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INTERNAL DOCUMENT 144.

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MEASUREMENTS OF THE DIRECTIONAL  
WAVE SPECTRUM OFF SOUTH UIST

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Internal Document No. 144

Prepared for the Wave Energy Steering Committee  
October 1981

## Measurements of the directional wave spectrum off South Uist

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### Introduction

The purpose of the work was to measure the directional wave spectrum and wave power at a number of stations off South Uist using a cloverleaf wave buoy. Previous measurements made by IOS of the frequency spectrum of the waves using two waverider buoys indicated that a considerable reduction in wave power occurred as the coast was approached. The mechanism for this reduction in wave power was thought to be through one or more of the following mechanisms: wave refraction, bottom dissipation or possible backscatter of the incident wave energy due to the rocky bottom off the Hebrides. Measurements of the directional spectrum at a number of positions offshore could possibly identify the mechanism responsible for most of the energy loss.

### Measurements

#### (a) Bottom topography

Measurements of the bottom topography were made with a Precision Echo Sounder with the ship steaming at about 4 knots along the track ABCD shown in Fig. 1. The lines AB and CD lie parallel and about 3 miles distant from the main line of wave measurements. (IOS, Taunton already have made depth measurements along the line of buoys). Wave conditions were slight during these measurements so that there was very little contamination of the bottom depth measurements due to ship motions. The following table gives details of the times and positions of these measurements.

Table 1

Station	Time (GMT)	Day(September) 1980	Position
A	2003	20	57°15.'9N, 7°30.'0W
B	2308	20	57°14.'8N, 7°54.'0W
C	0020	21	57°20.'6N, 7°54.'8W
D	0303	21	57°21.'8N, 7°31.'0W

#### (b) Wave measurements

The cloverleaf wave buoy was deployed from RRS Challenger at the positions shown in Fig. 1 and Table 2. (One measurement, C1, was made to test the equipment and procedure for deploying the buoy). The wave measurements were of 35 min. duration, care being taken to avoid situations where the ship shielded the buoy from the waves or the possibility of wave reflection off the ship's side.

The wave conditions consisted of swell waves with visually observed height of about 2m, wave period of 13 sec. propagating in an easterly direction. Local winds were less than 10 knots from the west.

Table 2

Station	Time (GMT)	Day (September) 1980	Position
C1	0932	22	57°46'N, 9°12'W
C2	0635	23	W
C3	1230	23	WR(O)
C4	1404	23	U
C5	1537	23	WR(M)
C6	1710	23	V
C7	1908	23	WR(I)
C8	0337	24	WR(I)
C9	0520	24	V
C10	0719	24	X

During Record C2 a component failed in one of the curvature channels. The other two curvature channels cannot be analysed into useful information on their own. It was not possible to repair the fault in the time available, so it was decided to continue the trial using the buoy in effect as a pitch-roll-heave acceleration system and the standard method of analysis as described by Longuet-Higgins, Cartwright and Smith (1963). This is not a serious limitation for the narrow-band swell encountered during this experiment since a pitch-roll system is able to resolve a narrow directional distribution adequately (Hasselmann, Dunkel and Ewing, 1981).

Record C10 was suspect due to low voltage in the internal battery supply of the buoy. The results in this report therefore only discuss records C2-C9.

Transmissions from the waveriders were recorded with equipment on the ship simultaneously with cloverleaf buoy measurements at waverider stations WR(O), WR(M) and WR(I). When the buoy was deployed at other stations the signal from the nearest waverider was usually monitored to give information on the spatial variability in wave energy. However, all the waverider data reported in this paper came from analyses of the records taken on land.

The following table sets out details of the recordings.

Table 3

Waverider Record	Time (GMT)	Day (September) 1980	Waverider buoy (WR)
1	1004	21	0
2	1227	21	M
3	1451	21	I
4	0620	23	O
5	1217	23	O
6	1349	23	O
7	1432	23	M
8	1539	23	M
9	1600	23	M
10	1849	23	I
11	1943	23	M
12	0320	24	I
13	0449	24	I
14	0700	24	M

Finally, the routine recording of waverider signals at the land station on South Uist was enhanced by providing measurements every 1½ hours from all three waverider stations. Each of these records was of 35 min. duration.

