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Multi-field inflation from holography

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Abstract. We initiate the study of multi-field inflation using holography. Bulk light scalar fields correspond to nearly marginal operators in the boundary theory and the dual quantum field theory is a deformation of a CFT by such operators. We compute the power spectra of adiabatic and entropy perturbations in a simple model and find that the adiabatic curvature perturbation is not conserved in the presence of entropy perturbations but becomes conserved when the entropy perturbations are set to zero or the model is effectively a single scalar model, in agreement with expectations from cosmological perturbation theory.

Keywords: inflation, quantum cosmology, cosmological perturbation theory

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1 Introduction

The physics of the very Early Universe provides a window to Planck scale physics where quantum gravity effects are expected to play an important role. Moreover, recent cosmological observations provide us with a unique opportunity to test theoretical ideas about the physics of this period and strongly suggest that the Universe underwent a period of rapid accelerated expansion at early times, a period of cosmic inflation. The underlying theoretical description of inflation is based on perturbative quantization around an accelerating Friedmann-Robertson-Walker (FRW) spacetime. This description however is in general sensitive to the UV completion of the theory and requires an embedding of inflation in a consistent quantum theory of gravity. Furthermore, the degrees of freedom of the Early Universe may have been strongly coupled requiring an altogether new framework for the description of their dynamics.

Holography offers a framework that could accommodate both conventional inflation and also new models describing a Universe that is non-geometric and strongly interacting at early times. Recall that in gauge/gravity duality [1–3] a strongly coupled phase of bulk gravity is dual to a weakly coupled quantum field theory (QFT) and vice versa. Thus conventional inflation could have a dual description in terms of a strongly coupled QFT while weakly coupled QFT could describe the dynamics of a non-geometric Universe.

The application of gauge/gravity correspondence to cosmology was initiated in refs. [4, 5] (see also ref. [6]) where a correspondence between de Sitter (dS) and conformal field theory (CFT) was discussed. The conjecture was further sharpened in ref. [7] where it was argued that the CFT partition function computes the late time behavior of the dS wavefunction upon a certain analytic continuation. Early related work on dS and quasi-dS (including discussions of cosmological perturbations) can be found in refs. [8–14] while more recent works include refs. [15–33]. See also other recent discussions in refs. [34–38].

While the connection between the bulk wavefunction and the CFT partition function is intuitive and well-motivated, the absence of string theory examples and a number of technical subtleties in the original formulation of the duality initially cast doubt about its validity. A different route to this duality was put forward in ref. [39]. This was based on a fact that FRW spacetimes of a theory with potential V are in correspondence with domain-wall space times of a theory with potential $-V$ [40, 41]. This correspondence links accelerating FRW spacetimes to domain-wall space times, either asymptotically AdS or asymptotic to non-conformal brane backgrounds, where standard gauge/gravity duality is better understood. Computing independently cosmological observables such as the spectra and bi-spectra and correlation functions of the QFT dual to the corresponding domain-wall spacetime one could then analyse whether there is any relation between the two. It turned out that in this way one could establish precise holographic formulae for the spectra and bi-spectra of general inflationary models in terms of a specific analytic continuation of 2- and 3-point functions of the energy momentum tensor of the dual QFT [39, 42–48]. This provides an effective holographic description of general single field inflationary model.

The holographic formulae relate cosmological in-in correlators with QFT correlation functions of the energy momentum tensor. While these formulae were obtained in the regime where gravity is valid, it was also conjectured [44, 46] that the formulae hold more generally: cosmological in-in correlators and the QFT correlators may receive quantum corrections but their relation should remain the same. Indeed, in a class of slow roll models one can establish the same formulae without assuming the gravity approximation but by using that the partition function of the dual QFT computes the wavefunction of the bulk theory [25].

This class of slow-roll models consists of single field models where the bulk potential has two near-by de-Sitter critical points [47]. Correspondingly the dual QFT has two near-by fixed points. One may use conformal perturbation theory to compute the QFT correlation functions at strong coupling and in this way the slow-roll results are exactly reproduced both for the spectra and bispectra (to the order checked) [47, 48]. For the same class of models, one can alternatively compute the bulk wavefunction from the QFT partition function by only assuming that the dual QFT has two near-by fixed points [25], recovering the holographic formulae derived in refs. [39, 44, 47] (after taking into account the relation between the analytic continuations we discuss in appendix A).¹

¹More generally, there is agreement between explicit computations of 2- and 3-point functions computed using the wavefunction such as those in refs. [17, 24] and the formulae in refs. [39, 44, 46] (after the analytic continuations are matched, see appendix A, and modulo the semi-local contributions which are often not computed using the wave function approach).

In more detail, the dual QFT was considered to be a deformation of a CFT by a marginally relevant operator of dimension $3 - \lambda$, where $0 < \lambda \ll 1$ and the strength of the deformation is also proportional to λ . Assuming that the coefficient in the 3-point function of the deforming operator is positive and of order one (relative to λ) one may show that there is a nearby fixed point. If in addition the UV CFT contains no other nearly marginal operator the RG flow between the UV fixed point to the IR fixed point is controlled by the deforming operator alone. In the bulk this means that we are considering a single field model.

The purpose of this paper is to relax some of these conditions. While we will still arrange for the theory to have two nearby fixed points we will now assume that there are a multitude of deforming scalar operators. Each of these is related to a light bulk field and we now have a multi-field inflationary model. While recent observations appear to favour single field inflation [49], the presence of many nearly marginal operators is natural from the perspective of the dual QFT.

Our main result is the computation of the power spectra for adiabatic and entropy perturbations. We will be able to carry out this computation completely for a simple model. One of the main differences between single field and multi-field models is about the time dependence of the adiabatic curvature perturbation ζ . According to standard cosmological perturbation theory, the adiabatic curvature perturbation ζ should be time independent at large scales in single field models of inflation but it should not be conserved in the presence of entropy perturbations. It was indeed confirmed in ref. [50] that the holographically computed power spectrum for ζ is conserved in the one field case (although some unresolved subtleties were pointed out concerning the conservation of the bispectrum). In this paper we will demonstrate that the holographically computed spectra are not conserved due to the influence of the entropy perturbation, as expected from cosmological perturbation theory (see for instance ref. [51]).

The outline of this paper is as follows. In section 2 we review and extend the holographic setup for inflation to the multi-scalar case. In section 3 we discuss the dual QFT and in section 4 we relate the holographic RG flow with cosmological evolution in the bulk. In section 5 we set up the computation of the spectra of primordial adiabatic and entropy perturbations using holography and we explicitly compute these spectra in a simple example in section 6. We present our concluding remarks in section 7. The paper also contains two appendices: in appendix A we review the analytic continuation from AdS to dS, which is a part of the holographic dictionary while appendix B contains an alternative derivation of the power spectra.

2 Preliminaries

In this section we will use the approach developed in refs. [25, 50] and extend the results of refs. [25, 47] to the case of A scalar fields.

2.1 Wave function

The cosmological spacetime metric can be given in ADM formalism as

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt). \quad (2.1)$$

Here we shall restrict attention to the situation where the metric is asymptotically de Sitter both in the far future and in the far past. I.e. we are describing the case of a period of slow roll inflation followed by Λ domination.

Our starting point is the assumption that the wave function of the bulk gravitational field is related to the generating functional of a boundary QFT

$$\psi_{\text{bulk}}[h, \phi] \propto Z_{\text{QFT}_+}[h, \phi], \quad (2.2)$$

where the generating functional Z_{QFT_+} is given by

$$Z_{\text{QFT}_+}[h, \phi] = e^{-W_{\text{QFT}_+}[h, \phi]} = \int D\chi \exp(-S_{\text{QFT}_+}[\chi, h, \phi]). \quad (2.3)$$

Here ψ_{bulk} denotes the wave function of the bulk and χ denotes boundary fields. The metric h_{ij} and the inflaton field (or fields) ϕ act as sources for corresponding gauge invariant operators constructed using χ (we will often suppress the indices of h_{ij}). $Z_{\text{QFT}_+}[h, \phi]$ is understood as the collection of correlation functions of gauge invariant operators. The QFT should admit a 't Hooft large N limit and is taken to be Euclidean (though this is not essential — the Euclidean theory may be related by Wick rotation to a Lorentzian QFT).

