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INTERNAL DOCUMENT 201.

I.O.S.

FINAL REPORT OF CONTRACT
E/5A/CON/1666/632
"THE VALIDATION AND INTERPRETATION
OF DIRECTIONAL WAVE OBSERVATIONS
WITH THE MAREX BUOY"

B.S. McCartney

NATURAL ENVIRONMENT RESEARCH COUNCIL
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February 1984

This work was carried out under contract to the Department of Energy.

Final Report of Contract E/5A/CON/1666/632

"The Validation and Interpretation of Directional Wave Observations with the Marex Buoy"

Executive Summary and Conclusions

1. This contract funded NERC (IOS) to assess the technical performance of a new directional wave buoy (S100) produced by Marex, and to interpret directional data obtained at its proposed location west of South Uist.

2. Early intercomparisons at Freshwater Bay, I.O.W., indicated inadequacies in the buoy slope response. Modifications were advised and made for the South Uist moorings.

3. Faults in the Marex buoy compass signals prevented a successful intercomparison with the IOS pitch-roll buoy at South Uist.

4. A thorough one-dimensional (heave) intercomparison between the Marex buoy and an IOS Datawell Waverider at the South Uist site was made over a 2 month period, in May to July 1983.

5. Further directional intercomparison trials were proposed but inevitably delayed by the need for compass repairs.

6. In view of the lateness for W.E.S.C. purposes, UKAEA terminated the Marex contract prematurely and consequently this NERC contract.

7. The transfer of directional wave buoy analysis software from IOS to Marex was completed satisfactorily.

8. In view of the almost developed state of the Marex buoy, and since this buoy has good potential as a medium capability directional wave buoy between the Datawell Wavec and the EMI DB2 types, recommendations for its repair, completion and further critical evaluation are made in Section 9.

1. Objective (as stated in Contract)

(a) To determine whether the Marex buoy produces reliable directional wave information. This will be done by comparisons with measurements made using proven independent systems.

(b) To examine and interpret data from the Marex buoy in its proposed location west of South Uist. The most important aims of this are to ascertain to what extent the presently used description of directional wave climate is accurate, to assess the directionality factor and energy flux at the deep water site and relate this to other sites, and generally to examine the results to look for unusual and potentially important factors.

2. Programme of Work (as intended in Contract)

(a) Validation

1. Use a pitch-roll buoy to make an early comparison with the Marex buoy during its initial trials in Freshwater Bay, Isle of Wight.
2. Using the first available month of directional observations from the South Uist Marex buoy (after they have passed through the Marex control procedure) compare the sea surface elevations estimated by the Marex buoy with the results from a Waverider buoy at South Uist.
3. Perform selected internal consistency checks on the direction results and resolve any discrepancies.
4. Advise the Project Officer on the results and propose further quality control measures if required.
5. After a satisfactory routine operation of the Marex buoy has been achieved, perform an intercomparison of measurements from the Marex buoy and an IOS Cloverleaf buoy. This comparison would extend over several days and the results would be analysed and a report presented.

(b) Interpretation

1. Obtain validated Marex directional observations in monthly batches.
2. Calculate directionality factors and energy flux, as a function of frequency and in toto.
3. Use a refraction model to determine changes in directionality at the other wave measurement depths and relate these to changes in energy flux.
4. Using directional wave observations in 3 monthly batches, check that distributions of directional width, mean power with direction and peak power direction are consistent with those presently used.
5. Report on the progress of the work to the Project Officer.

In the event, (a)1 to (a)4 were completed, (a)5 was attempted unsuccessfully, not using the Cloverleaf but a pitch-roll buoy for ease of handling; (b)1 was only completed for one month and (b)2-(b)4 were not attempted. This report completes (b)5.

3. Dates

1st November 1982	: Contract commenced.
28th November 1982	: Marex buoy deployed in Freshwater Bay, I.O.W.
21st December 1982	: Data format proposed by IOS (JAE) to Marex (see Section 5 below and Appendix 2).
20th January 1983	: First pitch-roll buoy intercomparison - failed, because Marex data quality inadequate.
10th February 1983	: Second pitch-roll buoy intercomparison - failed, gyro fault on IOS P/R buoy.
2nd March 1983	: Third pitch-roll buoy intercomparison - 5 successful 34 minute records (see Section 6 below).
15th March 1983	: Meeting at Marex, results discussed, modifications for South Uist mooring proposed.
29th March 1983	: 6 monthly progress report.
1st May 1983	: Marex buoy deployed at South Uist.
5th May 1983	: Shore receiving station operational.
24th May 1983	: Some directional data passed to IOS for comment.
7th June 1983	: IOS assessment indicates still possible mooring problems and suspect compass data.
6th July 1983	: End of heave data series (see Section 8 below).
22nd-27th July 1983	: IOS P/R buoy deployed from R/V Calanus, 18 recordings obtained.
31st August 1983	: Marex inform that their buoy compass was defective during July.
9th September 1983	: Meeting at Harwell, project future discussed, need to repeat intercomparisons.
31st October 1983	: Contract terminated.

