**A theoretical study of the fundamental torsional wave in buried pipes for pipeline condition assessment and monitoring**

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***Abstract:*** Waves that propagate at low frequencies in buried pipes are of considerable interest in a variety of practical scenarios, for example leak detection, remote pipe detection, and pipeline condition assessment and monitoring. Whilst there has been considerable research and commercial attention on the accurate location of pipe leakage for many years, the various causes of pipe failures and their identification, have not been well documented; moreover, there are still a number of gaps in the existing knowledge. Previous work has focused on two of the three axisymmetric wavetypes that can propagate: the *s*=1, fluid-dominated wave; and the *s*=2, shell-dominated wave. In this paper, the third axisymmetric wavetype, the *s*=0 torsional wave, is investigated. The effects of the surrounding soil on the characteristics of wave propagation and attenuation are analyzed for a compact pipe/soil interface for which there is no relative motion between the pipe wall and the surrounding soil. An analytical dispersion relationship is derived for the torsional wavenumber from which both the wavespeed and wave attenuation can be obtained. How torsional waves can subsequently radiate to the ground surface is then investigated. Analytical expressions are derived for the ground surface displacement above the pipe resulting from torsional wave motion within the pipe wall. A numerical model is also included, primarily in order to validate some of the assumptions made whilst developing the analytical solutions, but also so that some comparison in the results may be made. Example results are presented for both a cast iron pipe and an MDPE pipe buried in two typical soil types.

***Key words:*** Buried pipes, torsional wave, dispersion relationship, spiral fracture, ground surface vibration, condition monitoring, wave propagation

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# Introduction

The condition assessment of underground utilities is of paramount importance for engineers working in the realms of buried infrastructure and, indeed, for the society that they serve; quite apart from the essential services which the utilities provide to support 21st century ways of life, certain catastrophic accidents result directly from gas pipe leaks or broken water mains. In recent years there has been a move towards more remote and non-intrusive methods of condition assessment, alongside developments in continuous monitoring techniques. If the condition of the buried infrastructure is routinely assessed and monitored, proactive warning of impending failure might be achieved and thus the probability of the serious consequences of accidents caused by deterioration of the utility network can be effectively reduced.

Traditionally, for example, sewer surveys were carried out by sending out inspectors to ‘see and touch’ the defects inside those man-entry pipes along the network. However, this method, although highly effective at revealing the internal condition and providing certain clues about the external condition, suffers from inefficiency in terms of manpower, and it is obviously impractical for the majority of pipes and cables that make up the network.

Moreover, modern legislation restricted this approach of direct inspection by operators for health and safety reasons [1]. Remote (i.e. non-manual) techniques were developed to overcome this concern and greatly increase the inspection efficiency.

Over the past decade, much attention has been paid to remotely detecting buried pipes and cables in all ground conditions without the need for excavation, under the umbrella of a major UK initiative entitled “Mapping the Underworld” (MTU) [2]. A new research project, entitled “Assessing the Underworld” (ATU) [3], building on the highly successful outcomes from MTU, has recently commenced to take the research into a new sphere. This programme aims to use geophysical sensors deployed both on and beneath the ground surface to remotely determine the condition of these urban assets. Vibro-acoustics, as one of the four essential technologies in the MTU, has been proven to be suitable and highly successful for locating buried water pipes [4, 5]. In the ATU phase, some of the vibro-acoustic techniques developed in MTU will be extended from detecting pipes to assessing their condition. It is within this framework that the work presented here has been undertaken.

Failures in aging water mains are a serious problem for all water distribution systems. Whilst there has been considerable research and commercial attention on the accurate location of water leakage for many years [6-8], the various causes of pipe failures and their identification, have not been well documented; moreover, there are still a number of gaps in the existing knowledge. One mode of failure, about which there is very little in either the academic or industrial literature, is spiral fracture, occurring, for the most part, in cast iron pipes [9]. Beyond the obvious case of a spirally welded pipe, it is not altogether clear what mechanism might underlie such a failure. Steverding [10], investigating helical cracks in cylinders suggested that the spiral was caused by self-radiation of the moving crack, with the pitch being determined by the crack speed, this, in turn, being linked to the ductility of the fracture process. It is also possible that some kind of torsional excitation might initiate a spiral failure; although it is difficult to envisage how that could occur in practice, differential soil movement either side of the pipe might play a part.

Perhaps more tractable and undoubtedly of more relevance to the present study is the link between spiral failure (however initiated) and the wave motion set up within the pipe in consequence. There is a desire, particularly within the gas industry, to be able to detect and locate fracture events as they occur and, if possible, remotely confirm the likely mode of failure. For a vibro-acoustic technique to be effective, the acoustic characteristics of the dominant wave mode(s) associated with particular types of pipe failure must be known *a priori*. Furthermore, how these waves may radiate to the ground surface, where than can potentially be detected, is of considerable interest. In general, in buried water pipes, acoustic energy propagates at relatively low frequencies [11]. Of the four main energy carriers, three of them are axisymmetric (*n*=0) waves including a predominantly fluid-borne (s=1) wave, a compressional shell (*s*=2) wave, and a torsional (*s*=0) wave. Much of the present authors’ previous work has involved both theoretical and experimental investigations into the *s*=1, 2 waves [12-16]. The focus of the present paper is the *s*=0, torsional wave. Whilst, in the field, it has not been confirmed beyond reasonable doubt, it would seem logical to assume that, when a spiral fracture occurs in a pipe, torsional waves are excited and thence propagate along the pipe. Moreover, if it were possible to detect these waves from the ground surface, possibilities for the remote detection and monitoring of the fracture events open up.

