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INTERNAL DOCUMENT 243

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PRESENTATION AND INTERPRETATION
OF DIRECTIONAL WAVE DATA

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This work was carried out under contract to the Department of Energy

(1) Introduction

In recent years directional wave spectra have become increasingly available from data buoys and platform-mounted sensors. A central problem in working with routinely collected directional wave data is the presentation of such data in a convenient form for users.

This report discusses the analysis and presentation of directional data with reference to user requirements for both short term and long term statistics.

(2) Summary of systems used for determining wave direction

A variety of methods are available for the estimation of directional wave spectra. However, most of these methods are only useful for a limited series of measurements mainly for research purposes; only a few methods are suitable for the routine collection of directional wave data.

(a) Stereophotography

This method employs stereophotography using cameras carried in one or more aircraft to contour the wave surface. The first attempt at using this technique was in the early 1950's and the results were reported by Cote et al. (1960). Recently Holthuijsen (1983) has given the results from four observations of the sea surface under different meteorological conditions in the southern North Sea. The method has a high degree of directional resolution but, due to the expense of aircraft time and the laborious data processing, it is unsuitable for routine data collection.

(b) Arrays of wave recorders

Array techniques can be used in shallow water and have been employed in several oceanographic studies (for example, Barber, 1963; Munk et al. 1963). Th degree of directional resolution depends on the number of wave recorders. The types of wave recorder which can be used are platform mounted wave staffs and pressure recorders on the sea bed. Directional spectra collected from such systems are usually site specific as the recorded waves are influenced by refraction and dissipation in shallow water. The analysis methods are usually difficult to use and the interpretation of results is also complicated (see, for example, Davis and Regier, 1977).

(c) Current meters

Electromagnetic current meters which measure the two horizontal components of wave orbital velocity in conjunction with a pressure recorder or a wave staff can provide useful directional wave information (Nagata (1964), Bowden and White (1966)). These systems can be mounted on offshore structures and, provided the influence of the platform is not too great, wave directional information can be obtained (Forristall et al., 1978). Analysis methods are directly analogous to those used in the pitch-roll buoy to be discussed later in this report.

(d) Pitch-roll buoys

These systems measure the vertical acceleration and two components of surface slope using a surface following wave buoy. The method was fi¨st developed by Longuet-Higgins, Cartwright and Smith (1963) for use in the open ocean with the buoy either free-drifting with internal data recording or connected by a loose cable to an attendant ship. The moored pitch-roll buoy is now the most commonly used technique for the routine collection of directional wave data. Large met-ocean buoys such as the U.K. data buoys of UKOOA employ the principle of the pitch-roll buoy. Smaller systems are commercially available from Datawell, Nereides and Bergen Ocean Data amongst others. These buoys can be moored at a selected location and data transmitted to a shore based system or recorded on board.

The analysis methods for data from this system are well developed and straightforward to use.

The cloverleaf wave buoy, which measures the three curvatures of the surface in addition to heave acceleration and slopes, is mainly used for research purposes (Cartwright and Smith (1964), Ewing (1969), Mitsuyasu et al. (1975)) and is unsuitable for routine measurements.

(e) Remote sensing

Remote sensing techniques from satellites offer the possibility of world-wide coverage of directional spectra using Synthetic Aperture Radar (SAR). However there are, at present, difficulties in interpreting SAR images of the sea surface and the technique cannot therefore be considered to be available for routine observations. Advanced altimeter systems also have the potential for the estimation of the directional spectrum.

Another technique, with the potential for obtaining good resolution of the directional distribution, is the use of radar scattering from sea waves. The main advances using this technique have been made in the U.S.A. and U.K.

(3) Analysis methods

We only discuss methods relevant to the pitch-roll buoy. Panicker (1974) has given a general review of the methods available for other systems as well.

