INVESTIGATION OF SYSTEMATIC ERRORS ASSOCIATED WITH WAVE HEIGHT MEASURED IN UKOOA WEATHERSHIP PROGRAMME
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INVESTIGATION OF SYSTEMATIC ERRORS ASSOCIATED WITH WAVE HEIGHT MEASURED IN UKOEA WEATHERSHIP PROGRAMME

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INVESTIGATION OF SYSTEMATIC ERRORS ASSOCIATED WITH WAVE HEIGHT MEASURED IN THE UKOOA WEATHERSHIP PROGRAMME

1. INTRODUCTION

Since its development during the early 1950's the shipborne wave recorder (SBWR) has been used to collect a substantial proportion of the currently available wave records (Tucker 1956). More recently the commercially developed Waverider buoy system (WRB) has been used extensively for routine wave data collection. It is therefore of some concern that during the United Kingdom Offshore Operators Association (UKOOA) weathership programme discrepancies were reported between these two instruments. The work described below was undertaken to investigate sources of these discrepancies.

In 1973 the firm of Marine Exploration Ltd (Marex) was given a contract by the UKOOA to operate an environmental data gathering programme using small weatherships. This included running SBWR's in the ships and WRB's nearby while the ships were on station. IOS was at the same time given a contract by the Department of Trade and Industry to make sure that the data was suitable for government application. The Department also made a major financial contribution to the UKOOA programme. This arrangement was not entirely satisfactory because there was no provision in the Marex contract for them to cooperate with IOS in their validation work. In the circumstances relationships between IOS and Marex remained surprisingly cooperative, but Marex was understandably reluctant to undertake work or changes in procedure which involved them in significant expense which they had not foreseen when preparing their estimates for the contract. At the same time the Engineering Oceanography Group of IOS at Taunton which was responsible for the validation, did not have the staff available to undertake all the work which would have been desirable. This latter fact explains why the present investigation has only now been undertaken.

At each weathership station waves were recorded by both an IOS SEWR and a WRB deployed nearby. The two wave recording systems were used to provide records on paper chart rolls. The wave recorders were also interfaced to a FM analogue tape recorder which provided a backup in the event of a chart recorder failure. Ordinarily, chart roll records were analysed using the
Tucker/Draper method to estimate Hs (Tucker (1961), Draper (1966)). When the backup FM tape was used, a further stage of analysis was necessary in order to take into account the scaling factors introduced by the interface circuits and the tape recorder. Calibration voltages were recorded on the tape for this purpose, but various features of the interface design made their use unsatisfactory and a different method was used to determine FM tape scaling factors (Crabb (1976)). The FM tape was replayed into a UY recording galvonometer and the record so produced compared with available chart roll records. Thus by matching up the records immediately before and after a period of pen recorder failure an average scaling factor for FM tape data was deduced. As is standard practice the heights of the highest two positive and negative maxima were extracted from each record and were corrected for the appropriate instruments frequency response using an average frequency, \( f_{av} \), to describe each record. \( f_{av} \) is defined by the equation

\[
f_{av} = \frac{1}{T_z}
\]

where \( T_z \) is the mean zero upcross period of the record. Both instruments were recalibrated at six-monthly intervals and where calibration revealed significant instrumental drift appropriate corrects were applied retrospectively. Occasionally recalibration indicated a serious instrumental malfunction in which case the corresponding data were discarded.

Plots of Hs from the SBWR against those from the WRB showed consistent discrepancies typically of the order of 10% (Fig 1). A doubt of this size in the wave statistics has significant economic implications for the design of offshore structures. It should be pointed out that an accuracy of better than 10% has never been claimed for the SBWE, which is an inherently imprecise system for hydrodynamic reasons; its compensating virtues are its simplicity of operation and its reliability.

