Full-Diversity Dispersion Matrices From Algebraic Field Extensions for Differential Spatial Modulation

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***Abstract*—We consider differential spatial modulation (DSM) operating in a block fading environment and propose sparse unitary dispersion matrices (DMs) using algebraic field extensions. The proposed DM sets are capable of exploiting full transmit diversity and, in contrast to the existing schemes, can be constructed for systems having an arbitrary number of transmit antennas. More specifically, two schemes are proposed: 1) field-extension-based DSM (FE-DSM), where only a single conventional symbol is transmitted per space–time block; and 2) FE-DSM striking a diversity–rate tradeoff (FE-DSM-DR), where multiple symbols are transmitted in each space–time block at the cost of a reduced transmit diversity gain. Furthermore, the FE-DSM scheme is analytically shown to achieve full transmit diversity, and both proposed schemes are shown to impose decoding complexity, which is independent of the size of the signal set. It is observed from our simulation results that the proposed FE-DSM scheme suffers no performance loss compared with the existing DM-based DSM (DM-DSM) scheme, whereas FE-DSM-DR is observed to give a better bit-error-ratio performance at higher data rates than its DM-DSM counterpart. Specifically, at data rates of 2.25 and 2.75 bits per channel use, FE-DSM-DR is observed to achieve about 1- and 2-dB signal-to-noise ratio (SNR) gain with respect to its DM-DSM counterpart.**

***Index Terms*—Decoding complexity, differential spatial modulation (DSM), dispersion matrices (DMs), diversity, field extension.**

I. Introduction

It is widely recognized that multiple-input multiple-output (MIMO) communication systems provide significant spectral efficiency improvements compared with single-input–single-output systems, owing to their higher degrees of freedom [1]. However, the benefit of increased spectral efficiency comes at the cost of high decoding complexity at the receiver, since the transmitted symbols interfere with each other at the receiver due to the simultaneous activation of multiple transmit antennas (TAs). For instance, in the classic Vertical Bell Laboratories Layered Space–Time architecture [2], the decoding complexity of the maximum-likelihood (ML) receiver exponentially increases with the number of TAs. An additional overhead in MIMO systems is that of estimating the channel coefficients between each TA and receive antenna (RA) pair and tracking their changes over the entire transmission duration for coherent detection [4]. Spatial modulation (SM) [5]–[8] is a beneficial multiantenna scheme that overcomes some of these drawbacks. Unlike the conventional MIMO system, the SM system activates only a single TA in each symbol duration, thereby avoiding the interference of transmitted symbols with each other at the receiver. As a further substantial benefit, it only requires a single radio frequency (RF) chain, as opposed to N_t chains, albeit this potentially precludes having a transmit diversity gain. More specifically, the bitstream is divided into blocks of $\log_2(MN_t)$ bits, and in each block, $\log_2(M)$ bits are used to select a symbol from an M-ary alphabet to be transmitted from a TA chosen from N_t TAs based on $\log_2(N_t)$ bits.

The SM system has been extensively studied with regard to various system parameters, which include its transmit diversity order [9]–[12], low-complexity near-ML detection [13]–[17], TA subset selection for performance versus complexity enhancement [18]–[22], and the impact of channel estimation error on the attainable performance [23]–[25]. A significant research effort was spent on increasing the transmit diversity order of the SM system, since achieving transmit diversity gain in the SM system was not straightforward, owing to the constraint of a single RF chain at the transmitter. This problem was partly addressed by conceiving space–time-coded SM schemes [9]–[12], which operate in an open-loop scenario, and by employing TA subset selection [20], [21], which operate in a closed-loop scenario. Note that both these approaches require accurate channel estimation and tracking at the receiver. Furthermore, the SM system has been studied in noncoherent communication scenarios [26]–[29], where the high-complexity channel estimation and tracking are dispensed with by employing differential encoding of the transmitted symbols. Naturally, this complexity reduction is achieved at 3-dB performance loss. This scheme is referred to as differential SM (DSM) throughout this paper. More specifically, Bian *et al.* in [26] have extended the conventional SM to a noncoherent scenario by obtaining dispersion matrices (DMs) from a set of

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TABLE I
COMPARISON OF VARIOUS EXISTING DSM SCHEMES

	P-DSM [26]	**DM-DSM** [27]	**CS-DSM** [28]
No. of transmit RF chains required	1	1	1
Throughput (bpcu)	$\frac{\log_2(M^{N_t})+\log_2\lfloor(N_t!)\rfloor_{2^p}}{N_t}$	$\frac{\log_2(M^{N_t/d})+\log_2(Q)}{N_t}$	$\frac{\log_2(Q'Q)}{2}$
Achievable diversity order	N_r	dN_r $(1 \leq d \leq N_t)$	$2N_r$

bpcu : bits per channel use

$(N_t \times N_t)$ permutation matrices having only a single nonzero element in every row and column, where each nonzero element is drawn from an M-ary phase-shift keying (PSK) signal set. This scheme is referred to as permutation-based DSM (P-DSM). In [27], a fixed set of sparse complex-valued DMs is used in conjunction with a set of diagonal matrices, whose elements are drawn from an M-ary PSK signal set. In this scheme, a higher transmit diversity order is shown to be achievable, albeit at the cost of a reduced transmission rate. We refer to this scheme as DM-based DSM (DM-DSM). More recently, a DM set construction was specifically proposed for two TAs [28], where a transmit diversity order of 2 is guaranteed to be achieved. This scheme, which employs a cyclic signal structure based on diagonal matrices along with a set of fixed DMs, is referred to as cyclic-signaling-based DSM (CS-DSM) in this paper. Table I compares these schemes, where Q denotes the number of DMs, Q' represents the number of diagonal matrices used for signaling [28], d is the transmit diversity order, and $\lfloor a \rfloor_{2^p}$ denotes the largest integer that is a power of 2 and smaller than a, where d is assumed to divide N_t with a zero remainder.

It is clear from Table I that the DM-DSM achieves the same throughput as that of P-DSM for $d = 1$ and $Q = \lfloor N_t! \rfloor_{2^p}$, but this will not yield any diversity advantage. To achieve the same throughput as that of P-DSM with full diversity, Q should be equal to $M^{N_t - 1}\lfloor N_t! \rfloor_{2^p}$. Similarly, CS-DSM is capable of achieving the same throughput as that of P-DSM for $Q' = M^{N_t}$ and $Q = \log_2(\lfloor N_t! \rfloor_{2^p})$. However, CS-DSM is specifically designed for the $N_t = 2$ case, where Q has been restricted to 2 [28]. Furthermore, CS-DSM is different from DM-DSM in the sense that only matrices are used for encoding the information bits, which is in contrast to the DM-DSM, where a set of DMs and a conventional signal set are used for encoding the information bits. To the best of our knowledge, there is no systematic method of obtaining the number of DMs required to achieve a desired throughput and transmit diversity order in systems with arbitrary N_t. Hence, in this paper, we focus on constructing structured DMs for DM-DSM schemes.

Against this background, the contributions of this paper are as follows.

1) We propose a systematic method of obtaining the set of DMs for DSM systems for an arbitrary N_t by exploiting the related results from algebraic field extensions. More specifically, we show that the companion matrix of an irreducible polynomial over a certain base field will be unitary, when the base field is a cyclotomic field [30], and exploit these unitary companion matrices for constructing DMs to be used in DSM. Additionally, we analytically show that the proposed scheme is capable of achieving full transmit diversity.
2) Furthermore, we generalize the proposed field-extension-based DSM (FE-DSM) scheme to strike a flexible trade-off between attainable diversity and multiplexing gain.
3) Finally, we evaluate the decoding complexity of ML detection of the proposed schemes and show that they offer significantly reduced complexity, owing to the DM-based approach of encoding information by exploiting results from [34].

The rest of this paper is organized as follows. Section II provides the system model of DSM. In Section III, the proposed DM set construction, as well as the diversity analysis of the proposed scheme, are presented. Specifically, Section III-A gives a brief overview of algebraic field extensions. Section III-B provides the proposed DM construction and our diversity analysis. In Section III-C, we conceive the low-complexity decoding method for the proposed schemes. Section IV provides our simulation results, and Section V concludes this paper.

Notations: If S_1 and S_2 are two sets, then $S_3 = S_1 \times S_2$ represents the Cartesian product of sets S_1 and S_2. Lowercase and uppercase boldface letters represent vectors and matrices, respectively. Furthermore, $\|\cdot\|$ represents the 2-norm of a vector or the Frobenius norm of a matrix. The notations of $(\cdot)^T$ and $(\cdot)^H$ indicate the transpose and Hermitian transpose of a vector/matrix, respectively, whereas $|\cdot|$ represents the cardinality of a given set or the magnitude of a complex quantity. Furthermore, $\otimes$ defines the Kronecker product of two matrices. $\mathcal{CN}(\mu, \sigma^2)$ denotes a complex Gaussian random variable with mean μ and variance σ^2. $\mathbb{R}$ and $\mathbb{C}$ represent the field of real and complex numbers, respectively. If F is a field, then $F[X]$ represents the ring of polynomials in X over F. $\mathbf{A}([a:b],:)$ defines a matrix with rows $a, a+1, \ldots, b-1, b$ of $\mathbf{A}$, and $\mathbf{A}(:,[a:b])$ is a matrix with columns $a, a+1, \ldots, b-1, b$ of $\mathbf{A}$. $\mathbf{I}_n$ represents an $n \times n$ identity matrix. If $\mathbf{x}$ is an n-length vector, then $\mathrm{diag}(\mathbf{x})$ represents an $n \times n$ diagonal matrix whose (i,i)th element is $\mathbf{x}_i$.

II. Differential Spatial Modulation System

Consider a MIMO system having N_r RAs and N_t TAs operating in a Rayleigh flat-fading channel, which is characterized by

$$\mathbf{Y}_i = \sqrt{\rho}\mathbf{H}_i\mathbf{X}_i + \mathbf{N}_i \quad (1)$$

where $\mathbf{Y}_i \in \mathbb{C}^{N_r \times N_t}$ is the received space–time matrix (STM); $\mathbf{X}_i \in \mathbb{C}^{N_t \times N_t}$ is the transmitted STM; $\mathbf{N}_i \in \mathbb{C}^{N_r \times N_t}$ and $\mathbf{H}_i \in \mathbb{C}^{N_r \times N_t}$ are the noise and channel matrices, respectively, whose entries are from $\mathcal{CN}(0, 1)$; and ρ denotes the average signal-to-noise ratio (SNR) at each RA. The subscript i in all matrices indicates the block index.

A. DSM System

Differential encoding [31], [32] of the transmitted STM is given by

$$\mathbf{X}_i = \mathbf{X}_{i-1}\mathbf{S}_i$$

where $\mathbf{S}_i \in \mathbb{C}^{N_t \times N_t}$ is the *unitary* STM to be transmitted during the symbol period of the ith block. For the transmitted STM $\mathbf{X}_i$ to become unitary, it is sufficient to ensure that $\mathbf{X}_0$ be unitary. In this paper, we consider $\mathbf{X}_0$ to be $\mathbf{I}_{N_t}$. Furthermore, each column of $\mathbf{S}_i$ is assumed to have only a single nonzero element, since the SM system employs only a single RF chain at the transmitter. Assuming that the channel remains constant over a period of two successive blocks, we have

$$\mathbf{Y}_{i-1} = \sqrt{\rho}\mathbf{H}_i\mathbf{X}_{i-1} + \mathbf{N}_{i-1}$$

and hence, (1) can be written as

$$\mathbf{Y}_i = \mathbf{Y}_{i-1}\mathbf{S}_i + \mathbf{N}_i - \mathbf{N}_{i-1}\mathbf{S}_i.$$

Assuming that there is no channel state information at the receiver, the optimal differential receiver [31] is given by

$$\hat{\mathbf{S}}_i = \arg\min_{\mathbf{S}\in\mathcal{S}} \|\mathbf{Y}_i - \mathbf{Y}_{i-1}\mathbf{S}\|^2 \quad (2)$$

where $\mathcal{S}$ is the set of transmit STMs.

B. DM-DSM

In the case of DM-DSM, each transmitted STM is of the following form:

$$\mathbf{S}_i = \mathbf{D}(\mathbf{s})\mathbf{A}_q \quad (3)$$

where we have $\mathbf{s} = [s_1, s_2, \ldots, s_{N_t}]$, $\mathbf{D}(\mathbf{s}) \in \mathcal{D} = \{\text{diag}(\mathbf{s}) | s_i \in \mathcal{L}_i - \text{PSK}\}$, and $\mathbf{A}_q \in \mathcal{A}$, where $\mathcal{A} = [\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_Q]$ is the set of DMs. The rate achieved by DM-DSM is given by

$$R_{\text{DM-DSM}} = \frac{\log_2(Q \cdot \mathcal{L}_1 \cdots \mathcal{L}_{N_t})}{N_t}\text{bpcu}.$$

In the following section, we propose a method for constructing the set $\mathcal{D}$ having diagonal or block-diagonal matrices as its elements and the set of DMs $\mathcal{A}$, such that they enable the DSM scheme to achieve full transmit diversity.

C1: We emphasize the condition that each element of $\mathcal{A}$ should be a unitary matrix [32] and should have only a single nonzero element in each column and row. The latter condition is necessary since the SM system can transmit only one symbol in each channel use, owing to a single RF chain at the transmitter.