4. The Marex Directional Wave Buoy (S100)

The Marex S100 directional wave buoy is a development of the Marex oceanographic data buoy S066 which has been used operationally since 1976. The data buoy was designed to gather one dimensional wave data (wave height) together with meteorological data (wind speed and direction, barometric pressure and air temperature), sea temperature, current speed and direction at up to eight depths. To meet these requirements it was designed to follow vertical displacements of the sea surface up to a frequency of 0.3 Hz and to provide an anemometer platform at the 'obligatory' height of 10 metres above sea level. These features were achieved using a discus hull of 2.5 m diameter with a 'keel' similar in form to an inverted mast to achieve stability and self righting ability.

In late 1978, when the company was considering the possibility of developing a data buoy capable of measuring the 2-dimensional (directional) wave spectrum, it became apparent to members of IOS, who were invited to discuss this possibility, that Marex had underestimated the implications of such a development and were hoping to achieve a marketable system with relatively little modification of their standard data buoy hull configuration. Since IOS had considerable experience with directional wave buoys as a result of their involvement with DB1 design and operations, apart from the IOS pitch-roll buoys for scientific directional wave measurement, IOS doubts as to the likelihood of Marex obtaining good surface following with such a hull/mast mooring design were expressed at this meeting. Another point queried at the time was the compass performance. The compass, being horizontally gimballed 2 component type fitted high on the mast, would be subject to considerable accelerations in the horizontal plane and IOS raised the point that the outputs from the proposed fluxgate compass would need careful processing to obtain accurate, stable, buoy heading information. Marex held the view at the time that the compass had been used previously for wind direction reference satisfactorily on their data buoy (but they had never looked at the short period compass output variations). In the event, when the directional buoy was first tested at sea, this was proved to be a problem and Marex had to modify their processing considerably to achieve satisfactory heading data. It was also found that the slope following performance of the hull/mooring system was inadequate, as will be discussed below. These instances are quoted to demonstrate that although there were consultations during the early development phase, there was insufficient notice taken of some critical observations.

Another feature, which was queried during this contract, was the effect of windage of the mast in producing excessive mean tilt of the buoy. As the original Marex databuoy tilted as much as 26° in high winds, this was obviously of importance in maintaining slope following performance and resulted in the mast being approximately halved in height. Since the mast forms part of the aerial system, this necessitated electrical design changes and difficulties were later experienced in maintaining optimum aerial tuning.

A number of other points which arose during development were settled satisfactorily, and therefore will not be mentioned here. It should be recorded that relations between Marex and IOS remained good throughout.

5. Method of Analysis and Presentation

(a) Method of analysis

IOS advised Marex on the method of analysis to be used in processing wave data from a surface-following wave buoy.

The principle of the method has been described by Longuet-Higgins, Cartwright and Smith (1963; In "Ocean Wave Spectra", Prentice-Hall, 111-136). The directional wave spectrum is derived from estimating certain cross-spectra between the measured series of heave acceleration, pitch and roll, the latter being transferred to North-East axes using the compass signal. Appendix 1 gives details of the method.

The relevant parameters derived from the analysis are also given in Appendix 1 and form the basis for the statistics to be given in Marex data reports.

IOS asked Marex to check their software for the calculation of directional wave spectra by processing one of our pitch-roll buoy records as "standard". The comparison between IOS estimates of the cross-spectra and directional parameters and equivalent estimates made by Marex were considered to be entirely satisfactory and a validation of their computer program.

(b) Presentation of results from the Marex directional buoy

Two meetings were held with Marex about the format of their Quarterly Reports. Appendix 2 shows the quantities to be reported in the form of tabulations and graphs. The intention of this presentation was to allow users the opportunity of studying the characteristics of wave power and directional parameters at South Uist. For quantitative work it would be necessary to have access to the same data on magnetic tape; MIAS agreed to provide this service.

In addition to the above reports, it was agreed that Marex should provide MIAS with their analysed results in the form of nine cross-spectra. These data would then be archived in the "data buoy format" which has been adopted as a standard for reporting directional wave spectra by MIAS.