The pitch-roll buoy measures the vertical acceleration and two components of the sea surface slope (after correction of the pitch and roll signals with the compass reading) with respect to true North-East axes. If we denote heave acceleration by the subscript 1 and let subscripts 2 and 3 refer to the surface slopes in the North and East directions respectively, then the directional wave spectrum is derived from the 6 cross-spectral estimates C_{11}, C_{22}, C_{33}, Q_{12}, Q_{13}, C_{23} (the other 3 cross-spectra having expectancy zero for a surface following buoy).

The six cross-spectra completely define the results of a surface following system. From these cross-spectra it is useful to compute the one dimensional wave spectrum E(f), the four angular harmonics A_n, B_n ($n = 1,2$) and a check ratio, R (see Appendix 1).

(a) Conventional analysis

If the directional distribution is unimodal then two directional parameters can be obtained. These are the mean wave direction θ_1 and the spread parameter θ_2. For a narrow directional distribution θ_2 is the r.m.s. spread about the mean wave direction θ_1.

In the conventional analysis, Longuet-Higgins et al. (1963) propose a directional distribution of the form $\cos^{2s} 1/2(\theta-\theta_1)$ where the exponent s can be estimated from the first and second order angular harmonics. In some situations a bimodal directional distribution can be encountered, that is wave energy propagating in two distinct directions at the same frequency. Large differences between the two estimates of s (and also between θ_1 and θ_{12}) are indicative of bimodality. Kuik et al. (1984) have suggested two other parameters which indicate bimodality. Statistical criteria for the significance of these differences can be developed using the work of Long (1980).

In fetch-limited conditions when there is only one dominant wave direction present, Mitsuyasu et al. (1975) and Hasselmann et al. (1980) have proposed empirical directional distributions for the parameters as a function of wind speed and wave frequency in relation to the spectral peak frequency. There is, at present, no generally accepted formulation for bimodal directional distributions.

(b) Variational method

Long and Hasselmann (1979) have developed a new method of analysis for wave data from the pitch-roll buoy. In this technique a function called the "nastiness" function is minimized subject to a number of constraints. These contraints are that the directional spreading function is non-negative and that the difference between the estimated directional spectrum and some favoured model is minimized. Random variability in the cross-spectral estimates is also taken into account. The advantage of using this technique over the conventional analysis method is in its capability of resolving bimodal peaks in the directional distribution. Appendix 2 shows the result of using this method compared with the conventional analysis procedure for an artificial wave data set.

(4) Data presentation

A central problem in working with routine directional wave spectra is the presentation of such data in a convenient form to users without being too voluminous. Many questions on directionality are comparatively straightforward and should be answerable by reference to a report containing tables and graphs. Other more complicated questions will require access to the full data set on magnetic tape (see Chapter 5). Data presentation is also a good guide to the validity of the measurements and can usually indicate malfunction in the buoy system.

It is proposed that the results should be presented in tabular and graphical forms as follows:

Tabulations:

The following parameters are to be listed for each wave recording:

h_s (significant wave height $4\sqrt{\text{variance}}$)

T_z (mean zero-crossing period obtained from spectral moments $\equiv \sqrt{(m_o/m_2)}$)

T_p (wave period of the highest peak of the wave spectrum)

T_1 (wave period obtained from spectral moments $\equiv m_o/m_1$)

Average of 3 frequencies around T_p:
- (θ_1 (mean wave direction)
- (θ_2 (directional spread)
- (s_1 (exponent of cosine power distribution)
- (R (check ratio of heave accn. to combined slope spectra)

Average over high (θ_1
frequency range (θ_2
0.25-0.30 Hz (R

Graphs In the following graphs we assume θ_1 and θ_2 at T_p are obtained from an average over 3 frequencies, as defined in the tabulations.

(a) Time histories of

(i) h_s
(ii) θ_1 at T_p
(iii) θ_2 at T_p

(b) Scatter diagram of h_s vs. T_z

(c) Distribution of h_s vs. θ_1 (at T_p)

(d) Distribution of T_p vs. θ_1(at T_p)

(e) Joint distribution of h_s, T_p, θ_1 (at T_p) (see Appendix 3).

Tables corresponding to this joint distribution with 8 directional sectors each of 45° width.

