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as measured by the N.I.O. shipborne wave recorder installed in the R.V. "Atlantis" and the Woods Hole Oceanographic Institution wave pole

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ABSTRACT

This report is complementary to the Woods Hole Oceanographic Institution Report "Analysis of the performance of the N.I.O. Shipborne Wave Recorder installed in the R.V. 'Atlantis'", reference No. 55-64. The W.H.O.I. report describes statistical tests and comparisons on a wave-by-wave basis, whereas in the present report the responses are compared by measuring the spectra of the outputs of the two wave recorders. It is, of course, only possible to find the relative response of the two instruments. There appears to be a consistent ratio 1.4:1 between the amplitudes they give, but the variation of the ratio with wave period follows the theoretical curve. The results have some bearing on ship motion studies.

Introduction

In September 1955 a shipborne wave recorder designed by the National Institute of Oceanography was installed in the R.V. "Atlantis", an auxilliary ketch of about 300 tons displacement owned by the Woods Hole Oceanographic Institution. W.H.O.I. have developed a free-floating wave pole which can be fitted with either a capacitance sensing element or a resistance-wire sensing element.* The "Atlantis" was shortly to leave on a wave-measuring cruise, and it was decided to carry out a series of tests on the shipborne wave recorder and to compare its output with that of other wave-measuring devices. A series of tests was devised by Wilbur Marks, who has reported on the results in reference 1. His analyses and comparisons were partly wave-by-wave and partly statistical. The present report deals with comparisons of the spectra of the outputs of the two instruments, obtained by means of the N.I.O. Fourier Analyser.

The Shipborne Wave Recorder

This instrument measures the sea pressure at a point on the ship's hull to give the height of the water surface above the point; it adds this to the vertical displacement of the point, as measured by an accelerometer whose output is integrated twice electronically (figure 1). This combined signal is proportional to the wave-height (with known correction factors) and is virtually independent of the motion of the ship. In order to overcome the effects of wave reflexion from the side of the ship, two measuring heads are mounted opposite one another on the port and starboard sides of the ship and their mean output is recorded. A more detailed account of the instrument is contained in reference 2, and full details are contained in reference 3.

The pressure units have to be mounted sufficiently far below the waterline to ensure that they do not emerge when the ship rolls. This means that the very short waves do not reach down to them and are not measured; this constitutes one of the major limitations of the instrument. In most installations the measuring heads have to be fixed where there is accessible clear space on the hull, and often this is at a place where the hull is sloping. The effective depth in these circumstances is doubtful, but is probably the distance round the hull: in the "Atlantis" this is 13 ft., whereas the true depth is only 9 ft. The response curve shown in figure 3 is calculated on this assumption. There are other causes of uncertainty in the response to short waves, and the present test was designed largely to find out whether the actual response curve really followed the predicted curve.

The W.H.O.I. Wave Pole

A diagram of the wave pole is given in figure 2. The large tank between 75 and 95 feet acts as a stabilising inertia below the action of the shorter waves, so that these go up and down the comparatively stationary pole and are measured by the sensing element.

The calculation of the response of this system to waves of different periods is somewhat involved. The differential equation of the vertical motion of the pole is

$$f = M\frac{d^2z}{dt^2} + R\frac{dz}{dt} + Nz$$

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* An earlier version of the wave pole is described in reference 1. The resistance-wire sensing element was part of a slope-measuring device whose early development was carried out by the Georgia Institute of Technology.

Where f is the force due to the waves
z is the vertical linear displacement of the pole
M is the effective mass of the system ≃ the volume displacement
R is the coefficient of resistance to vertical motion (assumed to be indpendent of velocity)
N is the buoyancy coefficient = ρg . area of the pole.

The force f is the sum of the wave pressure x area on the four horizontal faces, due allowance being made for the exponented attenuation of wave pressure with depth, plus a component due to the fluid friction of the water moving past the pole, which can be neglected in the present calculations.

If the surface elevation is $A_0 \cos \omega t$, where $\omega = 2\pi/T$ and T is the wave period, then $f = F(T)\, A_0 \cos \omega t$ where $F(T)$ is a function of T and the solution of the differential equation is of the form

$$z = A_0 F(T)\,[G(T)\cos \omega t + H(T)\sin \omega t]$$

where the terms inside the square brackets represent a response of the normal resonant type with a resonant period of 36 seconds.

The output of the sensing element is $(A_0 \cos \omega t) - z$

$$= A_0\,[1 - F(T)\,G(T)]\cos \omega t - A_0 F(T)\,H(T)\sin \omega t$$

The relative amplitude of the output is thus

$$\sqrt{\{[(1 - F(T)\,G(T)]^2 + [F(T)\,H(T)]^2\}}$$

It is now necessary to estimate the value of R. Reference 1 gives $2R = \sqrt{NM}$ (0.25 critical damping) for a system rather similar to the one under consideration, and using this value it can be shown that $[F(T)\,H(T)]^2$ is negligible below about 12 seconds period, so that the relative amplitude response becomes $1 - F(T)G(T)$.