Torsional waves in pipes have received considerable attention in the literature at ultrasonic frequencies to support commercial testing systems that have been in use in industry for a number of years. They are exploited for the detection and characterization of cracks and other (small) pipe defects in both unburied and buried pipe, for example[17-24]. At these high frequencies, the fundamental torsional wave in a buried pipe is non-dispersive, propagating at the same speed as that in an *in-vacuo* pipe. Ultrasonic torsional waves have been used to measure the near-surface shear wave velocities of saturated soils [25], but no work has been done relating the soil response to that within the pipe wall. Little is available in the literature for the low-frequency regime, beyond the *in vacuo*, textbook solution. Kudlička [26] presented the dispersion characteristics of thick-walled pipes *in vacuo* and, moreover, investigated anisotropy. Parnes *et al* [27,28] considered torsional waves in a clad rod and examined the conditions under which waves could propagate in the rod. Initially, the rod was treated effectively as a shell, but in the later work variation over the rod cross-section was included. However, he framed the analysis in terms of wavespeed, rather than wavenumber, and then made no allowance for the possibility of complex (as opposed to purely real) solutions. This severely restricts the conditions for which wave propagation can occur. Moreover, the response within the cladding was not investigated in detail. Thurston [29] investigated a similar problem, formulated it in terms of wavenumber and allowed for the possibility of attenuating waves, but did not present analytical expressions for the solutions. To the authors’ knowledge no other work has, to date, been presented on the low-frequency behavior of buried pipes and the concomitant response within the soil, in particular, in terms of an analytical solution. In this paper, the dispersion characteristics of the fundamental torsional wave in a buried pipe at low (typically <1 kHz) frequencies is studied and analytical solutions presented. Additionally, the resultant response within the surrounding ground is examined.

The present paper is organized as follows:

* In Section 2, the equations of motion are derived, leading to the dispersion relationship for torsional motion; an analytical expression for the torsional wavenumber is presented, which encapsulates both the wavespeed and the wave attenuation.
* In Section 3, how such waves might radiate to the ground surface is considered; an approximate analytical expression for the displacement seen at the ground surface relative to the circumferential motion of the pipe wall is derived; this in turn can be related to the circumferential pipe wall motion at any point along its length.
* In Section 4, a numerical model is described and some results presented, primarily to provide an independent validation for a few of the approximations made in developing the analytical solutions but also to compare with the example results presented in Section 5.
* In Section 5, numerical examples are presented for both a cast iron and (for the sake of completeness) an MDPE pipe; two typical soil types are considered.
* In Section 6, the practical applications of this research are considered in more detail.
* Section 7 presents the conclusions and outlines proposed work to take this research forward.

# Dispersion relationship for the *n*=0, *s*=0, torsional wave


Fig. 1. The coordinate system for a buried, fluid-filled pipe

Here, the pipe equations for  axisymmetric wave motion are derived for a buried fluid-filled pipe. The surrounding soil is regarded as an infinite elastic medium which can sustain both compressional and shear waves. A semi-infinite, isotropic, cylindrical shell is shown in Fig. 1: the shell displacements are ,  and  in the axial (), circumferential (), and radial () directions, respectively; *ur*, *uθ* and *ux* denote the soil displacements in the *r*, *θ* and *x* directions respectively. The contained fluid imposes a pressure () on the pipe wall; the surrounding soil imposes normal stress ** and tangential stresses  and  at the pipe/soil interface. The pipe has a mean radius *a* and wall thickness *h*, and is assumed to be thin such that . It is noted that the pressure and stresses are evaluated at *r*=*a*. The following are simplified forms of Kennard’s equations for a thin-walled shell [30], with shell bending neglected, and so are only valid below the ring frequency.

### Vibration of the soil

Assuming the infinite elastic medium is homogeneous and isotropic, the displacement **u**(*r*, *θ*, *x*, *t*) of a point in the surrounding medium satisfies the equation of motion[31]

  (1)

where *λm* and *µm* (also referred to as shear modulus) are the Lamé’s first and second coefficients of the surrounding medium; *ρm* is the density of the surrounding medium; Δ is the dilatation and the rotation vector **ω**=1/2curl**u**. For axisymmetric torsional motion both the radial and axial displacements *ur* and *ux* vanish (*ur*=0 and *ux*=0), and *uθ* must be independent of *θ* (∂*uθ* /∂*θ=*0). Thus dilatation . Eq. (1) can be reduced to the equation of motion in the circumferential direction as

  (2)

The components of the rotation about the three orthogonal directions are given by

 , ,  (3a-c)

A travelling wave solution for the surrounding medium may be assumed of the form

  (4)

with the amplitude of the soil displacement in the circumferential direction *Vm* being a function of *r* but independent of *x* and *θ*; *k*0 is the torsional wavenumber for *n*=0, *s*=0, wave motion; and *ω* is the angular frequency. Substituting the travelling wave solution for *uθ* given by Eq. (4) into Eqs. (3a, b) gives

 ,  (5a, b)

Substituting Eqs. (4) and (5) into (2) gives

  (6)

where the surrounding medium shear (rotational) radial wavenumber is given by ; and *kr* is the shear wavenumber in the surrounding medium, which is given by . As expected, for pure torsional motion in the pipe, only a shear wave may be radiated into the surrounding soil. No compressional waves are excited. To represent an outgoing shear wave decaying to zero at infinity, the Hankel function of the second kind H1 of order one is used to solve the Bessel’s Eq. (6). Correspondingly, the solution for an external medium is

  (7)

where *B* is a constant. Thus, the travelling wave solution becomes

  (8)

For torsional motion (*n*=0, *s*=0), it is noted that the stresses ** and  vanish. According to Hooke’s Law, the tangential stress  is given by

  (9)

Substituting the travelling wave solution of *uθ* given by Eq. (8) into (9) (*ur*=0) results in

  (10)

where .

