

NATIONAL INSTITUTE OF OCEANOGRAPHY

WORMLEY, GODALMING, SURREY

**An investigation into the
large-wave-height response of
two Wave Recorders**

by

L. DRAPER and J. D. HUMPHERY

N.I.O. INTERNAL REPORT A. 63

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Introduction

Instrumental wave measurements have been made in many parts of the world, and until recently an accuracy of about $\pm 10\%$ has been quite adequate for the majority of engineering purposes. With the advent of oil exploration in British and adjacent waters, where severe conditions prevail, it has become necessary to try to reduce the error margins, because the cost of overdesign to cover such uncertainties run into millions of pounds per structure.

In the measurement of sea waves there are many possible sources of error, not the least of which is the ability of the equipment, whatever it may be, to follow the surface accurately. This experiment was designed simply to test the ability of the equipment to record its own wave-induced movements.

As far as was known, no wave recorder had been calibrated against a simulated wave greater than 3 metres in height, whereas waves seven times this height are commonly recorded and the results used to predict severe conditions of ten times this height. It was obvious that to improve confidence in the results it would be necessary to simulate waves of a much greater height. To achieve this, arrangements were made to use the Big Wheel (Ferris Wheel) at Southsea Fun Fair. The wheel has a diameter of 14 metres and can be rotated to give simulated wave periods of from about 13.6 seconds to 30 seconds.

Scope of the Experiment

The instruments tested were:

1. The I.O.S. Shipborne Wave Recorder (S.B.W.R.). This is the recorder which has produced virtually all the I.O.S. deep-water wave records, and one currently employed in the North Sea wave study.

2. The Waverider. This is a surface-following moored buoy, manufactured by Datawell of Haarlem, Netherlands. This instrument is in increasing use for environmental data collection and is likely to be the main source of such data in the future. The buoy used in the experiment was newly delivered from the manufacturers and had not been used at sea.

To ensure that the experiment was performed to the satisfaction of the manufacturers, Mr. P.L. Gerritzen of Datawell attended as an observer.

Waverider buoys are used by the Interservices Hovercraft Unit (I.H.U.) and the Hythe establishment of the National Physical Laboratory (N.P.L.). Both those bodies expressed interest in the experiment. Mr. B. Russell and Mr. J. Fairbrother of I.H.U. recorded the times of individual revolutions of the wheel, and provided a reserve Waverider receiver some eight miles distant from the test site. The records from this second receiver showed some evidence of unlocking however; this was probably due to two reasons:

- a) the use of a stub-aerial on the buoy probably caused a reduced signal to be radiated, and
- b) Armed Services establishments in the area probably caused strong interference signals to be present.

Mr. A. Wright of N.P.L. provided a spare Waverider buoy in case of trouble with the I.O.S. instrument.

The S.B.W.R. (see Tucker, M.J., 'A Shipborne Wave Recorder', Transactions of the Institution of Naval Architects, Vol.98, 1956) electronically sums acceleration and pressure signals produced by sensors on the ship's hull. As long as a steady pressure acts on the pressure sensors (during the experiment - atmospheric pressure) no disturbing influence is set up by not having the

sensors under water. Thus the deflexions traced out on the chart record in this experiment were produced only by the changes in vertical acceleration experienced by the two accelerometers.

The Waverider contains an accelerometer, processing circuitry and a radio transmitter. The doubly-integrated accelerometer signal is transmitted to a shore receiving station which demodulates the signal, and presents the wave-trace on a chart record.

Experimental Details

The diameter of the wheel was measured as accurately as possible by dropping a long tape measure from a point at the top of the wheel to a similar point at the bottom of the wheel.

The two S.B.W.R.s and the Waverider were placed in cars on the wheel such that they were approximately at the apices of an equilateral triangle. The instruments were lashed down carefully to prevent spurious vibrations from interfering with the action of the accelerometers. The wheel was then carefully balanced so that it rotated at a constant speed, i.e. to ensure that all parts of the wheel were subject to the same acceleration at any point relative to the ground.