III. Dispersion Matrix Set Construction

Here, we provide a brief overview of algebraic field extensions as required for our exposition on the proposed DM set construction. For further details, see [30] and [33].

A. Review of Field Extensions

Definitions: Let J be an extension of a field L and I be a subset of J, i.e., $I \subset J$. Field J is said to be generated by I if J is obtained by *adjoining*[1] the elements of I to L, and it is denoted by $J = L(I)$. If set I is finite, then the extension, which is denoted by J/L, is said to be *finitely generated*. If $\beta \in J$, then the *minimal polynomial* of β is the monic polynomial of least degree among the polynomials in $L[X]$ having β as a root. The extended field J can be viewed as a vector space, where its elements are considered as vectors, and the elements of L are viewed as scalars. The dimension of the vector space J is termed as the *degree of extension*, and it is denoted by $[J : L]$. Furthermore, the extension J/L is said to be an *algebraic extension*, if every element in J is a root of a nonzero polynomial with coefficients in L. An algebraic extension J/L is said to be *normal* if J is a *splitting field* of the family of polynomials $L[X]$, i.e., each polynomial in $L[X]$ splits or decomposes into linear factors over J. Furthermore, an algebraic extension H of J is said to be a *normal closure* of the algebraic extension J/L, if it is the only subfield of H that contains J and if a normal extension of L is H itself.

Let S be a conventional signal set, such as M-PSK, and $F = \mathbb{Q}(S)$ be the extended field of rationals over S. If α is a root of a minimal polynomial over F, which is given by

$$p(x) = x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_0 \quad (4)$$

then F can be extended by adjoining α to obtain $K = F(\alpha)$. The degree of extension $[K : F]$ is equal to n, since $p(x)$ is irreducible over F. Any element $k \in K$ can be expressed as $\sum_{i=0}^{n-1} f_i\alpha^i$, where $f_i \in F \,\forall\, 0 \le i \le n-1$. From [30, Sec. 7.3], there exists a natural mapping $k \mapsto \lambda_k \forall k \in K$ that embeds K in $\mathbf{M}_n(F)$, where λ_k is a linear transformation of K into itself. The *regular representation* of λ_k maps any $v \in K$ to kv. The linear transformation λ_α associated with α is given by

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & \cdots & 0 & -a_1 \\ 0 & 1 & \cdots & 0 & -a_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -a_{n-1} \end{bmatrix} \in F^{n\times n} \quad (5)$$

[1]The *adjoining* operation refers to including all the elements resulting from field operations considering the elements from the extended set $I \cup L$.

which is the companion matrix of $p(x)$. Thus, for any $k = \sum_{i=0}^{n-1} f_i\alpha^i \in K$, the associated λ_k is given by $\sum_{i=0}^{n-1} f_i\mathbf{M}^i$.

Lemma 1: Let K, F, and S be defined as above. For any $k = k_1 - k_2, k_1 \neq k_2 \in K, \lambda_k \in \mathbf{M}_n(F)$ is invertible.

Proof: The proof directly follows from K being a field, which guarantees the existence of the inverse for every nonzero element in K, and the fact that the natural mapping $k \mapsto \lambda_k$ is a one-to-one mapping. ∎

Lemma 2: If L is a normal closure of K/F and σ_i, $i = 0, 1, 2, \ldots, n-1$ are distinct F-homomorphisms from K to L; then, for any element $k \in K$, we have $\det(\lambda_k) = N_{K/F}(k) = \prod_{i=0}^{n-1} \sigma_i(k)$, where $N_{K/F}(k)$ is the norm of the element k from K to F [33, Th. 8].

B. Proposed DM Set for DSM

We propose to use the DM set given by

$$\mathcal{A} = \{\mathbf{I}_n, \mathbf{M}, \mathbf{M}^2, \ldots, \mathbf{M}^{n-1}\} \tag{6}$$

where $\mathbf{M}$ is as in (5), and n is chosen to be equal to N_t. However, to meet $C1$, every element of $\mathcal{A}$ has to be unitary. Note that it is sufficient to ensure that $\mathbf{M}$ is unitary for all the elements of $\mathcal{A}$ to be unitary. Hence, we have to satisfy the following equation:

$$\mathbf{M}\mathbf{M}^H = \mathbf{I}_n. \tag{7}$$

Note that $(\mathbf{M}\mathbf{M}^H)_{1,1} = |a_0|^2$ and $(\mathbf{M}\mathbf{M}^H)_{i,i} = 1 + |a_{i-1}|^2$ for $2 \leq i \leq n-1$. Thus, by choosing a_0 to be an element from the unit circle and $a_i = 0$ for $1 \leq i \leq n-1$, $C1$ can be satisfied. Thus, while constructing $\mathcal{A}$, we have to consider polynomials of the form $x^n + a_0$ with $|a_0| = 1$ values that are irreducible over F. Since we have $|\mathcal{A}| = n = N_t$, our construction results in a maximum of N_t DMs, i.e., $Q \leq N_t$. Furthermore, we assume that the set $\mathcal{D}$ has scaled identity matrices of the form $s\mathbf{I}_n$, where $s \in S$. Note that F should contain the specific signal set S from which s is chosen. Thus, the following conditions have to be met:

1) $S \subset F$; and
2) $p(x) = x^n + a_0$ with $|a_0| = 1$ should be irreducible over F.

We satisfy the given conditions by choosing $F = \mathbb{Q}(S, a_0)$, where a_0 is any transcendental element over $\mathbb{Q}(S)$ lying on the unit circle. In the following, we shall explain the method of constructing set $\mathcal{A}$ in detail.

Let S be a conventional M-PSK signal set denoted by $\{\omega_M^i\}_{i=0}^{M-1}$, where we have $\omega_M = e^{j2\pi/M}$ and $a_0 = -e^{ju_1}$, with u_1 being algebraic over $\mathbb{Q}$. For instance, u_1 can be $\sqrt{3}$, which is a root of the polynomial $x^2 - 3$. Note that a_0 is transcendental over $\mathbb{Q}(S)$, and we can choose $F = \mathbb{Q}(S, e^{ju_1})$. Thus, the polynomial $x^n + a_0 = x^n - e^{ju_1}$ (for any n) is irreducible over F. Therefore, we can have the extension $K = F(\alpha)$, where α is the primitive nth root of e^{ju_1}. Thus, the associated companion matrix is given by

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & \cdots & 0 & e^{ju_1} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \in F^{n \times n}. \tag{8}$$

Example 1: Consider $n = N_t = 4$ and $a_0 = -e^{j\sqrt{3}}$. Then, the elements of set $\mathcal{A}$ are given by $\mathbf{I}_4$

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & 0 & e^{j\sqrt{3}} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \mathbf{M}^2 = \begin{bmatrix} 0 & 0 & e^{j\sqrt{3}} & 0 \\ 0 & 0 & 0 & e^{j\sqrt{3}} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{M}^3 = \begin{bmatrix} 0 & e^{j\sqrt{3}} & 0 & 0 \\ 0 & 0 & e^{j\sqrt{3}} & 0 \\ 0 & 0 & 0 & e^{j\sqrt{3}} \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Remark 1: Note that a_0 has to be chosen in conjunction with the specific signal set S that maximizes a certain performance metric, such as the coding gain. This can be achieved by searching for an optimal a_0 over a large set of closely spaced transcendental elements on the unit circle.

We term the DSM scheme employing the proposed FE-DMs as an FE-DSM scheme. Since the set of transmit STMs is given by $\mathcal{S} = \mathcal{D} \times \mathcal{A}$, the rate achieved by the proposed scheme is

$$R_{\mathrm{FE-DSM}} = \frac{\log_2(|\mathcal{D}||\mathcal{A}|)}{N_t} = \frac{\log_2(MN_t)}{N_t} \text{ bpcu.}$$

1) Diversity Gain: The achievable transmit diversity order under differential detection [31, Sec. III-C] of (2) is given by

$$d = \min_{\mathbf{S}_1 \neq \mathbf{S}_2 \in \mathcal{S}} \mathrm{rank}(\mathbf{S}_1 - \mathbf{S}_2). \tag{9}$$

Proposition 1: The proposed FE-DSM scheme achieves a transmit diversity order of N_t, i.e., $d = N_t$.

Proof: The proof is given in Appendix A. ∎

2) Coding Gain: The coding gain of the proposed scheme is given by

$$G = \min_{\mathbf{S}_1 \neq \mathbf{S}_2 \in \mathcal{S}} \left|\det\left[(\mathbf{S}_1 - \mathbf{S}_2)(\mathbf{S}_1 - \mathbf{S}_2)^H\right]\right|^{\frac{1}{n}}. \tag{10}$$

In the following, we shall provide a simple expression for the determinant term in (10) that allows us to optimize the exponential a_0 in conjunction with an arbitrary M-PSK signal set to achieve a high coding gain.

Proposition 2: Consider an FE-DSM system using an M-PSK signal set and $N_t = n$ TAs. If $\mathbf{S} = e^{j(2\pi p/M)}\mathbf{M}^l$ and $\mathbf{S}_2 = e^{j(2\pi q/M)}\mathbf{M}^m$, where $0 \leq p, q \leq M-1$ and $0 \leq l, m \leq n-1$ such that $\mathbf{S}_1 \neq \mathbf{S}_2$, then $|\det[(\mathbf{S}_1 - \mathbf{S}_2)(\mathbf{S}_1 - \mathbf{S}_2)^H]|$ is given by

$$4^n \prod_{r=0}^{n-1} \sin^2\left(\frac{\pi(p-q)}{M} + \frac{(2\pi r + u_1)(m-l)}{2n}\right). \tag{11}$$

Proof: The proof is provided in Appendix B. ∎

In the following section, we provide a DM set construction based on two levels of field extensions, which facilitate a flexible tradeoff between the attainable transmit diversity and multiplexing gain.

C. FE-DSM With Diversity–Rate Tradeoff

The DM set construction presented in the previous section achieves a transmit diversity order of N_t, while transmitting only a single symbol from an M-PSK signal set. Note that when the channel conditions are good, it may not be necessary to exploit the full transmit diversity order. Under these conditions, we may aim at trading off the diversity gain for increasing the transmission rate. In the following, we shall provide a systematic method of constructing a DM set that achieves the desired diversity order and transmission rate. The DM set construction presented in the previous section may be viewed as a special case.

Let N_t be factored as $g \cdot h$. We construct a DM set that allows us to transmit h independent M-PSK symbols in each transmit STM and achieve transmit diversity order g. Considering $F = \mathbb{Q}(S, -e^{ju_1})$ as before and the extension $K = F(\alpha)$, where α is a primitive gth root of the polynomial $p_1(x) = x^g - e^{ju_1}$, we obtain the DM set given by

$$\mathcal{A}' = \{\mathbf{I}_g, \mathbf{M}, \mathbf{M}^2, \ldots, \mathbf{M}^{g-1}\} \tag{12}$$

where $\mathbf{M} \in F^{g\times g}$ is the companion matrix of $p_1(x)$. We define $\mathcal{D}$ to be a set of block-diagonal matrices given by

$$\mathcal{D}=\{\text{diag}(s_1\mathbf{A}_1,s_2\mathbf{A}_2,\ldots,s_h\mathbf{A}_h)|s_i \in M-\text{PSK},\mathbf{A}_i \in \mathcal{A}',\forall i\}. \tag{13}$$

Let us now consider the field extension $L = K(\beta)$ associated with the polynomial $p_2(x) = x^h - e^{ju_2}$, where e^{ju_2} is transcendental over K, and β is the primitive hth root of e^{ju_2}. Then, the regular representation of an element $l = \sum_{i=0}^{h-1} k_i\beta^i \in L$ is given by $\sum_{i=0}^{h-1} k_i\mathbf{N}^i$, where $k_i \in K$, $0 \le i \le h-1$, and $\mathbf{N} \in K^{h\times h}$ is the companion matrix of $p_2(x)$. We define the DM set as

$$\mathcal{A} = \left\{\mathbf{I}_n, \mathbf{N}', \mathbf{N}'^2, \ldots, \mathbf{N}'^{h-1}\right\} \tag{14}$$

where $\mathbf{N}' = \mathbf{N} \otimes \mathbf{I}_g$. The transmit STM set is given by $\mathcal{S} = \mathcal{D} \times \mathcal{A}$ as before. We refer to this scheme as the FE-DSM arrangement exhibiting a flexible diversity–rate tradeoff (FE-DSM-DR). Note that the DSM scheme requires each transmit STM to be unitary. The following proposition shows that this condition is satisfied.

Proposition 3: If $\mathcal{S}$ is the set of transmit STMs of FE-DSM-DR, then each element in $\mathcal{S}$ is unitary.

Proof: The proof is provided in Appendix C. ■

In the following, we shall provide an example construction to further illustrate the given set of points.

Since we have $|\mathcal{D}| = (Mg)^h$ and $|\mathcal{A}| = h$, the rate achieved by the FE-DSM-DR is given by

$$R_{\text{FE-DSM-DR}} = \frac{h\log_2(Mg) + \log_2(h)}{N_t} \text{ bpcu.} \tag{15}$$

Note that when we have $g = N_t$, FE-DSM-DR reduces to the FE-DSM scheme.

