Doppler Sonar Two Axis Current
Shear Meter
by
T. Crocker

[This document should not be cited in a published bibliography, and is supplied for the use of the recipient only].
INSTITUTE OF OCEANOGRAPHIC SCIENCES

Wormley, Godalming, Surrey, GU8 5UB.
(042-879-4141)
(Director: Dr. A. S. Laughton)

Bidston Observatory,
Birkenhead,
Merseyside, L43 7RA.
(051-652-2396)
(Assistant Director: Dr. D. E. Cartwright)

Crossway,
Taunton,
Somerset, TA1 2DW.
(0823-86211)
(Assistant Director: M.J. Tucker)
Doppler Sonar Two Axis Current
Shear Meter
by
T. Crocker

This work was supported in part by the Ship and Marine Technology Requirements Board

This document should not be cited in a published bibliography, and is supplied for the use of the recipient only

Institute of Oceanographic Sciences
Wormley
Godalming
Surrey  GU8 5UB
Doppler Sonar two axis current shear meter

I. Introduction

An instrument to measure vector current at various depths has been investigated and proposed by various institutions\(^1\),\(^2\). The engineering proposal which forms the latter half of this report has evolved on paper through several inputs of performance specification, and of engineering limitation. It is very much tailored to fulfilling the requirements of I.O.S., within the bounds of time and resource restrictions, and more positively trying to follow the course that maximises the use of 'in house' expertise. It is in this light that some parts of the specification may appear to have been taken for granted, although alternatives have been given some, if only cursory, consideration. It is somewhat reassuring that the final proposal has considerable similarity to those of the other institutions, differences arising only in response to our somewhat different required capability. The following pages follow the path to the proposed instrument, and alternatives are discussed with respect to this proposal. Some of the more detailed work to justify conclusions is only available in note form, and has been left out for the sake of brevity.
II. Basic Considerations

(a) Principle

The fundamental principle of utilising the doppler effect on an acoustic reflection to determine relative velocity is too well known to deserve space here. In this application it is assumed that there will be scatterers of sound in the water, and that these will be substantially (or at least statistically) stationery within their local mass of water. It is apparent that an acoustic beam angled down into the water will both penetrate to depth, and still 'see' a component of the scatterer's velocity. To determine the complete horizontal vector relative velocity one needs two such beams, preferably perpendicular to each other in plan view, and for reasons of accuracy, and to eliminate heave motion, it is desirable to look both forward and backward along each axis. This leads immediately to the commonly used configuration of Fig. 1. The four beams are mutually perpendicular in plan view, and angled at 8 to the horizontal. The best justification for this geometry comes from consideration of the effects of ship motion - and is discussed later. It is adequate now to state that if both transducers use the same frequency for transmission, then the difference in the received frequencies will be the best indicator of horizontal motion relative to the scatterers.
(b) Velocity Sensitivity

Taking a result from a later section, the simple solution for the horizontal velocity $U$ is

$$ U = \frac{c}{2\sqrt{2}} \left[ \frac{f_{AB}}{f_a} - \frac{f_{BB}}{f_B} \right] \quad \theta = 45^\circ $$

As an indication of the order of magnitude, 1 m/sec relative velocity will lead to a shift of frequency of 0.1% in each beam on reception, i.e. 70 Hz shift in the proposed 70 kHz.

(c) Depth Discrimination

In order to obtain velocity information about finite layers below the surface it is necessary to use a pulse of sonar energy that will, on reflection, define a cell at a particular depth. If the pulse length is $\tau$, then the insonified range that will contribute to the reverberation at any instant will be $\frac{c}{2} \tau$ ($c =$ velocity of sound), which limits the independence of depth cells to increments of $\frac{c}{2} \sin \theta$. Making the pulse length a finite $\tau$ secs leads to bandwidthing of the transmitted pulse to a spectral width of order $\frac{1}{\tau}$ Hz, and additionally there is uncertainty in frequency measurement upon
reception. One would then expect a single measurement of frequency to have a total uncertainty of $\frac{\sqrt{2}}{t}$ Hz. It can be seen that the precision of a single velocity measurement will be proportional to frequency, and proportional to pulse length. The precision in depth is however inversely proportional to pulse length, and additionally both frequency and pulse length appear in the 'Sonar Equations' which determine the range to which the device will operate.

The combination of parameters to make the device work is then a matter of some complication and interaction. All of these effects are treated separately in the following sections with respect to 'what has been chosen', and hopefully the values chosen are near optimum.

(d) Specification

The optimum parameters may be judged only in terms of the requested performance, and at this stage it is worth noting the original specification:

<table>
<thead>
<tr>
<th>cells of thickness 50 m down to 500 m depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot; &quot; &quot; 20 m &quot; &quot; 200 m &quot;</td>
</tr>
<tr>
<td>&quot; &quot; &quot; 10 m &quot; &quot; 100 m &quot;</td>
</tr>
</tbody>
</table>

with acquisition times of up to 30 minutes per set of velocity values, and overall accuracy of a few centimetres/sec. It has also been assumed (since it does not appear to be a difficult requirement) that the apparatus should be capable of working with the ship moving at speed. It would, however, seem from calculation, that excessive oscillatory ships motions, induced by bad seas, will lead to some degradation of performance. At the present time it is felt that the device should work under most weather conditions, but since the mechanical engineering is not available to provide complete transducer stabilisation, there will inevitably be some conditions under which the results will be unreliable. Experience alone will show whether or not this extra mechanical work is necessary, but it is hoped that the 'intelligence' of the proposed \( \mu \)-processor system will obviate this need by compensating for the induced errors.
III. Specific Considerations

(a) Transducer Angle

The most trivial decision is to pick the transducer angle $\theta$ of Fig. 1. If at any frequency one can define a range $R$ beyond which the signal to noise ratio is unacceptable, then the maximum operating depth is $R \sin \theta$. The component of any horizontal scatterer velocity seen along the beam will be proportional to $\cos \theta$, and if one takes operating depth x velocity sensitivity as an overall measure of merit, then this will be optimised for $\sin \theta \times \cos \theta$, i.e. $\theta = 45^\circ$.

