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WORMLEY, GODALMING, SURREY

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Introductory note (February 1972)

The following report was prepared in September 1968, and distributed only to certain interested parties, namely the Storm Tide Warning Service, the Hydrographic Department M.O.D., and the authors. When some interest in the results was recently revived, it was found that the few distributed copies of the report had been lost, presumably because the report had no obvious 'tag' for filing purposes. To remedy this situation, the principal author's copy is herewith re-typed, 3½ years later, and issued as a N.I.O. Internal Report.

This report is primarily the work of Cartwright; contributions from Rossiter were subsequently incorporated and the whole is submitted jointly to emphasise unanimity in respect of the conclusions. In reading the report is should be borne in mind that Southend is a port with moderate shallow water tides. Similarly investigations are in progress for other ports with different degrees of shallow water distortion.

This account supersedes an interim report of January 1968, which covered Southend predictions for 1962 and early 1966 only. The whole 'storm surge season' of 1967/68 is now included, with comparison of hourly values as well as High and Low Waters.

A comparative assessment is made of the results of three of the most advanced methods of tidal prediction available today.

1. Harmonic Shallow Water Correction (H.S.W.C.)

These predictions have traditionally been provided on a routine basis by the University of Liverpool Tidal Institute and Observatory (L.T.I.) and published in the Port of London Authority's Tide Tables. In this method, devised by Doodson, a primary tidal prediction of modest accuracy is made, then the times and heights of HW and LW are explicitly improved by constant additions and special sets of harmonic terms. As it stands the method does not provide for hourly height predictions; in principle it could be extended to do so, but this would involve much laborious interpretation of tidal records and an excessive amount of computation.

2. Extended Harmonic Method (E.H.M.)

E.H.M. predictions for Southend are also computed by L.T.I., for the special requirement of the Storm Tide Warning Service. The basic principle is the same as in standard harmonic prediction, but many more harmonic constituents are allowed for than were possible in the days of analogue predicting machines. Both Rossiter and Lennen (Ref. 1) and Zetler and Cummings (Ref. 2) independently arrived at an optimum number of 114 constituents, as opposed to 45-65 used in conventional harmonic predictions. Hourly heights are specifically predicted, and HW and LW values consistent with them are derived from the turning points of the function. This function has not yet been perfected for routine application. Some minor defects still require correction on the basis of experience with results from a variety of stations with different tidal regimes.

3. Improved Response Method (I.R.M.)

These are computed by Cartwright, primarily for research purposes, using programs described in Ref. 4. In the 'response method', developed by Munk and Cartwright (Ref. 3) the tide is expressed as a linear transformation of the gravity potentials of the Moon and Sun*, expanded in spherical harmonics, plus a simple series of expansions to account for shallow water effects. The principal improvements embodied in the method of Ref. 4 are the calculation of the potentials to an accuracy unprecedented in tidal work, and provision for a wide range of nonlinear terms, including seasonal modulations. (The positions of the Moon and Sun are defined with the precision of a golf-ball at one mile distance, and allowance is made for changes in the Earth's attitude and rotation in space). As for the E.H.M., this method predicts hourly heights, with subsidiary turning point calculations for HW and LW.

Ordinary Harmonic predictions for Southend (and Tower Pier) have been shown to be inferior to E.H.M. predictions by L.T.I. in A.C.O.M.R. paper 27 (iv), and to ordinary Response predictions by the writer in paper 25 (ii). They are therefore not considered further here.

Tide gauge data

Except for a short period in 1966, all data were read from P.L.A.'s tide gauge charts. These readings covered the 4-year period 1 January 1959 - 12 January 1963 and the 8 months 1 September 1967 - 9 May 1968. For 1 January - 14 April 1966 HW and LW values only were read from the S.T.W.S. repeater charts. However, the frequent appearance of 'flats' and the suspicion of a 'backlash' effect added to occasional reports of gauge silting made these readings rather uncertain, so these charts were not used again except for qualitative comparison.

Hourly series were carefully corrected for misreadings by computer scanning followed by re-check of all dubious values listed by the computer. Values of HW and LW times and heights were checked wherever they differed from all predictions by more than 3 feet or 30 minutes.

Prediction constants

The I.R.M. constants were derived from analysis of the 3-year period 1959-61, and are listed in Table 1, using the notation of Ref. 4. The E.H.M. constants were derived from the 18-month period October 1960 - March 1962, according to the Monaco Symposium (1967) version of Ref. 1, where they are listed.

