OPTICAL FIBRES

1. Introduction

The structure of optical fibre transmission lines takes the very simple [Suematsu & Iga 1982] form of a cylindrical glass core of refractive index n_1 surrounded by a cladding glass of refractive index n_2 where
$n_2 < n_1$. Normally most of the propagating energy is contained in the core but there is always a radially-decaying evanescent field in the cladding, which may extend over several wavelengths in the case of single-mode fibres. Both core and cladding materials must therefore have very low absorption and scattering losses.

As with waveguides, when the transverse dimensions of the guiding structure, in this case the core, are comparable with a wavelength then only a single mode can be supported, whereas for larger core diameters multimode operation prevails. Normally the wavelength of operation is in the region of $1\mu m$, corresponding to a frequency of 300,000GHz, so that single-mode fibres have a core diameter of 1 to $10\mu m$ while multimode fibres have standardised core diameters of between 50 and $60\mu m$. For practical convenience the outer diameter of telecommunication fibres is made $125\mu m$ in both cases. In step-index fibres the refractive indices are constant in both core and cladding, whereas in (ideal) graded-index fibres the refractive index is a maximum n_1 at the core centre but falls monotonically to that of the cladding n_2 at the core boundary. With fibres designed for long-distance transmission $(n_1-n_2) \leq n_1 \approx 1.5$ and the relative refractive-index $\Delta \approx (n_1-n_2)/n_1$ is about 1%.

At this point it is convenient to provide definitions of a few other basic quantities. Firstly, the maximum angle θ_m to the axis that light can enter a fibre at the input end from a medium of refractive index n_o is defined in terms of the numerical aperture NA as

$$n_o \sin \theta_m = NA = (n_1^2 - n_2^2)^{0.5} \approx n_1(2\Delta)^{0.5} \qquad (1)$$

If the medium is air, for which $n_o = 1$, and $\Delta = 0.01$ then $\theta_m \approx 12^o$. The core radius a is usually normalised to the free-space wavelength of operation λ through

$$V = (2\pi a/\lambda)(n_1{}^2 - n_2{}^2)^{0.5} \approx (2\pi a n_1/\lambda)(2\Delta)^{0.5} \qquad (2)$$

V is called the normalised frequency although it could equally well be referred to as the normalised core diameter or normalised wavelength.

Why, one might ask, and under what circumstances, are optical fibres preferred to other forms of transmission line? Some of their merits and drawbacks are discussed in the following pages but may be summarised as follows.

Advantages

1. Extremely low transmission loss (down to 0.15dB/km) giving distances between repeaters in a trunk network, or in underwater cables, of 100 to 200km and more, compared with 2km for coaxial cables.

2. Extremely large bandwidths of up to 1GHzkm for graded-index multimode fibres and 100 GHzkm for single-mode fibres, ~20MHzkm for coaxial cable.

3. Small size, low weight and high degree of flexibility.

4. Freedom from electromagnetic interference and earth-loop problems.

5. Fabricated from relatively abundant materials (silica, phosphorus, germanium, boron).

6. Zero cross-talk between closely-spaced lines.

7. Larger Young's modulus and resistance to crushing than copper.

Disadvantages

1. Glass is brittle and therefore breaks when the elastic limit is exceeded.

2. Long-term (20 years) mechanical stability under strain is unknown.

3. Demountable connectors and other, similar, components are expensive.

The properties of the principal types of optical fibre waveguide are summarised in Table 1.

Optical fibres can have a wide variety of applications and can be made from a range of optically "transparent" materials. However, the most important fibres at the present time are those used in telecommunications and most attention, in the first part of this chapter, will therefore be devoted to them, although the general principles apply to all types of fibre. The principal component of such fibres is silica, which can be produced in very pure form (only a few parts of impurity in 10^9 parts of silica).

The glassy form [Rawson 1967] of pure SiO_2 has an extremely low transmission loss in the optical and near-infra-red regions of the spectrum and can be drawn into long lengths with a high degree of precision. Silica, unfortunately, has the low refractive index of 1.45 and, when used as the core of a fibre, there are comparatively few compatible materials which have a sufficiently low refractive index to act as cladding. Possibilities are silica admixed with a small proportion of boron or fluorine and certain plastics such as silicone rubber. Conversely the index of silica can be raised by adding such oxides as P_2O_5, GeO_2 or TiO_2.

Telecommunication fibres are generally prepared from a preform which is drawn down into a fibre which can be many (10-20) kilometres in length. The preform may contain three regions: a central core of fibre core material (e.g. germanosilicate glass); a surrounding layer of cladding glass (e.g. phosphosilicate glass); and an outer layer, or substrate, of commercial silica tubing. In the resulting fibre only the core and cladding contribute to optical propagation and they must be of extremely high purity. There are several different methods of preform fabrication which are based on various forms of chemical vapour deposition [Gambling, Hartog & Ragdale 1981].