2. CHOICE OF RECORDS FOR COMPARISON

The choice of a suitable set of records for a detailed comparison presented difficulties. Towards the end of the UK00A Weathership programme the SBWR on the last ship in use (the MV Skaggerak) was replaced by a Mk II instrument with a different calibration, and it was thought best to avoid this period. In the early stages of the programme calibration methods for the sensors and recorders were unsatisfactory and records of exactly what happened are very incomplete; in fact, though they were steadily improved, both calibration methods and documentation never did become completely satisfactory. The calibration of the SBWR's often appeared to drift by more than 5% between 6-monthly calibrations, though this may be at least partly due to the inherent lack of precision of the calibration methods used, and it was
therefore desirable to choose a period when calibrations were more frequent than usual. The period should have good records with no known instrument malfunction, and the instruments should still be available for examination.

The best compromise appeared to be the records from the Skaggerak installation for the two periods 26 September to 16 December 1975 and 2 June to 21 August 1976. It was on station Boyle for both the periods. The hardware is still extant and was examined, but of course little confidence can be placed in tests made two years after the data were recorded.

The SEVR was calibrated on 16 September 1975 and was checked during the October crew change. The check revealed a 5% decrease in sensitivity and the accelerometer channels were adjusted accordingly. Recalibration during the December crew change showed a 4% increase in sensitivity. During this period two waverider buoys were deployed, buoy 6500 being used until the October crew change when it was replaced by buoy 6501. Buoy 6500 was first deployed on 20 February 1975. It was not Marex policy at that time to calibrate new buoys and no calibration was carried out. The buoy was due for a six-monthly calibration during August 1975 but this was not carried out at that time and subsequent damage to the buoy between 16 and 17 October made calibration impossible. The history of buoy 6501 is similar and it also was never calibrated by Marex as it was run down during its first six months of use.

For the second period the SEVR was calibrated on 2 June 1976 and tests carried out in mid-November 1976 showed an increase in sensitivity of 2% on the starboard accelerometer channel and 12% on the port accelerometer channel. From 2 June until 21 August 1976 buoy 6679 was used. It was first deployed without calibration during February 1976 and was calibrated at NMI during December 1976 at which time it was 5% less sensitive than specification. Appropriate corrections have been applied to the data recorded from the calibrated buoy and are presented in corrected form in Marex Annual Report No 351. Marex have also undertaken to ensure that this data will be placed on the Marine Information and Advisory Service's data bank in corrected form. This calibration was carried out using an NMI receiver No 1175, while all the measurements were recorded using Marex's own receiver No 1199.
SUMMARY OF CALIBRATIONS RELEVANT TO SELECTED DATA
Period 26 September - 16 December 1975

<table>
<thead>
<tr>
<th>Month</th>
<th>Day</th>
<th>SEWR</th>
<th>WRB</th>
</tr>
</thead>
<tbody>
<tr>
<td>September</td>
<td>16</td>
<td>SEWR calibrated</td>
<td>Buoy 6500 in use</td>
</tr>
<tr>
<td>October</td>
<td>16</td>
<td>SEWR checked, gain 5% low</td>
<td>Buoy 6500 replaced by Buoy 6501</td>
</tr>
<tr>
<td>December</td>
<td>17</td>
<td>SEWR checked, gain 16% high</td>
<td>Buoy 6501 in use</td>
</tr>
</tbody>
</table>

Period 2 June - 24 August 1976

<table>
<thead>
<tr>
<th>Month</th>
<th>Day</th>
<th>SEWR</th>
<th>WRB</th>
</tr>
</thead>
<tbody>
<tr>
<td>June</td>
<td>2</td>
<td>SEWR calibrated</td>
<td>Buoy 6679 in use</td>
</tr>
<tr>
<td>August</td>
<td>24</td>
<td></td>
<td>Buoy 6679 in use</td>
</tr>
<tr>
<td>November</td>
<td>17</td>
<td>SEWR checked, gain 7% high</td>
<td>Buoy 6679 calibrated at NMI</td>
</tr>
<tr>
<td>December</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Port accelerometer +12% starboard arc, +2% average + 7%

3. COMPARISON OF RECORDS
Owing to the random nature of waves, instant-by-instant comparison of wave recorders is only possible if they are effectively in the same place. Wave recorders with an appreciable separation can only be compared statistically; that is, the statistical parameters of the sea-state are estimated from each record and these parameters are compared. The estimates are subject to sampling errors which depend on the length of the record, the spectral composition of the wave pattern, and the method of estimation. In the present case, the rms sampling errors are of the order of 10% for Hs and of the order of 2% to 3% for Tz. However, when the results for many records are treated statistically, these random errors can be averaged out and the relationship between the two instrument responses can be established to a much higher degree of accuracy.