The whole response curve shown in figure 3 has been calculated using this simplified equation, since we are most interested in waves with periods below 12 seconds and the value of R is no more than a guess. In any case the resistance is turbulent and R is therefore not a constant. The curve is therefore only approximate for periods greater than about 12 seconds.

For short waves the wave force is downwards (negative) under a wave crest, but G(T) is also negative so that the relative response is less than 1. As the waves get longer the buoyancy, which is upwards under a wave crest, becomes more important and there is a period beyond the limit of ordinary waves at which the wave forces just balance and there is no resultant force on the pole. It is the approach of this condition which accounts for the upward trend of the response near 20 seconds period.

For the present comparison the resistance-wire sensing element was used. In this a stainless steel wire passes vertically through the water surface and the resistance between the top of the wire and sea is measured using high-frequency a.c. The wave pole is connected to the ship by a 2000 ft. cable which is buoyed at frequent intervals. In operation the device is put overboard and allowed to drift away from the ship while recording is in progress, cable being paid out continuously. When the limit of the cable is reached the ship steams slowly up to the pole and the cable is brought back inboard.

The records and the method of analysis

The output of the two wave recorders were recorded side by side on a Sanborn 4-channel recorder with a chart speed of 2.5 mm/sec (approx. 6 in./min). The sensitivity of the shipborne wave recorder was 4 mm/ft and that of the wave pole 4.84 mm/ft. Recording started at 1113 hours E.D.T. on 14th October 1955 and continued for about 45 minutes. The wave pole was sufficiently near the ship for a short time at the beginning of the recording to enable the the same wave to be seen on both records, but for most of the time there is no wave-by-wave correlation.

To convert the records into a suitable form for analysis on the N.I.O. wave analyser, early versions of which are described in references 4 and 5, they were first traced onto plain chart paper and then converted into an electric current using a photo-electric curve follower. This current was re-recorded photographically in the form required by the analyser.

The analyser gives the amplitudes of the Fourier harmonics of a record and will resolve up to the 100th harmonic. With the most satisfactory set-up at present available, analysis starts at the 30th harmonic. The period range of greatest interest for the present purpose is 3 to 10 seconds, and records of about 5 minutes duration were therefore required. 9 such records for each instrument could be obtained from the 45-minute original available. The harmonics on each analysis were divided into groups of 10, and the sum of the squares of their amplitudes were computed. These are the values of Σa^2 given in the tables, and represent the energy contained between the corresponding period limits in each record. The means of these values over the 9 records corrected for the instrument calibration have been plotted in figure 4. The square roots of the ratios of these spectral densities, corrected for the nominal sensitivities of the two instruments have been plotted in figure 5 and are compared with the theoretical variation of the relative amplitude response with wave period.

The analysis could usefully have been extended to about 15 seconds period, which would have allowed another point to be added to the measured responses. However, this would have involved making a further set of reproduction records on a contracted time scale and was not thought to be worth the considerable effort involved.

The statistics of the comparison

The wave records are effectively uncorrelated, and their comparison is therefore subject to random statistical errors which are, unfortunately, quite large. Although each a^2 in the table is the average of 90 harmonics, the standard statistical error is $1/\sqrt{90} = 10.5\%$ (assuming the spectral energy level is constant in the range under consideration: if it is not, the error is increased). The comparison of two figures each subject to this error has a standard error of $\sqrt{2} \times 10.5\% \simeq 15\%$. However, when the square roots of the mean squares are taken to given r.m.s. amplitudes, the proportional error is approximately halved and becomes about ±7.5%, so that the 95% confidence limits are approximately ±15% for the points in figures 5 and 6.

Discussion of the results

The ratios of the outputs of the two instruments at various periods are plotted in figure 5 and compared with the theoretical response curve. Too much reliance should not be placed on the lowest point since this is computed from very low spectral densities and may have been significantly influenced by noise introduced in the reproduction process.

It is immediately clear that there has been a major error in the calibration of one of the instruments, and that either the shipborne wave recorder was about 1.4 times more sensitive or the wave pole was 1/1.4 times less sensitive than stated. This need not have been an error in the absolute calibration (i.e. volts output per foot), but could have occurred in the transfer of this into chart divisions per foot. Marks' comparison of the sensitivity of the shipborne wave recorder with that of the capacitance wave pole showed that these two instruments agreed within 3%, but this is not conclusive since he was not able to make allowance for the different frequency responses of the two instruments.

A new curve has been drawn with this constant factor included, and the points now all lie on this within their confidence limits. The mean-square deviation from the line is 9%, which is not significantly greater than the theoretical standard error of 7.5%, and it can therefore be said that within experimental error the variation of the response with wave period has been confirmed.

