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N.7

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N.I.O. PROGRAM 61

Title Wind Profiles

Language EMA

Machine ATLAS I (original version was for Mercury only)

Purpose To compute, given wind speeds (u) and heights (z), the best fit to the equation u = alog + bz + c and u = alogz + c.

Input
0) Program
1) Job description and data

Program The program starts with

COMPILER EMA

SO02 NIO PROGRAM 61 WIND PROFILES 30/1/1968

The program ends with

***Z

Job description and data

JOB

Job No. and Title

INPUT

0 SO02 NIO PROGRAM 61 WIND PROFILES 30/1/1968

SELF = 1

OUTPUT

0 FIVE-HOLE PUNCH m BLOCKS

STORE 20/65 BLOCKS

COMPUTING i INSTRUCTIONS

where m = about $\frac{1}{2}$ block/case

i = 500 + 100/case

then follows

1) DATA
2) DATA TITLE
3) One line of title
4) Parameter t = 1, 2, or 3
5) Series of values of z, u where z is the height in cm. and u the wind speed in cm/sec.
6)a) If t = 1, 2
   - * asterisk
   - Δu difference in wind velocity between two heights
   - ΔT difference in temperature between same two heights (note one height is usually the ground or sea-surface)

   b) If t = 3
   - > (greater than)
7) The last case or set of data must be followed by

   DATA TITLE

   END OF DATA

   / (solidus)

   ***Z

Items 2 to 6 may be repeated as often as required.

Output

1) Data title
2) u_*/k (friction velocity/Von Karman constant) in cm/sec.
3) zo (roughness height) in cms. (floating point)
4) log zo
5) L (Monin-Obukhov length) in cms.
6) A ($\frac{Au_*}{Lk}$ is coefficient b in the fittedequation)

Note 5) and 6) are omitted if t = 3. If $\Delta T = 0$ then L is replaced by $L' = L/A$ and A is not printed.

7) At each observed height z there are printed
   a) z height in cms
   b) u computed velocity in cm/sec.
   c) Δu (observed-computed) velocity in cm/sec.
   d) $K \Delta u/u_*$
   e) z/L if $t \neq 3$ (If $\Delta T = 0$, z/L' is printed)
8) Standard deviation of observations from the fitted profile.
9) Computed velocities at 200 cms and 1000 cms.

When t = 2, two sets of output are printed as if t = 1 in the first case and t = 3 in the second case.

When $\Delta T = 0$ and t = 1, 2 then the words NEUTRAL STABILITY will be printed immediately below data title.

Failures

A division by zero or exponent overflow will cause the profile to be omitted and 'PROFILE OMITTED' will be printed.

A spurious character, other than *, >, / results in a print-out 'ERROR IN DATA'.

In both cases the program continues with next profile.

Cost

For six cases the cost was about £2 0s 0d.

Method

When t = 1, least squares fit of u = ax + bz + c where x = logz

when t = 3, least squaresfit of u = ax + c

when t = 2, least squares fit of u = ax + bz + c and u = ax + c.

Programmer

J. Crease

N.I.O. PROGRAMS 62/A, 62/B, AND 62/C ATLAS

Title — Calibration factors for the N.I.O. Shipborne Wave Recorder

Language — EMA

Machine — ATLAS I

Purpose — To compute the factors by which measured wave heights should be multiplied to give the true wave heights, where the measured heights are from an N.I.O. Shipborne Wave Recorder.

Inputs

0) Program
1) Job description

There is no data, the parameters being set within the program.

Program

The program starts with

COMPILER EMA

followed by the program title and main, auxiliary and dumps specifications. Two lines of comment follow describing the function of the program.

Then follows the program, ending with

***Z

Job description

This is as follows:-

JOB

Job number, data link number, job title

INPUT

```
0 SOC2 NIO PROGRAM 62/A ATLAS 3/11/1967 ⎫
                   62/B ATLAS 6/11/1967 ⎬ as appropriate
                   62/C ATLAS 7/11/1967 ⎭
```

SELF = 1

OUTPUT

0 FIVE-HOLE PUNCH n BLOCKS

STORE 15/60 BLOCKS

COMPUTING 2000 INSTRUCTIONS

The tape ends with

***Z

For the programs in their present forms, the number of blocks of output, n, is as follows:-

```
NIO PROGRAM 62/A ATLAS  3 blocks
            62/B ATLAS  4 blocks
            62/C ATLAS 14 blocks
```

Output

This is a table of ratios of true wave height to recorded wave height as a function of KD and T, where K is a factor which may vary but is usually taken to be 2·5, D is the mean depth of the two pressure units below mean water level (in feet), and T is the wave period in seconds.

Cost

For the programs in their present forms, the costs are as follows:-

```
NIO PROGRAM 62/A ATLAS £1. 13. 5.
 "     "    62/B   "   £1. 16. 6.
 "     "    62/C   "   £3. 17. 8.
```

Method

The ratio r evaluated, i.e. true wave height/recorded wave height, is that given by MOLLIE DARBYSHIRE in her paper "WAVES IN THE NORTH SEA", The Dock and Harbour Authority, Nov. 1960 No. 481, Vol. 41, pp. 225-228 and is

$$r = 0{\cdot}83\ [1 + (8{\cdot}8\mu)^{-2}]^{\frac{3}{2}} \exp(\mu^2 d/g)$$

where $\mu = 2\pi/$significant period, d is the mean depth below the water-line, and g is the acceleration of gravity.

NIO PROGRAM 62/A ATLAS 3/11/1967 is for the evaluation of r at equal increments of T, the wave period in seconds. In this version, T = 2 (1) 22 and KD has values 10 (1) 44.

NIO PROGRAM 62/B ATLAS 6/11/1967 is for the evaluation of r at equal increments of frequency F, although r is still listed as a function of period. The values of F used in this version are 0·03 (0·01) 0·25 where $T = \frac{1}{F}$ (giving values of T from 33·33 to 4·00); KD has values 22 (0·2) 28·8.

NIO PROGRAM 62/C ATLAS 7/11/1967 is for the evaluation of $\frac{1}{r}$, i.e. the inverses of the calibration factors, for equal increments of frequency. The values of F used in this version are 0·02 (0·01) 0·50 where $T = \frac{1}{F}$; KD has values 12 (0·2) 28·8.

Notes

1) The original version of NIO PROGRAM 62 was for use on MERCURY and has been superceded.

2) Parameters in the program can easily be altered to give different ranges of values of T and KD.

Programmer

Pamela Edwards

N.I.O PROGRAM 63

Title Q Factors

Language FORTRAN V

Machine ATLAS I

Purpose To calculate the pressure in decibars from the reading of $(T_u - T_w)$ given by an unprotected reversing thermometer.

Tapes
0) Program
1) Job description and data

Program Tape

COMPILER FORTRAN

S002 NIO PROGRAM 63 Q FACTORS 4/9/1967

then follows the rest of the program

***Z

Job description and data

JOB

Job number, job title

INPUT

0 S002 NIO PROGRAM 63 Q FACTORS 4/9/1967
(or date of appropriate version)

SELF = 1

OUTPUT

0 FIVE-HOLE PUNCH n BLOCKS

where n = 2 x (number of pages of output)

STORE 20/70 BLOCKS

COMPUTING 5000 INSTRUCTIONS

DATA

The Q factor numbers are punched in columns 1 - 6, FORMAT F6·4 and the maximum temperature numbers punched in columns 6 - 10, FORMAT I4. If the maximum temperature number is a two digit number, two spaces must be punched in columns 7 and 8.

The data is followed by

0·0000s̸s̸00 punched in columns 1 - 10

runout

***Z

Output The output consists of a table of values of pressure for temperatures from zero to the maximum temperature required in steps of 0·1°C. The pressure difference corresponding to temperature increments of 0·01°C are printed at the bottom of each page which may contain up to sixty integral temperature values.

Cost £2 0s 0d per page of output

Programmer Elizabeth Palethorpe

N.I.O. PROGRAM 65 (incomplete)

Title 24 Element Array.

Code CHLF 3/4.

Machine MERCURY.

Purpose Given a number, to compute the 12 values of current and the gain of an acoustical array, using Dolph shading.

Tape Test program and data.

Parameters None.

Data One number x.

Restrictions $-3.34 \quad x \quad +3.34$
x may not exceed 9 significant figures.

Operation
(1) Read in the program.
(2) Read in the data. The machine computes and prints the results.
(3) Operation (2) may then be repeated.

Output
(a) Title
(b) 12 sets of numbers:

$i \quad y_i \quad z_i$ where $i = 1(2)23$.

(c) 12 sets of numbers:

$k \quad a_k \quad b_k$ where $k = 12(-1)1$.

(d) Prints "SUM IK" followed by the value ΣI_k

(e) Prints "G" followed by the value $\dfrac{(\Sigma I_k)^2}{12\,\Sigma(I_k^2)}$

Time 1 minute per value of x.

Method The values x, x^3, x^5, ... x^{23} are computed in double length arithmetic and stored in $((y_i, z_i))$, where $i = 1(2)23$.
Then the values of I_k, where $k = 12(-1)1$, are calculated and stored in $((a_k, b_k))$.
Also computed are: ΣI_k, $\Sigma(I_k)^2$ and $G = \dfrac{(\Sigma I_k)^2}{12\,\Sigma(I_k^2)}$.