To date there is no string/M theory derivation of the dual QFT and there are questions whether such a theory exists non-perturbatively (see, for example, refs. [52, 53] for early discussions of this point in the context of dS/CFT). However, one can give an operational definition of the theory in large N perturbation theory and infer many its properties (within perturbation theory). In ref. [7] Maldacena used the bulk description in order to extract correlators of the putative dual CFT_+ of de Sitter and he showed that these correlators are related by analytic continuation to the correlation functions of the CFT_- dual to AdS,² see appendix A for a review of the analytic continuation. In refs. [39, 44, 46] it was shown that correlation functions of QFT_+ (the putative dual to general single field inflationary spacetimes) are related by analytic continuation to correlation functions of QFT_- (the QFT dual to the corresponding domain-wall space-time [40, 41]) in the large N limit. QFT_+ was dubbed “pseudo-QFT” in ref. [39] since it was only defined via analytic continuation from QFT_- . A duality where the CFT_+ has an a priori definition was proposed in ref. [18]. In this duality the CFT was conjectured to be dual to the dS solution of Vasiliev’s higher spin gravity rather than Einstein gravity. At the perturbative level the CFT_+ is related by analytic continuation to the CFT_- dual to AdS, as in ref. [39], but now one could analyse the theory non-perturbatively and such analyses were presented in refs. [21, 27].

Since the analytic continuation takes $N \rightarrow -N$, it is unclear whether the QFT_+ and QFT_- are related to each other by this analytic continuation non-perturbatively in $1/N$. A recent study [54] of a specific (highly supersymmetric) example showed that the partition function of QFT_- can be extended to the entire complex plane as a function N . However, the non-perturbative contributions become significant as $N \rightarrow -N$ and may potentially upset the agreement between the holographic and cosmological computations. On the other hand, one may have theories that coincide in large N perturbation theory and only differ non-perturbatively, as was recently shown in ref. [55]. Thus, the dual to cosmology may only agree with the analytic continuation of QFT_- in large N perturbation theory.

In this paper we will set aside such non-perturbative issues and focus only on the leading contribution in the large N limit. We will do computations in QFT_- and then analytically continue to obtain the correlators relevant for cosmology.

²Here and below we will use the subscripts $+$ and $-$ to refer to the QFT dual of the cosmology and domain-wall space-time, respectively. These spacetimes are linked via the domain-wall/cosmology correspondence [40, 41].

2.2 Correlators in the bulk

Once we are given the wave function ψ_{bulk} , we can compute the correlators for the bulk. We describe the perturbation of the spatial metric as

$$h_{ij} = a^2(t) e^{2\zeta(t, \mathbf{x})} \delta_{ij}, \quad (2.4)$$

where we neglected transverse traceless perturbations (gravitational waves). We will specify the time slicing later. In this paper, extending the analysis in ref. [25], we consider the case of A scalar fields in the bulk. Then, in addition to the curvature perturbation ζ , there are $(A - 1)$ scalar degrees of freedom, $s^{a'}$ with $a' = 2, \dots, A$, the entropy perturbations. In the following, we collect together the curvature perturbation ζ and the entropy perturbation $s^{a'}$ in Φ^a ($a = 1, \dots, A$), where $\Phi^1(t, \mathbf{x}) \equiv \zeta(t, \mathbf{x})$ and $\Phi^{a'}(t, \mathbf{x}) \equiv s^{a'}(t, \mathbf{x})$. The index a is raised and lowered by the Kronecker delta δ_{ab} .

As in the discussion in the previous subsection, the wave function in the bulk is related to the generating functional of the connected correlators of gauge invariant operators as

$$\psi_{\text{bulk}}[\Phi^a] = \mathcal{N} Z_{\text{QFT}_+}[\Phi^a] = \mathcal{N} e^{-W_{\text{QFT}_+}[\Phi^a]}, \quad (2.5)$$

with

$$W_{\text{QFT}_+}[\Phi^a] \equiv -\ln Z_{\text{QFT}_+}[\Phi^a], \quad (2.6)$$

where \mathcal{N} is a normalisation constant. Using the wave function $\psi_{\text{bulk}}[\Phi^a]$, the probability density function $P[\Phi^a]$ is given by

$$P[\Phi^a] = |\psi[\Phi^a]|^2 = |\mathcal{N}|^2 e^{-2\text{Re}[W_{\text{QFT}_+}[\Phi^a]]}. \quad (2.7)$$

Once we have the probability density $P[\Phi^a]$, we can calculate cosmological n -point functions as

$$\langle \Phi^{a_1}(\mathbf{x}_1) \Phi^{a_2}(\mathbf{x}_2) \dots \Phi^{a_n}(\mathbf{x}_n) \rangle = \int D\Phi P[\Phi^a] \Phi^{a_1}(\mathbf{x}_1) \Phi^{a_2}(\mathbf{x}_2) \dots \Phi^{a_n}(\mathbf{x}_n), \quad (2.8)$$

where $D\Phi$ denotes the integration measures for ζ and s , given by

$$D\Phi \equiv D\zeta \prod_{a'=1}^{A-1} Ds^{a'}. \quad (2.9)$$

The integration measure is not part of the boundary QFT data since the fields Φ^a are external fields and hence some additional input may be necessary. Changes in the measure may be represented by local terms in the integrand, which may be incorporated by a redefinition of W_{QFT} . Here we will assume that there are no non-trivial measure factors. In principle, one can determine the measure factors in the semi-classical regime using the derivation of the holographic formulae via the domain-wall/cosmology correspondence. In the single field case this indeed yields a trivial measure.

Eliminating the background contribution $W_{\text{QFT}_+}[\Phi^a = 0]$ by a redefinition of \mathcal{N} , the probability density function $P[\Phi^a]$ is given by

$$P[\Phi^a] = |\mathcal{N}|^2 e^{-\delta W[\Phi^a]}, \quad (2.10)$$

where we defined

$$\delta W[\Phi^a] \equiv 2\text{Re} [W_{\text{QFT}_+}[\Phi^a] - W_{\text{QFT}_+}[\Phi^a = 0]]. \quad (2.11)$$

Assuming that the wave function is normalizable, we then fix the normalization constant \mathcal{N} by

$$\int D\Phi P[\Phi^a] = 1. \quad (2.12)$$

We now expand $\delta W[\Phi^a]$ as

$$\delta W[\Phi^a] = \sum_{n=1}^A \frac{1}{n!} \int d^d \mathbf{x}_1 \cdots \int d^d \mathbf{x}_n W_{a_1 \cdots a_n}^{(n)}(\mathbf{x}_1, \cdots, \mathbf{x}_n) \Phi^{a_1}(\mathbf{x}_1) \cdots \Phi^{a_n}(\mathbf{x}_n), \quad (2.13)$$

where

$$W_{a_1 \cdots a_n}^{(n)}(\mathbf{x}_1, \cdots, \mathbf{x}_n) \equiv 2 \operatorname{Re} \left[\frac{\delta^n W_{\text{QFT}_+}[\Phi^a]}{\delta \Phi^{a_1}(\mathbf{x}_1) \cdots \delta \Phi^{a_n}(\mathbf{x}_n)} \Big|_{\Phi^a=0} \right]. \quad (2.14)$$

Once we obtain $W_{a_1 \cdots a_n}^{(n)}(\mathbf{x}_1, \cdots, \mathbf{x}_n)$, we can compute the n -point functions using Feynman rules [25]. In more detail, we view (2.13) as our action with Φ^a treated as elementary fields. Then the inverse of the quadratic part provides the propagator for the Feynman rules while the higher order terms provide the vertices.

For example, the two-point function on $\Phi^a(\mathbf{x})$ is given by

$$\langle \Phi_{a_1}(\mathbf{x}_1) \Phi_{a_2}(\mathbf{x}_2) \rangle = W_{a_1 a_2}^{(2)-1}(\mathbf{x}_1, \mathbf{x}_2), \quad (2.15)$$

where $W_{a_1 a_2}^{(2)-1}(\mathbf{x}_1, \mathbf{x}_2)$ denotes the inverse matrix of $W_{a_1 a_2}^{(2)}(\mathbf{x}_1, \mathbf{x}_2)$,

$$\sum_{a_2=1}^A \int d^d \mathbf{y} W_{a_1 a_2}^{(2)}(\mathbf{x}, \mathbf{y}) W_{a_2 a_3}^{(2)-1}(\mathbf{y}, \mathbf{z}) = \delta_{a_1 a_3} \delta(\mathbf{x} - \mathbf{z}). \quad (2.16)$$

Furthermore, $W_{a_1 a_2}^{(2)}(\mathbf{x}_1, \mathbf{x}_2)$ equal minus the two-point function of the operators \mathcal{O}_{Φ^a} dual to scalar Φ^a

$$W_{a_1 a_2}^{(2)}(\mathbf{x}_1, \mathbf{x}_2) = -2 \operatorname{Re} [\langle \mathcal{O}_{\Phi^{a_1}}(\mathbf{x}_1) \mathcal{O}_{\Phi^{a_2}}(\mathbf{x}_2) \rangle_+], \quad (2.17)$$

where $\langle \rangle_+$ indicates correlators of QFT_+ . (In the following sections we will often change the basis of operators we consider. In such cases one needs to take into account the Jacobian of the transformation to compute correctly the 2-point functions.)