Example 2: Let $n = N_t = 4$, $g = h = 2$, $u_1 = \sqrt{2}$, and $u_2 = \sqrt{3}$. The elements of set $\mathcal{D}$ are

$$\begin{bmatrix} s_1 & 0 & 0 & 0\\ 0 & s_1 & 0 & 0\\ 0 & 0 & s_2 & 0\\ 0 & 0 & 0 & s_2\end{bmatrix}, \begin{bmatrix} 0 & s_1e^{j\sqrt{2}} & 0 & 0\\ s_1 & 0 & 0 & 0\\ 0 & 0 & s_2 & 0\\ 0 & 0 & 0 & s_2\end{bmatrix}$$

$$\begin{bmatrix} 0 & s_1e^{j\sqrt{2}} & 0 & 0\\ s_1 & 0 & 0 & 0\\ 0 & 0 & 0 & s_2e^{j\sqrt{2}}\\ 0 & 0 & s_2 & 0\end{bmatrix}, \begin{bmatrix} s_1 & 0 & 0 & 0\\ 0 & s_1 & 0 & 0\\ 0 & 0 & 0 & s_2e^{j\sqrt{2}}\\ 0 & 0 & s_2 & 0\end{bmatrix}$$

where s_1 and s_2 are from the classic M-PSK signal set. The elements of the DM set $\mathcal{A}$ are

$$\begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}, \begin{bmatrix} 0 & 0 & e^{j\sqrt{3}} & 0\\ 0 & 0 & 0 & e^{j\sqrt{3}}\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}.$$

Remark 2: Note that e^{ju_1} and e^{ju_2} have to be optimized in conjunction with the signal set S to maximize the coding gain. Unlike FE-DSM, the STM matrices of the FE-DSM-DR scheme are not representations of field elements, and hence, no closed-form expression is derived for the determinant of the codeword difference matrix. We resort to numerical search to arrive at the optimal values of u_1 and u_2.

D. ML Decoding Complexity

Here, we evaluate the complexity order of ML decoding for the proposed schemes. We show that the ML decoding complexity of both proposed schemes is independent of the size of the signal set S.

1) FE-DSM: Let $\chi = \{s\mathbf{e}_i | 1 \le i \le Q, s \in S\}$, where $\mathbf{e}_i$ is the ith column of $\mathbf{I}_Q$. Furthermore, let $\mathbf{G} = [\text{vec}(\mathbf{A}_1), \text{vec}(\mathbf{A}_2), \ldots, \text{vec}(\mathbf{A}_Q)] \in \mathbb{C}^{N_t^2\times Q}$, where $\mathbf{A}_i$ values are the elements of $\mathcal{A}$. Considering the optimal detection rule of (2), we have

$$\hat{\mathbf{S}}_i = \arg\min_{\mathbf{S}\in\mathcal{S}} \|\mathbf{Y}_i - \mathbf{Y}_{i-1}\mathbf{S}\|^2 \tag{16}$$

$$\equiv \arg\min_{s\in S, \mathbf{A}_q\in\mathcal{A}} \|\mathbf{Y}_i - \mathbf{Y}_{i-1}(s\mathbf{A}_q)\|^2 \tag{17}$$

$$\equiv \arg\min_{\mathbf{s}\in\chi} \left\|\bar{\mathbf{Y}}_i - (\mathbf{I}_{N_t} \otimes \mathbf{Y}_{i-1})\mathbf{G}\mathbf{s}\right\|^2 \tag{18}$$

where $\bar{\mathbf{Y}}_i = \text{vec}(\mathbf{Y}_i) \in \mathbb{C}^{N_rN_t\times 1}$. Since we have $|\chi| = Q|S|$, the decoding complexity order is $\mathcal{O}(QM)$, when S is an M-PSK signal set. However, owing to the interference-free nature of transmit vectors, the decoding complexity can be reduced from $\mathcal{O}(QM)$ to $\mathcal{O}(Q)$ with the aid of hard-limiting (HL)-based detection [34]. In other words, the ML decoding complexity of the FE-DSM scheme does not scale with the size of the signal set. By contrast, the existing full-diversity DSM scheme in [28] does not allow such low decoding complexity.

2) FE-DSM-DR: The optimal detection rule of (2) yields

$$\hat{\mathbf{S}}_i = \arg\min_{\mathbf{S}\in\mathcal{S}} \|\mathbf{Y}_i - \mathbf{Y}_{i-1}\mathbf{S}\|^2 \tag{19}$$

$$\equiv \arg \min_{\mathbf{D}\in\mathcal{D},\mathbf{A}_q\in\mathcal{A}} \|\mathbf{Y}_i - \mathbf{Y}_{i-1}\mathbf{D}\mathbf{A}_q\|^2 \tag{20}$$

$$\equiv \arg \min_{0\le k\le h-1} \left\{ \min_{\mathbf{D}\in\mathcal{D}} \left\| \mathbf{Y}_i - \mathbf{Y}_{i-1}\mathbf{D}\mathbf{N}'^k \right\|^2 \right\} \tag{21}$$

$$\left(\hat{k}, \hat{\mathbf{D}}^{(\hat{k})}\right) \equiv \arg \min_{0\le k\le h-1} \left\| \mathbf{Z}_i^{(k)} - \mathbf{Y}_{i-1}\hat{\mathbf{D}}^{(k)} \right\|^2 \tag{22}$$

where $\hat{\mathbf{D}}^{(k)} = \min_{\mathbf{D}\in\mathcal{D}} \|\mathbf{Z}_i^{(k)} - \mathbf{Y}_{i-1}\mathbf{D}\|^2$, and $\mathbf{Z}_i^{(k)} = \mathbf{Y}_i (\mathbf{N}'^k)^H$ for $0 \le k \le h-1$. Since $\mathbf{D}$ is block diagonal, we have

$$\hat{\mathbf{D}}^{(k)} = \min_{\mathbf{D}\in\mathcal{D}} \left\| \mathbf{Z}_i^{(k)} - \mathbf{Y}_{i-1}\mathbf{D} \right\|^2 \tag{23}$$

$$\equiv \sum_{l=1}^{h} \min_{s_l\in S, \mathbf{A}_{i_l}\in\mathcal{A}'} \left\| \mathbf{Z}_i^{(k)}(:,\mathcal{I}_l) - \mathbf{Y}_{i-1}(:,\mathcal{I}_l)\left(s_l \mathbf{A}_{i_l}\right) \right\|^2 \tag{24}$$

where $\mathcal{I}_l = [g(l-1)+1 : gl]$. By invoking the HL-based detector in [34], the search complexity of the minimization problem, i.e.,

$$\min_{s_l\in S, \mathbf{A}_{i_l}\in\mathcal{A}'} \left\| \mathbf{Z}_i^{(k)}(:,\mathcal{I}_l) - \mathbf{Y}_{i-1}(:,\mathcal{I}_l)\left(s_l \mathbf{A}_{i_l}\right) \right\|^2 \tag{25}$$

can be reduced from $\mathcal{O}(|S||\mathcal{A}'|)$ to $\mathcal{O}(|\mathcal{A}'|) = \mathcal{O}(g)$. Specifically, this is achieved by converting (25) into an interference-free system analogous to (18) and then employing the detector in [34]. Thus, the ML decoding complexity order of FE-DSM-DR is independent of the size of the signal set, and it is given by $\mathcal{O}(|\mathcal{A}'||\mathcal{A}|) = \mathcal{O}(gh) = \mathcal{O}(N_t)$.

E. Computational Complexity

Here, we compare the computational complexity of the ML detector of various existing schemes with that of the proposed scheme. Specifically, we show that all the existing schemes essentially impose the same computational complexity when operating at a given rate. However, since the ML decoding complexity order of the proposed schemes does not scale with the signal set, the computational complexity involved in ML decoding remains constant, when the size of the signal set is increased to increase the transmission rate.

Considering the ML detection rule of (2), we have

$$\hat{\mathbf{S}}_i = \arg\min_{\mathbf{S}\in\mathcal{S}} \|\mathbf{Y}_i - \mathbf{Y}_{i-1}\mathbf{S}\|^2 \tag{26}$$

where $\mathcal{S}$ is the set of transmit STMs. The number of real-valued multiplications in evaluating (26) is $6N_rN_t|\mathcal{S}|$, where $|\mathcal{S}|$ is the cardinality of the set of transmit STMs. When the transmission rate is fixed, $|\mathcal{S}|$ is essentially the same across all the existing schemes [26]–[28]. The direct evaluation of (26) results in the same computational complexity across all the schemes, since the number of nonzero elements in each $\mathbf{S} \in \mathcal{S}$ is the same in all of them. However, the proposed FE-DSM (DM-DSM [27]) scheme has the property that $\mathcal{S} = S \times \mathcal{A}$, which makes it amenable to HL-based ML detection (HL-ML) [34]. The computational complexity imposed by the HL-ML detector can be shown to be $(10N_tN_r + 9)|\mathcal{A}|$.[2] In the following section, we compare the computational complexity imposed by the direct ML solution in (26) to that of the HL-ML solution [34] by considering various system parameters and transmission rates.

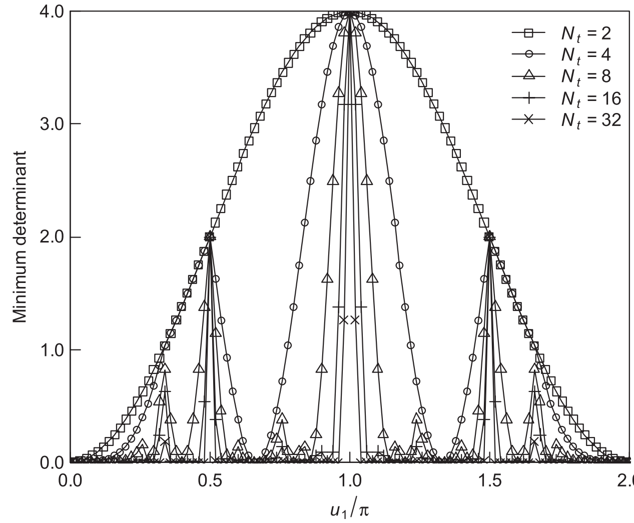

Fig. 1. Variation of coding gain as a function of u_1 in FE-DSM employing a BPSK signal set for various N_t values.

IV. SIMULATION RESULTS AND DISCUSSIONS

Simulation Parameters: In all our simulations, we have used block Rayleigh fading channels. In evaluating the bit error ratio (BER) of 10^{-t}, we have used at least 10^{t+2} bits. For DM-DSM schemes operating at different rates, the optimal DM sets are obtained by optimizing the coding gain over a large set of feasible matrices in conjunction with the associated M-PSK signal set. The parameter e^{ju_1} of FE-DSM and the parameters (e^{ju_1}, e^{ju_2}) of FE-DSM-DR are optimized in conjunction with the associated signal sets to obtain the optimal set of DMs. For the FE-DSM scheme using an M-PSK signal set, it is observed that $u_1 = 2\pi/M$ is optimal for any value of N_t. Fig. 1 shows the achievable coding gain of FE-DSM employing a binary phase-shift keying (BPSK) signal set. It is clear in Fig. 1 that the value of $u_1 = \pi$ remains optimal even when N_t is varied.

Fig. 2 compares the BER performance of the FE-DSM and DM-DSM schemes, both having $N_t = 2$ and employing 4-PSK as well as 16-PSK signal sets that achieve a throughput of 1.5 and 2.5 bpcu, respectively. The BER performance of P-DSM is also provided to highlight the transmit diversity gain achieved by the DM-DSM scheme. Furthermore, the BER performance of the proposed codebooks in the coherent scenario is also provided. Fig. 3 compares the BER performance of the FE-DSM and DM-DSM schemes, both having $N_t = 4$ and employing 4-PSK, as well as 16-PSK signal sets that achieve a throughput of 1 and 1.5 bpcu, respectively. It is clear in Figs. 2 and 3 that

[2]It takes $4N_tN_r|\mathcal{A}|$ multiplications to compute $(\mathbf{I}_{N_t} \otimes \mathbf{Y}_{i-1})\mathbf{G}$ and $(6N_tN_r + 9)|\mathcal{A}|$ multiplications to compute various decision metrics of the HL-ML detector [34]. For further details, see [34, Sec. IV-B].

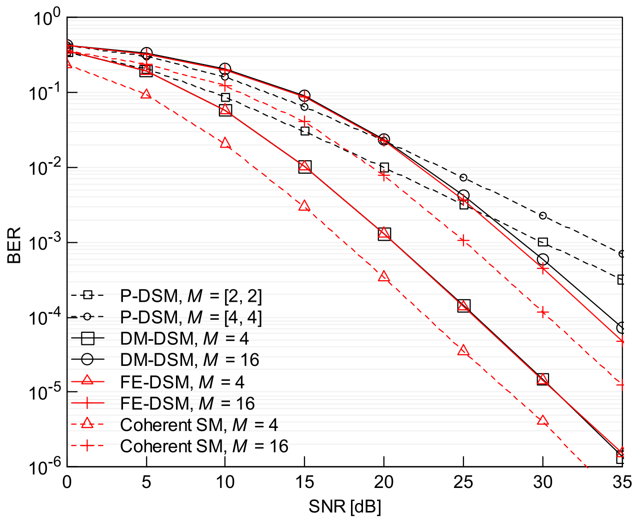

Fig. 2. BER performance of the DM-DSM and FE-DSM schemes, having $N_t = 2$ and employing 4-PSK and 16-PSK signal sets. The BER performance of the P-DSM scheme is provided to highlight the transmit diversity gain achieved in DM-DSM and FE-DSM.

