Determination of the frequency response of the SBWR installation at the Seven Stones Lightvessel

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<td>ABSTRACT</td>
<td>This report describes an experiment to measure the response of a modified shipborne wave recorder (SBWR) fitted to the Seven Stones Lightvessel by comparison with a Waverider buoy moored nearby. A brief discussion of the processing method is given and of the statistics of the comparison. The response was found to differ from previous comparisons involving the former type of SBWR installation.</td>
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INTRODUCTION

Trinity House has decided to convert its manned lightvessels (LV) to automatic (unmanned) operation. In the past it was the practice to drill a hole in each side of the vessel to provide communication between the instrument's pressure sensors and the sea. These were at a depth of 1 to 2 m and presented a slight risk of water ingress. However, so long as they were provided with isolating valves which could be operated by the crew in an emergency, this was considered acceptable. With the departure of the crews, Trinity House has insisted that the watertight integrity of the vessel's hull shall not be compromised and so a different method of communicating the pressure in the sea to the instrument has had to be devised. This consists of a pipe on each side of the ship running from the measurement position up the outside of the ship to deck level so that an aperture through the hull is no longer required.

In recent years studies have been undertaken to provide a better description of the frequency response of the SBWR, and five sets of measurements have been used to develop an empirical response formula. It has been found that when the frequency is scaled using the ship's length and the mean pressure sensor depth, the responses measured with different SBWR installations agree reasonably well.

It was therefore of some interest to determine the frequency response of the new SBWR installation and see to what extent it fitted in with earlier empirical results.

THE MEASUREMENT PROGRAMME

The response of the SBWR was determined by comparison with the measurements of a Datawell Waverider buoy (WR) moored nearby. It was intended that measurements should be made of the response of the original installation as well as the new one and so a WR was moored to the seabed about 1 km NW of the manned LV on June 30 1987. At the same time a visit was made to the LV and the SBWR and microcomputer were replaced and other servicing work undertaken. A receiving
site was set up at the Coastguard station at Gwennap Head to receive the WR signal direct and process and record the data in the usual way.

In mid-September 1987, the manned LV was withdrawn and replaced by an unmanned vessel. This was equipped with the 'new' (modified) SBWR installation and a modulator/transmitter to send an analogue of the SBWR output to the receiving site at Gwennap Head. Here the signal was received and the wave information was processed and recorded using a microcomputer in much the same way as it had previously been on board the LV.

The WR went adrift during the latter part of October 1987, the last observation being received on 17 October, and this brought the comparison exercise to an end. The buoy later went ashore in Eire from where it was eventually recovered.

DATA RETURN

The data return was poor throughout this period. The first disappointment was that there were no valid data from the 'old' LV installation before it was replaced by the unmanned vessel. The causes of this were technical failures combined with loss of morale among the LV crew, many of whom faced redundancy. As far as the 'new' installation was concerned, a combination of unreliable telemetry and recording failures reduced the data return to the point where simultaneous data from both sources were obtained only for a few days before the WR drifted off station. In total, 32 simultaneous observations were recorded.

CALCULATION OF SPECTRA

The SBWR data were processed on board the LV and at Gwennap Head using a microprocessor according to the standard IOSDL scheme. The wave observation was divided into two halves, each of 2048 samples measured at 0.5 second intervals. Each half was subjected to the following:
(1) Quality control: a number of range and rate of change checks were performed on the data, and depending on the outcome of these the record was either 'rejected' or 'accepted'.

(2) If the 1024 second sequence was rejected no further processing was undertaken. If accepted, the spectrum was calculated by standard FFT methods using the following parameters:

(a) A cosine taper was applied over 1/8 of the series at each end, and the scaling was adjusted to compensate for the resulting loss of variance.

(b) If both halves were accepted, these were averaged.

(c) The spectral moments were calculated and from these $H_g$ and $T_z$.

(d) The spectrum was averaged over 7 adjacent estimates to give a final resolution of 0.006836 Hz. If both halves were used the final spectrum was defined with 28 degrees of freedom, if only one half was available the spectrum was defined with 14 degrees of freedom.

The results were written to magnetic tape.

The WR transmissions were received at Gwennap Head and processed using the standard IOSDL microcomputer system. The method of processing was essentially the same as for the SBWR data except that the wave observation was processed in one sequence consisting of 4096 samples at 0.5 second intervals. Also, the elementary spectral estimates were averaged 15 at a time leading to a final resolution of 0.007324 Hz with 30 degrees of freedom.