The data recorded during the chosen periods show discrepancies of the kind
reported by Marex. An example is shown in Figure 1 where data recorded
during December 1975 are plotted. During this period the SEWR calibration
drifted by 1%. As there is no evidence that such drifts are linear with
time no correction has been applied for this. The mean gradient of
1.15 indicates that the SEWR gives a higher estimate of Hs than the WEB by
1%. Applying the 1% SEWR correction to all of this data would reduce this
figure to 11% which is typical of the Marex data. Attempts to correlate
the ratio

\[
\frac{H_s}{SEWR} \quad \frac{H_s}{WEB}
\]

with Hs and Tz were not statistically significant and it is concluded that
the discrepancy between SEWR and WEB is not sensitive to the sea state
being measured.

4. INVESTIGATION OF POSSIBLE SOURCES OF THE DISCREPANCY

4.1 Errors in measurement and processing of the data
Checks have been made which eliminate computational errors as a possible
source of the discrepancy. A 5% spot check procedure used by IOS when they
were responsible for quality control of this data indicates that uncertainties
involved in estimating wave heights were ± 1% and those involved in estimating
Tz were ± 2%. There was no obvious bias involved in these estimates so their
net effect averaged over a number of records should be negligible. Inspection
of the original chart rolls reveals peculiar features on some of the SEWR
records which may have been caused by fluctuating chart speed or an insecure
pen arm. These features are not obvious in the majority of the records
but are apparent during a period when, on average, the SEWR recorded shorter
zero crossing periods than the WEB. This latter behaviour is most
uncharacteristic of the two instruments and it is therefore felt that data
recorded from 1 - 5 December 1975 should be regarded with suspicion.
However, a much larger body of data shows the discrepancies under
investigation.

4.2 Calibration procedures
Both instruments' accelerometers are calibrated in a similar way. The
accelerometer is mounted on an arm of known radius which can be rotated in the
vertical plane at constant angular speed. The accelerometers are mounted
on gimbals so that they measure acceleration in the direction of the apparent
vertical (i.e. in the direction of the resultant acceleration). It is
assumed that the acceleration in this direction is sufficiently close to the
vertical acceleration that the doubly integrated output of the accelerometer will be sinusoidal with amplitude $2r$ where $r$ is the distance from the axis of rotation to the accelerometer. It can be shown that this approximation is accurate to better than 1% under the conditions in which both instruments are calibrated. There is therefore no reason to expect systematic errors from this technique.

In the case of the SEWR, calibration is carried out using a standard calibration set designed for the purpose. It incorporates an arm of 50cm radius driven by a synchronous motor to calibrate the accelerometer and a mercury manometer for calibration of the pressure sensors. Throughout the UKOOA program SEWR calibrations were carried out by IOS personnel. The integral nature of the SEWR combined with the calibration procedure used, ensures that all component parts of the wave recording system from transducer to chart recorder are included in the calibration.

The waverider system comprises of two distinct parts: the buoy itself and the Waver receiver and chart recorder. The buoys used by Marex during the UKOOA program were calibrated at the National Maritime Institute (NMI). It was their practice to use NMI's receiver for this purpose. Were all receivers equally sensitive this would not matter. There is little reason to believe that this is the case. Mr J D Humphery of IOS Taunton has reported to me that on one occasion at least, he has found that a 'standard' NMI receiver was 5% less sensitive than an IOS receiver which was being used to calibrate its associated buoy. Laboratory tests carried out later at IOS showed that this receiver was 4% less sensitive than specification. Thus with a buoy calibrated against the NMI receiver and used with a receiver which was adjusted to its specified sensitivity, the wave heights would be overestimated by 9%. Following this work NMI have carried out tests on their 'standard' receiver and report that they found its sensitivity to be that specified by the manufacturers within 1%. The lack of agreement is larger than the errors expected in measuring the sensitivity of a receiver and the matter remains unresolved.