Notes

1. Choice of a period parameter

Three wave periods are presently used in wave data analysis, T_z, T_p and T_1. In the past T_z has been used extensively in wave data collections because of its important role in statistical theory. To this extent it is retained in the scatter diagram (b) above, as used for many past non-directional compilations. However, when considering the joint distribution involving the directional parameter θ_1 it seems more logical to use the wave period T_p corresponding to the most energetic part of the wave spectrum. The mean wave direction θ_1 is furthermore most stable in the region of the spectral peak.

The wave period paramter T_1 is included in the tabulations since it is close to the period of the highest wave in a record.

2. Checks on the data analysis

(i) The mean wave direction θ_1 at high frequencies (in the range from 0.25 Hz to 0.30 Hz) should be close to the wind direction. It is important to note that both wave and wind directions are defined as those directions from which the waves and wind are coming from. Under strong winds θ_1 at the spectral peak and θ_1 at high frequencies should be approximately the same.

(ii) The check ratio, R, should lie between 0.9 and 1.1 unless there are strong currents present. Significantly greater deviations from unity are a possible indication of sensor malfunction. At high frequencies however such deviations are usually caused by lack of surface following.

(iii) θ_2 should be a minimum at frequencies near the spectral peak and increase with frequency.

3. Bimodal frequency spectra

The selection of T_p as the period parameter is appropriate only when there is one dominant peak in the wave spectrum. It is possible for both wind-sea and swell to be present with comparable energies. When this situation occurs it may be desirable to introduce additional parameters characterizing the energy, period and direction of wind-sea and swell. However there does not appear to be any accepted procedure for doing this at the present time.

(5) Data banking

It is recommended that directional wave spectra information should be stored in the MIAS format (Jones, 1980) which has been used extensively in the data banking of information from the UKOOA data buoys. The data to be banked consist of the co- and quad-spectra which define the measurement system. This general format allows for any number of cross-spectra from a variety of systems such as wave gauge arrays, pitch-roll buoys, electromagnetic current meters and stereophotography. For the pitch-roll buoy, nine cross spectra completely define the information from this system. The Fourier expansion of the directional wave spectrum in terms of its angular harmonics (see Appendix 1) is not stored within this file, neither are the directional parameters. These quantities are readily computed or can be obtained from tabulations in the data report.

(6) Interpretation of directional wave data

Uses of directional wave data have requirements on two time scales namely short term and long term.

(a) Short term description

For unimodal directional distributions use of the $\cos^{2s} 1/2(\theta-\theta_1)$ model is a good description of the waves as the parameters s and θ_1 can vary with frequency. It is therefore expected that the use of observational data can lead to improvements in engineering design as, at the present time, either the waves are assumed to be unidirectional or simple, frequency independent directional distributions (i.e. $\cos^2\theta$) are used by many engineers.

If there are indications of bimodality in the directional distribution then advanced techniques such as the maximum likelihood method or the method of Long and Hasselmann (1979) can give improved information but at the cost of considerable extra computing effort. It is not clear if the characteristics of bimodal distributions are required except in a few specialized studies.

(b) Long term description

In the long term description we refer to changes in the mean wave direction with time. At each instant of time there is, of course, a directional spread about each mean direction as described in the previous section. Torset and Olsen (1982) give a clear account of the use of both short term and long term directionality in the design of offshore structures.

The most important aspect of directional wave data in long term statistics is in the use and presentation of joint wave statistics such as wave height, period and direction. It is now generally recognized (Graham (1982), Haring et al. (1982)) that the proper use of joint wave statistics is in the evaluation of response statistics rather than in any attempt to fit conditional probability distributions. This use of encounter probability for extreme response values is essential when dealing with multi-parameter descriptions of the marine environment (see, for example, Battjes (1979)).

(7) Conclusions

An outline has been given of the various systems suitable for obtaining directional wave spectra. Systems based on the principle of the pitch-roll buoy seem to have the greatest potential for the routine collection of directional wave data.