6. Freshwater Bay Intercomparisons

In the course of development of the buoy, during a meeting of Marex and IOS representatives and of the UKAEA Project Officer, the problems of validating the directional measuring performance of the Marex buoy were discussed. It was agreed that useful information could be obtained, at relatively low cost, by an intercomparison of the Marex buoy and of an IOS pitch-roll buoy during the trial deployment of the former at a shallow mooring site off Freshwater Bay, I.O.W. This would then be followed by a more detailed intercomparison at the operational mooring site to the west of South Uist (57°18'N, 7°54'W).

The Freshwater Bay site was chosen on account of its reasonable exposure to waves combined with its accessibility from the Marex base: at this stage of the commissioning of the buoy, frequent visits were anticipated for the purpose of optimising hardware and software adjustments. The site depth was, however, only 20 m so that the performance of the mooring and its effect on the buoy's surface following could not be expected to be fully representative of the performance at the operational site. Nevertheless, it was hoped that it would reveal any gross deficiencies in the performance of the Marex system.

The Marex buoy was deployed on 28th November 1982. During December 1982 and January 1983, Marex were testing out the buoy and rectifying various deficiencies. On 20th January and 10th February 1983 intercomparisons with an IOS pitch-roll buoy were made. The IOS buoy was deployed from the Marex boat, M.V. Triton: the buoy was used in its internally recording configuration and was tethered to the M.V. Triton by a rope incorporating a compliant section. The IOS buoy's recording schedule was adjusted to coincide with the Marex sampling schedule. As far as was consistent with operational restrictions, comparisons were to be made at times when the current was low, near slack water. This was to minimise uncertainty in the intercomparisons due to the drift of the IOS buoy relative to the moored Marex buoy. Although the resulting Doppler shifts could be corrected for, it was desirable to keep such effects to a minimum. The first intercomparison was a failure due to inadequate Marex data quality. The second intercomparison was aborted due to failure of the gyro in the IOS buoy. A third intercomparison on 2nd March 1984 was successful and resulted in five records which were analysed by the respective buoy operators. Energy levels were quite low with significant wave height increasing from 0.55 to 0.77 metre during the day. Although full directional analyses were carried out, the data presented here (Appendix 3) is in the form of estimates, over 0.1 Hz frequency bands, which have 400 degrees of freedom: the 90% confidence limits are, therefore, 0.91/1.09.

Taking the one dimensional energy estimates first, it is clearly seen that the Marex buoy gave higher estimates than the IOS buoy in the bands 0.3-0.4 Hz (averaging 52% higher) and 0.4-0.5 Hz (averaging 18% higher). This is broadly in agreement with the results obtained by Marex, comparing their S066 buoy with a Datawell Waverider in 1981, and by IOS, comparing the S100 buoy with a Waverider at the South Uist site (see 8 below). The discrepancies are due to the underdamped heave resonance of the Marex buoy at approximately 0.37 Hz.

The directional information, given by the IOS (I) and Marex (M) S100 buoys at Freshwater Bay, is summarised in the form of the check ratio and mean direction (θ_1) averaged over each of the four frequency bands. As far as the mean direction is concerned, there is better agreement at low frequencies than at high frequencies. The difference in the IOS and Marex values of θ_1 at high frequencies will be partially due to tidal currents and there is some evidence of a systematic difference whose polarity reverses after slack water: this is consistent with waves coming from the south with a predominantly East-West tidal flow.

The check ratio, which is the ratio of the vertical displacement given by the heave sensor to that calculated from the slope sensors (with allowance made for the effect of the water depth upon the wavelength) should be sensibly unity for all of the frequency bands. As can be seen from Appendix 3, the check ratio for the Marex buoy data averages 1.45. In the frequency bands 0.3-0.4 and 0.4-0.5 Hz, the effect of the Marex buoy's heave resonance would be to give check ratios of 1.23 and 1.09. The major cause of the high check ratio was, therefore, inadequate slope following. This was confirmed by comparison of the slope spectra of the IOS and Marex buoys and, indeed, was visually apparent during the measurements. Following discussions, primitive model tests in the wave tank and calculations (A. Packwood, pers. comm.) at IOS it was concluded that neither weight distribution nor buoyancy contribution could account for this and that the pendulum chain and clump were the most likely cause. The purpose of these components in the mooring was to provide self righting in the event of a capsize. Marex agreed to increase the length of the pendulum chain and reduce clump weight somewhat and to alter the mooring slightly for the South Uist deployment. However, for reasons of time and/or cost Marex did not feel able to implement any of the three principal palliatives suggested, namely (i) a compliant 3 point mooring, (ii) a higher single point mooring attachment or (iii) replacement of chain by a tubular bar pivoted under the buoy with a much reduced clump weight.