2. Transmission Loss of Optical Fibres

The factors contributing to loss in an optical fibre transmission line include absorption, scattering due to inhomogeneities in the core refractive index (Rayleigh scattering), scattering due to irregularities at the boundary between core and cladding, bending loss, loss at joints and connectors and the coupling losses at the input and output.

Remarkable progress has been made in reducing the transmission loss, which is about two orders of magnitude lower than that of coaxial cables having a similar transmission bandwidth. The absorption loss at some wavelengths is almost negligible and below about 0.8μm scattering is the dominant factor.

The main cause of absorption is the presence of transition metals such as Fe, Cu (especially in multicomponent glasses), water in the form of OH^- ions and the intrinsic absorption of the pure glass itself. In order to reduce the absorption to an acceptable level, it is necessary to prevent a metal concentration of more than 1 in 10^9, and an OH radical concentration of more than 1 in 10^7, from occurring.

Another purely material effect is the scattering due to inhomogeneities in the refractive index. These fluctuations are on a scale which is smaller than the wavelength and the resulting Rayleigh scattering is inversely proportional to the fourth power of the wavelength (λ^{-4}), so that it becomes rapidly smaller at longer wavelengths.

The transmission of a single-mode fibre is shown in Figure 1 and exhibits the characteristic features of fibres made by the modified CVD process. In this particular case the core diameter is 10.5μm, the relative refractive-index difference is Δ = 0.17% and the cut-off wavelength of the second set of modes (TM_{01}, TE_{01} and HE_{21}) is 1.2μm. At short wavelengths the attenuation is inversely proportional to the fourth power of the wavelength and therefore confirms Rayleigh scattering.

The rise in attenuation beyond 1.7μm is attributed to the intrinsic infra-red absorption of the glass. The effect of OH^- impurities can be clearly seen but are at a much lower level than is normally observed. The fundamental vibration is at λ = 2.8μm in silica and there are overtones at 1.39μm and 1.24μm.

The transmission loss at 1.3μm is below 0.4dB/km and is 0.16dB/km at 1.55μm. See also [Miya et al 1979].

A mode conversion loss, and a loss due to radiation, occur if the fibre has small irregularities at the boundary between the core and cladding. However, this interface scattering, which is referred to as "microbending", can be reduced by increasing both the core radius and the index gradient in the core in order to minimise the light intensity at the core/cladding boundary.

Bends can also cause mode conversion to occur in addition to the energy loss due to radiation.

3. <u>Propagation in Single-Mode Fibres</u>

The analysis of optical fibres follows the same procedure as that for any other transmission line which guides electromagnetic waves. Thus solutions of Maxwell's equations and the corresponding wave equation are sought in terms of the appropriate boundary equations, using well-established techniques. For each of the propagating modes it is possible to deduce the spatial distribution of the electric and magnetic fields, the propagation constant, phase and group velocities, and so on, in the normal way. Optical fibres differ in degree only, and not in principle, from, say, hollow metal waveguides, in that they are designed for operation at frequencies higher by a factor of 10^4 and the guiding structure is fabricated entirely from dielectric materials since metals are very lossy at optical frequencies.

3.1 **Basic Concepts**

A dielectric waveguide supports a finite number of guided modes and an infinite number of radiation modes which together form a complete orthogonal set. Only guided modes are considered here.

To simplify the analysis of fibre waveguides the cladding may be assumed to be of infinite extent. In practice this simply means that the cladding diameter must be large enough for the field to decay to a negligible level at its outer edge. In single-mode fibres a significant proportion of the power is carried in the cladding which must have a diameter roughly seven times that of the core, say $30\mu m$.

An exact description of the modal fields is complicated, but the analysis can be simplified by making use of the fact that, in practice, $(n_1 - n_2) \leq n_1$, the well-known "weakly-guiding approximation". The approximate mode solutions derived in this way are very nearly linearly-polarised [Snyder 1969A] [Gloge 1971] and are denoted by $LP_{\nu\mu}$ where ν and μ denote the zeros of the field in the azimuthal, and radial, directions, respectively.

These linearly-polarised modes correspond to a superposition of the two modes $HE_{\nu+1,\mu}$ and $EH_{\nu-1,\mu}$ of the exact solution to Maxwell's equations. The exact modes are nearly degenerate and as $n_2 \rightarrow n_1$ their propagation constants become identical.