However, there is some slight indication that the response of the shipborne wave recorder may fall off even more quickly for short waves than the theoretical curve indicates: a curve fitted to the top four points would be above the dashed line, whereas a curve fitted to the lower four points would be below the dashed line.

In figure 6 the measured points are compared with a response curve computed on the assumption that the shipborne wave recorder short-wave attenuation is that corresponding to a depth of 9 ft., which is the vertical distance of the pressure units from the water level (see above in the description of the shipborne wave recorder). It is clear that the points do not fit this curve. This fact is of fundamental importance in the study of ship motion, since most calculations of the forces acting on a ship have in the past assumed that the wave pressure on the ship's hull at a point is given by the simple exponential formula using the true depth.

Conclusions

(1) There was a major error in the calibration of one of the instruments.

(2) The relative response of the two instruments varies with wave period in the predicted manner.

(3) The wave pressure on sloping parts of the ship's hull falls off considerably faster with depth than is given by the classical exponential law using the true depth.

Acknowledgement

The records analysed in this report were taken when the author was a guest aboard the "Atlantis". They are the property of the Woods Hole Oceanographic Institution, and the author is very grateful for the opportunity to analyse them. The testing programme was under the direction of Wilbur Marks, and Harlow Farmer was Chief Scientist on the "Atlantis" during the voyage.

References

1) Farmer, H.G., Marks, W., Walden, R.G., and Whitney, G.G. 1955 A technique for ocean wave measurement. Proc. 1st Conference on ships and waves, Chapter 2.

2) Tucker, M.J. 1952 A wave recorder for use in ships. Nature, Lond. 170, 657.

3) Tucker, M.J. 1954 A shipborne wave recorder. N.I.O. Internal Report No. A2.

4) Barber, N.F., Ursell, F., Darbyshire, J. and Tucker, M.J. 1946 A frequency analyser used in the study of ocean waves. Nature, Lond. 158, 329.

5) A.R.L. Internal Report ARL/R9/103.30/W 1950 Handbook for the A.R.L. Analyser Type 1.

The Wave Spectra Measured from the Analyses

The main blocks of figures are values of Σa^2, that is, the sums of the squares of the amplitudes of 10 adjacent harmonics. Harmonic amplitudes are measured in chart divisions.

	Mean Period (sec)							
	3.03	3.36	3.79	4.29	4.94	5.86	7.16	9.23
Analysis No.	(a) The W.H.O.I. Wave Pole							
1	13.7	25.0	24.4	89.9	272.0	46.4	29.7	37.0
2	6.8	29.5	37.1	119.1	194.0	37.3	34.2	64.9
3	25.4	22.7	46.4	89.1	226.1	28.3	24.6	39.6
4	10.5	17.8	26.6	25.6	189.0	60.2	18.6	42.2
5	7.7	13.3	27.2	84.9	237.3	37.0	25.4	35.3
6	10.8	25.9	32.6	71.2	140.4	99.0	35.0	33.3
7	10.9	12.8	20.2	53.4	115.2	56.3	19.6	21.6
8	14.2	18.5	42.9	94.9	166.9	48.1	31.6	30.3
9	10.4	23.3	33.9	61.9	182.4	53.0	18.6	35.8
$\overline{a^2}$	1.22	2.10	3.23	7.67	19.14	5.18	1.63	3.78
Analysis No.	(b) The Shipborne Wave Recorder							
1	0.59	0.77	4.5	44.6	54.3	58.9	83.4	93.3
2	0.30	1.90	3.0	13.2	90.1	63.7	49.9	143.9
3	0.78	3.40	11.3	19.2	128.8	75.2	47.7	108.4
4	0.76	3.46	8.7	28.2	248.2	65.5	84.5	96.6
5	1.28	2.38	5.0	12.6	92.9	108.9	20.5	131.8
6	0.85	3.72	4.1	41.9	81.0	54.1	70.9	79.8
7	1.28	1.45	6.3	22.3	78.7	62.7	23.9	111.6
8	0.91	1.55	3.7	26.2	130.4	53.0	67.6	95.7
9	0.73	1.84	7.7	36.5	191.4	52.7	63.2	84.6
$\overline{a^2}$	0.083	0.216	0.603	2.72	12.18	6.61	5.68	10.51
	(c) The ratios of the r.m.s. amplitudes allowing for the nominal sensitivities of the two instruments. Values are $1.21\sqrt{\dfrac{\overline{a^2}\,\text{S.B.W.R.}}{\overline{a^2}\,\text{wave pole}}}$							
	0.315	0.387	0.521	0.719	0.964	1.342	1.775	2.015

