**Seasonal variation of river and tide energy in the Yangtze estuary, China**

Min Zhang, Ian Townend, Yunxuan Zhou and Huayang Cai

**Abstract**

In many large estuaries there are significant variations in flow conditions due to the interaction between tide (with spring-neap changes) and river discharge (with wet-dry seasons), which is key to understanding the evolution of the morphology and the resultant equilibrium state. To explore whether there exists an equilibrium state, and what might control such a state in such a dynamic environment, both numerical and analytical methods have been used to investigate the relative importance of tide and river contributions to the work done locally and globally over a wide range of discharge conditions in the Yangtze estuary. In particular, we have quantified the contributions from the tidal flow, the river flow and the tide-river interaction in terms of energy and its dissipation under different river discharge conditions. Model results suggest that there is a state of minimum tidal work for the case representing the wet season, when river and tide are doing uniform work locally and minimum work globally, within the bi-directional tidal reach for tide and along the whole estuary for river. We also observe that, the system is not optimized for other conditions (peak discharge and low flows during the dry season), but the system would tend to do the minimum work possible given the constraints on the system (e.g., imposed forcing conditions and available sediment supply). Our results therefore are consistent with the use of these two energetic optimization principles, and the proposed method could be applicable to other alluvial estuaries.

Keywords: Yangtze estuary, river-tide interaction, hydraulics, energy, equilibrium

**1Introduction**

It has been suggested that an estuary is a self-organization system (Nield et al., 2005) and that an equilibrium is achieved subject to every constraint on the system (Pethick, 1984; Savenije, 2012; Townend, 2012). Generally, river and tide conditions are the basis for a generalized analysis of system equilibrium and adjustment (Dalrymple and Choi, 2007; Sassi and Hoitink, 2013). Understanding and quantifying the relative importance of tide and river contributions to the work done locally and globally over different states of riverine discharge is limited (Ensign et al., 2013), although it has been noted that the general configuration of a mature estuary may strive to adjust itself in response to varying constraints in such a manner that the system seeks to do minimum work as a whole, whilst also minimizing energy dissipation locally (Yang, 1971a; Yang et al., 1981; Nield et al., 2005).

Rivers derive their energy from precipitation on higher ground within the catchment, whereas estuaries derive their energy from the ocean tide. A further difference that is often made use of when analyzing these systems is that the river discharge can be considered constant over a given reach (Yang, 1971a). This being the case, it follows that the energy head must change exponentially if the system as a whole is to do minimum work (Leopold and Langbein, 1962). In contrast, many tidal estuaries are exponentially convergent (Pethick, 1984), and also exhibit an exponential variation in discharge, with approximately constant energy head, which again implies a condition of minimum work (Langbein, 1963). Field studies have produced some evidence compatible with the assumptions that have allowed some selective testing of these concepts (Langbein, 1963; Yang, 1971b; Cherkauer, 1973) . Subsequently, there are further developments (Rodríguez-Iturbe et al., 1992; Dun and Townend, 1998; Lanzoni and Seminara, 2002; Savenije, 2012; Kleidon et al., 2013; Huang et al., 2014) and criticisms (Davy and Davies, 1979; Griffiths, 1984) of the arguments surrounding the concept of minimum work and uniform energy dissipation. The criticisms mostly focus on:

1. whether it is legitimate to use an analogue to apply the thermodynamic principle to stream counterparts. Although Kleidon et al (2013) have since made clear that this is simply a function of how work is defined; and
2. the compatibility with observations when combining these extremal hypotheses with conventional sediment transport and flow resistance equations, although recent work by Huang et al (2014) demonstrated the applicability of these concepts to the fluvial reaches of the Yangtze.

In many systems either river or tide may dominate, while there are some systems where there is a strong interplay between river and tide, such as the Yangtze, where the river discharge changes dramatically from season to season (Chen and Zhao, 2001; Xu et al., 2005; Xu and Milliman, 2009), and river dominance changes to tide dominance cyclically from wet season to dry season (Dalrymple and Choi, 2007; Zhang et al., 2012). Such systems have strong signals from both river and tide, and so provide a useful basis for investigating how energy dissipation and concepts such as uniform energy density, or minimum work vary as the energy inputs to the system vary.

Some recent work has examined the hydraulic geometry of the middle-lower Yangtze (Huang et al., 2014) and demonstrated that equilibrium geometry (width/depth ratios) is consistent with conditions of minimum energy slope equivalent to maximum sediment transport. They attributed the deviation of theoretical predictions from observed equilibrium state to tidal influence. Attempts to determine the evolution of a longitudinal configuration of estuary based on the net sediment flux, i.e. “mass flux equilibrium” model were first proposed by Gilbert (1876) and developed by Ahnert (1994). Todeschini et al. (2005) and Guo et al. (2014) applied it to estuary and argued that when the system achieves equilibrium, the net sediment flux over a tidal cycle is constant throughout the estuary and equals to the constant sediment flux delivered by the river. Others argued that bed equilibrium can be reached without river discharge, in the presence of either fixed channel width (Lanzoni and Seminara, 2002; van der Wegen et al., 2008; Seminara et al., 2010; Toffolon and Lanzoni, 2010), or the channel width resulting from the requirement that the shear stress is everywhere lower than the critical value for bed erosion (Lanzoni and D'Alpaos, 2015). Research by Davies (1964) and Pethick (1984) and more recently by Guo et al. (2014) and Bolla Pittaluga et al. (2015) also emphasizes the coupling effect of river and tide on the development of an equilibrium estuary form. Of course there are other definitions adopted for estuaries, such as the net balance of erosion and deposition over a tidal cycle (Townend, 2012) and the controlling effect of local geological constrains (Dalrymple and Choi, 2007;Davies and Woodroffe, 2010).

Based on energetic minimization principles, Nield et al. (2005) argued the optimal channel form may not be unique, but the system is able to reside in one of several compatible states, without having to increase energy expenditure temporarily in the process of approaching a more energy efficient system. Similarly, Townend and Pethick (2002) used the term “goal seeking” to characterize the energetic optimal principle as a “dynamic process” in seeking the most efficient system. For a system, initially in equilibrium, a change in the external forcing, such as a decrease in river discharge, may give rise to a new equilibrium (target) and the estuary should seek to adjust to the new equilibrium in some way (such as infilling with sediment). However, whether this target can be achieved is strongly dependent on both the rate of such external changes and the response time of the estuary. If river discharge varies monotonicly and at a rate slower than that of the estuary response, then the estuary should follow external changes closely, with only a small perturbation around the equilibrium level. In contrast, under rapid change it may not be possible for the system to adjust and so the system remains out of equilibrium until the change abates to something more compatible with the system response time.

Thus, how quickly a system can respond to external changes is regarded as crucial to determining the basis of dynamic equilibrium over a suitable time interval (note, dynamic equilibrium is used here in the sense of a balance of processes consistent with environments and constraints as discussed by Thorn and Welford (1994) in the form of “metastable equilibrium” and as distinct from the mass flux equilibrium or steady state). This necessitates a careful distinction of timescales, where we look at the “state” integrated over a tidal cycle so that the boundary condition is approximately constant and at distinct river flow conditions or states, again so that for the interval of interest the boundary condition is constant (Prigogine, 1955) .

The river discharge from the Yangtze catchment changes periodically with wet and dry seasons following the annual precipitation cycle. These changes are relatively rapid and it is anticipated that the morphological response of the Yangtze (the fluvial system) is unlikely to keep pace with such frequent changes, because of the comparatively slow sedimentation response (Xiong, 1996; Yang and Yang, 2014). However, there can be expected to be a similarly rapid response of tidal propagation to any changes in river discharge. This has the potential to mitigate some of the influence of the fluvial dynamics helping to maintain the system in, or at least close to, some form of equilibrium state. The adaptation of the Yangtze estuary to the very substantial variations in fluvial influence and how this relates to concepts such as minimum work and uniform energy dissipation are explored in this paper.

To explore the relative importance of tide and river contributions to the work done locally and globally, a range of discharge conditions are considered for the tidal reach of the Yangtze estuary. The seasonal variation of energy flux and the equilibrium mechanisms due to the interplay between river and tide are examined. To explore these interactions in more detail, under controlled experimental conditions, hydrodynamic models are set-up to cover the river and estuary from a location 550km upstream and extending seaward to the mouth and out into the East China Sea.

Two models are used for the analysis, one solving the depth averaged shallow water equations in 2D and applied to the actual bathymetry and the other a quasi-analytical solution applied to a highly simplified representation of the estuary geometry. Both are calibrated and validated using extensive contemporaneous field data collected over spring and neap cycles. The use of two models helps initially to confirm that the dominant processes are being represented and the residual differences between the models, notably in energy flux, help to identify the importance of tidal asymmetry, which is further explored in detail in this paper. The key aspects of the models are briefly outlined but the focus of the paper is on a description of the energy and energy flux under different scenarios and the subsequent interpretation of how river and tide interacts and what this implies about energy dissipation and concepts such as uniform energy density, or minimum work vary as the energy inputs to the system vary.

**2 Overview of the Yangtze estuary**

The middle and lower Yangtze river runs through a vast low and flat alluvial plain formed by the main Yangtze river and its funnel shaped mouth. The Yangtze estuary is located downstream from Datong, which is approximately the tidal limit during the dry season (Figure 1). The estuary has a length of approximate 600 km and width of 90 km near the mouth. The channel pattern of this reach is mostly weakly meandering, while a relatively big turn is identified at around 250km from the mouth. Such a high sinuosity, due to the un-erodible basement, has significantly influenced the tidal propagation further upstream (Zhang et al., 2015). At several places along the estuary large islands emerge, forming two separate channels. Downstream from Xuliujing, the estuary branches into the North Branch and South Branch. Further downstream, the South Branch branches into the North Channel and the South Channel.