Each S.B.W.R. was powered from two 12-volt lead acid accumulators, connected in parallel. The necessary 240 volts a.c. were produced with 100 watt inverters. This arrangement produced a reasonably steady supply - better than that which could be taken from the Big Wheel lighting supply. The Waverider buoy contains its own batteries.

The S.B.W.R.s were calibrated so that full scale deflexion on the chart record corresponded to a displacement of 60 feet. This was greater than the measured wheel diameter, but as the

instruments are normally calibrated with a nominal sensitivity of only 0.833 times the absolute sensitivity, a deflexion on the chart record greater than the wheel diameter would result. Corresponding corrections are made in computing the results.

The Waverider receiver is calibrated by the manufacturer to give two ranges, 5-0-5 metres and 10-0-10 metres. The larger range was used for the experiment. The receiver was set up in a vehicle close to the base of the wheel, and was run from a 240 volts 50 Hz supply derived from a portable generator.

The usual transmitting aerial used with the buoy was too long and could have caught in the wheel bracing. Hence a much shorter stub aerial with a loading coil was made and used in the experiment.

The instruments were started and the wheel rotated at speeds between the maximum (period of approximately $13\frac{1}{2}$ sec) and the minimum (period of approximately 30 sec). The time taken for ten revolutions was noted at each speed, and the S.B.W.R. oscillator voltages were measured at the end of each run.

Measurements:-

1. S.B.W.R.:-

In the table of results below, Table No.1, the figures were obtained as follows:-

- (a) Run number - the order in which the measurements were made.
- (b) Average Period (seconds) - the average period of wheel rotation measured over ten revolutions. Individual revolutions were also timed, showing that some speed variation was exhibited by the wheel; however average periods were used throughout the calculations.

$$\text{Average period} = \frac{\text{Time taken for 10 revolutions}}{10} \text{ seconds.}$$

(c) Average chart deflexion (feet) - the average deflexion taken from the chart records, measured over ten complete cycles.

$$\text{Average deflexion} = \frac{\sum (10 \text{ deflexions})}{10} \text{ feet}$$

(d) Oscillator voltage (volts) - the S.B.W.R. oscillator voltage, measured at the end of each run with an Electronic Avo, model EA113.

(e) Deflexion corrected for oscillator voltage - this corrects the deflexion in column (c) for any incorrect S.B.W.R. oscillator voltage. (The sensitivity of the instrument is directly proportional to the oscillator voltage, to within small limits; hence if the oscillator voltage is 1% low, then the recording will also be 1% low.)

Corrected deflexion

$$\begin{aligned} &= \text{Chart deflexion} \quad \times \quad \frac{\text{Nominal oscillator volts}}{\text{Actual oscillator volts}} \\ &= \frac{\text{Column (c)} \times 80}{\text{Column (d)}} \text{ feet.} \end{aligned}$$

(f) Deflexion corrected for calibration. After the experiment was performed, the calibration of each S.B.W.R. was checked. Recorder No.1 was found to be correct, but Recorder No.2 was found to be some 7% low, due to an arithmetical error in the calibration performed before the experiment. This column thus gives the corrected "wave height" as recorded by the instrument.

Corrected deflexion

$$= \frac{\text{Column (e)}}{0.928} \text{ feet, for Recorder No.2 only.}$$

(g) Attenuation corrected for nominal sensitivity. The S.B.W.R. is calibrated for a nominal sensitivity of 0.833 times the absolute sensitivity. It is therefore

necessary to calculate values of (attenuation coefficient $\div 0.833$). (The values of attenuation coefficient at any period of acceleration applied to the instrument between four and twenty five seconds can be obtained from the S.B.W.R. handbook).

(h) The last column gives the ratio of

$$\frac{(\text{corrected wheel diameter (as measured by the instrument)})}{(\text{true wheel diameter})}$$

For Recorder No.1

$$\text{Column h} = \frac{\text{Column (e)}}{45.92}$$

For Recorder No.2

$$\text{Column h} = \frac{\text{Column (f)}}{45.92}$$

Thus the relative response, both theoretical (column g) and measured (column h) can be plotted against period.

