AN INVESTIGATION INTO THE RELIABILITY OF THE PROCEDURE USED TO ESTIMATE THE DIRECTIONAL WAVE CLIMATE AT SOUTH UIST

Internal Document 101

J A Crabb
July 1980

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AN INVESTIGATION INTO THE RELIABILITY OF THE
PROCEDURE USED TO ESTIMATE THE DIRECTIONAL
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SUMMARY

The procedure used to estimate the directional properties of waves at South Uist has been tested in an experiment conducted to the west of the Scilly Isles. An aircraft fitted with a radar altimeter was used to measure sea surface profiles on a number of occasions in the vicinity of a moored Waverider buoy. On each occasion the directional wave spectrum was estimated from the Waverider data using the method previously used at South Uist, and a prediction was made of the expected spectrum of each aircraft profile run, the so called encounter spectrum. The measured and predicted encounter spectra have been compared in an attempt to determine the accuracy of estimated directional spectra. It became apparent that, due to low frequency noise in the aircraft measurements, satisfactory comparisons could not be made at these frequencies. In addition a series of simple tests revealed that the form of the encounter spectrum was not very sensitive to changes in the properties of the corresponding directional spectrum.

As a consequence of these two factors no definite conclusions on the quality of the directional spectra, and thus on the efficacy of the estimation procedure, could be drawn on the basis of those comparisons alone.

Further comparisons were therefore made of some properties of the estimated directional spectra with those measured simultaneously at DB1, moored some 260km to the south west of the Scilly Isles. Mean wave directions were found to be in broad agreement, but there was some indication that the estimated directional spreads tended to be less than those measured.

The average properties of the selected set directional spectra were also found to compare well with the results of the Meteorological Office wind-wave model averaged over the years 1978 and 1979.

On the basis of these considerations it is concluded that the selected set directional spectra represent a reasonable first estimate of the South Uist wave climate.
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BACKGROUND AND INTRODUCTION

The need for information on the directional characteristics of waves was recognised at an early stage in the investigation into the feasibility of the large scale extraction of power from ocean waves. A specification of the directional wave climate was required to allow both a detailed evaluation of the available resource and to aid in the design of individual devices. Such a requirement could have been satisfied if long series of measured directional data had been available. In lieu of such data the Institute of Oceanographic Sciences was contracted to conduct a study aimed at providing the Wave Energy Steering Committee with an estimate of the long term directional wave climate at two sites of interest using only data already available.

Work started in May 1977 and data summarising the estimated one-dimensional (i.e. wave energy and frequency but not direction) climate at South Uist were released early in 1978 (Ref 1). Information on the estimated directional wave climate at South Uist, comprising a representative set of 399 directional wave spectra, and information on the wave climate at the St Gowan Light Vessel are now being prepared for release. The method used for the detailed estimation of the directional characteristics of waves at South Uist (Ref 2, and outlined in the next section) could not, for lack of relevant data, be repeated at St Gowan. In view of the low annual mean power level now expected at that site it has been agreed with the Department of Energy's Project Officer that a detailed investigation of directional characteristics is not justified.

The desirability of being able to check the efficacy of the method used to ascribe properties was recognised at the outset. It was to this end that a contract was placed (November 1977) with Flight Refuelling Ltd, to measure a series of sea surface profiles in the vicinity of a Waverider buoy moored in 99 metres of water some 28km to the west of the Scilly Isles. The shape of such profiles, obtained by flying a recording radar altimeter at low altitude over the surface, depends upon the directional characteristics of the waves. It was thus expected that comparisons of the sea surface properties measured by the aircraft with the same quantities derived from the directional wave spectra estimated from the Waverider measurements, would allow the validity of the estimation method to be assessed. A further opportunity for comparison of estimated with measured data arose with the mooring in June 1978 of the UK Data Buoy 1 (DB1) at position 48° 42'N, 08° 58'W some 260km south west of the Scilly Isles Waverider. This data buoy is capable of measuring some simple directional properties of the waves.
A series of digital wave recordings covering a whole year, with only a few gaps, was available at the South Uist site. Each record was 10,000 seconds long and comprised samples of instantaneous water surface elevation taken at 0.5 second intervals. One such record was taken every 3 hours.

Records of hourly mean wind speed and direction were available from the Meteorological Office for the nearby Benbecula Anemograph Station, as well as the long term frequency distributions of these quantities.

