HF Radar: An assessment of its limitations for measuring waves

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Note: This assessment is in two parts: the assessment itself for general readers, and an appendix giving detailed justification for some of the statements made in the assessment: this is necessary since some of the arguments appear to break new scientific ground.

The assessment applies specifically to ground-wave radar. Sky-wave radar will have the same limitations plus those due to propagation problems.
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1. INTRODUCTION

A number of circumstances recently combined to make me feel that I had to try to make a thorough assessment of the present position with regard to the use of HF radar as a practical tool for the routine measurement of waves. One of the circumstances is a meeting of the Science and Engineering Research Council Panel concerned with the SERC-supported HF Radar project and this has set a severe time limit. Thus, I have not been able to do more than glance at a small proportion of the relevant literature on this occasion. However, I have had a long and helpful talk with Dr Lucy Wyatt at Birmingham University and so am reasonably confident that what is said below is not at variance with published knowledge. (The views expressed below are entirely my own.)

A further incentive is a proposal by CODAR Inc for a major development programme to produce a platform-mounted HF radar for the measurement of waves and currents. Support for this is being solicited in the UK as well as elsewhere under the sponsorship of the Gulf Research and Development Corporation. Another commercial instrument is being developed by the French firm Syminex with the support of IFP.

The inescapable conclusion from the present study is surprising: the most important limitations concern the properties of the first-order Bragg resonance. Not only is the relevant theory apparently not understood, but little attention has been devoted to the problem and the rather sparse empirical data is in many respects confusing and contradictory.

2. THE LIMITING FACTORS FOR THE DETERMINATION OF WAVEHEIGHT AND 1-D SPECTRA

2.1 The most promising technique for getting information about waves is interpretation of that part of the Doppler spectrum arising from non-linear interactions. This technique has been developed mainly by Barrick and Lipa. In a limited region near the first order Bragg lines (whose frequency is $f_B$) the energy in the Doppler spectrum at a frequency $f_B + \Delta f$ arises from water wave components with frequencies $f_w = \Delta f$. It is possible to "invert" this part of the Doppler spectrum into the corresponding wave spectrum with some (but not complete) confidence. The region which can be used at present is limited as follows:

(a) At low $\Delta f$ by the skirts of the first order Bragg line (see below).
(b) At high $\Delta f$ by linearising assumptions in the equations used. The limit quoted in the CODAR Inc proposal is $\Delta f \approx 0.4 f_B$. This seems to be optimistic, but even if valid represents a serious limitation. It seems to be possible in principle to extend this limit by the use of more sophisticated algorithms, such as CODAR Inc propose to develop.
Figure 6. The structure of features in the second-order portion of the Doppler spectrum (more heavily shaded) shows the dominant periods and strengths of wind waves and longer period swell.
2.2 The absolute values of waveheight and spectral density are obtained by using the energy $E_B$ of the first-order Bragg line as a reference. When integrating the spectrum to measure $H_s$, this is the factor limiting the accuracy and it is a serious limitation (confidence limits for $H_s$ typically = ±10% even when integrating over a 3 hour record; see Section A1).

2.3 With a radar frequency of 25 MHz, typical Doppler spectra such as that shown in Fig 6(b) of the CODAR Inc proposal (reproduced here) give the point at which the skirts of the first-order Bragg line intersect the second-order spectrum as $\Delta f = \pm 0.05$ Hz. However, these "typical" values seem to correspond to modest waveheight ($H_s$ of 1 to 2 m).

It seems to have been established empirically that the width of the first-order Bragg line is roughly proportional to the significant waveheight $H_s$ (though the evidence is by no means consistent: see A3 and A4). This has been proposed as one way of obtaining $H_s$, but is in fact not a good measure (see section A2). As $H_s$ increases, the increased width of the first-order Bragg line more than compensates for the increased energy of the second-order Doppler spectrum and the point of intersection moves to higher $\Delta f$. This effect is thus a serious limitation on the ability of a 25 MHz radar to measure low frequency waves.

Simple physical arguments supported by some measurements (see section A3) show that the width of the first-order Bragg line increases with radar frequency $f_R$, and probably as $f_R^{3/2}$. Thus CODAR Inc's proposal to move to VHF in order to improve the limitation in 2(b) above seems doomed to failure.

It will be seen that whereas limitation 2(a) favours lower radar frequencies, limitation 2(b) favours higher radar frequencies. The limitation mentioned in section 2.2 due to random sampling errors in $E_B$ also favours higher frequencies.

3. DIRECTIONAL SPECTRA

Clearly the amplitude information is subject to the limitations discussed above, but perhaps there is useful information to be obtained about the relative directional distributions.