Analysis

(a) Bottom topography

The bottom profile measurements were passed to IOS, Taunton to be incorporated into their series of measurements at this site.

(b) Wave measurements

Cross-spectral analysis of the vertical acceleration and two components of wave slope (derived from the measurements of pitch and roll after correction to North-East axes using the compass signal) were made using the Fast Fourier Transform (FFT). Each signal of length 2000s. was digitized at 0.5s. interval and transformed using the FFT. The resulting cross-spectra were then averaged over 20 successive harmonics to produce smoothed cross-spectra defined at 0.01 Hz interval with 40 degrees of freedom.

The wave height spectrum was obtained by dividing the acceleration spectrum by (frequency)<sup>4</sup> before smoothing over successive harmonics.

Directional parameters,  $\theta_1$  and  $\theta_2$ , were obtained using standard methods of analysis (Cartwright, 1963).  $\theta_1$  is the mean wave direction towards which the waves are propagating;  $\theta_2$  is a measure of the spread of energy about the mean wave direction and is equal to the r.m.s. spread for a narrow directional distribution.

Waverider estimates of the frequency spectrum were obtained using standard analysis techniques developed at IOS, Taunton (Fortnum, Humphery and Pitt, 1979).

## Results

### (a) Intercomparison of waverider and cloverleaf wave buoy

Comparisons of significant wave height  $h$  and wave power  $P$  from the cloverleaf buoy (C/B) and waverider (W/R) measurements are shown in Table 4.

Table 4

Station	Recorder	Time (GMT)/Day Sept. 1980	$h$ (m)	$P$ (KW/m)
WR(O)	W/R	1200/23	2.9	55
	W/R	1330/23	3.4	71
	C/B(C3)	1230/23	2.7	50
WR(M)	W/R	1500/23	2.7	48
	W/R	1630/23	2.9	52
	C/B(C5)	1537/23	2.8	54
WR(I)	W/R	1800/23	3.1	62
	W/R	1930/23	2.8	51
	C/B(C7)	1908/23	2.9	57
WR(I)	W/R	0300/24	2.7	42
	W/R	0430/24	2.6	40
	C/B(C8)	0337/24	2.6	42

The agreement between estimates of significant wave height from the two measuring systems is within the random sampling error. For example, it can be shown (Tucker, 1957) that the proportional standard error in estimates of the variance for record C3 and the corresponding outer station WR(O) are 0.12 and 0.10 respectively. The proportional standard error of the ratio of the variances of these two records is therefore

0.16, giving the proportional standard error in the ratio of significant wave heights as about 8%.

Fig. 2 compares estimates of wave power from the three waveriders with the cloverleaf buoy measurements. The wave power levels increased rapidly during the period 0600 to 1300 hrs on 23 September. Although the power levels at the two inner stations remain about the same during the main period of the measurement, there is some indication of higher power levels at the outer waverider station.

The reason for the attenuation of power from the outer to the middle station cannot be explained within the limited measurement period of this study. But, in view of the water depths at these stations, it seems unlikely that most of this attenuation can be due to refraction. (See Table 6 of next section). Routine measurements of the directional wave spectrum would help in understanding the processes governing changes in wave power at this site.

(b) Cloverleaf buoy measurements

The main set of results from the measurements are given in Table 5. The significant wave height,  $h$ , and wave power,  $P$ , were computed by integration over the frequency range 0.045 - 0.325 Hz. (The depth dependence of the group velocity was used in computations of  $P$ ). Values of the mean wave direction,  $\theta_1$ , are given at three frequencies near the spectral peak;  $\theta_2$  is given as the minimum value of the directional spread at these three frequencies.  $\theta_1$  is in the direction of wave propagation.

Fig. 3 and 4 show the wave height spectrum for records C2 - C7 (i.e. from 0635 hrs - 1900 hrs on 23 September). The last two records, C8 and C9, which were separated from the earlier measurements by over 8 hrs., show a slow change in the wave conditions towards the end of the experiment.