Notes (1) This program has not been fully tested, and is not completely correct. It was used in Job No. 1996 for which the answers obtained were sufficiently correct for the purpose.
(2) The whole program is written in Double Length Arithmetic to try to maintain a greater degree of accuracy, and yet the accuracy of the results is known not to exceed 6 decimal places.

Programmer Mrs. [illegible]. Wilson.

N.I.O. PROGRAM 66

Title Curve fitting with point restraint

Language EMA

Machine Atlas I

Purpose To fit polynomials $y = f(x)$ having degrees up to ℓ to a set of n points, not necessarily of equal weight. The polynomial satisfies r restrictions at x_0, that $f(x_0)$, $f'(x_0)$.......... $f^{r-1}(x_0)$ shall have specified values.

Tapes Input 0: Program
Input 1: Job description and data

Program Tape COMPILER EMA

S001 NIO PROGRAM 66 CURVE FITTING WITH POINT RESTRAINT 8/6/1966 (or date of appropriate version)

MAIN 1530

AUXILIARY (0, p) where p = 20(n+13)

DUMPS 0

then follows the rest of the program ending with

*** Z

Job description and data

JOB

Job number, job title

INPUT

0 S001 NIO PROGRAM 66 CURVE FITTING WITH POINT RESTRAINT 8/6/1966

SELF = 1

OUTPUT

[Either tape or lineprinter output may be obtained. Up to 250 lines, or 4 blocks of tape, per set of points will be required. Output 0 should be used]

STORE 40/70 BLOCKS

COMPUTING q INSTRUCTIONS where q = 2000 + n l

DATA

1) DATA TITLE
2) One line of title
3) One integer, n, the number of points
4) One integer, l, the maximum degree required
5) One integer, r, the number of restrictions at x_0
6) p equal to zero for equal weights, 1 otherwise
7) q equal to 1 if all residuals are required, 0 otherwise
8) The value of x_0
9) List of r restrictions i.e. the values of $f(x_0)$, $f'(x_0)$ etc.
10) Next comes the list of the n points with the x co-ordinate of each point first followed by the y co-ordinate.
11) Finally, if p=1, a list of n weights; these need not be integers.

Each new set of data begins with

DATA TITLE

and the last set is followed by the sequence

DATA TITLE

END OF DATA

/

and then

* * x Z

Output

The polynomial coefficients and the residual of maximum modulus are printed for each degree of polynomial from r up to l. The sum of squares of the residuals after fitting the polynomial of degree ℓ is printed and can be compared with a check value derived in the orthogonal fitting process. If they disagree, this indicates appreciable loss of significant figures in the polynomial evaluation. Provided the maximum residual is acceptable this is of no importance. Otherwise the origin for x should be shifted to somewhere near the middle of the range of x values, and the program run again.

The component of the sum of squares is printed at each degree and the residual at any degree, m, can be found by adding to the sum of squares of residuals the components at degrees (m+1), (m+2) ...... l. If this and the maximum residual are acceptable, the coefficients at degree m can be used.

Finally the residual at each point for the polynomial of degree l are printed (if q=1).

Restrictions

$n \leq 500$

$p \leq 10751$

[n and p may be increased if the A, B and W directives are increased to n+12, n+12 and n-1 respectively. The main store allocation should also be increased.]

$0 \leq r \leq l \leq 12$

Cost

For n = 40, l = 2, cost is under £1
For n = 150, l = 12, cost is under £3

Notes

This program is based on the R.A.E. program 146/A written by J.H. Cadwell. It differs from that program in the layout of the auxiliary store, in the layout of the data for the n points, in the use of a data title and in the method of terminating the data. All Mercury autocode instructions such as 620,14 and CAPTION have been replaced by EMA instructions.

Programmers

JAMES CREASE
BRIAN HINDE

N.I.O. PROGRAM 67

Title Cross-spectral analysis.

Code EMA.

Machine Atlas I (University of London, A.C.S.).

Purpose To compute:- means, variances, normalised auto-covariances, and auto-spectra of 1-7 time series; and also, normalised cross-covariances, co- and quad-spectra, phase lags, intensities, and coherences for 2-7 time series, for all combinations of pairs.

Tapes 1) Program tape, which starts with the job description and ends

* * * T

2) Data tape, which includes parameters, and has a specified ending, with a final terminator of

* * * Z

N.B. The job, program and data are all one document.

Program tape This is as follows:- (capitals denote actual punching, lower case describes what is required).

JOB
job No., provided by A.C.S., followed by:
N.I.O. PROGRAM 67. CROSS-SPECTRAL ANALYSIS. Job title.
(All this on one line)

OUTPUT

0 LINEPRINTER m LINES

STORE s BLOCKS

COMPUTING t INSTRUCTIONS

This comprises the job description.

COMPILER EMA

Then comes the store allocation as follows:-

MAIN → q

DUMPS 0

AUXILIARY (0, p)

Then come some instructions, followed by the heading:-

PROGRAMME -1

This is followed by Chapter 1 of Programme -1 and the rest of the program. Program parameters m, s, t, q and p are described below. Note that 0 is zero.

The program tape must end with

* * * T (i.e., 3 asterisks, letter T).

Program parameters The directives for chapters 1 and 2 must be set as described at the beginning of the relevant chapters; directives for chapter 0 need not be altered.

The number of lines of output

$$m = 150 + L + M + 2L\,(\text{int.pt. } K/2) + (M + 6)\left[\frac{K(K-1)}{2}\right]$$

where K is the number of series, L the number of lags, and M the number of frequencies.

The store s blocks = 10 + $\left(\frac{\text{main store + auxiliary store + 6000}}{512}\right)$

Computing t instructions where t is an integer and

$t \sim 4000 + K^2NL/_{80}$, where N is number of terms/series.

The main store requirement q is found as follows:- Find the total number of main variables (directives) in each of the three chapters (remembering that the count starts at zero, so that $X \rightarrow 1000$ allocates 1001 storage locations); then q must be greater than or equal to the largest of these three totals.

The auxiliary store requirement p is greater than or equal to:- $D' + (L + M + 2)K^2$ or $D' + NK$, whichever is greater, where $D' \geq K\ (2M + 6)$.

For more than one set of data, see note 2.

Data and data parameters These may physically be on the same tape as, and after, the program or may be physically separate from the program - both cases need exactly the same specifications, except that if they are on the same tape * * * T is omitted after the program.
Data must be in the usual Mercury format. The data starts as follows (capitals denoting actual punching, as before):-

1) DATA TITLE
one teleprinter line of descriptive title must immediately follow the CR LF which follow the words "Data Title"; the title may contain words and/or numbers;

2) an integer equal to the number of series (K);

3) " " " " " " " covariance lags required (L);

4) " " " " " " " frequencies required (M where $M \leq L$);

5) the K numbers required to convert the K series to physical units (each number being a ratio of physical/computing units);

6) the K numbers being the time lags of each series relative to the first, in decimal fractions of the time between successive terms of the same series. Thus the first number will usually be zero, and if all the series are sampled simultaneously all the numbers will be zero;

7) T, the sampling interval;

then follows the data, the K series of numbers being arranged sequentially, i.e., with the first terms of all series first, the second terms next, and so on.

The data must be terminated with sp. sp. > (i.e., immediately after the last digit punch "space space > ").

The whole sequence, beginning DATA TITLE and ending >, may be repeated for another set of data.

The last set of data must be followed by:-

DATA TITLE
END OF DATA
/

* * * Z (i.e., 3 asterisks, letter Z)

Operation This is done by the staff of the Atlas Computing Service.

Output Data title.

Number of terms in each series.

Time lags (decimal fractions of time between successive terms of the same series).

X, the means of the series.

X′, the means converted to physical units.

V_x, the variances of the series.

V_x', the variances converted to physical units.

Ψ_{xx}, the lagged normalised auto-covariances (preceded by the value r) for r = 0(1)L, where L is the number of lags specified.

V_{xy}, V_{xy}', the un-normalised cross-covariances at zero lag, firstly in computing units and secondly in physical units.

Ψ_{xy}, the lagged normalised cross-covariances for each pair of series (preceded by the value r) for r = -L(1)L.

in cycles/time unit.
between successive terms of the original series.

E_{xx}, auto-spectra, in (physical units)²/cycle/time unit (preceded by s, where s = 0(1)M).

Cross-spectra for each pair of series, preceded by s and consisting of:-

E_{xy}, E_{xy}^*, co and quad-spectra (in physical units²/cycle/time unit).

I, intensity.

ϕ, phase (in degrees).

γ^2, coherence.

If input had any spurious characters other than those specified for ending data, the words

ERROR IN DATA, TERM -

will be punched, with the actual term number given.

Restrictions $1 \leq K \leq 7$ for layout as at present arranged.

Number of terms per series ≤ 5,000.

$1 \leq M \leq L$.

Cost See the Atlas Computing Service formula and output logging for computing the cost.