In an attempt to overcome the bias inherent in one year's wave measurements, a sub-set of records more representative of the long term conditions was extracted from the available data. This was effected by choosing records whose associated wind records were distributed in approximately the same way as the long term winds. A total of 399 records were selected in this way and a fast fourier transform routine used to calculate the one-dimensional energy spectrum of each. This set of records and associated spectra were then considered as statistically representative of the long-term one-dimensional wave climate and were made available to users, as well as being analysed to give various statistical parameters of the wave climate (Ref 1).

A procedure was then developed by which directional characteristics could be estimated for each of these one dimensional spectra, thus converting them to directional or two-dimensional spectra. It is the ability of this procedure to produce realistic estimates of the directional spectrum which it was desired to test.

The first step in the estimation procedure was to divide each one-dimensional spectrum into regions according to the source of the wave energy present. The regions, in descending order of frequency, were: wind sea, waves under active generation by the local wind; old wind sea, waves still propagating through the site after the local wind responsible for their generation had changed speed or direction; swell, waves not generated by local winds but propagated in from other generating centres. It was not common to find all three categories present in the same spectrum. The directional distribution of energy within each of these regions was modelled as the product of the measured one-dimensional spectrum and an angular spreading function \( h(\theta) \) or \( h'(\phi, \theta) \) (normalised to preserve the original energy). Each of these spreading functions had the form \( \cos^{2s} \left( \frac{\theta - \bar{\theta}}{s} \right) \), where \( \bar{\theta} \) was the estimated mean direction of the waves. The parameter \( S \), which controls the width of the distribution was calculated in a manner appropriate to each region. In the wind sea region \( S \) was calculated according to an empirical formula due to Mitsuyasu (Ref 9), which is a function of frequency as well as the angle, and has wind speed as a parameter. In this region \( \bar{\theta} \) was taken to be the mean wind direction. In the
old windsea region $S$ was given a fixed value of 6 (approximately equivalent to $n = 2$ in the alternative $\cos^n(\theta - \bar{\theta})$ model), $\bar{\theta}$ was again the mean direction of the wind responsible for generating the waves. In the swell region $S$ was set so as to force the width of the resulting distribution to be equal to the angle subtended at the site by the swell generating wind field. This latter required that the position and size of the generating windfield for each observed swell be identified on the meteorological charts, $\bar{\theta}$ was then taken to be the great circle bearing of the wind field centre.

The identification of these wind fields was accomplished (Ref 2) by presenting the routinely collected wave data as a contoured time series of spectral densities, Fig 1. In this representation an arriving swell train was manifest as a ridge feature in the energy contours; such a ridge has positive slope indicating, in accordance with the dispersion relation, the arrival of low frequency energy followed by progressively higher frequencies. The slope of this line is inversely proportional to the distance of the generating wind field, and the intercept on the time axis indicates the time of generation. This information allowed (in most cases) the swell generating wind field to be identified.

Having thus estimated the directional distribution for each region of the spectrum the estimated directional spectrum was formed by recombining the separate regions and evaluating the angular spreading functions at ten degree intervals.

It was intended to repeat this procedure, as closely as the available data allowed, at the Scilly Isles site and to check the resulting directional spectrum in the manner described in Section 1 and expanded in the following.
3 THE MEASUREMENTS

3.1 Waverider

The installation of the Waverider buoy and shore receiving station at the Scilly Isles have been fully reported on by the contractor, Flight Refuelling Ltd (ref 3). The Waverider measures the instantaneous water surface elevation and transmits this information continuously to shore. At the receiving station the data were recorded for 10448 seconds every three hours, both in digital form and on paper chart roll. The digital logger at this station was fitted with a switch which allowed the routine three hourly recording sequence to be interrupted and a record of arbitrary length to be taken, on both the digital logger and the paper chart roll, at the time of an aircraft measurement.

3.2 Aircraft

A towed target body was fitted with a radar altimeter and attached to the towing winch of a Canberra aircraft by a combined towing and data link cable. The target also carried an accelerometer and both the vertical acceleration of the target and the target height above the sea surface were recorded on an analogue FM tape recorder aboard the towing aircraft. On a typical sortie over the Waverider buoy the target was towed at an altitude of approximately 15 metres at a speed of 125 metres/sec. An 80 second record of the sea surface profile was taken in each of 8 different directions.