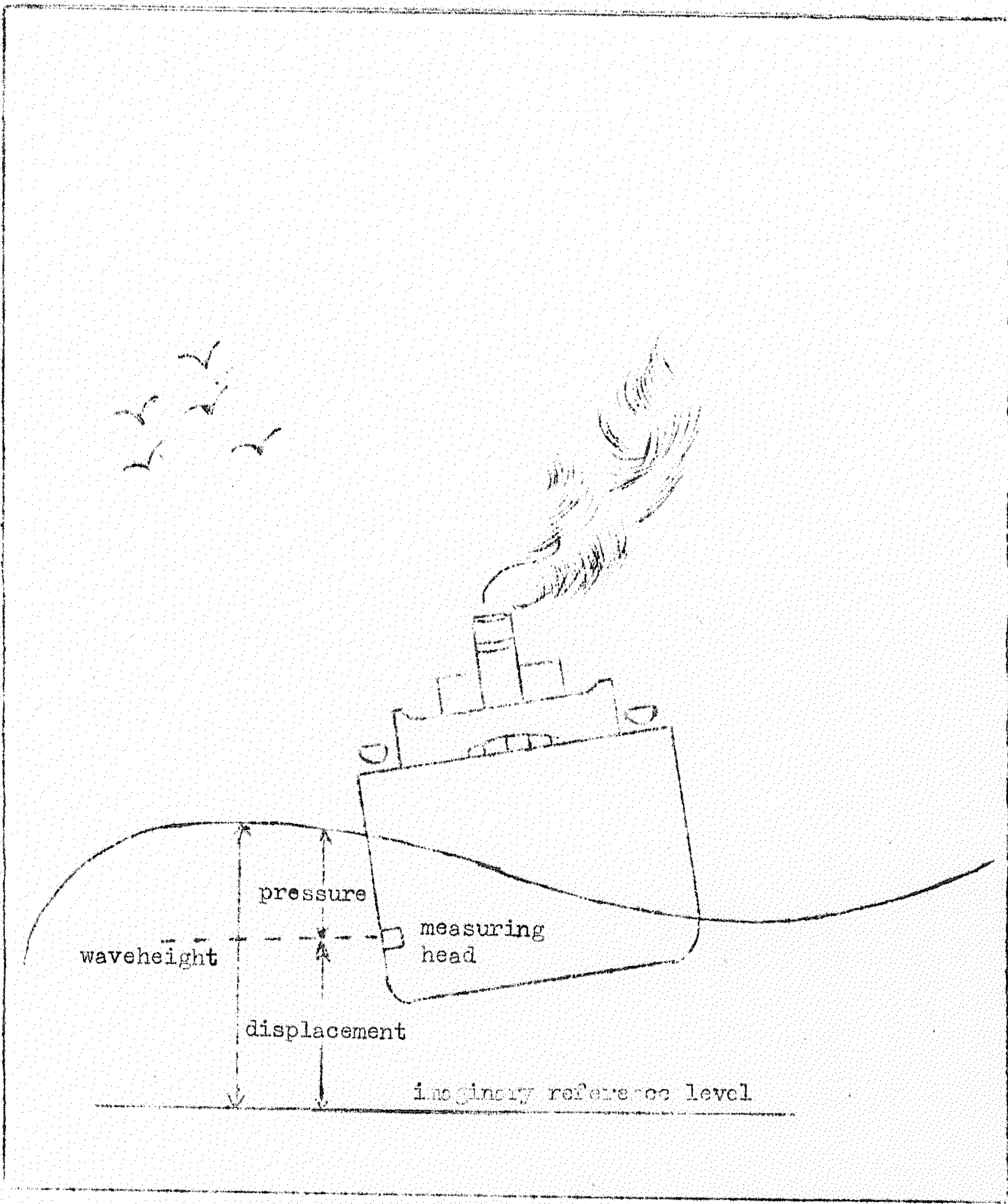


Figure 1 The shipborne wave recorder

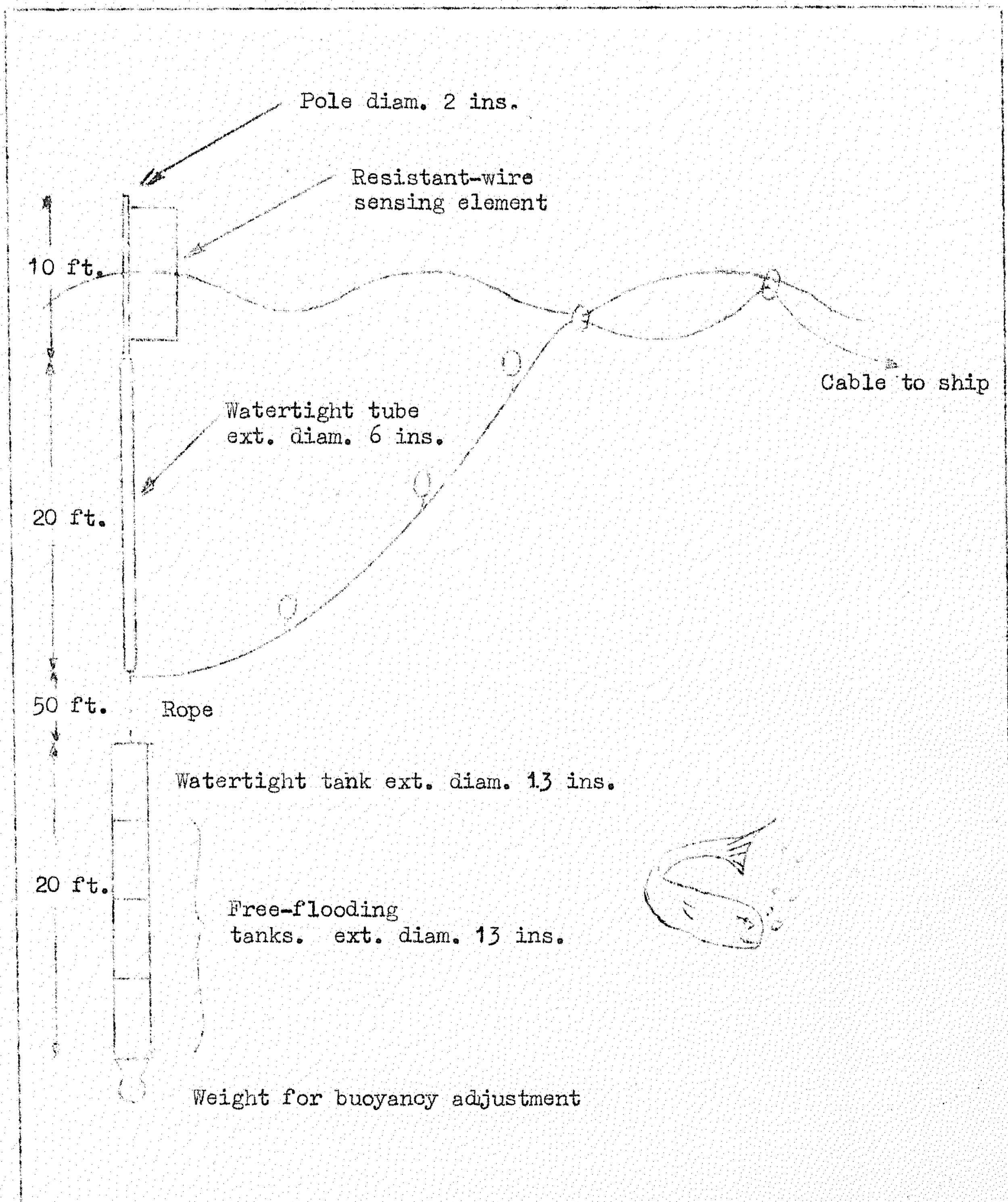


Figure 2 The W.H.O.I. wave pole