This is done in Figure 1.

Table No.1

Column	a	b	c	d	e	f	g	h
	Run Number	Av.Period (seconds)	Av.Chart deflexion (feet)	Oscillator voltage (volts)	Deflexion corrected for osc. volts(feet)	Deflexion corrected for calibrat. (feet)	Attenuation corrected for nominal sensitivity	Corrected deflexion Wheel diameter
Recorder No.1	1	13.96	50.25	80.1	50.19		1.101	1.093
	2	16.57	49.14	80.4	48.90	Not	1.064	1.065
	3	22.13	45.34	80.1	45.28	Applicable	0.971	0.986
	4	25.27	43.13	80.0	43.13	to	0.912	0.939
	5	29.79	39.17	80.1	39.12	Recorder	0.828	0.852
	6	21.60	45.97	80.1	45.91	No. 1	0.981	0.999
	7	13.60	50.83	80.0	50.83		1.104	1.107
Recorder No.2	1	13.96	45.93	80.0	45.93	49.49	1.101	1.078
	2	16.57	44.66	80.7	44.27	47.70	1.064	1.039
	3	22.13	41.06	81.0	40.55	43.70	0.971	0.952
	4	25.27	38.83	79.8	38.93	41.95	0.912	0.914
	5	29.79	35.45	79.6	35.63	38.39	0.828	0.836
	6	21.60	41.51	79.8	41.61	44.84	0.981	0.976
	7	13.60	46.06	80.0	46.06	49.63	1.104	1.081

2. Waverider:-

The results for the Waverider are shown in Table No. 2

(a) Run Number, the order in which the measurements were made.

(b) Period, seconds - the average period of wheel rotation, measured over 10 cycles.

(c) $|A|$ theoretical - this is calculated as follows:

The transfer characteristic for the Waverider buoy is given in the Handbook as

$$|A| = \frac{1}{1 - j\sqrt{2}p - p^2} \cdot \frac{1}{(1 - jq)^3},$$

from which

$$|A| = \frac{1}{\sqrt{(1 - p^2)^2 + 2p^2} \cdot (1 + q^2)^3}$$

$$= |A|_{\text{theoretical}},$$

$$\text{where } p = \frac{T}{T_{o1}} \quad \text{and } q = \frac{T}{T_{o2}},$$

where T = period of applied acceleration.

$$\begin{array}{ll} T_{o1} = 30.8 \text{ seconds} & \left. \begin{array}{l} \text{Taken from} \\ \text{Waverider} \\ \text{Handbook} \end{array} \right\} \\ T_{o2} = 460 \text{ seconds} & \end{array}$$

This gives a result close to unity for periods up to 10 seconds, and unity is of course the ideal result.

(d) $|A|$ experimental - this is calculated from

$$|A|_{\text{experimental}} = \frac{\text{Chart deflexion}}{\text{Diameter of Wheel}} = \frac{\text{deflexion (metres)}}{13.96}$$

The value of chart deflexion was the average of 10 cycles, and was measured direct from the chart, i.e. not using the interpolation scale provided by the Waverider receiver manufacturer. Note that the receiver circuitry takes account of any non-linearity in the chart recorder, and that Datawell arrange for the recorder to be linear to within 1% of the half-scale deflexion over 90% of the full span.

(e) % Disagreement - this shows the percentage disagreement of the experimental results from the theoretical results, and is calculated from

$$\frac{|A|_{\text{theory}} - |A|_{\text{exp.}}}{|A|_{\text{theory}}} \times 100\%$$

Table No.2

Results for Waverider Buoy

a	b	c	d	e
Run Number	Period seconds	$ A _{\text{theory}}$	$ A _{\text{exp.}}$	$\frac{ A _{\text{theory}} - A _{\text{exp.}}}{ A _{\text{theory}}} \times 100\%$
1	20.66	0.9092	0.905	0.46
2	13.85	0.9788	0.971	0.79
3	16.99	0.9547	0.945	1.02
4	30.76	0.7033	0.685	2.60
5	26.11	0.8082	0.795	1.63
6	21.64	0.8937	0.879	1.64

It should be noted that the final results for the Waverider were not made at the same time as the final results for the S.B.W.R.