### Vibration of the contained fluid

For torsional wave motion, the fluid is considered to be uncoupled from the pipe motion because a fluid cannot support shear.

### Vibration of the pipe wall

With reference to Fig. 2, for torsional motion, each cross-section remains in its own plane and rotates about the *x* axis. Noting that the surrounding soil only exerts a tangential stress, , at the boundary *r*=*a*, equilibrium of forces in the circumferential direction gives

  (11)

This can further reduce to

  (12)

where *ρp* is the density of the shell material; and *σxθ* is the tangential stress in the shell. The general form of the stress-strain relationship (Hooke’s Law) is [31]

  (13)

where *Gp* is the shear modulus of the shell material and given by *Gp*=*Ep*/2(1+*vp*); *Ep* and *vp* are the corresponding Young’s modulus and Poisson ratio, respectively; and the strains

  (14)

where

 ,  (15a,b)

Substituting Eqs. (14) into (13) gives the tangential stress  by

  (16)

Substituting into Eq. (12) gives

  (17)

Similar to the soil displacements, a travelling wave solution of the form  is used to describe the pipe wall displacement, where *Vs* is its amplitude in the circumferential direction.



Fig. 2. Schematic diagram showing stresses on the shell element of radius *a* and thickness *h* relevant to motion in the circumferential direction *v*.

### Fully coupled equations

The coefficient *B* in Eq. (10) may be determined from the conditions at the pipe/soil interface. In previous studies, for the *s*=1 and *s*=2 waves we have considered two coupling extremes: that of lubricated contact, for which the shear stress at the pipe wall/soil interface vanishes; and that of compact contact, for which there is continuity of displacements. For torsional motion the lubricated contact case is trivial in the sense that, if the shear stress at the pipe wall/soil boundary is set to zero, the torsional wave will be completely uncoupled from the surrounding soil and the will behave as if the pipe were *in vacuo*. The compact coupling case is of considerably more interest, and it is that which is investigated here. Moreover, except when very large shear strains are expected, as might be the case for earthquake-excited motion, the compact coupling case is more realistic from a physical standpoint. This is explained in more detail in the Appendix.

Under this coupling condition, at the boundary *r*=*a*, the displacement of the soil in the circumferential direction matches that of the shell, i.e., *Vm*=*Vs*. Thus Eq. (7) becomes

  (18)

From Eq. (10) this then gives the shear stress at the pipe wall as

  (19)

Here it should be noted that Eq. (19) is only strictly valid if the maximum allowable shear stress at the pipe/soil interface is not exceeded, thus ensuring compact coupling. Should the displacement in the shell exceed the corresponding limit, , the form of Eq. (19) remains the same, with  replacing . For a thin walled pipe for which *h*<<*a*, substituting into Eq. (17) and adopting the travelling wave solution for the pipe wall displacement, *v*, gives the torsional wavenumber as

  (20)

where *kT* is given by .

The individual terms contributing to the wavenumber expression can be readily identified as:

* the *in-vacuo* torsional wavenumber, *kT*;
* a pipe wall mass component, *ω2ρph*;
* a soil shear stiffness component, *μm*/*a*;
* and a shear wave radiation component associated with the Hankel function ratio,

With this in mind, the wavenumber given in Eq. (20) can be re-expressed in terms of component impedances, viz.

  (21)

where  is the inertial impedance of the pipe wall and  is the total shear wave radiation impedance .

Eq. (20) can now be examined in a little more detail. In non-dimensional form, and substituting for **, it becomes

  (22)

Using the formula for Hankel function derivatives [32],

  (23)

giving

  (24)

Eq. (22) becomes

  (25)

This form of the wavenumber expression without derivatives is more amenable to investigation both analytically and numerically.

* At high frequencies, when asymptotic forms of the Hankel functions for large arguments can be used [32], Eq. (25) becomes

  (26)

Taking the square root and retaining only the first two terms in the series expansion gives, after some manipulation

  (27)

This shows that, at high frequencies, the real part of the *s*=0 wavenumber, related to the wavespeed, tends, as expected, to the *in vacuo* value. Substituting for  using  and approximating as then gives the imaginary component of the *s*=0 wavenumber as

  (28)

When the *in-vacuo* torsional wavespeed in the pipe is much greater than the shear wavespeed in the soil (), Eq. (28) becomes

  (29)

It can be seen that the imaginary part of the wavenumber comprises two terms: one associated with losses within the pipe wall; and the second associated with radiation losses.