Data tapes were returned to the CI Data Centre for processing along the lines reported by Machin (Ref 4). The accelerometer signal was doubly integrated to yield the vertical displacement of the target, this was subtracted from the target height signal to yield the sea surface profile. Previous experience with this system (Ref 4) had led to the conclusion that some residual target motion still contaminated the signal at this point, and the signal was thus filtered with a digital high pass \( \omega_c = 0.15 \text{ Hz} \) to remove it. Unfortunately, this process undoubtedly removed some legitimate wave energy at the same time. The consequences of this are discussed in a later section. Data were finally presented to IOS as magnetic tape files of sea surface profile digitized at 0.05 second intervals.
A total of 9 successful sorties were made as detailed below:

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<td>25</td>
<td>330°</td>
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ESTIMATING THE DIRECTIONAL SPECTRA AT THE SCILLY ISLES SITE

4.1 Data available

Three hourly digital recordings of the Waverider signal were obtained routinely during April, but a failure of the digital recorder, not discovered until the tape was read, meant that paper chart records only were available during March. Individual records corresponding to the aircraft sorties were extracted from the charts and rendered into digital form. The record taken on 24 July 1978 was not used, and the record for 29 March 1979 could not be digitized because of an intermittent ink trace. Thus a total of seven from a possible nine measurements were used in this analysis. Because of the lack of routine data for March it was not possible to produce a contoured spectral plot to aid swell identification for this period. One-dimensional spectral data were, however, available from the DB1 location and these were used to produce a substitute plot.

The data available for the analysis are summarised in the following chart.

<table>
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4.2 Forming the directional spectrum estimates

To render the estimation procedure as used at South Uist suitable for use at the Scilly Isles site, it was necessary to 're-tune' the procedure used to divide the spectra into windsea and swell regions (Ref 2). This modification entailed plotting all the one-dimensional spectra from the Scilly Isles buoy for April and using those displaying clearly defined windsea and swell peaks to adjust the algorithm.

The Waverider records taken at the time of each overflight were divided into 1024 second sections and transformed section by section into energy spectra. These records had been taken by site personnel (HM Coastguard) switching the recorder on when the aircraft arrived in the area and switching it off when it left. Consequently the records were of variable length, comprising from one to three 1024 second sections. All data available for each overflight were used.
to produce a mean one-dimensional spectrum for each occasion.

Each spectrum was divided into wind sea and swell regions and directional properties ascribed as previously described.

It is unfortunate for the generality of this series of checks that the above procedure revealed that no significant swell component was present on any of the overflight occasions. Although this may be regarded as a significant shortcoming it does mean that the lack of routine wave data for March is not in practice of any importance.
5 THE COMPARISONS

The comparisons made between estimated and measured quantities are presented below and the results discussed.

It is perhaps worth stating at the outset that a problem common to all these comparisons is the difficulty in establishing the degree of legitimate variation to be expected between separate samples from the same population. That is to say, in establishing whether or not the observed difference between two values of a quantity indicates a significant difference between the underlying processes from which they were drawn.

5.1 Comparison of measured and estimated encounter spectra

The measured sea surface profiles, as supplied by the CI Data Centre, were each plotted on a VDU screen and visually inspected for the presence of spurious spikes. Large amplitude spikes were found in some records and were edited out. It must be accepted that such an inspection procedure could have overlooked other errors which, due to a lack of familiarity with such data, were not recognised. A Fourier spectrum was then calculated for an 80 second section of each profile record. Some records were not as long as this and were consequently not considered. These spectra are the measured encounter spectra.

Estimated encounter spectra for comparison with these measured spectra were then calculated from the directional spectra estimated for the time and date of each overflight.

The encounter spectrum is derived from the directional spectrum (Ref 5) using the fact that each element $E (f_i, \Theta_c)$ of the directional spectrum has an apparent frequency to an aircraft flying at speed $V$ and direction $\phi$, of:

$$f_i' = f_i - \frac{V \cos(\Theta_i - \phi)}{\lambda_i}$$

where $\lambda_i$ is the wave length calculated according to the dispersion relation.