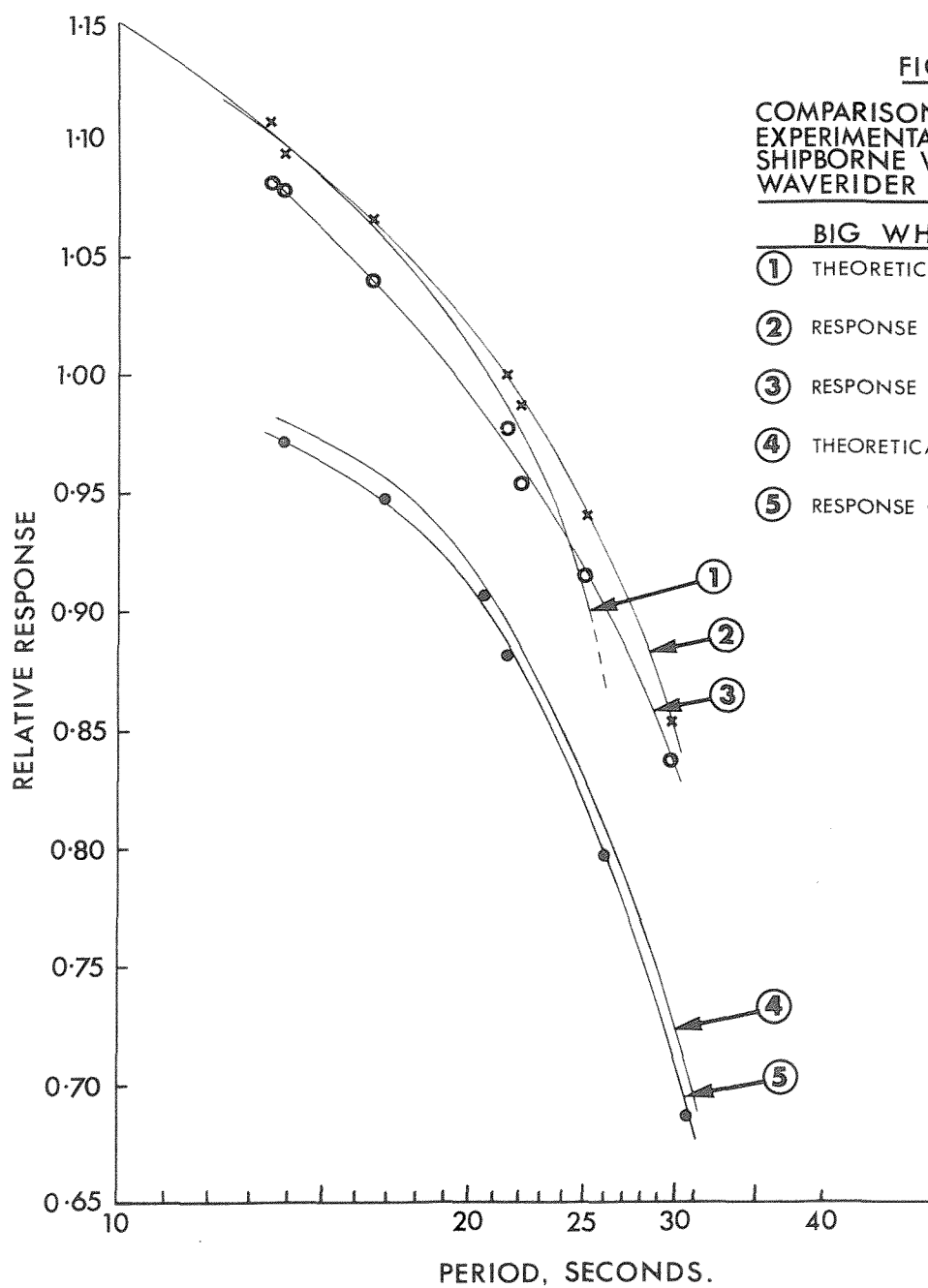


FIGURE 1.