Examples of cost:-

1) For 1 series of 1184 terms, with 50 lags and 50 frequencies required - £3. 5. 9d.
2) For 2 series of 1184 terms each, 50 lags and frequencies - £5. 2. 6d.
3) For 5 series of 460 terms each, with 60 lags and frequencies - £11. 8. 0d.
4) For 7 series of 500 terms each, with 50 lags and frequencies - about £18. 0. 0d.
5) For 7 pairs of series of about 1200 terms/series, 50 lags and frequencies - about £26. 0. 0.

Definitions Let the first series of data x_i be $x_0, x_1, x_2, x_3 \ldots x_{N-1}$, and the second series of data y_i be

$y_0, y_1, y_2, y_3 \ldots y_{N-1}$, etc.

Let F, G, etc., be the respective conversion factors from computing to physical units.

Means:

$$X = \frac{1}{N} \sum_{i=0}^{N-1} x_i \quad \text{for each series of N terms.}$$

$$X' = FX$$

Variances:

$$V_x = \frac{1}{N-1} \sum_{i=0}^{N-1} x_i'^2$$

where

$$x_i' = x_i - X$$

$$V_x' = F^2 V_x$$

Lagged Normalised auto-covariances:

$$\Psi_{xx}(r) = \left[(N-r-1)V_x \right]^{-1} \sum_{i=0}^{N-r-1} x_{i+r}' \; x_i'$$

Cross-covariances:

$$V_{xy} = \frac{1}{N-1} \sum_{i=0}^{N-1} x_i' \; y_i'$$

$$V_{xy}' = FG \; V_{xy}$$

Lagged Normalised cross-covariances:

Negative lags:-

$$\Psi_{xy}(r) = \left[(N-r-1)\sqrt{V_x V_y} \right]^{-1} \sum_{i=0}^{N-r-1} x_{i+r}' \; y_i'$$

Positive lags:-

$$\Psi_{yx}(r) = \left[(N-r-1)\sqrt{V_x V_y} \right]^{-1} \sum_{i=0}^{N-r-1} x_i' \; y_{i+r}'$$

Frequency increment between successive spectral estimates is

0.5/LT cycles/time unit (e.g., c/sec.)

i.e., 0.5/L cycles/time interval (T).

Auto-spectra:

$$E_{xx}(s) = 4TV_x' \sum_{r=0}^{L}{}'' \phi_{xx}(r) \cos\left(\frac{rs\pi}{L}\right)$$

where $\sum''$ means the sum with the first and last terms halved,

and $\phi_{xx}(r) = \Psi_{xx}(r) \cos^2\left(\frac{\pi r}{2L}\right)$

Cross-spectra:

Co-spectra:

$$E_{xy}(s) = 4\,FG\,\sqrt{V_x\,V_y}\;\sum_{r=0}^{L}{}''\;\tfrac{1}{2}\left[\phi_{yx}(r) + \phi_{xy}(r)\right]\cos\left(\frac{rs\pi}{L}\right)$$

where $\phi_{yx}(r) = \Psi_{yx}(r)\cos^2\left(\frac{\pi r}{2L}\right)$

and $\phi_{xy}(r)$ is similarly defined.

Quad-spectra:

$$E_{xy}^*(s) = 4TFG\,\sqrt{V_x\,V_y}\;\sum_{r=0}^{L}{}''\;\tfrac{1}{2}\left[\phi_{yx}(r) - \phi_{xy}(r)\right]\sin\left(\frac{rs\pi}{L}\right)$$

The Cross-spectrum is then defined as $E_{xy} + iE_{xy}^*$.

Intensity: $I = \sqrt{E_{xy}^2 + E_{xy}^{*2}}$

Phase lag: $\phi = \tan^{-1}\dfrac{E_{xy}^*}{E_{xy}}$

If ϕ is positive series x leads series y in time.

Coherence: $\gamma^2 = \dfrac{I^2}{E_{xx}\,E_{yy}}$

Corrections for time lags between series

If there exists a time lag xT between corresponding terms of series n and series 1 (n being an integer with values from 2 to 7 inclusive, and x being a decimal having values between 0 and 1 inclusive), then the cross-spectral estimates $E_{xy}(s)$ and $E_{xy}^*(s)$ are modified as follows to $F_{xy}(s)$ and $F_{xy}^*(s)$ where:-

$$F_{xy}(s) = E_{xy}(s)\cos\left(\frac{\pi sx}{L}\right) - E_{xy}^*(s)\sin\left(\frac{\pi sx}{L}\right)$$

and $$F_{xy}^*(s) = -E_{xy}^*(s)\cos\left(\frac{\pi sx}{L}\right) - E_{xy}(s)\sin\left(\frac{\pi sx}{L}\right)$$

Then the computation of intensity, phase lag and coherence proceeds using $F_{xy}(s)$ and $F_{xy}^*(s)$ in place of $E_{xy}(s)$ and $E_{xy}^*(s)$.

Notes

1) There is no error detection (apart from spurious characters) or correction in this program; it is intended for use with 'clean' data.

2) For more than one set of data to be run at one time, the program parameters are the same as for a single set of data (1 - 7 series) except that the number of output lines and computing instructions specified must be for the total output; in the case of computing instructions, the extra number of instructions needed for each set of

Notes (continued)

data after the first is t - 1550 (i.e., the total number required for one set of data minus the approximate number required to compile this program.

3) In calculating the spectra the lagged covariances have been multiplied by $\cos^2\left(\frac{\pi r}{2L}\right)$; this is equivalent to the "Hanning" process used in N.I.O. Program 60.

Programmer: JAMES CREASE

N.I.O. PROGRAMS 68, 68/A and 68/B

Title — Analysis of Cloverleaf Buoy records

Language — EMA

Machine — Atlas 1

Purpose — To compute the directional spectrum of sea waves from the output of the NIO cloverleaf buoy.

Outline of Program

The cloverleaf buoy records five signals proportional to the slopes and curvatures of the sea surface relative to axes which are fixed relative to the buoy. The vertical acceleration and magnetic heading of the buoy are also recorded, together with some other ancillary data such as the output of the shipborne wave recorder. From the details of the data sequence given later it can be seen that the buoy heading and SBWR are logged on alternative cycles of the slope, acceleration and curvature data.

The data is logged sequentially from each channel as a positive 3 figure b.c.d. number on 5 track paper tape. The analysis begins by detecting data errors and replacing them by the mean values of the series in which they occur. The mean values are then subtracted and the series converted to physical units. The slope records in the two perpendicular directions are combined to form two series of slopes referred to magnetic north and east. The three curvature records are treated similarly.

The six records of slope, curvature and vertical acceleration are then input to NIO programme-1 which computes their auto and cross spectra from which the directional spectrum of the sea waves is found.

The power spectrum of the SBWR is also calculated and compared with the doubly integrated spectrum of vertical buoy acceleration.

There are three distinct programs, and their differences are as follows:-

NIO 68 assumes a 64 channel cycle of data, lists the ancilliary data once every cycle and assumes that the buoy compass was not functioning correctly and that the visual observations of buoy heading at 1 sec. intervals will be read in separately.

NIO 68/A assumes a 32 chennel cycle of data, does not list any ancilliary data and accepts the compass data separately as in NIO 68.

NIO 68/B assumes a 32 channel cycle of data, does not list any ancilliary data but accepts the compass readings as part of the main data tape.

Definitions

n or N	Number of terms per series (Note: SBWR and compass series have n/2 terms)
l or L	Number of lags in covariance analysis of buoy data.
m or M	Number of frequencies in spectral analysis of buoy data.
l′ or L′	Number of lags in covariance analysis of SBWR data.
m′ or M′	Number of frequencies in spectral analysis of SBWR data.
C_{ij}	Co-spectrum of series i, j.
Q_{ij}	Quad-spectrum of series i, j.
X_i	i = O(1)7 refers to one of the eight original series possibly in a modified form.
Y_i	i = O(1)6 refers to one of the six series required for programme -1.
a_i	i = O(1)6; proportionality factors for converting the first seven series to physical units

a_7, b_7	Factors converting buoy headings X_7 to radious $(a_7X_7 + b_7)$.
$E(\sigma,\Psi)$	Directional spectrum at frequency σ and direction Ψ clockwise from north (magnetic north for 68/B).
$E(\sigma)$	One dimensional wave spectrum.
$G(\sigma,\Psi)$	Angular distribution of energy. [Note: $E(\sigma,\Psi)=E(\sigma)G(\sigma,\Psi)$].
A_n, B_n	Cosine and sine components of angular spectrum.
e	The time interval in seconds between successive terms of each of series 1-6.
Block	Always refers to a block of 64 numbers of data. The main series X_i repeat in cycles of 8 numbers so there are 8 cycles/block in NIO 68 and 4 cycles/block in NIO 68/A and B. X_0 and X_7 repeat in cycles of 16
ζ	Wave height.

Tapes

1) Program tape
2) First part of data
3) Second part of data
4) Third part of data
5) Job description and parameters (also buoy headings in NIO 68 and 68/B).