H.S.W.C. constants are averages based upon analyses for 1952, 1956 and 1960, although terms representing mean values (high and low water luni-tidal intervals, high and low water mean heights) are based upon all data between 1952 and 1960.

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*The anomalous effects of solar radiation are also allowed for.

Periods of comparison

In order to avoid artificial 'data-fitting' predictions were compared only for periods later than those described above. 1962 was the only complete year used for comparison, but since the E.H.M. predictions were not available, only HW/LW values were used. (Hourly I.R.M. residuals for 1962 were analysed spectrally, but the results are not relevant to this report.)

All three HW/LW predictions were compared for the first 104 days of 1966, but owing to the dubious nature of the data, mentioned above, it was not considered worth the trouble to read and compare hourly values.

Finally, for the 236 day period 1 September 1967 - 23 April 1968, all HW/LW values and hourly heights were fully compared.

Comparison of HW and LW predictions

Table 2 shows means and standard deviations of Observed-Predicted by H.S.W.C. and I.R.M. for all 1440 turning points (720 HW and 720 LW) occurring in the period 1 January 1962-8 January 1963).

Table 2. High and Low Waters, 1962

(Times in minutes, heights in 0.1 ft. units)

| | | HIGH WATERS | | LOW WATERS | |
|---|---|---|---|---|---|
| Prediction | | Time | Height | Time | Height |
| H.S.W.C. | Mean | -0.5 | -0.9 | -3.9 | 0.1 |
| | S.D. | 8.5 | 6.8 | 11.7 | 7.7 |
| I.R.M. | Mean | 2.6 | -0.6 | 2.7 | -1.7 |
| | S.D. | 8.2 | 6.9 | 11.1 | 7.9 |

S.D.s (r.m.s. deviation from mean, rather from zero) are given because there may well be justification for adding the mean error to future predictions as a 'shallow water correction'.

Clearly, there is little to choose between the predictions, H.S.W.C. tends to be a little late in times, I.R.M. tends to be a little early. Height comparisons are similar and satisfactory.

Table 3 compares the same quantities with E.H.M. predictions added for all 400 turning points in the period 1 January - 14 April, 1966, using the S.T.W.S. repeater charts for data.

Table 3. High and Low Waters 'Winter' 1966

(Times in minutes, heights in 0.1 ft. units)

| | | HIGH WATERS | | LOW WATERS | |
|---|---|---|---|---|---|
| Prediction | | Time | Height | Time | Height |
| H.S.W.C. | Mean | 1.3 | 5.5 | 0.9 | -0.1 |
| | S.D. | 8.1 | 6.3 | 10.7 | 6.2 |
| E.H.M. | Mean | -0.9 | 4.1 | 2.2 | -1.5 |
| | S.D. | 11.3 | 7.0 | 12.1 | 6.6 |
| I.R.M. | Mean | 4.2 | 2.1 | 9.8 | 1.8 |
| | S.D. | 9.3 | 7.2 | 9.6 | 7.0 |

It is seen that, although the H.S.W.C. values tend to be on the whole a little better than the other two, most of the mean values are rather poor. It now seems clear that the data themselves were below par, as already mentioned, so that gauge silting and backlash produced an artificial retardation to the times, and probably affected the heights also. However, the results as they stand prompted Cartwright to investigate two possible sources of error in the I.R.M. predictions.

One was in deriving the times of maxima and minima. The method used is cubic interpolation between predicted heights and first derivatives at two consecutive hours spanning the point of zero slope. Commander C.T. Suthons suggested that quintic interpolation may be necessary for very shallow water. The relevant program was therefore extended to compute second derivatives as well as the first, and hence to use quintic interpolation. The results for Southend were almost identical, with only rare differences of 1 minute. (The quintic method may well be worthwhile for say Tower Pier, but other errors may arise there due to neglect of high order interaction terms).*

The other queried source of error was whether random meteorological effects of the winter months might produce a consistent bias one way or another in the times and heights of HW or LW. (This is just possible in theory). The 1962 series described in Table 2 were therefore divided into a 'winter' series and a 'summer' series, consisting of the first and second 118 days of the year, and analysed separately. Apart from the expected reduction of S.D.s in summer, the results were fairly similar, and did not suggest any bias as sought.

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* The method used by Liverpol Tidal Institute for all ports is one of iteration; starting with gradients 3 hours apart, 8 iterations will always produce a turning point with a maximum error of one minute.