As reviewed in the previous subsection, we are currently lacking a proper definition of QFT_+ . However, we know that in the large N limit we can compute the correlators in QFT_- (which is a standard QFT) and then analytically continue the result according to the rules discussed in appendix A. All QFT computation in this paper are done in standard QFT and they are about QFT_- . To obtain the final cosmological results proper care was taken to implement the necessary analytic continuation. Note that the derivation of the relation between bulk and boundary correlators given in section 5 applies irrespective of how QFT_+ is provided. In the following sections to simplify the presentation we will suppress the distinction between QFT_- and QFT_+ and simply refer to the dual QFT. Hopefully this will not cause any confusion.

3 Dual quantum field theory

In this section, we discuss the d -dimensional quantum field theory which would be dual to the $(d+1)$ -dimensional inflationary spacetime. We shall assume that d is odd to avoid

complications due to the gravitational conformal anomalies (our main interest is four bulk dimensions, so $d = 3$). Since we are considering space-times that are asymptotically de Sitter, the dual theory is a deformation of a CFT,

$$S_{\text{QFT}}[\chi] = S_{\text{CFT}}[\chi] + \sum_{a=1}^A \int d\Omega_d u^a O_a(\mathbf{x}), \quad (3.1)$$

where $d\Omega_d$ is the d -dimensional invariant volume and S_{CFT} is the action at the UV fixed point.³ This CFT is deformed by gauge invariant operators $O_a(\mathbf{x})$ of dimension $\Delta_a < d$ and u^a are the corresponding coupling constants. Hereafter, we suppress the summation over a . In this paper, we assume that the field space for u^a is flat and hence we raise and lower indices a by the Kronecker delta. In this section, we solve the RG flow in the flat space and determine the renormalisation scale dependence of the coupling constants u^a .

3.1 RG flow

The relevant operators in eq. (3.1) induce an RG flow which we will study in this section. As long as the deformation parameters are small one can study the RG flow using conformal perturbation theory. To prepare for this computation let us summarise standard results for a CFT on flat \mathbb{R}^d . Conformal symmetry determines the two-point function and the three point function up to constant parameters as

$$\langle O_a(\mathbf{x}) O_b(\mathbf{y}) \rangle_{\text{CFT}} = \frac{c_a \delta_{ab}}{|\mathbf{x} - \mathbf{y}|^{2\Delta_a}}, \quad (3.2)$$

and

$$\langle O_a(\mathbf{x}) O_b(\mathbf{y}) O_c(\mathbf{z}) \rangle_{\text{CFT}} = \frac{C_{abc}}{|\mathbf{x} - \mathbf{y}|^{\Delta_a + \Delta_b - \Delta_c} |\mathbf{y} - \mathbf{z}|^{\Delta_b + \Delta_c - \Delta_a} |\mathbf{z} - \mathbf{x}|^{\Delta_c + \Delta_a - \Delta_b}}, \quad (3.3)$$

where Δ_a is the scaling dimension of the operator O_a . These imply the operator product expansion (OPE)

$$O_a(\mathbf{x}) O_b(\mathbf{y}) = \frac{c_a \delta_{ab}}{|\mathbf{x} - \mathbf{y}|^{2\Delta_a}} + \delta^{cd} \frac{C_{abd}}{c_c} \frac{O_c(\mathbf{x})}{|\mathbf{x} - \mathbf{y}|^{\Delta_a + \Delta_b - \Delta_c}} + \dots, \quad (3.4)$$

where the dots denote less singular terms as $\mathbf{x} \rightarrow \mathbf{y}$.

Following ref. [56], we now study the RG flow for the theory with action (3.1). The generating function for the QFT is given by

$$Z_{\text{QFT}} = \int D\chi \exp \left(- S_{\text{CFT}} - \int d^d \mathbf{x} u^a O_a(\mathbf{x}) \right). \quad (3.5)$$

Let us introduce a UV cutoff μ_0 . In the regulated theory the correlators are given by

$$\begin{aligned} & \langle O_{a_1}(\mathbf{x}_1) \cdots O_{a_n}(\mathbf{x}_n) \rangle_{\mu_0} \\ &= \frac{1}{Z_{\text{QFT}}} \int D\chi O_{a_1}(\mathbf{x}_1) \cdots O_{a_n}(\mathbf{x}_n) \exp \left(- S_{\text{CFT}} - \int d^d \mathbf{x} u_0^a O_a(\mathbf{x}) \right), \end{aligned} \quad (3.6)$$

³The dual QFT takes a similar form also near the IR fixed point.

where we denote the coupling constants u^a at $\mu = \mu_0$ by

$$u_0^a \equiv u^a(\mu_0), \quad (3.7)$$

and we consider all points \mathbf{x}_i ($i = 1, \dots, n$) separated from each other by distance more than $1/\mu_0$. We can now express the correlation functions of the QFT in terms of CFT correlators with the cutoff by expanding the exponential factor in the generating functional as

$$\begin{aligned} & e^{-S_{\text{CFT}} - \int d^d \mathbf{x} u_0^a O_a(\mathbf{x})} \\ &= \left[1 - \int d^d \mathbf{x} u_0^a O_a(\mathbf{x}) + \frac{1}{2} \int d^d \mathbf{x} \int_{|\mathbf{x}-\mathbf{y}| > 1/\mu_0} d^d \mathbf{y} u_0^a u_0^b O_a(\mathbf{x}) O_b(\mathbf{y}) + \dots \right] e^{-S_{\text{CFT}}}. \end{aligned} \quad (3.8)$$

Let us now integrate out the modes between μ and μ_0 . Using the OPE (3.4) in the third term of eq. (3.8) we obtain

$$\begin{aligned} & \frac{1}{2} \int d^d \mathbf{x} \int_{1/\mu_0 < |\mathbf{x}-\mathbf{y}| < 1/\mu} d^d \mathbf{y} u_0^a u_0^b O_a(\mathbf{x}) O_b(\mathbf{y}) \\ &= \frac{1}{2} u_0^b u_0^c \delta^{ad} \frac{C_{bcd}}{c_a} \int d^d \mathbf{x} O_a(\mathbf{x}) \int_{1/\mu_0 < |\mathbf{x}-\mathbf{y}| < 1/\mu} d^d \mathbf{y} \frac{1}{|\mathbf{x}-\mathbf{y}|^{\Delta_b(\mu_0) + \Delta_c(\mu_0) - \Delta_a(\mu_0)}} + \dots \\ &= -\frac{1}{2} u_0^b u_0^c \tilde{C}_{bc}^a \frac{\mu^{\lambda_a - \lambda_b - \lambda_c} - \mu_0^{\lambda_a - \lambda_b - \lambda_c}}{\lambda_a - \lambda_b - \lambda_c} \int d^d \mathbf{x} O_a(\mathbf{x}) + \dots, \end{aligned} \quad (3.9)$$

where

$$\lambda_a \equiv d - \Delta_a(\mu_0). \quad (3.10)$$

Here we introduced

$$\tilde{C}_{cb}^a \equiv \text{Vol}(S^{d-1}) \delta^{ad} \frac{C_{bcd}}{c_a} = \frac{2\pi^{d/2}}{\Gamma(d/2)} \delta^{ad} \frac{C_{bcd}}{c_a}, \quad (3.11)$$

where $\text{Vol}(S^{d-1})$ is the volume of the round $(d-1)$ -dimensional sphere. The integration of the UV modes gives rise to the running of the coupling constant u^a as

$$u^a(\mu) = u_0^a + \frac{1}{2} u_0^b u_0^c \tilde{C}_{bc}^a \frac{\mu^{\lambda_a - \lambda_b - \lambda_c} - \mu_0^{\lambda_a - \lambda_b - \lambda_c}}{\lambda_a - \lambda_b - \lambda_c} + \mathcal{O}(u_0^3). \quad (3.12)$$

In the second and third lines of eq. (3.9), we included only the terms which contribute to the running of the coupling constants u^a .

Let us now introduce the dimensionless coupling constants g^a as

$$g^a(\mu) \equiv \mu^{-\lambda_a} u^a(\mu). \quad (3.13)$$

By using eq. (3.12), the running of $g^a(\mu)$ is given by

$$g^a(\mu) = \left(\frac{\mu}{\mu_0} \right)^{-\lambda_a} g_0^a + \frac{1}{2} g_0^b g_0^c \tilde{C}_{bc}^a \frac{\left(\frac{\mu}{\mu_0} \right)^{-(\lambda_b + \lambda_c)} - \left(\frac{\mu}{\mu_0} \right)^{-\lambda_a}}{\lambda_a - \lambda_b - \lambda_c} + \mathcal{O}(g_0^3), \quad (3.14)$$

where $g_0^a \equiv \mu_0^{-\lambda_a} u^a(\mu_0)$. The beta function

$$\beta^a(\mu) \equiv \frac{dg^a(\mu)}{d \ln \mu} \quad (3.15)$$

can now be computed by inserting eq. (3.14) into eq. (3.15), yielding

$$\beta^a(\mu) = -\lambda_a g^a(\mu) + \frac{1}{2} g^b(\mu) g^c(\mu) \tilde{C}_{bc}^a + \mathcal{O}(g^3) \quad (3.16)$$

(there is no sum over a in the first term and there are implicit sums over b and c in the second term). The second term stems from the new short distance singularities due to the composite operators and this leads to the deviation from the classical scaling. This analysis is valid as long as g^a is small and hence we can solve the RG flow perturbatively. Note that the beta function does not depend on the UV cutoff μ_0 explicitly, so we can send it to the infinity.