When losses in the pipe wall dominate, such as is the case for plastic pipes, the imaginary part of the wavenumber varies linearly with frequency; when the pipe wall losses are small, such as is the case for metal pipes, the imaginary part of the pipe wavenumber is independent of frequency. Eq. (29), whilst strictly not valid above the ring frequency, then provides an insight into the rationale behind the research presented in [26] to estimate soil shear wave velocities from the measured attenuation of the *n*=0 torsional wave in buried pipes at ultrasonic frequencies. It is also clear now that the pipe does not simply become uncoupled from the surrounding medium at high frequencies; it is rather that the radiation loading becomes that of radiation damping only.

* At low frequencies, when the pipe wall impedance is much smaller than the shear radiation impedance, Eq. (21) simply becomes

  (30)

Reducing Eq. (21) to this form is effectively neglecting the smaller variation of the tangential stress in the shell along the axial direction  in Eq. (12), compared with the shear stress imposed by the surrounding soil,  which dominates at low frequencies.

It can be seen from Eq. (21) that the torsional wavenumber is dependent on both the shear wave radiation impedance and the pipe wall impedance. The real part of the wavenumber gives the wave propagation speed, and the imaginary part gives the wave attenuation. The radiation impedance will either be: predominantly mass-controlled, with radiation damping, in which case the torsional wavenumber will increase relative to the *in-vacuo* case; or predominantly stiffness-controlled with radiation damping, in which case whether the wavenumber increases or decreases relative to the *in-vacuo* case will depend on the ratio of the radiation impedance to the pipe wall impedance. This will be examined further in the numerical examples presented in Section 5.

As expected, when the shear modulus of the surrounding medium ** and , i.e. when the surrounding medium is a fluid, Eq. (20) shows that the torsional wavenumber . Under these conditions the torsional wave is, as anticipated, uncoupled from the medium.

# Radiation from the pipe to the ground surface

In the preceding analysis, we have included the effects of a surrounding medium on the axisymmetric torsional waves propagating in a buried pipe. However, the medium was considered to be of infinite extent, with no free surface being included in the analysis. Here, we wish to see how such waves propagating in a buried pipe might radiate to a free surface and what resultant displacements might be seen at that surface. A comprehensive analysis of the fully coupled system, including the ground surface, would be extremely complex and beyond the scope of the present paper. What is offered here is a somewhat simplified analysis, in order to gain some understanding of the physical processes in play, and which makes the following assumptions:

* The effects of the soil on the pipe and the effects of the waves propagating in the pipe on the soil can be considered independently; what this means in practice is that, in the calculation of the dispersion characteristics of the torsional wave, the free ground surface (along with the concomitant wave reflections) is neglected – it is only included once the waves in the pipe have already, so to speak, been set up. Because of the large attenuation in most soils, this is only likely to become problematic at extremely low frequencies when the number of shear wavelengths between the pipe and the ground surface becomes very small. This was, indeed, confirmed by Thurston [29] when investigating torsional modes in a rod with finite cladding: except at very low frequencies, he found the rod modes to closely resemble those of the infinitely clad rod.
* Once the waves radiating from the pipe reach the ground surface, they can be considered to be in the far field and undergo a plane wave treatment. This limits the lower frequency bound for which the analysis is valid, in a similar way to the assumption described above.
* Only excitation of the ground directly over the pipe is considered. Analytical description of the interaction of cylindrical and conical waves with a planar surface is complex. For example, at a lateral distance from the pipe axis comparable to the pipe depth, incident shear cylindrical or conical waves can excite surface waves of various kinds in addition to the reflected bulk waves [33, 34]. Directly over the pipe, which is the main region of interest for this study, surface waves do not develop, and only reflected shear waves need to be considered. A similar approach was adopted by Jette and Parker [35] when studying the effects on the ground surface of a fluid-borne wave propagating in a steel gas pipe. What might be considered to be “directly over the pipe” is discussed in more detail in the following paragraphs.

For the torsional, *n*=0, *s*=0, wave propagating in a pipe, the excitation in the surrounding soil is given by Eqs. (8) and (18) as

  (31)

This equation represents conical shear waves radiating from the pipe into the soil. In the far field, for large values of , this becomes [32]

  (32)

Correspondingly, the far field wave impedance, , becomes

  (33)

Eq. (32) shows that, in the far field, the incident wave field becomes quasi-planar, with a complex-exponential dependence moderated by the cylindrical spreading term. That the field may be treated in a planar fashion is substantiated by the fact that the radiation impedance given by Eq. (33) tends to a purely real value. However, the wavefront is still curved, so the plane wave treatment warrants further consideration. At this stage it is more convenient to adopt a Cartesian coordinate system in preference to a cylindrical one, with *z* denoting the direction normal to the ground surface, *xy*, plane, and the origin located on the pipe centre line, as shown in Fig. 3.


Fig. 3. Pipe cross-section depicting wavefronts as they reach the ground surface.

Consider the curved wavefront reaching the surface as shown in the figure. The particle motion at each point along the wavefront will be parallel to the wavefront and in the plane of the figure, noting that this will be the case regardless of the axial (*x*-wise) dependence. At each point along the wavefront, the direction of particle motion will equate to the polarization of the wave at that point. This means that directly above the pipe, the wave reaching the ground surface will be purely shear horizontal (SH); at lateral distances from this mid-point, the particle motion will be at an angle to the horizontal, having both horizontal (SH) and vertical components (SV), which may be considered separately. SH waves undergo no mode conversion on reflection from a free surface, whereas the SV component will lose a portion of its energy to compressional (P) waves [36]. Thus, directly above the pipe, the waves undergo a simple reflection at the surface, analogous to a wave in a fluid.