Rossiter has compared luni-tidal intervals at several Thames gauges during the years 1960/66 and found marked gauge-to-gauge correlations. These may suggest long term changes in the tidal characteristics of the estuary not attributable to individual gauge errors. However, from Rossiter's figures, 1966 as a whole does not seem to be an unusual year for luni-tidal intervals, so such trends cannot account for the time errors in Table 3.

The results for the 'storm-surge season' of 1967/68 are again comparable with 1962, thus confirming the suspicions about the 1966 data. Table 4 lists the same quantities for all 912 turning points in the period 1 September 1967 - 23 April 1968. (The period of 236 days was chosen deliberately to suit the spectral analyses of hourly values, considered later).

Table 4a High and Low Waters, September 1967 - April 1968

(Times in minutes, heights in 0.1 ft. units)

| Prediction | | HIGH WATERS Time | HIGH WATERS Height | LOW WATERS Time | LOW WATERS Height |
|---|---|---|---|---|---|
| H.S.W.C. | Mean | -1.5 (-1.1) | 3.6 (3.7) | -6.6 (-4.9) | -1.6 (-1.8) |
| | S.D. | 9.9 (9.3) | 7.7 (7.1) | 15.4 (11.6) | 8.5 (7.7) |
| E.H.M. | Mean | -2.0 (-1.1) | 3.1 (3.2) | -1.3 (-0.6) | -0.8 (-1.1) |
| | S.D. | 12.3 (10.7) | 8.3 (7.7) | 19.1 (12.9) | 9.7 (9.0) |
| I.R.M. | Mean | 2.2 (2.3) | 0.2 (0.4) | 3.6 (3.2) | 0.1 (-0.2) |
| | S.D. | 9.8 (9.4) | 8.2 (7.6) | 13.9 (10.8) | 8.9 (8.1) |

Here, we repeat the pattern : H.S.W.C. late, I.R.M. early, in mean time predictions. The E.H.M. mean time error for LW is better than either, but I.R.M. has the smallest standard deviations of all three in times. I.R.M. also has the best record in mean heights and has better S.D.s than E.H.M., but not quite as good as H.S.W.C.

The bracketed figures are the corresponding values when all time errors > 30 minutes and height errors > 3.0 feet are removed from the comparison. This was done because some people feel that one should not include very bad weather disturbances when comparing tidal predictions. However, the figures are not remarkably different.

As an adjunct to the bracketed values, Table 4b lists the numbers of turning points included or excluded by the above restrictions.

Table 4b Numbers of turning points included and excluded by restricting magnitudes of errors to 30 units.

| Prediction | Number | HIGH WATERS Time | HIGH WATERS Height | LOW WATERS Time | LOW WATERS Height |
|---|---|---|---|---|---|
| H.S.W.C. | included | 451 | 453 | 425 | 454 |
| | excluded | 5 | 3 | 31 | 2 |
| E.H.M. | included | 442 | 453 | 406 | 454 |
| | excluded | 14 | 3 | 50 | 2 |
| I.R.M. | included | 454 | 453 | 434 | 454 |
| | excluded | 2 | 3 | 22 | 2 |

These numbers are quite revealing, in showing that large errors are much much frequent in LW times than HW times. They also show that I.R.M. predictions have the least score all round in numbers of large time errors. Numbers of height errors are surprisingly consistent.

Comparison of hourly predictions

The hourly predictions by E.H.M. and I.R.M. covering the same period in the 1967/68 season were compared by dividing the period into four consecutive 59-day stretches (hence the 236 days), and computing Fourier power spectra of 'Observed' and 'Obs. - Pred.' for each stretch. For the sake of economy, the spectra were confined to 9 c/month bands (19 consecutive harmonics) centred on the tidal species 1-6, and the band 0-2 c/m representing the low frequency tides and seasonal effects. There is tidal energy at frequencies greater than 6 c/day, for example about 5 $cm^2$ in the 8 c/d species, but spectral comparison here seems hardly worth the candle.

Table 5 compares the average variance budgets of the four analyses. Variances in $centimetres^2$ are given for the 7 spectral bands, their exact total, and for the overall variances without spectral discrimination.