(b) Sonar Equation

(i) The derivation of the relevant sonar equation is given in Appendix 1. It is only necessary here to state the result, using the convenient logarithmic power level notation (Urick, ref. 3). From this equation the operating frequency, power level, etc. may be determined.

\[
10 \log \frac{S}{N} = SL - NL + DI + 10 \log \frac{c}{1000} - 10 \log r - 2a + SV + 10 \log \frac{cT}{2} - 10 \log \frac{c}{1000}
\]

(ii) $\frac{S}{N}$ is the power signal to noise in the receiver, within the receiver bandwidth (perfect receiver), and is the best measure of how reliably the frequency of the signal may be determined.

$SL$ is the source level of the transducer on axis expressed in ratio to the reference level ($RMS$ pressure $1\mu Pa$).

$NL$ is the ambient noise power level per Hz of bandwidth with respect to the reference level.

$DI$ is the receiving directivity index.

$V$ is an equivalent solid angle defining the volume from which reverberation will be received.

$r$ is range in yds.

$a$ is the attenuation coefficient in dB/Kyd.

$SV$ is the volume back-scattering parameter in dB.

$c$ is the velocity of sound in water (yds/sec).
\( \tau \) is the pulse length.

\( B \) is the receiver bandwidth.

(iii) Quick calculation shows that there is very little spare signal return to meet the original specification, and that the various parameters will have to be carefully chosen. Taking the terms individually:

(a) SL is a 'product' of transmitter power and directivity. A lower limit on the beamwidth and therefore on directivity is set by ships motion (III.e.iv. a & c). An optimum would appear to be \( 3^\circ \) half angle to half power, although this would not be achievable at low frequencies, due to practical limitations on transducer size.

(b) Noise level (NL) is a reasonably well known function of frequency, but should be considered here in conjunction with receiver bandwidth. The ambient noise power (per Hz) has a minimum at 80-100 kHz, with roughly equal rates of increase of 5-6 dB/octave either side (Urick, p.188). The minimum bandwidth that any receiver may have must be sufficient to contain the range of 'velocity noise' over the period of measurement. It is concluded, from calculations of the ships motion (III.e.iii) that a variation of \( \pm75 \) cm/sec should be available. In this case the receiver bandwidth will be proportional to the operating frequency. It can be argued however that as long as the absolute signal to noise in the band is adequate, then the higher frequencies have the advantage of being more sensitive to velocity. In the absence of other frequency dependent effects one could have two different types of merit figure to optimise. The more optimistic would go for lowest spectrum (per Hz) noise level arguing that the disadvantage in widening the receiver bandwidth exactly offsets the advantage of higher velocity sensitivity. The alternative argument is that one has to have adequate signal to noise in the receiver before any data can be recorded usefully. Any lack of velocity sensitivity can then be made up by extending the recording period and enhancing the data by multiple measurements. The first argument is undoubtedly correct in abstract, considering only total information content. The second argument is to be borne
well in mind when designing practical systems near the signal to noise limit. This is the essential difference between plots on Fig. 2 and Fig. 3.

(c) The directivity index is determined by beam pattern, and as in (a) above is fairly tightly constrained by ships motion considerations. In that there is some room for variation, there is an advantage for a given transducer size, in increasing frequency, and hence obtaining better directivity. This can be seen by lumping all the beam dependent effects together. Disregarding all other effects, a tighter beam improves transmitter directivity, improves receiver directivity, but reduces the solid angle \( \Psi \) of scatterers insonified. The net effect is then an improvement with frequency of once times the directivity index.

(d) \( \alpha \), the attenuation coefficient, given historically in dB/Kyd, is a very strong function of frequency, and becomes dominant in designs at the high frequency end of the considered band. Its effect on performance can only be appreciated for a defined range \( r \). In choosing the operating frequency, attenuation is combined with directivity and noise considerations in Fig. 2 and Fig. 3, for a range of 750 m (depth \( \sim 500 \) m). The figures used are from Fisher and Simmons, ref. 4.

(e) \( SV \), the volume scattering strength parameter, is a measure of the expected level of return from a scatterer, expressed as the ratio in dB of the reflected intensity per unit volume of scatterers to the incident intensity on those scatterers. The value of \( SV \) is not well known, and those values that do exist (e.g. ref. 5 and 6) suggest that it is highly variable with location. There is some evidence that at a given location the value of \( SV \) is approximately constant over the frequency range 10 kHz to 100 kHz. Figs. 2 and 3 were drawn on an assumed worst case value for \( SV \) of -100 dB. The proposed French system (ref. 2) has a somewhat more optimistic value of -80 dB, but at 300 kHz.

