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A Group of Computer Programs for Tidal Analysis and Prediction by the "Response Method"

by

D. E. CARTWRIGHT

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General description

The programs enable general application and investigation of the "response method" (Ref. 1) in an improved and extended form. They have been written primarily for use with the IBM 7094 computer, but could easily be adapted for use with any large machine which accepts "Fortran 4" language. (The author intends to adapt them for the IBM 360/65 some time during 1968.)

Previously, the only similar programs available have been in the version included in an annex to the BOMM system, (special tape, in "Fortran 63"). The BOMM version accords precisely with the description in Appendix A of ref. 1, but is less accurate than the present method, and is not designed for inclusion of nonlinear terms without a lot of extra programming. The chief advantages of the present programs are the high precision of their Ephemeris for the Moon and Sun, and their provision for a wide arbitrary choice of nonlinear variables. The basic principles, however, remain as outlined in ref. 1, and acquaintance with that paper is assumed in what follows.

At least two separate operations are involved. The first, which need be performed only once for all subsequent work on a given period of time, is the parent program, MONSUN. This computes the 12-hourly lunar and daily solar ephemeris for an arbitrary span of days, and writes it on the user's private tape in BCD format.

Subsequent programs in the main group all read from the tape written by MONSUN, interpolate its elements to an arbitrary subdivision of 12 hours, and compute from them the requested gravitational and radiational potentials and associated nonlinear forms. POTA merely prints out the potentials etc. POTC reads hourly tidal series from another tape, and derives response weights and predicted variances. POTE and POTF read a set of given response weights and produce predictions from them, at equal intervals of time (POTE), or as times and amplitudes of maxima and minima (POTF). Other variants in the same group (e.g. POTD) perform special analyses of observed and predicted series.

Finally, there are two subsidiary programs. ZTOW reads H and g values for a small selection of major harmonic tidal constituents and derives from them an approximate set of response weights and their admittance curves. (Such weights may be useful to initiate the nonlinear terms in POTC). WTOZ defines the linear admittance curves from a given set of genuine response weights.

All programs have passed rigorous tests, by comparison with published ephemerides and with computations by the BOMM process, and by analysis of tidal data. However, it is possible that certain "bugs" still remain, and the author would appreciate any complaints from users which may lead to their elimination.

Details and operating instructions

MONSUN computes:

$\bar{R}/R$ = Sine Equatorial Parallax in units of its mean value,

CD, SD = cos, sin (Apparent * Declination), and

CL, SL = cos, sin (Apparent * Hour Angle relative to Greenwich Ephemeris Meridian), at 0h Ephemeris Time for both Moon and Sun, and at 12h E.T. for the Moon only, for any sequence of days between 1801 and 2099 AD (inclusive). The above quantities, together with ND = number of days from 1900 Jan 1 and ΔT = correction from E.T. to U.T. in seconds interpolated between the given yearly values, are written on tape unit 11 and also on the systems output tape for printing. They are written and printed in the order:

ND, ΔT, $(CD, SD, CL, SL, \bar{R}/R)_{Moon\ 0h}$, $(CD, SD, CL, SL, \bar{R}/R)_{Sun\ 0h}$, $(CD, SD, CL, SL, \bar{R}/R)_{Moon\ 12h}$ with the Format:

(1X, I6, F8.3, 1X, 5F10.7, 1X, 5F10.7/16X, 5F10.7)

In addition, at 0h E.T. of the first day requested and at subsequent 10-day intervals, certain quantities involved in the calculation are merely printed, in a form suitable for checking against a published ephemeris (see Astronomical Notes). These are, in order of printing:

apparent ecliptic longitude and latitude of the Moon;

apparent * " " " " " " Sun;

(predictable) nutation of the Earth in longitude and obliquity;
apparent sidereal time in ephemeris seconds.

The angular quantities are expressed in seconds of arc, and refer to the Ecliptic and Mean Equinox of Date

A brief account of the method and its accuracy is given in the Astronomical Notes at the end of this report. The rest of this account merely describes the form of data input necessary to operate the program.

* For the Sun, the term "apparent" differs from its usual astronomical meaning in the omission of Aberration.

(MONSUN continued)

Data cards

1. A pack of 134 standard data cards, numbered 000 to 133.

2. $Year_1$, $Month_1$, Day_1, $Year_2$, $Month_2$, Day_2, NR, Format (7I6).

The suffices 1 and 2 refer to the first and last complete days of the series, although for the purpose of interpolation, a record for 0h only of the day following Day_2 is also computed and written. Years, months and days are in the usual notation, (e.g. 1926 10 21 means 21st October 1926 AD), except that there is no restriction to the day number, (e.g. 1964 3 3 and 1964 2 32 are synonymous; so are 1958 12 21 and 1959 1 -10).

NR denotes the number of BCD records already on tape 11 to be skipped before writing the new series. NR must be 0 for a blank tape, or for series after the first when several different series are written in one operation of the program. The number of records written on tape 11 in any one series is $(2ND_2 - 2ND_1 + 3)$, where ND is the actual number of days from 1900·0.

It should be remembered that the maximum period of analysis or prediction which can be performed from a MONSUN series starting and ending on ND_1 and ND_2 respectively (including 0h of $ND_2 + 1$), is from $(ND_1 + 7)$ to $(ND_2 - 7)$.