The Yangtze estuary experiences an irregular semi-diurnal tide, with a mean and maximum tidal range of 2.67 m and 4.62 m, respectively. The average flood and ebb duration are 5 and 7.5 hours, respectively (Zhang et al., 2012). Most of the river-born material is composed of silt and clay (with Mass-median-diameter D50=0.027mm), half of which settles in the river mouth area (Yang et al., 2000). Except for several extremely wet or dry years, the water discharge has remained relatively constant since the 1950s, while the sediment discharge has fallen from ~490 Mt/y in the 1950s and 1960s to ~150 Mt/y after the closure of the Three Gorges Dam in 2003 (Xu and Milliman, 2009). Most of the catchment is affected by the southeast monsoon in summer, so the precipitation occurs mostly from May to October. According to data collected from 1956 to 2009, the average rate of fresh water discharge is 29,000m3/s, and the monthly mean discharge reached a maximum of 66,200 m3/s in July, 1983, and a minimum of 7,600 m3/s in February, 1987. The South Branch delivers >99% of fresh water to the sea, whereas the channel of the North Branch is almost perpendicular to the main channel and discharges only a small proportion of the total flow (Zhang et al., 2012). Typical river discharges for the dry and wet season are 15,000 m3/s and 35,000 m3/s respectively. This suggests an Estuary number (or Canter-Cremers number, N=, where *T* is the tidal period, is the tidal prism and is the river discharge) during a mean spring tide of ~0.1 for the dry season, and ~0.5 for the wet season at the mouth. The large variation of N indicates that the Yangtze estuary is a marine well-mixed channel in the dry season (Zhang et al., 2012) while it becomes totally river dominant in the wet season (Xu and Milliman, 2009).

**3Methodology**

**3.1Numerical method**

**3.1.1 The TELEMAC model**

A detailed hydrodynamic model using the open-source software TELEMAC (Hervouet, 2007) was set-up to cover the Yangtze estuary extending out into the East China Sea, and driven by tidal boundary conditions obtained from TPXO[[1]](#footnote-1) and a river boundary based on daily recordings at Datong (some 600km from the mouth). The model set-up has been described elsewhere (Zhang et al., 2015) , and only a brief summary is given here. The model solves the shallow water equations, resulting in a detailed description of the flow field (i.e., velocities and water levels) over the model domain. Navigation charts extending from the -20m-isobath of the outer delta, to 600km upstream at Datong were collected to define the bottom boundary. The GEBCO (http://www.gebco.net) database information was used to define the continental shelf and parts of Hangzhou bay that are not covered by navigation charts. A digital elevation model (DEM) was generated by the assimilation of heterogeneous data types having corrected all of them to mean sea level of Huanghai1985 datum. They were then projected to UTM Zone 51 coordinate and interpolated to 200×200m grid DEM with Kriging Interpolation in ArcGIS. Shoreline positions were set at dikes or land-sea boundary lines at spring tide extracted from archived satellite images (<http://glovis.usgs.gov/>). They were assigned with the elevation of high water (Wu et al., 2002) to help define the intertidal zone.

Details of the model validation at the mouth area in terms of 10 anchored surveys are given in (Zhang et al., 2015) . The observations at both neap tides (22–24 July, 2007) and spring tides (29–31 July, 2007) were obtained synchronously and semi-synchronously at the measuring sites with 5 boats. The data were used to determine some of the common model parameters for all simulations (e.g., the Manning friction coefficient). The comparison generally presents good agreement and the root mean square errors were considered to be sufficiently small to justify further use of the model for the range of conditions planned. Further validation has examined the upstream mean water levels based on the data measured at five hydrological stations, i.e. Datong, Wuhu, Maanshan, Nanjing and Zhenjiang (Figure 1). For the wet season case the difference between modelling result and measured data is within 5-10% at the five stations. The dry season validation is within 5% at Zhenjiang and Datong, 10-15% at Maanshan and Wuhu and less than 20% at Nanjing (see Figure 3). It is possible that the complex topography of the Yangtze has a greater effect during low flow conditions and further refinement of the bed representation is needed to improve the model performance further.

**3.1.2 Scenario simulations**

In order to investigate the response of tidal wave propagation to river discharge under controlled and therefore well-defined conditions, a number of scenario simulations were performed.

The tides in the Yangtze estuary are approximately the same over each spring-neap tidal cycle. Possible causes of longer term fluctuations such as the 18.61 year lunar nodal cycle and the 8.85 year cycle of lunar perigee are small and have minimal influence in this part of East China Sea (Haigh et al., 2011). Therefore, a representative tide for a spring-neap tidal cycle was derived using the method of (Latteux, 1995) and used to define representative harmonic constants. This representative tide allowed all the simulations to be conducted using this same tide to force the open seaside boundary.

Although the magnitude of monthly-averaged discharges varies significantly from wet season to dry season (Xu and Milliman, 2009), the fluvial speeds generally vary only slowly, typically at a weekly to monthly time scale in response to precipitation across the catchment. Therefore, the river currents can in most cases be considered to be effectively constant compared with the semi-diurnal variation in current speeds that characterize the tidal system. As a result, constant river discharge rates of 15,000 m3/s, representing a typical dry season, and 35,000 m3/s, representing a typical wet season, were adopted. This allowed the interactions of river and tide to be explored, whilst avoiding the need to interpret the additional complexity of time series data, with fluctuations at a number of time scales. For some more extreme dry season conditions the fluvial influence is insignificant compared with the tidal currents and so a condition close to the no runoff situation (1,000 m3/s) was also included. This also provides an estimate of the discharge and work done purely by the tide. At some spate conditions, peak river flows can be much larger than the typical wet season flow rate and so a peak river flow case (60,000m3/s) was also included. In order to ensure model integrity, all other model calibration parameters were kept constant for the various scenarios examined.

The scenarios modelled are shown in Table 1. The peak discharge case is dominated by the river flow, with the tidal influence only detectable over the outer reach of some 130km. Even at the mouth the discharge attributable to the tide is only about 65% of the river discharge. The river only case (without tidal forcing) confirms that the discharge remains constant along the length of the estuary. The other three cases lie between these two extreme cases and provide a basis for assessing how varying river flows influence the energy flux and associated dissipation of energy within the estuary. The analysis therefore focuses on the seasonal means and the ‘so called’ no-runoff case.

**3.2Analytical method**

**3.2.1** **The CST model**

The bathymetry of the Yangtze estuary is complex with numerous islands, narrows, gorges and deeps. Thus, the detailed model output provided by the TELEMAC model, which reflects this high degree of variability, can be difficult to interpret. Therefore, a much simpler analytical model was also used for comparison and as a tool for isolating key parameters. The model adopted is the solution proposed by Cai, Savenije and Toffolon, which is well suited and has already been applied to the Yangtze estuary (Cai et al., 2014). , derived from the conventional 1-dimensional conservation of mass and momentum equations (Savenije, 2012)ant,, The model reproduces the main tidal dynamics (i.e., tidal amplitude, velocity amplitude, wave celerity, and phase lag between elevation and velocity) along the estuary axis. This is referred to here as the CST model. For a full description of the model, readers can refer to Cai et al. (2014) and related papers. The model was also applied to the scenarios outlined in Table 1 and the results are compared with those obtained from the TELEMAC model.

**3.2.2** **Model setup**

The selected tide and river parameters used in the CST model were the same as those for the TELEMAC model. The additional parameters required are the hydraulic geometry, the storage width ratio and the bed friction.

In many alluvial estuaries the geometry profile of Cross Sectional Area (CSA) and width vary exponentially along the channel (e.g. Savenije (2012)):

(1)

where is the CSA (or width) at the mouth, *x* the distance along the estuary (directed in landward direction) and *L* the convergence length, which determines the rate of exponential decay such that the parameter halves for every multiple of *x=L.ln(2)* where *ln* is the natural logarithm.

For the CSA and width, it is possible to fit an exponential function to the seaward reach, while a linear regression provides a better representation of the landward part of the estuary. The results for the wet season model output are shown in Zhang et al. (2015). Results for the other cases are similar in form but the regression results for convergence length (L) and the distance to the transition point (distance from mouth to where the funnel estuary becomes a prismatic channel) are slightly different (see Table 2). The hydraulic geometry of the estuary shows some variation over the wet and dry season. In the wet season the CSA is exponential in character over a shorter length when compared to the dry season length and has a longer convergence length, again in comparison to the dry season.

A unified representation of funnel-prismatic estuary is more convenient for analytical modelling. In this paper, we assume the CSA and width can be described by the following exponential function:

(2)

where is the river CSA (or width) and is the adjusted convergence length. The adjusted convergence length retains the rate of convergence in the seaward part of the estuary, whilst accounting for the transition from funnel-shape reach to prismatic reach. The hydraulic geometry of the values at mouth, river and for the different scenarios are presented in Table 2.