3.2 Separable case

Having obtained the beta functions one may now investigate the dynamics along the RG flow and possible IR fixed points. For cases where g^a are small throughout the RG flow from the UV to the IR fixed point one may use conformal perturbation theory to compute the correlators and from those extract the cosmological prediction. The dynamics in such cases is essentially controlled by the form of the OPE. Leaving a general study for future work, in this paper we focus on the simplest possibility: the A operators do not couple to each other at leading order, i.e. three point functions involving more than one type of operator are zero. This implies that the structure constants C_{abc} and \tilde{C}_{bc}^a satisfy

$$C_{abc} = C_a \delta_{ab} \delta_{bc}, \quad \tilde{C}_{bc}^a = \tilde{C}_a \delta^{ac} \delta_{bc}, \quad (3.17)$$

where $\tilde{C}_a = (2\pi^{d/2}/\Gamma(d/2))(C_a/c_a)$ (no sum over a). In this case, the RG equation becomes separable as

$$\beta^a(\mu) = -\lambda_a g^a(\mu) + \frac{1}{2} \tilde{C}_a (g^a(\mu))^2 + \mathcal{O}(g^3). \quad (3.18)$$

We essentially have A decoupled copies of the case of a single deforming operator. One may thus directly borrow the results obtained for this case [47, 48, 56].

Assuming $g^a > 0$ and $\tilde{C}_a > 0$ we find that there is an IR fixed point at

$$g_*^a = \frac{2\lambda_a}{\tilde{C}_a}. \quad (3.19)$$

In this case, eq. (3.18) can be solved as

$$g^a(\mu) = \frac{2g^a(\mu_a)}{1 + (\mu/\mu_a)^{\lambda_a}}, \quad g^a(\mu_a) \equiv \frac{\lambda_a}{\tilde{C}_a}, \quad (3.20)$$

where a pivot scale μ_a is introduced as an integration constant. For the coupling to remain small throughout the RG flow we need $\lambda_a/\tilde{C}_a \ll 1$. In the UV, $\mu \gg \mu_a$, $g^a(\mu) \rightarrow 0$, while in the IR, $\mu \ll \mu_a$, $g^a(\mu) \rightarrow g_*^a = 2g^a(\mu_a)$.

4 RG flow and cosmological evolution

In the previous section, we derived the RG equation for the coupling constant g^a . In this section, we discuss the bulk evolution of the corresponding bulk scalar fields ϕ^a .

4.1 Superpotential

We consider gravity coupled to A scalar fields governed by the action

$$S = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{-g} [R - (\partial\phi^a)^2 - 2\kappa^2 V(\phi^a)] . \quad (4.1)$$

The scalar field equations in a $(d+1)$ -dimensional FRW universe are given by

$$\ddot{\phi}^a + dH\dot{\phi}^a + \kappa^2 \frac{\partial V(\phi)}{\partial \phi^a} = 0 , \quad (4.2)$$

where H is the Hubble parameter, which satisfies

$$H^2 = \frac{1}{d(d-1)} \left[\sum_{a=1}^A (\dot{\phi}^a)^2 + 2\kappa^2 V(\phi) \right] , \quad (4.3)$$

where $\kappa^2 \equiv 8\pi G$ is the gravitational constant. Here, ϕ denotes the set of variables ϕ^a ($a = 1, \dots, A$).

One may trade (under some assumptions) the second order field equations (4.2)–(4.3) by first order equation:

$$\dot{\phi}^a = \frac{\partial W(\phi)}{\partial \phi^a} , \quad (4.4)$$

$$H = -\frac{1}{d-1} W(\phi) , \quad (4.5)$$

where $W(\phi)$ is the so-called superpotential, which is related to the potential $V(\phi)$ by

$$2\kappa^2 V(\phi) = \frac{d}{d-1} W^2(\phi) - \sum_{a=1}^A \left(\frac{\partial W(\phi)}{\partial \phi^a} \right)^2 . \quad (4.6)$$

This reformulation was first obtained in cosmology using Hamilton-Jacobi theory in ref. [57]. It was rediscovered in gauge/gravity duality in refs. [40, 58–60] where it was linked with (fake) supersymmetry and gravitational stability.

One can verify that any solution of the first order equations also solves the second order equations (4.2) and eq. (4.3). Indeed taking the partial derivative of eq. (4.6) with respect to ϕ^a we obtain

$$\kappa^2 \frac{\partial V(\phi)}{\partial \phi^a} = \frac{d}{d-1} W(\phi) \frac{\partial W(\phi)}{\partial \phi^a} - \sum_{a=1}^A \sum_{b=1}^A \frac{\partial W(\phi)}{\partial \phi^b} \frac{\partial^2 W(\phi)}{\partial \phi^a \partial \phi^b} .$$

Using eqs. (4.4) and (4.5) we find that the first term of the right hand side is $-dH\dot{\phi}^a$ and the second term is $-\dot{\phi}^a$. Meanwhile, eq. (4.6) with eq. (4.5) is nothing but the Friedmann equation (4.3). The converse (i.e. going from eqs. (4.2)–(4.3) to eqs. (4.4)–(4.5)–(4.6)) is also true but under some additional assumptions, see ref. [40].

4.2 Potential for the solvable RG flow

We would now like to link the field theory discussion with bulk dynamics. The specific example we are considering is A decoupled copies of the single field case studied in refs. [25, 47, 48, 50] so one may borrow many of the results we need for our case. The properties of the potential near the critical point are fixed from those of the CFT. In particular, the spectrum of operators and the OPE coefficients fix the form of the potential through cubic order in the fields [47]. We actually know the QFT along the entire RG flow as an expansion in the λ_a . This fixes the superpotential (and thus the potential) as an expansion in λ_a . It was shown in ref. [48] that in the single field case the superpotential in $d = 3$ (and in our notation) is given by

$$W(\phi) = -2 \exp \left(-\frac{1}{2} \int_0^\phi dg \beta(g) \right) + \mathcal{O}(\lambda^7). \quad (4.7)$$

This is equivalent to saying that there is a QFT scheme such that [48]

$$-2W'(\phi)/W(\phi) = \beta(\phi) + \mathcal{O}(\lambda^6). \quad (4.8)$$

One should compare this expression with the proposal [61] that the QFT beta function is given exactly by (now for general d and many couplings)

$$\beta^a = -(d-1) \frac{\partial}{\partial \phi^a} \ln W(\phi^a). \quad (4.9)$$

This scheme was also the choice used in early works on holographic cosmology [10–12]. One should note however that in cases where the RG flow is driven by supersymmetric operators, eq. (4.9) is not the correct QFT beta function [62, 63], so one should use eq. (4.9) with caution.

Standard holographic methods compute renormalised correlators [64]. On the cosmology side we correspondingly obtain the cosmological observables at the end of the inflationary period ($t \rightarrow \infty$). Here however we would like to study one of the main properties that distinguish single vs multi-scalar inflation, namely whether the adiabatic curvature perturbation is conserved outside the horizon or not. To do this we need to understand the time dependence of the cosmological perturbations. Following general expectations, we assume that the coupling constant g^a is related to the scalar field ϕ^a as

$$g^a(\mu, \mathbf{x}) = \phi^a(t(\mu), \mathbf{x}), \quad (4.10)$$

and the renormalization scale is related to the scale factor by

$$\frac{\mu}{\mu_0} = \frac{a}{a_0}. \quad (4.11)$$

(See also refs. [47, 50, 65, 66].) With this choice, the power spectrum of the curvature perturbation ζ is conserved for single field model [25]. One should note however that this prescription does not produce conserved bispectrum. This may be related with the well-known difficulties with realising Wilsonian RG using holography [67]. In any case, in this paper we only discuss the power spectra and for those the prescription is adequate. As discussed in ref. [50], changing the linear relation (4.10) to a more general (local) function $g^a[\phi]$ can be understood simply as a field redefinition.