(iv) Optimisation of Sonar Equation Parameters

Figs. 2 and 3 allow visual optimisation of the various parameters, according to the two ideas of overall merit mentioned above. They both apply to an instrument to operate to 500 m depth in 50 m depth cells.
The solid lines labelled A show the advantage gained by directivity when the frequency is increased for a given transducer size. Both the top and bottom horizontal scales may be used with these lines, to show absolute receiver directivity index in dB, or equivalent half angle beamwidth. The solid curve B shows the disadvantage due to noise level and absorption (and bandwidth in Fig. 3) in relative dB, against frequency. The dashed lines C are the sum of A and B, and therefore represent the overall relative performance of a given size of transducer against frequency. The difference between Fig. 2 and 3 is the shape of the disadvantage curve B, as discussed above. It can be seen from Fig. 2 that the optimum frequency is around 32 kHz, and from Fig. 3 is in the region 20-30 kHz.

The final frequency chosen, of 70 kHz, is considerably higher than either of these two values, for several reasons.

(i) The 3° half power half angle beamwidth deemed to be necessary would require impractically large transducers at the lower frequencies.

(ii) As will be seen later each measurement at such frequencies would be very insensitive to velocity, and intuition suggests that it would be better to sacrifice some range capability in return for greater velocity precision at closer ranges.

(iii) The later proposal to limit range in exchange for greater depth precision puts obvious advantage on higher frequencies.

(iv) Transducer designs are available within I.O.S. that have almost ideal beam patterns, and reportedly very good temporal response.

(v) Calculations have been made to determine the usefulness of higher frequencies (above about 70 kHz it is necessary to pick frequencies for which transducers are available from commercial sources). Doubling frequency to 150 kHz carries a very heavy penalty in increased absorption for only a two-fold increase in velocity sensitivity. Whilst one might obtain useful results out to 100 m, 200 m would probably not be within range. Also a set of commercial
transducers has to be costed at £3000.

(vi) Against the performance at limited range of the 150 kHz system, the 70 kHz system should provide excellent signal to noise out to 200 m. Under these circumstances it should be possible to get good enhancement of the poor velocity precision of the short pulses (20 m and 10 m depth cells) by statistical means.

Actual figures for the performance expected are given in appendix I.

(c) Velocity Calculations

The arithmetic necessary to derive velocity from frequency change is given in Appendix II, and the justification for the transducer geometry in the next section. The frequencies that are used in these expressions are however not measured over an infinite time, so they have only a finite accuracy, leading to uncertainty in velocity. If, as proposed, one uses a counting technique to determine the returned frequency, then for a pulse length (and therefore counting time) of $\tau$ see the accuracy of a single frequency measurement will be $\sim \frac{1}{\tau}$ Hz. The corresponding uncertainty in velocity $\Delta U$ is, from A.II(i)

$$\Delta U = \frac{c}{2\sqrt{2} \frac{1}{f\tau}}$$

$c =$ velocity of sound  \hspace{1cm} \hspace{1cm} f =$ transmitted frequency

At 70 kHz $\Delta U \approx 8$ cm/sec for $\tau = 100$ ms (≈ 50 m)

$\approx 20$ cm/sec for $\tau = 40$ ms (≈ 20 m)

$\approx 40$ cm/sec for $\tau = 20$ ms (≈ 10 m)

The velocity resolution clearly needs to be enhanced by averaging several values. For the 100 m sec pulses, only a few values will be needed per set, but it can be seen that at least a factor of 10 improvement is needed for the short pulses. This implies a set of at least 100 measurements (the improvement will presumably go as $\sqrt{n}$) for each velocity value of the smaller depth cells. It can be seen however that with the expected values of signal to noise at these shorter ranges such enhancement should be possible. In the proposed $\mu$-processor based system the choice of number of elements to a set will be under program control, and will be completely flexible.
(d) Signal Detection Techniques

The simple trade-off between velocity information (\(\omega_1\)) and depth information (\(\omega_\frac{1}{4}\)) is fundamental, but not as rigid as it has been presented. In radar and sonar it is quite normal to use more complicated pulse forms, frequency swept pulses for instance, and then to recover the information content that has apparently been lost by processing in the receiver. None of these techniques can give an improvement in overall merit (velocity discrimination \(\times\) depth discrimination) over a measurement with a single frequency pulse unless either the total pulse time is increased, or the signal to noise is so good that it may be traded for improved discrimination.

There are very many good reasons for wanting to use a single frequency pulse in this system, not least of which is the obvious simplicity. It can be seen from the preceding sections that this equipment will only just be able to achieve the required performance at present, so some time has been spent checking that the complication of more complex pulse forms would not be worthwhile overall. Some points that arise are:

(i) The signal to noise ratio that can be expected at 500 m depth and 70 kHz looks (Appendix I) to be just about usable. As a result, the only improvement in velocity discrimination would be achieved by increasing the pulse length. It would be possible to recover the lost range discrimination by use of a suitable pulse and signal processing, but the increase in pulse length itself produces problems;

(a) The platform will have to be better stabilised in order that the receiving beam does not move out of the insonificd volume.

(b) The same transducer may now not be used for reception and transmission, since otherwise the nearer depth cells would be lost.

(ii) The converse of (i) above would be to increase the signal to noise ratio by decreasing the operating frequency, in the hope that the loss of velocity sensitivity would be more than compensated for by an increase in frequency resolution, obtained by using a more complex pulse type, and trading
in some of the improved signal to noise by processing. It has been seen earlier that the optimum frequency for information return is around 30 kHz.

However the transducer size needed to meet the beamwidth requirements at this frequency would pose a severe practical problem.