Program Tape

(Capitals denote actual punching)

COMPILER EMA

Either: S001 NIO PROGRAM 68 CLOVERLEAF BUOY ANALYSIS 10/9/1965

or: S001 NIO PROGRAM 68/A CLOVERLEAF BUOY ANALYSIS 10/9/1965

or: S001 NIO PROGRAM 68/B CLOVERLEAF BUOY ANALYSIS 10/9/1965

In some early version of program 68 used for a special job the title is as follows:-

SD04 CREASE, NIO, CLOVERLEAF BUOY ANALYSIS NIO PROG 68

The date of the version in use should be substituted in the title in place of 10/9/1965

MAIN q
AUXILIARY (O, p) } see next section
DUMPS O

Then follows the program, ending with

*** Z

Program parameters

In a program of this length it is worth making reasonable attempts to economise on space as costs can be considerably reduced. The following lists give minimum specifications for these directives for different amounts of data and output. Whenever a directive is changed the program date should be amended to avoid confusion.

Chapter 1

X → 64

B → 600

C → 600

D → 600

[600 is the maximum number of errors allowed in the data and can be altered if required. Only half this number are allowed in the SBWR and compass channels].

Y → 64

Z → > N/2

The instruction M'= (an integer) occurring 5 lines after label 20 should be only slightly greater than the number of blocks of buoy data.

Chapter 2

X ⟶ > Max. of 4L, N/2

Y ⟶ > Max. of N(M+2)/4L, L

Z ⟶ > N(M+2)/4L

F ⟶ > M

Programme -1, Chapter 1

X ⟶ ≥ 8(L+1) and as large as possible within the bounds of the MAIN specification of other chapters.

Y ⟶ ≥ 36(L+1)

Π ⟶ 8

U ⟶ 8

V ⟶ 8

C ⟶ 8

W ⟶ 8

A ⟶ 8

The instruction A' = (an integer) occurring just after label 8 at start of chapter should be 10 less than the directive X.

Programme -1, Chapter 2

Y ⟶ > 36(L+1)

Z ⟶ > 36(M+1)

U ⟶ 8

V ⟶ 8

F ⟶ > 6(M+1)

G ⟶ > 6(M+1)

Chapter 0

X ⟶ > Max. of [N, Min.(4096, 8N)]

H ⟶ 1

Y ⟶ 511

Π ⟶ 10

V ⟶ 10

A ⟶ ≥ 4(M+1)

B ⟶ ≥ 4(M+1)

C ⟶ 2

Z ⟶ 64

The sum of the X and H variables must be a multiple of 512 (remember that the number of variables is 1 greater than the number in the directive) and X must be set before H, followed by Y.

The instruction ⟶ 25, (integer) ≥ N should have the integer set less than the X directive setting.

The MAIN directive q is equal to or greater than the maximum number of main variables in any one chapter. Remember that the number of variables is 1 greater than the number in the directives.

The AUXILIARY directive p is given by

$$p = \text{Max. of } D' + 6N,\ D' + 36(M+L+2),\ 64M'$$

where $D' = 1{\cdot}5N + 12M + 50$

and M' has the value set in Chapter 1.

Data Tapes

The tape from the data logger should be split (or copied) into three sections, the first two ending with a channel 64 "999" (see specification of channels below). This means that for NIO 68/A or 68/B the terms must be counted beginning with the term following the first 999 to determine the correct place for splitting the data.

The data must consist of three figure integers in the following format:-

NIO 68

Cycle	Series → 0	1	2	3	4	5	6	7
1	SBWR	ζ_x	ζ_y	$2\sqrt{3}\zeta_{xx}$	ζ_{tt}	$2\zeta_{xy}$	$2\zeta_{yy}$	(compass)
2	Wind speed	"	"	"	"	"	"	000
3	SBWR	"	"	"	"	"	"	(compass)
4	Ship's head	"	"	"	"	"	"	999
5	SBWR	"	"	"	"	"	"	(compass)
6	Wind Dirn.	"	"	"	"	"	"	Supply volts (500)
7	SBWR	"	"	"	"	"	"	(compass)
8	Ship's head	"	"	"	"	"	"	999

The compass readings are presumed faulty and are hence read in separately on INPUT 4

NIO 68/A and B

Cycle	Series → 0	1	2	3	4	5	6	7
1	SBWR	ζ_x	ζ_y	$2\sqrt{3}\zeta_{xx}$	ζ_{tt}	$2\zeta_{xy}$	$2\zeta_{yy}$	Compass
2	Serial No.	"	"	"	"	"	"	500
3	SBWR	"	"	"	"	"	"	Compass
4	Ship's head	"	"	"	"	"	"	999

The compass readings are presumed faulty in NIO 68/A and are hence read in separately on INPUT 4.

The serial No. and ship's head may be replaced by any other parameter required since no processing is carried out on these channels.

The scanning rate for both formats is taken to be 8/e channels/second. The numbers should be separated with two space characters and CRLF characters can conveniently occur after every 8 channels.

In the above tables:-

ζ_x is signal X, the X tilt to the buoy framework.

ζ_y is signal Y, the Y tilt of the buoy framework.

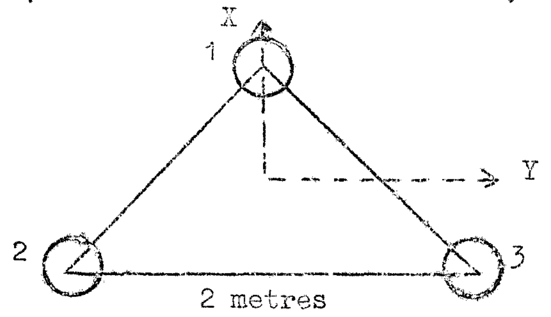
$2\sqrt{3}\zeta_{xx}$ is signal $2X_1 - (X_2 + X_3)$

ζ_{tt} is the buoy vertical acceleration.

$2\zeta_{xy}$ is signal $X_3 - X_2$

N.7 $2\zeta_{yy}$ is signal $Y_3 - Y_2$

where X_1 is the X tilt of float 1 etc.



The numerical factors arise from the dimensions of the buoy framework and imply units of metres for length. ζ_{tt} has units of g and t has units of seconds.

NIO 68 assumes an error in the SBWR series such that alternate terms are high by a fixed unknown amount. This is calculated and subtracted in the analysis. If the data is recorded correctly, this section of the program will have no effect.

First part of data

1) DATA
2) S001 RUN 27 PART 1 (or other title, exactly as used in the job description)
3) DATA TITLE
4) Title of data to be printed on output (one line only)
5) An integer (S') slightly less than the number of blocks of data in the full buoy record.
6) First part of the data

   followed by the terminator on a new line:-
7) 012345678

   then 12" runout and

***Z

Second part of data

1) DATA
2) S001 RUN 27 PART 2 (or other title, exactly as used in the job description)
3) Second part of the data

   followed by the terminator on a new line:-
4) 0123456789

   then 12" runout and

***Z

Third part of data

1) DATA
2) S001 RUN 27 PART 3 (or other title, exactly as used in the job description)
3) Third part of the data

   followed by the terminator on a new line:-
4) 1234567

   then 12" runout and

***Z

N.7

Job description and parameters

JOB

Job Number , Job Title

INPUT

0 S001 NIO PROGRAM 68 CLOVERLEAF BUOY ANALYSIS 10/9/1965
(or appropriate program number and date)

1 S001 RUN 27 PART 1

2 S001 RUN 27 PART 2

3 S001 RUN 27 PART 3
(or other titles exactly as on data tapes)

SELF = 4

OUTPUT

0 LINEPRINTER a LINES

STORE b BLOCKS

COMPUTING c INSTRUCTIONS

TAPE COMMON

0

a, b and c are calculated as follows:-

$a = 7(9L/8 + 3M + 100) + N/8 + 200$

$b = 11 + (\text{Main} + \text{Auxiliary} + 16000)/512$

$c = 6000 + NL/2 + 200M + 20N$

DATA

1) [NIO 68 and 68/A only] Series of compass readings to replace those on main data tapes. The number of terms in the series should be greater than the number of original compass readings and must terminate with

ƀ ƀ /

The compass readings should synchronise with the rest of the data by beginning after the first 999 in NIO 68/A or after the 999 of the first channel 64 in NIO 68.

2) Nine constants a_0 ...... a_7, b_7, to convert the six buoy outputs, SBWR and compass outputs to physical units. The first seven series are SBWR, ζ_x, ζ_y, ζ_{xx}, ζ_{tt}, ζ_{xy}, ζ_{yy}. Each constant should be the ratio of physical/computing units. The eighth and ninth factors convert the compass readings x_7 to radians by the formula $y_7 = a_7 x_7 + b_7$. Note that in NIO 68/B b_7 should include the local magnetic variation in radians.

3) An integer (L) equal to the number of covariance lags required in the buoy analysis.

4) An integer (M) equal to the number of frequencies required.

5) Six conversion factors to physical units of the series Y_j, $j = 1(1)6$ for programme -1. These will all be unity since conversion is done by 2) above. The series enter programme -1 in the order: ζ_{tt}, ζ_x, ζ_y, ζ_{xx}, ζ_{yy}, ζ_{xy}.