The scheme (4.10)–(4.11) implies (4.9). Indeed, using eqs. (4.10) and (4.11), we obtain

$$\beta^a = \frac{\dot{\phi}^a}{H} = -(d-1) \frac{\partial}{\partial \phi^a} \ln W(\phi). \quad (4.12)$$

Inserting this expression into eq. (4.6), we can compute the potential as

$$2\kappa^2 V(\phi) = \left(\frac{W(\phi)}{d-1} \right)^2 \left[d(d-1) - \sum_{a=1}^A (\beta^a)^2 \right]. \quad (4.13)$$

In our example in section 3.2, the RG equation for g^a is separable, see eq. (3.18). In this case, the corresponding superpotential takes the form

$$W(\phi) = W_0 \prod_{a=1}^A W_a(\phi^a), \quad (4.14)$$

where W_a is a function of ϕ^a only. It follows from eq. (4.9)

$$W_a(\phi^a) = \exp \left[-\frac{1}{d-1} \int dg^a \beta^a(g^a) \right], \quad (4.15)$$

where W_0 in eq. (4.14) is a constant that originates from the integration constants when we integrate the beta functions in eq. (4.15). Using eqs. (4.5), (4.13), (4.14) and (4.15), we obtain the Hubble parameter $H(\phi)$ as

$$H(\phi) = -\frac{W_0}{d-1} \prod_{a=1}^A \exp \left[-\frac{1}{2(d-1)} \left(-\lambda_a (\phi^a)^2 + \frac{\tilde{C}_a}{3} (\phi^a)^3 + \mathcal{O}((\phi^a)^4) \right) \right], \quad (4.16)$$

which implies that W_0 determines the de Sitter radius of the dS critical point at $\phi^a = 0$. The potential $V(\phi)$ is now equal to

$$V(\phi) = \frac{H^2(\phi)}{2\kappa^2} \left[d(d-1) - \sum_{a=1}^A \left(-\lambda_a \phi^a + \frac{\tilde{C}_a}{2} (\phi^a)^2 + \mathcal{O}((\phi^a)^3) \right)^2 \right]. \quad (4.17)$$

While the beta functions β^a are in the separable form for each a , the Klein-Gordon equation for ϕ^a is not separable because $\partial V(\phi)/\partial \phi^a$ also contains ϕ^b with $b \neq a$.

To get an intuition about the potential, we show in figure 1 the contour plot of the potential for the two-field case, $V(\phi_1, \phi_2)$, with $\lambda_1 = 0.02$ and $\lambda_2 = 0.05$. Brighter regions denote larger values of the potential.

5 Primordial perturbations

In this section we introduce adiabatic and entropy perturbations and we derive the relation between $W_{ab}^{(2)}(\mathbf{x}_1, \mathbf{x}_2)$ and the two point functions of the boundary operators.

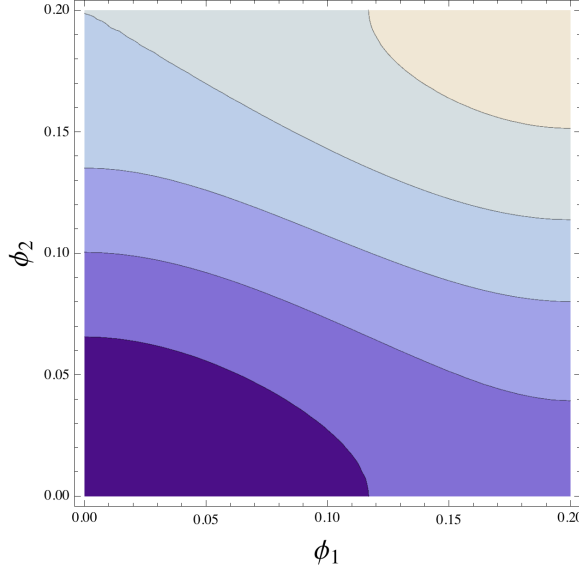


Figure 1. Contour plot of the potential for the two-field case with $\lambda_1 = 0.02$, $\tilde{C}_1 = 0.2$, $\lambda_2 = 0.05$, $\tilde{C}_2 = 0.5$, $d = 3$. In this case, ϕ_1 and ϕ_2 vanish in the UV fixed point and they take the same value 0.2 in the IR fixed point. The brighter region shows the range of ϕ_1 and ϕ_2 with larger values of $V(\phi_1, \phi_2)$.

5.1 Boundary QFT

In this section, we further study the case of A deformation operators,

$$S_{\text{QFT}}[g^a, \chi] = S_{\text{CFT}} + \sum_{a=1}^A \int d\Omega g^a(\mu, \mathbf{x}) \mathcal{O}_a(\mu, \mathbf{x}), \quad (5.1)$$

where we introduced

$$g^a(\mu, \mathbf{x}) \equiv \mu^{-\lambda_a} u^a(\mu, \mathbf{x}), \quad (5.2)$$

$$\mathcal{O}_a(\mu, \mathbf{x}) \equiv \mu^{\lambda_a} O_a(\mu, \mathbf{x}). \quad (5.3)$$

We consider couplings $g^a(\mu, \mathbf{x})$ of the form

$$g^a(\mu, \mathbf{x}) = g^a(\mu) + \delta g^a(\mu, \mathbf{x}), \quad (5.4)$$

where $g^a(\mu)$ is the background value, corresponding to the FRW as described in the previous section. In the following only the background value of $g^a(\mu)$ and the fluctuation $\delta g^a(\mu, \mathbf{x})$ appear, so to simplify the notation we will suppress the arguments and (with abuse of notation) will denote the background value by g^a .

5.2 Adiabatic and entropy perturbations in cosmological perturbation theory

We decompose the fluctuations of the scalar field $\delta\phi^a(t, \mathbf{x})$ into a fluctuation $\delta\sigma$ tangent to the background trajectory (adiabatic perturbations) and fluctuations δs^a transverse to the trajectory (entropy perturbations) as [51]

$$\delta\sigma(t, \mathbf{x}) \equiv e_a(t) \delta\phi^a(t, \mathbf{x}), \quad (5.5)$$

$$s^a(t, \mathbf{x}) \equiv P_b^a(t) \delta\phi^b(t, \mathbf{x}), \quad (5.6)$$

where $e_a(t)$ is the unit vector defined by

$$e^a \equiv \frac{\dot{\phi}^a}{\dot{\sigma}}, \quad \dot{\sigma} = \sqrt{\dot{\phi}^a \dot{\phi}_a}, \quad (5.7)$$

and $P_{ab}(t)$ is the projection tensor defined by

$$P_{ab} \equiv \delta_{ab} - e_a e_b. \quad (5.8)$$

Note that since $e_a s^a = 0$, there are only $A - 1$ independent entropy perturbations.

It follows from the definition of $\delta\sigma$ that under a change of the time coordinate, $t \rightarrow \tilde{t} = t + \delta t$, and to linear order in perturbation it transforms as

$$\delta\sigma \rightarrow \delta\tilde{\sigma} = \delta\sigma - \dot{\sigma}\delta t. \quad (5.9)$$

One may construct a corresponding gauge invariant variable by

$$\zeta = \mathcal{R} - \frac{H}{\dot{\sigma}}\delta\sigma, \quad (5.10)$$

where \mathcal{R} is the curvature perturbation. Then ζ can be understood either as the curvature perturbation in the gauge $\delta\sigma = 0$ or as being related to $\delta\sigma$ in the flat gauge, $\mathcal{R} = 0$,

$$\zeta = -\frac{H}{\dot{\sigma}}\delta\sigma_f, \quad (5.11)$$

where

$$\delta\sigma_f(\mu, \mathbf{x}) \equiv \delta\sigma(\mu, \mathbf{x})|_{\mathcal{R}=0}. \quad (5.12)$$

One may also consider a similar construction with $\delta\sigma$ replaced by $\delta\phi^a$. Then the corresponding gauge invariant variable in the flat gauge, $\mathcal{R} = 0$, is given by

$$\zeta^a \equiv -\frac{H}{\dot{\phi}^a}\delta\phi_f^a. \quad (5.13)$$

Using ζ^a we can express ζ and s^a as

$$\zeta = \sum_{a=1}^A \left(\frac{\dot{\phi}^a}{\dot{\sigma}} \right)^2 \zeta^a, \quad (5.14)$$

$$s^a = -\left(\delta_b^a - \frac{\dot{\phi}^a \dot{\phi}_b}{\dot{\sigma}^2} \right) \frac{\dot{\phi}^b}{H} \zeta^b. \quad (5.15)$$

When the background trajectory is predominantly determined by one of the A scalar fields, $\dot{\phi}^a$, satisfying $|\dot{\phi}^a| \gg |\dot{\phi}^b|$ for $b \neq a$, the adiabatic curvature perturbation ζ approximately agrees with ζ^a .