COMPARISON OF THEORETICAL AND EXPERIMENTAL RESPONSES FOR THE SHIPBORNE WAVE RECORDERS AND WAVERIDER BUOY.

BIG WHEEL EXPERIMENT

- ① THEORETICAL RESPONSE OF S.B.W. R.
- ② RESPONSE OF S.B.W.R. № 1. (CROSSES)
- ③ RESPONSE OF S.B.W.R. № 2. (LARGE CIRCLES)
- ④ THEORETICAL RESPONSE OF WAVERIDER
- ⑤ RESPONSE OF WAVERIDER (BLACK DOTS)

Low wave-height tests in the laboratory

1. Shipborne Wave Recorder

The main weakness in the Big Wheel Experiment is that the minimum period of the applied acceleration was about 13.6 seconds - considerably longer than the average wave period likely to be encountered. Low-height experiments were performed on the instruments in the laboratory to check their responses over the whole period range likely to occur at sea.

The S.B.W.R. accelerometers were rotated in the vertical plane on the 3 ft. diameter arm developed for calibrating the instruments. The gears available allowed the periods of applied acceleration to be varied between 6 and 36 seconds. A D.C. amplifier was used to produce a reasonable deflexion on the chart records.

The instrument's response at 12 seconds is taken to be 0.935 times the maximum theoretical deflexion which would be obtained without any computer circuit attenuation. Hence

$$\frac{\text{Deflexion at 12 seconds}}{0.935} = \text{maximum theoretical deflexion}$$

and maximum theoretical deflexion \approx 3 feet in this case.

(Note that only a comparison between theoretical and actual response is being made, hence no correction is necessary for nominal sensitivity).

The results are shown in Table 3, as follows:-

- (a) Period (seconds) - the average period of rotation measured over 10 cycles.
- (b) Deflexion (divisions) - the average peak-peak deflexion over 10 cycles, taken from the chart record.
- (c) Experimental response - calculated from:

Relative response at 12 seconds assumed to be 0.935.

$$\begin{aligned} \text{Maximum theoretical response} &= \frac{\text{Response at 12 seconds}}{0.935} \\ &= R_{\text{max. th.}} = 33.73 \text{ divisions} \end{aligned}$$

$$\begin{aligned} \text{Experimental response at any period} \\ &= \frac{\text{Response at that period}}{R_{\text{max. th.}}} \\ &= \frac{\text{Response}}{33.73} \end{aligned}$$

(d) Theoretical response - the theoretical response taken from the S.B.W.R. handbook.