Maxwell's equations with the weakly-guiding approximation give the scalar wave equation as:

$$\frac{d^2\psi}{dr^2} + \frac{1}{r}\frac{d\psi}{dr} + \frac{1}{r^2}\frac{d^2\psi}{d\phi^2}\,[n^2(r)k^2 - \beta^2]\psi = 0 \qquad (3)$$

where ψ is the field (E or H), $k = 2\pi/\lambda$ is the free-space wave number, $n(r)$ is the radial variation of the refractive index and r, ϕ are the cylindrical co-ordinates. The propagation constant β of a guided mode obviously lies between the limits $n_2k < \beta < n_1k$. The fibre is circular in cross-section and the solutions of the wave equation are separable, having the form:

$$\psi = E(r) \cos \nu\phi \exp\,[j(\omega t - \beta z)] \qquad (4)$$

For simplicity the factor $\exp[j(\omega t-\beta z)]$ will be omitted from later equations.

In single-mode fibres only the fundamental LP_{01} mode propagates and it has no azimuthal dependence, i.e. $\nu = 0$. It corresponds to the HE_{11} mode derived from the exact analysis. For this fundamental mode equation (3) reduces to

$$\frac{d^2E}{dr^2} + \frac{1}{r}\frac{dE}{dr} + [n^2(r)k - \beta^2]E = 0 \qquad (5)$$

In a step-index fibre, i.e. one with a constant refractive index n_1 in the core, equation (5) is Bessel's differential equation and the solutions are cylinder functions. The field must be finite at $r = 0$ and therefore in the core region the solution is a Bessel function J_ν. Similarly the field must vanish as $r \rightarrow \infty$ so that the solution in the cladding is a modified Bessel function K_ν. For the fundamental LP_{01} mode polarised in either the x or y direction the field is therefore [Snyder 1969A]

$$E(r) = AJ(UR) \qquad R<1 \text{ (core)}$$

$$= \quad AJ_o(U)\,\frac{K_o(WR)}{K_o(W)} \qquad R>1 \text{ (cladding)} \tag{6}$$

where R = r/a is the normalised radial coordinate and A is the amplitude coefficient. U and W are the eigenvalues in the core, and cladding, respectively, and are defined by

$$U^2 = a^2(n_1{}^2k^2 - \beta^2) \tag{7}$$

$$W^2 = a^2(\beta^2 - n_2{}^2k^2)$$

therefore

$$V^2 = a^2k^2(n_1{}^2 - n_2{}^2) = U^2 + W^2$$

Related to these parameters is the normalised propagation constant b, defined as [Gloge 1971]

$$b = [(\beta/k)^2 - n_2{}^2]/2n_1{}^2\Delta = 1 - \frac{U^2}{V^2} \tag{8}$$

where

$$\Delta = \frac{n_1{}^2 - n_2{}^2}{2n_1{}^2} \approx \frac{n_1 - n_2}{n_1} \leq 1$$

Since, for a guided mode, the limits of β are n_2k and n_1k then b must lie between 0 and 1.

The field expressions in equation (6) are normalised so as to have the same value at $r = a$. In addition the tangential electric field components must be continuous at this point, leading to the following eigenvalue equation for the LP_{01} mode:

$$\frac{UJ_1(U)}{J_o(U)} + \frac{WK_1(W)}{K_o(W)} \qquad (9)$$

It should be noted that it is only because of the weak-guidance approximation that the boundary conditions of the magnetic field components are also satisfied by this condition.

By solving equations (7) and (9) the eigenvalue U, and hence β, can be calculated as a function of the normalised frequency V. Therefore the dependence of the propagation characteristics of the mode on the wavelength and fibre parameters can be determined.

At the lower limit of $\beta = n_2k$ the mode phase velocity equals the velocity of light in the cladding and the wave is no longer guided, the mode is cut off and the eigenvalue $W = 0$ (equation (7)). As β increases, less power is carried in the cladding and at $\beta = n_1k$ all the power is confined to the core.

The limit of single-mode operation is determined by the wavelength at which the propagation constant of the second LP_{11}, mode equals n_2k. For a step-index fibre this cut-off condition is given by

$$J_o(V_c) = 0$$

where V_c denotes the cut-off value of V which, for the LP_{11} mode, is equal to 2.405. The fundamental mode has no cut-off, hence single-mode operation is possible for $0 \leq V \leq 2.4$.

4. Dispersion in Single-Mode Fibres

The bandwidth of optical fibres is limited by broadening of the propagating light pulse which has a finite spectral width due to (i) the spectral width of the source, and (ii) the modulation sidebands of the signal. If, therefore, the fibre waveguide is dispersive the different frequency components will travel at different velocities resulting in pulse distortion.

The transit time for a light pulse propagating along a fibre of length L is

$$\tau = \frac{L}{c}\frac{d\beta}{dk} \tag{10}$$

where c is the velocity of light.

If β varies non-linearly with wavelength the fibre will be dispersive. From equation (8) we have

$$\beta^2 = k^2 n_1{}^2[1 - 2\Delta(1 - b)] \tag{11}$$

Thus β is a function of the refractive indices of the core and cladding materials and of b. Equation (8) shows that b is a function of V so that pulse dispersion arises from the variation of b with the ratio a/λ. In addition, the refractive index of the fibre material varies non-linearly with wavelength and this also gives rise to pulse dispersion.