The complexity of the channel in terms of islands, narrows, gorges and deeps cannot be adequately represented by a smooth exponential convergence. One way of representing this in the CST model is to adopt an “effective friction”. Estimates of the effective friction were obtained using the local slope, s, the hydraulic depth, h, and the cross-sectionally averaged velocity, u, in Manning’s friction equation:

(3)

where K=1/n is Manning-Strickler friction coefficient (m1/3s-1), with n being Manning’s coefficient. The estimates were used to guide the subdivision of the estuary into three reaches (0-176km, 176-245km, 245-550km). Friction was introduced so that it is linearly interpolated within each reach. Initial values were based on the analysis of K values derived from the TELEMAC modelling. These were then adjusted so that a reasonable fit was achieved for mean water level, tidal amplitude and tidal velocity. The resultant friction factors adopted are detailed in Table 3.

**3.3 Computational framework for analysis of energy flux**

**3.3.1 Hydraulic parameters**

In order to quantify the seasonal variance of hydrology and energy along the Yangtze estuary, 97 cross-sections at intervals of 5,500m were extracted using the method proposed by Davies and Woodroffe (2010), (see Figure 1). Free surface elevations and mean velocity over a cross section were used to define surfaces based on High and Low Water elevations (HW and LW) for each grid cell and the surfaces at the time of slack water– High and Low Water Slacks (HWS and LWS), again for each grid cell. These, in-turn, were used with the bathymetric data to compute the discharge and energy flux on both flood and ebb periods, net and total over a cycle. In order to detail the effect of tide, it is necessary to decompose the total velocity into tide () and river () contributions. Simplistically, river velocity could be determined by assuming a constant river discharge and predefined CSA (), and the tidal velocity could be given by taking the river flow from the total velocity. However, this overlooks the influence of Stokes drift, which is caused by a phase lead or lag between tidal elevation and tidal currents, when velocities are obtained at fixed cross-sections in an Eulerian framework (Savenije, 2012). As Stokes velocity can be determined from the tidal velocity (Savenije, 2012), where c is the wave celerity, the separation of tidal and Stokes components from the model output involves an iterative procedure to extract both values.

**3.3.2 Energy and energy flux over a tidal cycle**

Following the definition by Richards (1982), the mechanical energy head per unit width (J/m2) in a combined tide-river framework relative to a zero datum () can be written as Zhang et al. (2015):

(4)

where is the elevation of the mean water surface relative to zero datum (m), is the free surface variation (m), is the tidal velocity (m/s) in a Lagrangian framework, the river flow velocity, , is negative as it is directed seaward, ρ is the water density (kg/m3), g is gravitational acceleration (m/s2), with being the channel bed elevation (m). is often associated with the mechanical energy of kinetic energy, tidal wave potential energy and riverine potential energy (Knight, 1981; Pedlosky, 2003; Huang et al., 2004; Nield et al., 2005; Ensign et al., 2013), since it is directly related to river-tide hydraulic gradient variance and energy dissipation.

Consistent with the method proposed by Tulin (2007), Buschman et al. (2009) and Sassi and Hoitink (2013), equation (4) can be decomposed into contributions from

(i) tidal mechanical energy (J/m2), comprising the tidal range potential and kinetic energy

, where (5)

(ii) fluvial mechanical energy (J/m2), comprising the residual water level potential and kinetic energy

, where (6)

(iii) river-tide interactions of kinetic energy and potential energy .

In addition to the mechanical energy comprising kinetic and potential components, the energy head includes a pressure term, , where , Z is the free water surface, which could be the mean water surface, , in a river or the elevation of the tidal wave, , in an estuary. As a result the total energy head (per unit width) for river (J/m2), tide (J/m2) and river-tide combination (J/m2) can be defined as:

(7)

(8)

(9)

With these definitions, energy head density (also called the total head, H, J/m3) can be easily obtained by dividing by the relevant water depth (, or ).

Ignoring any second order interactions between river and tidal flows, a simple superposition of flow velocity, flow width (m) and energy per unit width gives the time dependent energy flux over width (W or J/s):

(10)

where is the mechanical energy flux () or total energy flux () for the tidal component, river component or river-tide combination, which are calculated by using the relevant velocity (, or ) for u, and the relevant mechanical energy per unit width () or total energy per unit width () for , respectively. For estuaries with large intertidal area this would tend to overestimate the cross-sectional values because the same energy density is being applied across the entire width, while in reality additional dissipation is likely to occur over the shallows of the intertidal. However, in some cases, such as the Yangtze, this simplification is reasonable as the intertidal area is minimal.

The energy flux is a vector and is time dependent, but the integral over a tidal cycle should be constant from cycle to cycle, when the modelling is performed with a fixed seaward boundary condition. In this study we integrate the energy flux, equation (10), with respect to time over a complete tidal cycle to give the net (, Ws or J) or total (, Ws or J) energy passing through a section by taking the modulus (Townend and Dun, 2000; Ensign et al., 2013), where represents the difference of energy flux between flood and ebb tide, and represents the sum of energy flux for both the flood and ebb fide, for mechanical energy flux over a tidal cycle, and for total energy flux over a tidal cycle. Estimates can be further separated into components from tide, river and river-tide combination. The time interval can also be subdivided, for example, to consider the energy flux over the flood period (, Ws or J) or ebb period (, Ws or J) by integrating over a flood tide (LWS to HWS) or ebb tide (HWS to LWS). In summary, we make use of the following integral parameters

and (11)

**4Results**

**4.1Modelling results and comparisons**

Following setup and validation of the two models, a direct comparison of the results was made. Given that the friction values for the CST model were established using some of the TELEMAC outputs, it is to be expected that the two models should be reasonably consistent. Comparisons of mean water level, tidal amplitude, and velocity amplitude are shown in Figure 3. Overall the CST model is able to reproduce the main characteristics of the flow. The main discrepancies from the numerical model are:

1. for the no runoff case, where there is an apparent difference in mean water levels; however the friction used in the model may be unrealistic for this case because the model has been calibrated and validated for river flows characteristic of wet and dry season discharges.
2. for the peak flow case, where the velocity appears to be a little underestimated by the CST model in comparison to the TELEMAC estimates. However, neither model has been validated against measured data for this condition.

There are some rapid changes in the TELEMAC model results reflecting the complexity of the bathymetry, which is represented in considerably more detail in the TELEMAC model. The presence of many islands, narrow and deep gorges and the rapid transitions make the upstream tidal reaches particularly challenging and more extensive measurements of elevation and velocity in this region would be needed to validate these models if changes over such local scales are the focus of interest. However for this application the primary interest is the interaction of river and tide at an estuary scale.

Overall the CST model provides a reasonable representation of the tidal conditions under different river discharge conditions and this should be sufficient to allow a meaningful comparison of the system energy and dissipation along the estuary.

**4.2 River-tide hydraulics and their contribution to the system in terms of energy**

The hydraulic slope along the estuary is the result of tidal propagation and the interaction of river and tide, which gives rise to a backwater effect (Godin, 1999) . At the mouth the tidal range is approximately the same given different river discharge conditions (reflecting the use of constant tidal boundary conditions for all cases) and the MWL is close to 0m (Figure 4a, b, c, d). As tide propagates landwards, the MWL gradually increases and the tidal wave is damped (Figure 4a, b, c). In the no runoff case, the water level changes more gradually due to the lack of river-tide interactions (Figure 4a), and provides a lower bound point of reference which is beyond the observed range of flow conditions. The two obvious water level setups at around 250km and 290km are the result of geological effects, as identified in Zhang et al. (2015). For example, in wet season the gorge at 250km sets up MWL by 1.1 meter where the tidal range is reduced significantly from 2.5 meters to 1 meter.

Tidal mechanical energy, as given by the tidal energy flux over a tidal cycle, decreases more or less exponentially along the estuary (blue line in Figure 4e, f, g and h). At the mouth, MWL is almost at 0 m, tidal range is around 4 m and the current velocity is around 1 m/s, so the tidal energy is mostly composed of tidal range potential energy and kinetic energy. Fluvial mechanical energy, as given by the river energy flux over the same period, by contrast, decreases in the opposite direction (red line in Figure 4f, g and h), mostly because of the residual water level decreases as the river approaches the sea. The relatively constant current velocity, tidal range and exponential convergence width for the estuary, and relatively constant current velocity, flow depth and exponential decreasing elevation for the river is compatible with equilibrium conditions for an ideal estuary (Leopold and Langbein, 1962; Savenije, 2012; Bolla Pittaluga et al., 2015). Similarly, the results for the "system total energy" show similar trends with the largest values, at the mouth caused by tide and at the landward boundary produced by river discharge. In between there exists a minimum energy for the combined river and tide energy (Figure 4j, k and l), where the energy contributions from river and tide are equal. One important difference, compared with the system mechanical energy, is that the total energy for the river transitions from an exponential growth over the tidal reach to a more linear trend further landwards. This effect becomes more pronounced as the river discharge increases.

The fluvial kinetic energy () over a tidal cycle is relatively constant over the upper reaches (green line in Figure 4f, g and h) where the channel has a prismatic shape, and the fluvial velocity is approximately constant (~1m/s). In contrast, towards the mouth it decreases as the fluvial velocity decreases because of the expanding cross-sectional area (). However the contribution of river-tide interaction to the kinetic energy (, where is the mean ebb velocity, m/s) can potentially enhance the contribution of the fluvial process. If this energy is included, the constant kinetic energy contribution from fresh water discharge is extended further downstream to the mouth. Overall this supports the assumption of constant kinetic energy contribution from the river, which is often adopted for tidal rivers (Ensign et al., 2013). However, the potential energy of the river is totally dominant upstream, especially for larger river discharges.