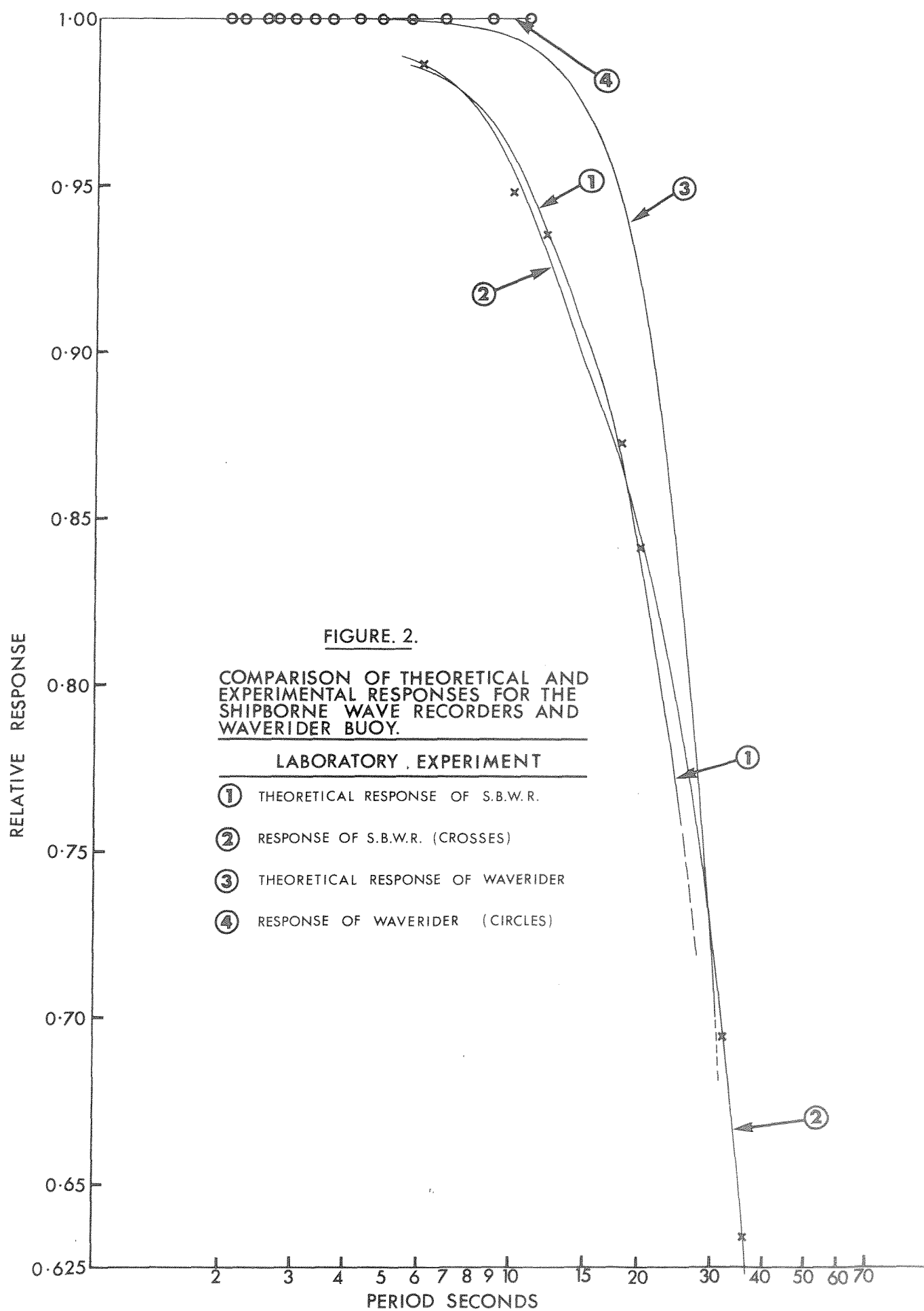
Table 3

Period seconds	Deflexion divisions	Relative response	Theoretical response
6	33.23	0.985	0.984
10	31.97	0.948	0.958
12	31.54	0.935	0.935
15	-	-	0.904
18	29.43	0.873	0.866
20	28.38	0.841	0.840
22	-	-	0.807
25	-	-	0.764
32	23.43	0.695	-
36	21.41	0.635	-

The experimental and theoretical responses are compared in Figure 2.

2. Waverider

The Waverider system was taken to N.P.L., Hythe, on 16th April, 1973 for testing on the calibration rig built there. The rig carries the buoy through a 3 metre diameter vertical circle, and the buoy is maintained in a vertical position throughout by chain drives.



The period of rotation can be varied from 2 seconds to 15 seconds approximately, although the I.O.S. buoy was tested rigorously only up to 10.8 seconds.

During the test, the record showed a steady output corresponding to a 3 metre deflexion at all rotation speeds, (read direct from the chart to an accuracy of about 0.4%). Only at the fastest speed (period of 2.09 seconds) did any irregularity occur; the mean line of the trace started to wander away from the zero line by as much as $\pm \frac{1}{2}$ metre. This phenomenon is presumably due to instability in the accelerometer system at these short periods and relatively large amplitudes. However, a 3 metre sea wave with a two second period does not occur under natural conditions. The minimum period which can occur with a natural 3 metre wave may be calculated from

$$\lambda = 1.56T^2$$

where λ is the wavelength in metres

T is the period of the wave in seconds,

and wavelength_{minimum} = 7x wave height.