(iii) Since the magnitude of the 'velocity noise', due mainly to ship's motion, will probably be a few times larger than the uncertainty due to a single frequency measurement (at least for 50 m cells) it seems highly doubtful that there will be any practical gain in the averaged velocity accuracy if the frequency discrimination alone is increased.

(e) Frequency Measurement Techniques

(i) One is now left with the choice of method of frequency measurement.

We know that the signal received will be noisy both in amplitude and frequency. Clearly the receiver should add as little as possible to this noise, but since averaging is to be used to enhance data, a more important requirement is that the receiver should not systematically add any error in frequency. The need to average, and to hold the data for different depths, makes digital manipulation of the data almost mandatory. This gives counting an initial edge, but other techniques could be used, and these were investigated.

(ii) There is one additional consideration which impinges on the choice of technique; receiver bandwidth. The purpose of the instrument is to collect current velocity shear information, and transducers on an athwartships axis will do this directly. However transducers on a fore- and-aft axis will have the ship's speed superimposed on to the fore-and-aft component of the current. If the receiver bandwidth is opened enough to accommodate this velocity range, then the signal to noise ratio in the receiver will suffer considerably. A better scheme is to make a bandpass window track the ship's speed, the window being only wide enough to accommodate the expected short term velocity noise and the actual shear variation. Practical tracking filters are difficult, and this, combined with the sort of bandpass required (100 Hz in 70 kHz, i.e. Q ~ 700), makes a heterodyning receiver an obvious choice. The main
receiver filter, and the frequency measuring device now operate at audio frequencies, and coarse steps in velocity are managed by changing the local oscillator frequency. As a result the actual frequency measurements are over a very small range. Putting figures to this: at 70 kHz, 100 Hz bandwidth is equivalent to \( \pm 1.5 \) knots variation. With a 100 ms pulse (\( \pm 50 \) m depth cell) signal/bandwidth \( \sim 10 \) Hz so one only has 10 discriminable steps in the band.

The individual techniques available are:

(iii) \textbf{Fourier transform of received signal}

This is almost guaranteed to produce some results (e.g. ref. 7) but does not lend itself to a simple real-time instrument. One needs to digitise the returning signal (4 channels), and then process this into a set of spectra, one for each pulse and depth cell. These individual spectra may then be added, across frequency, to produce enhanced records for each depth cell. Once this average spectrum has been obtained the peak value, or possibly an average, is chosen to be converted into a velocity. The great advantage is that no information is lost, and the quality of results depends only on the receiver and digitiser. The disadvantage is a high sophistication system, requiring at least a minicomputer and fast fourier transform 'black box'.

(ii) \textbf{Filter Bank}

Analogue filters are fed, in parallel, with the input signal, each filter with its centre frequency \( \frac{1}{4} \) Hz from the next. The outputs are then all sampled at a time corresponding to the bottom of a depth cell, and the samples added over a set of records to provide an enhanced but still discrete spectrum for each cell. The frequency taken can then be chosen either as the frequency of the filter with the highest average output, or better by interpolating between filter steps on the averaged spectrum. For practical purposes it is more convenient to use a variation on this design. The filters are all made identical to each other, and are each fed from
the output of a mixer, each mixer having a local oscillator frequency spaced \( \frac{1}{T} \) Hz from the next. This allows the local oscillator frequencies to be generated from a crystal source, giving greater consistency to the effective filter frequencies. It should be noted that the filters cannot usefully be put closer than \( \frac{1}{T} \) Hz apart, since any filter with this bandwidth has an impulse response of at least 1 sec. This means that if the filter discrimination is made tighter then the depth cells become less independent, since the filter will still have significant energy from the previous depth cell. This system has the advantage of simplicity, and also of predictable performance in poor signal to noise conditions. The disadvantages are purely practical. Each channel would need at least 10 good quality filters and mixers, and 10 individual frequency division chains. Also the transmitted pulse length is effectively fixed by the filter bandwidth, and different pulse lengths would need different filters, or switchable components. In all this offers a straightforward and workable solution, but not one with any room for variation of parameters. The final instrument would be very large.

(iii) Counting

The counting technique appeals because of its ease of interface to digital circuits, and the flexibility that this brings. Since this is the technique that has been chosen the make-up of the receiver is described in section IV. It is worth noting, however, some of the theory of counting in noise, since much of it is not obvious or well known. This theory gives some guide to the performance that can be expected.

It is evident that counting, like other techniques, should give an inaccurate measurement of frequency in the presence of noise. It is not intuitively obvious how this effect will manifest itself. The theory has been partially worked out, and a good review of the subject is ref. 8 by Bendat. All of the current work deals with the number of zero-crossings
in a finite period, although one could perhaps count peaks*. The situation
is further complicated since some degree of hysteresis must be included in
a practical counter to avoid counting high frequency internal noise.

The theory given by Bendat can be used to derive the following simple
result for the special case of signals and noise in a narrow band (Q \gtrsim 10).

\[ \mathcal{f}_o = f - \frac{\delta f}{p+1} \]

where \( \mathcal{f}_o \) = observed frequency (counted),
\( f \) is the true frequency of the signal,
\( \delta f \) is the deviation of \( f \) from the centre band frequency \( f_c \)

i.e. \( \delta f = \mathcal{f}_o - f_c \),

\( p \) is the power signal to noise ratio.