It should be noted, as detailed in Appendix 1, that perfect slope following is not of itself essential for estimates of mean wave direction, θ_1, and directional spread, θ_2, providing (a) the response is linear with magnitude of slope and (b) the pitch and roll responses are identical, without directional or phase differences. Intercomparisons are essential to check these features, especially the effects of practical moorings at sites with currents present.

7. South Uist Intercomparisons (57°17'N, 7°53'W)

(a) Narrative

R.V. Calanus was chartered from the Scottish Marine Biological Association, Oban, for the intercomparisons off South Uist. The vessel left Dunstaffnage on 21st July 1983 and sailed for Castlebay, Barra, which was to be the ship's base. It was the intention, due to the ship's small size and limited endurance, to make measurements over a period of about 2 to 3 days at a time, weather conditions permitting. Two measurement periods were made. During the first period from 22nd-24th July, the wave conditions consisted of swell waves of height about 1.5 m decaying to less than 0.5 m wave height: local winds were less than 10 knots. This set of 15 wave mesurements of swell wave conditions is noted in Appendix 4.

On return to Castlebay, Barra the weather charts and forecasts were noted so as to return to the site for measurements of wave spectra during conditions of active wind-wave generation.

On 26th July the ship left Castlebay for the Marex buoy site in view of the forecasts of force 4-5 winds. Measurements commenced at 0100 on 21st July and were continued until later that day until it was impossible to deploy the pitch-roll buoy due to the extreme motions of 'Calanus' in waves of about 3 m height. The ship returned to Dunstaffnage on 27th July due to the deteriorating weather conditions.

(b) Pitch-roll buoy analysis

Appendix 4 gives details of the wave measurements made by IOS. We have fully analysed records 1 and 17 to confirm that our wave buoy records are of good quality.

As the Marex data buoy did not provide directional wave spectra over the period 22nd-27th July the remaining sixteen records have not been processed. These magnetic tape records will be kept for possible future use.

(c) Directional wave data from Marex buoy in May, June

Marex experienced radio interference with the HF telemetry reducing the analysable data during this period to about 50% of the maximum. From their reduced data the IOS view was that the buoy was measuring the directional wave spectrum, albeit not perfectly. The θ_1 estimate at high wave frequencies (0.3-0.35 Hz) seemed to be about 30° different from the wind direction measured at Benbecula. The winds were generally low however and the differences may not be unreasonable, in view of the separation distance of the wind and wave measurements. The main problem remained the reduced sensitivity to pitch/roll; though the check ratio was lower than during the Freshwater Bay trials indicating that the weight change had been beneficial, it was still too high; what is more it remained high to longer wave periods (9 seconds, c.f. 6 secs in Freshwater Bay), and it is tempting to argue that this is the result of lengthening the pendulum chain length from 10 metres to 20 metres.

8. Heave Response Comparison with Waverider

(a) The Marex directional wave buoy was moored at a distance of approximately 200 m from the Institute of Oceanographic Sciences Waverider buoy at 057°17'N, 007°53'W, to the West of South Uist. The water depth at this location is approximately 97 m.

Both buoys employ the same principle for the measurement of wave height: the vertical accelerations suffered by a passively stabilised accelerometer are transformed to displacement by double integration. In the case of the Waverider this is achieved electronically within the buoy and information on buoy's vertical displacement is telemetered continuously to a shore station. The Marex buoy transmits the acceleration signal directly.

The transmitted data are processed according to the schemes given below.

(i) IOS Waverider

Analysis of these data is achieved at the receiving site using a dedicated microcomputer. The data are subjected to an automatic validation procedure and only those records passed as acceptable are used in the subsequent comparisons.

An individual wave record comprises 4096 displacement values sampled at 0.5 s intervals. Each record is cosine tapered over 512 points at each end and then Fourier transformed. The resulting spectral estimates are scaled to restore the variance to the total record variance before tapering, and then adjusted to compensate for the known response of the measuring system. The final spectrum is formed by averaging estimates in groups of 15 to give final frequency resolution of 0.00732 Hz. A spectrum is obtained every one and a half hours.

(ii) Marex buoy

The transmitted acceleration signal is received at the shore station where it is sampled and recorded in digital form. Data tapes are returned regularly to Marex for analysis.

An individual record, comprising 4096 half second values of acceleration, is divided into 4 equal length sub-sections. Each sub-section is Fourier analysed and converted to a spectrum of displacement variance by dividing each spectral estimate, at frequency f_i, by $(2\pi f_i)^4$. The resulting estimates are averaged in groups of 5 and then averaged over all 4 sub-section spectra to give a final spectrum with a frequency resolution of 0.00976 Hz. One spectrum is obtained every 3 hours.