A computer program was written to transform the energy of each element of the estimated directional spectrum to the appropriate point on the encounter frequency axis. The performance of this program was checked against an analytical solution for the simple case of a Pierson-Moskowitz spectrum spread by a $\cos^3(\Theta - \Theta_0)$ function (Ref 6), and was found to compare well except for a slight excess energy allocated by the program to the lowest encounter frequency. Using this program, encounter spectra were calculated for each overflight using values of $V$ and $\phi$ (corrected for wind drift) reported by the flight crew.
Figures 2 to 25 show pairs of estimated and measured encounter spectra for each low level run analysed. Note that, for economy in drawing, only the first of these plots is fully annotated but the details are the same for all the remaining figures.

5.2 Initial discussion of the comparison

When originally plotted a consistent discrepancy was apparent between measured and estimated spectral densities at mid and high frequencies. It was assumed that a systematic error in one or both of the measurement systems was responsible. The discrepancy ( -1.1dB) is clearly seen in Figure 26 which plots the average value of the ratio of aircraft to estimated spectral density at each frequency. The average value of this ratio was determined over a range of mid-band frequencies and the aircraft spectra were multiplied by a factor of 1.29 to compensate for the discrepancy.

Implicit here is the assumption that the Waverider measurements, on which the estimates are based, provide a more reliable indication of wave height than do the altimeter results. Figures 27 and 28 show how the mean square waveheight (Mo) measured on each aircraft run compares with the same quantity measured simultaneously by the Waverider.

Figures 2 to 25, which incorporate the adjustment to the aircraft spectra, show a fairly consistent deficiency in the aircraft spectra at low frequencies, this again is clearly seen in Figure 26. It was mentioned in Section 3.2 that a high pass filter was used to remove residual target motion from the aircraft data. The empirically determined shape of the filter response is shown superimposed upon the data points of Figure 26*, and it appears probable that the application of this filter was largely responsible for the observed deficiency.

*The filter response curve as actually determined was displaced by +0.68dB relative to the curve shown in Figure 26. The curve did thus not level out at 0dB as might be expected, but at -0.12dB. The filter was therefore a contributing factor to the overall -1.1dB discrepancy noted above, the remainder being presumably due to a calibration inconsistency. The comparison of the filter response and the data is correctly shown in the Figure, since the curve position chosen is equivalent to eliminating the effect of the supposed calibration error. The shape of the filter response was initially determined empirically by evaluating its effect on the spectrum of a random number series. Confirmation of the shape was later obtained analytically by my colleague Dr G N Crisp.
The presence of the spurious target motion which the filter was intended to remove is also apparent at the two lowest frequencies, where the measured to estimated spectral density ratios exceed those which would be predicted by the filter response alone. The low frequency region of the measured encounter spectra appears then to contain both wave energy and energy due to residual target motion, both of which have been reduced by the application of the filter. Some of the increased variability, as evidenced by the width of the error bars, at low frequencies may be due to this fact. It is not possible under the present scheme to separate these two components. Given this difficulty it is not possible to determine the true degree of agreement between the estimated and measured encounter spectra at low frequencies. An improved scheme for removing target motion from the measured profiles has recently been suggested (Ref 7) and this could be applied to the raw data. It would appear, however, that the difficulty is fundamental and any comparison of encounter spectra is bound to be as much a test of the procedure used to remove target motion as of the procedure used to estimate one of the compared quantities. At frequencies outside this lower frequency range the agreement is seen to be generally good. Confidence limits are readily calculable for the aircraft spectra and are marked on the first of the plots. They are not so readily determined for the estimated spectra and this point will be more fully discussed later. The significance which may be attached to the degree of agreement obtained was investigated by a series of calculations designed to demonstrate empirically the sensitivity of the encounter spectrum to changes in various properties of the associated directional spectrum, the results of these calculations are described in the following sections.

5.3 Effect of errors in estimating mean wave direction

An estimated directional spectrum was chosen at random for this test and encounter spectra corresponding to ten different profiles at 10° intervals were calculated. A selection of these is plotted in Figure 29. The result of wrongly estimating the mean wave direction would be to ascribe a direction to each estimated encounter spectrum which did not correspond with the true surface direction. Each encounter spectrum measured by the aircraft, where the true direction is known, would then be compared with an estimated spectrum calculated for a different direction. The likely effect of such a discrepancy may be seen by inspecting the difference between encounter spectra for different directions. It is apparent that the difference between spectra separated by 20° is not great, and consequently it must be concluded that, in this example...
at least, the estimated encounter spectra are not very sensitive to the accuracy with which the mean wave direction is estimated. The effect of deviating from the mean direction is to move energy progressively to lower frequencies. The effect on spectral shape is not great when the spectra are initially rich in low frequency energy. The effect would be more marked if the encounter spectrum in the mean direction contained little energy at these frequencies.