COMPARISON OF THE SPECTRA

Suppose the spectrum in the region of the sea in which both measurements are made is $S_i$, where $i$ is the frequency index. The sample estimate of the spectrum measured, for example, by the WR will differ from the 'population' spectrum by a random error which is proportional to the true spectrum:
If \( s_{wi} \) is estimated with \( v_w \) degrees of freedom and if the error \( \varepsilon \) arises entirely from the intrinsic variability of the (Gaussian) sea surface, \( v_w(1 + \varepsilon_{wi}) \) is Chi-square distributed with \( v_w \) degrees of freedom.

In the case of the SBWR we must also consider the frequency-dependent instrument scaling factor \( R_i \), so that:

\[
S_{bi} = R_i s_i (1 + \varepsilon_{bi})
\]

Forming the ratio of the measured estimates we have:

\[
\frac{S_{bi}}{S_{wi}} = \frac{R_i (1 + \varepsilon_{bi})}{(1 + \varepsilon_{wi})} \tag{1}
\]

This is the ratio of two Chi-square variables divided by their degrees of freedom and is distributed as \( F_{bi, wi} \). We may estimate \( R_i \) by

\[
R_{iA} = \frac{1}{N} \sum_{j=1}^{N} \frac{S_{bij}}{S_{wij}} \tag{2}
\]

where \( j \) is the observation index and \( N \) is the total number of observations. This estimator and its statistics are discussed by Carter (1988), where he points out that an unbiased estimator can be defined by

\[
R_{iU} = \frac{v_w - 2}{v_w} \left( 1 - \frac{2}{N} \sum_{j=1}^{N} \frac{S_{bij}}{S_{wij}} \right) \tag{3}
\]

Now suppose that \( \varepsilon \) no longer arises entirely from the 'intrinsic' variability, but includes contributions from 'external' variability such as small scale charges in the wind field, stability of the atmosphere and non-linear...
processes such as wave-breaking. Then the ratio \((1)\) may no longer be \(F\) distributed and may occasionally be very large. Such occasions could substantially bias the mean \((2)\).

Let us rewrite \((1)\) as \(S_{bi}(1 + \epsilon_{wi}) = R_iS_{wi}(1 + \epsilon_{bi})\), and summing over the observations we get:

\[
\sum_{j=1}^{N} S_{bij} + \sum_{j=1}^{N} S_{bij} \epsilon_{wij} = R_i \sum_{j=1}^{N} S_{wij} + R_i \sum_{j=1}^{N} S_{wij} \epsilon_{bij}
\]

The second sums on each side tend to zero for large \(N\), but in any case large variations will tend to be smoothed out. We may thus define another estimator of \(R_i\),

\[
R_i = \frac{\sum_{j=1}^{N} S_{bij}}{\sum_{j=1}^{N} S_{wij}} \quad (4)
\]

and expect it to be more robust in the presence of highly variable data.

Unfortunately, the distribution of \(R_{iB}\) is not available theoretically, but the essential statistics can be found by computer simulation using a Monte-Carlo approach. This was undertaken and \(R_{iB}\) was found to be unbiased with a 90% confidence interval of about ±11% in the middle range of frequencies, becoming wider at low frequencies.

**RESULTS**

Since the spectra from the WR and SBWR were calculated using different parameters, the estimates were evaluated at different frequencies. The first task, therefore, was to linearly interpolate the WR spectrum to the set of SBWR frequencies.
The response function \( R^g \) was estimated from the data and is plotted against non-dimensional frequency in Fig.1. The asterisks represent this data set, while the other five symbols represent the other five sets of data which were used to develop the empirical response function - this is drawn as a smooth continuous line.

The response represents the relationship between the pressure measured at the position of the pressure sensors and the surface waves. Thus from physical reasoning it should approach unity at low frequency, and the empirical response formula is constructed to reflect this.

However, the present response estimates appear to be about 15-20\% too high at the lower frequencies, and this discrepancy persists at higher frequencies as well.

In addition there is a secondary maximum at about 0.4 which is absent from the other data sets, or much smaller.

CONCLUSIONS

We have estimated the frequency response of a modified SBWR installation using eqn. (4). Carter (1988) shows that use of a different estimator (eqn. (3)) leads to rather similar results. The response obtained is rather variable and seems to be biased by +15\% or so overall. In addition there is a secondary maximum which may be related to enhanced non-linear effects associated with the new installation.

In view of these important differences from earlier empirical results it would be prudent to repeat the experiment and attempt to record many more comparisons than the 32 obtained in the present exercise.
The work described in this report was undertaken as part of the IOSDL Wave Climate Project which is supported financially by the Department of Energy.
REFERENCE

CARTER, D.J.T. 1988 Estimating the Ratios of Spectral Components. WACAS Note No.118. 8pp. (Unpublished manuscript)
Figure 1.

RESPONSE $M^*/M^*2$ vs FREQUENCY

-227 Hz

-453 Hz