Here it is appropriate to quantify what lateral span above the pipe might, in practice, be considered to be “directly above the pipe”.

1. With reference to Fig. 3, provided that the angle of incidence, **, is small enough, we can choose to neglect the vertical component in the analysis; it is clear that waves that reach the surface at an angle less than 45 degrees to the vertical will be dominated by the SH component. This immediately sets one limit on the span over which we can consider only simple SH wave reflections. In the limit we must ensure that

  (34)

 giving

  (35)

1. The second constraint on the region of interest is that the phase difference on arrival at the surface across the span of interest (*l* in Fig. 3) is small. Specifically, we would like to ensure that

  (36)

giving

  (37)

This effectively constrains the span, *l*, by

  (38)

It can be seen that at high frequencies, when becomes large, this upper bound for *l/d* tends to zero. Combining Eqs. (35) and (38), we have

  (39)

We now see that the second condition effectively sets an upper frequency limit for which the span is sufficiently large. These conditions will be examined further when example results are presented in Section 5.

Returning now to Eq. (32), when the waves radiating from the pipe are incident upon the ground surface in the region “directly above the pipe” (now as plane waves), the travelling wave solutions for the soil displacements can now be expressed as

  (40)

where the incident amplitude *V*+ is given by

  (41)

and the reflected amplitude *V*- is to be determined by the boundary conditions at the ground surface.

At the ground surface, the condition that the shear stress is zero (*zy*=0. Given that the vertical component of displacement at the ground surface is also zero, the shear stress is given by

  (42)

For a pipe buried at depth, *d*, this gives the reflected displacement amplitude *V*- as

  (43)

i.e. a doubling of the response is seen at the free surface, analogous to the pure acoustic case. Here, it should be noted that, for the SH wave component (which is assumed to dominate) this doubling will occur regardless of the angle of incidence of the wave (i.e. it is independent of both the *x*- and *y*-dependences). Substituting Eqs. (41) and (43) into Eq. (40) gives the soil displacement at the ground surface as

  (44)

Eq. (44) relates the soil displacement at the surface, in the direction perpendicular to the pipe, to the circumferential displacement of the pipe shell wall directly beneath. The variation with frequency is not straightforward, with both the magnitude and the phase of the response at one location depending on the non-dimensional radial wavenumbers, and . However, the solution presented above may be examined in a little more detail before recourse to numerical examples is required.

* At very low frequencies (), the solution presented is not valid as the waves at the ground surface cannot be considered plane.
* At high frequencies (,), adopting the large argument approximation for the Hankel function, Eq. (44) becomes

  (45)

This shows that here the pipe behaves as a plane wave source with the plane waves following a cylindrical spreading law.

* At mid frequencies (,) the full form of Eq. (44) must be employed.

The numerical examples presented in the following section serve to illustrate the expected behaviour in a variety of situations.

# Numerical model

In the preceding sections, a number of simplifying assumptions have been made in order to obtain the analytical results presented, both for the torsional wave dispersion relationship (Eq. (20) and for the ground surface response arising from a torsional wave propagating in the pipe wall (Eq. (44). To provide an independent validation of some of these assumptions, a numerical (finite element) model has been developed, against which some of the results obtained can, in specific cases, be compared.

The numerical validation was performed using the semi-analytical finite element (SAFE) method [37-39]. The key concept of the method lies in utilising a finite element-like procedure for discretisation of the cross-section of the waveguide and assuming a space-harmonic variation of the displacement field in the propagation direction. The assembled system matrices form a generalised eigenvalue problem with the wavenumber as the eigenvalue which can be solved using any standard eigensolver. This approach is well known and is widely used in the structural dynamics and acoustic community with applications dating back to the 1970s, for example in [40].

SAFE can be conveniently implemented in a commercial finite element package (COMSOL) using its partial differential equation formalism as described by Predoi *et al*. [41]. This allows for performing calculations based on SAFE without developing a dedicated computer code and offers ready-to-use tools for visualisation of the results. Modelling of wave propagation along a waveguide embedded in an infinite or semi-infinite space using SAFE can be realised using a few approaches; here we chose to use gradually absorbing damping layers [42] as these can be implemented in COMSOL directly. An infinite space or semi-infinite half space is approximated with a layer along which damping, modelled using a constant loss factor, increases gradually away from the region of interest.

Two cases, namely a cast iron pipe embedded in an infinite sandy soil (see Table 1) and a cast iron pipe embedded in a sandy half-space were considered, initially to evaluate the effect of the free surface on the dispersion curves and then for comparison with the analytical results[[1]](#footnote-1)♣. Figs. 4 and 5 depict the salient features of the models. The pipe radius was 10cm (measured from the centre line to the mid-thickness of the pipe wall) and the pipe wall was 1cm, mirroring the pipe parameters used in Section 5. In each model the pipe was first embedded in the cylindrical layer of soil (with no loss factor variation) of 0.3m radius and then in the absorbing medium (see Fig. 4). Calculations were performed in the frequency range from 1 Hz to 5 kHz.



Fig. 4. SAFE mode of the pipe embedded in an infinite soil: (a) mesh around the pipe for the low frequency model; (b) schematic representation of the model.