4.3 Interfacing to the FM tape recorder
Most analyses were made from the chart records and the calibration methods described above apply to these. As explained in the introduction, owing to doubts in the transfer of the calibration to the FM tape recorder, the records from this were calibration by direct comparison with simultaneous chart records.

All of the data investigated in this report was derived directly from chart records, and it is therefore evident that the fidelity of the FM recording
system is not relevant to the present investigation. However, the interface circuits were connected to the recording instruments at all times and it is important to ascertain their effect, if any, upon the performance of the wave recorders. For this reason the interface arrangements between the wave recorders and the FM tape recorder will be described.

The interface electronics between the wave recorders and the FM tape recorder were designed by Marex. The signal from the Warep receiver for the waverider system was derived from an analogue output which is a standard part of the Warep receiver unit. The SEWR signal on the other hand was derived directly from the recording galvanometer which was modified for this purpose. This was necessary as an analogue output is not a standard feature of the Mark I SEWR.

Three interface units were built, one for each of the weatherships used in the UKOAA program. It is clear that modifications were subsequently made. The documentation of the interface units was minimal and there are no records as to what these modifications were, why they were necessary, nor when they were made. The engineers responsible for the interface unit have now left Marex and the interface unit's history is irretrievable. The information presented below has been reconstructed from undated engineers' sketches and inspections of the interface unit after it was removed from MV Skaggerak on 20 June 1977. It should be borne in mind that it is not known for how long prior to 20 June 1977 the interface was in its present form.

The relevant parts of the interface circuit diagram are shown in Figures 2 and 3.

The waverider interface consists of a potential divider and voltage follower with unit gain. This circuit presents a load of approximately 800k to the Warep receiver. The analogue output of the Warep is in a feedback loop and loading the output has the surprising effect of increasing the amplitude of the chart recorder deflections. With a load of 800k the effect is small and a simple calculation shows that the chart recorder amplitude would be increased by only 0.7%.

The SEWR interface is more complicated. The reason for this is that the valve electronics of the SEWR provide a differential output which is some 80V above ground potential. The SEWR interface provides a load of approximately 1M across the output of the SEWR which has an impedance from
cathode followers of less than 1K. Again, the error introduced by the interface is negligible.

It is interesting to note that if at any time the input impedance of either interface circuit were not large enough compared with the recorder's output impedances this would result in an increase in Waverider heights compared with those recorded by the SBWR. This is the opposite of the effect under investigation.

4.4 Corrections for the frequency responses of the sensors

(a) The Waverider Buoy
This measures vertical acceleration using an inertia stabilised platform. The output is integrated twice to give displacement. The electronics filter out very low frequencies and give an overall response of:

$$|A(f)| = \left| \left(1 + \frac{\rho}{30.8 f} \right) \left(1 + \frac{\eta}{4.60 f} \right) \right|^{-1/2}$$

where $\rho = \frac{1}{30.8 f}$ and $\eta = \frac{1}{4.60 f}$. The response of the radio receiver combined with the graphic recorder is assumed to be uniform over the frequency range of interest. $|A(f)|$ is plotted in Figure 8.

In principle, when correcting a wave record for this response, a spectral technique should be employed so that the measured spectral density at each frequency $f$ is multiplied by $1/|A(f)|^2$. In practice the amplitude of a record is corrected by using an average measure of the record's frequency content, $f_{av}$ where $f_{av}$ is defined by the equation

$$f_{av} = 1/T_z$$

where $T_z$ is the mean zero upcross period.

Rice (1974) has shown that $T_z = \sqrt{M_0/M_2}$ where $M_0$ and $M_2$ are respectively the zero and second moments of the wave spectrum. The height of the waves extracted from each record are scaled by the factor $1/|A(f_{av})|$ when such parameters as $H_s$ are calculated.