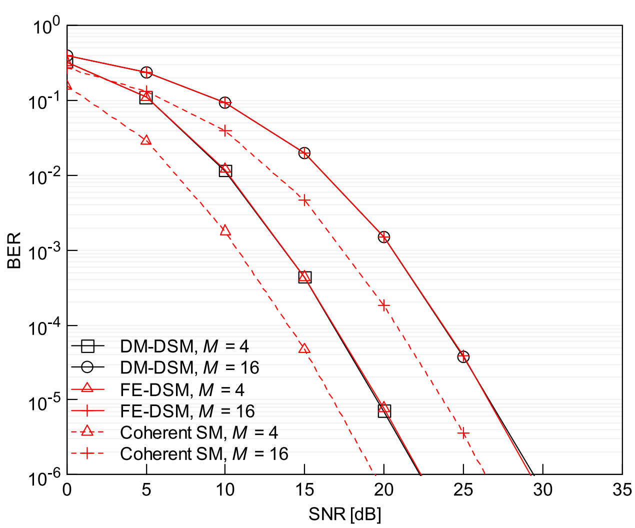

Fig. 3. BER performance of the DM-DSM and FE-DSM schemes, having $N_t = 4$ and employing M-PSK signal sets.

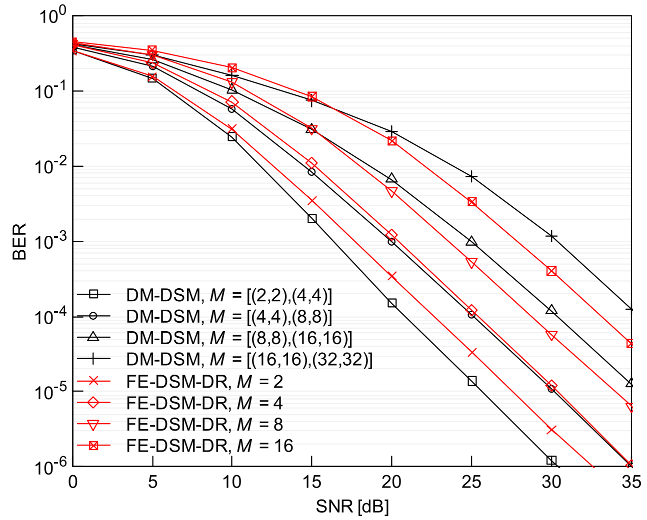

Fig. 4. BER performance of the DM-DSM and FE-DSM-DR schemes, having $N_t = 4$ and employing M-PSK signal sets.

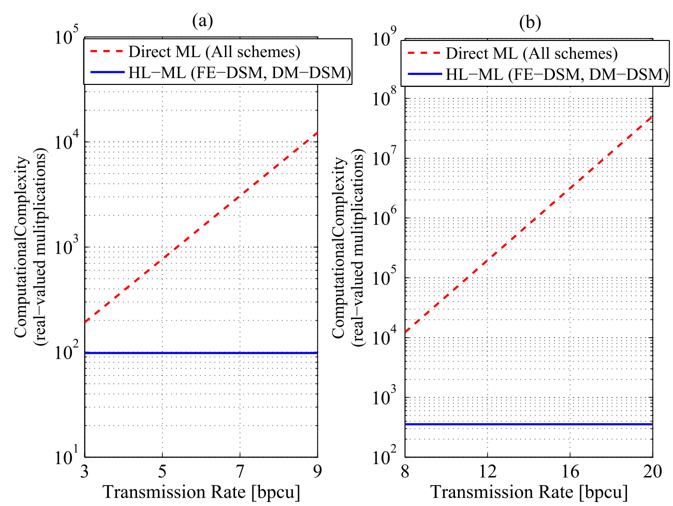

Fig. 5. Comparison of the computational complexity imposed by the ML detector in the existing schemes with that of the proposed scheme in systems having $N_t = 2, 4$ and $N_r = 2$ and employing various transmission rates.

the proposed FE-DSM scheme suffers from no performance loss compared with the DM-DSM scheme, and there is a 3-dB performance loss with respect to the coherent counterparts, which is as expected.

Fig. 4 compares the BER performance of FE-DSM-DR and DM-DSM that trades off diversity gain against throughput. Both the systems are assumed to have $N_t = 4$. Specifically, four data rates are considered for comparison. For $h = g = 2$, FE-DSM-DR achieves throughput values of 1.25, 1.75, 2.25, and 2.75 bpcu when employing BPSK, quaternary phase-shift keying, 8-PSK, and 16-PSK signal sets, respectively. The DM-DSM scheme is assumed to have a set of four DMs as proposed in [27] and employs $M = [(L_1, L_1), (L_2, L_2)]$, where L_1 and L_2 correspond to the sizes of the PSK signal sets encoding symbols s_1 and s_2, respectively. To elaborate, L_1 and L_2 are chosen so that the rates achieved by the proposed scheme and the FE-DSM scheme are the same. When operating at 1.25 bpcu, it can be observed in Fig. 4 that FE-DSM-DR suffers from a 1.5-dB SNR loss at a BER of 10^{-4} compared with DM-DSM. However, as the rate is increased from 1.25 to 2.75 bpcu, the performance of FE-DSM-DR improves, which is evident in Fig. 4. Specifically, when operating at 2.25 and 2.75 bpcu, it is observed that the FE-DSM-DR scheme achieves an SNR gain of about 1 dB and about 2 dB at a BER of 10^{-4}, respectively.

Fig. 5 gives the computational complexity imposed by the ML detector in various existing schemes along with that of the proposed FE-DSM scheme in systems having $N_t = 2, 4$ and $N_r = 2$ and employing various transmission rates. In the case of $N_t = 2$, the P-DSM scheme is assumed to employ BPSK, 4-PSK, 8-PSK, and 16-PSK to achieve a transmission rate of 3, 5, 7, and 9 bpcu, respectively, and in the case of $N_t = 4$, the same signal sets achieve a transmission rate of 8, 12, 16, and 20 bpcu, respectively. In the case of FE-DSM, the size of the DM set is fixed to 2 and 4 in the case of $N_t = 2$ and $N_t = 4$, respectively, and the size of the signal set (or the number of cyclic matrices in the case of CS-DSM [26] when $N_t = 2$) is assumed to vary to increase the transmission rate. It is clear in Fig. 5 that the HL-ML detector results in significant complexity reductions over the direct ML solution. Specifically, at a transmission rate of 5 and 8 bpcu, a reduction of about 670 multiplications in the case of $N_t = 2$ and about 11934 multiplications in the case of $N_t = 4$ is observed, respectively.

Future Work: While the proposed DM set constructions cater to the requirements of the SM scheme relying on differential encoding, it would be interesting to study the feasibility of extending the proposed schemes to generalized SM (GSM) [35], where more than one TAs are activated during each channel use. Note, however, that this is not straightforward, since

1) the transmitted STMs in GSM may not be unitary in general; and
2) the product of any two distinct transmit STMs does not satisfy the sparsity constraint analogous to condition C1 given in Section II-B.

As inferred from Fig. 4, the FE-DSM-DR-based DMs are not optimal at low rates (unlike the FE-DSM scheme). Furthermore, considering algebraic structures for designing DM sets would enable us to quantify the achievable diversity order and the coding gain in addition to attaining the benefits of a simple and systematic encoding at the transmitter. Thus, worth investigating are other representations of algebraic structures such as division algebras for their suitability in obtaining sparse, full-diversity, and optimal DM sets for differential SM/GSM schemes.

V. Conclusion

We have proposed a systematic method for obtaining a DM set for DSM with the aid of algebraic field extensions. It was analytically shown that the proposed FE-DSM achieves full transmit diversity. Furthermore, a closed-form expression was derived for the determinant of the codeword difference matrix. The proposed FE-DSM scheme was then further extended to FE-DSM-DR, which stroke a flexible tradeoff between diversity gain and throughput. Both the proposed schemes were shown to offer ML decoding complexity, which is independent of the size of the signal set. Our simulation results have shown that the FE-DSM scheme achieves the same BER performance as the DM-DSM scheme, whereas FE-DSM-DR is observed to give a better BER performance at higher rates compared with its DM-DSM counterpart.

Appendix A
Proof of Proposition 1

Let $\mathbf{S}_1 = s\mathbf{M}^i$ and $\mathbf{S}_2 = s'\mathbf{M}^j$, where $s, s' \in S \subset F = \mathbb{Q}(S, a_0)$. Recall that $\mathbf{S}_1$ and $\mathbf{S}_2$ are regular representations of $k_1 = s\alpha^i$ and $k_2 = s'\alpha^j$, respectively. Then, we have $\lambda_k = \mathbf{S}_1 - \mathbf{S}_2$, where $k = k_1 - k_2$. From Lemma 1, we see that λ_k is invertible for all $k_1, k_2 \in K$ and $k_1 \neq k_2$. Thus, we have $\mathbf{S}_1 - \mathbf{S}_2$ as invertible. In other words, we have $\text{rank}(\mathbf{S}_1 - \mathbf{S}_2) = n = N_t, \forall\, \mathbf{S}_1 \neq \mathbf{S}_2 \in \mathcal{S}$. Thus, we have $\min_{\mathbf{S}_1 \neq \mathbf{S}_2 \in \mathcal{S}} \text{rank}(\mathbf{S}_1 - \mathbf{S}_2) = N_t$. This concludes the proof.

Appendix B
Proof of Proposition 2

If $\lambda_k = \mathbf{S}_1 - \mathbf{S}_2$, then the element k associated with λ_k is given by $s\alpha^m - s'\alpha^l$, where $s = e^{j(2\pi p/M)}$, $s' = e^{j(2\pi q/M)}$, and α is the primitive nth root of $-a_0$. From Lemma 2, we have $\det(\lambda_k) = \det(\mathbf{S}_1 - \mathbf{S}_2) = \prod_{r=0}^{n-1} \sigma_r(k)$, where we have $\sigma_r : \alpha \mapsto \alpha_r$, such that α_r, $0 \leq r \leq n-1$ are the nth roots of $-a_0 = e^{ju_1}$. Therefore, we have

$$\det(\mathbf{S}_1 - \mathbf{S}_2) = \prod_{r=0}^{n-1} \sigma_r(k) = \prod_{r=0}^{n-1} \sigma_r(s\alpha^m - s'\alpha^l) \tag{27}$$

$$= \prod_{r=0}^{n-1} \left(s\left(\sigma_r(\alpha)\right)^m - s'\left(\sigma_r(\alpha)\right)^l \right) \tag{28}$$

$$= \prod_{r=0}^{n-1} \left(e^{j\frac{2\pi p}{M} + \frac{(2\pi r + u_1)m}{n}} - e^{j\frac{2\pi q}{M} + \frac{(2\pi r + u_1)l}{n}} \right). \tag{29}$$

Thus, we have

$$\det\left[(\mathbf{S}_1 - \mathbf{S}_2)(\mathbf{S}_1 - \mathbf{S}_2)^H\right]$$

$$= \left| \prod_{r=0}^{n-1} \left(e^{j\frac{2\pi p}{M} + \frac{(2\pi r + u_1)m}{n}} - e^{j\frac{2\pi q}{M} + \frac{(2\pi r + u_1)l}{n}} \right) \right|^2 \tag{30}$$

$$= 4^n \prod_{r=0}^{n-1} \sin^2\left(\frac{\pi(p-q)}{M} + \frac{(2\pi r + u_1)(m-l)}{2n} \right). \tag{31}$$

This concludes the proof.

Appendix C
Proof of Proposition 3

Let $\mathbf{S} \in \mathcal{S}$ such that $\mathbf{S} = \text{diag}(s_1\mathbf{A}_1, s_2\mathbf{A}_2, \ldots, s_h\mathbf{A}_h)\mathbf{N}'^k$ for some $0 \leq k \leq h-1$, $s_i \in M$-PSK signal set, $\mathbf{A}_i \in \mathcal{A}'$ for $1 \leq i \leq h$. Consider $\mathbf{S}\mathbf{S}^H = \mathbf{D}\mathbf{N}'^k\mathbf{N}'^{kH}\mathbf{D}^H$, where $\mathbf{D} = \text{diag}(s_1\mathbf{A}_1, s_2\mathbf{A}_2, \ldots, s_h\mathbf{A}_h)$. Since $\mathbf{N}'^k = (\mathbf{N} \otimes \mathbf{I}_g)^k = \mathbf{N}^k \otimes \mathbf{I}_g$, we have

$$\mathbf{N}'^k\mathbf{N}'^{kH} = (\mathbf{N}^k \otimes \mathbf{I}_g)(\mathbf{N}^k \otimes \mathbf{I}_g)^H$$

$$= \left(\mathbf{N}^k\mathbf{N}^{kH} \otimes \mathbf{I}_g\right)$$

$$= \mathbf{I}_h \otimes \mathbf{I}_g = \mathbf{I}_n.$$

Thus, we have

$$\mathbf{S}\mathbf{S}^H = \mathbf{D}\mathbf{D}^H$$

$$= \text{diag}\left(\mathbf{A}_1\mathbf{A}_1^H, \mathbf{A}_2\mathbf{A}_2^H, \ldots, \mathbf{A}_h\mathbf{A}_h^H\right)$$

$$= \text{diag}(\mathbf{I}_g, \mathbf{I}_g, \ldots, \mathbf{I}_g) = \mathbf{I}_n.$$

This concludes the proof.