The pulse spreading caused by dispersion is given by the derivative of the group delay with respect to wavelength [Payne & Gambling 1979]

$$\text{pulse spread} = \left|\delta\lambda \frac{d\tau}{d\lambda}\right| L = \frac{L}{c}\frac{2\pi}{\lambda^2}\frac{d^2\beta}{dk^2}\,\delta\lambda \tag{12}$$

where $\delta\lambda$ is the spectral width of the source. Substituting equation (11) into equation (12), and differentiating with respect to k, gives the dependence of the pulse spreading on the material properties and the mode parameter b. The dependence on the refractive index is given in terms of the material dispersion parameter $-(\lambda/c)(d^2n/d\lambda^2)$ where $n = n_1$ or n_2 and the dependence on b is given by the mode dispersion parameter defined as $V(d^2(bV)/dV^2)$. In addition, a third term, which is proportional to $d\Delta/d\lambda$, arises from the differentiation in equation (12).

The preceding three effects are inter-related in a complicated manner, but [Gambling, Matsumura & Ragdale 1979A] show that the expression for pulse spreading can be separated into three composite dispersion components in such a way that one of the effects dominates each term. For example, a composite material dispersion term can be defined which has a dependence on both b and $d^2n/d\lambda^2$, however it becomes zero when $d^2n/d\lambda^2$ is zero.

In multimode fibres the majority of the modes are far from cut off and most of the power is carried in the core. In this case the composite dispersion components simplify to terms which depend on either material or mode dispersion, and the two effects can be separated. In addition, in step-index multimode fibres the effect of $d\Delta/d\lambda$ can be neglected.

Material and mode dispersion also have a dominant effect in single-mode fibres but the effect of $d\Delta/d\lambda$ can no longer be neglected [Gambling, Matsumura & Ragdale 1979A].

In the absence of material dispersion the pulse spreading is controlled by the mode parameter $Vd^2(bV)dV^2$ which is shown in Figure 2(a) as a function of V for the LP_{01} mode. In the single-mode regime, i.e. $V < 2.4$, the mode dispersion is always positive and reaches a maximum at $V = 1.15$. It is seen that a change in any of the waveguide parameters, e.g. core radius or wavelength, changes V and hence the mode dispersion.

The material dispersion parameter, $(\lambda/c)(d^2n/d\lambda^2)$ is plotted as a function of wavelength in Figure 2(b) for a germanophospho-silicate glass fibre with NA = 0.2. At most wavelengths the material dispersion exceeds mode dispersion, but at 1.29μm the material dispersion is zero [Payne & Gambling 1975] (i.e. $d\tau/d\lambda = 0$). Thus at wavelengths near this value the bandwidth is limited by mode dispersion.

The total dispersion of a single-mode fibre arises from the combined effects of material dispersion, mode dispersion and $d\Delta/d\lambda$ terms. As shown in Figure 2(b) the material dispersion function changes sign at a wavelength of approximately 1.29μm, whereas mode dispersion always has the same sign in the single-mode regime. Therefore the effects of material dispersion, $d\Delta/d\lambda$ and mode dispersion can be balanced to give zero first-order dispersion at a given wavelength [Gambling, Matsumura & Ragdale 1979A]. Hence extremely large bandwidths can, in theory and practice, be achieved in single-mode fibres.

F2b

Since the dispersive properties of the fibre depend on both the fibre core dimensions and the fibre materials the total dispersion can be altered by changing either of these parameters. The wavelength λ_o at which the first-order dispersion is zero can therefore be tuned by appropriate choice of the core diameter or of NA. The total dispersion is plotted in Figure 3 as a function of wavelength for different core diameters and a fixed NA of 0.23.

The range of wavelengths over which λ_o can be tuned is limited. The maximum value depends on the usable value of NA, while the minimum value is approximately the wavelength at which material dispersion is zero (~1.3μm).

If a fibre is designed to operate with zero first-order dispersion the limitations imposed on the bandwidth by secondary effects must be considered. For example, birefringence arising from ellipticity or stress in the core causes the two orthogonally-polarised modes of the "single-mode fibres" to become distinguishable, i.e. they are no longer degenerate as in the scalar approximation [Adams et al 1979] [Love et al 1979]. The modes have different propagation constants which results in pulse dispersion. The dispersion caused by a difference between the major and minor axes of about 5% is less than 2 ps/km and can therefore be neglected [Adams et al 1979]. On the other hand the pulse dispersion arising from stress birefringence may be as high as 40ps/km if the expansion coefficients between the fibre core and cladding materials are not matched [Norman et al 1979].