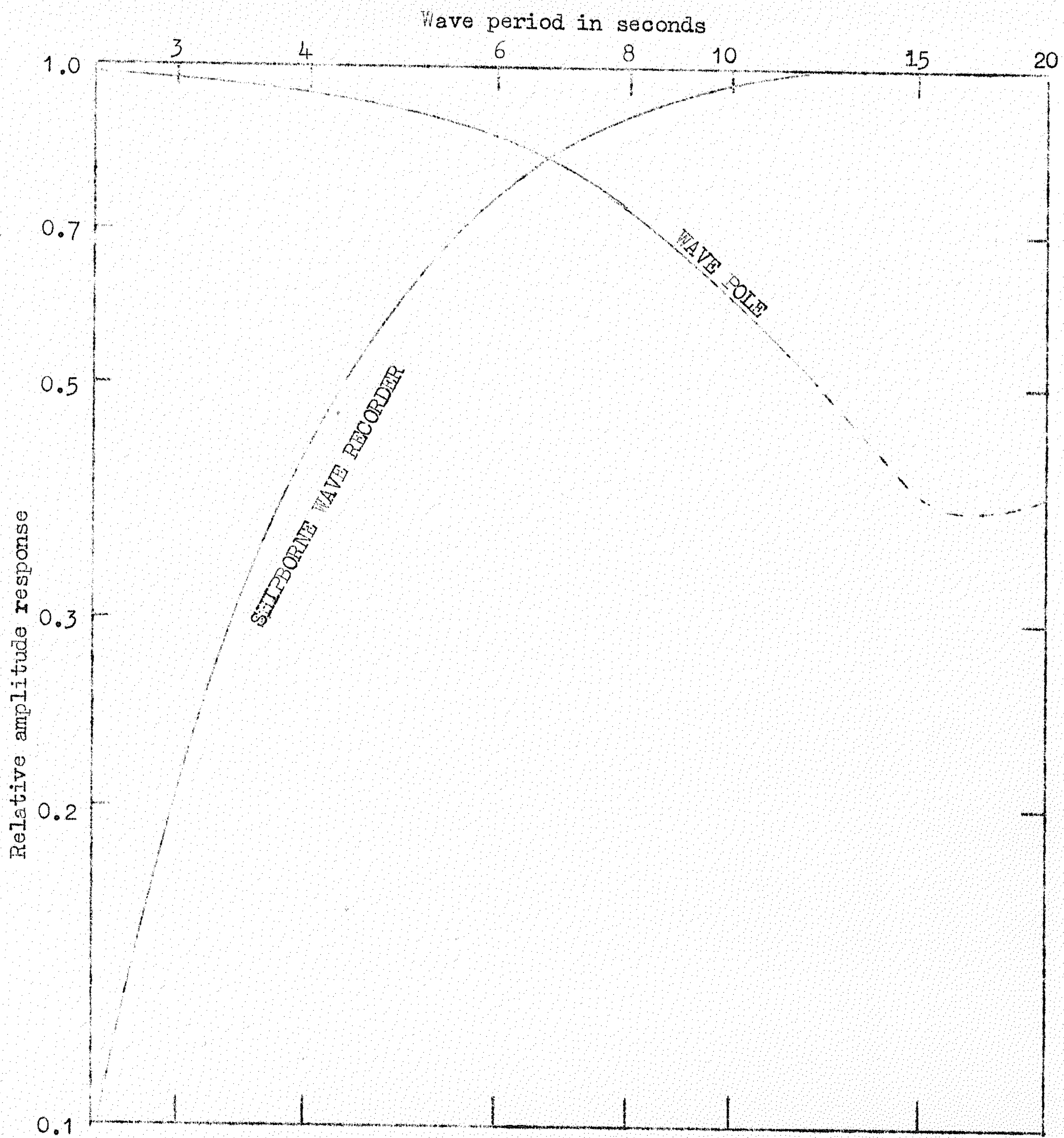


Figure 3 The theoretical responses of the wave recorders. The wave pole response is only approximate for periods above 12 seconds.