5.4 Effect of errors in estimating directional spread

The effect of errors in the estimated width of the angular distribution of wave energy were investigated by producing a simple model directional spectrum; in this case a Pierson-Moskowitz one-dimensional spectrum spread by a $\cos^n \Theta$ function, where $n$ was given values from 2 to 20. The resulting encounter spectra for a range of $n$ values, at a fixed direction of 60° from the mean, are shown in Figure 30. Again it may be seen that the encounter spectrum is not very sensitive to the angular spread of wave energy.

5.5 Effect of variability in the measured Waverider spectra

A further source of possible disagreement between measured and estimated encounter spectra is to be found in the statistical variability of the Waverider spectra used as the basis for the estimates. As described in Section 4.2 the original Waverider records were not of a fixed length and were divided into a number of 1024 second sections for the calculation of the spectra. Up to three such spectra were available for each flight and a mean of all those available was used. Using the component spectra individually, however, led to differences in the resulting encounter spectra as shown in Figure 31. Thus the sampling variability in the Waverider spectra is to a certain extent manifest in the estimated encounter spectra, and in considering the plots an allowance must be made for possible legitimate differences from this cause.

5.6 Discussion

Inspection of the plots of Figures 2 to 25 reveals that over the mid and high frequency ranges, the agreement between estimated and measured encounter spectra is generally good within the limits of the expected statistical variability. In the light of the simple sensitivity tests previously described, it must be said that this degree of agreement does not reveal a great deal concerning the accuracy of the estimated directional spectra from which the encounter spectra were derived. The difficulty involved in interpreting the results at low frequency has already been discussed in Section 5.2. In view of the inconclusive nature of these comparisons other ways were sought to establish the accuracy of the estimated directional spectra; these are described in Sections 6 and 7.
The results of the encounter spectra comparisons and the sensitivity tests do, however, indicate that the encounter spectrum is probably a very robust quantity and not oversensitive to moderate changes in the directional properties of the waves. If, as indeed seems likely, the encounter spectrum and the closely related wave number spectrum prove to be important quantities for device design, we may expect to be able to measure or estimate its form to a relatively high degree of accuracy using simple techniques. In this sense the insensitivity of the encounter spectrum to the detailed directional properties of the waves is advantageous as far as its use in engineering design is concerned. The questions which remain to be addressed are, which properties of the encounter spectrum are relevant in this application and are these suitably stable also?
A separate attempt to determine the quality of the estimated directional spectra was made by comparing mean wave directions and directional spreads with those measured by DB1. Although DB1 is moored some 260 km to the south west of the Scilly Isles site, it is reasonable to expect that, under suitable meteorological conditions, directional properties there would be a reasonable indication of those likely to be experienced to the west of the Scilly Isles. This would not be the case when an incoming swell train had arrived at DB1 but not reached the Scillies. Likewise a meteorological front positioned between the two sites would imply different wind and thus wave directions. Wave conditions resulting from winds blowing from the arc 60° to 90°, where the fetch at the Scillies is very limited, would also be dissimilar at the two sites. There is no indication that any of the above limitations apply on the occasions for which the comparisons were made.

Figures 32 and 33 show comparisons between the mean wave directions measured at each frequency by DB1 and those estimated at the Scillies site. The region within which the majority of the spectral energy lies is marked on these plots by dashed vertical lines. It may be seen that the agreement is generally good to within 20° or 30°. It is relevant to note that the sensitivity test described in Section 5.3 would indicate that even if this degree of discrepancy represented actual errors in estimated mean wave direction, and not just a real difference in conditions at the two sites, the effect on the encounter spectra would be small.

Figures 34 to 39 show comparisons between the spreading index, S, estimated at each frequency for the Scilly Isles spectra, and those measured by DB1. Again, the region of significant energy is marked.