The overall trend of system mechanical energy and total energy is similar for both the TELEMAC and CST model (solid and dotted lines in Figure 4e, f, g, h, i, j, k and l respectively). However, the tidal energy estimated using the CST model is consistently larger than that of the TELEMAC model as the river discharge increases. This difference is further explored in section 5.4. The large fluctuation in fluvial energy upstream of 250km in the TELEMAC model results is due to the complexity of the channel geometry. Local constraints in terms of islands, narrows, gorges and deeps result in localized increases in water level, thus increasing the potential energy which cannot be dissipated by friction in the space and time scales of the local system.

**4.3 Seasonal variation of tidal mechanical energy during the flood and ebb**

It has been suggested that river discharge attenuates the tidal motion (Sassi and Hoitink, 2013), thus the flood discharge volume () decreases as river discharge rate increases (Figure 5a, solid lines). The locations I, IIand III in Figure 5a mark the tidal current limit for the peak, wet and dry season respectively. Further upstream bi-directional tidal dominance becomes unidirectional river dominance. It is worth noting that the flood duration (LWS-HWS) and ebb duration (HWS-LWS) are not fixed along the estuary. As river discharge gains influence upstream, the ebb duration increases. Therefore, at a point where LWS and HWS coincides the flood duration is reduced to zero and further upstream the flow is always seaward. If one removes the river contribution to the discharge, , from the total ebb volume (), the ebb tidal volume ( is almost equal to at each cross section for all cases examined (Zhang et al., 2015).

Mechanical energy is directly related to the energy dissipation. We can see from Figure 5b that the energy distribution during the flood and ebb periods shows good resemblance with the corresponding discharged water volume (Figure 5a). Both of them show an exponential decrease in the tide-dominant reach of the estuary, while a roughly constant value in the river-dominant reach. In addition, the fresh water discharge attenuates the distribution on the flood limb while reinforcing the distribution on the ebb limb. Importantly we observe that after filtering the estimated riverine kinetic energy contribution () from the ebb total mechanical energy (), the ebb tidal mechanical energy () is found to be approximately equal to the flood mechanical energy () for the wet season case (m3/s, m3/s). In other cases there is a residual tidal mechanical energy between flood limb and ebb limb for peak (m3/s, m3/s), dry (m3/s, m3/s) and no runoff scenario (m3/s, m3/s). Furthermore, the ebb tidal mechanical energy shows almost the same distribution under different river discharge conditions, suggesting that river-tide interaction varies in such a way that the ebb tidal mechanical energy is comparable at different river discharge conditions (Figure 6a).

As plotted the ebb tidal mechanical energy in the wet season is still a little larger than the flood mechanical energy (Figure 6b, red points). This is mainly due to river-tide interactions (), which reduces the flood energy and reinforces the ebb tidal energy. If this contribution is removed, the ebb tidal energy shows a much better agreement with the flood energy for the wet season. By noting that the ebb tidal energy in the no runoff case is just the ebb tidal energy without river disturbance and plotting this against the flood energy for the wet season (Figure 6b, black points) this equivalence is clearly demonstrated. As the energy flux in and out of the estuary is approximately equal, this implies that the tide does as little work as possible while the system energy dissipation is mainly introduced by the river flow.

**4.4 Entropy production**

Leopold and Langbein (1962) suggested that an exponential decay of the energy head is the most probable state to achieve equal entropy production per unit discharge, which is proportional to the longitudinal gradient of energy dissipation (, dS is entropy production, dH is change in energy head over distance dx). Considering that estuaries are governed by time and directionally varying discharges over the tide, following Townend (1999) we suggest longitudinal energy dissipation per unit volume in a combined river and tide system can be expressed by evaluating the total energy flux (including the pressure energy) over a tidal cycle () instead of the energy head (). When it is factored by energy density (), the rate of entropy production per unit volume is given by:

(12)

An alternative proposition is that the system does uniform work along the system. The study by van der Wegen et al. (2008) analyzed model simulations of an 80 km basin and concluded that the tidal energy decreased linearly along the channel, once the system achieved equilibrium after some 3,200 years. This implies uniform work per unit volume:

(13)

Our results for the Yangtze estuary show that both tidal energy and river energy exhibit a reasonable exponential fit within the tidal current reaches (outer 300km) in all discharge conditions, while further upstream river energy shows more of a linear trend (Figure 4 i, j, k, l). To investigate the development of the assumed equilibrium states, equations (12) and (13) for both river and tide at various conditions are illustrated in Figure 7. The dotted lines show the mean of the non-zero values for tide and river components.

The results suggest that tidal energy dissipation per unit volume varies about a mean value to a reasonable degree in all cases (), while the river is too noisy to make any conclusion. The anomaly of energy dissipation is believed to be a consequence of the local geological constraints. These local factors, mostly in the form of non-erodible bed rock, can be considered as a physical constraint of the system, preventing it from evolving toward a most probable state with a more gradual and uniform distribution of energy dissipation. The constant tidal energy dissipation per unit discharge reflects the linear character of tidal energy head density, while the total energy is dominated by the along basin discharge profile, which decreases exponentially. When considering the energy flux gradient relative to the energy available of that location (), values for the tide from the mouth to the tidal current limit (130km for peak case, 170km for wet season case, 250km for dry season case and 300km for no runoff case) are relatively constant. A similar result is also obtained for the river from the head down to the tidal current limit. As one should expect, this quotient () works for both river and tide because the energy flux combines the approximately constant variable (energy head density for tide and discharge for river) with the exponentially decaying variable (discharge for tide and energy head density for river). Therefore, the results support the use of as the entropy production in a combined river-tide system, which can be collapsed to principle components for river and tide, namely for tide and for river, respectively. The constant entropy production for both river and tide support the criteria of minimum total work in the system as a whole. Equation (12) can also be seen as the energy dissipation percentage with distance for the river () and the tide (). The resulting averaged global energy dissipation percentage for the Yangtze estuary is 6.67% (no runoff), 5.23% (dry season) and 5.12% (wet season and peak flow). These results are therefore reasonably consistent with uniform energy dissipation locally. The morphological implication is that the tidal channel section will vary so that the discharge varies exponentially and the surface elevation within a river will similarly vary exponentially along the channel.

**5Discussions**

**5.1 Variation of hydraulic geometry**

Over the landward half of the Yangtze estuary, the scaling behavior of the channel geometry coincides with the scaling observed in river deltas (Savenije, 2012), and has a prismatic channel. Over the seaward part, the channel geometry resembles that of funnel shaped estuarine channels (Davies and Woodroffe, 2010), reflecting the importance of the tidal discharge relative to the river discharge in channel forming processes (Pethick, 1984). There is an intermediate zone between the reaches without clear river or tide dominance. The location of this zone moves in response to changes in forcing, both river and tide, but in the Yangtze the movement is controlled by the seasonal changes in river discharge (Figure 5a). This is also reflected in the overall estuary shape which is seen to vary between wet and dry season (Table2). The extent of the funnel shaped portion of the channel reduces as the river discharge increases (distance to transition point in Table 2). By taking account of the changing length of the prismatic (riverine) section, as used in the CST model, the convergence lengths () also gets shorter with increasing discharge (Table 2). Thus the rate of convergence is more rapid in the wet season than the dry season largely because the extent of the riverine reach extends seawards.

Another feature of the Yangtze is a number of very dramatic changes in the actual form of the cross-section from deep and narrow to broad and shallow. One consequence of this is that for some of the sections used in the analysis the actual cross-sectional areas are not representative of the effective hydraulic section. The conventional way to obtain the river velocity simply uses the defined river discharge and the measured cross-sectional areas (ur = qr/A), as mentioned above. However, when this estimate is compared with the velocity obtained by running the TELEMAC model for the same river discharge with no tidal forcing, as shown there is a lot of noise and a rather poor agreement (Figure 8a). This was addressed by using the tidal mean of the total velocity (river and tide, ie ) to estimate the residual velocity, which was found to provide a far better agreement, as shown by the “+” symbols in Figure 8a. Therefore, the tidally averaged of the total velocity has been used to define the river velocity and to separate the tidal velocity from the total values in the model. Using the resultant river velocity, the “effective” cross-sectional area was calculated (effective CSA= qr/ur) and compared with the actual mean-tide CSA, as shown in Figure 8b. The disagreement highlights the difference between the actual CSA section and the effective hydraulic section, which is particularly the case in narrow gorges and branches, and the deeps that are often found immediately downstream (where the CSA in some cases is more than 60,000m2). In these areas, changes in hydraulic gradient can be very rapid and this may not be properly represented in the model because of the rather coarse specification of bed friction, which is unable to capture such local variations.

**5.2 Tidal storage variation**

The tidal storage volume (or tidal prism) is defined as half the total discharge less the river discharge (i.e. ) (Zhang et al., 2015). This is essentially the water stored in the system over a tidal cycle and is derived by assuming that the river discharge is approximately constant over the length of the estuary and that the tidal flood and ebb discharge are equal. This tidal storage volume has been shown to be equivalent to the geometric volume given by the volume between the slack water surfaces over the reach where the flow reverses plus the volume between high and low water surfaces further upstream. More remarkably this volume remains approximately constant for the various river discharge conditions considered (when imposed with an invariable tidal forcing) (Zhang et al., 2015). When there is little or no river flow, this volume is occupied by the tide. As the river flow increases, a backwater effect is established reducing the tidal volume and replacing it with stored river water during the flood and evacuating this stored volume on the ebb. This is illustrated schematically in Figure 9.