 (a) (b)
Fig. 5. A SAFE model for the pipe embedded in a half-space (including ground surface): (a) schematic representation of the model; (b) shear loss modulus distribution demonstrating the absorbing layer setup.

# Example results

Examples are presented for typical cast iron and MDPE pipes buried in two soil types, representing a sandy soil and a clay soil, respectively. The relevant pipe and soil parameters are shown in Table 1.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | MDPE | Cast iron | Sandy soil | Clay soil |
| Density (kg/m3) | 2000 | 7800 | 2000 | 2000 |
| Shear modulus (GN/m2) | 0.6 | 65 | 0.02 | 0.18 |
| Poisson’s ratio | 0.4 | 0.2 |  |  |
| Loss factor | 0.065 | 0.001 | 0.1 | 0.1 |
| Pipe thickness/radius | 0.1 | 0.1 |  |  |
| Pipe depth/radius | 10 | 10 |  |  |

Table 1
Pipe and soil elastic properties

### Analytical dispersion relationships

Figs. 6(a) and (b) show the real parts of the wavenumber and the wave attenuation (presented as loss in dB per unit propagation distance - measured in pipe radii) respectively for both types of pipe in each soil. For the real parts (Fig. 6a), the *in-vacuo* values are also shown; for the imaginary parts, the high frequency approximations given by Eq. (29) are included. Eq. (25) is used in order to present the results in non-dimensional form; the non-dimensional wavenumbers are plotted against the free-field non-dimensional wavenumber for water, *kfa*. This is done for two reasons: firstly to enable the results for both pipes to be presented in a consistent manner so that they may be compared; and secondly so that the results may be compared with those presented previously for the *s*=1, 2 wavetypes [12, 15]. (Note that, for a pipe of radius 100mm, the maximum frequency presented in the figures is approximately 5kHz, with the ring frequencies being approximately 7.3kHz for a cast iron pipe and 1.5kHz for an MDPE pipe. Furthermore, the ring frequency will occur at a value of  for a cast iron pipe and  for an MDPE pipe, regardless of the pipe radius.) It should also be noted that, as for the *s*=1, 2 wavetypes, the solutions to Eq. (25) must be found recursively, also using the relationship ; moreover, a choice must be made as to the sign of the square root of . Here, the choice is made based on the assertion that the pipe waveumber, *k*0, must have a negative imaginary part, so that waves travelling along the pipe decay, rather than grow (this differs slightly from the *s*=1, 2 cases presented previously in that, for these, both choices of square root resulted in a decaying pipe wave, so a further constraint was required in order to decide which root should be used).

Fig. 6(a) shows that, for the cast iron pipe, the real part of the wavenumber (and hence the wavespeed) is very close to the *in-vacuo* value at all the frequencies considered, for both soil types. Even at low frequencies, the effect of the soil is small. Close inspection reveals there to be a very slight decrease in the values relative to the *in-vacuo* case, with this effect being slightly greater in the clay soil. This indicates that, the soil exerts a small stiffness effect on the pipe. For the MDPE pipe, the same may be said of the sandy soil. However, for the clay soil, due to the larger shear stiffness of clay, the real part of the wavenumber is increased relative to the *in-vacuo* case, indicating that the effect of the soil is one of mass loading; the effect is also much more marked than for the other cases. Moreover, the wavenumber curve does not pass through the origin, suggesting that the wavespeed tends to zero at very low frequencies and that the wave may be cut-off. Wavenumbers were also computed assuming no loss in the soils, in order to evaluate the effect of the soil damping. As anticipated, it was found that the soil damping had a negligible effect on the real part of the wavenumbers.

Fig. 6(b) shows that, for the cast iron pipe, the wave attenuation is of the order of 1-2dB/a at low frequencies and decreases with frequency, reaching the constant value predicted by Eq. (29). The attenuation is larger for the clay soil, as might be expected, given the higher shear stiffness of clay. For the MDPE pipe, for both soil types, the attenuation is much greater than for the cast iron pipe, this being largely due to radiation into the surrounding soil. Moreover, the attenuation generally increases with frequency; this effect is a result of the higher losses within the pipe wall. For the sandy soil, at higher frequencies, the attenuation approaches that given by Eq. (29); for the clay soil, there is still a marked difference between the results predicted by Eqs. (25) and (29). This is due to the assumption related to Eq. (28) not being valid in this case; as the wave attenuation is large, approximating the pipe wavenumber to the *in-vacuo* value in Eq. (27) leads to large errors. As for the real part of the wavenumber, the effect of soil damping on the wave attenuation was found to be extremely small for all four pipe/soil combinations. Finally, comparing with the results presented in [15], for the *s*=1 wave in an MDPE pipe in the same soil types (not repeated here), it is noted that, for the same frequency range the attenuation values for the *s*=0 wave are significantly greater.


(a)


(b)

Fig. 6. Predicted wavenumbers for cast iron and MDPE pipes buried in sand/clay: (a) real part; (b) attenuation.

### Results from the numerical model

Figs. 7(a) and (b) show the real parts of the wavenumber and the wave attenuation (again presented as loss in dB per unit propagation distance - measured in pipe radii) respectively for the cast iron pipe in sandy soil, as obtained using the numerical model described in Section 4. Also included are the corresponding results from the analytical model. Fig 7(a) shows that, except at very low frequencies, the real part of the wavenumber is very close to the *in-vacuo* value, as expected. Moreover, whether or not the free surface is included in the model has negligible effect. The results show good agreement with results from the analytical model.