In this case, wave height = 3 metres,

$$\therefore \lambda_{\min} = 21 \text{ metres}$$

$$\text{and } 21 = 1.56T^2$$

from which T = 3.7 seconds, approximately.

This is almost double the period at which instability was exhibited by the buoy, and so there is no reason to suspect the buoy performance under natural conditions.

Accuracy

It is difficult to obtain a figure for the accuracy of the Big Wheel Experiment as a whole. However, all significant errors arise from reading the chart records.

If it is assumed that the chart drive motors do not vary sensibly during the period of a run (chart speed variation should not exceed 1%), then an estimate of wheel speed variation may be made from the records. Generally, the faster the wheel ran, then the more constant was its period. Thus for the S.B.W.R. results the maximum period variations, compared with the average values are:-

Run No.	Average Period seconds	Maximum % deviation from average period
7	13.60	2.3
1	13.96	2.2
6	21.60	2.8
4	25.27	3.3
5	29.79	4.8

Not all runs have been calculated, but the trend is clear enough. Similar speed variations can be expected for the Waverider results.

The percentage deviations given above are themselves misleading. The accuracy to which a single wave period can be measured is approximately $\pm 1.5\%$, and if this error is subtracted from the maximum deviation, then only a small net deviation results. The real speed variations which were exhibited by the wheel were mainly caused by wind speed variations. Due to the proximity of high buildings, the wind had a greater effect at the top of the wheel than at the bottom, causing a net moment to be applied to the wheel.

These speed variations cause small variations in the response of the recorders. As both instruments have a gradually decreasing sensitivity for increasing period, a change in speed will result in a slight change in response.

The deflexion variations of the two S.B.W.R.s and the Waverider are shown below in Table No.4.

../..

Table No.4

Run No.	Average Period seconds	Average deflexion feet	Max. % deflexion variation
S.B.W.R. No.1			
1	13.96	50.25	0.3%
2	16.57	49.14	0.5%
3	22.13	45.34	1.8%
4	25.27	43.13	2.4%
5	29.79	39.17	2.4%
6	21.60	45.97	2.1%
7	13.60	50.83	0.4%
S.B.W.R. No.2			
1	13.96	45.93	0.5%
2	16.57	44.66	0.8%
3	22.13	41.06	0.8%
4	25.27	38.83	4.8%
5	29.79	35.45	1.8%
6	21.60	41.51	1.2%
7	13.60	46.06	1.4%
'Waverider'		metres	
1	20.66	12.616	1.5%
2	13.85	13.544	0.5%
3	16.99	13.204	0.9%
4	30.76	9.564	4.1%
5	26.11	11.076	5.7%
6	21.64	12.276	4.2%

The errors in reading the chart deflexions are of the order of $\pm \frac{0.2}{50}$, i.e. $\pm 0.4\%$, for both the Waverider and the S.B.W.R.s. It should be repeated here that the interpolation scale provided with the Waverider receiver was not used to read the Waverider traces.

A small error is probably introduced into the measurement by the Big Wheel cars swinging outwards near the horizontal axis of the wheel. Theoretical calculation and cine-film evidence show that the seats fly out so that the apparent 'vertical' seat axis is some 8° to 9° from the true vertical. This 'fly-out' is progressively reduced as the cars near the vertical axis through the wheel hub. Unfortunately, due to friction at the car bearings, the cars tended to oscillate about the theoretical angle (this was shown up clearly by film measurements), making it impossible to calculate any error introduced by this effect. The cars, and the S.B.W.R. accelerometers, set themselves to the apparent vertical, but the time constant of the Waverider is such that these oscillations of the buoy do not have a significant effect on the instrument's performance. For the purposes of this experiment, the errors introduced in this way must be ignored.