The presentation of wave data in a form suitable for users has been discussed. Many engineering design problems can be solved by access to tabulated data of the mean wave direction and directional spread or joint probability distributions. For other more detailed or specific studies, it is suggested that the basic cross-spectral information be stored in the format devised by MIAS. Analysis using more sophisticated methods (Long and Hasselmann (1979)) may then be used to derive additional directional information, for example, on the energy in bimodal distributions.

The short and long term requirements for directional wave data need different interpretations. In the short term it would appear that use of measured data should lead to improvements in engineering design since, at the present time, either a unidirectional or a frequency independent directional distribution is used by engineers. The parametrization of wind-sea directional distributions is presently based on limited information from Mitsuyasu et al. (1975) and Hasselmann, Dunkel and Ewing (1980). An important future research problem is to investigate the form of the directional distribution in a wide variety of wave conditions including severe storms.

For long term statistics the presentation of joint probability distributions is important. The evaluation of response statistics through the proper use of joint probability statistics seems to be appropriate for engineering design.

APPENDIX 1

The determination of the directional spectrum of the sea surface follows the method first suggested in Longuet-Higgins et al. (1963).

The pitch, roll and compass channels are used to derive the two components of surface slope with respect to North-East axes. Let subscripts 2 and 3 denote the components of surface slope in the North and East directions respectively and let subscript 1 refer to the series for vertical acceleration. Then it can be shown that the six cross-spectra derived from the wave buoy system are given by

$$
\begin{aligned}
C_{11}(f) &= \int_0^{2\pi} (2\pi f)^4 F(f,\theta)\,d\theta, \\
C_{22}(f) &= \int_0^{2\pi} k^2 \cos^2\theta\, F(f,\theta)\,d\theta, \\
C_{33}(f) &= \int_0^{2\pi} k^2 \sin^2\theta\, F(f,\theta)\,d\theta, \\
Q_{12}(f) &= \int_0^{2\pi} k(2\pi f)^2 \cos\theta\, F(f,\theta)\,d\theta, \\
Q_{13}(f) &= \int_0^{2\pi} k(2\pi f)^2 \sin\theta\, F(f,\theta)\,d\theta, \\
Q_{23}(f) &= \int_0^{2\pi} k^2 \sin\theta \cos\theta\, F(f,\theta)\,d\theta,
\end{aligned}
\qquad \ldots (1)
$$

where C_{ij} and Q_{ij} are the co- and quadrature spectra of series i with j. $F(f,\theta)$ is the directional wave spectrum with respect to frequency f and direction of propagation θ. k is the wave number. Only five of the cross-spectra are independent. This allows estimation of the five Fourier coefficients in the expansion of $F(f,\theta)$, namely,

$$a_n + i b_n = \frac{1}{\pi}\int_0^{2\pi} e^{in\theta} F(f,\theta)\,d\theta \;, \quad n = 0, 1, 2 \qquad \ldots (2)$$

In the calculations it is convenient to compute the normalised angular harmonics $A_1 = a_1/a_0$ etc. where a_0 is the one-dimensional spectrum obtained by integrating $F(f,\theta)$ over all directions. Thus

$$A_1 = \frac{Q_{12}}{\sqrt{[C_{11}(C_{22}+C_{33})]}} \;, \qquad B_1 = \frac{Q_{13}}{\sqrt{[C_{11}(C_{22}+C_{33})]}}$$

$$A_2 = \frac{C_{22}-C_{33}}{C_{22}+C_{33}} \;, \qquad B_2 = \frac{2C_{23}}{C_{22}+C_{33}} \qquad (3)$$

In the above we make use of the relation

$$\left(\frac{C_{11}}{C_{22}+C_{33}}\right)^{\frac{1}{2}} = \frac{(2\pi f)^2}{gk} = \tanh kh \qquad (4)$$

which is the dispersion relation for waves of small amplitude in water of depth

h. The quantity $R = \frac{1}{\tanh kh}\left(\frac{C_{11}}{C_{22} + C_{33}}\right)^{\frac{1}{2}}$ provides a check on the correct functioning of the wave buoy system and on the analysis. R is called the check ratio and should be unity in value for a surface following buoy.