F4 Figure 4 shows the bandwidth of a single-mode fibre designed for $\lambda_o = 1.3\mu m$. In the absence of second-order effects the bandwidth is usually limited by the spectral width of the source. Thus the solid line shows the available bandwidth with a source of linewidth 1nm. On the other hand if stress birefringence introduces a pulse dispersion of 10ps/km the bandwidth in the vicinity of λ_o is considerably reduced (dotted curve). In the absence of polarisation dispersion the bandwidth near λ_o would be determined by higher-order effects.

Measurements with a narrow-linewidth laser source over a 20km length have revealed a pulse dispersion of less than 4ps/km in a typical fibre.

5. Spot Size

The spot size of the fundamental mode is one of the most important parameters in single-mode fibre design since it largely determines the launching efficiency, jointing loss and bending loss. Usually the spot size ω_o is defined as the width to 1/e intensity of the LP_{01} mode or, alternatively, in terms of the spot size of an incident Gaussian beam which gives maximum launching efficiency. The latter definition arises from the fact that the LP_{01} mode has almost a Gaussian distribution.

The spot size is a function of both V and NA, although the dependence on V is only slight [Gambling & Matsumura 1977] (ω_o changes by only 2% over the range V = 1.8 to 2.4). The numerical aperture, on the other hand, has a strong effect since a large NA increases the guidance effect and more of the power in the LP_{01} mode is confined to the core, so that the spot size decreases.

6. Launching Efficiency

The ratio of power accepted by the fibre to the power in an incident beam is defined as the launching efficiency and can be calculated by integrating the product of the incident and propagating modes over the fibre cross-section [Snyder 1969B].

Thus launching efficiency

$$= \frac{1}{n_1} \left| \int^A E_{inc}.E \, dA \right|^2 \int^A E^2{}_{inc} dA \int^A E^2 \, dA \tag{13}$$

where E_{inc} is the electric field distribution of the incident mode.

Maximum power is launched into the fibre when the spot size of the LP_{01} mode is matched to the waist of the incident Gaussian beam. In practice, however, the launching efficiency of the LP_{01} mode decreases if the input beam is offset or tilted.

7. Joint Loss

The efficiency with which power can be coupled between two fibres is determined by the extent to which the mode patterns of the incoming and outgoing fibres can be matched. Therefore angular or lateral misalignment can considerably increase the loss at a joint. While longitudinal separation between the ends of the fibres can also occur, its effect on loss in practical joints is small enough to be neglected.

If it is assumed that the spot sizes of the modes of the two fibres are the same then the joint loss can be derived simply in terms of spot size. In the absence of angular misalignment the loss caused by lateral offset is [Gambling, Matsumura & Ragdale 1978]

$$T_1 = 2.17(D/\omega_o)^2 dB \tag{14}$$

where D is the offset. The offset loss is thus inversely proportional to the square of the spot size, ω_0. On the other hand the loss caused by an angular misalignment α is

$$T_a = 2.17\ (\alpha \omega_0 nV/aNA)^2\ \text{dB} \tag{15}$$

and hence angular misalignment loss is directly proportional to the square of the spot size. Thus for a given loss there is a trade-off between the spot size, and hence NA, required for low offset loss and that required for low angular misalignment loss.

When angular and lateral misalignments occur together the combined effect is complicated [Gambling, Matsumura & Ragdale 1978], but if the total loss is small it can be approximated by the sum of equations (14) and (15).

8. <u>Bending Loss</u>

Radiation at bends in single-mode fibres can significantly increase the transmission loss [Petermann 1977] [Gambling, Matsumura & Ragdale 1979B]. The bending loss can arise either from curvature of the fibre axis or microbending, i.e. small inhomogeneities in the fibre such as diameter variations, which can arise during coating and cabling.

There are two different mechanisms giving rise to bend loss in single-mode fibres, namely transition loss and pure-bend loss [Gambling, Matsumura & Ragdale 1979B] [Gambling, Matsumura, Ragdale & Sammut 1978]. The transition loss is oscillatory and arises because power is lost by coupling between the fundamental mode and the radiation modes. In other words the power distribution in the HE_{11} mode of the straight fibre is different from that of the corresponding mode in the curved fibre and power is lost at the interface between the two due to this mismatch.

The second mechanism, pure-bend loss, represents a loss of energy from the pure mode of the curved fibre and can be explained as follows. At a bend the phase fronts are no longer parallel and at a sufficiently large distance from the centre of curvature the increased distance between the phase fronts corresponds to a phase velocity greater than c/n_2. This part of the wave is no longer guided and radiates away from the fibre. As the curvature is increased the radius at which the phase velocity equals the velocity of light decreases, hence more energy is lost. The amount of power radiated depends on the spot size. If the spot size is reduced the power is more tightly guided in the core and the pure-bend loss decreases.

Both the transition loss and pure bend loss are strongly dependent on the NA (and hence the spot size). It is therefore possible to reduce bending loss to a negligible level by increasing the numerical aperture of the fibre.