Conclusions

1. Shipborne Wave Recorders:

As may be seen from the response curves, (Figure 1), the response tends to be low at the shorter periods, but this improves as the period increases, to such an extent that the response is considerably greater than the extrapolated theoretical value at 30 seconds.

Departures from the theoretical values are:

Recorder 1 : 0% at 14 seconds
 4% high at 25 seconds

Recorder 2 : 2% low at 17 seconds
 0% at 24.5 seconds

Chart reading errors are of the order of 0.5%. Unfortunately, periods shorter than 13 seconds could not be attained with the big wheel. No investigation was made into phase angles due to the difficulty of correlating chart records with wheel position.

2. Waverider:

The Waverider theoretical response curve, computed from the expression given in the section on "measurements", is shown in Figure 2. This takes no account of any amplitude distortion generated within the receiving system, which is claimed in the Waverider handbook to be 1% at 2.5 seconds period. No other information is given.

The theoretical and actual response of the buoy and receiver system is shown in Figure 1. The discrepancy between theoretical and actual response is:

At 13.5 seconds	0.77% low
At 30 seconds	2.4% low.

Again, no attempt was made by I.O.S. staff to measure the phase of the chart records relative to the phase of the applied acceleration. However, Mr. Gerritzen made a quick visual check at a period of 31 seconds approximately, and he found that the phase shift "was slightly over 90° lead," but that he experienced difficulty in measuring this value. The Waverider used was new when tested - it had not been put into the sea. Also, care had

been exercised in handling the equipment prior to the experiment. Thus the only possible time when damage could have been caused was during shipment and delivery. It is unknown what effects time of operation, battery charge-state, or rough handling have on the accuracy of calibration, - these can only be evaluated by experience. Also, the buoy mooring must exert an influence on the buoy motion while it is actually in the water, despite the fact that Datawell have taken every possible care in the mooring design.

3. S.B.W.R. laboratory experiment:

Figure 2 shows even closer agreement between theoretical and experimental response values for the S.B.W.R. than does Figure 1. The discrepancies between the curves are:

at 6 seconds	0.2%	high	} experimental curve relative to theoretical curve
at 15 seconds	1%	low	
at 25 seconds	3.3%	high	

These figures agree very closely with the Big Wheel results. No attempt was made to measure phase differences, and chart reading errors can be expected to be about 0.5% again.

Only one accelerometer was subject to these small scale laboratory tests, but the closeness of agreement between the Big Wheel and laboratory results suggests that this experimental approach is sound.

4. Waverider laboratory experiment:

The 3-metre test on the Waverider showed that the response was sensibly correct over the range of periods tested, to within the errors expected in chart reading. Thus the experimental response curve in Figure 2 is a straight line at a relative

response value of one. The test was performed on the 5-0-5 metre scale of the receiver. No information is given in the handbooks as to how the accuracy of readings is changed when the range is changed, and so no direct comparison can be made between the curves in Figures 1 and 2.

Acknowledgements:

The Wave Data Section wishes to thank the Home Office for helping to locate a suitable Big Wheel; Mr. Billing Manning, owner of the Clarence Pier Fun-Fair, Southsea, for his generosity in allowing us the use of the Big Wheel, without charge, for the experiment. Thanks are also due to Mr. A.A. Grubb, of R.V. Stokes and Co., Solicitors, of Victoria Road South, Southsea for handling the arrangements with Mr. Manning, to Mr. A. Madgwick of the I.O.S. photographic department for filming the experiment and to Mr. J.A. Ewing for advice on the theoretical 'fly-out' of the cars.

The Section also wishes to thank Mr. B. Russell and Mr. J. Fairbrother of I.H.U. for their help with timing, and for providing a reserve Waverider receiver, and to Mr. A. Wright of N.P.L. for providing a reserve buoy.

Finally, we wish to thank Datawell and Mr. P.L. Gerritzen for their cooperation and advice in setting up the experiment and interpreting the results.

