. & Degrande, G. (2014) *The fictitious force method for efficient calculation of vibration from a tunnel embedded in a multi-layered half-space*. Journal of Sound and Vibration (in press)