9. Arbitrary Profiles

In the discussions above only a stepped refractive-index profile has been considered. In practice, however, the real profiles of single-mode fibres have a dip in the centre and some grading of the core/cladding boundary is caused by diffusion of dopants during the fabrication process. In addition the refractive index in the cladding is not usually constant. The field distribution and propagation characteristics of the LP_{01} mode are thus different quantitatively, although not qualitatively, from those in the step-index fibre. Hence all of the properties discussed above will be different for fibres with different profiles.

There are two ways of dealing with this problem. The first is to use the measured refractive-index profile to produce a numerical solution [Matsumura et al 1980] of the scalar wave equation (equation (3)). The second method involves matching the mode field of the real fibre to that of a fibre with equivalent step-index distribution [Matsumura et al 1980] [Snyder & Sammut

1979]. This simplifies the problem since the analytical expressions for a step-index fibre can be used; however, it is not very accurate for fibre profiles which depart too far from a stepped distribution.

10. <u>Propagation in Multimode Fibres</u>

10.1 Basic Concepts

Multimode fibres have larger core diameters and numerical apertures than single-mode fibres and, as a result, can be coupled more easily to optical sources. In particular, light-emitting diodes, which are cheaper and more reliable than lasers, can be used to drive multimode fibre links. Moreover jointing and splicing losses are much lower than with single-mode fibres since the dimensions are larger and hence the alignment tolerances are much less stringent. Finally, multimode fibres are less susceptible to microbending losses. However single-mode fibres have far higher bandwidths than those offered by multimode fibres.

Typical multimode fibres have a core diameter of $50\mu m$, a numerical aperture of 0.2 (i.e. a relative index difference Δ of slightly less than 1%) and an outer diameter of $125\mu m$. At a wavelength of $0.85\mu m$ (the emission wavelength of GaAs devices) the corresponding normalised frequency is $V = 37$ and the number of guided modes (approximately $V^2/4$ for graded-core fibres is ~ 340).

In general, therefore, power is launched into a large number of modes having different spatial field distributions, propagation constants, chromatic dispersion and so on. In an ideal fibre, having properties (e.g. core size, index difference, refractive-index profile) which are independent of distance, then the power launched into a given mode remains in that mode and travels independently of the power launched into other modes. In addition most of the modes are operated far from cut-off and their properties are, therefore, relatively independent of wavelength. This behaviour contrasts with single-mode operation where the mode parameters, such as normalised propagation constant or power confinement factor, vary rapidly with wavelength.

Since the majority of modes operate far from cut-off, and are thus well confined, most of the power carried by multimode fibres travels in the core region. The properties of the cladding therefore only significantly affect those modes which are near cut-off and whose electromagnetic fields extend appreciably beyond the core.

11. Dispersion in Multimode Fibres

The existence of several hundred modes, each having its own propagation constant, causes a form of pulse distortion which does not exist in single-mode fibres, namely intermodal dispersion. The energy of an impulse launched into a multimode fibre is therefore spread over a time interval corresponding to the range of propagation delays of the modes. The number of signal pulses which may be transmitted in a given period, and hence the information-carrying capacity of the fibre, is therefore reduced. Since, in the absence of mode filtering or mode conversion, the pulse spreading increases linearly with fibre length, the bandwidth is inversely proportional to distance. The product of bandwidth B and distance L is therefore a figure of merit for the information capacity of an optical fibre. The BxL product for a step-index fibre is typically 20MHz/km. As indicated in the Introduction, a careful choice of the radial variation of the refractive index enables the transit-times of the modes to be almost equalised so that BxL products of 10-20GHz/km have been predicted but cannot be achieved in practice. The power-law, or α, class of refractive-index profiles, given by

$$n^2(r) = n_1^2 \left[1-2\Delta \left(\frac{r}{a}\right)^{\alpha}\right] \qquad r < a$$

$$n^2(r) = n_1^2[1-2\Delta] = n_2^2 \qquad r > a \tag{16}$$

has been used extensively to model the grading function of multimode fibres. The profile is optimised by a suitable choice of α. It may be shown [Gloge & Marcatili 1973] [Olshanksy & Keck 1976] that, neglecting the dispersive properties of the glasses forming the waveguide, the value of α which minimises the r.m.s. pulse broadening is given by

$$\alpha_{opt} = 2 - \frac{12\Delta}{5} \qquad (17)$$

and the r.m.s. output pulse width produced by a unit impulse at the input is then

$$\sigma_{opt} = \frac{L}{c} \frac{n_1}{20\sqrt{3}} \Delta^2 \qquad (18)$$

The intermodal dispersion is, however, an extremely sensitive function of the index-profile. Minute departures of refractive index from the power law, or an incorrect design of the profile, lead to a much lower bandwidth than is theoretically achievable. For example, an error in α of ~ 1% degrades the bandwidth by a factor of two. The central dip caused by dopant evaporation in the high-temperature collapse stage of the CVD process, and the step-like structure caused by the deposition of individual glass layers, have been shown to contribute significantly to the pulse broadening.