6) Six numbers being the time lags of the series relative to the first on entry to programme -1, in decimal fractions of a cycle (8 channels). These will be

0·625 0·3125 0·3125 0·75 0·75 0·75

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7) (6M + 1) amplitude and phase reponse factors at each frequency, starting at zero, for each of the six series. i.e. for series j, a unit input at frequency s/2le cycles/second has a response

$$a_s^j \exp(i\phi_s^j) \qquad [\phi \text{ in degrees}]$$

The series are in the order given in 5) above.

The responses are listed as follows:-

$$\begin{array}{llllll} a_0^1 & \phi_0^1 & a_0^2 & \phi_0^2 & \ldots\ldots\ldots\ldots & a_0^6 \quad \phi_0^6 \\ a_1^1 & \phi_1^1 & \ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & \\ \vdots & & & & & \\ a_m^1 & \phi_m^1 & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & a_m^6 \quad \phi_m^6 \end{array}$$

8) The time interval (e) in seconds between successive terms of any one of the six series Y_j,

e.g. 0·5

9) An integer (L′) equal to the number of covariance lags required in the SBWR analysis. This value of L′ should normally be half that specified in 3) above for L.

10) An integer (M′) equal to the number of frequencies required in the SBWR analysis.

11) A conversion factor to physical units for Y_1, equal to unity since conversion is done by 2) above.

12) (M + 1) amplitude response factors for the buoy at frequencies s/4le, s = 0(1)M. [i.e. a_0^1 to a_m^1 as in 7)].

13) A conversion factor to physical units for the SBWR series, equal to unity since conversion is done by 2) above.

14) (M′ + 1) amplitude response factors for the SBWR.

12" runout followed by

* * * Z

Output

1) Data title

2) (MIO 68 only) Listing of wind speed and direction, ship's heading and supply voltage; there is one set of readings for every 8 cycles of main data.

3) Detected errors in input data.
These are listed by block number, channel number (1-64) and a code indicating the type of fault as follows:

-1, a faulty 999
5 -2, a 5 digit number
7 -2, a 7 digit number
8 -2, an 8 digit number
1, 2 or 4, for 1, 2 or 4 digit numbers
and 3 for a 6 digit number

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3

4) The number of terms in each of series Y_i.

5) The time lag of each series Y_i in units of the cycle time of the original data.

6) The means and variances of each series in physical units.

7) The lagged normalised auto covariances of each series Y_i.

8) The cross covariances at zero lag for each pair of series.

9) The lagged normalised cross covariances for each pair of series for lags -L(1)L.

10) The frequency increment 1/2le between successive spectral estimates in cycles/second.

11) Auto spectra of each series Y_i in (physical units)^2/cycle/second.

12) Cross spectra for each pair of series consisting of

a) Co-spectra C_{ij}
b) Quad-spectra Q_{ij} } in (physical units)^2/cycle/second
c) Intensity $(C^2_{ij} + Q^2_{ij})^{\frac{1}{2}}$

d) Coherence $\dfrac{C_{ij}^2 + Q_{ij}^2}{C_{ii} \cdot C_{jj}}$

e) Phase $\tan^{-1} \dfrac{Q_{ij}}{C_{ij}}$ in degrees

13) The first four angular harmonics of $G(\sigma, \psi)$ (cosine, A_j; sine, B_j.) are printed at each frequency together with the ratios R_1, R_2, R_3 (see Method).

14) The normalised angular distribution of $G(\sigma, \psi)$ at intervals of 15° is printed at each frequency.

15) Comparison of the SBWR and doubly integrated acceleration record from the buoy.

a) Frequency increment for this section 1/2l'(2e) c/s.
b) Number of terms used in the SBWR analysis.
c) The SBWR auto spectrum.
d) The doubly integrated acceleration spectrum, $E(\sigma)$
e) The ratio of d) to c).

The lowest frequency printed is at [int. part (l'/12)]/4l'e c/s.

The highest frequency printed is at m'/4l'e c/s.

Restrictions

$N < 3{,}000$ when $M' = 375$ (Chapter 1)

$< 3{,}800$ when $M' = 475$

M' may be increased to a maximum of 600 if required ($N < 4{,}800$) corresponding to 25 mins. and 31 mins. of data respectively.

Maximum number of errors = 600 for channels with sampling interval e.
and 300 for channels with sampling interval 2e

$1 \leqslant M \leqslant L \leqslant N/2$

$1 \leqslant M' \leqslant L' \leqslant$ integral part $\left(\frac{N}{4}\right)$

Cost

For N = 2400, L = 100 M = 50, L' = M' = 50	→	N.I.O. 68/B	£100
N = 2520, L = 100 M = 50, L' = M' = 50	→	N.I.O. 68	£110

Method

1) The first step is entry to Chapter 1 which reads and detects errors in the series. The types of error detected are

a) A number not comprising three digits
b) A spurious character
c) 999 not occurring at the correct place in the recording sequence.

(a) and (b) are bracketed together on the page.

Faulty values in the data are temporarily replaced by -100. The data is read in on the "Sunvic logger" routine one character at a time. This routine enables each number to be tested for spurious characters and the number of digits it contains to be recorded. The block number and channel number in which any fault occurs are recorded. The data tape must be divided into three parts. This necessitates the use of three terminators. The first tape (on INPUT 1) is terminated by a 9 digit number, the second by a 10 digit number and the third by a 7 digit number. In case there should be a 7 digit number within the record, a facility has been included which causes the Sunvic logger routine to ignore a 7 digit number as a terminator unless it occurs after a specified length of record. The parameter S' is used for this purpose and should be made equal to an integer slightly less than the expected number of blocks in the record.

The series are separated and stored behind each other in the auxiliary store, some ancillary data being printed out in NIO 68 and not stored.

Next the list of detected errors is printed, or the words "NO FAULTS".

2) From now on the program is concerned with the eight input series X_i, $i = 0(1)7$ which represent the SBWR, ζ_x, ζ_y, ζ_{xx}, ζ_{tt}, ζ_{xy}, ζ_{yy} and buoy heading respectively. (The SBWR and buoy heading occur only on odd cycles, starting at cycle 1). The program halts if N is greater than 8M', set in Chapter 1.

3) [NIO 68 and 68/A only]. The compass readings in X_7 are incorrect so the visual observations of the buoy are read in on INPUT 4 and substituted for the existing series. The program halts if there are insufficient visual compass readings to overwrite all the original series.

4) Each of the eight series X_i are now examined for errors (-100) and the series means found. Means are subtracted from series $i = 0(1)6$ to form new series X_i. Errors are then replaced by zero, and finally the X_i, $i = 0(1)6$ are multiplied by factors a_i to convert them to physical units, and the buoy headings x_7 are converted to radians by the

I.7

linear transformation $a_7 x_7 + b_7$. The buoy headings are linearly interpolated to the previously unoccupied even numbered cycles.

On each cycle, the two slopes X_1 and X_2 must be referred to a common time before rotation of the axes. Thus X_1 is advanced 1/16 cycle and X_2 lagged 1/16 cycle by linear interpolation. Similarly the curvature X_i is advanced 1/4 cycle and X_6 lagged 1/8 cycle. X_5 and X_4 are unchanged. It is not necessary at this stage to refer the slope series to the same time as the curvature series. The series continue to be stored in auxiliary store and are still referred to as X_i.

5) Magnetic tape is now used to rearrange data sequentially so that the first cycle X_0, X_1 ...... X_7 is followed by the second cycle, etc.

6) Blocks of 512 terms (i.e. 64 of each series) are brought down from the tape and X_1, X_2 combined to form new series Y_2, Y_3 referred to left-handed axes in directions north and east (Magnetic north in NIO 68/B):-

$$Y_2 + iY_3 = (X_1 + iX_2) \exp (i\theta)$$

where θ is the buoy heading at the common time of X_1 and X_2, interpolated from adjacent values of X_7.

Similiarly, the curvature series X_3, X_5, X_6 are combined to form Y_4, Y_5, Y_6 the curvatures referred to the fixed axes, by solving

$$Y_2 + Y_5 = X_3 + X_6$$

and

$$Y_4 - Y_5 + 2iY_6 = (X_3 - X_6 + 2iX_5) \exp (2i\theta)$$

where θ is the buoy heading at the common time of X_3, X_5, X_6

X_4 (acceleration) is placed unchanged in Y_1.

Thus the Y_i contain the following data referred to axes X (north) and Y (east):-

Y_1 - ζ_{tt}
Y_2 - ζ_x
Y_3 - ζ_y
Y_4 - ζ_{xx}
Y_5 - ζ_{yy}
Y_6 - ζ_{xy}

These are stored in the auxiliary store, overwriting the series X_i.

7) The auto and cross spectra of $Y_i, i = 1(1)6$ are now found using NIO programme -1 (This is outlined in the program descriptions of NIO 67 and NIO 74). Various parameters for this programme require to be set or read in before entry to it. There are a few special points with regard to these parameters:-

a) Parameters are required for conversion from digital to physical units. Since this is done elsewhere, these parameters will all be unity.

b) Parameters are required to describe the amplitude and frequency response of each series at each of the (m+1) frequencies for which the spectra are found. Now as Y_2, Y_3 and Y_4, Y_5, Y_6 are derived by combination of the original series X_1, X_2 and X_3, X_5, X_6 it follows that the amplitude and phase responses of the series Y_2, Y_3 and similiarly Y_4, Y_5, Y_6 must be the same. The analysis could have been done without this assumption but would have been much more costly. In fact the buoy has been designed to approach this ideal of uniform response of all channels.