3. n values of ΔT in Format (10 F7·2),

$$\text{where } n = Year_2 - Year_1 + 3.$$

The 1st value should be that for $(Year_1 - 1)$ and the last value for $(Year_2 + 1)$. See Astronomical Notes and Table 2 for values of ΔT, n may be greater than 10, in which case more than one card is used.

4. The sequence 2, 3 may be repeated indefinitely with other parameters. The most efficient may to stop the program is to end with card 2 containing $Year_1 \leqslant 1800$.

Tape

The output series is written on Unit 11 (A6).

Time

Computing time on the 7094 is approximately 0·93 minutes per year of ephemeris.

POTA, POTC, POTE, POTF (general)

All these programs read from the tape written by MONSUN, interpolate the lunar and solar elements to the subdivision of 12h required, correct them to refer to Universal Time (i.e. G.M.T.) and to the true meridian of Greenwich, then proceed in various ways, according to the program and data input. Everett's formula is used for interpolation, up to 8th order for the Moon, up to 4th order for the Sun, thereby retaining the 7 d.p. accuracy of the daily values.

In order to allow for interpolation and various time lags and leads, the MONSUN series must include at least 7 days before and 7 days after the time series computed in any of the present programs. (See also note about skipping records, under POTA data card no. 2).

All linear gravity and radiation potentials and non-linear input forms which may be used in the programs are listed in Table 1. The low frequency variables (species 0), if requested, are always treated separately within the program and in output lists, because they nearly always require subtraction of constant (mean) values. For the benefit of the analysis program, POTC, the other variables are also treated under their listed species, 1, 2, 3, 4, or (5+), unless one of the following conditions hold :

1. If the total numbers of complex variables in species 1, 2 and 3 combined is 15 or less, then all these variables are grouped nominally under species 1.

2. If the total numbers of complex variables in species 4 and higher is 15 or less, all these variables are grouped nominally under species 4, (except in the following case).

3. If the total number in all species other than 0 is 15 or less, then all these variables are grouped nominally under species 1.

The principal reason for the above groupings is so that:

(a) in the analysis of a long time series (e.g. 355 days), many variables can be included without using unduly large covariance matrices, since in this case the inter-species covariances can be regarded as zero;

(b) in the analysis of short time series (e.g. less than 29 days), for which one should not try to resolve weights for very many variables, one may include all or nearly all the relevant covariances in one or two grouped matrices.

Most of the output figures are accompanied by alpha-numeric statements. Only output groups which are not self-explanatory are noted in what follows.

POTA

merely prints the "real" and "imaginary" parts of any linear potentials or nonlinear input forms listed in Table 1 for an arbitrary sequence of times. The output format is (5(2X, 2F11.6)), repeated for each species, (possibly regrouped - see previous paragraphs), at every time.

Data cards

1. A standard card containing the following 13 numbers:

-1 30 58 89 119 150 180 211 242 272 303 333 364

Format (13I4)

These numbers are reproduced in the output list, as a check.

2. $Year_1$, $Month_1$, Day_1, $Hour_1$, $Year_2$, $Month_2$, Day_2, $Hour_2$, NP12, NR, I   Format (11I6)

Years, months, days, and hours refer to the start (1) and end (2) times - U.T. - of the series to be computed.

NP12 is the number of time intervals per 12 hours. At present 1≤NP12≤12, but the program is set to handle values up to 24 (i.e. half hourly intervals), provided one alters the dimension statement at the beginning of the program to read

POT (576, 21) instead of POT (288, 21).

(However, the extra 6K of core space required may exceed the permissible limit for the larger program variants such as POTC)* .

NR is the number of BCD records (2 per day) to be skipped on input tape 11 before starting a new cycle. The first record read after skipping must be 0h (i.e. not 12h) of a day number between 7 and 107 (inclusive) before Day_1. Between these limits, the computer will adjust to start reading at the proper day number. In normal use, NR will be an even number or 0, but NR must be odd if either (a) on first entry, or with I>0, the required MONSUN series is preceded by an odd number of other similar complete series, or (b) I = 0, and part of the same MONSUN series has just been read in the preceding cycle.

I is an index determining whether the statement "Rewind 11" is executed (I>0) or not (I = 0), before second and subsequent cycles of the program. (Tape 11 is rewound automatically on first entry, whatever the first value of I may be). NR must be adjusted accordingly.

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* In case of storage trouble, the "Do loop" ending with statement 50 could be separated into a subroutine.

(POTA continued)

3. Five cards, as follows:

```
0,  (K(N), N = 1, 11)
1,  (K(N), N = 12, 26)
2,  (K(N), N = 27, 41)
3,  (K(N), N = 42, 48)
4,  (K(N), N = 49, 58),  (K(N), N = 59, 71)
```

Format (I4, 4X, 11I2/I4, 4X, 15I2/I4, 4X, 15I2/I4, 4X, 7I2/

I4, 4X, 10I2, 4X, 13I2).

In the above notation, values of N are the input variable numbers as in Table 1.

K(N) = 0 causes variable number N to be ignored

K(N) > 0 causes variable number N to be included in the computations.

Any selection of zero and non zero **K** may be used, but note the list of variables which depend on others for proper execution, under Table 1.

In POTA, and all variants except POTC, the precise values of K(N) > 0 are insignificant, (e.g. K(16) = 9 has the same effect as K(16) = 1), but in POTC the numbers used have a special meaning - see description.