In order to test the validity of this procedure a numerical calculation has been performed in which a Pierson Moskowitz spectrum was multiplied by $|A(f)|^2$. This modified spectrum simulates the spectrum...
recorded by a Waverider when the sea state is adequately described by a Pierson Moskowitz spectrum. A numerical integration procedure was used to evaluate the zero and second-order moments of the modified spectrum, and hence a value of $T_z$ as would have been recorded was deduced. The effect of applying the correction $1/\tilde{A}(f)A(f)\tilde{A}(f)\tilde{A}(f)|^2$ to the modified spectrum's zero moment was then computed. The square root of the zero order moment of the spectrum is a proportional measure of $H_s$. It was therefore possible to compare the value of $H_s$ describing the assumed sea state with the estimate of $H_s$ which would be derived by non-spectral analysis of the chart records. For all reasonable values of the parameters describing the original spectrum, the agreement was very good; the error due to applying an average frequency correction being less than $1\%$ for values of $T_z$ varying from 4 to 14 seconds. This is not surprising as $\tilde{A}(f)$ is close to unity for a wide range of frequencies.

Thus, the procedure for correcting for the frequency response of WRB is unlikely to contribute a significant error.

(b) The Shipborne Wave Recorder

Here the situation is more complex. The vertical accelerometers are mounted on short period pendulums, which introduces spurious low frequency signals which must be filtered out (Tucker 1959). The electronic arrangements to do this introduce considerable attenuation in the wave-frequency band (Fig 5). There is also considerable hydrodynamic attenuation of the high frequency waves due to the pressure measuring sensors having to be mounted well below the waterline (this is shown in Fig 6). The way in which these attenuations should be combined is open to some doubt, but there are plausible arguments indicating that multiplying the attenuation should give approximately the correct answer.

The transfer function used for the Mk 1 SEWR is

$$|\tilde{A}(f)| = \left| \frac{1 + (6.8 \sigma)^2}{\sigma^2} \right|^{-3/2} e^{-2.5 \sigma^2 d/g} \quad \text{[8]}$$

where $\sigma = 2\pi f$, $d =$ depth of sensor, $g =$ acceleration due to gravity

This function does not approach close to unity even at the peak response (Fig 7) and to allow a quick appreciation of the sea-state from the chart records, a nominal calibration factor is used so that the chart shows
approximately the correct wave height over the typical range of wave periods. However, when the charts undergo systematic routine analysis, this factor is taken out again and Hs corrected by the calculated factor corresponding to the measured Tz. As in the case of the WBR, we must ask whether this process introduces significant errors compared with a spectral correction process, and in the case of the SBWR we must also ask whether there is significant doubt about the accuracy of the calculated response function.

Looking at the first of these questions a similar analysis to that carried out with the Waverider transfer function using the SBWR transfer function shows that the heights of the waves are underestimated by the SBWR as shown in the table below. A value of 1.88m was assumed for the depth of the pressure sensors.

<table>
<thead>
<tr>
<th>True zero crossing period (seconds)</th>
<th>Measured zero crossing period (seconds)</th>
<th>Error in Hs(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>5.4</td>
<td>14.6</td>
</tr>
<tr>
<td>5.0</td>
<td>6.4</td>
<td>8.8</td>
</tr>
<tr>
<td>6.0</td>
<td>7.4</td>
<td>5.8</td>
</tr>
<tr>
<td>7.0</td>
<td>8.4</td>
<td>5.2</td>
</tr>
<tr>
<td>8.0</td>
<td>9.4</td>
<td>3.4</td>
</tr>
<tr>
<td>9.0</td>
<td>10.3</td>
<td>3.0</td>
</tr>
<tr>
<td>10.0</td>
<td>11.2</td>
<td>3.0</td>
</tr>
<tr>
<td>11.0</td>
<td>12.1</td>
<td>3.2</td>
</tr>
<tr>
<td>12.0</td>
<td>13.0</td>
<td>3.2</td>
</tr>
<tr>
<td>13.0</td>
<td>13.9</td>
<td>3.7</td>
</tr>
<tr>
<td>14.0</td>
<td>14.8</td>
<td>4.2</td>
</tr>
</tbody>
</table>

The reason that these errors arise is clear: the poor high frequency response of the instrument results in an underestimate of the high frequency content of the record. Thus an insufficiently large correction factor is used and the heights of the waves are underestimated. Similar reasoning explains the increase in the errors at low frequency. The large errors at short zero crossing periods might be expected to depend on the shape of the high frequency end of the wave spectrum. This is indeed the case and further calculations, using real wave spectra in the place of the theoretical Pierson-Moskowitz spectrum shows that with real spectra the correction technique is even less accurate particularly when the sea state has a flat or bimodal spectrum. The results for L7 spectra, recorded at South Uist, together with the results derived from a Pierson-Moskowitz spectrum, are plotted in Fig. 4.