Table 5

Record no.	Station	Water depth (m)	Time (GMT) Sept.	Day 1980	h (m)	P KW/m	$\theta_1$ (deg)			$\theta_2$ (deg)
							.065	.075 (Hz)	.085	
C2	W	127	0635	23	2.0	25	84	71	65	19
C3	WR(O)	98	1230	23	2.7	50	78	74	75	22
C4	U	75	1404	23	2.8	54	75	70	76	22
C5	WR(M)	51	1537	23	2.8	54	70	75	72	19
C6	V	31	1710	23	2.8	56	90	75	80	29
C7	WR(I)	24	1908	23	2.9	57	92	89	79	19
C8	WR(I)	24	0337	24	2.6	42	98	88	86	23
C9	V	31	0520	24	2.6	43	108	94	95	26

## Discussion of results

### (a) Influence of wave refraction and bottom dissipation

Table 5 and Fig. 3 show that there is little change in significant wave height and wave power from the outermost station WRO in water of depth 98m to the inner station WRI, where the depth is about 24m.

We can estimate the change in wave height in the case of parallel bottom topography using Plate C-6 of the Shore Protection Manual (Vol. III of U.S. Army Coastal Engineering Research Center, 1975). The following table sets out the changes due to refraction and shoaling for waves of 13s. period as a function of the deep water wave direction: (0 degrees is normal to the bathymetry).

Table 6

Station	Depth (m)	Deep water wave direction		
		0°	10°	20°
WR(O)	98	0.97	0.97	0.97
WR(M)	51	0.92	0.91	0.91
WR(I)	24	0.95	0.94	0.93

This table shows that the effect of wave refraction and shoaling on wave height is less than 9% and is relatively insensitive to the deep water wave direction within  $\pm 20^\circ$  to the normal to the bathymetry.

Wave refraction calculations have been made by the Hydraulics Research Station (P.H. Bellamy: personal communication) assuming characteristics of the offshore directional spectrum to be those for record C5, as given in Table 5. The calculations for the resulting inshore wave spectra, using the best available bathymetry, are compared with the measured wave spectra C6 and C7 in Fig 5. There is reasonable agreement with C7 but there are larger differences from C6. The inclusion of bottom friction decreases the agreement, especially at C7.

We therefore conclude that wave refraction and shoaling are sufficient to explain the minor changes in the form of the wave power when the deep water wave direction is within about  $20^\circ$  to the normal to the bathymetry.

Changes in the mean wave direction,  $\theta_1$ , during the experiment are due to the northerly progression of the storm which generated the swell waves on 21 and 22 September 1980. The centre of the storm travelled about 220n. miles during the 12 hours of the measurements and, being about 450n. miles distant from South Uist, gave an angular change of about  $20^\circ$  in agreement with the change in  $\theta_1$  shown in Table 5.

### (b) The directional distribution

Estimates of the normalized directional distribution,  $G(\theta)$ , formed

from the weighted sum of the five angular harmonics are shown in Fig. 6 for the three records C3, C5 and C7 at a frequency of 0.065Hz. These estimates indicate a small amount of energy travelling in the offshore direction ( $\theta = 270^\circ$ ). This energy increases from about 1% at station WR(I) to about 8% at station WR(O). This is in order of magnitude agreement with theoretical estimates of backscatter due to bottom roughness. (Further studies are in progress to investigate this effect but they are not part of this contract).

### Conclusions

Measurements of the directional wave spectrum of swell approaching South Uist from the west have been made for a limited period of about one day. During the period of the cloverleaf buoy measurements, there was less than 5% change in wave height and about 10% change in wave power as the depth decreased to about 24 m. It appears that, for the period of the measurements, wave refraction and shoaling can account for these changes in the wave characteristics without the need to appeal to additional bottom friction or to backscatter of the incident wave energy.

Estimates of significant wave height from the cloverleaf buoy and the three waverider buoys agree within the expected random sampling error.

There is some indication that wave power levels at the outer waverider station are greater than those at the middle station. The reasons for the attenuation of wave power from the outer to the middle stations cannot be explained within the limited set of measurements made in this study. Routine measurements of the directional wave spectrum would however help in understanding the processes governing the variability in wave power at South Uist.

The results of this work imply that almost all of the energy loss to swell waves approaching South Uist from the west occurs in the nearshore zone where the depth is less than about 24 m.

Wave refraction and shoaling could be important for long waves approaching from the north and south since then the angle of approach to the bathymetry can lead to considerable refraction with large spatial differences in the nearshore wave characteristics.