8) The co- and quad-spectra (C_{ij} and Q_{ij}) of the ith, jth series are now combined to give the directional spectra $E(\sigma, \psi)$

If $E(\sigma, \psi) = E(\sigma)\, G(\sigma, \psi)$

and $G(\sigma, \psi) = \frac{1}{\Pi}\left[\frac{1}{2} + \sum_n (A_n(\sigma) \cos n\psi + B_n(\sigma) \sin n\psi)\right]$

then
$$A_1 = -Q_{12}/C_{11}$$
$$A_2 = (C_{22} - C_{33})/(C_{22} + C_{33})$$
$$A_3 = (Q_{24} - 3Q_{25})/(C_{14} + C_{15})$$
$$A_4 = (C_{44} - 6C_{45} + C_{55})/(C_{44} + 2C_{45} + C_{55})$$
$$B_1 = -Q_{13}/C_{11}$$
$$B_2 = 2C_{23}/(C_{22} + C_{33})$$
$$B_3 = (3Q_{34} - Q_{35})/(C_{14} + C_{15})$$
and
$$B_4 = 4(C_{46} - C_{56})/(C_{44} + 2C_{45} + C_{55})$$

The following ratios are also found
$$R_1 = C_{11}/(C_{22} + C_{33})$$
$$R_2 = H\, C_{11}/(C_{14} + C_{15})$$
$$R_3 = H^2 C_{11}/(C_{44} + 2C_{45} + C_{55})$$

where $H = \dfrac{\Pi^2 s^2}{9{\cdot}8 L^2 e^2}$ $(s = 1(1)m)$

These should, for small amplitude gravity waves recorded by the buoy, be unity. Departures might indicate errors in frequency response or a low signal-to-noise ratio and might help in a selection of the useful part of the spectrum.

$G_8(\sigma, \psi)$ is then evaluated using weights W_n as follows:

$$G_8(\sigma, \psi) = \frac{1}{\Pi}\left[\frac{1}{2} + \sum_{n=1}^{4} W_n(A_n \cos n\psi + B_n \sin n\psi)\right]$$

where
$$W_1 = 0{\cdot}8889$$
$$W_2 = 0{\cdot}6222$$
$$W_3 = 0{\cdot}3394$$
$$W_4 = 0{\cdot}1414$$

N.7 giving a filter approximately proportional to $\cos^{16\frac{1}{2}}\psi$ which has r.m.s. width ± 29°.

9) The final section compares the doubly integrated acceleration spectrum with the SBWR spectrum. The acceleration data is first fourier analysed and divided by the square of the frequency. The sum of squares of cosine and sine components is then smoothed over a number of harmonics to give approximately the same bandwidth as the power spectrum of the SBWR.

Since the SBWR is recorded at time intervals of 2e seconds, the frequency increment in the power spectrum is 1/2l'(2e) cycles/second. If there are (n+1) terms in the series for buoy acceleration then the fourier series increments are 1/ne cycles/second.

For the two increments to be commensurate ne/4l'e must be an integer K' say. Thus the smoothing should produce values of energy at intervals of K'/ne cycles/second. To satisfy these requirements n may have to be reduced. The power spectrum of the SBWR is found in the usual way at frequencies s/4l'e s = j'(1)m' and the analysis is started at j' = integral part of l'/12.

For the two spectra to be comparable the bandwidth should be approximately the same. This is achieved by applying a triangular smoothing function symmetrical about the required frequencies to the fourier components of the doubly integrated acceletation record. It extends 2K' harmonics on either side of the central harmonics.

Thus, the modulus of the individual harmonics of the acceleration record is

$$C_p = \left| \frac{2}{n} \sum_{r=0}^{n} y_r \exp\left(\frac{2\pi i r p}{n}\right) \right|$$

where p is the harmonic and y_r is the series of accelerations.

The modulus of the displacement spectrum C_p' is

$$C_p' = C_p \Big/ \left(\frac{2\pi p}{ne}\right)^2 \quad \text{energy/cycle/frequency increment.}$$

Smoothing with the triangular filter yields the smoothed spectrum

$$E(p) = \frac{g^2}{4K'^2} \sum_{j=-2K'}^{2K'} (2K' - |j|) \; C'^2_{(p+j)} \; ne$$

where g (gravity) is required since the acceleration record is in units of g (9·8 metres/sec^2) and the term ne converts the spectrum to units of energy/cycle/second.

Thus

$$E(p) = \frac{g^2 n^3 e^5}{16\pi^4 K'^2} \sum_{j=-2K'}^{2K'} (2K' - |j|) \sum_{r=0}^{n} \left| \frac{y_r \exp\left(\frac{2\pi i [p+j] r}{n}\right)}{(p+j)^2} \right|^2$$

and E(p) is to be evaluated from j'K' to mK' at intervals of K' harmonics, but because of the smoothing the original harmonics are required from p = (j' − 2)K' to p = (m'+2)K'.

E(p), and similarly the power spectrum of the SBWR, estimates E(σ) at frequencies s/4l'e, s = j' (1)K'.

Note that on entering this section of the program, l and m may be reset to l′ and m′ to enable the frequency increment of the spectra to be the same as that used in the cross spectral analysis.

Notes

Chapter 1 of this program was written by staff of the British Ship Research Association.

If a version of MIO 68 is required with compass readings read from the main data tape the following modifications should be made:-

a) Item 1) under Job description and parameters should be omitted.

b) The group of instructions in Chapter 0 starting at label 25 and ending with END immediately before label 21 should be removed.

c) The instruction —> 25, ≥N should be altered to —> 21, ≥N.

d) In Chapter 1 the last block of Ø7 transfers to auxiliary store should be altered to

Ø7 (B′ + 48M′)X_8, 1

Ø7 (B′ + 48M′ + 2) X_{24}, 1

Ø7 (B′ + 48M′ + 4) X_{40}, 1

Ø7 (B′ + 48M′ + 6) X_{56}, 1

Programmer: JAMES CREASE

N.I.O. PROGRAM 68/C

Title	Analysis of Cloverleaf buoy records
Language	EMA
Machine	ATLAS 1

Purpose

To compute the directional spectrum of sea waves from the output of the N.I.O. Cloverleaf buoy. This program is similar to N.I.O. 68/B, the differences being outlined below (page references are to the program description of N.I.O. 68/B).

a) p.4. The data sequence is assumed to be:

Series / Cycle	0	1	2	3	4	5	6	7
1	SBWR	ζ_x	ζ_y	$2\sqrt{3}\zeta_{xx}$	ζ_{tt}	$2\zeta_{xy}$	$2\zeta_{yy}$	Compass

b) p.2 and p.8. The maximum number of errors allowed is 6 per series (or 3 in SBWR series). If there are more errors than this the program will halt after listing all the errors in the data.

c) p.3. An additional directive is present in chapter 0 i.e. D $\rightarrow$ 16.

d) p.4. $D' = 1{\cdot}5N + 12M + 65$.

e) p.5. The data is assumed to be on one input document, normally a magnetic tape. The terminator should be a seven-digit integer, and the data title and S' are now read from the parameter tape, input 2.

f) p.1. The compass readings are assumed to be correct, and part of the main data tape.

g) p.1 and p.3. Programme -5 is used in place of programme -1 to compute the auto- and cross-spectra. The description of programme -5 should be referred to for full details of the differences but certain directives are now increased:

Chapter 1 $X \geq 8(2L+1)$

also B $\rightarrow$ 15 is required.

Chapter 2 B $\rightarrow$ 15 is required.

Larger directives than those specified may be used in which case the MAIN store requirement need only be as large as necessary for the particular data parameters in use.

h) p.8. Cross-spectra of series 1,6 2,6 and 3,6 are omitted.

i) Certain quantities are output to magnetic tape in addition to the lineprinter output. See 'output' below.

Inputs

(Assuming program on paper tape)

0) Program
1) Data
2) Job description and parameters

(Assuming program on magnetic tape)

0) Job description
1) Data
2) Parameters

Program

COMPILER EMA

S001 NIO PROGRAM 68/C CLOVERLEAF BUOY ANALYSIS 21/6/1967
(or date of appropriate version)

MAIN 70000

AUXILIARY (0,32500)

DUMPS 0

Then follows the program, ending with

*** Z

Values of MAIN and AUXILIARY given above are for up to 31 minutes of data when L = 100, M = 50 and the sampling interval is $\frac{1}{2}$ second.

Job description

(Assuming program and data on magnetic tape)

e.g. JOB

S001054301, CLOVERLEAF BUOY ANALYSIS

INPUT

2 S001 RUN 17/1 PARAMETERS

INPUT TAPE 1/e/0 — e = starting block on input tape

1 S001 RUN 17/1 — a as defined in NIO 68/B

OUTPUT — d = starting block on output tape.