In this series of plots the values of S for the Scilly Isles have been estimated as previously described. The analysis of DB1 data, however, gives rise to two separate quantities, S₁ and S₂. Both are functions of the coefficients of the angular harmonics of the directional distribution (Ref 8). If the unimodal cos⁴⁵(θ-θ₀) model, adopted in both the standard DB1 analysis and in the present procedure, is a good representation of the actual directional distribution these two quantities should be equal. Both S₁ and S₂ are shown in the figures, and it is the mean of these two quantities which should be compared with the estimated S, though for higher values of S, S₂ is generally the more reliable value.
In considering these comparisons it is again not clear what degree of similarity one may expect at these diverse locations. A complicating factor is the fact that windspeed is used to determine the value of $S$ in the estimated case and, during the period considered, the wind speed sensors on DB1 were inoperative. The quantity $S$ is a sensitive function ($\propto u^{-2.7}$) of wind speed (Ref 9), and for these comparisons it would have been more appropriate to use windspeeds measured at DB1 rather than those at the Scilly Isles in its determination. Part, if not all, of the observed discrepancy may be due to the difference in windspeeds as measured onshore and those actually experienced at sea. Ewing (Ref 10) has found that offshore winds in the region of the Hebrides were greater than those measured at Benbecula by an average factor of 1.29. Since the uncorrected shore based winds were actually used in the estimation of wind sea directional widths in the main Hebrides study, the discrepancies apparent in the plots presented here may also be inherent in the spectra of the selected set. There is then, some reason to suspect that the values of wind sea directional spreads incorporated in this set may be too narrow.

Furthermore, the simple way in which directional properties were ascribed to the swell portions of the selected set spectra may also give rise to over narrow estimates of directional spread. This is because such relevant factors as the movement of generating storms whilst transmitting swell to the site, and the possibility of the presence of a number of low intensity sources on any given occasion were ignored.

The simplest way to at least partially compensate for this unquantifiable tendency to narrowness, would appear to be to abandon the directional resolution of 10° to which the spectra were originally evaluated in favour of a 30° resolution. The smoothing effect of this procedure will render the directional distributions less narrow in appearance. Preliminary discussion with some potential users have indicated that this would in any case be their preferred resolution. Consequently it is proposed that the selected set spectra will be presented this way.
The main conclusions drawn from the comparisons previously presented are firstly that the intended prime test quantity, the encounter spectrum, has not proved to be a sufficiently sensitive function of the directional properties of the wave spectrum. This, combined with the difficulties experienced with the measurements at low frequencies, has meant that no conclusive statement on the quality of the estimated directional spectra can be made on the basis of these comparisons alone. Secondly, comparisons made with the DB1 measurements suggest that the estimated mean wave directions would be in good agreement with those which might have been measured at the site, but that the procedure has resulted in a tendency towards over narrow distributions of energy with direction. A simple strategy whereby the worst effects of this error may be mitigated has been suggested.

The one obvious shortcoming in this evaluation procedure has been the lack of observed swell. This means that the ability of the procedure to estimate the directional properties of this most significant portion of the wave spectrum remains largely untested. A separate consideration of a 'swell ridge' appearing in the DB1 contoured spectral data on the 16 and 17 March produced an estimated mean direction of $270^\circ$ compared with $273^\circ$ obtained from the low frequency DB1 measurements. The estimated directional width on this occasion was $15^\circ$ compared with a measured width of $58^\circ$. This isolated example is of course not a reliable general indication of a trend but, in so far as it goes, tends to reinforce the previous conclusion regarding wind sea mean directions and widths.

A separate approach to the evaluation of this set as an indicator of the main properties of the South Uist directional wave climate is to compare it with estimates from other sources. There are admittedly few such estimates with which to compare, but even so the results which are available all tend to point to similar conclusions and an increasingly convincing picture of the climate is emerging.

The main characteristics of the wave climate are summarised by the mean annual wave power, the mean distribution of wave power with wave frequency and the mean distribution of wave power with wave direction. These quantities are estimated for the South Uist location by forming averages across the selected set spectra and in the following paragraphs are compared with estimates of similar quantities from other studies.
Mean annual power:

The mean annual power at South Uist predicted by the present procedure is 47.8kw/m. Mollison (Ref 11) has used all the data from the first two years of measurements at this site to predict the same quantity. He ascribed unequal weights to each of the 2h months data according to the level of wind energy input and formed a weighted average of all the data. The resulting prediction was 50.3kw/m. Both of the foregoing estimates are based on interpretations of the South Uist measurements. Some indication that these are not atypical of the region is given by an analysis (Ref 12) of the results produced by the Meteorological Office wind-wave model (Ref 13) during its routine operational use in the years 1978 and 1979. The average power at a location close to the South Uist buoy was 46kw/m. A further broad confirmation of the representativeness of these results was obtained by a consideration of data from OWS India (Ref 14) which suggested that the mean annual power at South Uist should lie in the range 45.7 to 54.0kw/m.