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Full-Diversity Dispersion Matrices From Algebraic Field Extensions for Differential Spatial Modulation

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***Abstract*—We consider differential spatial modulation (DSM) operating in a block fading environment and propose sparse unitary dispersion matrices (DMs) using algebraic field extensions. The proposed DM sets are capable of exploiting full transmit diversity and, in contrast to the existing schemes, can be constructed for systems having an arbitrary number of transmit antennas. More specifically, two schemes are proposed: 1) field-extension-based DSM (FE-DSM), where only a single conventional symbol is transmitted per space–time block; and 2) FE-DSM striking a diversity–rate tradeoff (FE-DSM-DR), where multiple symbols are transmitted in each space–time block at the cost of a reduced transmit diversity gain. Furthermore, the FE-DSM scheme is analytically shown to achieve full transmit diversity, and both proposed schemes are shown to impose decoding complexity, which is independent of the size of the signal set. It is observed from our simulation results that the proposed FE-DSM scheme suffers no performance loss compared with the existing DM-based DSM (DM-DSM) scheme, whereas FE-DSM-DR is observed to give a better bit-error-ratio performance at higher data rates than its DM-DSM counterpart. Specifically, at data rates of 2.25 and 2.75 bits per channel use, FE-DSM-DR is observed to achieve about 1- and 2-dB signal-to-noise ratio (SNR) gain with respect to its DM-DSM counterpart.**

***Index Terms*—Decoding complexity, differential spatial modulation (DSM), dispersion matrices (DMs), diversity, field extension.**

I. Introduction

It is widely recognized that multiple-input multiple-output (MIMO) communication systems provide significant spectral efficiency improvements compared with single-input–single-output systems, owing to their higher degrees of freedom [1]. However, the benefit of increased spectral efficiency comes at the cost of high decoding complexity at the receiver, since the transmitted symbols interfere with each other at the receiver due to the simultaneous activation of multiple transmit antennas (TAs). For instance, in the classic Vertical Bell Laboratories Layered Space–Time architecture [2], the decoding complexity of the maximum-likelihood (ML) receiver exponentially increases with the number of TAs. An additional overhead in MIMO systems is that of estimating the channel coefficients between each TA and receive antenna (RA) pair and tracking their changes over the entire transmission duration for coherent detection [4]. Spatial modulation (SM) [5]–[8] is a beneficial multiantenna scheme that overcomes some of these drawbacks. Unlike the conventional MIMO system, the SM system activates only a single TA in each symbol duration, thereby avoiding the interference of transmitted symbols with each other at the receiver. As a further substantial benefit, it only requires a single radio frequency (RF) chain, as opposed to N_t chains, albeit this potentially precludes having a transmit diversity gain. More specifically, the bitstream is divided into blocks of $\log_2(MN_t)$ bits, and in each block, $\log_2(M)$ bits are used to select a symbol from an M-ary alphabet to be transmitted from a TA chosen from N_t TAs based on $\log_2(N_t)$ bits.

The SM system has been extensively studied with regard to various system parameters, which include its transmit diversity order [9]–[12], low-complexity near-ML detection [13]–[17], TA subset selection for performance versus complexity enhancement [18]–[22], and the impact of channel estimation error on the attainable performance [23]–[25]. A significant research effort was spent on increasing the transmit diversity order of the SM system, since achieving transmit diversity gain in the SM system was not straightforward, owing to the constraint of a single RF chain at the transmitter. This problem was partly addressed by conceiving space–time-coded SM schemes [9]–[12], which operate in an open-loop scenario, and by employing TA subset selection [20], [21], which operate in a closed-loop scenario. Note that both these approaches require accurate channel estimation and tracking at the receiver. Furthermore, the SM system has been studied in noncoherent communication scenarios [26]–[29], where the high-complexity channel estimation and tracking are dispensed with by employing differential encoding of the transmitted symbols. Naturally, this complexity reduction is achieved at 3-dB performance loss. This scheme is referred to as differential SM (DSM) throughout this paper. More specifically, Bian *et al.* in [26] have extended the conventional SM to a noncoherent scenario by obtaining dispersion matrices (DMs) from a set of

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TABLE I
COMPARISON OF VARIOUS EXISTING DSM SCHEMES

	P-DSM [26]	**DM-DSM** [27]	**CS-DSM** [28]
No. of transmit RF chains required	1	1	1
Throughput (bpcu)	$\frac{\log_2(M^{N_t})+\log_2\lfloor(N_t!)\rfloor_{2^p}}{N_t}$	$\frac{\log_2(M^{N_t/d})+\log_2(Q)}{N_t}$	$\frac{\log_2(Q'Q)}{2}$
Achievable diversity order	N_r	dN_r $(1 \leq d \leq N_t)$	$2N_r$

bpcu : bits per channel use

$(N_t \times N_t)$ permutation matrices having only a single nonzero element in every row and column, where each nonzero element is drawn from an M-ary phase-shift keying (PSK) signal set. This scheme is referred to as permutation-based DSM (P-DSM). In [27], a fixed set of sparse complex-valued DMs is used in conjunction with a set of diagonal matrices, whose elements are drawn from an M-ary PSK signal set. In this scheme, a higher transmit diversity order is shown to be achievable, albeit at the cost of a reduced transmission rate. We refer to this scheme as DM-based DSM (DM-DSM). More recently, a DM set construction was specifically proposed for two TAs [28], where a transmit diversity order of 2 is guaranteed to be achieved. This scheme, which employs a cyclic signal structure based on diagonal matrices along with a set of fixed DMs, is referred to as cyclic-signaling-based DSM (CS-DSM) in this paper. Table I compares these schemes, where Q denotes the number of DMs, Q' represents the number of diagonal matrices used for signaling [28], d is the transmit diversity order, and $\lfloor a \rfloor_{2^p}$ denotes the largest integer that is a power of 2 and smaller than a, where d is assumed to divide N_t with a zero remainder.

It is clear from Table I that the DM-DSM achieves the same throughput as that of P-DSM for $d = 1$ and $Q = \lfloor N_t! \rfloor_{2^p}$, but this will not yield any diversity advantage. To achieve the same throughput as that of P-DSM with full diversity, Q should be equal to $M^{N_t-1}\lfloor N_t! \rfloor_{2^p}$. Similarly, CS-DSM is capable of achieving the same throughput as that of P-DSM for $Q' = M^{N_t}$ and $Q = \log_2(\lfloor N_t! \rfloor_{2^p})$. However, CS-DSM is specifically designed for the $N_t = 2$ case, where Q has been restricted to 2 [28]. Furthermore, CS-DSM is different from DM-DSM in the sense that only matrices are used for encoding the information bits, which is in contrast to the DM-DSM, where a set of DMs and a conventional signal set are used for encoding the information bits. To the best of our knowledge, there is no systematic method of obtaining the number of DMs required to achieve a desired throughput and transmit diversity order in systems with arbitrary N_t. Hence, in this paper, we focus on constructing structured DMs for DM-DSM schemes.

Against this background, the contributions of this paper are as follows.

1) We propose a systematic method of obtaining the set of DMs for DSM systems for an arbitrary N_t by exploiting the related results from algebraic field extensions. More specifically, we show that the companion matrix of an irreducible polynomial over a certain base field will be unitary, when the base field is a cyclotomic field [30], and exploit these unitary companion matrices for constructing DMs to be used in DSM. Additionally, we analytically show that the proposed scheme is capable of achieving full transmit diversity.
2) Furthermore, we generalize the proposed field-extension-based DSM (FE-DSM) scheme to strike a flexible trade-off between attainable diversity and multiplexing gain.
3) Finally, we evaluate the decoding complexity of ML detection of the proposed schemes and show that they offer significantly reduced complexity, owing to the DM-based approach of encoding information by exploiting results from [34].

The rest of this paper is organized as follows. Section II provides the system model of DSM. In Section III, the proposed DM set construction, as well as the diversity analysis of the proposed scheme, are presented. Specifically, Section III-A gives a brief overview of algebraic field extensions. Section III-B provides the proposed DM construction and our diversity analysis. In Section III-C, we conceive the low-complexity decoding method for the proposed schemes. Section IV provides our simulation results, and Section V concludes this paper.

Notations: If S_1 and S_2 are two sets, then $S_3 = S_1 \times S_2$ represents the Cartesian product of sets S_1 and S_2. Lowercase and uppercase boldface letters represent vectors and matrices, respectively. Furthermore, $\|\cdot\|$ represents the 2-norm of a vector or the Frobenius norm of a matrix. The notations of $(\cdot)^T$ and $(\cdot)^H$ indicate the transpose and Hermitian transpose of a vector/matrix, respectively, whereas $|\cdot|$ represents the cardinality of a given set or the magnitude of a complex quantity. Furthermore, $\otimes$ defines the Kronecker product of two matrices. $\mathcal{CN}(\mu, \sigma^2)$ denotes a complex Gaussian random variable with mean μ and variance σ^2. $\mathbb{R}$ and $\mathbb{C}$ represent the field of real and complex numbers, respectively. If F is a field, then $F[X]$ represents the ring of polynomials in X over F. $\mathbf{A}([a:b],:)$ defines a matrix with rows $a, a+1, \ldots, b-1, b$ of $\mathbf{A}$, and $\mathbf{A}(:,[a:b])$ is a matrix with columns $a, a+1, \ldots, b-1, b$ of $\mathbf{A}$. $\mathbf{I}_n$ represents an $n \times n$ identity matrix. If $\mathbf{x}$ is an n-length vector, then $\mathrm{diag}(\mathbf{x})$ represents an $n \times n$ diagonal matrix whose (i,i)th element is $\mathbf{x}_i$.

II. Differential Spatial Modulation System

Consider a MIMO system having N_r RAs and N_t TAs operating in a Rayleigh flat-fading channel, which is characterized by

$$\mathbf{Y}_i = \sqrt{\rho}\mathbf{H}_i\mathbf{X}_i + \mathbf{N}_i \tag{1}$$

where $\mathbf{Y}_i \in \mathbb{C}^{N_r \times N_t}$ is the received space–time matrix (STM); $\mathbf{X}_i \in \mathbb{C}^{N_t \times N_t}$ is the transmitted STM; $\mathbf{N}_i \in \mathbb{C}^{N_r \times N_t}$ and $\mathbf{H}_i \in \mathbb{C}^{N_r \times N_t}$ are the noise and channel matrices, respectively, whose entries are from $\mathcal{CN}(0,1)$; and ρ denotes the average signal-to-noise ratio (SNR) at each RA. The subscript i in all matrices indicates the block index.

A. DSM System

Differential encoding [31], [32] of the transmitted STM is given by

$$\mathbf{X}_i = \mathbf{X}_{i-1}\mathbf{S}_i$$

where $\mathbf{S}_i \in \mathbb{C}^{N_t \times N_t}$ is the *unitary* STM to be transmitted during the symbol period of the ith block. For the transmitted STM $\mathbf{X}_i$ to become unitary, it is sufficient to ensure that $\mathbf{X}_0$ be unitary. In this paper, we consider $\mathbf{X}_0$ to be $\mathbf{I}_{N_t}$. Furthermore, each column of $\mathbf{S}_i$ is assumed to have only a single nonzero element, since the SM system employs only a single RF chain at the transmitter. Assuming that the channel remains constant over a period of two successive blocks, we have

$$\mathbf{Y}_{i-1} = \sqrt{\rho}\mathbf{H}_i\mathbf{X}_{i-1} + \mathbf{N}_{i-1}$$

and hence, (1) can be written as

$$\mathbf{Y}_i = \mathbf{Y}_{i-1}\mathbf{S}_i + \mathbf{N}_i - \mathbf{N}_{i-1}\mathbf{S}_i.$$

Assuming that there is no channel state information at the receiver, the optimal differential receiver [31] is given by

$$\hat{\mathbf{S}}_i = \arg\min_{\mathbf{S}\in\mathcal{S}} \|\mathbf{Y}_i - \mathbf{Y}_{i-1}\mathbf{S}\|^2 \tag{2}$$

where $\mathcal{S}$ is the set of transmit STMs.

B. DM-DSM

In the case of DM-DSM, each transmitted STM is of the following form:

$$\mathbf{S}_i = \mathbf{D}(\mathbf{s})\mathbf{A}_q \tag{3}$$

where we have $\mathbf{s} = [s_1, s_2, \ldots, s_{N_t}]$, $\mathbf{D}(\mathbf{s}) \in \mathcal{D} = \{\mathrm{diag}(\mathbf{s}) | s_i \in \mathcal{L}_i - \mathrm{PSK}\}$, and $\mathbf{A}_q \in \mathcal{A}$, where $\mathcal{A} = [\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_Q]$ is the set of DMs. The rate achieved by DM-DSM is given by

$$R_{\mathrm{DM-DSM}} = \frac{\log_2(Q \cdot \mathcal{L}_1 \cdots \mathcal{L}_{N_t})}{N_t}\text{bpcu}.$$

In the following section, we propose a method for constructing the set $\mathcal{D}$ having diagonal or block-diagonal matrices as its elements and the set of DMs $\mathcal{A}$, such that they enable the DSM scheme to achieve full transmit diversity.

C1: We emphasize the condition that each element of $\mathcal{A}$ should be a unitary matrix [32] and should have only a single nonzero element in each column and row. The latter condition is necessary since the SM system can transmit only one symbol in each channel use, owing to a single RF chain at the transmitter.