5.3 Adiabatic and entropy perturbations from holography

We will now adapt the action (5.1) to reflect the split into adiabatic and entropy perturbations. Using $\beta^a = \dot{\phi}^a/H$ we obtain,

$$e^a = \frac{\beta^a}{\beta}, \quad \beta \equiv \sqrt{\beta^a \beta_a} = \frac{\dot{\sigma}}{H}. \quad (5.16)$$

Introducing

$$\mathcal{O}_\sigma(\mu, \mathbf{x}) \equiv e^a \mathcal{O}_a(\mu, \mathbf{x}), \quad (5.17)$$

$$\mathcal{O}_{s,a}(\mu, \mathbf{x}) \equiv P_a^b \mathcal{O}_b(\mu, \mathbf{x}), \quad (5.18)$$

we may re-express the deformation term as (repeated indices indicate summation)

$$\begin{aligned} g^a(\mu, \mathbf{x}) \mathcal{O}_a(\mu, \mathbf{x}) &= g^a(\mu) \mathcal{O}_a(\mu, \mathbf{x}) + \delta\sigma(\mu, \mathbf{x}) \mathcal{O}_\sigma(\mu, \mathbf{x}) + s^a(\mu, \mathbf{x}) \mathcal{O}_{s,a}(\mu, \mathbf{x}), \\ &= g^a(\mu) \mathcal{O}_a(\mu, \mathbf{x}) + \delta\sigma(\mu, \mathbf{x}) \mathcal{O}_\sigma(\mu, \mathbf{x}) + s^{a'}(\mu, \mathbf{x}) \hat{\mathcal{O}}_{a'}(\mu, \mathbf{x}), \end{aligned} \quad (5.19)$$

where $a' = 2, \dots, A$. Here, we eliminated s^1 using $e_a s^a = 0$ and

$$\hat{\mathcal{O}}_{a'}(\mu, \mathbf{x}) \equiv \mathcal{O}_{s,a'}(\mu, \mathbf{x}) - \frac{e_{a'}}{e_1} \mathcal{O}_{s,1}(\mu, \mathbf{x}) = \sum_{b=1}^A \left(\delta_{a'}^b - \frac{e_{a'}}{e_1} \delta_1^b \right) \mathcal{O}_b(\mu, \mathbf{x}). \quad (5.20)$$

We have thus traded the original operators \mathcal{O}_a for operators \mathcal{O}_σ and $\hat{\mathcal{O}}_{a'}$ that couple directly to the adiabatic perturbation $\delta\sigma$ and the $A - 1$ entropy perturbations $s^{a'}$,

$$S_{\text{QFT}}[\chi, g^a; \delta\sigma_f, s^{a'}] = S_{\text{QFT}}[\chi, g^a] + \int d\Omega (\delta\sigma_f(\mu, \mathbf{x}) \mathcal{O}_\sigma(\mu, \mathbf{x}) + s^{a'}(\mu, \mathbf{x}) \hat{\mathcal{O}}_{a'}(\mu, \mathbf{x})), \quad (5.21)$$

where $S_{\text{QFT}}[\chi, g^a]$ depends only on the background $g^a(\mu)$. Since the QFT lives in flat space we replaced $\delta\sigma$ by $\delta\sigma_f$ to express the action in terms of gauge invariant sources (see (5.12)).

5.4 Power spectra

We are now ready to obtain the relation between the bulk 2-point functions of the adiabatic and entropy perturbations and 2-point functions of the dual QFT.

Using eq. (5.11) we may express ζ derivatives in terms of σ_f ones:

$$\frac{\delta W_{\text{QFT}}}{\delta \zeta(\mathbf{x})} = -\beta(\mu) \frac{\delta W_{\text{QFT}}}{\delta \sigma_f(\mathbf{x})}. \quad (5.22)$$

Taking further derivatives and using eq. (2.17) and the results in the previous subsection we get

$$\begin{aligned} W_{11}^{(2)}(\mathbf{x}_1, \mathbf{x}_2) &= \frac{\delta^2 W_{\text{QFT}}}{\delta \zeta(\mathbf{x}_1) \delta \zeta(\mathbf{x}_2)} = \beta^2(\mu) \frac{\delta^2 W_{\text{QFT}}}{\delta \sigma_f(\mathbf{x}_1) \delta \sigma_f(\mathbf{x}_2)} \\ &= 2\beta^a(\mu) \beta^b(\mu) \text{Re} [\langle \mathcal{O}_a(\mathbf{x}_1) \mathcal{O}_b(\mathbf{x}_2) \rangle_\mu], \end{aligned} \quad (5.23)$$

$$\begin{aligned} W_{1a'}^{(2)}(\mathbf{x}_1, \mathbf{x}_2) &= \frac{\delta^2 W_{\text{QFT}}}{\delta \zeta(\mathbf{x}_1) \delta s^{a'}(\mathbf{x}_2)} = -\beta(\mu) \frac{\delta^2 W_{\text{QFT}}}{\delta \sigma_f(\mathbf{x}_1) \delta s^{a'}(\mathbf{x}_2)} \\ &= -2\beta^a(\mu) \left(\delta_{a'}^b - \frac{\beta_{a'}}{\beta_1} \delta_1^b \right) \text{Re} [\langle \mathcal{O}_a(\mathbf{x}_1) \mathcal{O}_b(\mathbf{x}_2) \rangle_\mu], \end{aligned} \quad (5.24)$$

$$\begin{aligned} W_{a'b'}^{(2)}(\mathbf{x}_1, \mathbf{x}_2) &= \frac{\delta^2 W_{\text{QFT}}}{\delta s^{a'}(\mathbf{x}_1) \delta s^{b'}(\mathbf{x}_2)} \\ &= 2 \left(\delta_{a'}^a - \frac{\beta_{a'}}{\beta_1} \delta_1^a \right) \left(\delta_{b'}^b - \frac{\beta_{b'}}{\beta_1} \delta_1^b \right) \text{Re} [\langle \mathcal{O}_a(\mathbf{x}_1) \mathcal{O}_b(\mathbf{x}_2) \rangle_\mu]. \end{aligned} \quad (5.25)$$

Thus the two-point function of \mathcal{O}_{Φ^a} , which is dual to Φ^a , is now expressed by the two-point function of \mathcal{O}_a . An additional minus sign is introduced by taking into account the relative minus sign between the two-point functions in QFT₊ and QFT₋.

The (equal time) 2-point function of ζ and $s^{a'}$ can be obtained from the inverse matrix of $W_{ab}^{(2)}(\mathbf{x}_1, \mathbf{x}_2)$, see eq. (2.15). In Fourier space and restricted to $d = 3$ from now on

$$\langle \Phi_a(\mathbf{k}_1) \Phi_b(\mathbf{k}_2) \rangle_{\text{conn}} = (2\pi)^3 \frac{2\pi^2}{k_1^3} \mathcal{P}_{ab}(k_1) \delta(\mathbf{k}_1 + \mathbf{k}_2) \quad (5.26)$$

where

$$\hat{W}_{ac}^{(2)-1}(k) \hat{W}_{cb}^{(2)}(k) = \delta_{ab}, \quad (5.27)$$

and

$$W_{ab}^{(2)}(\mathbf{x}_1, \mathbf{x}_2) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k} \cdot (\mathbf{x}_1 - \mathbf{x}_2)} \hat{W}_{ab}^{(2)}(k). \quad (5.28)$$

The scalar power spectra are now given by

$$\mathcal{P}_{ab}(k) = \frac{k^3}{2\pi^2} \hat{W}_{ab}^{(2)-1}(k). \quad (5.29)$$

Once the beta function β^a and the correlators of \mathcal{O}_a are derived by solving the RG flow, these formulae may be used to compute the power spectra for the adiabatic and entropy perturbations.

6 Power spectra for the separable example

In this section we explicitly compute the power spectra for the separable example.

6.1 Formulae for the separable case

The beta functions $\beta^a(\mu)$ may be readily computed from eq. (3.20),

$$\beta^a(\mu) = \frac{4(\mu/\mu_a)^{\lambda_a}}{[1 + (\mu/\mu_a)^{\lambda_a}]^2} \beta^a(\mu_a), \quad \beta^a(\mu_a) = -\frac{\lambda_a^2}{2\tilde{C}_a}. \quad (6.1)$$

The two-point function of the operators \mathcal{O}_a may be computed using the same steps as in the single field case [47, 48, 50] leading to

$$\begin{aligned} \langle \mathcal{O}_a(\mathbf{x}_1) \mathcal{O}_b(\mathbf{x}_2) \rangle_\mu &= \delta_{ab} \left[1 + \left(\frac{\mu}{\mu_a} \right)^{-\lambda_a} \right]^4 [1 + (\mu_a r)^{\lambda_a}]^{-4} \frac{c_a \mu^{2\lambda_a}}{r^{2(3-\lambda_a)}} \\ &= \delta_{ab} \left[\frac{\beta^a(\mu_a)}{\beta^a(\mu)} \right]^2 \mathcal{F}_a(r), \end{aligned} \quad (6.2)$$

where we introduced

$$\mathcal{F}_a(r) \equiv \frac{16c_a \mu_a^{2\lambda_a}}{r^{2(3-\lambda_a)} [1 + (\mu_a r)^{\lambda_a}]^4}. \quad (6.3)$$

Here, r denotes $r \equiv |\mathbf{x}_1 - \mathbf{x}_2|$. In obtaining this result we fixed a number of integration constants by requiring the correlators behave appropriately in the limit $\lambda_a \rightarrow 0$, see the discussion in section 2.2 of ref. [47].