11.1 Effect of the Wavelength Dependence of Refractive Index

The variation of refractive index with wavelength also causes the transmitted pulses to broaden, as we have seen in the case of single-mode fibres. With multimode fibres an additional, more subtle, effect exists since the index dispersion $dn/d\lambda$ also alters the relative transit-times of the modes and hence, the intermodal dispersion [Olshansky & Keck 1976]. This is normally referred to as "profile dispersion" and is a result of the difference which exists between the group index $N = n-\lambda(dn/d\lambda)$ (which determines the pulse transit time) and the refractive index n.

Since $dn/d\lambda$ is, in general, a function of glass composition it varies across the core of a graded-index fibre. Hence each mode is affected differently by dispersion since the spatial distribution of power is not the same for all modes. For example, low-order modes travel, on average, in a medium of higher dopant concentration than do higher-order modes. Thus a correction to the optimum profile parameter is required.

11.2 **Material Dispersion in Multimode Fibres**

The power carried by multimode fibres travels almost entirely in the core region. Because, in addition, most modes are operated far from cut-off they are almost free of waveguide dispersion. The pulse delay in multimode fibres is thus given, to first order, by [Gloge 1971]

$$\tau = \frac{L}{c} N_1 = \frac{L}{c} \left(n_1 - \lambda \frac{dn_1}{d\lambda} \right) \tag{19}$$

where N_1 is the group index of the core material. (For graded-index fibres, N_1 represents a value of group index averaged over the core area).

Semiconductor sources used in optical communications systems radiate over a finite range of wavelengths and, from equation (19), each spectral component travels at a different group velocity. The resulting pulse broadening σ_m is known as material dispersion.

For a source of r.m.s. spectral width σ_s and mean wavelength λ_s, σ_m may be evaluated by expanding equation (19) in a Taylor series about λ_s:

$$\sigma_m = \sigma_s \frac{d\tau}{d\lambda} + \frac{\sigma_s^2}{2!} \frac{d^2\tau}{d\lambda^2} + \ldots \qquad (20)$$

The first term normally dominates, particularly for sources operating in the 0.8 to 0.9μm wavelength region. Thus for the material illustrated in Figure 2(b) the material dispersion parameter $M = (1/L)\ (d\tau/d\lambda)$ is ~ 100ps nm^{-1} km^{-1} at 0.85μm. For a typical light-emitting diode having σ_s = 18nm and λ_s = 850nm the resulting pulse broadening is 1.8ns/km^{-1} which limits the BxL product to 100MHz/km. This level of dispersion is an order of magnitude greater than the intermodal dispersion. Even with semiconductor lasers (having spectral widths of, typically, 1nm r.m.s.) material dispersion sets an ultimate limit on the capacity of multimode fibre systems.

Figure 2(b) shows that a wavelength region exists where the material dispersion parameter is negligible. The wavelength λ_m of zero material dispersion is found to vary according to the glass composition [Payne & Hartog 1977], but for silica-based fibres, is always in the vicinity of 1.3μm. Operation in this wavelength region substantially reduces the bandwidth limitations arising from material dispersion and greater BxL products are available, even with light-emitting diodes. It may be seen from Figure 1 that, for silica-based glasses, the fibre attenuation is also extremely low (0.38dB/km^{-1}).

As shown in Figure 5, the bandwidth available from multimode fibres increases rapidly as the wavelength of operation is increased from 0.85μm (emission of GaAlAs devices) to the region of negligible material dispersion (~ 1.3μm). Thus for a laser having a spectral width of 1nm r.m.s. (dashed curve), the information-carrying capacity is limited in the region of 1.3μmm, by residual intermodal dispersion. For a numerical aperture of 0.2 the maximum available bandwidth is ~ 13GHz/km, providing the refractive-index profile is correctly designed.

Material dispersion does not completely vanish, however, even at the wavelength where the first derivative of group delay $d\tau/d\lambda$ is zero. According to equation (22) higher-order terms must then be taken into account [Kapron 1977]. The pulse broadening resulting from second-order material dispersion is proportional to the square of the source linewidth, and for silica-based fibres is of the order of $0.1 ps/nm^{-2}km^{-1}$. With laser sources second-order material dispersion is not a serious limitation.

The linewidth of light-emitting diodes increases as the square of the operation wavelength [Gloge et al 1980] and, at $1.3\mu m$, values of 100nm f.w.h.m. (42nm r.m.s. spectral width) have been reported. With such broad spectral widths, second-order material dispersion limits the bandwidth [Adams et al 1978] to F5 about $2GHz/km^{-1}$, see Figure 5, solid curve. Although the bandwidth of the best multimode fibres can, in principle, be limited in this way, in practice a counteractive effect, namely wavelength filtering, takes place. The latter effect results from even minor variations in the loss/wavelength characteristic of the fibre, which are enhanced by transmission over long distances (e.g. 20-30km for typical $1.3\mu m$ systems). Thus, the effective wavelength spread is dictated more by the attenuation characteristics of the fibre than by the spectral width of the source. In addition the wavelength at which the received power is a maximum may not coincide with the peak source wavelength.