This simply means that all observed frequencies will be pulled towards the
centre of the band by noise, and also gives a good indication of the signal
to noise that one needs to obtain given accuracy. In the proposed system
the bandwidth of the receiver will be \( \sim 100 \text{ Hz} \), and the local oscillator
will have steps of \( \sim 55 \text{ Hz} \). The maximum distance from the centre will be
a mean of \( \sim 25 \text{ Hz} \). A 10 dB signal to noise ratio will then give an error
of 2.5 Hz, equivalent to 3.5 cm/sec in velocity.

It can be seen that lower signal to noise than this will produce
significant errors. However the effect of the noise is predicted by the
above expression, and should eventually be susceptible to improvement by
processing.

(iv) Tracking Filters/Phase Locked Loops

There are a few other possible frequency measuring techniques. One
apparent possibility is to take say two of the filters in (ii) above, and

*The converse technique, measuring time for a fixed number of cycles of the
signal, appears to give arbitrarily high precision, and certainly would do
so with a perfect signal to noise ratio, and a continuous signal. In the
case of a reverberated signal return, the frequency uncertainty in the
time limited transmission is present in the water, with returns from
different parts of the pulse arriving at the same time. It is likely
then that the intrinsic uncertainty of the signal frequency will still
limit the performance of the receiver.
to make their outputs steer the local oscillator. If this local oscillator kept to \( \frac{1}{10} \) Hz steps, then its frequency could be recorded. However, such small steps are only available from a crystal by division and then multiplication of the crystal frequency, and the settling time of this last element is very significant. The feedback control of this circuit is then highly critical and could take a considerable period to develop, and is not seen to have any advantage over other more straightforward methods. If the digital division train is replaced by a voltage controlled oscillator, then either a phase locked loop or a tracking filter could be employed. The purpose of such a technique is then somewhat dubious, because the only way to find the local oscillator frequency accurately (stability and linearity of the VCO control voltage is not that good) is to count it. This has the advantage over straight counting of moving the noise problem to the PLL. Analysis of the performance of such a system has not been attempted, since once again it does not offer an obvious advantage in return for considerable complexity.

(f) Ship's Motion Effects

(i) The determination of the relative velocity of a shipborne transducer to the water is obviously complicated by the movement of the ship. The basic assumption behind the present proposal is that over the period necessary to obtain one set of data, an input will be available, from navigation, of the ship's true motion over the ground. All of the oscillatory movements are then presumed to have averaged out. The justification of this appears in the following analysis of possible effects.

(ii) The first set of effects are the superimposed velocities. Of the three axes of ship's translation, the two horizontal components are the parameters being measured, and any noise must be rejected by averaging. The vertical, heave component is neatly removed by the geometry of the transducer pairs, and this is of course the main purpose of this design (Appendix 2). It is interesting that this heave rejection also works for
quite large angular motions of the ship, and the working for this is in Appendix 3.

(iii) In addition to pure translation of the whole ship there are three \( r^\theta \) type motions due to pitch, roll and yaw \( (\dot{\theta}) \) about the radius \( r \) from the centre(s) of motion of the ship. Any vertical component will be removed, as above, by the transducer pair, but rough calculations (data is not available in detail) specifically about the proposed mounting on RRS Discovery, suggest that horizontal velocities could be quite high, with amplitudes of order 50 cm/sec in quite modest seas. This figure has been used as the basis for a guess at the best filter bandwidth to use in the receiver, although this is something that will have to be decided by experience. The present proposal effectively ignores these motions mathematically, in the hope that, like other motional noise, averaging will reduce the effect. An alternative approach, which is possible, but requires greater sophistication, is to use real time correction for these velocities by calculating a correction from measured values of \( \theta \) or \( \dot{\theta} \).

The system proposed would be able to accept such data.

(iv) A second group of ship's motion effects are the disturbances to the geometry of the system. There are really three subgroups here:

(a) One is the alteration of the acoustic path, in that a narrow beam transducer may not, on reception, be looking at the volume insonified in transmission. In the proposed deployment on RRS Discovery the roll and azimuth stabilised platform will be used, so that pitching motions are the only ones of concern. Rough calculation (note 4) indicates that the maximum angular motion of the ship in a 1 second transmit/receive time, should be about equal to the \( \frac{1}{2} \) beamwidth of the proposed transducer \( (\sim 3^\circ) \).

(b) The second subgroup concerns the geometry of the velocity equations. No attempt is being made in the present proposal to calculate horizontal velocity from the actual angle of the transducer pair.
Rather the calculations in Appendix 3 are used to justify the assertion that pitching motions of less than 8° peak will produce acceptable errors of ±1%. Once again angular data could be accepted if it was found to be necessary.

(c) The third effect is the error in depth that the acoustic system is looking at, when there is an angular displacement of the ship. Consideration of this requires the trivial calculation (note 4) of the maximum beamwidth allowable in order that the specified depth cells are independent at maximum depth. Conveniently this also turns out to give a ½ angle beamwidth of 3°. Quite clearly a pitch of 8° will move the beam completely out of the cell at maximum range. One proposal to limit the error is to record velocity measurements only when the angular displacement of the ship from the horizontal is less than say 3°. The problem with this is that it will also select the measurements with the largest magnitude of r6 velocity noise. If there is any asymmetry in the pitching motion of the ship then these r6 motions could introduce a substantial error. The problem is one that will only occur under some weather conditions, and it is alleviated by the proposed angling of the transducers 45° off axis (Sec. IV). It remains a problem however and will probably only be solved by proper angular corrections. Experience with the basic instrument will be the best guide to what is needed.
IV. Engineering Proposal

(a) Electronics

The preceding sections give some idea of the problems, and indicate the specifications of the acoustic system. Specifically the apparent transducer needs appear to be very well met, at modest cost, by the 70 kHz line and cone units last used on the SOND cruise. It is proposed that four of these are built and mounted on the ASDIC platform of the RRS Discovery. Practically, they cannot be aligned in the fore and aft and athwartships directions, so the two axes will be at 45° to the ship's head. The platform allows stabilisation in roll and azimuth, so the geometry of the axes will be steady even if the ship's heading wanders during a set of measurements.