The matrices used for obtaining the directional information are ill-conditioned and can only be solved in practice by limiting the information obtained: so far as I can see, this amounts to smoothing. I have not had time to understand the process in detail, but from statements by Lipa it
looks as though she solves the matrices for the lower order directional harmonics (zero, first and second). From some rather superficial thinking I would expect even the estimates of these to be unstable, but this subject is at the frontiers of research at the moment.

Present techniques also involve either the use of two radars looking at the sea in different directions, or the assumption of a homogeneous wave field over a fairly wide area.

4. CONCLUSIONS

At present we do not know enough about the characteristics of the first-order Bragg line either theoretically or empirically to establish with any degree of certainty the quantitative limits of the ability of HF radar to measure waves, and how these vary with radar frequency. We are thus also unable to choose the optimum frequency.

With present knowledge and techniques the limitations are serious enough to make HF radar a tool of doubtful practical value for measuring waves.

The most urgent requirement is for research into the properties of the first-order Bragg line.

While directional information is present in the radar Doppler spectra, its extraction in useful form is not yet convincingly established.

5. REFERENCES


APPENDIX

JUSTIFICATION FOR SOME OF THE STATEMENTS MADE IN THE ASSESSMENT

(A1) Sampling errors

If a random signal is sampled for a time T, the accuracy to which the energy in a range of frequencies $\delta f$ can be determined is $1/(T \delta f)^{1/2}$ (this is the rms value of the proportional sampling error, assuming the spectral density is uniform in the interval $\delta f$). Thus, since in the region of interest the contribution to the Doppler spectrum from a wave of frequency $f_W$ is at $f_B + f_W$, $\delta f$ in the Doppler spectrum equals $\delta f$ in the wave spectrum and if only one sideband is used, the sampling error is identical to that for a single wave buoy recorded for the same period of time.

By the same argument*, the error in determining $E_B$, the energy of the Bragg line, is $1/(T \delta f_B)^{1/2}$. The 3 db width of the Bragg line for a 25 M Hz radar is typically 0.01 Hz. So even for a 3 hour record ($T \approx 10^4$), the rms proportional sampling error in $E_B$ is approximately 10%, giving confidence limits of approximately ± 20% (Equivalent to approximately half these values in terms of wave height.)

(A2) Variation of $\delta f_B$ with waveheight

A model capable of giving a feel for the factors involved in the broadening of the first-order Bragg line is what SAR scientists call the two scale model. In this, the Bragg resonant waves are carried about and modulated by the much longer waves carrying the main wave energy.

For example, using 25 M Hz, the Bragg resonant waves have a wavelength of approximately 6 m, whereas in a 15 m/s wind the dominant wavelength is approximately 150 m with a corresponding period of 10 s. The phase velocity of a 6 m wave is approximately 3 m/s. Thus, to first-order with these parameters, the phase of the Bragg resonant wave is modulated by the horizontal displacements in the longer waves, which are equal in magnitude to the vertical displacements.

Note that since the Bragg resonant waves are long-crested waves moving in the range direction, the relevant particle displacements in the long waves are the components in the range direction. Thus, the broadening $\delta f_B$ of the Bragg line is given by

$$(\delta f_B)^2 \propto \int S(f, \phi) \cos^2 \phi \, df \, d\phi$$

whereas by definition $H_s^2 = 16 \int S(f, \phi) \, df \, d\phi$

Thus, $\delta f_B$ is not a good measure of $H_s$.

*It is an approximate argument in this case: see section A4.
It should be noted that this model is not valid for longer radar wavelengths or shorter seas, and that it seems to be by no means certain that a process of this type is even the main contributor to the broadening of the Bragg line (see section A5).

(A3) Variation of $\delta f_B$ with radar frequency

For a given long-wave particle displacement the phase change of the Bragg resonant waves in radians is inversely proportion to the Bragg resonant wavelength $\lambda_B$. Thus, so far as I can see, the proportional broadening of the Bragg line should be proportional to $1/\lambda_B$. Since $f_B^2 = g/2\pi \lambda_B$, this gives $\delta f_B \propto \sqrt[3]{\lambda_B^{3/2}}$, or $\delta f_B \propto f_B^{3/2}$. I am not absolutely sure of the validity of this argument, but it seems plausible.