0 LINEPRINTER (a-150) LINES

OUTPUT 1 — e = 8M

TAPE d/LINEPRINTER e LINES

OCEANS L0094*WRITE PERMIT — Output tape title for results

STORE b/80 BLOCKS — b = 95 (25 mins. data) or 105 (31 mins.)

COMPUTING c INSTRUCTIONS

EXECUTION $\left(\frac{c}{250\text{x}60} + 4\right)$ MINUTES — c = 120000 (25 mins.) or 190000 (31 mins.)

TAPE COMMON

0

TAPE

1 OCEANS L0095*WRITE INHIBIT — Input tape with data

TAPE

99 SLOAD2 LS169*WRITE INHIBIT — Tape containing program

COMPILER LOAD

009/NIO 68/C — Program file number

***Z (or ***C)

Data

Input stream 1:-

The data on magnetic tape commencing with an SBWR reading and ending with a compass reading followed by a seven-digit terminator (e.g. 1234567). [For compatibility with other programs the terminator >> or)) may also be present between the end of the data and the seven-digit integer]. The data begins in block e of the input magnetic tape.

Input stream 2:-

DATA

S001 RUN 17/1 PARAMETERS (e.g.)

DATA TITLE

One line of title

1) An integer (S′) (see p.5)

2) Nine constants a_0 .... a_7, b_7, to convert etc. (as on p.6 to 7) (Item 7 line 1 should read "(6M+6) amplitude and ......")

The data ends

*** Z

<u>Output</u>

Stream 0:-

As for N.I.O. 68/B except that the cross-spectra of series 1,6 2,6 and 3,6 are omitted.

Stream 1:-

On magnetic tape beginning at block d.

1) Frequency number (s) followed by the coefficients a_1-a_4 and on the next line the coefficients b_1-b_4.

2) Frequency number (s) followed by the angular distribution (normalised) at intervals of 15° beginning at 0° (line 1) and 180° (line 2).

3) Frequency number (s, where **s** = INTPT [L/12+0·01] (1) M) followed by the doubly integrated heave spectrum, $E(\sigma)$.

<u>Programmers</u>

Chapter 1: British Ship Research Association

Chapter 2: James Crease

Programme -5 and chapter 0: James Crease and Brian Hinde.

Title	Power spectrum of a single series.
Code	Mercury CHLF 3/4
Machine	MERCURY
Purpose	Given a time series, sampled at intervals of t seconds to compute the mean, variance, auto-covariances with lags 0-L, and from these to compute the auto-spectra for frequencies 0 to M/2Lt.
Tapes	(1) Program. (2) Parameters and data.

Parameters

(a) X = Run number. (Two decimal places).

(b) 4 integers:

J,K where 100J + K is the number of terms in the series

L = Largest lag required for covariances (Spectral analysis is in terms of harmonics of the basic frequency 0·5/Lt).

M = Largest order of harmonic in auto-spectra, frequency 0·5M/Lt.

(c) 2 numbers (fixed decimal point):

D = Maximum allowable difference between consecutive values of the series.

F = Calibration factor, physical units per digit of the series.

T = Sampling interval, t units of time.

Data

A series of numbers, positive or negative, with or without decimal point:

$x_0, x_1, \ldots\ldots\ x_{n-1}$. where n = 100J + K.

(At present, the form of print-out for the digital mean and variance assumes the terms lie between -1000 and +1000 , but this is not essential to the program as a whole).

Restrictions

$J \geq 1$ ($n \nless 100$)

$0 \leq K \leq 99$

$100J + K + 2L \leq 20{,}476$ with CHLF 4
$\qquad\qquad\qquad\quad 12{,}284$ with CHLF 3

$T < 100$

$2 \leq L \leq 240$

$1 \leq M \leq L$

Operation

(1) Read in program.

(2) Read in parameters and data.

As it reads in the data tape the computer tests each difference $(x_r - x_{r-1})$, and prints out the value of r in the form j,k where r = j + k for any difference exceeding the given D. All such values of r are printed and unless there is none the computation will not proceed any further, but returns to ask for another set of parameters and data, starting with X. If there are no "errors" in the data, the computations and print-out follow in due course. Operation (2) may then be repeated starting with parameter X.

Output

(a) Title

(b) Run number

(c) Error numbers r (described above).

(d) If there is nothing in (c):-

(i) Number of terms in the series (100J + K).

(ii) Mean value in digital units.

(iii) Variance in digital units, variance in physical units.

(e) For s = O(1)L prints (1 line for each s):-

s

Ψ_{11} = Normalised auto-covariance with s lags.

(f) (i) Frequency increment 0·5/Lt, cycles/unit time.

(ii) For s = O(1)M prints (1 line for each s):-

s

E_{11} = Energy spectrum (spectrum of variance) in physical units^2/cycle/unit time.

Time

2 minutes to read in the program tape.
The following is a guide to the computation time:

With J = 12, K = O, L = M = 50, time = 4 minutes (D not exceeded).

Method

Mean value: $X = \frac{1}{n} \Sigma x$

Each value of x is then replaced by

$$x' = x - X$$

and the variance computed:

$V_{11} = \frac{1}{n-1} \Sigma x'^2$ (x F^2 for physical units)

Normalised auto-covariances:

$$(s),\ \Psi_{11}(s) = [(n-s-1)V_{11}]^{-1} \sum_{r=o}^{n-s-1} x'_{r+s}\ x'_r$$

where s is the lag number

Auto-spectra:

$$E_{11}(s) = \tfrac{1}{4}e_{11}\ (s-1) + \tfrac{1}{2}e_{11}(s) + \tfrac{1}{4}\ e_{11}(s+1),\ s \neq O$$

$$E_{11}(O) = \tfrac{1}{2}e_{11}\ (O) + \tfrac{1}{2}e_{11}(1)$$

where $e_{11}(s) = 4F^2TV_{11} - \sum_{r=o}^{L}{}'' \ \Psi_{11}(r) \cos\left(\frac{rs\pi}{L}\right)$

and Σ'' means the sum with the first and last terms halved, and s is the harmonic number.

Notes

This program is based on N.I.O. Program 60 - Cross spectrum of two series, written by D. E. Cartwright.

Programmer

Mrs. W. Wilson

N.I.O. PROGRAM 70

Title Revision of Matthews Tables (1, Mediterranean Sea)

Language EMA

Machine ATLAS I

Purpose An echo-sounding machine is set to a standard sound velocity, e.g. 1500 m/sec., and gives readings of uncorrected depth to which Matthews corrections are applied to yield a 'true' or 'corrected' depth.

The object of this project was to determine the corrections to be applied to sound velocity values at various locations and depths in order to investigate the accuracy of the existing Matthews Tables and new methods of presentation of the data.

In this instance the program was used with both new and existing (i.e. already punched) hydrographic data for the Mediterranean Sea; a study of the output was made and a report written by H. Charnock and J. Crease; this was published as NIO Internal Report No. A26, JULY 1966.

Inputs

1) Program
2) Data
3) Job description, parameters and data titles.

Program This starts

COMPILER EMA

SOO2 NIO PROGRAM 70 MATTHEWS REVISION 1, 17/11/1967

MAIN 700

AUXILIARY (0, 0)

DUMPS 0

Then follows the program, ending with

CLOSE

followed by basic data, which consists of 101 numbers, and ending

***Z

Data This starts with the word

DATA

and is followed by one line of title; this will usually start SOO2 and be followed by a description of the data. Then follow the parameters and data, exactly as described in the descriptions of the various versions of NIO PROGRAM 58 (in the section labelled "Parameters and Data"). [In practice, the second parameter m and the group of parameters a_i are not used from here but from input 3; they must appear in the data section (i.e. input 2) because previously - punched data will have them here, but as they may not be the required parameters for this analysis they are superceded by those in input 3.]

The data ends with

END OF DATA

>

***Z

Job description, parameters and data titles

This input starts

JOB

Job number, data link number and job title

INPUT

1 SO02 NIO PROGRAM 70 MATTHEWS REVISION 1, 17/11/1967

2 SO02 data title, exactly as on input 2

SELF = 3

OUTPUT

0 LINEPRINTER l LINES

STORE 25/70 BLOCKS

COMPUTING q INSTRUCTIONS

where l = 200 + 70 (no. of stations)

and q = 1500 + 150 (no. of stations)

The job description is followed by blank tape and then the word

DATA

Then follow m and a_i as defined in NIO PROGRAM 58, a_i being the standard pressures and intervals for which interpolation is required.

Then comes

DATA TITLE

followed by one line of descriptive title, such as ship's name and station number, position and date. On a new line is punched an integer, being the nearest integer latitude for that station.

The sequence from DATA TITLE to the integer value of latitude is repeated for as many stations as are in the data, and in the same order. This is the source of the titles which are printed on the output before the output at observed depths. The tape ends

***Z

Output

This is basically as for NIO PROGRAM 58 (see that program description for details) with some quantities omitted. The output for each station consists of:-

station title
a) results at observed depths
b) results at standard depths

Sections a) and b) are on separate pages on the line printer output.