Table 5 Spectral variances ($cm^2$) of hourly data and residuals for 236 day period 1 September 1967 - 23 April 1968

| Tidal species | Observed data | E.H.M. residual | I.R.M. residual |
|---|---|---|---|
| 0 | 135 | 111 | 111 |
| 1 | 241 | 72 | 50 |
| 2 | 23063 | 217 | 92 |
| 3 | 42 | 11 | 8 |
| 4 | 93 | 32 | 26 |
| 5 | 2 | 1 | 1 |
| 6 | 34 | 4 | 3 |
| TOTAL | 23608 | 449 | 291 |
| OVERALL | 24204 | 1035 | 876 |

It is clear that the I.R.M. residuals are better in all important respects, except for the noisy low frequency band (species 0) and the trivial 5th diurnal band (species 5) where the two methods give identical results. The I.R.M. total is 158 $cm^2$ (0.170 $ft.^2$) lower than the E.H.M. total. The fact that the 'overall' figures differ by 159 $cm^2$ shows that the two predictions do equally well in the tidal species greater than 6.

Examination of the individual spectral details (not reproduced here) shows that E.H.M. tends to have surprisingly large residuals in the $M_1$ and $L_2$ groups, not evident in I.R.M. residuals. They tend to be larger elsewhere in species 1 and 2 of course, but it is difficult to specify any other consistent culprits.

It has been common practice to ignore those modulations arising from terms in the potential having a different geodetic coefficient to the principal line, and this is almost certainly one reason for the existence of residuals at $M_1$ and $L_2$ in 1968. In the case of $L_2$ a further complication exists; here the fundamental frequency coincides with that of the interaction term $2MN_2$ which is associated with quite a different set of nodal corrections. Using equilibrium theory, the term $L_2$ at Southend should be defined by H = 0.2 ft, g = 24°; the analytical results are H = 0.48 ft, g = 20°. The difference must indicate the presence of a large $2MN_2$.

The species 2 residuals tend to occur in 'bursts' effecting all groups within the species simultaneously. This accords with the now familiar tidal modulations in the estuary, affecting the semi-diurnal tide as a whole rather than selective details of the astronomical spectrum. The cause is outside the scope of ordinary tidal prediction, or of current methods of surge prediction. Refs. 1 and 5 shed some light on this phenomenon of tidal modulation, but do not solve it.

Both species 4 residuals are high in comparison with species 3, 5 and 6. This also seems to be a notable property of Southend, and possibly other Thames gauges, to which Cartwright called attention in Ref. 5. It may well be a symptom related to other peculiar effects in the estuary. The residuals are fairly evenly spread over the principal tidal groups, but the group which includes $MN_4$ is consistently high in residuals from both methods.

In general, these rather large tidal residuals make Southend (and the Thames) a 'difficult' locality to predict, even though its main tidal curve is not nearly so complicated as say Portsmouth. Cartwright has analysed Portsmouth similarly, and found a typical winter species 2 residual of only 20 $cm^2$, an order of magnitude less than at Southend.

## Concluding remarks

On the basis of HW and LW predictions alone, there is little to choose between the three prediction methods for accuracy. Each is 'best' in certain figures for certain periods, but one suspects that comparison over a very long period would prove them all, roughly speaking, equal.

There is some evidence to suggest that high and low water time predictions prepared by any one of three methods could be improved by the application of constant corrections of the order of a few minutes; the sign and magnitude would depend upon which method is considered. This would be in accordance with the principle of Doodson's shallow water expansion. The times of turning points can be very sensitive to higher harmonics than can be included in either the E.H. or I.R. methods even though negligible in any ordinary analysis of height residuals. Their presence can however be accommodated to a first order by addition of a constant to these special times (but not of course to the times of hourly heights). Cartwright's experience with Portsmouth and Rossiter's with Tower Pier and other shallow ports tend to confirm this view. In cases of extreme distortion (e.g. Tower Pier and Goole) Rossiter has noted that the correction would need to be different for high and low waters. Moreover, such corrections would need to be determined from quite a number of years data, as the luni-tidal investigation referred to earlier has shown. Finally, the application of such corrections would destroy the compatibility between high and low water prediction and hourly height predictions.

The mean predicted HW/LW heights could be similarly adjusted, although this involves also the different question of mean sea level prediction, which is outside the scope of this report. It will be noticed however that the I.R.M. mean heights tend to be not only very small but also nearly equal at HW and LW. This does not apply to H.S.W.C. or E.H.M., implying that these perhaps require a change in tidal amplitude, a rather more difficult matter.