These errors are significant, but are too small and of the wrong sense to account for the observed errors.
The accuracy of the calculated response function will now be considered.

The 8BWE combines two signals (pressure and double integrated acceleration) in order to reconstruct a signal proportional to the elevation of the sea surface above the mean. The analysis of its transfer function therefore breaks down into three separate parts; the analysis of the transfer functions of the pressure and accelerometer channels and analysis of the way in which the individual channel responses should be combined.

The accelerometer channel signal is straightforward; the frequency response being equivalent to that of three RC filters. The combined response is given by

\[ |A(f)| = \sum_{i=1}^{3} \frac{1}{s + \frac{2}{\tau_i}} \quad (4) \]

This expression has been experimentally verified by Draper et al, in an experiment in which the large amplitude linearity of the acc channel was also verified by mounting the accelerometers on a fairground wheel of 14m diameter.

The pressure channel signal is a measure of the elevation of the sea surface relative to the ship, signals from sensors on both sides of the vessel being averaged to allow for the surface elevation varying from one side of the hull to the other. This technique also compensates for the reflection of waves by the ship's hull. The pressure recorded at depth \( d \) below the sea surface as a wave passes over a pressure gauge is not the hydrostatic pressure \( \rho g \Delta H \) because of the dynamical nature of waves. In open water the pressure disturbance due to a surface wave decays exponentially with depth \( d \), the pressure disturbance being given by the equation

\[ p = p_0 e^{-\frac{2\pi d}{\lambda}} \quad (5) \]

where \( p_0 \) is the hydrostatic pressure disturbance which would be expected from a hydrostatic change in surface elevation equal to the wave elevation.
In terms of wave frequency this equation can be rewritten
\[ \rho = \rho_0 e^{-\sigma^2 c^2/\gamma} \]  \hspace{1cm} (c)
where \( \sigma = 2\pi f \)

The SBWR pressure transducers however measure the pressure variations on
the ship's hull and the above equation no longer applies as the ship's
hull produces a large perturbation on the motion of the water in its
vicinity. A realistic calculation of the pressure variation on the ship's
hull is extremely complicated. Korvin - Kroukovsky has shown that
for a circular hull the attenuation is the square of the factor given by
equation (c)
That is:
\[ \rho = \rho_0 e^{-2\sigma^2 c^2/\gamma} \]

Attempts have been made to approximate the pressure distribution for real
hulls using the expression
\[ \rho = \rho_0 e^{-k \sigma^2 c^2/\gamma} \]
where \( k \) is an adjustable parameter determined experimentally.

Cartwright (1963) made comparisons between directional spectra estimated
using a pitch and roll buoy and spectra of encounter measured by a SBWR
mounted on a ship steaming on various courses. His results are summarised
below:

<table>
<thead>
<tr>
<th>Speed Kt</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>2.61</td>
</tr>
<tr>
<td>10</td>
<td>2.42</td>
</tr>
<tr>
<td>7</td>
<td>2.60</td>
</tr>
<tr>
<td>0</td>
<td>2.27</td>
</tr>
</tbody>
</table>

The average value of \( k \) is 2.48 which is very close to the value 2.5 used
in the analysis of the UKOOA data.

Derbyshire (1961) reports comparisons of spectra recorded on Discovery, a
light ship equipped with SBWR, and an accelerometer buoy. As in the
previous experiment the buoy was allowed to drift freely during the
measurements and the combined results from both experiments give a measure
of the variability of the factor \( k \) from ship to ship.