To summarise the requirements put on the electronics, the instrument must

(i) be able to measure returned frequencies from successive depth cells,
(ii) keep some sort of average of a set of values,
(iii) translate these into velocities,
(iv) sequence the transmission of signals and the subsequent measurement cycles,
(v) keep noise in the receiver bandwidth to a minimum by tracking a narrow filter to include only a range of velocities.

The system chosen to do this is blocked out in Fig. 4. The specifications of the main parts of the analogue system are shown in the diagram, and follow the conclusions of the previous sections. The processing and digital part is specific to this proposal and needs some description. The heart of the system is a 6000 μ-processor built into a μ-computer system by Midwest Technical Instruments. This will run programs in BASIC and machine code loaded from the cassette. In a final system it is envisaged that the processor will accept commands from the Teletype specifying the parameters of the measurement to be made. At the beginning of a measurement cycle it will
select a suitable mixer local oscillator frequency to bring each returned signal into the filter band. The \( \mu \)-processor will then enable the power amplifiers to transmit for the required period. The counting of the returned signals, after passage through the analogue circuits will be done by the processor itself. It will start to sample the four input channels, which are presented to it in a clipped parallel form. By making a comparison with the previous sample it will determine whether or not there has been a zero-crossing between samples. If there has, then it adds one to the memory location (initially zeroed) reserved for that channel and depth cell. It will continue to do this until the sample count reaches that for the first complete depth cell. It will then credit further zero-crossings to the succeeding cell's memory location. After all the signal returns have come from the final depth cell, the processor will command a new transmitted pulse, and then start again, adding cumulatively the zero-crossing counts for the depth cells to their correct memory locations. In this way a table of the total returned counts is built up in the machine's memory, corresponding to each of four channels and all the depth cells. In addition the cycle may interlace measurements with different sized depth cells, using different tables in memory, to give the required depth detail.

The numbers in each memory location are just twice the quantities 
\[ E N_{ai} \text{ of App. II.ii} \]
and \[ E N_{bi} \]
since both positive and negative zero-crossings have been counted. The \( \mu \)-processor can then calculate the velocities according to A.II.ii. These can then be either stored on cassette tape, or printed out to the teletype.

(b) Trials

It can be seen that the \( \mu \)-processor has control of all the parameters other than transmitter frequency and transducer geometry, and this is of course done with the intention of making the system as flexible as possible. Until some experiments are done it will be impossible to determine how well the theory
above will match with practice, and it is inevitable that changes will be made. Given that the transducer geometry is correct, everything else is variable. The electronics should cover the range 30 kHz to 150 kHz with only minor changes, and only the ceramic elements in the transducers need be changed, at least to move the frequency upwards.

The initial aim is to test the system using only two of the four transducers, using relatively simple software for the μ-processor. If this works well then the other two channels can be added, and more sophisticated software can be written, to provide a simple system for the operator, and such features as data logging on cassette. At this stage experience with the ship's motion problems will have been gained, and if necessary interfaces to provide angular information to the processor could be built.
Appendix 1

Derivation of Sonar Equations for Volume Scattering Target

Take the source to have an intensity $I_0$ at unit distance (1 yd) on axis, and beam pattern $b(\theta, \phi)$. Working in yds, with $\alpha$ attenuation coefficient in dB/kyd, $r$ range in yds,

$$\text{Intensity at scatterer} = \frac{1}{r^2} b(\theta, \phi) I_0 \times 10^{10} \times \frac{-\alpha \times r}{1000}$$

Backscattered intensity, 1 yd from scattering volume defined by elemental solid angle $d\Omega$, pulse length $T$ and velocity of sound $c$

$$= \frac{1}{r^2} b(\theta, \phi) I_0 \times 10^{10} \times 1000 \times \frac{d\Omega}{2 \pi} \times \frac{\frac{CT}{2} \sigma V}{2}$$

where $\sigma V$ is the scattering strength parameter.

$$\text{Intensity at transducer due to scatterer} = \frac{a}{2\pi} b^2(\theta, \phi) I_0 \times 10^{10} \times \frac{1}{1000} \times \frac{d\Omega}{2 \pi} \times 10 \times \frac{\frac{CT}{2} \sigma V}{2}$$

therefore the power in the receiver due to all the scattering elements

$$= k \int b^2(\theta, \phi) I_0 \times 10^{10} \times 1000 \times \frac{d\Omega}{2 \pi}$$

where $k$ is a constant of the transducer, and all contributions have been weighted by the receiver beam pattern $b(\theta, \phi)$.