Fig. 7(b) shows that the surrounding soil has a measurable effect on the wave attenuation but here, again, the effect of the free surface is negligible. There is, however, a small quantitative difference (approximately 10% in the high frequency asymptote) between the predictions from the analytical model when compared with the numerical model, although the trends are identical. The attenuation prediction is particularly sensitive to the pipe thickness to radius ratio, *h/a*, and the observed differences in the results can be attributed to two factors in the development of the analytical model, associated with assuming that the shell is thin-walled: firstly, the term *h/a* in Eq. (17) is neglected in the derivation of Eq. (20); and secondly, it is assumed that the pipe/soil boundary conditions are satisfied at *r=a* , which is strictly the mean radius. It is felt that this is a small compromise in accuracy for the sake of the compact analytical representation obtained.



(a)


(b)
Fig. 7. Predicted wavenumbers for cast iron pipe buried in sand, as obtained using the numerical model: (a) real part; (b) attenuation

### Response at the ground surface

Figs 8(a) and (b) show the displacement response at the ground surface relative to the pipe wall displacement for both types of pipe, embedded in each type of soil. The responses presented are derived from Eq. (44); the approximate responses at high frequency, given by Eq. (45) are also included. Frequencies for which data are considered invalid, by virtue of the responses not being in the far field, have been excluded from the plots. Examining Fig 8(a), it can be seen that, for the cast iron pipe, the response is consistently less at the ground surface than on the pipe, becoming a linear variation at higher frequencies. This is as might be expected, given that the waves undergo both attenuation due to soil damping and cylindrical spreading in order to reach the surface. Here, large differences between the two soil types can be seen, with the attenuation in the clay soil being much less than in the sandy soil. This is largely due to the soil damping itself which has a much greater effect on the smaller wavelengths present in the sandy soil compared with those propagating in the clay soil. Almost none of the frequency range has been excluded for not meeting the far field condition at the surface, suggesting that the non-dimensional wavenumber, , is greater than unity even at very low frequencies. This, again, is as expected, given that the wavenumber in the pipe approximates the *in-vacuo* value and, in turn, is much smaller than the free-field wavenumber in the soil. What is perhaps more surprising are the values observed for the MDPE pipe - significantly larger than those for the cast iron pipe and which, for the clay soil suggest a wave amplification. At first sight, this might seem non-physical. However, it must be remembered that waves radiating to the surface do so at an angle to the pipe (determined by the Snell’s law), and that waves arriving at the ground surface have originated from a location on the pipe somewhat ‘upstream’ axially; over that distance, the waves in the pipe will have suffered large attenuations. For all the cases considered, the high frequency approximations are valid for all but very low frequencies for the cast iron pipe and valid throughout the whole frequency range for the MDPE pipe.

Fig. 8(b) shows the phase of the ground surface displacement relative to that seen on the pipe wall. Again, the differences result from the phase lag of a wave travelling a certain distance along the pipe compared with a wave travelling a longer distance in the surrounding soil. The high-frequency approximations are, here, indistinguishable from the full solution over the whole frequency range.


(a)


(b)
Fig. 8. Displacement response at ground surface, relative to pipe wall displacement for cast iron and MDPE pipes buried in sand/clay: (a) magnitude (dB); (b) unwrapped phase (rad).

Fig. 9 shows the span above the pipe which can be considered to be “directly above the pipe” for each pipe soil combination as given by Eq. (39). Here it can be seen that, except at very low frequencies for the cast iron pipe, the maximum span is constrained by the requirement that the phase difference across the span at the surface is small, rather than that the angle of incidence at the surface should be less than 45 degrees. The span decreases, as expected, with frequency and, in this case, becomes of the order of the pipe radius (*a/d*=1) at high frequencies.


Fig. 9. Span “directly over the pipe”.

# Practical Considerations

The modelling presented here shows that, if torsional waves are excited in a buried pipe, they can indeed be potentially detected at the ground surface. In practice, for plastic pipes, the wave attenuation is so large that it is unlikely that the waves could even be considered to propagate more than a few pipe radii; however, for metal pipes such detection might be feasible, particularly if lower frequencies are considered (at higher frequencies, the soil attenuation would be prohibitive). In particular, for pure torsional motion in an ideal medium, the resulting displacement at the ground surface directly above the pipe is confined to one direction: perpendicular to the pipe and in the plane of the surface. This is in contrast to the ground surface displacements resulting from either *s*=1 (fluid-dominated) or s=*2* (shell-dominated) axisymmetric wave motion in the pipe; under ideal conditions these displacements are confined to being a combination of vertical and horizontal, in-line with the pipe [43]. These differences may be brought to bear when attempting to determine, from the ground surface, which type of wave motion has occurred in the pipe, for example when a fracture event occurs.