It will be noted that the two systems produce spectra with estimates at different frequencies.

The spectral data used in the comparisons reported here were supplied directly by Marex. Some initial difficulties were experienced due to the contents of the magnetic tapes being incorrectly described in the accompanying documentation.

(b) Data availability

Staffing and electricity supply problems, which occurred at the shore station during the period of joint deployment, affected the collection of data by both systems. Difficulty was experienced in acquiring sufficient simultaneous data to allow a definitive comparison of heave response. Of the data collected over the period 5 May to 6 July 1983, 239 usable simultaneous measurements were obtained; this is equivalent to 29⅞ days of three hourly wave recordings.

Three measurements taken by the Marex buoy were rejected as the values of significant wave height calculated for these spectra were clearly anomalous.

(c) Comparison procedure

The available simultaneous data were used to establish the relative response of the two systems over the whole of the frequency range. As the systems gave spectral estimates at different frequencies, the first task was to interpolate linearly between the Waverider estimates to give values for direct comparison with the Marex spectra.

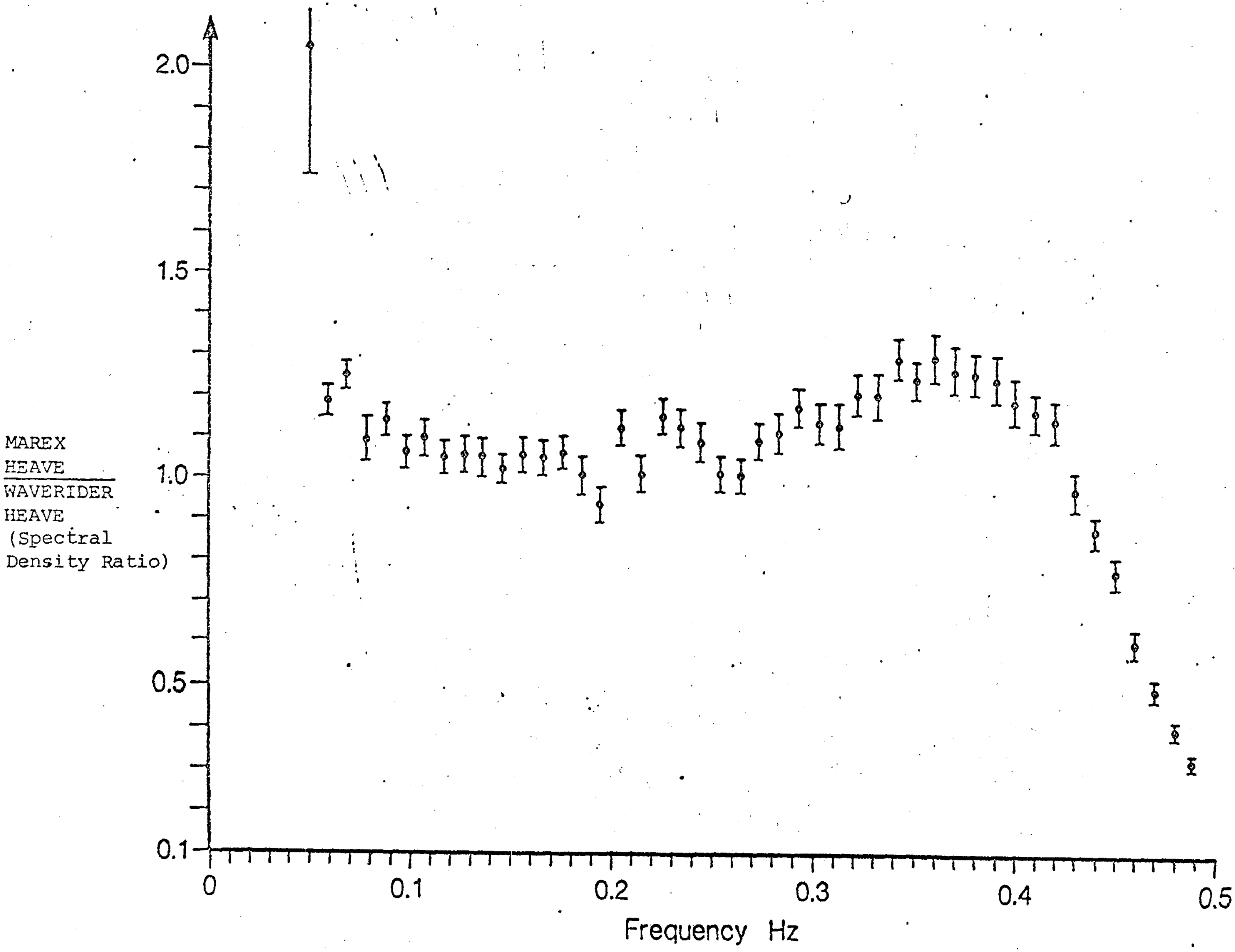
MAREX HEAVE / WAVERIDER HEAVE (Spectral Density Ratio)
2.0
1.5
1.0
0.5
0.1
0
0.1
0.2
0.3
0.4
0.5
Frequency Hz