If the equivalent plane wave noise level is $(NL)$ in watts/sq yd Hz and $B$ is the receiver bandwidth (Hz) then the noise power in the receiver is

$$k(NL) B \frac{\int b(\theta, \phi) d\Omega}{4\pi}$$
the signal to noise ratio in the receiver

\[ \frac{S}{N} = I_0 \cdot \frac{1}{r^2} \cdot \frac{SV \cdot CT}{2} \cdot 10 \frac{\alpha}{10} \frac{2r}{1000} \times 4\pi \frac{\int_{4\pi} \beta^2(\theta,\phi) \, d\Omega}{\int_{4\pi} \beta(\theta,\phi) \, d\Omega} \]

taking logs gives

\[ 10 \log \frac{S}{N} = 10 \log I_0 - 20 \log r - \frac{2\alpha r}{1000} + 10 \log \frac{CT}{2} + SV \]

\[ + 10 \log \frac{4\pi}{\int_{4\pi} \beta(\theta,\phi) \, d\Omega} + 10 \log \int_{4\pi} \beta^2(\theta,\phi) \, d\Omega - 10 \log (NL) \]

\[ - 10 \log B \]

where SV is the logarithmic scattering strength parameter. If \( I_0 \) and \( \text{(NL)} \) are now expressed in the same units or with respect to the source reference level, then they can be replaced by \( SL \) (source level) and \( NL \) (noise level/Hz) their conventional logarithmic representations.

\[ 10 \log \frac{4\pi}{\int_{4\pi} \beta(\theta,\phi) \, d\Omega} \] is merely the directivity index, conventionally \( DI \).

\[ 10 \log \int_{4\pi} \beta^2(\theta,\phi) \, d\Omega \] is the weighted two way beam pattern response, which is normally replaced by \( 10 \log \Psi \), where \( \Psi \) is a solid angle within which a value of \( \beta^2(\theta,\phi) = 1 \) with \( \beta^2(\theta,\phi) = 0 \) elsewhere, has the same integrated value.

Hence

\[ 10 \log \frac{S}{N} = SL - 20 \log r - 2 \frac{\alpha r}{1000} + 10 \log \frac{CT}{2} + SV + 10 \log \Psi + DI - NL - B. \]

This expression can be obtained 'from the book', for instance Urick, however the physical derivation was included here because it was felt that it illustrated the results for reverberation somewhat better.

Results applied to proposed system

\[ SL = 100 \, \text{W into} \, 3^\circ \text{ half angle} \rightarrow SL = 222 \, \text{dB re} \, 1\mu\text{Pa rms} \]

\[ NL = \sim 30 \, \text{dB re} \, 1\mu\text{Pa at sea state 3} \]

\[ DI = 30 \, \text{dB} \]
\[ 10 \log Y = -22 \text{ dB} \]

\[ r \text{ is 750 m, 300 m, or 150 m} \]

\[ \alpha = 18 \text{ dB/kyd (Fisher & Simmons)} \]

\[ SV = -100 \text{ dB} \]

\[ B = 100 \text{ Hz} \]

\[ 10 \log \frac{ct}{2} \text{ is 100 ms, 40 ms or 20 ms approx.} \]

\[ \therefore 10 \log \frac{S}{N} = 80 - 20 \log r - 2 \frac{\alpha r}{1000} + 10 \log \frac{ct}{2} \]

\[ \therefore \text{for } \tau = 100 \text{ ms } r = 750 \text{ m } 10 \log \frac{S}{N} = 13 \text{ dB} \]

\[ \text{for } \tau = 40 \text{ ms } r = 300 \text{ m } 10 \log \frac{S}{N} = 34 \text{ dB} \]

\[ \text{for } \tau = 20 \text{ ms } r = 150 \text{ m } 10 \log \frac{S}{N} = 43 \text{ dB} \]
Derivation of velocity from frequency measurements

fa and fb are frequencies transmitted, and faB and fbB are the frequencies received.

Doppler shift expected gives

L.H.S. \[ fa_B = \frac{c + |\vec{V}_1|}{c - |\vec{V}_1|} \]

R.H.S. \[ fb_B = \frac{c + |\vec{V}_2|}{c - |\vec{V}_2|} \]

where terms in \( \frac{|\vec{V}|^2}{c^2} \) and higher have been neglected. These terms would be marginally significant on their own (\(~1\%\) error at full speed) but even lower terms in fact cancel in differencing, so first error is \( \sim \frac{|\vec{V}|^3}{c^3} \)

\[ \frac{fa_B}{fa} - \frac{fb_B}{fb} = \frac{2}{c} (|\vec{V}_1| - |\vec{V}_2|) \]

\[ = \frac{2}{c} (w \sin \theta + u \cos \theta - (w \sin \theta - u \cos \theta)) \]

\[ = \frac{4}{c} u \cos \theta \quad (A.II.i) \]

\[ \therefore u = \frac{c}{4 \cos \theta} \left( \frac{fa_B}{fa} - \frac{fb_B}{fb} \right) \]

However with a heterodyning receiver faB and fbB are not directly measured.