4. Five cards, containing weights to be used for approximate prediction of species 1 and 2 tides for computations of nonlinear variables, as follows:

U_1, V_1, U_2, V_2	for time	argument		(t - 4 days)
ditto	"	"	"	(t - 2 days)
ditto	"	"	"	t
ditto	"	"	"	(t + 2 days)
ditto	"	"	"	(t + 4 days)

Format (5X, 4F12.6)

All 20 numbers must be present, even if no nonlinear variables are called, but zeros are permissible.

The combination $U_m = 1$, $V_m = 0$ for agrument t, with all other weights zero, causes the linear tide potential $c^m_2(t)$ to be used as "primary tide". Otherwise, appropriate weights may be obtained from a preliminary (possibly short) run of POTC, or values for arguments t and (t ± 2) only may be calculated from known (H, g) values, using program ZTOW.

The weights are read in units giving predictions in cms, but are multiplied by 0·01 within the program, in order to limit the numerical magnitudes of the derived nonlinear forms.

(POTA continued)

5. Any number of sequences 2, 3, 4 may follow. Card no. 2 with $Year_1 \leq 1800$ steps the program.

<u>Subroutines</u> POTPOT only

<u>Tapes</u> Unit 11 = A6 (File Protect), containing Ephemeris written by MONSUN.

<u>POTB</u>

is a special purpose program of research interest only. It computes fourier spectra of the input variables. The author will supply details on personal request.

<u>POTC</u>

performs a covariance analysis between the specified input variables and a given sequence of tidal data (e.g. sea levels) read from magnetic tape, and computes prediction weights for arbitrary combinations of variables.

The program is set up to read hourly data in Format (8X, 24I3) or (8X, 24F3.0), but could be adapted fairly easily to other time intervals or formats by altering a few cards. (Author will advise on request). Since no input variable in Table 1 involves frequencies greater than about 10 c/d, there is no advantage in trying to adapt to a time interval less than 1 hour. If raw data are given at shorter intervals, the best procedure would be to convert them to a smoothed hourly series, using a suitable lowpass filter (see BOMM manual, for example).

The input data must be continuous, so all short gaps should be filled by interpolations. However large time gaps could also be overcome by adapting the program to sum the covariances over a number of discrete time intervals, and compute weights only from the overall totals.

The output list includes the following easily identifiable quantities:

(a) The mean and variance of the data series, with given constant subtracted and the variances when the 1st order predictions (from cards 4) are successively subtracted from same. These provide a useful check on the accuracy of the 1st order predictions, and whether the times are properly specified.

(b) The mean values of the species 0 input variables. The program uses these values to reduce the covariances to zero means, and they are also necessary for future predictions.

(c) The non redundant part of the covariance matrix for each species or group of species. These are presented in Format (I3, 8F14.6/3X, 8F14.6) in order of variable number, M as follows:

(POTC continued)

M, $(COV(A_L, A_M)$, $L = M...N)$, $COV(X, A_M)$

M, $(COV(A_L, B_M)$, $L = M...N)$,

M, $(COV(B_L, B_M)$, $L = M...N)$, $COV(X, B_M)$

M, $(COV(B_L, A_M)$, $L = M...N)$.

where N is the highest variable number in the group,

$L = M...N$ is the sequence of variable numbers for which $K(L) > 0$,

A_L, B_L are the real and imaginary parts of input variable number L, and X is the data series in cms. For species 0, only the first line for each M is given.

(d) The computed weights for various groups of variables, in Format (I3, 2F14·6), and their "predicted variance" with the given data: Weights for nonlinear variables of order s are given in the units 10^{-2s} cm^{1-s}.

Data cards

1. As for POTA.

2. As for POTA. In present use, NP12 must be 12, and $Hour_1$ (not necessarily 0) must be the GMT of the first number in each data record.

2′ NS, SUB, SCALE, in Format (I6, F8·2, F8·4).

NS = number of BCD records (days) of data to be skipped on Tape 13, before record starting in Day_1.

SUB and SCALE are such that

$$X = (X' - SUB) \times SCALE$$

converts the read series X′ to a transformed series X in centimetres, whose mean value is small compared with the tidal range (say < 1/10). The exact value of SUB is unimportant, and in principle the results are independent of it, except perhaps for records of less than about 14 days' duration.

3. Five cards as for POTA. Here, the actual values K(N) determine the number of times variable N is included in the weight calculation, as follows. All variables with $K(N) > 0$ are included in the covariance matrices, and in the first calculation of prediction weights. Then, weights are evaluated by selecting only those variables with $K(N) > 1$, then only those with $K(N) > 2$, and so on, until the highest K(N) is reached.

For example, if the 4th card of (3) is punched:

3 0 1 3 1 3 2 0

the output for species 3 (if not combined with species 1) would consist of:

(a) The covariances for variables 43, 44, 45, 46, 47

(b) Weights and prediction variance for variables 43, 44, 45, 46, 47.

(POTC continued)

(c) Weights and prediction variance for variables 44, 46, 47 only.

(d) Weights and prediction variance for variables 44, 46 only.

4. Five cards as in POTA.

5. Any number of sequences 2, 2′, 3, 4.
Year$_1$ ≤ 180 stops the program.

Subroutines (i) POTPOT
(ii) MATINV - the standard library subroutine, which must however be dimensioned as follows:

DIMENSION IPIVOT (30), A (30, 30), B (30, 2), INDEX (30, 2), PIVOT (30).