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### Acknowledgements

This paper was prepared for the Wave Energy Steering Committee and was supported by the Department of Energy.

I thank K. Birch, N. Smith and B. Woodley for refitting the cloverleaf buoy and for their work at sea. The help of R. Gleason, E. Pitt and A. Salkield who organized and analysed the waverider buoy measurements is also acknowledged. The officers and crew of RRS Challenger are thanked for their cooperation.

October, 1981

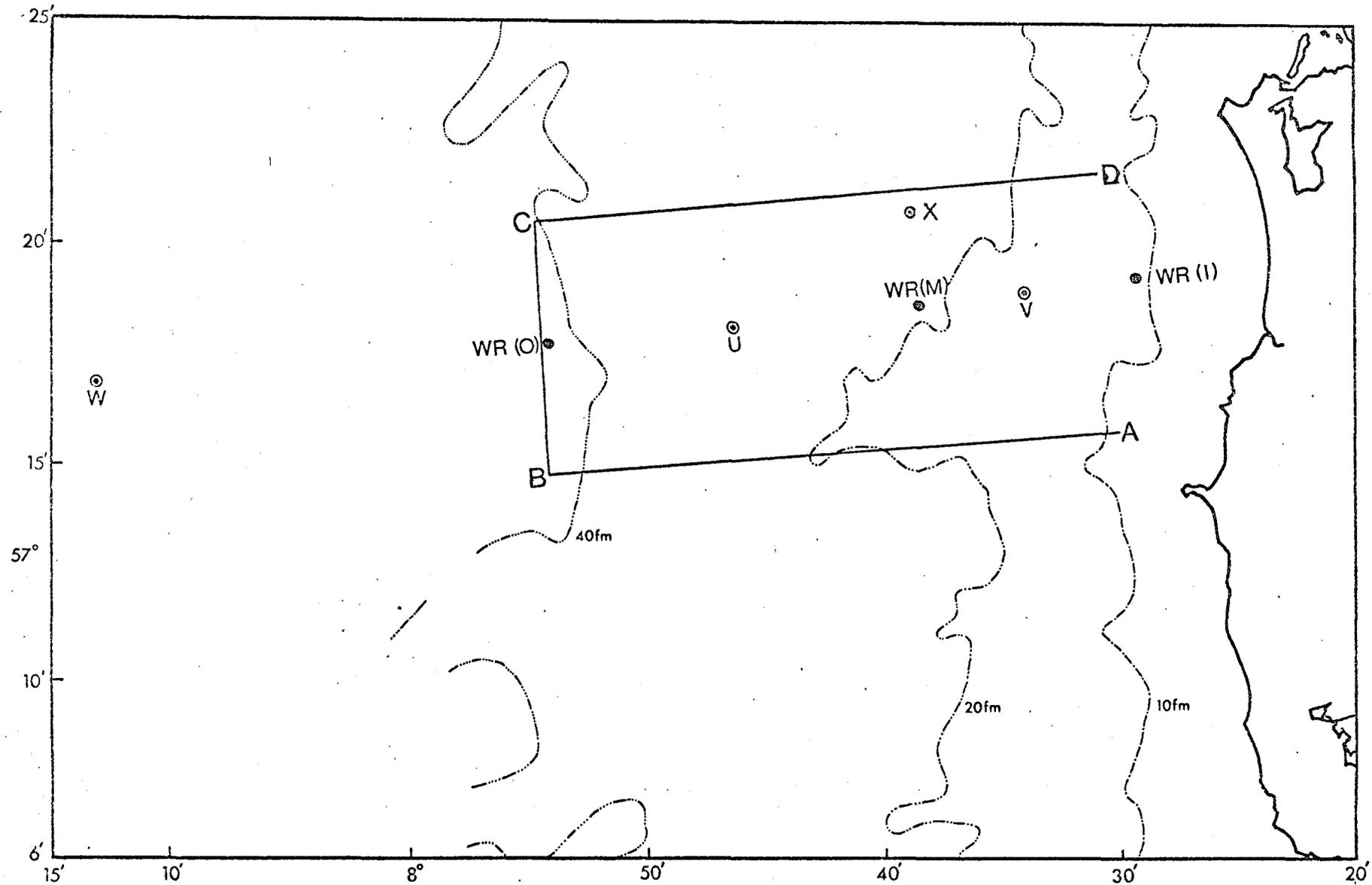


Fig. 1 Map of area west of South Uist showing track of ship A, B, C, D during echo sounder measurements and location of wave buoy stations. Waverider stations are shown with a solid circle.