The analysis techniques used in published papers do not allow adequate resolution of the width of the Bragg resonant line, but Barrick and Snider (1977) have measured the decorrelation time $\tau_D$ of the peak (which is approximately the inverse of the bandwidth) at a range of radar frequencies. Using the maximum height of the higher of the two peaks (the advancing peak in this case) their results are as follows:

<table>
<thead>
<tr>
<th>Frequency: M Hz</th>
<th>$\tau_D$:s</th>
<th>Deduced $\delta f_B$: Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.41</td>
<td>140</td>
<td>.007</td>
</tr>
<tr>
<td>4.54</td>
<td>85</td>
<td>.012</td>
</tr>
<tr>
<td>6.92</td>
<td>35</td>
<td>.029</td>
</tr>
<tr>
<td>9.40</td>
<td>20</td>
<td>.050</td>
</tr>
<tr>
<td>13.41</td>
<td>20</td>
<td>.050</td>
</tr>
</tbody>
</table>

The resolution of their measurement of $\tau_D$ was not good enough to measure $\tau_D$ accurately at the higher frequencies, but the trend is clear.

They do not state the waveheight at the time, but they do state earlier in the paper that it did not exceed 4 m for any of their measurements.

They could detect no change in $\tau_D$ with waveheight: see the section A5 for comment on this.

(A4) The importance of the shape of the Bragg resonant band

The precise equation for the proportional standard deviation $\sigma$ of the random sampling error in the estimate of the energy in a band of frequencies is

$$\sigma^2 = \frac{1}{T} \frac{\int s^2(f) \, df}{\left(\int s(f) \, df\right)^2},$$

where $T$ is the duration of observation, and the integral is taken over the band under consideration.
For most circumstances this will be not very different from the simpler equation quoted in section A1 using $\delta f = 3 \text{ dB bandwidth}$. However, the bandwidth controlling the limit to which the non-linear spectrum can be used is more like the 25 dB bandwidth. It is therefore important to understand the factors controlling the shape of the skirts of the first-order Bragg peak, as well as its 3 dB width.

The relationship between the decorrelation time and the bandwidth also depends on precise definition, but again, for approximate purposes the 3 dB bandwidth is appropriate.

At this point one must emphasise the importance of a properly designed spectral analysis procedure. Some methods in current use have poor characteristics in terms of the width of the lower skirts of their response functions.

(A5) The bandwidth of the first order Bragg line in the absence of phase modulation, and the mechanisms for broadening it.

If the length of the radar pulse on the sea (that is, its range resolution) is $L$, then the number of Bragg resonant waves in it is $N = L/\lambda_B$.

If we consider a component of the wave system with $n$ waves in $L$, then the value of $n$ at which coherence is lost is $n = N + 1$ (assuming a sharp-edged square pulse).

The corresponding wavelength is $\lambda_n$ given by

$$\frac{1}{\lambda_n} = \frac{N+1}{L} = \frac{1}{\lambda_B} + \frac{1}{L}$$

The corresponding wave frequencies are

$$\frac{\omega^2}{2\pi g} = \frac{\omega_B^2}{2\pi g} + \frac{1}{L}$$

Putting $\omega = \omega_B \pm \Delta \omega$, where we assume $\Delta \omega \ll \omega_B$ gives

$$\Delta \omega = \frac{\pi g}{\omega_B} \cdot \frac{1}{L}$$

or

$$\Delta f = \frac{g}{4\pi f_B^2} \cdot \frac{1}{L}$$

$\Delta f$ here is the difference between the centre frequency and first zero, but this is also approximately the bandwidth between 3 dB points.

For example, for a 25 M Hz radar ($f_B = 0.5$ Hz) with a resolution of 10 km, $\Delta f = 1.6 \times 10^{-4}$ Hz.
The above calculation is only approximate, and if the 3 db bandwidth were calculated more exactly taking account of the range of directions over which coherence is achieved, then a slightly narrower bandwidth would be produced.

It is clear that measured bandwidths (for ground-wave radar) are much wider than this, so there is some mechanism in the sea broadening them. It is by no means certain that phase modulation due to the longer waves as discussed above is the main cause. For example, variations in surface current within the resolution cell would produce a Doppler spread and the rms variation would need to be only a few cm/s to produce the observed spreads. This seems quite possible, particularly in coastal waters, and could explain Barrick and Snider's observation of no dependence on waveheight. Assuming that the current pattern changes comparatively slowly so that a Doppler analysis is appropriate, the bandwidth is given by

$$\delta f_B = 2 \frac{V_{rms}}{\lambda_B}$$

So in this case the bandwidth is proportional to radar frequency, which actually agrees better with Barrick and Snider's observations.

I am, in fact, by no means certain in my own mind that the process of phase modulation by longer waves is not included in the non-linear analysis of Barrick and Lipa. If it is, then a very important conclusion follows: for higher radar frequencies (which includes 25 MHz) the third and higher order interaction terms are very important, because wave particle displacements are comparable with or exceed the radar wavelength.

To illustrate this point, at microwave frequencies the Doppler broadening of returns from the sea surface is several tens of Hertz.