Also, it is advisable to remove the statement:

320 DETERM = DETERM * PIVOT (I)

from the MATINV source deck, because the determinant is not required in the main program, and its evaluation often causes an "overflow" statement.

Tapes

Unit 11 = A6 (File Protect) - Ephemeris
Unit 13 = A7 (File Protect) - Data series.

Time

34 values of K(N) > 0 take about 5·5 minutes per year of hourly data. 11 values, about 3·0 min/year. (Compiling, tape mounting and initial winding not included).

POTD

is of reasearch interest only. It computes hourly tide predictions, subtracts them from a corresponding data series, and computes raw and residual spectra and variances at 59-day intervals. The author will supply details on personal request.

POTE

lists predictions computed from a given set of weights. The present Format lists 14 days of hourly predictions with dates, preceded by an alphanumeric heading, on each sheet. The headings include the name of the station, and refer to the units used. Format statement number 547 should be altered according to actual requirements.

Data cards

1. As for POTA.

2. As for POTA. NP12 = 12, unless other adjustments are made.

(POTE continued)

2′ N, ADD, SCALE. Format (I6, F8·2, F8·4)

N should be zero, and is included only for uniformity with other programs.

ADD and SCALE are numerically similar to the quantities used in POTC, except that they refer to the reverse transformation from prediction X′ in centimetres to printed predictions in desired units:

$$X = ADD + (X'/SCALE)$$

In present use, SCALE should have the value 30·4800.

3. Five cards as for POTA.

4. Five cards as for POTA.

5. NK cards, where NK is the total number of values K(N) > 0.

Each card contains

$M \quad U_M \quad V_M$ in Format (I4, 2F12·6),

where M is the variable number, and for all species except O, U_M and V_M are the prediction weights.

For species O, U_M is the sole prediction weight, and V_M is the appropriate mean value, to be subtracted from A_M, as listed in the output to POTC. It will be appreciated that some of the species O inputs vary with very low frequencies, so that their mean value evaluated over only a year or two is not constant. However, it may be shown that the mean values derived from the period of analysis however short should also be used in any other prediction period, provided the constant "ADD" corresponds with the mean value of the data during the period of analysis (except possibly for a change of datum).

6. Any number of sequences 2, 2′, 3, 4, 5.

$Year_1 \leq 1800$ stops the program.

Subroutines POTPOT only.

Tapes

Unit 11 = A6 (File Protect) - Ephemeris
(A private tape for storing the output could be included by a trivial addition to the program.)

Time

About 3·5 min per year, with NK = 34.

POTF

computes and lists a sequence of times and amplitudes of turning points (e.g. High and Low Waters) of a prediction from a given set of weights. The output is printed, and written

(POTF continued)

on tape unit A5 in a form suitable for card reproduction, in records containing numbers relative to 8 consecutive turning points as follows:

KODE, KARD, (IH(N), IM(N), IP(N), N = 1, 8)

in Format (1X, I3, I4, 8 (I3, I2, I4)).

KODE is an arbitrary identification number, read from input card no. 2″.

KARD is a sequence of card numbers, starting with 1.

IH, IM, IP are time in hours and minutes, and value of each turning point. IP is a rounded integer, in (say) 1/10 ft. units. Any extrema computed after the last complete block of 8 are printed in an incomplete record.

The method used is essentially that outlined in ref. 2 with some refinements. In brief, the prediction and its derivative are computed at hourly intervals until two consecutive derivatives differ in sign, when the value at zero derivative is located by fitting a cubic curve to the two neighbouring predictions and derivatives. The program then steps on to a time shortly before the next required extremum. The step is arbitrary (see card 2″), so that certain sequences of extrema, (e.g. minima) can be ignored. Accuracy is quite sufficient for linear or moderately nonlinear tides as at Southend or Rosyth. If high accuracy is required for extremely nonlinear ports such as Southampton, it may be necessary to extend the program, either by applying the present process to half hourly time intervals, or else by computing second derivatives and hence using quintic interpolation.

Input cards

1. As for POTA.

2. As for POTA with NP12 = 12. The starting time should be between 2 and 3 hours before the expected time of the first turning point required, and the end time between 2 and 3 hours after the last.

2″. KODE, ADD, SCALE, NSTEP in Format (3X, I3, F8·2, F8·4, I6).

KODE is an arbitrary identification number for the card output.

ADD and SCALE have the same meanings as in POTE, except that since the amplitudes are printed as rounded integers, one would normally use one tenth the value of SCALE used in POTE.

NSTEP = (LCH - 4), where LCH is the least number of complete hours expected between consecutive turning points of the type required. Its object is to save unnecessary computation of predictions which are not close to the turning points, and also to enable one to skip over certain extrema. NSTEP = 1, (in some cases 2), will locate all maxima and minima at most semi diurnal ports, but the Hook of Holland, for example, would require NSTEP = 0. If maxima and minima are required at less than 4 hour intervals,

(POTF continued)

some minor alterations to the program logic are necessary, (consult author).

3, 4, 5. all card sequences as in POTE.

6. Any number of sequences 2, 2″, 3, 4, 5 may follow. $Year_1 \leq 1800$ stops the program.

Subroutines POTPOT only.