From the above we can conclude that the volume stored during the flood is equal to the volume of river inflow over the same period. The ebb discharge includes this stored volume and the river inflow over the ebb. There is therefore a mass balance of river waters. Once corrected for river discharge and the volume stored on the flood, the actual flood and ebb tidal discharges are symmetric resulting in a mass balance of tidal waters. When the river inflow over the period is added on the flood (or subtracted on the ebb), the tidal storage volume is obtained. This volume includes the contribution from the river, which is stored on the flood and released on the ebb, and remains approximately constant regardless of changes in discharge. As to be expected, tidal storage volume is larger than the tidal discharge when the river flow is significant, but converge to be equivalent, as the river flow reduces to zero. The fact that tidal storage volume for given tidal conditions, is invariant with river discharge conditions suggests that the tidal propagation, tidal asymmetry and water surface slope adjust in such a way that the volume stored over a tidal period remains constant. The constant tidal storage volume implies a constant ebb tidal energy, as shown in Figure 6a, because the same amount of water is released completely over the ebb, regardless of the discharge rate.

**5.3 The mechanism of energy dissipation**

As a tidal wave propagates into a shallow estuary, it is modified due to many influences, including along channel changes in width, variations in propagation speed due to the influence of intertidal storage, varying bed forms (and hence bed friction conditions), the blocking (on the flood) and enhancing (on the ebb) effect of any river discharge. As well as determining the character of the tidal wave (propagating, standing or more likely some combination) these various mechanisms contribute to the spatial adjustment of the tidal asymmetry.

The research by Guo et al. (2014) showed that the decomposition of the total sediment transport can be analyzed by tidal asymmetry under the effect of river discharge and overtide in a 1-D model study. They identified three major components, namely: residual sediment transport from Eulerian net transport (ENT), Euler-induced asymmetry (EIA) and tide-induced asymmetry (TIA).The spatial variation of these three components were used to explain the process of sediment transport in terms of the relative significance of river discharge and tidal asymmetry.

To explore how tidal asymmetry contributes to the distribution of energy under different forcing conditions, we decompose the total current and surface elevation into residual current and water level, and periodic M2 and M4 tidal currents and elevation in a similar manner to Guo et al. (2014), as:

(14)

(15)

in which is the residual current, is the residual water level, , and , are the amplitude of the M2 and M4 current and elevation respectively, and are the M2 tidal frequency and wave number, and are the 2M2-M4 phase difference of velocity and elevation, respectively. Another internally generated overtide, M6, is smaller in amplitude compared to M4 and was not included in this study.

The residual current can be induced by Stokes drift and/or river discharge (if it is present). Stokes drift is comparatively small in magnitude (order of a few cm/s) and exhibits a slightly decreasing trend with increasing river discharge (see 3.3.1 for details). Generally Stokes drift is directed landward (Stokes, 1847; Guo et al., 2014), however the landward accumulation of water and momentum will result in an enhanced seaward return flow due to mass balance, which is termed “Stokes return flow”. The Stokes return flow is responsible for the tidal energy dissipation, however the river discharge enlarges the seaward residual current significantly, and the Stokes drift becomes relatively minor and was ignored for this analysis.

The mechanical energy is defined as the energy associated with the motion and position of an object. This is therefore the sum of the potential and kinetic contributions, it is often referred as the true wave energy (Pedlosky, 2003; Huang et al., 2004; Nield et al., 2005; Ensign et al., 2013), and in this case reflects the water body’s ability to do work. At any given location the pressure energy exists but has no capacity to do work. Therefore energy dissipation is directly related to mechanical energy. Based on equation (4) and (10), we propose that the kinetic energy flux is proportional to the current velocity to a power of 3, and potential energy flux is proportional to current velocity and tidal elevation squared. Integrating over the whole cross-section gives the mechanical energy flux:

(16)

Three major contributors to the mechanical energy flux can be identified from equation (16). The first term relates to the energy of the Eulerian residual current and residual water level due to back water effect. As the Stokes drift is small compared with the seaward discharge-induced residual current from the river, this is referred to as the river net energy (RNE). The second term represents the interactions between the river residual current with tidal current (kinetic) and tidal range (potential), which has the same direction as the residual current from the river, and is referred to as the river-induced asymmetry (RIA). The third term is induced by the interaction between M2 and M4 tides, reflecting a tide-induced asymmetry (TIA). The direction is determined by the relative phase difference between the horizontal and vertical M2 and M4 tides.

We estimate the mechanical energy components using equation (16) based on the modeled residual current and elevation, M2 and M4 current, elevation and phase difference (Figure 10 a, b, c). The results show that over the landward half of the Yangtze estuary elevation potential energy is dominant and both elevation potential energy and kinetic energy show large fluctuations, while towards the mouth the elevation potential energy is much smaller and the kinetic energy is almost constant (Figure 10d, e). The RNE is dominant upstream whereas the TIA and RIA gain importance downstream. With increasing river discharge, we see an increase of RNE, RIA and TIA (Figure 10d, e, f).

Though the M2 tide is symmetric and the net energy flux over a tidal cycle is zero, its interaction with river (RIA) and M4 tide (TIA) increase the net mechanical energy flux. As shown in Figure 10f, the net energy flux produced by river induced asymmetry (RIA) is seaward (negative) and has a positive gradient landward, while the net energy flux produced by tide induced asymmetry (TIA) is landward (positive) and has a negative gradient landward. A larger river discharge is able to reverse the net tidal energy from flood dominance (due to TIA) to ebb dominance (due to RIA). As a result, the TIA is larger in magnitude than the RIA for no runoff and dry season cases, and the RIA is larger in magnitude than the TIA in peak case, while in the wet season the TIA almost equals to the RIA, which results in a balance of the tidal energy flux over the flood and ebb. This may explain why the tide effectively does no work in the wet season. Overall, the net energy flux in the estuary is still ebb dominant due to the presence of a significant river discharge (RNE).

**5.4 The importance tidal asymmetry in tidal energy dissipation**

Using the same assumptions and calculation method, the tidal mechanical energy over a cycle from the TELEMAC model and the CST model are compared in Figure 11a. The overall behavior of the models is similar, with the flood tidal energy varying over the outer 150 km in response to changes in river discharge, whilst the ebb tidal energy remains similar, regardless of river discharge. However the CST model clearly has a somewhat larger ebb tidal energy in comparison to the TELEMAC model.

The numerical solution considered retains all higher-order nonlinear terms to represent the tidal propagation over a complex bathymetry (Hervouet, 2007). Whereas, the CST model (Cai et al., 2014) formulates fluvial influence on tidal damping for a much simplified representation of the estuary shape and does not include tidal asymmetry either at the tidal boundary, or as a result of propagation. Thus the analytical solution provides a simplified representation without the noise of the more detailed model and the difference between the two highlights the influence of tidal asymmetry in the system response to tide and river flow.

In order to understand the difference between the numerical and analytical solution, the behavior of flow velocity, width integrated mechanical energy and width integrated energy flux over a cycle were examined. This is illustrated for a cross-section at x=11 km from the mouth in Figure 11b, c, d. The river discharge attenuates the flood velocity but reinforces the ebb (Sassi and Hoitink, 2013; Zhang et al., 2015). As a result, both models show a vertical shift in the velocity-time graph (Figure 11b), and a decrease in the flood energy peak and increase in the ebb energy peak (Figure 11c). However, a significant difference lies in the fact that the superposition of river discharge with tidal flows slow down the peak velocity and enlarges the time span on the ebb more than on flood so that the wave velocities from the TELEMAC model are highly asymmetric. In contrast the CST model uses a symmetric wave and does not account for the tidal asymmetry delivered to, or generated within, the estuary. This imbalance in the ebb and flood durations, and maximum flow velocities, is generated by the interaction of the various tidal constituents especially M2 and M4, shortening the rising tide and reducing ebb velocities. Dalrymple and Choi (2007) attributed the deformation of incoming symmetric tidal wave to the difference in friction at different water depths, which slows the trough more than the crest in shallow water. Savenije (2012) took account of the influence of residual slope on tidal wave propagation, demonstrating that the effect of river discharge on wave damping is similar to that of bottom friction. The flood tide asymmetry results in a smaller tidal energy flux on the ebb because of the weaker ebb tidal current (cubic relationship for energy flux), even though the ebb tidal period is longer. As a result, flow velocities from the TELEMAC model are deformed and have a smaller magnitude on the ebb with longer duration (Figure 11b). This results in a smaller magnitude of energy (Figure 11c) and energy flux (Figure 11d) than those of the CST model. Thus, the shorter duration, higher peak values, wave in the CST model gives a larger energy flux on the ebb than the longer duration, lower peak, wave in the TELEMAC model.

This illustration, at 11km, is relatively close to the mouth and further upstream the asymmetric gradient () between wet and dry season tends to increase along estuary. As shown in Figure 12, the longitudinal variation of the M4/M2 amplitude ratio for elevation and velocity shows a broadly similar pattern for the different river discharge conditions. For very low flows (no runoff case) the ratios are relatively constant. A modest increase of river discharge (the dry season case with =15,000m3/s) results in a pronounced increase of the ratio particularly between ~150-400 km. As river discharge increases, the peak value of the amplitude ratios moves seawards and the values in the upstream part reduce, reflecting the fact that the tide is progressively constrained within a more limited downstream reach. This reinforces the suggestion that tidal discharge and energy are changing between wet and dry season, not only due to water level but also because the tidal asymmetry is changing. A larger transmission of discharge and energy becomes possible if the tidal asymmetry is enhanced. This also implies that adjusting the asymmetry of the tidal wave is one of the mechanisms to enable the system to accommodate varying discharges and maintain a system that is at, or close to, a state of minimum work.