III. Dispersion Matrix Set Construction

Here, we provide a brief overview of algebraic field extensions as required for our exposition on the proposed DM set construction. For further details, see [30] and [33].

A. Review of Field Extensions

Definitions: Let J be an extension of a field L and I be a subset of J, i.e., $I \subset J$. Field J is said to be generated by I if J is obtained by *adjoining*[1] the elements of I to L, and it is denoted by $J = L(I)$. If set I is finite, then the extension, which is denoted by J/L, is said to be *finitely generated*. If $\beta \in J$, then the *minimal polynomial* of β is the monic polynomial of least degree among the polynomials in $L[X]$ having β as a root. The extended field J can be viewed as a vector space, where its elements are considered as vectors, and the elements of L are viewed as scalars. The dimension of the vector space J is termed as the *degree of extension*, and it is denoted by $[J : L]$. Furthermore, the extension J/L is said to be an *algebraic extension*, if every element in J is a root of a nonzero polynomial with coefficients in L. An algebraic extension J/L is said to be *normal* if J is a *splitting field* of the family of polynomials $L[X]$, i.e., each polynomial in $L[X]$ splits or decomposes into linear factors over J. Furthermore, an algebraic extension H of J is said to be a *normal closure* of the algebraic extension J/L, if it is the only subfield of H that contains J and if a normal extension of L is H itself.

Let S be a conventional signal set, such as M-PSK, and $F = \mathbb{Q}(S)$ be the extended field of rationals over S. If α is a root of a minimal polynomial over F, which is given by

$$p(x) = x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_0 \tag{4}$$

then F can be extended by adjoining α to obtain $K = F(\alpha)$. The degree of extension $[K : F]$ is equal to n, since $p(x)$ is irreducible over F. Any element $k \in K$ can be expressed as $\sum_{i=0}^{n-1} f_i\alpha^i$, where $f_i \in F \, \forall \, 0 \le i \le n-1$. From [30, Sec. 7.3], there exists a natural mapping $k \mapsto \lambda_k \forall k \in K$ that embeds K in $\mathbf{M}_n(F)$, where λ_k is a linear transformation of K into itself. The *regular representation* of λ_k maps any $v \in K$ to kv. The linear transformation λ_α associated with α is given by

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & \cdots & 0 & -a_1 \\ 0 & 1 & \cdots & 0 & -a_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -a_{n-1} \end{bmatrix} \in F^{n \times n} \tag{5}$$

[1]The *adjoining* operation refers to including all the elements resulting from field operations considering the elements from the extended set $I \cup L$.

which is the companion matrix of $p(x)$. Thus, for any $k = \sum_{i=0}^{n-1} f_i \alpha^i \in K$, the associated λ_k is given by $\sum_{i=0}^{n-1} f_i \mathbf{M}^i$.

Lemma 1: Let K, F, and S be defined as above. For any $k = k_1 - k_2, k_1 \neq k_2 \in K$, $\lambda_k \in \mathbf{M}_n(F)$ is invertible.

Proof: The proof directly follows from K being a field, which guarantees the existence of the inverse for every nonzero element in K, and the fact that the natural mapping $k \mapsto \lambda_k$ is a one-to-one mapping. ■

Lemma 2: If L is a normal closure of K/F and σ_i, $i = 0, 1, 2, \ldots, n-1$ are distinct F-homomorphisms from K to L; then, for any element $k \in K$, we have $\det(\lambda_k) = N_{K/F}(k) = \prod_{i=0}^{n-1} \sigma_i(k)$, where $N_{K/F}(k)$ is the norm of the element k from K to F [33, Th. 8].

B. Proposed DM Set for DSM

We propose to use the DM set given by

$$\mathcal{A} = \{\mathbf{I}_n, \mathbf{M}, \mathbf{M}^2, \ldots, \mathbf{M}^{n-1}\} \tag{6}$$

where $\mathbf{M}$ is as in (5), and n is chosen to be equal to N_t. However, to meet $C1$, every element of $\mathcal{A}$ has to be unitary. Note that it is sufficient to ensure that $\mathbf{M}$ is unitary for all the elements of $\mathcal{A}$ to be unitary. Hence, we have to satisfy the following equation:

$$\mathbf{M}\mathbf{M}^H = \mathbf{I}_n. \tag{7}$$

Note that $(\mathbf{M}\mathbf{M}^H)_{1,1} = |a_0|^2$ and $(\mathbf{M}\mathbf{M}^H)_{i,i} = 1 + |a_{i-1}|^2$ for $2 \leq i \leq n-1$. Thus, by choosing a_0 to be an element from the unit circle and $a_i = 0$ for $1 \leq i \leq n-1$, $C1$ can be satisfied. Thus, while constructing $\mathcal{A}$, we have to consider polynomials of the form $x^n + a_0$ with $|a_0| = 1$ values that are irreducible over F. Since we have $|\mathcal{A}| = n = N_t$, our construction results in a maximum of N_t DMs, i.e., $Q \leq N_t$. Furthermore, we assume that the set $\mathcal{D}$ has scaled identity matrices of the form $s\mathbf{I}_n$, where $s \in S$. Note that F should contain the specific signal set S from which s is chosen. Thus, the following conditions have to be met:

1) $S \subset F$; and
2) $p(x) = x^n + a_0$ with $|a_0| = 1$ should be irreducible over F.

We satisfy the given conditions by choosing $F = \mathbb{Q}(S, a_0)$, where a_0 is any transcendental element over $\mathbb{Q}(S)$ lying on the unit circle. In the following, we shall explain the method of constructing set $\mathcal{A}$ in detail.

Let S be a conventional M-PSK signal set denoted by $\{\omega_M^i\}_{i=0}^{M-1}$, where we have $\omega_M = e^{j2\pi/M}$ and $a_0 = -e^{ju_1}$, with u_1 being algebraic over $\mathbb{Q}$. For instance, u_1 can be $\sqrt{3}$, which is a root of the polynomial $x^2 - 3$. Note that a_0 is transcendental over $\mathbb{Q}(S)$, and we can choose $F = \mathbb{Q}(S, e^{ju_1})$. Thus, the polynomial $x^n + a_0 = x^n - e^{ju_1}$ (for any n) is irreducible over F. Therefore, we can have the extension $K = F(\alpha)$, where α is the primitive nth root of e^{ju_1}. Thus, the associated companion matrix is given by

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & \cdots & 0 & e^{ju_1} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \in F^{n \times n}. \tag{8}$$

Example 1: Consider $n = N_t = 4$ and $a_0 = -e^{j\sqrt{3}}$. Then, the elements of set $\mathcal{A}$ are given by $\mathbf{I}_4$

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & 0 & e^{j\sqrt{3}} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \mathbf{M}^2 = \begin{bmatrix} 0 & 0 & e^{j\sqrt{3}} & 0 \\ 0 & 0 & 0 & e^{j\sqrt{3}} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{M}^3 = \begin{bmatrix} 0 & e^{j\sqrt{3}} & 0 & 0 \\ 0 & 0 & e^{j\sqrt{3}} & 0 \\ 0 & 0 & 0 & e^{j\sqrt{3}} \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Remark 1: Note that a_0 has to be chosen in conjunction with the specific signal set S that maximizes a certain performance metric, such as the coding gain. This can be achieved by searching for an optimal a_0 over a large set of closely spaced transcendental elements on the unit circle.

We term the DSM scheme employing the proposed FE-DMs as an FE-DSM scheme. Since the set of transmit STMs is given by $\mathcal{S} = \mathcal{D} \times \mathcal{A}$, the rate achieved by the proposed scheme is

$$R_{\mathrm{FE-DSM}} = \frac{\log_2(|\mathcal{D}||\mathcal{A}|)}{N_t} = \frac{\log_2(MN_t)}{N_t} \text{ bpcu}.$$

1) Diversity Gain: The achievable transmit diversity order under differential detection [31, Sec. III-C] of (2) is given by

$$d = \min_{\mathbf{S}_1 \neq \mathbf{S}_2 \in \mathcal{S}} \mathrm{rank}(\mathbf{S}_1 - \mathbf{S}_2). \tag{9}$$

Proposition 1: The proposed FE-DSM scheme achieves a transmit diversity order of N_t, i.e., $d = N_t$.

Proof: The proof is given in Appendix A. ■

2) Coding Gain: The coding gain of the proposed scheme is given by

$$G = \min_{\mathbf{S}_1 \neq \mathbf{S}_2 \in \mathcal{S}} \left|\det\left[(\mathbf{S}_1 - \mathbf{S}_2)(\mathbf{S}_1 - \mathbf{S}_2)^H\right]\right|^{\frac{1}{n}}. \tag{10}$$

In the following, we shall provide a simple expression for the determinant term in (10) that allows us to optimize the exponential a_0 in conjunction with an arbitrary M-PSK signal set to achieve a high coding gain.

Proposition 2: Consider an FE-DSM system using an M-PSK signal set and $N_t = n$ TAs. If $\mathbf{S} = e^{j(2\pi p/M)}\mathbf{M}^l$ and $\mathbf{S}_2 = e^{j(2\pi q/M)}\mathbf{M}^m$, where $0 \leq p, q \leq M-1$ and $0 \leq l, m \leq n-1$ such that $\mathbf{S}_1 \neq \mathbf{S}_2$, then $|\det[(\mathbf{S}_1 - \mathbf{S}_2)(\mathbf{S}_1 - \mathbf{S}_2)^H]|$ is given by

$$4^n \prod_{r=0}^{n-1} \sin^2\left(\frac{\pi(p-q)}{M} + \frac{(2\pi r + u_1)(m-l)}{2n}\right). \tag{11}$$

Proof: The proof is provided in Appendix B. ■

In the following section, we provide a DM set construction based on two levels of field extensions, which facilitate a flexible tradeoff between the attainable transmit diversity and multiplexing gain.

C. FE-DSM With Diversity–Rate Tradeoff

The DM set construction presented in the previous section achieves a transmit diversity order of N_t, while transmitting only a single symbol from an M-PSK signal set. Note that when the channel conditions are good, it may not be necessary to exploit the full transmit diversity order. Under these conditions, we may aim at trading off the diversity gain for increasing the transmission rate. In the following, we shall provide a systematic method of constructing a DM set that achieves the desired diversity order and transmission rate. The DM set construction presented in the previous section may be viewed as a special case.

Let N_t be factored as $g \cdot h$. We construct a DM set that allows us to transmit h independent M-PSK symbols in each transmit STM and achieve transmit diversity order g. Considering $F = \mathbb{Q}(S, -e^{ju_1})$ as before and the extension $K = F(\alpha)$, where α is a primitive gth root of the polynomial $p_1(x) = x^g - e^{ju_1}$, we obtain the DM set given by

$$\mathcal{A}' = \{\mathbf{I}_g, \mathbf{M}, \mathbf{M}^2, \ldots, \mathbf{M}^{g-1}\} \tag{12}$$

where $\mathbf{M} \in F^{g\times g}$ is the companion matrix of $p_1(x)$. We define $\mathcal{D}$ to be a set of block-diagonal matrices given by

$$\mathcal{D}=\{\text{diag}(s_1\mathbf{A}_1,s_2\mathbf{A}_2,\ldots,s_h\mathbf{A}_h)|s_i\in M-\text{PSK},\mathbf{A}_i\in\mathcal{A}',\forall i\}. \tag{13}$$

Let us now consider the field extension $L = K(\beta)$ associated with the polynomial $p_2(x) = x^h - e^{ju_2}$, where e^{ju_2} is transcendental over K, and β is the primitive hth root of e^{ju_2}. Then, the regular representation of an element $l = \sum_{i=0}^{h-1} k_i\beta^i \in L$ is given by $\sum_{i=0}^{h-1} k_i\mathbf{N}^i$, where $k_i \in K$, $0 \le i \le h-1$, and $\mathbf{N} \in K^{h\times h}$ is the companion matrix of $p_2(x)$. We define the DM set as

$$\mathcal{A} = \left\{\mathbf{I}_n, \mathbf{N}', \mathbf{N}'^2, \ldots, \mathbf{N}'^{h-1}\right\} \tag{14}$$

where $\mathbf{N}' = \mathbf{N} \otimes \mathbf{I}_g$. The transmit STM set is given by $\mathcal{S} = \mathcal{D} \times \mathcal{A}$ as before. We refer to this scheme as the FE-DSM arrangement exhibiting a flexible diversity–rate tradeoff (FE-DSM-DR). Note that the DSM scheme requires each transmit STM to be unitary. The following proposition shows that this condition is satisfied.

Proposition 3: If $\mathcal{S}$ is the set of transmit STMs of FE-DSM-DR, then each element in $\mathcal{S}$ is unitary.

Proof: The proof is provided in Appendix C. ■

In the following, we shall provide an example construction to further illustrate the given set of points.

Since we have $|\mathcal{D}| = (Mg)^h$ and $|\mathcal{A}| = h$, the rate achieved by the FE-DSM-DR is given by

$$R_{\text{FE-DSM-DR}} = \frac{h\log_2(Mg) + \log_2(h)}{N_t} \text{ bpcu}. \tag{15}$$

Note that when we have $g = N_t$, FE-DSM-DR reduces to the FE-DSM scheme.