Derived parameters

If the directional distribution is unimodal and of the form $\cos^{2s} \frac{1}{2}(\theta-\theta_1)$, it is useful to derive the following directional parameters:

(i) mean direction from the first order angular harmonics
$\theta_1 = \arctan (B_1/A_1)$

(ii) mean direction from the second order angular harmonics
$\theta_{12} = {}^1/2 \arctan (B_2/A_2)$. This direction is ambiguous to 180 degrees.

(iii) spread parameter from the first order angular harmonics

$$S_1 = \frac{C_1}{1 - C_1} \quad , \text{ where } C_1^2 = A_1^2 + B_1^2$$

(iv) spread parameter from the second order angular harmonics

$$S_2 = [(1+3C_2+\sqrt{(1+14C_2+C_2^2)}]/(2[1-C_2])$$

where $C_2^2 = A_2^2 + B_2^2$

(v) spread parameter $\theta_2 = \sqrt{(2-2C_1)}$. For a narrow directional distribution θ_2 is the r.m.s. spread about the mean direction θ_1.

Note: The definition of the angular harmonics A_1 and B_1 involves the ratios of terms containing heave and surface slopes such that A_1 and B_1 are relatively insensitive to the response amplitude characteristics of the slope measurements. In particular, $\theta_1 = \arctan (Q_{13}/Q_{12})$, so even if, at high frequencies, the slope response is not unity, the estimate of θ_1 can still be useful. If, however, there is a phase shift in addition to an amplitude response, then θ_1 cannot be considered reliable.

The second order angular harmonics A_2 and B_2 involve ratios of the slope spectra and are independent of heave. In this case, provided both pitch and roll have identical amplitude and phase responses, it should be possible to derive some directional information at frequencies where the buoy does not follow the wave slope.

As an indication of the analysis procedure to be adopted, for the IOS pitch-roll buoy measured data are at 0.5 s intervals. Each recording for the pitch-roll buoy is of 2048 s (34 min) length and, by use of the Fast Fourier Transform (FFT), we obtain estimates of the relevant cross-spectra by standard smoothing procedures to provide spectral estimates at 0.01 Hz interval with 40 degrees of freedom. The wave height spectrum is obtained by dividing the acceleration spectrum by $(\text{frequency})^4$ before smoothing.

APPENDIX 2

A series of calculations were made using the variational method for a set of data consisting of idealized bimodal distributions. These distributions were formed from the sum of two Gaussian distributions of equal energy but with increasing separation of the two dominant directions.

That is

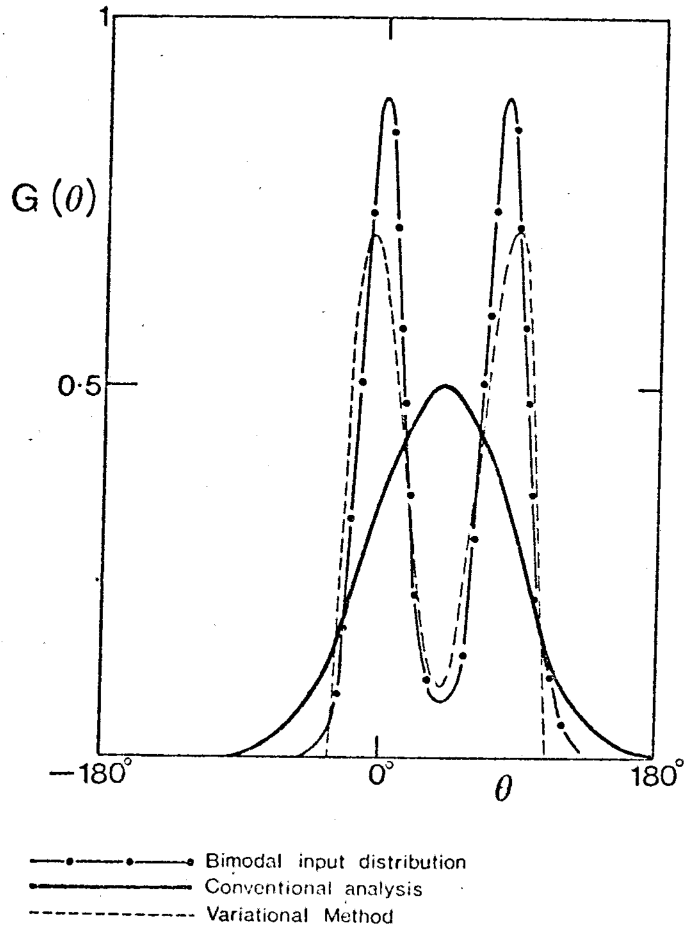
$$G(\theta) = N\exp\left[-\frac{1}{4}s(\theta-\theta_{1a})^2\right] + N\exp\left[-\frac{1}{4}s(\theta-\theta_{1b})^2\right]$$