**5.5Morphodynamic equilibrium**

The distributions of energy and dissipation have been widely used to make statements about the most probable geometry in rivers and estuaries (Langbein, 1963; Nield et al., 2005; van der Wegen et al., 2008). Langbein (1963) suggested that the most probable rate of convergence is subject to two competing principles: (i) energy dissipation is as uniformly distributed along the estuary (a local condition) as is consistent with the tendency that (ii) the total rate of work in the estuary as a whole is a minimum (a global condition). In the following analysis, we explore whether these two principles are consistent with the patterns observed in the Yangtze estuary.

**5.5.1Energy dissipation locally and globally**

The ratio analysis of energy and discharge in section 4.4 shows that the tide is doing both uniform work and uniform entropy production per unit volume within the tidal reach under various river discharge conditions. Further upstream, energy dissipation from the river shows some anomaly but the entropy production still remains relatively constant (see Figure 7). Thus, both the river and tide achieve equilibrium if we use the criteria of equal entropy production.

An alternative approach that is consistent with the entropy based argument, examines the difference between total energy entering and exiting the tidal channel and, defines this as the global energy contribution to the work done (Nield et al., 2005). This is equivalent to the net energy flux over a cycle () as defined for this study. The longitudinal variation of under different scenarios is presented in Figure 13.

For an open system Thorn and Welford (1994) argued that a dynamic system must fulfill the energy conservation law:

(17)

where equals the rate of energy dissipation, and represents the total energy available to the system and there are no additional sources or sinks of energy. is energy exchange with the environment. When a system is in a state of dynamic equilibrium, the rate of energy dissipation should be a minimum subject to the constraints on the system, and the rate of change of energy tends to zero, ie . When this is the case, the divergence of the flux, , is equivalent to the energy dissipation, . For the one dimensional approach used in this study, the divergence of energy flux over a cycle is equivalent to the gradient of the net energy flux, . As a result, energy dissipation rate can be explored in terms of the gradient adaptation of to the variable river discharges. The dissipation of net energy flux from river and tide is relatively uniform for the entire estuary in both the wet season and peak flow cases (Figure 13b). In contrast, this is not the case for the no runoff and dry season cases (Figure 13a). The linear response of in hydro-morphology indicates that the system must be in a near-equilibrium state (Gu, 1987).

For the no runoff and dry season cases, the flood tidal energy is larger than the ebb tidal energy, which results in some tidal energy dissipation over a tidal cycle. Thus the response of the net energy flux over the tidal reach shows an exponential decrease (Figure 13a). However as the river discharge increases, the hydraulic slope of the tide is overwhelmed by the hydraulic slope of the river and the ebb tidal energy is balanced by the flood tidal energy at wet season and peak case, making tidal propagation appear to be effectively frictionless over a tidal cycle. As a result, the very energetic river flow extends the uniform energy dissipation further seaward to the mouth, and the propagation of the tide seemingly does no work, with the total energy dissipation of the system being attributed to the river.

The global energy dissipation rate per unit reach of combined river-tide system for the wet season case is J/m, which is obviously smaller than peak case of J/m. During peak flow conditions, the flow will overtop the natural levees and spill over at some locations (but this is not included in the model). Such over flow will do much more work than flow in channels (Kleidon et al., 2013), and so can be expected to do even more work than predicted here. Thus the Yangtze estuary bathymetry is well adapted for discharges typical of the wet season so that energy dissipation is minimal in this high energy condition with the consequence that there is some tidal energy dissipation under conditions more typical of the dry season, during the winter. Overall the system appears to keep energy dissipation as small as possible subject to the imposed constraints, so ensuring maximum energy efficiency over the year as a whole and perhaps reflecting the relatively mature state of the Yangtze estuary.

With respect to mechanical energy, minimum work for the tide is achieved in the wet season case. During the wet season, the river discharge reinforces the dissipated tidal energy through river-tide interactions ( and ), and results in the flood tidal energy equaling the ebb tidal energy. The analysis of tidal asymmetry also concluded that the net energy flux induced by TIA balanced the net energy flux induced by the RIA in the wet season case, making the tidal propagation effectively frictionless over a tidal cycle when the river potential energy dominates. This suggests that the wet season case is a condition that is close to the equilibrium state that we are looking for, i.e. the tide does minimum work (almost none), while the river and tide together are doing uniform work locally and minimum work globally.

**5.5.2 Criteria for equilibrium in a tidal river**

As already stated, it has been postulated that an estuary should seek the most probable state by following two energy optimization principles (locally and/or globally work). Employing a number of hydrodynamic approximations and assuming that width B, depth, velocity and water surface slope are power functions of tidal discharge Q, (Langbein, 1963) argued explicitly that principles (i) and (ii) were inconsistent with each other under a simple ‘ideal’ estuary assumption. The former implies , while the latter implies , cannot be achieved at the same time in a specific estuary geometry.

However, Nield et al. (2005) applied similar energy dissipation concepts to the prediction of the optimal tidal channel and concluded that this did minimise energy dissipation both locally and globally. Locally, the rate of energy dissipation per unit channel area was made as uniform as possible subject to constraints. Globally, the total rate of energy dissipation in the system was minimised. Similarly, the work by Rodríguez-Iturbe et al. (1992) and Molnar and Ramirez (1998) in river networks clearly show that these two criteria work together towards the formation of an optimal channel network in dynamic equilibrium.

Our results for the Yangtze estuary support the principle of equal entropy production locally over a range of freshwater discharges. Furthermore for the wet season case the principle of ‘minimum work’ seems to also be met, at least for the tidal component. One possible explanation for the difference between Langbein’s derivation and real situation on the achievement of minimum work is the difference in the imposed wave shape. In reality, tidal asymmetry and river influence are important degrees of freedom in the system dynamics that allow the system to adjust and seek to do minimum work.

The analytical solution by Langbein (1963) formulates tidal wave propagation in a highly schematized estuary, using a simple geometric form, a sinusoidal tide and without considering the effect of river discharge. The more sophisticated solution provided by the CST model does take account of the river influence and friction but is again forced by a symmetric tide. The comparison between the CST model and the TELEMAC model in 5.4 shows that the asymmetric tidal wave is an adjustment mechanism that enables the system to do minimum work in the wet season case, while the CST model achieves a condition of minimum work at a much lower river discharge (dry season). The detailed analysis of asymmetry also demonstrated that the energy flux produced by the tidal induced asymmetry (TIA) and river induced asymmetry (RIA) are balanced for the wet season case. The wave distortion in the TELEMAC model, as a result of wave propagation and interaction with the river discharge, reproduces the tidal asymmetry and thereby illustrates how the system is able to minimize the work done for the tide and river as a whole.

**5.6 Implications of system efficiency over wet and dry season**

From the cases examined, we suggest that the seasonal variations in system efficiency, are a mechanism for delivering minimum work overall. This implies that the combination of doing minimum work in the more energetic conditions of the wet season and having to do some work at other times is the combination that overall means the system does as little work as possible. Alternative strategies are likely to result in higher rates of dissipation and work done in the system as a whole.

In the river reaches of the Yangtze, the bankfull river discharge is generally taken to be a useful indicator of the forcing conditions that determines the maximum channel capacity (Riggs, 1974), whereas in the tidal part the dominant forcing is more likely to be the spring tide discharge (Pethick, 1984). According to Savenije (2012) high discharges (bankfull) are necessary to maintain channel morphology but extremes events will tend to go out of bank, unless prevented from doing so by flood protection levees. Spring tide occurs every fortnight, while bankfull river flow is a relatively rare event, with a return period that is typically 1.5-2.5 year (Riggs, 1974; Yu et al., 2009). As a result, the most probable hydro-morphology with minimum work may be synchronous with the maximum channel capacity maintained by spring tide but it is asynchronous with the maximum channel capacity maintained by bankfull river discharge. For a system under varying fluvial influence, we conclude that the most probable channel morphology tends to be compatible with mean wet season discharge, which is smaller than bankfull river discharge, and occurs for several months over the summer every year.

In the wet season, the river flow makes the system very energetic, and the system adjusts to do minimum work by removing sediment to establish a channel that is in balance with the flow regime. When the river flow reduces in the dry season, the channel will initially be too large relative to the flow conditions to be in regime. However, the sediment transport capacity also diminishes, so that the rate at which the system can adjust towards a new equilibrium takes longer and may not be possible within the timescale of the dry season (Xu and Milliman, 2009), This has potentially been exacerbated by the construction of several upstream dams, including the Three Gorges Dam, resulting in a regulated river discharge and the annual sediment discharge being reduced (Xu and Milliman, 2009).

Considering the system as a whole, over a full yearly cycle, we suggest that the estuary does minimum work for the most probable flows during the more energetic wet season and then seeks to accommodate the less energetic flows of the dry and the much less frequent extreme flood flows by doing minimum work subject to the constraints, which include the sediment supply. This expectation is based on the fact that a smaller system which is doing minimum work during the dry season will do a lot more work in the wet season (Kleidon et al., 2013), while a larger system which is doing minimum work in the wet season will do some work in the dry season, but will still have the ability to adjust itself to do the least work possible, given the constraints on the system. The fact that during the dry season the sediment supply is not sufficient to allow a rapid readjustment of the channel (as far as we know from the survey data available), means that the dynamic response of the estuary is to adjust the extent of tidal propagation and the wave asymmetry to achieve a state approaching that of minimum work.