Example 2: Let $n = N_t = 4$, $g = h = 2$, $u_1 = \sqrt{2}$, and $u_2 = \sqrt{3}$. The elements of set $\mathcal{D}$ are

$$\begin{bmatrix} s_1 & 0 & 0 & 0 \\ 0 & s_1 & 0 & 0 \\ 0 & 0 & s_2 & 0 \\ 0 & 0 & 0 & s_2 \end{bmatrix}, \begin{bmatrix} 0 & s_1e^{j\sqrt{2}} & 0 & 0 \\ s_1 & 0 & 0 & 0 \\ 0 & 0 & s_2 & 0 \\ 0 & 0 & 0 & s_2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & s_1e^{j\sqrt{2}} & 0 & 0 \\ s_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & s_2e^{j\sqrt{2}} \\ 0 & 0 & s_2 & 0 \end{bmatrix}, \begin{bmatrix} s_1 & 0 & 0 & 0 \\ 0 & s_1 & 0 & 0 \\ 0 & 0 & 0 & s_2e^{j\sqrt{2}} \\ 0 & 0 & s_2 & 0 \end{bmatrix}$$

where s_1 and s_2 are from the classic M-PSK signal set. The elements of the DM set $\mathcal{A}$ are

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & e^{j\sqrt{3}} & 0 \\ 0 & 0 & 0 & e^{j\sqrt{3}} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Remark 2: Note that e^{ju_1} and e^{ju_2} have to be optimized in conjunction with the signal set S to maximize the coding gain. Unlike FE-DSM, the STM matrices of the FE-DSM-DR scheme are not representations of field elements, and hence, no closed-form expression is derived for the determinant of the codeword difference matrix. We resort to numerical search to arrive at the optimal values of u_1 and u_2.

D. ML Decoding Complexity

Here, we evaluate the complexity order of ML decoding for the proposed schemes. We show that the ML decoding complexity of both proposed schemes is independent of the size of the signal set S.

1) FE-DSM: Let $\chi = \{s\mathbf{e}_i | 1 \le i \le Q, s \in S\}$, where $\mathbf{e}_i$ is the ith column of $\mathbf{I}_Q$. Furthermore, let $\mathbf{G} = [\text{vec}(\mathbf{A}_1), \text{vec}(\mathbf{A}_2), \ldots, \text{vec}(\mathbf{A}_Q)] \in \mathbb{C}^{N_t^2\times Q}$, where $\mathbf{A}_i$ values are the elements of $\mathcal{A}$. Considering the optimal detection rule of (2), we have

$$\hat{\mathbf{S}}_i = \arg\min_{\mathbf{S}\in\mathcal{S}} \|\mathbf{Y}_i - \mathbf{Y}_{i-1}\mathbf{S}\|^2 \tag{16}$$

$$\equiv \arg\min_{s\in S, \mathbf{A}_q\in\mathcal{A}} \|\mathbf{Y}_i - \mathbf{Y}_{i-1}(s\mathbf{A}_q)\|^2 \tag{17}$$

$$\equiv \arg\min_{\mathbf{s}\in\chi} \left\|\bar{\mathbf{Y}}_i - (\mathbf{I}_{N_t} \otimes \mathbf{Y}_{i-1})\mathbf{G}\mathbf{s}\right\|^2 \tag{18}$$

where $\bar{\mathbf{Y}}_i = \text{vec}(\mathbf{Y}_i) \in \mathbb{C}^{N_rN_t\times 1}$. Since we have $|\chi| = Q|S|$, the decoding complexity order is $\mathcal{O}(QM)$, when S is an M-PSK signal set. However, owing to the interference-free nature of transmit vectors, the decoding complexity can be reduced from $\mathcal{O}(QM)$ to $\mathcal{O}(Q)$ with the aid of hard-limiting (HL)-based detection [34]. In other words, the ML decoding complexity of the FE-DSM scheme does not scale with the size of the signal set. By contrast, the existing full-diversity DSM scheme in [28] does not allow such low decoding complexity.

2) FE-DSM-DR: The optimal detection rule of (2) yields

$$\hat{\mathbf{S}}_i = \arg\min_{\mathbf{S}\in\mathcal{S}} \|\mathbf{Y}_i - \mathbf{Y}_{i-1}\mathbf{S}\|^2 \quad (19)$$

$$\equiv \arg\min_{\mathbf{D}\in\mathcal{D},\mathbf{A}_q\in\mathcal{A}} \|\mathbf{Y}_i - \mathbf{Y}_{i-1}\mathbf{D}\mathbf{A}_q\|^2 \quad (20)$$

$$\equiv \arg\min_{0\le k\le h-1} \left\{ \min_{\mathbf{D}\in\mathcal{D}} \left\|\mathbf{Y}_i - \mathbf{Y}_{i-1}\mathbf{D}\mathbf{N}'^k\right\|^2 \right\} \quad (21)$$

$$\left(\hat{k}, \hat{\mathbf{D}}^{(\hat{k})}\right) \equiv \arg\min_{0\le k\le h-1} \left\|\mathbf{Z}_i^{(k)} - \mathbf{Y}_{i-1}\hat{\mathbf{D}}^{(k)}\right\|^2 \quad (22)$$

where $\hat{\mathbf{D}}^{(k)} = \min_{\mathbf{D}\in\mathcal{D}} \|\mathbf{Z}_i^{(k)} - \mathbf{Y}_{i-1}\mathbf{D}\|^2$, and $\mathbf{Z}_i^{(k)} = \mathbf{Y}_i(\mathbf{N}'^k)^H$ for $0 \le k \le h-1$. Since $\mathbf{D}$ is block diagonal, we have

$$\hat{\mathbf{D}}^{(k)} = \min_{\mathbf{D}\in\mathcal{D}} \left\|\mathbf{Z}_i^{(k)} - \mathbf{Y}_{i-1}\mathbf{D}\right\|^2 \quad (23)$$

$$\equiv \sum_{l=1}^{h} \min_{s_l\in S,\mathbf{A}_{i_l}\in\mathcal{A}'} \left\|\mathbf{Z}_i^{(k)}(:,\mathcal{I}_l) - \mathbf{Y}_{i-1}(:,\mathcal{I}_l)\left(s_l\mathbf{A}_{i_l}\right)\right\|^2 \quad (24)$$

where $\mathcal{I}_l = [g(l-1)+1 : gl]$. By invoking the HL-based detector in [34], the search complexity of the minimization problem, i.e.,

$$\min_{s_l\in S,\mathbf{A}_{i_l}\in\mathcal{A}'} \left\|\mathbf{Z}_i^{(k)}(:,\mathcal{I}_l) - \mathbf{Y}_{i-1}(:,\mathcal{I}_l)\left(s_l\mathbf{A}_{i_l}\right)\right\|^2 \quad (25)$$

can be reduced from $\mathcal{O}(|S||\mathcal{A}'|)$ to $\mathcal{O}(|\mathcal{A}'|) = \mathcal{O}(g)$. Specifically, this is achieved by converting (25) into an interference-free system analogous to (18) and then employing the detector in [34]. Thus, the ML decoding complexity order of FE-DSM-DR is independent of the size of the signal set, and it is given by $\mathcal{O}(|\mathcal{A}'||\mathcal{A}|) = \mathcal{O}(gh) = \mathcal{O}(N_t)$.

E. Computational Complexity

Here, we compare the computational complexity of the ML detector of various existing schemes with that of the proposed scheme. Specifically, we show that all the existing schemes essentially impose the same computational complexity when operating at a given rate. However, since the ML decoding complexity order of the proposed schemes does not scale with the signal set, the computational complexity involved in ML decoding remains constant, when the size of the signal set is increased to increase the transmission rate.

Considering the ML detection rule of (2), we have

$$\hat{\mathbf{S}}_i = \arg\min_{\mathbf{S}\in\mathcal{S}} \|\mathbf{Y}_i - \mathbf{Y}_{i-1}\mathbf{S}\|^2 \quad (26)$$

where $\mathcal{S}$ is the set of transmit STMs. The number of real-valued multiplications in evaluating (26) is $6N_rN_t|\mathcal{S}|$, where $|\mathcal{S}|$ is the cardinality of the set of transmit STMs. When the transmission rate is fixed, $|\mathcal{S}|$ is essentially the same across all the existing schemes [26]–[28]. The direct evaluation of (26) results in the same computational complexity across all the schemes, since the number of nonzero elements in each $\mathbf{S} \in \mathcal{S}$ is the same in all of them. However, the proposed FE-DSM (DM-DSM [27]) scheme has the property that $\mathcal{S} = S \times \mathcal{A}$, which makes it amenable to HL-based ML detection (HL-ML) [34]. The computational complexity imposed by the HL-ML detector can be shown to be $(10N_tN_r + 9)|\mathcal{A}|$.[2] In the following section, we compare the computational complexity imposed by the direct ML solution in (26) to that of the HL-ML solution [34] by considering various system parameters and transmission rates.

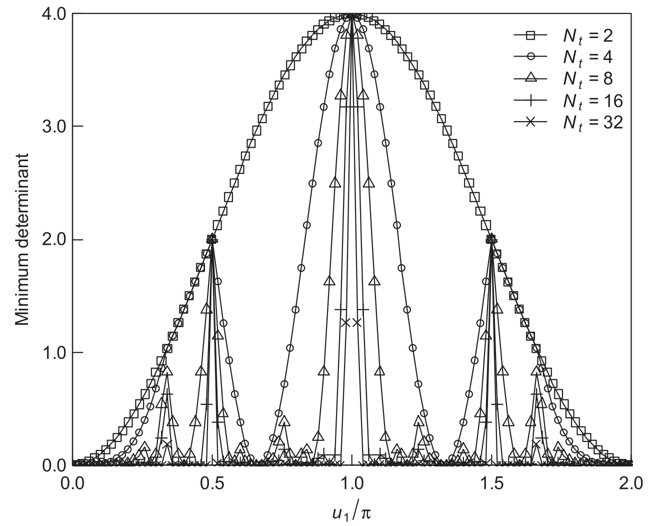

Fig. 1. Variation of coding gain as a function of u_1 in FE-DSM employing a BPSK signal set for various N_t values.

IV. Simulation Results and Discussions

Simulation Parameters: In all our simulations, we have used block Rayleigh fading channels. In evaluating the bit error ratio (BER) of 10^{-t}, we have used at least 10^{t+2} bits. For DM-DSM schemes operating at different rates, the optimal DM sets are obtained by optimizing the coding gain over a large set of feasible matrices in conjunction with the associated M-PSK signal set. The parameter e^{ju_1} of FE-DSM and the parameters (e^{ju_1}, e^{ju_2}) of FE-DSM-DR are optimized in conjunction with the associated signal sets to obtain the optimal set of DMs. For the FE-DSM scheme using an M-PSK signal set, it is observed that $u_1 = 2\pi/M$ is optimal for any value of N_t. Fig. 1 shows the achievable coding gain of FE-DSM employing a binary phase-shift keying (BPSK) signal set. It is clear in Fig. 1 that the value of $u_1 = \pi$ remains optimal even when N_t is varied.

Fig. 2 compares the BER performance of the FE-DSM and DM-DSM schemes, both having $N_t = 2$ and employing 4-PSK as well as 16-PSK signal sets that achieve a throughput of 1.5 and 2.5 bpcu, respectively. The BER performance of P-DSM is also provided to highlight the transmit diversity gain achieved by the DM-DSM scheme. Furthermore, the BER performance of the proposed codebooks in the coherent scenario is also provided. Fig. 3 compares the BER performance of the FE-DSM and DM-DSM schemes, both having $N_t = 4$ and employing 4-PSK, as well as 16-PSK signal sets that achieve a throughput of 1 and 1.5 bpcu, respectively. It is clear in Figs. 2 and 3 that

[2]It takes $4N_tN_r|\mathcal{A}|$ multiplications to compute $(\mathbf{I}_{N_t} \otimes \mathbf{Y}_{i-1})\mathbf{G}$ and $(6N_tN_r + 9)|\mathcal{A}|$ multiplications to compute various decision metrics of the HL-ML detector [34]. For further details, see [34, Sec. IV-B].

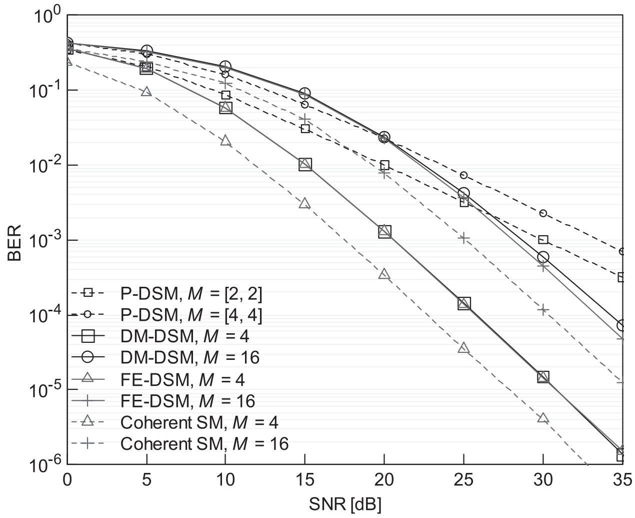

Fig. 2. BER performance of the DM-DSM and FE-DSM schemes, having $N_t = 2$ and employing 4-PSK and 16-PSK signal sets. The BER performance of the P-DSM scheme is provided to highlight the transmit diversity gain achieved in DM-DSM and FE-DSM.