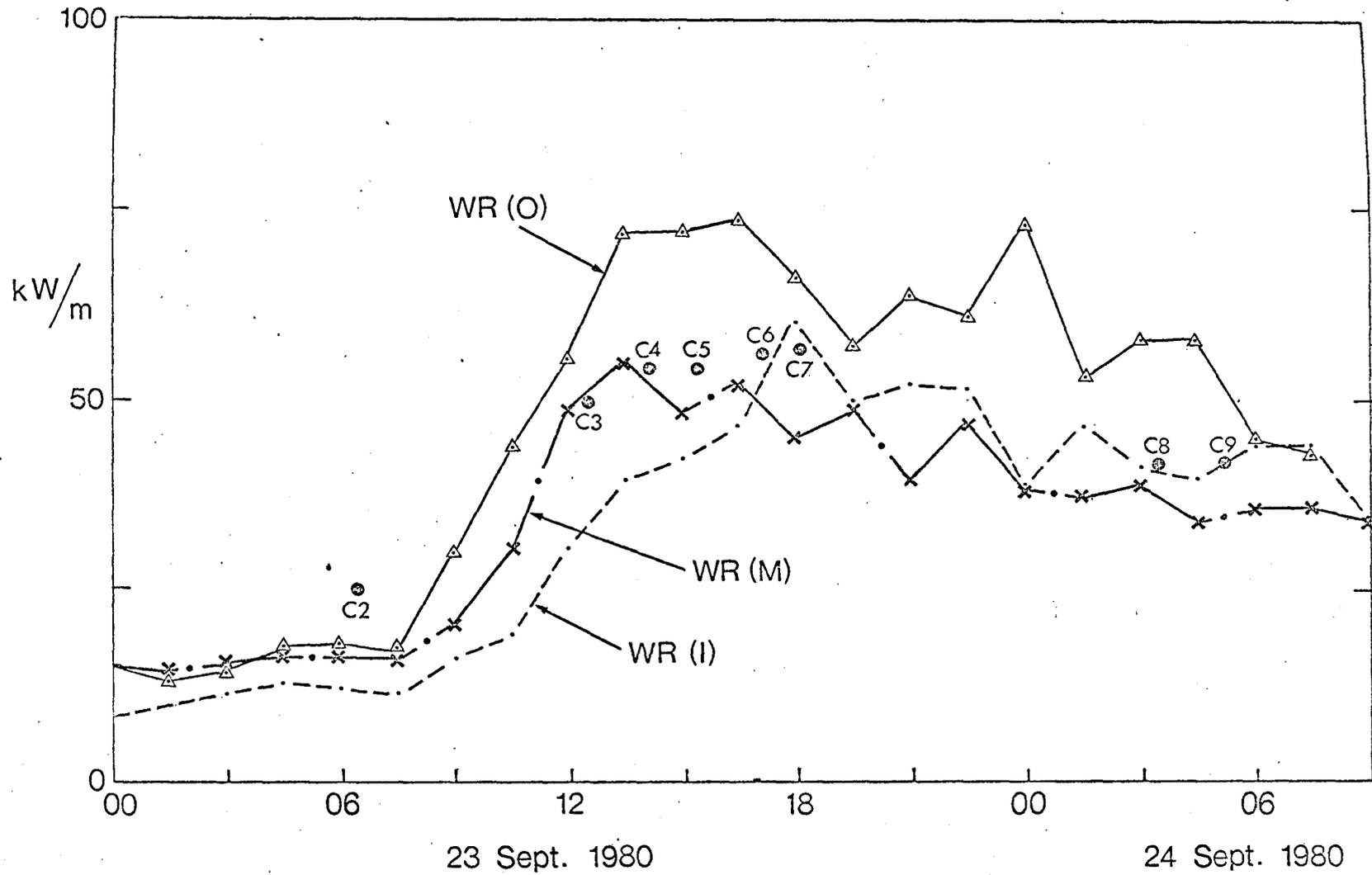


Fig. 2 Time histories of wave power from three waverider buoys compared with estimates from the cloverleaf buoy(●).