Tapes

Unit 11 = A6 (File Protect) - Ephemeris.
Unit 9 = A5 - Output for cards.

If card output is required, a special library routine such as TRANS9 has to be added after the EOF card. If cards are not required, some time is saved if the 4 statements referring to tape unit 9 (rewind, and three write statements) are removed from the program, so that this tape does not have to be mounted.

Time

About 3·5 minutes per year, using NSTEP = 1, NK = 34.

Subsidiary programs

ZTOW

computes approximate response weights for insertion in POTC, (card sequence 4), given 10 pairs (H, g) of harmonic constituents. The coefficients from Doodson's expansion of the tide potential are used, and it is assumed that the constituents are linearly related to the gravity potentials represented by c_2^1 and c_2^2 by an admittance function which can be defined by three pairs of weights, corresponding to time lags 2, 0, -2 days respectively. The weights are derived by least squares fourier analysis of the admittances, the squared residuals being weighted proportionally to H^2.

The output list consists of

(a) Frequency (c/d), H (cm), g (deg), X, Y; where (X, Y) is the equivalent admittance corresponding to the given constituent.

(b) The six pairs of computed weights for arguments (t - 2), t, (t + 2) respectively.
N.B. When used in POTC, zeros have to be added for (t ± 4).

(c) Weighted variances of X and Y values before and after least squares "fit".

(d) Admittances corresponding to the weights, in steps of 0·01 c/d.

<u>Input cards</u> (ZTOW continued)

1. Two cards of standard data, containing frequencies (deg/hr) and Doodson coefficients. Reproduced at head of output list.

2. SCALE in Format (2X, F8·4),

to convert the given H values to centimetres.
If H are in feet, SCALE = 30·4800.

3. 10 H values in Format (10 F8·3).

4. 10 g values in degrees, Format (10 F8·1). Symbols and order of constituents must be as follows:

Q_1, O_1, P_1, K_1, J_1, $2N_2$, N_2, M_2, S_2, K_2

5. Any number of sequences 2, 3, 4 may follow.

Scale ≤ 0 stops the program.

<u>Subroutine</u> None.

<u>Tapes</u> None.

<u>Time</u> A few seconds.

<u>WTOZ</u>

computes linear admittances in steps of 0·01 c/d corresponding to given sets of weights. The weights may refer to any gravity potential of order 0, 1, 2, or 3, and degree 2 or 3. Any number of lags may be used, and the lags are not restricted to multiples of 2 days. The output is self-explanatory.

<u>Input cards</u>

1. M, L, AMP, PHA in Format (2I3, F9·5, F6·1).

M = species number, i.e. order of spherical harmonic.
L = number of pairs of weights to be given. L≤9.
AMP = factor by which admittance is to be divided.
PHA = phase (degrees) by which it is to be reduced.
AMP = 1·0, PHASE = 0·0 is normal, but other values would be used if one wants the admittances to refer to the local equilibrium tide, instead of to the normalised spherical harmonic amplitude. (For normalising factor, see ref. 1, page 574).

2. L cards, each containing
M, U_M, V_M, LAG in Format (I5, 2 F12·6, I5).

M is the variable number, as used in the POT programs.
U_M, V_M are the response weights. If M = 0, the program puts V_M = 0 regardless of the values on the cards.
LAG is the time lag in hours.

N.B. LAG = + 96 refers to the argument (t - 4 days).

(WTOZ continued)

The program does not refer to Table 1, so M may be fictitious, if for example lags other than 2m days are given.

3. Any number of sequences 1, 2.

$M < 0$ stops the program.

Subroutine None.

Tapes None.

Time A few seconds.

Astronomical Notes

The method of calculating the position of the Moon and Sun, described in Appendix A of ref. 1 and executed in the TIDPOT program of BOMM (annex), is probably adequate for nearly all tidal work. However, its accuracy is poor by astronomers' standards, and sensible errors may occur in certain research applications with high spectral resolution. In starting afresh with the present programs, I decided to remove all possibility of error by using Brown's expansion for the Moon (with modern revisions) and Newcomb's for the Sun, as in the preparation of published astronomical ephemerides. The calculations are a good deal more elaborate than in the BOMM method, but computing time is minimised by careful programming.

E. W. Brown's expressions for the Moon, checked by computer and with some errors corrected, are conveniently summarised in pp. 283-363 of ref. 3. In brief, one first defines the fundamental arguments:

L = Mean longitude of the Moon,
L' = " " " " Sun,
ϖ = " " " Moon's perigee,
Ω = " " " " node,
ϖ' = " " " Sun's perigee,

all expressible in terms of Ephemeris Time T in the form

$$A_0 + A_1T + A_2T^2 + A_3T^3 + \sum_n c_n \sin(a_n + b_nT) \qquad (1)$$

where the A_k are given constants, recently revised. The last terms are low frequency variations, the largest of which, due to the planet Venus, has an amplitude of 14"3 and a period of 271 years.