6.2 Power spectra for the adiabatic and entropy perturbations

Inserting eqs. (6.1) and (6.2) into eqs. (5.23)–(5.25) we may now compute $W_{ab}^{(2)}(x_1, x_2)$. Then, computing the inverse matrix $\hat{W}_{ab}^{(2)-1}(k)$ leads to the power spectra for the adiabatic and entropy perturbations for the separable RG flow. We relegate this computation to appendix B. Since the RG equations are separable the power spectra for the adiabatic and entropy perturbations may be derived more easily by using the power spectra of ζ^a .

Repeating a similar computation to the one for the correlator of ζ , we obtain the correlator of $\zeta^a(\mathbf{x})$ as

$$\langle \zeta^a(\mathbf{x}_1) \zeta^b(\mathbf{x}_2) \rangle = W_{\zeta^a \zeta^b}^{(2)-1}(\mathbf{x}_1, \mathbf{x}_2), \quad (6.4)$$

with

$$W_{\zeta^a \zeta^b}^{(2)}(\mathbf{x}_1, \mathbf{x}_2) = 2 \operatorname{Re} [\beta^a(\mu) \beta^b(\mu) \langle \mathcal{O}_a(\mathbf{x}_1) \mathcal{O}_b(\mathbf{x}_2) \rangle_\mu]. \quad (6.5)$$

Since the correlators of \mathcal{O}_a are diagonal (see (6.2)), one may readily compute the inverse matrix $W_{\zeta^a \zeta^b}^{(2)-1}(\mathbf{x}_1, \mathbf{x}_2)$ to obtain the power spectrum of ζ^a ,⁴

$$\mathcal{P}_{\zeta^a \zeta^a}(k) \simeq \frac{3}{16\pi^4} \frac{1}{(\beta^a(\mu_a))^2 c_a} \left(\frac{k}{\mu_a} \right)^{-2\lambda_a} \left[1 + \left(\frac{k}{\mu_a} \right)^{\lambda_a} \right]^4 (1 + \mathcal{O}(\lambda_a)). \quad (6.6)$$

In the single field case this is the scalar power spectrum. Note that the μ dependence cancels out between the explicit $\beta^a(\mu)$ factors and the $1/(\beta^a(\mu))^2$ factor in (6.2). Thus the power spectrum in the single field case is indeed conserved. One may check that this agrees exactly with corresponding result in cosmological perturbation theory provided one uses the standard AdS/CFT normalisation for the operator (i.e. set $c_a = 12/(\pi^2 \kappa^2)$) [47].

We may now compute the auto-correlation of ζ using eq. (5.14):

$$\mathcal{P}_{11}(k) = \sum_{a=1}^A \left(\frac{\beta^a(\mu)}{\beta(\mu)} \right)^4 \mathcal{P}_{\zeta^a \zeta^a}(k). \quad (6.7)$$

Similarly, using eq. (5.15), we may obtain the cross-correlation between ζ and $s^{a'}$ and the auto-correlation of $s^{a'}$. For instance, the auto-correlation for $s^{a'}$ is given by

$$\mathcal{P}_{a'a'}(k) = (\beta^{a'}(\mu))^2 \left[\left\{ 1 - \left(\frac{\beta^{a'}(\mu)}{\beta(\mu)} \right)^2 \right\}^2 \mathcal{P}_{\zeta^{a'} \zeta^{a'}}(k) + \sum_{c \neq a'} \left(\frac{\beta^c(\mu)}{\beta(\mu)} \right)^4 \mathcal{P}_{\zeta^c \zeta^c}(k) \right]. \quad (6.8)$$

These results agree with the ones obtained by directly inverting $W_{ab}^{(2)}(k)$, see appendix B, as it should be.

We now come to our main point: the power spectrum (6.7) depends on μ and thus depends on time in cosmology. One should contrast this with the case of a deformation by

⁴To compute the Fourier transform of (6.5) we expand the r -dependent part of $\mathcal{F}_a(r)$ as

$$r^{-2(3-\lambda_a)} [1 + (\mu_a r)^{\lambda_a}]^{-4} = \sum_{m=0}^{\infty} \frac{(m+3)!}{3! m!} (-\mu_a)^{\lambda_a} r^{-6+\lambda_a(m+2)},$$

and use

$$\int d^3 \mathbf{x} r^{-6+(m+2)\lambda_a} e^{-i\mathbf{k} \cdot \mathbf{x}} = \frac{\pi^2}{12} k^{3-(m+2)\lambda_a} (1 + \mathcal{O}(\lambda_a)).$$

a single operator where (as we mentioned above) the power spectrum is μ independent [50]. This is consistent with the results obtained by standard cosmological perturbation theory which show a time variation of ζ at large scales in the presence of the entropy perturbation [51]. The δN formalism [57, 68–71] provides a powerful way to compute the primordial curvature perturbation ζ for multi-field models. Using this formalism, the curvature perturbation ζ has been computed in ref. [72] for the cosmology dual to the RG flow (3.18) and the power spectrum of ζ has indeed been found to agree exactly with eq. (6.7).

When λ_1 is smaller than $\lambda_{a'}$ with $a' \neq 1$, $|\beta^1(\mu)|$ becomes much bigger than $|\beta^{a'}(\mu)|$ in the UV. Then, the curvature perturbation ζ agrees with ζ^1 and the power spectrum $\mathcal{P}_{11}(k)$ is approximately given by the constant value,

$$\mathcal{P}_{\zeta\zeta}(k) \simeq \mathcal{P}_{\zeta^1\zeta^1}(k).$$

By contrast, the power spectrum $\mathcal{P}_{a'a'}(k)$ still depends on μ in this limit and vanishes as

$$\mathcal{P}_{a'a'}(k) \simeq (\beta^{a'}(\mu))^2 [\mathcal{P}_{\zeta^1\zeta^1}(k) + \mathcal{P}_{\zeta^{a'}\zeta^{a'}}(k)], \quad \mu \rightarrow \infty.$$

Notice that in the case all $\beta^a(\mu)$ are proportional to each other, the power spectrum of ζ is conserved. This is because the evolution of ϕ^a can be described by a single degree of freedom after an appropriate rotation in the field space.

As in the single field case [47], the power spectrum is red-tilted, where the spectral index for ζ^a is given by $n_{s,a} - 1 = -2\lambda_a$ in the long wavelength limit, $k \ll \mu_a$, while it is blue-tilted, $n_{s,a} - 1 = 2\lambda_a$ in the opposite limit, $k \gg \mu_a$. This is due to the fact that the potential given in eq. (4.17) has negative curvature near the IR fixed point (which corresponds to the early times in cosmology) while it has positive curvature near the UV fixed point (which corresponds to the late times in cosmology).

7 Concluding remarks

In this paper we initiated the study of multi-field inflation using holography. We studied a class of models where the Universe evolves from one de Sitter phase to another. Cosmological evolution maps to inverse RG flow in holography and thus the dual quantum field theory for this class of models has two fixed points. Multi-field inflation corresponds to an RG flow driven by a multitude of nearly marginal operators. In this paper we analysed a special case: the deformation operators do not couple to each at leading order, i.e. there are no non-zero three-point functions that involve more than one type of operator. Under this condition one can compute the relevant QFT correlators exactly in the same way as in the corresponding single field models.

We explicitly computed holographically the power spectra of the adiabatic curvature and entropy perturbations for the case of several deformation operators and recovered one of the hallmarks of multi-field inflation: the curvature perturbation ζ is not conserved in the presence of entropy perturbations. The curvature perturbation is conserved when the entropy perturbations are set to zero or when the model can be described by a single field after a rotation in field space. Furthermore, in the UV (= late time) limit the power spectrum approaches a constant value and it is solely determined by the operator which is closest to be marginal (i.e. has the minimum value of λ_a).

In this paper, we computed only the power spectra. It would be interesting to extend the analysis to higher point functions. However, as pointed out in ref. [50], already the single

field case is subtle. The curvature perturbation ζ in such models should be conserved at large scales as a consequence of general covariance but the computation of the bispectrum in [50] yielded a result that depended on μ . One would need to first resolve this issue before moving to higher point function in the multi-field case.

Here, we focused on the simplest (but very special) QFT model with many nearly marginal operators, namely the case where there is no operator mixing along the RG flow. It would be very interesting to extend the analysis to the generic case. It would also be interesting to present holographic versions of multi-field models that are already discussed in the cosmology literature. We leave such studies for future work.

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A Analytic continuation

We review in this appendix the analytic continuations needed using an example the case of a massless scalar field in four (bulk) dimensions. This example was discussed in detail in ref. [7] so we will simply borrow the results from there. This appendix also serves to illustrate that the analytic continuations in ref. [7] and ref. [39], despite apparent differences, are equivalent.