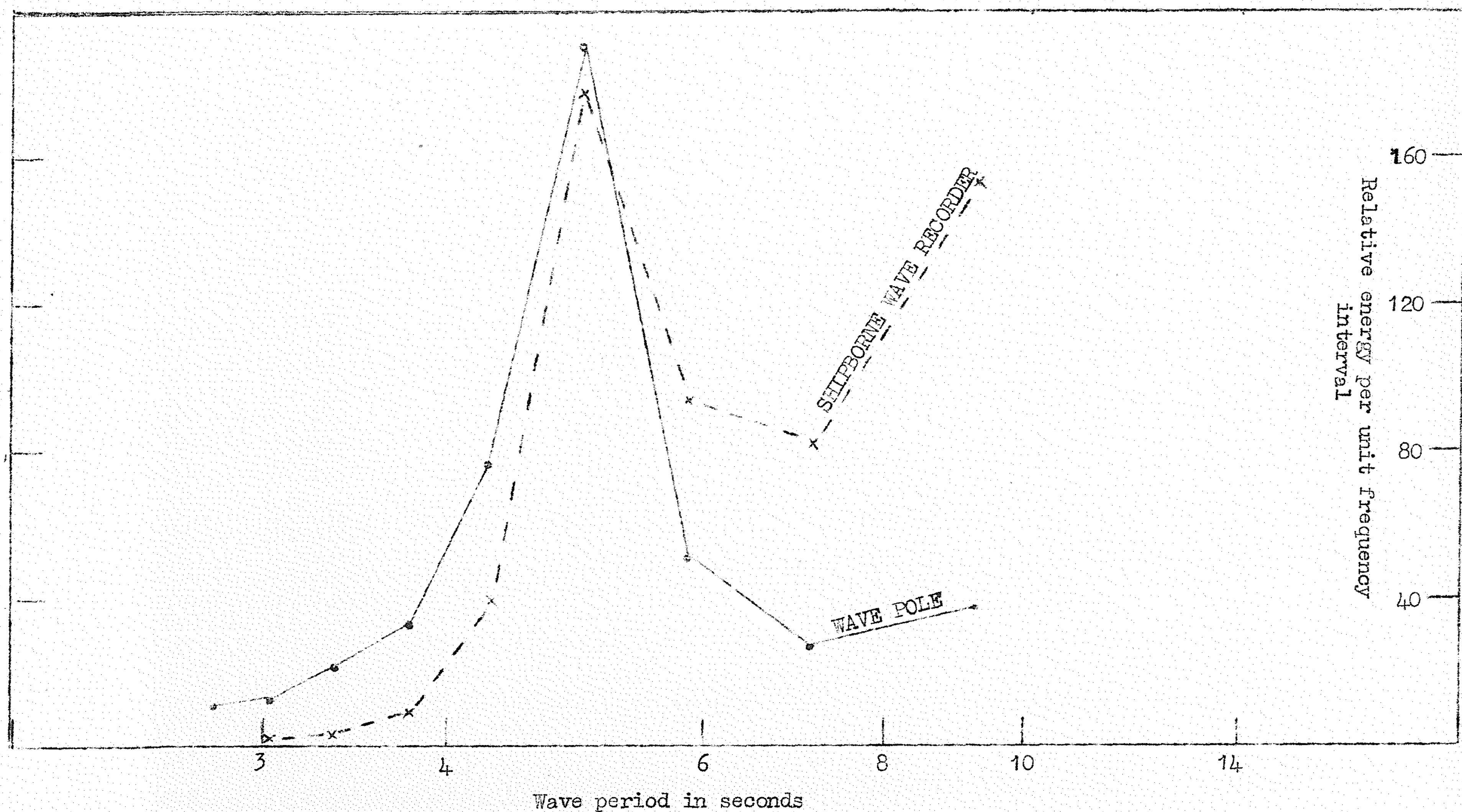


Figure 4 The wave spectrum as measured by the two instruments before correction 11.13 hrs. 14 October, 1955.