The True Longitude and Sine Equatorial Parallax are then expressed in the form

$$\text{Mean value} \;+\; \sum_n \mu_n \tau_n \begin{smallmatrix}\sin\\ \cos\end{smallmatrix} (i_n l + j_n l' + k_n F + m_n D)$$

$$+ \; \sum_n \rho_n \begin{smallmatrix}\sin\\ \cos\end{smallmatrix} (\alpha_n + \beta_n T) \qquad (2)$$

where
$$l = L - \varpi,$$
$$l' = L' - \varpi',$$
$$F = L - \Omega,$$
$$D = L - L',$$

and the i, j, k, m are sets of integers, mostly between ±4 (inclusive). τ_n and ρ_n are sets of tabulated amplitudes, the former due to solar perturbations, the latter due to the action of planets, principally Venus and Jupiter. The μ_n are multipliers, slightly greater than 1, varying with the solar eccentricity (6).

For astronomical purposes, the Sine Parallax is usually converted to the arc by adding a term proportional to its cube, but this is unnecessary here, because the sine is strictly the quantity involved in the gravitational potential. In program MONSUN, it is divided by Brown's mean value 3422″70, to give ($\bar{R}$/R). The Potential programs POTA etc. use the modern Mean Sine Parallax, 3422″451. *[1]

Ref. 1 assumes that the Moon's Latitude bears a simple geometrical relation to F and the inclination of the mean orbit. This is inaccurate by present standards. Brown's expression for the Latitude has the form

$$(1 + C)\ (\gamma_1 \sin S + \gamma_2 \sin 3S + N), \qquad (3)$$

where γ_1 and γ_2 are constants related to the orbital inclination, and (γ_1C), S, and N are the sums of periodic terms, similar to those occuring in (2).

Ref. 3 lists over 1600 harmonic terms for the summations in 1, 2 and 3 which are required for an accuracy of 0″01 in Longitude and Latitude and 0″001 in Parallax. I have aimed at an accuracy of 10^{-5} radians (2") in Longitude and Latitude and 10^{-5} in ($\bar{R}$/R). Since it appears that errors may occur up to about ten times the largest neglected amplitudes, program MONSUN therefore selects all amplitudes ≥0″20 in Longitude and Latitude and ≥0″0020 in Parallax. A few terms less than these limits are also included if their arguments are already used for other terms. In all, MONSUN uses 111 terms in Longitude, 82 terms in Latitude, and 95 terms in Sine Parallax, together with some 20 harmonic terms involved in equations 1. Comparison of 5 years (1959 - 1963) of 10-daily computations with the British "Astronomical Ephemeris"*[2] showed maximum errors of 2″5 in Longitude, 1″0 in Latitude, and 0″018 in Sine Parallax. Most of the errors were of course considerably smaller, with mean values near zero.

*[1] Published Ephemerides, however, use another value, 3422″54, for the sake of continuity with earlier tables.

*[2] Identical with "The American Ephemeris and Nautical Almanac".

To the present precision, we have also to take account of the change in the Moon's apparent position due to the forced Nutation of the Earth itself. This is resolved into a component additive to the longitude with variations of order 18″, and a component additive to the Obliquity of the Ecliptic, with variations of order 10″. They are expressed in the form

$$\sum_n (R_n + S_n T) \begin{matrix}\sin\\ \cos\end{matrix} (i_n l + j_n l' + k_n F + m_n D + p_n \Omega)$$

with coefficients given on page ix of ref. 3. Program MONSUN uses only the four largest terms for each component, but some other minor terms in longitude with $p_n = 0$ are added to the amplitudes in true longitude with the same argument, since this does not involve any extra computation. Comparison with the "Astronomical Ephemeris" showed errors $\leqslant 0''.2$.

The Mean Obliquity of the Ecliptic is given by

$$\epsilon = 84428''.26 - 46''.85\ T \tag{4}$$

where T is in Julian Centuries since 1900 Jan 0·5. After correcting ϵ to "True Obliquity", using the Nutation, the Apparent Right Ascension α and Apparent Declination δ relative to the True Equinox of Date are calculated from the Apparent Longitude λ and Latitude β from the well known relations:

$$\begin{aligned} \cos\delta\cos\alpha &= \cos\beta\cos\lambda \\ \cos\delta\sin\alpha &= \cos\beta\sin\lambda\cos\epsilon - \sin\beta\sin\epsilon \\ \sin\delta &= \cos\beta\sin\lambda\sin\epsilon + \sin\beta\cos\epsilon \end{aligned} \tag{5}$$

Finally cos α and sin α are converted to refer to geocentric longitude relative to Greenwich Ephemeris Meridian (or at 12h, its inverse) by effectively subtracting the Ephemeris Sidereal Time referred to the True Equinox, namely

$$23925{\cdot}836 + 8640184{\cdot}542\ T + 0{\cdot}093\ T^2 + \delta L \cos\epsilon \text{ seconds,} \tag{6}$$

where δL is the Nutation in Longitude.

For S. Newcomb's solar formulae, I have used ref. 4*. The standard elliptic formulae quoted in ref. 1 are of course a good approximation, but we here also take account of the secular variation in eccentricity,

$$e = 0{\cdot}01675104 - 0{\cdot}00004180\ T - 0{\cdot}000000126\ T^2, \tag{7}$$

various planetary perturbations, and the changes in apparent position due to Nutation and to the Earth's "orbit" about the Earth-Moon centre of gravity.