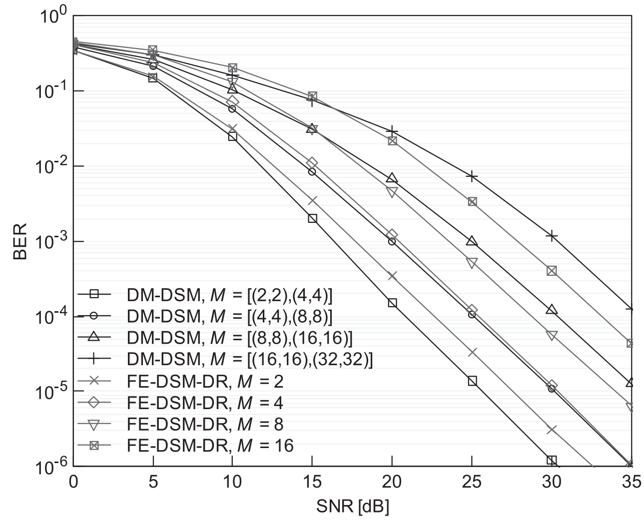

Fig. 4. BER performance of the DM-DSM and FE-DSM-DR schemes, having $N_t = 4$ and employing M-PSK signal sets.

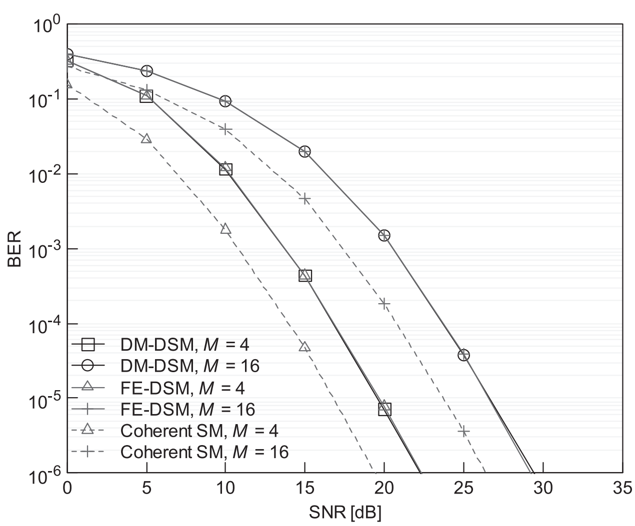

Fig. 3. BER performance of the DM-DSM and FE-DSM schemes, having $N_t = 4$ and employing M-PSK signal sets.

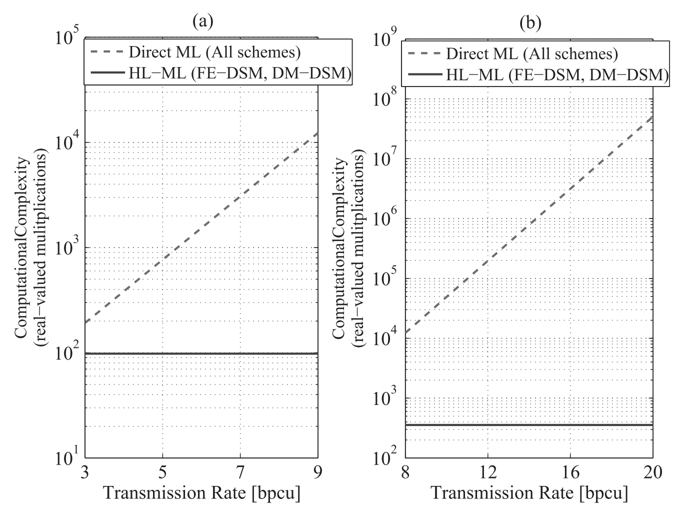

Fig. 5. Comparison of the computational complexity imposed by the ML detector in the existing schemes with that of the proposed scheme in systems having $N_t = 2, 4$ and $N_r = 2$ and employing various transmission rates.

the proposed FE-DSM scheme suffers from no performance loss compared with the DM-DSM scheme, and there is a 3-dB performance loss with respect to the coherent counterparts, which is as expected.

Fig. 4 compares the BER performance of FE-DSM-DR and DM-DSM that trades off diversity gain against throughput. Both the systems are assumed to have $N_t = 4$. Specifically, four data rates are considered for comparison. For $h = g = 2$, FE-DSM-DR achieves throughput values of 1.25, 1.75, 2.25, and 2.75 bpcu when employing BPSK, quaternary phase-shift keying, 8-PSK, and 16-PSK signal sets, respectively. The DM-DSM scheme is assumed to have a set of four DMs as proposed in [27] and employs $M = [(L_1, L_1), (L_2, L_2)]$, where L_1 and L_2 correspond to the sizes of the PSK signal sets encoding symbols s_1 and s_2, respectively. To elaborate, L_1 and L_2 are chosen so that the rates achieved by the proposed scheme and the FE-DSM scheme are the same. When operating at 1.25 bpcu, it can be observed in Fig. 4 that FE-DSM-DR suffers from a 1.5-dB SNR loss at a BER of 10^{-4} compared with DM-DSM. However, as the rate is increased from 1.25 to 2.75 bpcu, the performance of FE-DSM-DR improves, which is evident in Fig. 4. Specifically, when operating at 2.25 and 2.75 bpcu, it is observed that the FE-DSM-DR scheme achieves an SNR gain of about 1 dB and about 2 dB at a BER of 10^{-4}, respectively.

Fig. 5 gives the computational complexity imposed by the ML detector in various existing schemes along with that of the proposed FE-DSM scheme in systems having $N_t = 2, 4$ and $N_r = 2$ and employing various transmission rates. In the case of $N_t = 2$, the P-DSM scheme is assumed to employ BPSK, 4-PSK, 8-PSK, and 16-PSK to achieve a transmission rate of 3, 5, 7, and 9 bpcu, respectively, and in the case of $N_t = 4$, the same signal sets achieve a transmission rate of 8, 12, 16, and 20 bpcu, respectively. In the case of FE-DSM, the size of the DM set is fixed to 2 and 4 in the case of $N_t = 2$ and $N_t = 4$, respectively, and the size of the signal set (or the number of cyclic matrices in the case of CS-DSM [26] when $N_t = 2$) is assumed to vary to increase the transmission rate. It is clear in Fig. 5 that the HL-ML detector results in significant complexity reductions over the direct ML solution. Specifically, at a transmission rate of 5 and 8 bpcu, a reduction of about 670 multiplications in the case of $N_t = 2$ and about 11934 multiplications in the case of $N_t = 4$ is observed, respectively.

Future Work: While the proposed DM set constructions cater to the requirements of the SM scheme relying on differential encoding, it would be interesting to study the feasibility of extending the proposed schemes to generalized SM (GSM) [35], where more than one TAs are activated during each channel use. Note, however, that this is not straightforward, since

1) the transmitted STMs in GSM may not be unitary in general; and
2) the product of any two distinct transmit STMs does not satisfy the sparsity constraint analogous to condition C1 given in Section II-B.

As inferred from Fig. 4, the FE-DSM-DR-based DMs are not optimal at low rates (unlike the FE-DSM scheme). Furthermore, considering algebraic structures for designing DM sets would enable us to quantify the achievable diversity order and the coding gain in addition to attaining the benefits of a simple and systematic encoding at the transmitter. Thus, worth investigating are other representations of algebraic structures such as division algebras for their suitability in obtaining sparse, full-diversity, and optimal DM sets for differential SM/GSM schemes.

V. Conclusion

We have proposed a systematic method for obtaining a DM set for DSM with the aid of algebraic field extensions. It was analytically shown that the proposed FE-DSM achieves full transmit diversity. Furthermore, a closed-form expression was derived for the determinant of the codeword difference matrix. The proposed FE-DSM scheme was then further extended to FE-DSM-DR, which stroke a flexible tradeoff between diversity gain and throughput. Both the proposed schemes were shown to offer ML decoding complexity, which is independent of the size of the signal set. Our simulation results have shown that the FE-DSM scheme achieves the same BER performance as the DM-DSM scheme, whereas FE-DSM-DR is observed to give a better BER performance at higher rates compared with its DM-DSM counterpart.

Appendix A
Proof of Proposition 1

Let $\mathbf{S}_1 = s\mathbf{M}^i$ and $\mathbf{S}_2 = s'\mathbf{M}^j$, where $s, s' \in S \subset F = \mathbb{Q}(S, a_0)$. Recall that $\mathbf{S}_1$ and $\mathbf{S}_2$ are regular representations of $k_1 = s\alpha^i$ and $k_2 = s'\alpha^j$, respectively. Then, we have $\lambda_k = \mathbf{S}_1 - \mathbf{S}_2$, where $k = k_1 - k_2$. From Lemma 1, we see that λ_k is invertible for all $k_1, k_2 \in K$ and $k_1 \neq k_2$. Thus, we have $\mathbf{S}_1 - \mathbf{S}_2$ as invertible. In other words, we have $\text{rank}(\mathbf{S}_1 - \mathbf{S}_2) = n = N_t, \forall\, \mathbf{S}_1 \neq \mathbf{S}_2 \in \mathcal{S}$. Thus, we have $\min_{\mathbf{S}_1 \neq \mathbf{S}_2 \in \mathcal{S}} \text{rank}(\mathbf{S}_1 - \mathbf{S}_2) = N_t$. This concludes the proof.

Appendix B
Proof of Proposition 2

If $\lambda_k = \mathbf{S}_1 - \mathbf{S}_2$, then the element k associated with λ_k is given by $s\alpha^m - s'\alpha^l$, where $s = e^{j(2\pi p/M)}$, $s' = e^{j(2\pi q/M)}$, and α is the primitive nth root of $-a_0$. From Lemma 2, we have $\det(\lambda_k) = \det(\mathbf{S}_1 - \mathbf{S}_2) = \prod_{r=0}^{n-1} \sigma_r(k)$, where we have $\sigma_r : \alpha \mapsto \alpha_r$, such that α_r, $0 \leq r \leq n-1$ are the nth roots of $-a_0 = e^{ju_1}$. Therefore, we have

$$\det(\mathbf{S}_1 - \mathbf{S}_2) = \prod_{r=0}^{n-1} \sigma_r(k) = \prod_{r=0}^{n-1} \sigma_r(s\alpha^m - s'\alpha^l) \tag{27}$$

$$= \prod_{r=0}^{n-1} \left(s\left(\sigma_r(\alpha)\right)^m - s'\left(\sigma_r(\alpha)\right)^l \right) \tag{28}$$

$$= \prod_{r=0}^{n-1} \left(e^{j\frac{2\pi p}{M} + \frac{(2\pi r + u_1)m}{n}} - e^{j\frac{2\pi q}{M} + \frac{(2\pi r + u_1)l}{n}} \right). \tag{29}$$

Thus, we have

$$\det\left[(\mathbf{S}_1 - \mathbf{S}_2)(\mathbf{S}_1 - \mathbf{S}_2)^H\right]$$

$$= \left| \prod_{r=0}^{n-1} \left(e^{j\frac{2\pi p}{M} + \frac{(2\pi r + u_1)m}{n}} - e^{j\frac{2\pi q}{M} + \frac{(2\pi r + u_1)l}{n}} \right) \right|^2 \tag{30}$$

$$= 4^n \prod_{r=0}^{n-1} \sin^2\left(\frac{\pi(p-q)}{M} + \frac{(2\pi r + u_1)(m-l)}{2n} \right). \tag{31}$$

This concludes the proof.

Appendix C
Proof of Proposition 3

Let $\mathbf{S} \in \mathcal{S}$ such that $\mathbf{S} = \text{diag}(s_1\mathbf{A}_1, s_2\mathbf{A}_2, \ldots, s_h\mathbf{A}_h)\mathbf{N}'^k$ for some $0 \leq k \leq h-1$, $s_i \in M$-PSK signal set, $\mathbf{A}_i \in \mathcal{A}'$ for $1 \leq i \leq h$. Consider $\mathbf{S}\mathbf{S}^H = \mathbf{D}\mathbf{N}'^k\mathbf{N}'^{k^H}\mathbf{D}^H$, where $\mathbf{D} = \text{diag}(s_1\mathbf{A}_1, s_2\mathbf{A}_2, \ldots, s_h\mathbf{A}_h)$. Since $\mathbf{N}'^k = (\mathbf{N} \otimes \mathbf{I}_g)^k = \mathbf{N}^k \otimes \mathbf{I}_g$, we have

$$\mathbf{N}'^k\mathbf{N}'^{k^H} = (\mathbf{N}^k \otimes \mathbf{I}_g)(\mathbf{N}^k \otimes \mathbf{I}_g)^H$$

$$= \left(\mathbf{N}^k\mathbf{N}^{k^H} \otimes \mathbf{I}_g\right)$$

$$= \mathbf{I}_h \otimes \mathbf{I}_g = \mathbf{I}_n.$$

Thus, we have

$$\mathbf{S}\mathbf{S}^H = \mathbf{D}\mathbf{D}^H$$

$$= \text{diag}\left(\mathbf{A}_1\mathbf{A}_1^H, \mathbf{A}_2\mathbf{A}_2^H, \ldots, \mathbf{A}_h\mathbf{A}_h^H\right)$$

$$= \text{diag}(\mathbf{I}_g, \mathbf{I}_g, \ldots, \mathbf{I}_g) = \mathbf{I}_n.$$

This concludes the proof.

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