These comparisons, indicating as they do close agreement between independent estimates of power level, suggest that the selection procedure used in the present study has resulted in a set of sea states reasonably representative of the long term conditions.

Mean distribution of power with frequency:

Given the apparently reliable nature of the mean power estimate, it may reasonably be assumed that this distribution is well represented by the mean of the selected set measured power spectra. Close agreement is apparent between the mean power spectrum for the selected set and that for the results of the Meteorological Office model obtained during 1978/1979 (Ref 12), Figure 4.0.

Mean distribution of power with direction:

An estimate of wave directional properties is the most important addition to present information on the South Uist climate that the processed selected set has to offer. Comparison between the mean distribution calculated for the selected set and that for the 1978/1979 results from the Meteorological Office model reveals an encouraging degree of agreement, Figure 4.1.
8 CONCLUSION

On the basis of the comparisons and considerations presented above it is concluded that the selected set directional spectra, as they will be presented to users, represent an acceptable first estimate of the long term characteristics of waves at the South Uist site.

This is not to say that on every occasion individual spectra reproduce exactly the conditions which might have been measured but that, given the inbuilt constraints of the procedure each individual spectrum is an inherently reasonable possible outcome of prevailing conditions. Some support for this contention is provided by the comparisons with other independent estimates of the climate parameters.

It is hoped that the use of these spectra in assessing device performance and the magnitude of the overall resource will be seen as a significant improvement over the approach hitherto necessary. Their use should help not only in design and resource calculations, but also in enabling both scientists and engineers to gain experience in using this relatively novel tool.

It must however be remembered at all times that these results are no more than an estimate of climatic conditions and that the collection of a long series of measured directional data is essential for a reliable picture of the climate.

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Figure 1 Contoured time series of spectral densities
Figure 2
Figure 3
Figure 4
Figure 5
Figure 6
Figure 7
Figure 8
Figure 9
Figure 10
Figure 11
Figure 12
Figure 13
Figure 14
Figure 15

Graphs showing data for angles 345° and 114°.
Figure 17
Figure 19
Figure 20
Figure 21
Figure 22
Figure 23
Figure 24
Figure 25
Figure 26: Average ratio (dB) of encounter spectral densities derived from aircraft measurements to those derived from the estimated directional spectra. Some typical standard error bars are shown.
Figure 27 Mean square wave height \( (M_o) \) measured on each aircraft run (x) compared with \( M_o \) measured at the same time by the Waverider (---)
Figure 28 Mean square wave height ($M_o$) measured on each aircraft run (×) compared with $M_o$ measured at the same time by the Waverider (-----)
Figure 29  Encounter spectra calculated at various angles from mean direction to show effect on spectral shape of errors in estimating mean direction
Figure 30 Encounter spectra at $60^\circ$ mean direction derived from a model directional spectrum with various values of spreading index $n$ to show effect of errors in estimating $n$
Figure 31  Encounter spectra at $60^\circ$ to mean direction derived from consecutive 1024 second Waverider records
Figure 32 Mean wave directions (x) at each frequency measured by DB1 compared with mean directions estimated at the Scilly Isles Buoy (---). Vertical dashed lines demark region of significant spectral energy.
Figure 33 Mean wave direction (x) at each frequency measured by DB1 compared with mean directions estimated at the Scilly Isles buoy (—-). Vertical dashed lines demark region of significant spectral energy.
Figure 3: Estimated angular spreading index $S$ compared with $S_1$ and $S_2$ measured by DB1. Vertical dashed lines demark region of significant spectral energy.
Figure 35 Estimated angular spreading index $S$ compared with $S_1$ and $S_2$ measured by DB1. Vertical dashed lines demark region of significant spectral energy.
Figure 36 Legend as Figure 3b.
Figure 37 Legend as Figure 34
Figure 38 Legend as Figure 3h.
Figure 39 Legend as Figure 3b.
Figure 1.0 Selected set mean power spectrum compared with Meteorological Office model mean power spectrum for 1978 and 1979.
Figure 4.1 Distribution of power with direction for the selected set compared with results of Meteorological Office model for 1978 and 1979.