The planetary perturbations in Longitude include some low frequency variations as in (1), the largest of which has an amplitude of 6″.4 and a period of 1782 years. In addition, both longitude and Radius Vector include terms of the form

$$\sum_n \rho_n \cos(i_n\theta + j_n l' + \phi_n)$$

*Ref. 4 also includes approximate formulae for the Moon, which, though very instructive, are not quite as accurate as the method adopted here.

where θ is the difference in mean longitudes of Earth and a planet, i_n and j_n are small integers, and ρ_n, ϕ_n given constants. The planets involved, with the number of terms in longitude included in MONSUN are: Venus (8), Mars (6), Jupiter (8), and Saturn (2). Numbers in Radius Vector are about the same, in most cases using the same values i_n, j_n as for longitude. Four similar perturbations to the Sun's Latitude are also included.

It is worth noting that while these planetary perturbations have an appreciable effect on the Sun's elements (of order 10″ in longitude, 2×10^{-5} in $\bar{R}/R$), their net effect on the tides is incomplete, since the direct tidal potential of the planets has not been included in the program. However, the possibility of resolving such small effects in tidal data seems very remote, so there is little point in burdening the program any further.

The increments due to the Earth-Moon motion are geometrical, and are expressed as

$$\delta\lambda' = 2506'' \,\eta \cos\beta \sin(\lambda - \lambda')$$
$$\delta\beta' = 2506'' \,\eta \sin\beta$$
$$\delta R' = 0{\cdot}01215 \,\eta \cos\beta \cos(\lambda - \lambda'),$$

where $'$ refers to the Sun's elements, and η denotes the ratio of the parallaxes of the Sun and Moon, of order 0·0026. The effect is interesting in that it introduces lunar frequencies into the solar potentials, but the amplitudes are of course very small.

For astronomical work, a final correction amounting to $20''{\cdot}45\,R'$ is subtracted from λ' to allow for aberration, due to the finite velocity of light. This is inappropriate here, since gravity is supposed to act instantaneously, and so aberration is omitted from the program. The radiation potential, however, should strictly include aberration, but the program refers that potential to the same solar elements as the gravity potentials, for convenience. Apart from missing a minute annual term, of amplitude $0''{\cdot}3$, the only effect of its omission is effectively to refer the radiation potential to a meridian $20''{\cdot}45$ East of Greenwich.

The 5-year comparison with the Astronomical Ephemeris, mentioned above, gave the following maximum errors for the Sun: Longitude $1''{\cdot}1$, Latitude $0''{\cdot}15$, Radius Vector $2{\cdot}9 \times 10^{-6}$.

Since the Radius Vector is expressed in astronomical units, therefore $1/R'$ is precisely the solar parallax in units of the mean parallax, and is computed as such. The conversion to Apparent Declination and Greenwich Hour Angle is made in the same way as for the Moon.

The basic constants used in programs POTA etc. are the latest figures, approved by the I.A.U. at their General Assembly in 1964. They are listed in ref. 5 and include:

Earth/Moon mass ratio = 81·30
Moon's mean sine equatorial parallax = 3422″451
Sun's mean equatorial parallax = 8″794
Earth's equatorial radius = 6378160 metres.

The radiational potential is expressed in units of the Solar Constant, as in the BOMM program (although this is not obvious from the description in ref. 1). The actual figure for the Solar Constant, at present 1·99 cal/cm²/min, therefore does not appear in the computation.

Correction from E.T. to U.T.

Ephemeris Time is a perfectly uniform measure of time concordant with Newcomb's formulae for the Sun, introduced officially in 1960. It differs from Universal Time by a quantity

$$\Delta T = E.T. - U.T.,$$

which varies unpredictably according to the vagaries of the Earth's rate of rotation. Therefore ΔT is only known precisely for past years, after all observations have been collected and processed. The Astronomical Ephemeris at present tabulates accurate values of ΔT for all years from 1901 to about 4 years before the dates of publication, and tentative figures and extrapolations to the nearest second up to the current year. Table 67a of ref. 4 lists all known values back to AD 1681, and reasonable extrapolations up to AD 2000. For convenience, I reproduce the latest available figures since 1930 in Table 2. They are given as always for a time halfway through the year. Program MONSUN arbitrarily defines the half year as

$$1900 \text{ July } 2{\cdot}0 + 365{\cdot}25 \, (Y - 1900)$$

and to avoid discontinuities, makes a linear interpolation between successive values of ΔT.

Obviously, for the purpose of genuine tidal predictions, an error of order 1 sec in the values of ΔT used in these programs will be quite negligible. However, proper values should be used where possible, especially for analyses extending over many decades.

The correction of lunar and solar elements to the time E.T. + ΔT is made by straightforward linear interpolation, which is well within the general order of accuracy for $|\Delta T| \leqslant 200$ sec. Declination and Parallax need no further correction, but the reference of Hour Angle to sidereal time in E.T. units (equation 6) implies an origin at the "Ephemeris Meridian", situated

$$\Delta L = 1{\cdot}002738 \, \Delta T$$

east of Greenwich. ΔL is the hour angle through which the Earth rotates in time ΔT, so that (6) defines Greenwich correctly only when T is expressed in U.T.

Consider a prediction of lunar hour angle at 0h E.T. of some day in January 1952. The angle is measured eastwards from a direction in space which coincides instantaneously with a meridian at about 30 sec (see Table 2) or 450″E, the meridian of Woolwich, say. 0h U.T. occurs 30 sec later, and in the interval Greenwich is rotated to where Woolwich was at 0h E.T., so the meridional error has almost corrected itself. However, the origin also moves eastward, at the rate of one revolution per tropical year, that is by

$$\Delta L_2 = 0{\cdot}002738\ \Delta T,$$

which in this example is 1″2. Therefore ΔL_2 is the correction which is made to both hour angles in programs POTA etc to refer then to the geographical meridian of Greenwich.