There are two local oscillator frequencies faL and fbL which give two counted frequencies fac and fbc
so \( f_{BB} = f_{ac} + f_{aL} \)

\( f_{BB} = f_{bc} + f_{bL} \)

and \( u = \frac{c}{4 \cos \theta} \left( \frac{f_{ac}}{f_{a}} + \frac{f_{aL}}{f_{a}} - \frac{f_{bc}}{f_{b}} - \frac{f_{bL}}{f_{b}} \right) \)

if the mixed-down signals \( f_{ac} \) and \( f_{bc} \) are counted over an observation time \( \tau \), two counts are obtained, \( na \) and \( nb \)

\( so \ f_{ac} = \frac{na}{\tau} \)

\( f_{bc} = \frac{nb}{\tau} \)

This expression may be used with cumulative results of \( N \) counting periods to give an enhanced result

\[
\frac{1}{N} \sum_{i=1}^{N} u_i = \frac{c}{4 \cos \theta} \left( \frac{1}{N\tau} \left( \sum_{i=1}^{N} \frac{na_i}{fa} - \sum_{i=1}^{N} \frac{nbi}{fb} \right) + \frac{f_{aL}}{fa} - \frac{f_{bL}}{fb} \right) \]
Appendix III

Effect of angular displacement on velocity calculations

Assume for simplicity that the same frequency is used for transmission on both beams

_then \( f_{AB} = \frac{2f}{c} (u \cos \theta_a + w \sin \theta_a) \)

\( f_{BB} = \frac{2f}{c} (-u \cos \theta_b + w \sin \theta_b) \)

Then subtracting the frequencies in the normal way

\[
\frac{f_{AB} - f_{BB}}{f} = 2u (\cos \theta_a + \cos \theta_b) + 2w (\sin \theta_a - \sin \theta_b)
\]

\[
= 4u \left( \cos \left( \frac{\theta_a + \theta_b}{2} \right) \cos \left( \frac{\theta_a - \theta_b}{2} \right) \right) + 4w \left( \sin \left( \frac{\theta_a - \theta_b}{2} \right) \cos \frac{\theta_a + \theta_b}{2} \right)
\]

for the geometry suggested \( \theta_a + \theta_b = 90^\circ \)

\( \theta_a - \theta_b = 2\Delta \theta \)

where \( \Delta \theta \) is the displacement of the ship from the horizontal.

\[
\therefore \frac{f_{AB} - f_{BB}}{f} = 2\sqrt{2} \ (u \cos \Delta \theta + w \sin \Delta \theta)
\]

since \( \Delta \theta \) will have both positive and negative values the error due to heave should average to zero. The average value of \( \cos \Delta \theta \) is however always less than unity, and this represents a systematic error over the results calculated by A.II.ii. However \( \cos^{-1} 0.99 = 8^\circ \), so that whatever the detailed motion, a limit of \( 8^\circ \) peak amplitude will certainly produce less than 1% error in velocity.
Appendix IV

Derivation of expression for the expected number of zero-crossings for a signal in noise

Bendat gives an expression for the expected number of zero-crossings per unit time for a signal process in band limited Gaussian noise. The signal process is significant, since it gives a simpler answer than does a simple fixed amplitude sine wave, and the model used has precisely the properties that we would expect of a reverberated signal return. The process is described as a single frequency sinusoid, but over an ensemble of observations on the signal the amplitudes are Rayleigh distributed, and the phases random.

The expression he gives (his 10-72)

\[
\overline{N_0} = \frac{\omega_0}{\pi} \left( \frac{\rho + (a^2 + ab + b^2)/3}{\rho + 1} \right)^{1/2}
\]

where \(\omega_0\) is the signal frequency, \(a_0\) and \(b_0\) define the limits of the band, \(\rho\) is the power signal to noise and \(\overline{N_0}\) is the mean of the expected rate of zero crossings, both positive and negative.

Now define the geometric band centre as \(\omega_c\) so that \(a_0 = \frac{\omega_c}{k}\) and \(b_0 = k\omega_c\) where \(k\) is the geometric half width of the filter.

Putting this in Bendat's expression gives

\[
\overline{N_0} = \frac{1}{\pi} \left( \frac{\rho \omega_c^2 + \omega_c(1 + k^2 + \frac{1}{k})/3}{\rho + 1} \right)^{1/2}
\]

For narrow bands, \(k < 1.05\), \((1 + k^2 + \frac{1}{k^2})/3 = 1\)

so

\[
\overline{N_0} = \frac{1}{\pi} \left( \frac{\rho \omega_c^2 + \omega_c^2}{\rho + 1} \right)^{1/2}
\]

Now write \(\omega_0 = \omega_c + \delta\omega\) \(\omega_c = \omega_0 - \delta\omega\)

\[
\overline{N_0} = \frac{1}{\pi} \left( \frac{\rho \omega_c^2 + \omega_c^2 - 2\delta\omega \omega_c + \delta\omega^2}{\rho + 1} \right)^{1/2}
\]

\[
= \frac{\omega_c}{\pi} \left( 1 + \frac{\delta\omega^2}{(\rho + 1)\omega_c^2} \right)^{1/2}
\]

expanding, in the narrow band case where \(\frac{\delta\omega^2}{\omega_c\rho} (\rho + 1) \ll 1\).

\[
\overline{N_0} = \frac{1}{\pi} \omega_0 \left( 1 - \frac{\delta\omega}{\rho + 1} \right)
\]

\[
= \frac{1}{\pi} \left( \omega_0 - \frac{\delta\omega}{\rho + 1} \right)
\]
Writing in terms of cycle frequencies

\[ f_{\text{observed}} = f_{\text{true}} - \frac{\delta f}{\rho+1} \] where \( \delta f \) is the distance from the band centre.
References

(1) Regier & Stommel. Draft proposal "Mapping the Horizontal Structure of Near Surface Currents during Polymode local dynamics experiment. Meteorology Dept., M.I.T.


(5) Stockhausen, J.H. Volume scattering strength v depth and frequency in the eastern Atlantic Ocean and the western Mediterranean Sea. SACLANTCEN Memorandum SM 60, 1/2/75.


(8) Bendat, J.S. Principles and Applications of Random Noise Theory.