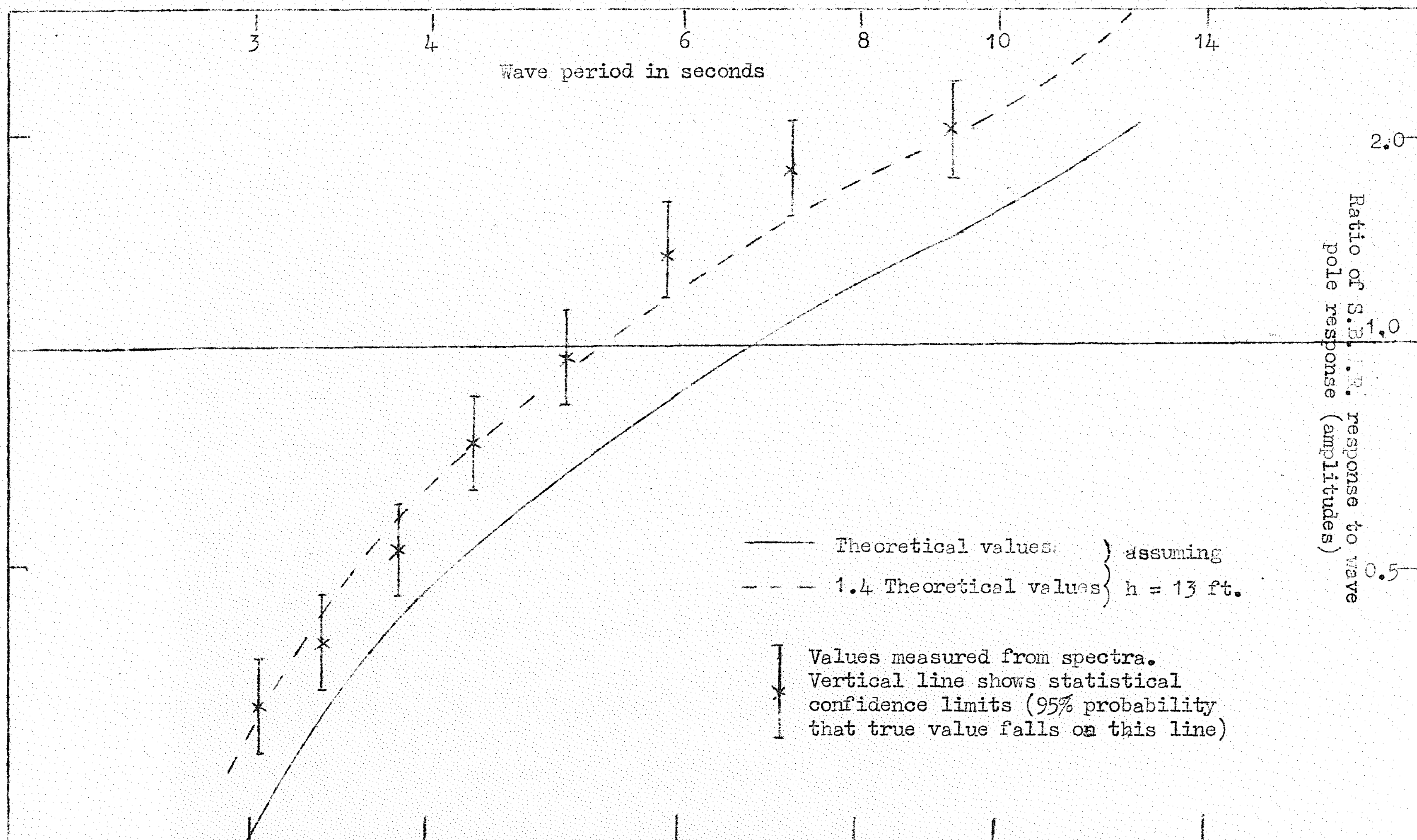
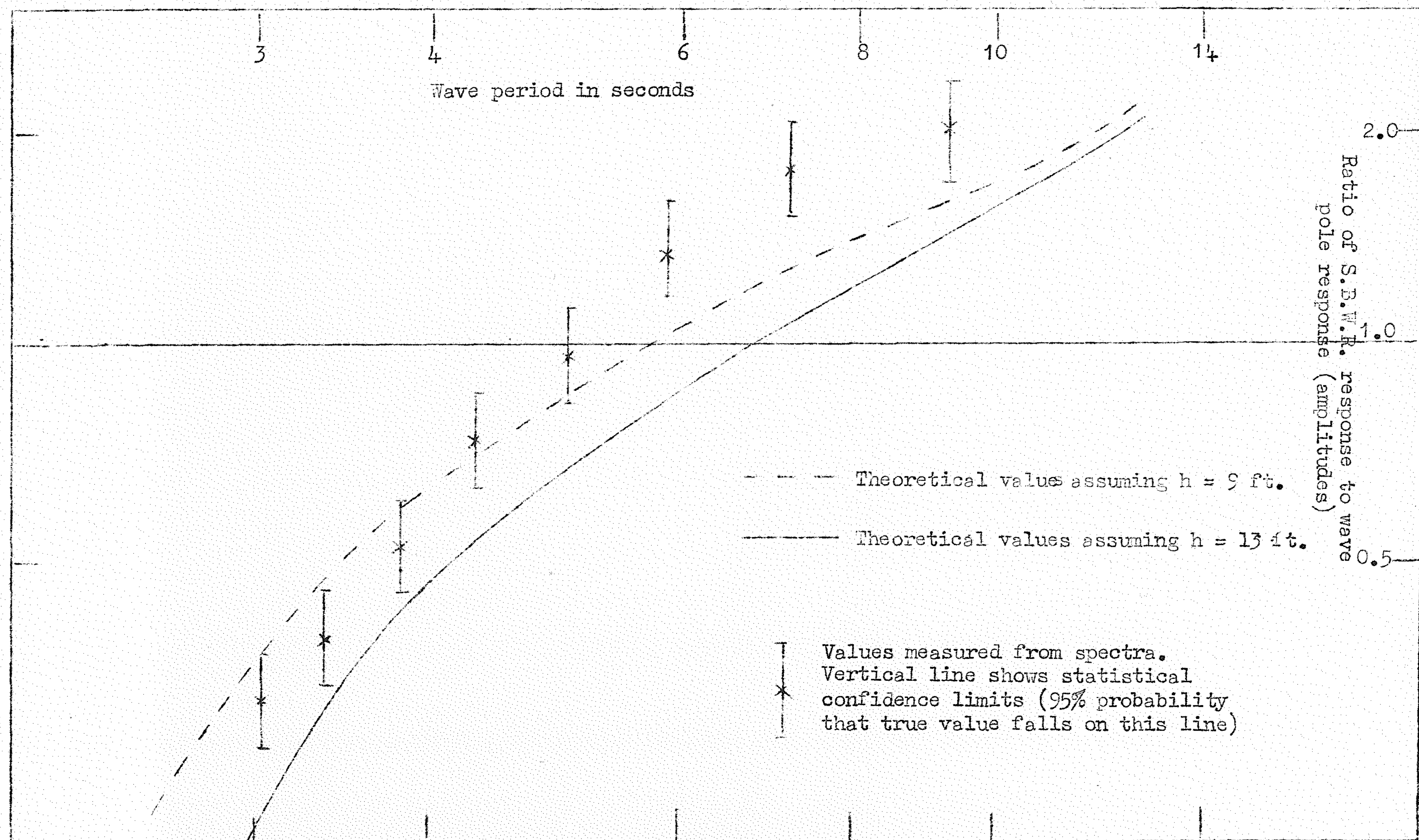

3
4
6
8
10
14
Wave period in seconds
2.0
1.0
0.5
Ratio of S.B.R. response to wave pole response (amplitudes)
Theoretical values
1.4 Theoretical values
assuming h = 13 ft.
Values measured from spectra.
Vertical line shows statistical confidence limits (95% probability that true value falls on this line)

Figure 5



3
4
6
8
10
14
Wave period in seconds
2.0
1.0
0.5
Ratio of S.B.W.R. response to wave pole response (amplitudes)
Theoretical values assuming h = 9 ft.
Theoretical values assuming h = 13 ft.
Values measured from spectra.
Vertical line shows statistical confidence limits (95% probability that true value falls on this line)

Figure 6