FIGURE 1

At each frequency, therefore, a population of 239 simultaneous pairs of spectral estimates was obtained. The relationship between the two buoys at each frequency was established by determining the form of the straight line which, when superimposed on the scatter plot of simultaneous values, resulted in a minimum value for the sum of the squares of the perpendicular distances of the points from the line. The slope of this major-axis line (constrained also to pass through 0,0) is taken as the measure of relative spectral response.

The 95% confidence limits on the slope were also calculated, and these are shown with the slope values in figure 1.

In addition to the spectral comparison, values of significant wave height and mean zero crossing period have been compared in the same way. Values of Hs and Tz were not supplied for the Marex buoy and had to be calculated from the spectra for this purpose. The results of the major axis analysis were:-

Hs (MAREX) = 1.035 x Hs (WAVERIDER)

Tz (MAREX) = 1.044 x Tz (WAVERIDER)

(d) Discussion

The relative spectral response plotted in figure 1 shows significant departures from unity over almost the whole range of frequencies. These are particularly marked, however, at frequencies below 0.1 Hz and between 0.27 and 0.43 Hz. At these frequencies the response of the Marex buoy exceeds that of the Waverider, whereas at frequencies above 0.43 Hz the Marex response falls rapidly below that of the Waverider.

The result of these differences on the integrated spectral properties is that the Marex buoy returns Hs values some 3.5% higher than the Waverider, and Tz values 4.5% higher.

The significant fluctuations of relative response in adjacent bands near 0.2 Hz is difficult to explain. The damped peak response at 0.36 Hz on the other hand is confidently believed to be due to the Marex buoy heave resonance.

9. Recommendations

The Institute has been involved in wave recording for over 30 years and in directional wave recording for over 20 years, for research purposes and with attended buoys. Why then has there been no commercially available autonomous directional wave data buoy until the 1980s? The reasons are many and varied, but at the risk of over-simplification are basically that it is a very difficult technical problem and that the market is small and recent. The wave buoy design is difficult because the need to survive the violent sea surface motions conflicts with the need to follow the surface, not only in heave but in slope also; practical buoy response functions are too complex to be calculated with accuracy so intercomparisons are essential; vast amounts of data are soon accumulated by the multiple sensor package and careful quality control preceding data reduction is essential.

Marex experienced problems with practically every aspect of the buoy: sensors (compass), structural design (mast), dynamical design (slope following), telemetry (HF interference), irregular data flow (causing critical delays in early July). Gradually many of these problems were being solved, but too late for the Wave Energy Programme.

Thus WESC are frustrated by late delivery and lack of data, Marex are frustrated by having a nearly developed but unproven buoy and IOS are frustrated, their desire to transfer the technology unfulfilled. What therefore can be saved? A remaining UK government market for such a buoy may be the Wave Climate Programme for another area of D. of Energy, namely PED, in connection with offshore platform requirements. It is therefore recommended that the buoy be used by PED since it is a potential contributor to their Wave Climate Programme. The grounds for this are that its costs are intermediate between the UKOOA (DB2, DB3) buoys and the lower capability Datawell Wavec buoy (since the Marex buoy can be further instrumented with standard Metocean sensors). It does assume too that Marex can quickly solve the compass and data flow problems and that the slope following insensitivity proved to be consistent and linear. As there is still need for intercalibration, it is strongly recommended that Marex be encouraged to take part in the exercise off Norway being organised by IKU later in 1984.

10. Acknowledgements

The following IOS staff contributed both to the project and to the report and their assistance is much appreciated - Mr M.J. Tucker, Mr J.A. Ewing, Dr C.H. Clayson, Dr J.A. Crabbe and Mr E.G. Pitt.

APPENDIX 1

ANALYSIS OF A PITCH-ROLL BUOY SYSTEM

The Marex data buoy employs the Datawell HIPPY-40 sensor to measure heave, pitch and roll. As this sensor introduces a phase shift between heave displacement and pitch or roll it was considered desirable to record acceleration directly since no phase shift is present with this signal.