with θ_{1a} = 0° and θ_{1b} = 0° (10°) 180°. N is a normalizing factor. The individual widths of the distributions was chosen with s = 40 (θ_2 = 12.6°).

It was found that the variational method was able to resolve the two distributons when $\theta_{1b} \geqslant 80°$ but not for smaller angular separations.

The following figure shows the results of these calculations when θ_{1b} = 80° compared with those using the conventional method.

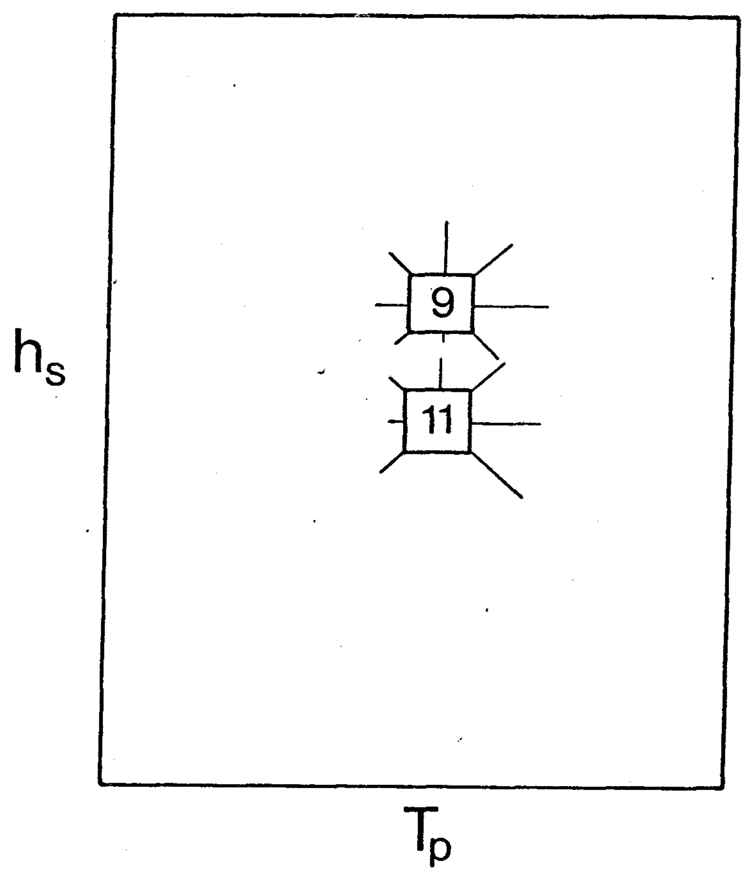
The variational method of Long and Hasselmann (1979) is clearly a powerful method for investigating the bimodality of directional distributions. Further work will however be necessary to establish its applicability in analysing real observational data.



APPENDIX 3

The conventional scatter diagram of significant wave height, h_s, versus wave period at the spectral peak, T_p, could be extended by the use of "rosettes" of mean wave direction in, say, 45 degree sectors.

The following figure shows a possible arrangement of the results. The numbers in each cell give the joint probability of h_s with T_p in parts per thousand.



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