References

1. W. H. Munk and D. E. Cartwright (1966) "Tidal Spectroscopy and Prediction". Phil. Trans. Roy. Soc. A259, 533-581.

2. D. E. Cartwright (1967) "Some further results of the response method of tidal analysis". Monaco Symposium on Tides, April 1967. Int. Hyd. Rev. (in press).

3. Improved Lunar Ephemeris 1952-1959. A joint supplement to the American Ephemeris and the (British) Nautical Almanac. U.S. Govt. Printing Office, 1954.

4. Jean Meeus (1962) "Tables of Moon and Sun" Kesselberg Sterrenwacht, Kessel-Lo, Belgium.

5. G. A. Wilkins (1965). The System of Astronomical Constants. Q.J.R.A.S. 5, 23-31 (Part I) and 6, 70-73 (Part II).

TABLE 1. Variables which may be used in POTA ... POTF

$(I)^{i\pm j}$ refers to the product of primary tide predictions, species i and j, conjugate for - sign.

$(I)^{k\pm 0}$ refers to the product of primary species k multiplied by an annual modulation $(\alpha_1^0 \pm i\ \alpha_1^0)$.

Numbers in brackets are time lags in 2 day units.

Species 0	1	2	3	4	(≥ 5)
N	N	N	N	N	N
1 α_1^0	12 χ_1^1	27 χ_2^2	42 χ_4^3	49 χ_4^4	59 $(I)^{2+2+2}(0,0,0)$
2 $\alpha_1^{0'}$	13 χ_2^1	28 χ_4^2	43 $C_3^3(1)$	50 $(I)^{2+2}(0,0)$	60 $(0,0,\frac{1}{2})$
3 α_2^0	14 $C_2^1(2)$	29 $C_2^2(2)$	44 (0)	51 $(0,\frac{1}{2})$	61 $(0,\frac{1}{2},\frac{1}{2})$
4 $\alpha_2^{0'}$	15 (1)	30 (1)	45 (-1)	52 $(\frac{1}{2},\frac{1}{2})$	62 $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$
5 $\alpha_2^0(2)$	16 (0)	31 (0)	46 $(I)^{2+1}$	53 $(0,-\frac{1}{2})$	63 $(I)^{2+2+2-0}$
6 (1)	17 (-1)	32 (-1)	47 $(I)^{2+2-1}$	54 $(-\frac{1}{2},-\frac{1}{2})$	64 $(I)^{2+2+2+0}$
7 (0)	18 (-2)	33 (-2)	48 $(I)^{1+1+1}$	55 $(I)^{2+2-0}$	65 $(I)^{2+2+1}$
8 $\alpha_3^0(1)$	19 $C_3^1(1)$	34 $C_3^2(1)$		56 $(I)^{2+2+0}$	66 $(I)^{2+2+2-1}$
9 (0)	20 (0)	35 (0)		57 $(I)^{2+1+1}$	67 $(I)^{2+2+2+1}$
10 $(I)^{1-1}$	21 (-1)	36 (-1)		58 $(I)^{2+2+2-2}$	68 $(I)^{2+2+2+2}$
11 $(I)^{2-2}$	22 $(I)^{1-0}$	37 $(I)^{2-0}$			69 $(I)^{2+2+2+2-1}$
	23 $(I)^{1+0}$	38 $(I)^{2+0}$			70 $(I)^{2+2+2+2+1}$
	24 $(I)^{2-1}$	39 $(I)^{1+1}$			71 $(I)^{2+2+2+2+2}$
	25 $(I)^{1+1-1}$	40 $(I)^{2+1-1}$			
	26 $(I)^{1+2-2}$	41 $(I)^{2+2-2}$			

N.B. To avoid redundant computations, some variables depend on others having been computed, and will otherwise be wrong:

4 depends on 2
22, 23, 37, 38, 55, 56, 63, 64 depend on both 1 and 2.
42 and 49 depend on 28.
61 and 62 depend on 52.
63, 64, 66, 67 depend on 59.
69, 70, 71 can only be taken together, preceded by 68.
All other variables are calculated independently.

TABLE 2. Values of ΔT available 1967

(Values without decimal point are extrapolations)

Year	ΔT	Year	ΔT	Year	ΔT	Year	ΔT
1930·5	23·18	1940·5	24·30	1950·5	29·42	1960·5	33·16
1931·5	23·34	1941·5	24·71	1951·5	29·66	1961·5	33·59
1932·5	23·50	1942·5	25·15	1952·5	30·29	1962·5	34·23
1933·5	23·60	1943·5	25·61	1953·5	30·96	1963·5	34·52
1934·5	23·64	1944·5	26·08	1954·5	31·09	1964·5	35·39
1935·5	23·63	1945·5	26·57	1955·5	31·59	1965·5	36·0
1936·5	23·58	1946·5	27·08	1956·5	32·06	1966·5	37·1
1937·5	23·63	1947·5	27·61	1957·5	31·82	1967·5	38
1938·5	23·76	1948·5	28·15	1958·5	32·69	1968·5	39
1939·5	23·99	1949·5	28·94	1959·5	33·05	1969·5	41

N.11