If we denote heave acceleration (in units of "g") by the subscript 1, and let subscripts 2 and 3 refer to surface slopes in the North and East directions respectively, then the directional wave spectrum is derived from the 6 cross-spectral estimates C_{11}, C_{22}, C_{33}, Q_{12}, Q_{13}, C_{23} (the other 3 cross-spectra having expectancy zero for a surface following buoy).

The six cross-spectra completely define the results of a surface-following pitch-roll buoy system. If results are required at high wave frequencies where the buoy motion may not follow the wave surface, then the 3 cross-spectra C_{12}, C_{13}, Q_{13} can be helpful in correcting the response. All nine cross-spectra are therefore computed and archived in the "data buoy format" of MIAS.

From the 6 fundamental cross-spectra it is useful to compute the following 6 quantities:

(a) the one-dimensional wave spectrum, $E(f) = \frac{g^2 C_{11}}{(2\pi f)^4}$ where f is the wave frequency in Hertz;

(b) the first and second order, normalized angular harmonics

$$A_1 = \frac{Q_{12}}{\sqrt{[C_{11}(C_{22} + C_{33})]}} \quad : \quad B_1 = \frac{Q_{13}}{\sqrt{[C_{11}(C_{22} + C_{33})]}}$$

$$A_2 = \frac{C_{22} - C_{33}}{C_{22} + C_{33}} \quad : \quad B_2 = \frac{2C_{23}}{C_{22} + C_{33}}$$

(c) the check ratio $R = \frac{1}{\tanh kh} \left(\frac{C_{11}}{C_{22} + C_{33}}\right)$

where $\tanh kh \rightarrow 1$ in deep water; h is the water depth, k the wave number. (Note: the normalization of the angular harmonics removes any dependence on the wavenumber or dispersion relation). The check ratio R should therefore be unity and provides an independent check on the correct functioning of the system and on the analysis.

Derived parameters

If the directional distribution is unimodal and of the form $\cos^{2s}\tfrac{1}{2}(\theta - \theta_1)$, it is useful to derive the following directional parameters:

(i) mean direction from the first order angular harmonics

$$\theta_1 = \arctan (B_1/A_1)$$

(ii) mean direction from the second order angular harmonics

$\theta_{12} = \tfrac{1}{2} \arctan (B_2/A_2)$. This direction is ambiguous to 180 degrees.

(iii) spread parameter from the first order angular harmonics

$$S_1 = \frac{C_1}{1 - C_1} \text{ where } C_1^2 = A_1^2 + B_1^2$$

(iv) spread parameter from the second order angular harmonics

$$S_2 = \left[(1 + 3C_2) + \sqrt{1 + 14C_2 + C_2^2}\right]/(2\left[1 - C_2\right])$$

where $C_2^2 = A_2^2 + B_2^2$.

(v) spread parameter $\theta_2 = \sqrt{[2 - 2C_1]}$. For a narrow directional distribution θ_2 is the r.m.s. spread about the mean direction θ_1.

Note: The definition of the angular harmonics A_1 and B_1 involves the ratios of terms containing heave and surface slopes such that A_1 and B_1 are relatively insensitive to the response amplitude characteristics of the slope measurements. In particular, $\theta_1 = \arctan (Q_{13}/Q_{12})$, so even if, at high frequencies, the slope response is not unity, the estimate of θ_1 can still be useful. If, however, there is a phase shift in addition to an amplitude response, then θ_1 cannot be considered reliable.

The second order angular harmonics A_2 and B_2 involve ratios of the slope spectra and are independent of heave. In this case, provided both pitch and roll have identical amplitude and phase responses, it should be possible to derive some directional information at frequencies where the buoy does not follow the wave slope.

APPENDIX 2

PROPOSED PRESENTATION OF RESULTS FROM THE MAREX DIRECTIONAL BUOY IN QUARTERLY REPORTS

(A) Tabulations

The following parameters are to be tabulated for each wave recording:

h_s (significant wave height = $4\sqrt{mo}$)

P (wave power computed from $P = \rho g \int c_g(h)E(f)df$ where ρ is the water density, c_g is the group velocity of waves of frequency f in water of depth h, and E is the wave spectrum)

T_z (mean zero-crossing period = $(mo/m2)^{\frac{1}{2}}$)

T_e (wave energy period = m-1/mo)

T_p (peak of the wave power spectrum)

at T_p:

- θ_1 (mean wave direction from the 1st order angular harmonics)
- θ_2 (r.m.s. spread from the 1st order angular harmonics)
- S_1 (exponent of cosine distribution, $\cos^{2S_1}\frac{1}{2}(\theta-\theta_1)$)
- E (wave spectral density)
- R (check ratio of heave accn. to combined slope spectra)
- θ_1 (average over the high frequency range 0.25 - 0.30 Hz)

(B) Graphs

(a) One dimensional wave spectra represented in "seismic form" for each record.