**6 Conclusions**

In this study, we have investigated the role of river and tide hydraulics in terms of energy and its dissipation under various fresh water discharge conditions (peak discharge, wet season, dry season, no runoff), making use of both numerical modelling (TELEMAC) and an analytical solution (CST). The main results are summarized as follows.

(1)The hydraulic geometry for the Yangtze estuary shows some variability between the wet and dry season cases. The overall estuary shape changes to be more prismatic in the wet season, while in the dry season it changes to be more convergent. In addition, at some locations the cross-sectional area is not representative of the effective hydraulic section, due to the very dramatic changes in the actual form of the cross-section. Despite these variations in geometry, the tidal storage volume remains approximately constant regardless of the changes in river discharge. The constant tidal storage volume implies a constant tidal energy on the ebb.

(2) The comparison of tidal mechanical energy on the flood and ebb was found to be similar for the wet season case, with the implication that there is “effectively” no tidal dissipation. The mechanical energy due to river-tide interactions was revealed to be an important feedback mechanism that allows the system to adjust and achieve this condition. This was also demonstrated by examining the mechanical energy flux due to river induced asymmetry (RIA), which balances the tide induce asymmetry (TIA) for the wet season case. The results from the CST model also lend support to this conclusion. Whilst generally comparing well with TELEMAC model results, the tidal mechanical energy is quite different because the model does not include tidal asymmetry.

(3) We relate the total system energy to the energetic optimization principles of estuary evolution and balance. Locally the results show uniform dissipation of tidal energy and uniform entropy production for both tide and river. Globally, the system energy exhibits uniform dissipation when the river discharge is dominant in the wet and peak cases. The anomaly is due to the local geological constraints. The minimum energy dissipation rate for the wet season case is J/m. For the no runoff and dry season case the river does less work globally because the energy input is less, but both the dissipation of tidal energy and global energy dissipation increase as a percentage of the energy input (relative to the wet and peak season cases).

(4) The results demonstrate that equal energy density locally and minimum work globally are consistent principles when applied to the Yangtze estuary under river discharge that is typical for the wet season. Langbein (1963) considered an ideal estuary in a simplified way, driven by sinusoidal tidal wave and without fluvial influence, whereas the current research describes a dynamic response of a tidal river that allows tidal prism and tidal asymmetry to adjust on the flood and ebb in response to varying river influence. As a result the net energy flux also exhibits a transition from TIA dominance in the no runoff and dry cases, to RIA dominance in the peak case. All these factors enable the system to adjust in order to achieve minimum work under constraints. This is a complex feedback mechanism between river, tide and morphology that has not been described before from the perspective of energy flux and river-tide interactions.

Finally, we argue that the Yangtze estuary bathymetry is most consistent with the concept of minimum work and uniform dissipation for the more energetic conditions of the wet season. As a consequence extreme events (peak and no runoff) are likely to be perturbations from this condition, while the sort of conditions that prevail in the dry season give rise to more energy dissipation as a percentage of the energy input, reflecting the extent to which the system is able to adjust. In the dry season, the size of the estuary needs to be smaller to do minimum work. However we surmise that the channel cannot adjust fast enough to attain a condition of minimum work, due to the limited sediment supply. A detailed study of sediment dynamics over the wet and dry season is required to examine this conjecture in more detail.

**Acknowledgements** This study was supported by the National Basic Research Program of China (973 Program) (No.2010CB951204), National Natural Science Foundation of China (NSFC) (44107180) and the 111 project (No. B08022). The authors acknowledge their indebtedness to the Shanghai Hydrological Administration and Changjiang Waterway Bureau for kindly providing the bathymetry and hydrology data. Michael Turnbull, from HR Wallingford Ltd., UK, is thanked for his technical support on numerical modelling. Alan Cooper and Jeremy Spearman, from HR Wallingford Ltd., UK, are thanked for providing their guidance on data processing and analysis.

**References:**

Ahnert, F., 1994. Equilibrium, scale and inheritance in geomorphology. Geomorphology, 11(2): 125-140.

Bolla Pittaluga, M. et al., 2015. Where river and tide meet: The morphodynamic equilibrium of alluvial estuaries. Journal of Geophysical Research: Earth Surface, 120(1): 2014JF003233.

Buschman, F.A., Hoitink, A.J.F., van der Vegt, M. and Hoekstra, P., 2009. Subtidal water level variation controlled by river flow and tides. Water Resources Research, 45(10): W10420.

Chen, Z.Y. and Zhao, Y.W., 2001. Impact on the Yangtze (Changjiang) Estuary from its drainage basin: Sediment load and discharge. Chinese Science Bulletin, 46S: 73-80.

Cherkauer, D.S., 1973. Minimization of power expenditure in a riffle-pool alluvial channel. Water Resources Research, 9(6): 1613-1628.

Dalrymple, R.W. and Choi, K., 2007. Morphologic and facies trends through the fluvial–marine transition in tide-dominated depositional systems: A schematic framework for environmental and sequence-stratigraphic interpretation. Earth-Science Reviews, 81(3–4): 135-174.

Davies, G. and Woodroffe, C.D., 2010. Tidal estuary width convergence: Theory and form in North Australian estuaries. Earth Surface Processes and Landforms, 35(7): 737-749.

Davies, J.L., 1964. A morphogenic approach to world shorelines. Zeitschrift fur Geomorphologie, 127(8): 42.

Davy, B.W. and Davies, T.R.H., 1979. Entropy Concepts in Fluvial Geomorphology: A Reevaluation. Water Resources Research, 15(1): 103-105.

Dun, R. and Townend, I., 1998. Contemporary Estuary Morphology and Long-term Change, Littoral ‘98, Barcelona, Spain, pp. 14-17.

Ensign, S.H., Doyle, M.W. and Piehler, M.F., 2013. The effect of tide on the hydrology and morphology of a freshwater river. Earth Surface Processes and Landforms, 38(6): 655-660.

Gilbert, G.K., 1876. The Colorado plateau province as a field for geological study. American Journal of Science, Series 3 Vol. 12(67): 16 -24.

Godin, G., 1999. The propagation of tides up rivers with special considerations on the upper Saint Lawrence River. Estuarine, Coastal and Shelf Science, 48(3): 307-324.

Griffiths, G.A., 1984. Extremal Hypotheses for River Regime: An Illusion of Progress. Water Resources Research, 20(1): 113-118.

Guo, L., van der Wegen, M., Roelvink, J.A. and He, Q., 2014. The role of river flow and tidal asymmetry on 1-D estuarine morphodynamics. Journal of Geophysical Research: Earth Surface, 119(11): 2014JF003110.

Haigh, I.D., Eliot, M. and Pattiaratchi, C., 2011. Global influences of the 18.61 year nodal cycle and 8.85 year cycle of lunar perigee on high tidal levels. Journal of Geophysical Research: Oceans, 116(C6): C06025.

Hervouet, J.M., 2007. Hydrodynamics of free surface flows: Modelling with the finite element method. John Wiley & Sons, INC, New York.

Huang, H., Chang, H.H. and Nanson, G.C., 2004. Minimum energy as the general form of critical flow and maximum flow efficiency and for explaining variations in river channel pattern. Water Resources Research, 40(W045024).

Huang, H.Q. et al., 2014. A test of equilibrium theory and a demonstration of its practical application for predicting the morphodynamics of the Yangtze River. Earth Surface Processes and Landforms, 39(5): 669-675.

Kleidon, A., Zehe, E., Ehret, U. and Scherer, U., 2013. Thermodynamics, maximum power, and the dynamics of preferential river flow structures at the continental scale. Hydrol. Earth Syst. Sci., 17(1): 225-251.

Knight, D.W., 1981. Some field measurements concerned with the behaviour of resistance coefficients in a tidal channel. Estuarine, Coastal and Shelf Science, 12(3): 303-322.

Langbein, W.B., 1963. The hydraulic geometry of a shallow estuary. Bulletin of Int Assoc Sci Hydrology(8): 84-94.

Lanzoni, S. and D'Alpaos, A., 2015. On funneling of tidal channels. Journal of Geophysical Research: Earth Surface, 120(3): 2014JF003203.

Lanzoni, S. and Seminara, G., 2002. Long-term evolution and morphodynamic equilibrium of tidal channels. Journal of Geophysical Research: Oceans, 107(C1): 1-1-1-13.

Latteux, B., 1995. Techniques for long-term morphological simulation under tidal action. Marine Geology, 126(1-4): 129-141.

Leopold, L.B. and Langbein, W.B., 1962. The concept of entropy in landscape evolution. Theoretical papers in the hydrologic and geomorphic sciences, Geological Survey Professional Paper 500-A.

Molnar, P. and Ramirez, J.A., 1998. Energy dissipation theories and optimal channel characteristics of river networks. Water Resources Research, 34(7): 1809-1818.

Nield, J.M., Walker, D.J. and Lambert, M.F., 2005. Two-dimensional equilibrium morphological modelling of a tidal inlet: an entropy based approach. Ocean Dynamics, 55(5-6): 549-558.

Pedlosky, J., 2003. Waves in the ocean and atmosphere: Introduction to wave dynamics. Springer-Verlag, Berlin, Heidelberg.

Pethick, J., 1984. An introduction to coastal geomorphology. Edward Arnold Publ., London, UK.

Prigogine, I., 1955. Introduction to thermodynamics of irreversible processes. John Wiley & Sons, London.

Richards, K., 1982. Rivers, form and process in alluvial channels, Methuen & Co Ltd, London.

Riggs, H., 1974. Flash food potential from channel measurements, Symposium on Flash Floods. IAHS Publ, pp. 52-56.