<table>
<thead>
<tr>
<th>Ship</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>OWS</td>
<td>2.4</td>
</tr>
<tr>
<td>Discovery</td>
<td>3.0</td>
</tr>
<tr>
<td>Lightship</td>
<td>2.25</td>
</tr>
</tbody>
</table>
From the above measurements it would seem likely that k is no less than 2.24. This would lead to a frequency dependent error given by

\[
\frac{\delta P}{P} = \frac{2}{\delta k} \delta k \times \frac{1}{P} = \sigma^2 \frac{d}{\Omega} \delta k \quad \text{where} \quad \delta k = 0.25
\]

<table>
<thead>
<tr>
<th>Period</th>
<th>(\frac{\delta P}{P})</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>7.5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5.2</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3.8</td>
<td></td>
</tr>
</tbody>
</table>

While these errors are of the correct sense to explain the effect reported by Marex their effect when averaged over frequency is not large enough to explain the results obtained during the UKOA programme. Values of \(k > 2.5\) would result in higher estimates of wave heights.

The approximation used for the whole instrument's transfer function is of the form

\[
\delta(\sigma\tau) = P(\sigma\tau) A(\sigma\tau)
\]

where \(P(\sigma\tau)\) and \(A(\sigma\tau)\) are the transfer functions of the pressure and accelerometer channels. In the limits \(\sigma \to \infty\) and \(\sigma \to 0\) this is obviously accurate, since \(P(\sigma) \to 0\) as \(\sigma \to \infty\) and \(A(\sigma) \to 0\) as \(\sigma \to 0\). For intermediate frequencies however the contributions of each channel to the total signal will depend on the ship's heave response, \(B(\sigma)\). Calculations due to Pitt et al using wave tank data to deduce \(B(\sigma)\) show that the ship's response results in a significant correction for frequencies greater than 0.125 Hz. However these calculations were made for a drilling ship of 20,000 Ton displacement operating in head seas. The ships used in the UKOA programme are much smaller than this and are usually operated broadside on to the waves. It is therefore not possible to use these results in order to predict the effect of the ship's response in the case of a weather ship.

5. CONCLUSION

It is clear that measurements made during the UKOA weathership programme show discrepancies between wave heights recorded by SEWR and WRB. These discrepancies are 10% with the SEWR giving larger values for the wave heights than the WRB. No obvious reason for the discrepancies has come to light during the analysis of the data and measurement techniques described in this report. Such other comparisons between accelerometer buoys and SEWR's as have been reported do not indicate that the SEWR's give larger estimates.
than buoys (Cartwright (1963), Derbyshire (1961), van Aken et al (1974)). These other measurements however are not directly comparable. The size and sea-keeping characteristics of the vessel on which the SBWR is installed may be important, and all other comparisons have used freely-floating buoys attached to a ship by a slack, floating line. It is possible that the restraint upon the WRB’s motion provided by a mooring may alter the buoy’s ability to follow the sea surface faithfully. It should also be noted that the comparison described by van Aken (1974) was carried out using a SBWR which was calibrated in an unconventional manner. As the weather ship program has now been completed further investigations are not possible in order to resolve this problem. There remains some doubt as to the calibrations of the Waverider Buoys used in the UKOA programme.

In order to clarify the situation it is desirable that a carefully designed experiment be performed. Such an experiment should investigate the effect, if any, of buoy moorings and should if practicable investigate the effect of the ship’s orientation with respect to the predominant wave direction. An investigation of the stability of Waverider receivers and of the way in which their sensitivity is determined should also be undertaken in order to reduce the possible errors involved with Waverider buoy calibrations.

REFERENCES


Crabb, J.A. (1976). Appraisal of the Marex FM Data recording system, as used at Shell Brent B oil production platform. *10S Internal Document No. 5.*


FIGURE 1
Comparison of wave heights recorded by SBWR and WRB, the solid line has a slope of unity
**Fig 2**

The Datawell Interface

![Circuit Diagram for Fig 2]

**Fig 3**

The S.B.W.R. Interface

![Circuit Diagram for Fig 3]
FIGURE 4

Percentage by which non-spectral analysis of data might be expected to underestimate wave height v.s. Tz measured by SBWR. The solid line was calculated using a Pierson Moskowitz spectrum to describe the sea state. The points were calculated using a set of measured sea spectra to describe the sea state.
Figs 5 and 6 show the frequency responses of the SBWR accelerometer and pressure channels respectively.
Figs 7 and 8 show the frequency responses of the SBWR and WRB wave recording systems.