Restricting our attention now to metal pipes, monitoring torsional waves directly on the pipe may also be beneficial, and not only for detecting the presence or not of torsional wave motion. As stated earlier, the *n*=0, *s*=0 torsional wave, such as is described in this paper, is frequently exploited for the detection of faults and cracks in pipes, see for example [17-24]. Mainly carried out at ultrasonic frequencies, inspections of this sort are possible because, at high frequencies, the wave is non-dispersive; moreover, the attenuation is atypically low and invariant with frequency in this frequency regime. The investigations presented here show that these trends do, in fact, persist at much lower frequencies, well into the audible range. Knowledge of this would allow pipe inspections to be carried out at these lower frequencies. Whilst the disadvantages of this approach are immediately apparent (the scale of the faults would need to be much greater and the excitation devices correspondingly larger), there would also be advantages of monitoring torsional waves in this way: sampling rates on acquisition and analysis equipment could be much lower, thus potentially reducing costs dramatically; larger, potentially more catastrophic, defects could be more readily exposed (more important, perhaps where repair and replacement prioritization is an issue, such as in the water industry); and finally, leak detection equipment generally operates in the audible frequency range so possibilities might open up for incorporating torsional wave monitoring into existing equipment.

# Conclusions

In this paper, axisymmetric torsional wave motion in a buried pipe has been studied. At the pipe/soil interface, compact contact has been assumed, for which continuity of circumferential displacement is maintained. An analytical dispersion relationship has been derived for the *n*=0, *s*=0, torsional wave. This wave is uncoupled from any contained fluid but, for the purposes of deriving the dispersion relationships, a surrounding medium of infinite extent was presumed. These expressions, along with low and high-frequency approximations, permitted insights to be gained into the physical mechanisms at play. Furthermore, expressions were derived for the ground surface displacement directly above the pipe, resulting from torsional wave motion within the pipe wall. Numerical examples were then presented for both a cast iron and an MDPE pipe. A numerical model developed in tandem with the analytical solutions has confirmed that neglecting the free surface in obtaining the dispersion characteristics is a valid approach.

It was found that, for the cast iron pipe, the effect of the soil (for either soil type) on the real part of the wavenumber, and hence wavespeed, was small. The small differences between the soil-loaded cases and the *in-vacuo* case demonstrated that, at low frequencies, the soil exerts a small stiffness effect. The same effect can be seen for the MDPE pipe in sandy soil. For the MDPE pipe in clay soil, the reverse is true, with the soil exerting a relatively large mass effect. Contrastingly, the effect of the surrounding soil on the wave attenuation can be considerable, particularly for the MDPE pipe, for which the torsional wave is unlikely to propagate more than a few pipe radii. Comparison with the results from the numerical model showed good agreement with the real part of the wavenumber. A small difference was observed in the wave attenuation predictions, attributable to the thin-walled pipe assumptions.

The torsional wave propagating in a buried pipe will radiate into the surrounding soil purely as a shear wave, which might be detectable at the ground surface. For the MDPE pipe, it was found that the torsional wave attenuation was such that ground surface detection would seem unlikely. For the cast iron pipe, detection at the ground surface might be possible.

The findings presented here open up new possibilities for both pipe damage detection and for the remote detection of pipe fracture in cast iron pipes, at audio frequencies. Future work will address whether such approaches are viable in practice.

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# Appendix: Discussion on the pipe/soil coupling: compact contact

Consider a pipe buried at a depth *d*. The normal static loading on the pipe, *pstat*, assuming unsaturated soil, is given by

  (A1)

where *g* is the acceleration due to gravity. If the soil is saturated, the soil density will increase accordingly but, in addition, the pipe will now be subjected to the atmospheric pressure*, patm*. In this case, Eq. (A1) becomes

  (A2)

For static friction to be maintained between the pipe wall and the surrounding soil (i.e. a condition of compact contact), the shear stress at the pipe wall must satisfy

  (A3)

where *s* is the coefficient of friction between the pipe wall and the surrounding soil, typically of order unity, and only likely to be significantly less if the contact surface is intentionally lubricated via a thin layer of oil, for example. This effectively sets a limit on the circumferential strain in the pipe wall before slippage starts to occur. The maximum shear strain in the soil, , for slippage not to occur, will then be of the order of

  (A4)

(Here, ** is the shear modulus in the soil.) Using the values given in Table 1 for sandy soil, and assuming a minimum burial depth of 1m, this gives maximum shear strain values of the order of 0.1% for dry sand or 0.5% for saturated sand. These are large strains, considerably greater than what would typically be considered to be within the linear region [44], and more commonly seen in earthquake conditions. Moreover such high strains can induce liquefaction [45].

In summary, for the levels of strain which are likely to be seen outside of the aforementioned extreme scenarios, it is reasonable to assume that compact coupling will exist between a buried pipe and the surrounding soil.

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1. ♣ Additional cases had not been included here for a number of reasons. Constraints on both the required thickness of the absorbing layer and the element size at high frequencies gave rise to a model which was found to be extremely computationally intensive. Moreover, in the frequency range considered there are only few propagating waves which are of interest. Extracting a small number of eigenvalues from a very large eigenvalue set requires a precise definition of eigenvalue search regions or conditions. Consequently, to obtain the solutions of interest the simulation needed to be run a number of times, optimising various solution parameters. It was also observed that for cases were the waveguide (pipe) was very well coupled to the surrounding medium (soil), guided waves in the strict sense were found not to exist. Wave energy propagates predominantly along the soil, instead. For such models (e. g. MDPE pipe in clay) it was very difficult to find torsional wave solutions without tuning model parameters for nearly every frequency step; proceeding further with such cases was felt, for the purposes of this paper, to be unwarranted. [↑](#footnote-ref-1)