The renormalised on-shell action for a massless scalar in EAdS is given by

$$\begin{aligned} S_{\text{AdS ren}} &= -\frac{1}{\kappa^2} \int \frac{d^3\mathbf{k}}{2\pi^3} \frac{1}{2} R_{\text{AdS}}^2 k^3 \phi_0(-k) \phi_0(k) \\ &= - \int d^3\mathbf{k} \langle O(k) O(-k) \rangle \phi_0(-k) \phi_0(k), \end{aligned} \quad (\text{A.1})$$

where ϕ_0 is value of ϕ at the conformal boundary of AdS and the source for the operator O . This is obtained by directly evaluating the on-shell action and using holographic renormalisation [64] (see eq. (5.4) of ref. [7]). Here (as in the main text) we adopt the “supergravity normalisation” where there is an overall factor of $1/\kappa^2$ in front of the action. The 2-point function of dual operator is then given by

$$\langle O(k) O(-k) \rangle = -\frac{\delta^2 S_{\text{AdS ren}}}{\delta \phi_0(k) \delta \phi_0(-k)} = \frac{1}{2} \frac{R_{\text{AdS}}^2}{\kappa^2} k^3. \quad (\text{A.2})$$

A direct evaluation of the on-shell action in de Sitter (again normalised with an overall $1/\kappa^2$) yields [7]

$$\text{Re}(iS_{\text{dS}}) = \frac{1}{\kappa^2} \int \frac{d^3\mathbf{k}}{2\pi^3} \frac{1}{2} R_{\text{dS}}^2 k^3 \phi_0(-k) \phi_0(k). \quad (\text{A.3})$$

This is related to (A.1) by

$$R_{\text{AdS}} = -iR_{\text{dS}}. \quad (\text{A.4})$$

Then one can compute the bulk 2-point function using the wave function $\psi_{\text{bulk}}[\phi_0] = \exp(iS_{\text{dS}})$ to obtain

$$\langle \phi(k)\phi(-k) \rangle = -\frac{1}{2\text{Re}\langle O(k)O(-k) \rangle} \Big|_{R_{\text{AdS}}=-iR_{\text{dS}}} = \frac{\kappa^2}{R_{\text{dS}}^2} \frac{1}{k^3}. \quad (\text{A.5})$$

In ref. [39] the bulk in-in correlators were computed using the in-in formalism and compared with the corresponding correlators in the dual QFT. In ref. [39], the AdS and dS radii were set to one and the following holographic formula was obtained

$$\langle \phi(p)\phi(-p) \rangle = -\frac{1}{2\text{Im}\langle O(-ip)O(ip) \rangle} \Big|_{\kappa^2 \rightarrow -\kappa^2} = \kappa^2 \frac{1}{\text{Im}(-ip)^3} = \kappa^2 \frac{1}{p^3}, \quad (\text{A.6})$$

which indeed agrees with eq. (A.5) upon setting $R_{\text{dS}} = 1$ there.

The apparent difference between the two prescriptions is due to fact that the three dimensional correlator in eq. (A.2) is on \mathbb{R}^3 with metric $ds^2 = R_{\text{AdS}}^2 dx_i dx_i$, while that in eq. (A.6) is on $ds^2 = dx_i dx_i$. This implies that $p = R_{\text{AdS}} k$ and under the continuation in eq. (A.4), $p \rightarrow -ip$. This then converts the real part in eq. (A.5) to the imaginary part in eq. (A.6). Once we set $R_{\text{dS}} = 1$ the overall minus in the correlator due to the overall factor of R_{dS}^2 in eq. (A.2) is now accounted for by taking $\kappa^2 \rightarrow -\kappa^2$.

In this example, the entire effect of the analytic continuation is to produce an overall minus sign. In more general cases the action of the analytic continuation is more non-trivial and produces additional signs, see ref. [48] for an example. If one uses the conventions of ref. [7] one must keep explicitly the factors of R_{dS} in order to do correctly the analytic continuation. In the cases we discuss in this paper where we only compute the leading order terms (in the deformation parameters) the effect of the analytic continuation is to only produce an overall sign.

Finally, we note that the analytically continued correlators that enter the holographic formulae of the power spectrum also have a meaning in the original QFT_ without any analytic continuation: they are related with the spectral density associated with the 2-point function of the energy momentum tensor [48].

B Inverting $W_{ab}^{(2)}$

In this appendix, we derive the power spectra for the adiabatic and entropy perturbations by computing the inverse matrix $\hat{W}_{ab}^{(2)-1}(k)$. For simplicity, we consider the two field case where the structure constant C_{abc} satisfies eq. (3.17) and hence the RG equation becomes separable. Inserting eqs. (6.1) and (6.2) into eqs. (5.23)–(5.25), we obtain

$$\hat{W}_{11}^{(2)}(k) = 2 \sum_{a=1}^2 (\beta^a(\mu_a))^2 \hat{\mathcal{F}}_a(k), \quad (\text{B.1})$$

$$\hat{W}_{12}^{(2)}(k) = 2\beta_2(\mu) \left[\left(\frac{\beta^1(\mu_1)}{\beta^1(\mu)} \right)^2 \hat{\mathcal{F}}_1(k) - \left(\frac{\beta^2(\mu_2)}{\beta^2(\mu)} \right)^2 \hat{\mathcal{F}}_2(k) \right], \quad (\text{B.2})$$

$$\hat{W}_{22}^{(2)}(k) = 2 \left[\left(\frac{\beta^2(\mu)}{\beta^1(\mu)} \right)^2 \left(\frac{\beta^1(\mu_1)}{\beta^1(\mu)} \right)^2 \hat{\mathcal{F}}_1(k) + \left(\frac{\beta^2(\mu_2)}{\beta^2(\mu)} \right)^2 \hat{\mathcal{F}}_2(k) \right], \quad (\text{B.3})$$

where $\hat{\mathcal{F}}_a(k)$ denotes the Fourier mode of $\mathcal{F}_a(r)$, given by

$$\hat{\mathcal{F}}_a(k) = \frac{4}{3}\pi^2 c_a k^3 \left(\frac{k}{\mu_a}\right)^{2\lambda_a} \left[1 + \left(\frac{k}{\mu_a}\right)^{\lambda_a}\right]^{-4} (1 + \mathcal{O}(\lambda_a)). \quad (\text{B.4})$$

Using $\hat{W}_{ab}^{(2)}(k)$, the 2×2 matrix $\hat{W}_{ab}^{(2)-1}(k)$ is given by

$$\hat{W}_{11}^{(2)-1}(k) = \frac{\hat{W}_{22}^{(2)}(k)}{\det \hat{W}^{(2)}(k)}, \quad (\text{B.5})$$

$$\hat{W}_{22}^{(2)-1}(k) = \frac{\hat{W}_{11}^{(2)}(k)}{\det \hat{W}^{(2)}(k)}, \quad (\text{B.6})$$

$$\hat{W}_{12}^{(2)-1}(k) = \hat{W}_{21}^{(2)-1}(k) = -\frac{\hat{W}_{12}^{(2)}(k)}{\det \hat{W}^{(2)}(k)}, \quad (\text{B.7})$$

with

$$\det \hat{W}^{(2)}(k) \equiv \hat{W}_{11}^{(2)}(k)\hat{W}_{22}^{(2)}(k) - (\hat{W}_{12}^{(2)}(k))^2, \quad (\text{B.8})$$

where we used $\hat{W}_{12}^{(2)}(k) = \hat{W}_{21}^{(2)}(k)$.

Inserting eqs. (B.1)–(B.3) into eq. (B.8) we obtain

$$\det \hat{W}^{(2)}(k) = 4\hat{\mathcal{F}}_1(k)\hat{\mathcal{F}}_2(k) \left(\frac{\beta^1(\mu_1)\beta^2(\mu_2)}{\beta^2(\mu)}\right)^2 \left(\frac{\beta(\mu)}{\beta^1(\mu)}\right)^4. \quad (\text{B.9})$$

Using eqs. (B.1), (B.3), and (B.9), we obtain the auto-correlations of the adiabatic and entropy perturbations as

$$\mathcal{P}_{11}(k) = \frac{k^3}{4\pi^2} \sum_{a=1}^2 \left(\frac{\beta^a(\mu)}{\beta(\mu)}\right)^4 \frac{1}{(\beta^a(\mu_a))^2 \hat{\mathcal{F}}^a(k)}, \quad (\text{B.10})$$

$$\mathcal{P}_{22}(k) = \frac{k^3}{4\pi^2} \frac{(\beta^1(\mu))^4 (\beta^2(\mu))^2}{(\beta(\mu))^4} \sum_{a=1}^2 \frac{1}{(\beta^a(\mu_a))^2 \hat{\mathcal{F}}^a(k)}, \quad (\text{B.11})$$

which agree with eqs. (6.7) and (6.8), respectively.

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