If we disregard the winter of 1966 comparison as using suspect data, and consider the 1967/68 comparison to be typical, then the I.R.M. figures are better than or as good as the E.H.M. hourly predictions in all respects, and the HW/LW values in most respects. However, the improvement is small by most standards, and may be further decreased by improving the nodal representation of terms of the $L_2/2MN_2$ type. Nor is a financial criterion helpful. On various computers H.S.W.C. and E.H.M. predictions for one year take between one and three minutes computer time; the corresponding time for I.R.M. predictions is about 5 minutes; the corresponding cost differential is small. Experience shows that routine computational processes should not be changed, for purposes of economy, until a factor of not less than 10 is involved.

The view is expressed that the only practical criterion for deciding on the optimum method of predicting tides in the future is that it should be generally applicable to all tidal regimes, including those with highly distorted profiles. The standard of accuracy should ideally be similar to that indicated for Southend in this paper. Further attention must therefore be focused on stations with maximum shallow water distortion in order to arrive at a decision. However, it is possible that these require a new approach different from any considered here.

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Table 1. The complete set of I.R.M. prediction constants for Southend.

The following correspond to card sets 3, 4 and 5 described on pp. 6 and 10 of Ref. 4.

The sequence of number 1-68 refer to the variables listed in Table 1 of Ref. 4.

The 'constant' sea level according with the low frequency variables is 0.321 feet above O.D. (Newlyn), which is the mean value of the data analysed in 1959-61.

| | | | | |
|---|---|---|---|---|
| 0 | 1 1 1 1 0 1 1 0 0 0 1 | | | |
| 1 | 0 0 1 1 1 1 1 0 1 0 0 0 1 0 1 | | | |
| 2 | 1 0 1 1 1 1 1 0 1 0 1 1 0 0 1 | | | |
| 3 | 0 0 1 0 1 1 0 | | | |
| 4 | 0 1 1 1 0 0 1 1 0 1 | | 1 0 0 0 0 0 1 0 0 1 0 0 0 | |
| 96 | -1.821116 | -0.268065 | -0.944538 | -2.067579 |
| 48 | 2.826564 | 1.433413 | 5.655226 | 2.663706 |
| 0 | -1.947395 | -2.267380 | -6.128242 | -10.732193 |
| -48 | 0.672375 | 1.816937 | 0.155826 | 12.085083 |
| -96 | -0.049608 | -0.716251 | 2.569019 | -3.464215 |
| 1 | -1.711366 | 0.0044 | | |
| 2 | -8.631176 | 0.0000 | | |
| 3 | -38.438531 | -0.1890 | | |
| 4 | 27.690623 | 0.0000 | | |
| 6 | 0.664706 | -34.1000 | | |
| 7 | -0.238510 | -34.1000 | | |
| 11 | 1.251072 | 4.9028 | | |
| 14 | -1.491998 | -0.378526 | | |
| 15 | 2.343338 | 1.019761 | | |
| 16 | -1.610912 | -1.702764 | | |
| 17 | 0.303405 | 1.469142 | | |
| 18 | 0.118399 | -0.475867 | | |
| 20 | -1.765017 | -1.732756 | | |
| 24 | 1.098706 | 5.394218 | | |
| 26 | -2.928209 | -3.216206 | | |
| 27 | -6.713215 | 28.571197 | | |
| 29 | -0.724463 | -1.991142 | | |
| 30 | 3.367446 | 1.972877 | | |
| 31 | 0.536443 | -6.317605 | | |
| 32 | -2.046166 | 4.973747 | | |
| 33 | 1.291170 | -1.343246 | | |
| 35 | 3.580668 | 0.612422 | | |
| 37 | 0.657760 | -3.882583 | | |
| 38 | -1.020800 | -2.176515 | | |
| 41 | -6.358808 | -1.691015 | | |
| 44 | 0.479912 | 2.194178 | | |
| 46 | 6.012952 | 10.793544 | | |
| 47 | -1.967081 | -2.041627 | | |
| 50 | -2.494923 | -10.412937 | | |
| 51 | 19.302587 | 10.961556 | | |
| 52 | -11.327230 | -1.786534 | | |
| 55 | -0.516469 | -0.295422 | | |
| 56 | -0.024198 | -0.325275 | | |
| 58 | -0.369294 | 0.240919 | | |
| 59 | 0.371331 | -0.464074 | | |
| 65 | -1.479441 | 0.036765 | | |
| 68 | 0.069371 | 0.051462 | | |