(b) Time histories of

(i) P

(ii) θ_1 and θ_2 at T_p

(iii) θ_1 at high frequencies

(c) Scatter diagram of h_s vs. T_e

(d) Directional scatter diagram of Wave power P (kW/m) vs. θ_1 (at T_p). The bin size for θ_1 to be 20°.

APPENDIX 3

IOS/MAREX INTERCOMPARISONS: 2 MARCH 1983

Frequency Band (Hz)	E (m^2/Hz) IOS	E (m^2/Hz) MAREX	Ratio IOS	Ratio MAREX	θ_1 (°T) IOS	θ_1 (°T) MAREX	
0.1-0.2	0.0936	0.0981	0.96	1.46	225	220	10.00 hrs
0.2-0.3	0.0213	0.0248	0.97	1.61	206	190	
0.3-0.4	0.0328	0.0440	1.01	1.63	213	192	
0.4-0.5	-	0.0216	-	1.11	-	250	

h_s(m) = 0.55   0.58

	E I	E M	Ratio I	Ratio M	θ_1 (°T) I	θ_1 (°T) M	
0.1-0.2	0.1210	0.1000	0.95	1.45	216	217	11.00 hrs
0.2-0.3	0.0235	0.0293	0.90	1.62	195	191	
0.3-0.4	0.0362	0.0590	0.97	1.73	180	179	
0.4-0.5	0.0171	0.0182	0.99	1.17	150	237	

h_s(m) = 0.65   0.66

	E I	E M	Ratio I	Ratio M	θ_1 (°T) I	θ_1 (°T) M	
0.1-0.2	0.1320	0.1350	0.94	1.41	213	216	
0.2-0.3	0.0396	0.0369	0.94	1.56	201	202	
0.3-0.4	0.0471	0.0703	0.99	1.67	171	174	12.00 hrs
0.4-0.5	0.0135	0.0141	0.95	1.12	133	187	Slack Water

h_s(m) = 0.73   0.76

	E I	E M	Ratio I	Ratio M	θ_1 (°T) I	θ_1 (°T) M	
0.1-0.2	0.1433	0.1537	0.93	1.29	206	230	13.00 hrs
0.2-0.3	0.0626	0.0575	0.98	1.50	194	221	
0.3-0.4	0.0458	0.0723	0.93	1.85	157	183	
0.4-0.5	0.0169	0.0196	0.98	1.30	139	166	

h_s(m) = 0.77   0.80

	E I	E M	Ratio I	Ratio M	θ_1 (°T) I	θ_1 (°T) M	
0.1-0.2	0.1015	0.1163	0.88	1.11	208	226	14.00 hrs
0.2-0.3	0.1029	0.0792	1.00	1.43	192	221	
0.3-0.4	0.0612	0.0941	1.01	1.84	156	188	
0.4-0.5	0.0158	0.0230	0.96	1.20	155	146	

h_s(m) = 0.77   0.77

APPENDIX 4

SUMMARY OF MEASUREMENTS MADE OF SOUTH UIST WITH THE IOS PITCH-ROLL BUOY AT 57°18'N, 7°54'W IN CONJUNCTION WITH THE MAREX WAVE DIRECTIONAL BUOY

Calanus Cruise : July 1983

Run Number	Day (July 1983)	Start time (BST)	Wind speed (kts) and direction* (°)	Visual wave observations height (m), period (s), direction* (°)
1	22	1300	<10, -	2, ~10, 280
2	22	1600	<10, -	2, ~10, 280
3	22	1900	<5, -	1½, ~10, 285
4	22	2200	10, 060	1½, ~10, 285
5	23	0100	10, 000	no observation
6	23	0400	12, 045	no observation
7	23	0700	<10, 090	½-1, ~10, 280
8	23	1000	<10, -	1, ~10, 175
9	23	1300	8, 025	<1, ~6, 045
10	23	1600	7, 025	<1, ~6, 000
11	23	1900	<5, -	½, ~6, 000
12	23	2200	very light	<½, -, 280
13	24	0100	very light	no observation
14	24	0400	very light	no observation
15	24	0700	5, 060	½, -, 270
16	27	0100	20, 340	1½, ~7, 340
17	27	0400	22, 340	2-3, ~7, 340
18	27	0739	20, 340	2-3, ~7, 340

*Direction (from)