13. <u>References</u>

Adams, M.J., Payne, D.N. and Ragdale, C.M.:
Birefringence in optical fibres with elliptical cross section,
Electronics Letters, 14, 1979, pp. 298-299.

Adams, M.J., Payne, D.N., Sladen, F.M.E. and Hartog, A.H.:
Optimum operating wavelength for chromatic equalisation in multimode optical fibres,
Electronics Letters, 14, 1978, pp. 64-66.

Gambling, W.A., Hartog, A.H. and Ragdale, C.M.:
Optical fibre transmission lines,
The Radio & Electronic Engineer, 51, 1982, pp 313-325.

Gambling, W.A. and Matsumura, H.:
Simple characterisation factor for practical single-mode fibres,
Electronics Letters, 13, 1977, pp. 691-693.

Gambling, W.A., Matsumura, H. and Ragdale, C.M.:
Curvature and microbending losses in single-mode optical fibre,
Optical & Quantum Electronics, 11, 1979B, pp. 43-59.

Gambling, W.A., Matsumura, H. and Ragdale, C.M.:
Joint loss in single-mode fibres,
Electronics Letters, 14, 1978, pp. 491-493.

Gambling W.A., Matsumura, H. and Ragdale, C.M.:
Mode dispersion, material dispersion and profile dispersion in grade-index single-mode fibres,
IEE J. Microwaves, Optics & Acoustics, 3, 1979A, pp. 239-246.

Gambling, W.A., Matsumura, H., Ragdale, C.M. and Sammut, R.A.:
Measurement of radiation loss in curved single-mode fibres,
IEE J. Microwaves, Optics & Acoustics, 2, 1978, pp. 134-140.

Gloge, D.:
Weakly guiding fibres,
Applied Optics, 10, 1971, pp. 2252-2258.

Gloge, D. and Marcatili, E.A.J.:
Multimode theory of graded-core fibres,
Bell Syst. Tech. J., 52, 1973, pp. 1563-1578.

Gloge, D., Ogawa, K. and Cohen, L.G.:
Baseband characteristics of long-wavelength led systems",
Electonics Letters, 16, 1980, pp. 366-367.

Kapron, F.P.:
Maximum information capacity of fibre-optic waveguides,
Electronics Letters, 13, 1977, pp. 627-629.

Love, J.D., Sammut, R.A. and Snyder, A.W.:
Birefringence in elliptically deformed optical fibres,
Electronics Letters, 15, 1979, pp. 615-616.

Matsumura, H., Suganuma, T. and Katsuyama, T.:
Simple normalization of single-mode fibres with arbitrary index profile,
Proc. 6th European Conference on Optical Communication, York, 1980.

Miya, T., Teranuma, Y., Hosaka, T. and Miyashita, T.:
Ultimate low-loss single-mode fibre at 1.55μm,
Electronics Letters, 15, 1979, pp. 106-108.

Norman, S.R., Payne, D.N., Adams, M.J. and Smith, A.M.:
Fabrication of single-mode fibres exhibiting extremely low polarization birefringence,
Electronics Letters, 15, 1979, pp. 309-311.

Olshansky, R. and Keck, D.B.:
Pulse broadening in graded-index optical fibres,
Applied Optics, 15, 1976, pp. 483-491.

Payne, D.N. and Gambling W.A.:
Zero material dispersion in optical fibres,
Electronics Letters, 11, 1975, pp. 176-178.

Payne, D.N. and Hartog, A.H.:
Determination of the wavelength of zero material dispersion in optical fibres by pulse-delay measurements,
Electronics Letters, 13, 1977, pp. 627-629.

Petermann, K.:
Fundamental mode microbending loss in graded-index and W fibres,
Optical & Quantum Electronics, 9, 1977, pp. 167-179.

Rawson, H.:
Inorganic glass-forming systems,
Academic Press, London, 1967, p. 317.

Snyder, A.W.:
Asymptotic expressions for eigenfunctions and eigenvalues of a dielectric optical waveguide,
IEEE Trans. on Microwave Theory & Techniques, MTT-17, 1969A, pp. 1130-1138.

Snyder, A.W. and Sammut, R.A.:
Fundamental (HE_{11}) modes of graded optical fibres,
J. Opt. Soc. Am., 69, 1979, pp. 1663-1671.

Suematsu, Y. and Iga, K.-I.:
Introduction to optical fibre communications,
Translation edited by W.A. Gambling,
John Wiley & Sons, New York 1982, p. 208.