Sassi, M.G. and Hoitink, A.J.F., 2013. River flow controls on tides and tide-mean water level profiles in a tidal freshwater river. Journal of Geophysical Research: Oceans, 118(9): 4139-4151.

Savenije, H.H.G., 2012. Salinity and Tides in Alluvial Estuaries. Elsevier Science, New York.

Seminara, G., Lanzoni, S., Tambroni, N. and Toffolon, M., 2010. How long are tidal channels? Journal of Fluid Mechanics, 643: 479-494.

Stokes, G.G., 1847. On the theory of oscillatory waves. Trans. Cambridge Philos. Soc.(8): 441–455.

Thorn, C.E. and Welford, M.R., 1994. The equilibrium concept in geomorphology. Annals of the Association of American Geographers, 84(4): 666-696.

Todeschini, I., Toffolon, M. and Tubino, M., 2005. Long-term evolution of self-formed estuarine channels, River, Coastal and Estuarine Morphodynamics: RCEM 2005. Parker G, Garcia M (eds.) Taylor & Francis Group, London, pp. 161-170.

Toffolon, M. and Lanzoni, S., 2010. Morphological equilibrium of short channels dissecting the tidal flats of coastal lagoons. Journal of Geophysical Research: Earth Surface, 115(F4): F04036.

Townend, I., 2012. The estimation of estuary dimensions using a simplified form model and the exogenous controls. Earth Surface Processes and Landforms, 37(15): 1573-1583.

Townend, I. and Dun, R., 2000. A diagnostic tool to study long-term changes in estuary morphology. Geological Society, London, Special Publications 2000, 175: 75-86.

Townend, I. and Pethick, J., 2002. Estuarine flooding and managed retreat. Philosophical Transactions of the Royal Society A-Mathematical Physical and Eegineering Sciences, 360(1796): 1477-1495.

Townend, I.H., 1999. Long-term changes in estuary morphology using the entropy method, International Association of Hydraulic Engineering and Research Symposium on River, Coastal and Estuarine Morphodynamics. Int. Assoc. of Hydraul. Eng. and Res., Genova, Italy.

Tulin, M., 2007. On the transport of energy in water waves. Journal of Engineering Mathematics, 58(1-4): 339-350.

van der Wegen, M., Wang, Z.B., Savenije, H.H.G. and Roelvink, J.A., 2008. Long-term morphodynamic evolution and energy dissipation in a coastal plain, tidal embayment. Journal of Geophysical Research: Earth Surface, 113(F3): F03001.

Wu, H.L., Shen, H.T. and Wu, J.X., 2002. Relationships among depth datum levels in the Yangtze Estuary. The Ocean Engineering, 20(1): 69-74.

Xiong, Z.P., 1996. A forecast study on changes in water and sediment discharges induced by Three Gorges Reservoir and related influences on the lower reaches. Sediment Inf. (5-12).(in Chinese)

Xu, K. et al., 2005. Simulated sediment flux during 1998 big-flood of the Yangtze (Changjiang) River, China. Journal of Hydrology, 313(3-4): 221-233.

Xu, K. and Milliman, J.D., 2009. Seasonal variations of sediment discharge from the Yangtze River before and after impoundment of the Three Gorges Dam. Geomorphology, 104(3–4): 276-283.

Yang, C.T., 1971a. Potential Energy and Stream Morphology. Water Resources Research, 7(2): 311-322.

Yang, C.T., 1971b. Formation of riffles and pools. Water Resources Research, 7(6): 1567-1574.

Yang, C.T., Song, C.C.S. and Woldenberg, M.J., 1981. Hydraulic geometry and minimum rate of energy dissipation. Water Resources Research, 17(4): 1041-1018.

Yang, S.L. and Yang, H.F., 2014. Temporal variations inwater and sediment discharge from the Changjiang (Yangtze River) and downstream sedimentary responses. In: Zhang, J.(Ed.), Land–Sea Interaction in the Changjiang Estuary and Adjacent Seas, Springer.

Yang, S.L., Eisma, D. and Ding, P.X., 2000. Sedimentary processes on an estuarine marsh island in the turbidity maximum zone of the Yangtze estuary. Geo-Marine Letters, 20(2000): 87-92.

Yu, F., Chen, Z., Ren, X. and Yang, G., 2009. Analysis of historical floods on the Yangtze River, China: Characteristics and explanations. Geomorphology, 113(3-4): 210-216.

Zhang, E.F., Savenije, H.H.G., Chen, S.L. and Mao, X.H., 2012. An analytical solution for tidal propagation in the Yangtze Estuary, China. Hydrology and Earth System Sciences, 9(16): 3327-3339.

Zhang, M., Townend, I.H., Cai, H. and Zhou, Y., 2015. Seasonal variation of tidal prism and energy in the Yangtze estuary: A numerical study. Chinese Journal of Oceanology and Limnology. DOI: 10.1007/s00343-015-4302-8

Table 1. Seasonal Scenarios Defined by Varying River Discharge and Tidal Boundary Conditions

|  |  |  |  |
| --- | --- | --- | --- |
| Group | Scenarios | Discharge (m3/s) | Tide |
| 1 | Peak flow | 60,000 | Mean spring tide |
| 2 | Averaged wet season | 35,000 | Mean spring tide |
| 3 | Averaged dry season | 15,000 | Mean spring tide |
| 4 | No runoff case | 1,000 | Mean spring tide |
| 5 | River only cases | 15,000/35,000/60,000 | No tidal forcing |

Table 2– Hydraulic geometry used in the CST model

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Scenario | Property | Unconstrained convergence length, L (m) | Distance to transition point (m) | Value at mouth, (m2) | Adjusted convergence length, (m) | Value for river, (m2) |
| No runoff | CSA (m2) | 153,900 | 297,500 | 161,600 | 99,000 | 13,300 |
| Width (m) | 113,000 | 250,000 | 18,000 | 60,000 | 2,500 |
| Dry season | CSA (m2) | 161,100 | 275,500 | 163,000 | 97,000 | 24,300 |
| Width (m) | 114,000 | 250,000 | 18,000 | 56,000 | 2,900 |
| Wet season | CSA (m2) | 167,900 | 236,500 | 165,400 | 88,000 | 40,500 |
| Width (m) | 114,000 | 250,000 | 18,000 | 57,000 | 2,900 |
| Peak flow | CSA (m2) | 175,200 | 177,500 | 170,600 | 77,000 | 55,600 |
| Width (m) | 113,900 | 250,000 | 18,000 | 55,000 | 2,900 |

Table 3–Hydraulic parameters used in the CST model. Friction coefficients are the values at the start and end of each of three reaches (0-176, 176-245, 245-550km).

|  |  |  |  |
| --- | --- | --- | --- |
| Scenario | Tidal amplitude (m) | River discharge (m3/s) | Manning’s friction coefficient (K=1/n) |
| No runoff | 1.52 | 1,000 | 60,40,20,20 |
| Dry season | 1.50 | 15,000 | 60,50,30,20 |
| Wet season | 1.46 | 35,000 | 70,50,15,15 |
| Peak flow | 1.42 | 60,000 | 90,30,10,15 |

**Captions of Figures**

Figure 1. Satellite image of the Yangtze estuary displaying, location of cross-sections and the estuary bathymetry

Figure 2. Definition sketch of tidal river

Figure 3. CST model results plotted against TELEMAC results for the Yangtze estuary using parameters derived from hydraulic geometry (see Tables 2 and 3). Black points are the field measured data.

Figure 4. The tidally averaged hydraulic profile (a, b, c, d) and the decomposed mechanical energy (e, f, g, h) and total energy (i, j, k, l) from both river and tide along the Yangtze estuary

Figure 5. The distribution of flood (positive values) and ebb (negative values) discharge volume (a) and mechanical energy (b) along the Yangtze estuary for different fresh water discharge conditions.

Figure 6. The distribution (A) and ratio (B) of flood and ebb tidal mechanical energy under different fluvial discharges. The ebb tidal mechanical energy is derived by subtracting a constant river kinetic energy flux () from the total ebb mechanical energy flux (). The black points compare the flood tidal mechanical energy for the wet season case, with the ebb tidal mechanical energy for the no runoff case.

Figure 7. Energy dissipation and entropy production per unit volume from tide component and river component

Figure 8. Comparison of (a) river velocity calculated from discharge (river only) and by taking the mean of the total velocity (river and tide); and (b) the actual cross sectional area compared to the effective hydraulic cross sectional area

Figure 9. Schematic to show the balance of tidal discharge and the storage or release of river flow over a tidal cycle under different river flow conditions

Figure 10. Along channel variation of tidal constituents and asymmetry for (a) tidal velocity amplitude; (b) elevation; (c) phase difference; (d) decomposed mechanical energy flux contributions from RNE potential energy; (e) RNE kinetic energy; and (f) the two components of asymmetry: TIA and RIA

Figure 11. Comparison between TELEMAC and CST modelling results for the wet and dry season cases, (a) along estuary tidal energy distribution on the flood and ebb; and the variation over a tidal cycle of (b) width-averaged flow velocity; (c) width-integrated mechanical energy; and (d) width-integrated mechanical energy flux, all at a section around 11km from the mouth.

Figure 12. Wave asymmetry () along the channel of the Yangtze estuary: (a) velocity, (b) elevation

Figure 13. The net energy flux integrating over a tidal cycle () for (a) the no runoff and dry cases; and (b) the wet and peak cases

1. A global data set derived from the TOPEX/POSEIDON global tidal model [↑](#footnote-ref-1)