The output for a) (at observed depths) consists of:-

sample number

depth, metres

correction, i.e. the Matthews correction (given rather inconveniently in this program as a function of true depth)

pressure, db

sound velocity, metres/second

sounding velocity, metres/second

potential temperature, °C

N.7. potential density, i.e. σ_θ

The output for b) (at standard depths) consists of:-
depth, metres
correction (as above)
interpolation error
sound velocity, metres/second
interpolation error
sounding velocity, metres/second
interpolation error

Cost	About 5/- per station
Method	As for NIO PROGRAM 58
Programmer	James Crease

Title Current meter observations (using paired current meters).

Language EMA

Machine ATLAS I

Purpose To resolve current meter observations into northerly and easterly components, and then to calculate the components at the deeper meter relative to those at the shallow meter. They are then compounded into a velocity and direction.

Inputs
0) Program
1) Job and data

Program The program starts

COMPILER EMA

SC72 NIO PROGRAM 71 CURRENT METER OBSERVATIONS 7/9/1966
(or date of appropriate version)

and ends with

CLOSE

***Z

Job description In the job description, the number of lines of output on channel 0 = (200 + total number of sets of observations + 10 (number of stations)),

the store required = 15/70 blocks,

the number of computing instructions = 200 + 4 (total number of observations).

Data A block of data consists of:-

station number

number of sets of observations at that station

the relevant sets of observations, where a set of observations consists of:-

deep meter velocity in knots

deep meter direction (true)

shallow meter velocity in knots

shallow meter direction (true)

New blocks of data may follow; the last block must be followed by

>

and ***Z

Output There are six headed columns as follows:-

the easterly and northerly components of the deep meter readings

the easterly and northerly components relative to the readings at the shallow meter,

the compounded velocity and direction relative to the shallow meter readings.

Programmers Stewart Kempster, James Crease.

N.I.O. PROGRAM 72

Title Interpolation of gap up to 100 hrs. in hourly tidal series.

Code Mercury Autocode Machine MERCURY

Purpose To replace missing hourly tide gauge data by a harmonic development with polynomial coefficients based on about 8 days of good data surrounding the gap.

Tapes (1) Program (2) Parameters and data.

Parameters and data Define a "check number" as any negative number < -999 (e.g. -1000, -1001, -2000, -3158, ... may be used as check numbers).

An input group consists of:

Integer parameters I, J, K,
a sequence of I readings, preceding the gap,
a "check number", see above,
a sequence of K readings, following the gap,
another "check number".

I and K must be one of the following five pairs:

I = 45	K = 145
I = 70	K = 120
I = 95	K = 95
I = 120	K = 70
I = 145	K = 45

of which I = 95 = K is recommended if possible.

J is the number of missing readings, and should be in the range $4 \leqslant J \leqslant 100$. The program will work formally for values of J up to 120, but the real accuracy of the interpolation may be doubtful for large values of J. The two sets of data are hourly readings in chronological order. They may be integers or decimal, positive or negative, but never below -999. Since the interpolated values are printed in the Autocode format "4,2" it may be advisable to multiply the original data by a power of 10 so as to be integers $< 10^4$, or to have $\leqslant 2$ d.p.

End indication Program and data tapes end with →

Operation Read program, then data tape. The results for one input group are punched, then the next group is read, or called for. Any number of input groups may be used.

Output (1) I, J, K, and check numbers (see "Failures"),

(2) 6 "mean sea levels", and their average value, F^1,

(3) 6 weights for self-prediction 1 hr. ahead, variance of "observed" and of "obs-pred", and predictions (less F^1) 1 hr. ahead of I block and K block,

(4)-(6) As (3) for self predictions 2 hr. ahead, 1 hr. behind, 2 hrs. behind,

(7)-(12) Central time and harmonic coefficients $A_0 - A_6$, $B_0 - B_6$, for 6 50-hr. analyses,

(13) 1st two and last two self predictions within gap (repeated from (3), with F^1 added),

(14) Complete set of J interpolations, 6 values per line.

Restrictions See "parameters and data".

Failures 777777 printed after (1) means that I and K do not correspond to one of the 5 pairs specified above.

888888 printed after (1) means that the block of data before the first check number consists of more or less than I numbers.

999999 printed after (1) means that the 2nd block of data consists of more or less than K numbers.

If either of these 3 faults occur, the program skips through the appropriate check numbers, and re-starts at (1) with the next input group.

N.B. If any of the check numbers is missing or invalid, or if the data contains a number < -999, the program goes berserk and fails to deal properly with any subsequent data.

Time About 1·3 + 1·7 n minutes, where n is the number of input groups.

Method Calling the data F(t), with the first value of t at 2 hours, we refer to six times T_1 --- T_6 defined by

t = 24, then steps of 25 h. to I-21

and (I+J+24), then steps of 25 h. to (I+J+K-21).

We also use central time, defined as

$t = T_0$ = integral part of $\frac{1}{2}(I+J+K+3)$, and write t^1 for $t-T_0$.

The six "mean sea levels" (2) are the results of the Doodson "Z_0 filter" applied at T_1 ---T_6. Their average F^1 is subtracted from all data, and cross covariances with lags 0 - 7 h. computed from 150 times, to derive least-square self-prediction formulae, (F(t) now written for $F(t)-F^1$):

$$F(t+n) \sim \sum_{i=0}^{5} W_i^{(n)} F(t-i), \quad F(t-n) \sim \sum_{i=0}^{5} W_i^{(-n)} F(t+i), \quad n=1,\ 2 \text{ hours}$$

The four sets of weights W are printed in (3) - (6) with appropriate variances and the formulae used to supply "pseudo" values of F(t) at

t = 0,1; I+2, I+3; I+J+1; I+J+K+2, I+J+K+3

The harmonic coefficients (7) - (12) are computed from 49-hour segments of data with a "cosine taper" to reduce sideband effects:

$$A_n(T_p^1) + i\,B_n(T_p^1) = (1/25)\exp(in\Omega T_p^1)\sum_{r=-24}^{24} K_r F(T_p+r)\exp(in\Omega r)$$

(n = 1(1)6, half this value for n = 0).

where $T_p^1 = T_p - T_0$, $\Omega = 2\pi/25$, and $K_r = 1 + \cos\frac{1}{2}r\Omega$.

$r = \pm 25$ is not used because $K_r = 0$ for these values.

The computation method is "Watt iteration" outwards from r = 0.

It is now assumed that $A_n(t)$, $B_n(t)$ vary but slowly over the permitted time range, as is valid for tidal data whose frequencies are concentrated around (n/25) cph, n = 0(1)6. Accordingly, the coefficients of a quintic polynominal in $x = 0{\cdot}01(t-T_0)$ are evaluated to fit each of the 13 sets of 6 estimates $A_n(t^1)$ or $B_n(t^1)$, exactly. Then, for each value of

t' within the "gap" the values of A_n and B_n are computed from the polynomials, and the interpolation

$$F(t') = F'+A_0(t') + \sum_{n=1}^{6}\left[A_n(t')\cos n\Omega t' + B_n(t')\sin n\Omega t'\right],$$

for $t' = I+2-T_0$ to $I+J+1-T_0$,

printed as (14).

Notes

(1) The two "self-predictions" at each end of the gap are intended for comparison with the corresponding harmonic interpolations (14), as a check whether the data immediately preceding or following the gap is seriously disturbed by meteorology. In this case, the self-predictions may be more accurate than those depending on A_n and B_n at the nearest value of T_p, since the latter are averages over about 50 hours, scarcely affected by the end data because of the smallness of their K_r values. A reasonable compromise, avoiding the discontinuity, is to take the selfprediction as the first interpolated value, the average of self-prediction and (14) for the second, and of course (14) for subsequent interpolations, with a similar adjustment at the other end of the gap.

(2) The interpolations are of course valueless if J does not correspond exactly to the number of hourly times between the given data.

(3) The input data should be checked graphically for smoothness.

(4) The program is no use for interpolating gaps in time series other than tidal, or for tidal data at intervals other than hourly.

Programmer D.E. Cartwright.

N.I.O. PROGRAM 72 (ATLAS)

Title	Interpolation of gap up to 100 hrs. in hourly tidal series.
Language	EMA
Machine	ATLAS 1
Purpose	This program is identical in purpose and method to the Mercury version of NIO 72.
Inputs	0) Program 1) Job description and data

Program

Begins:

```
COMPILER EMA
S002 NIO PROGRAM 72 (ATLAS) TIDAL GAPS 20/9/1966
```

and ends

```
* * * Z
```

Job description and data

```
JOB
Job number, Job title
INPUT
0 S002 NIO PROGRAM 72 (ATLAS) TIDAL GAPS 20/9/1966
                    (or date of appropriate version)
SELF = 1
OUTPUT
0 FIVE-HOLE  PUNCH n BLOCKS
STORE 20/60 BLOCKS
COMPUTING p INSTRUCTIONS
```

where n = the number of input groups

and p = 1000 + 200n

```
DATA
```

then follows the data, the last group ending with / on a new line and

```
* * * Z
```

Cost — About £1 per input group

The output and method are identical with NIO 72.