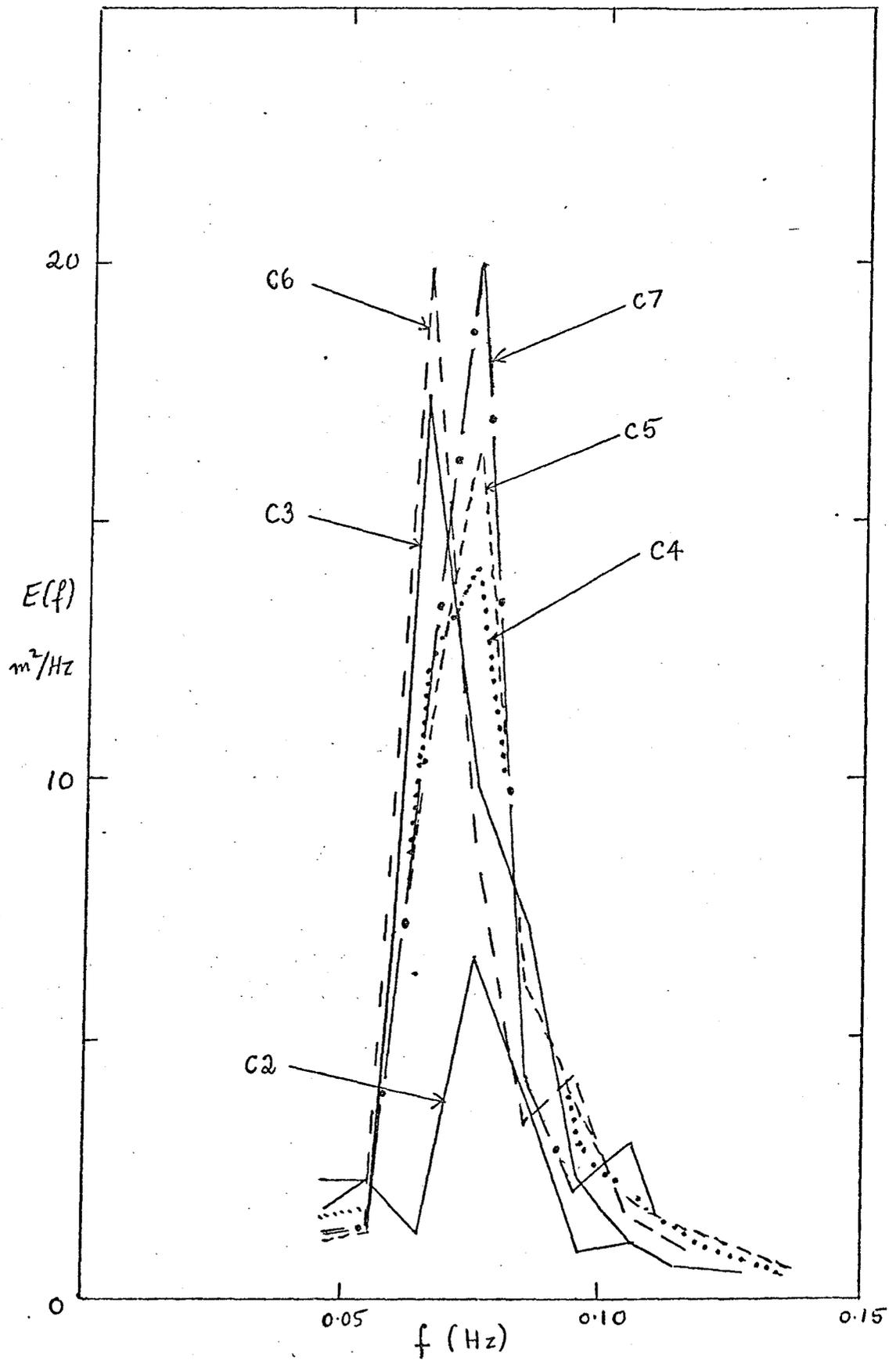


Fig. 3 One dimensional wave spectra measured during cloverleaf buoy measurements C2-C7.

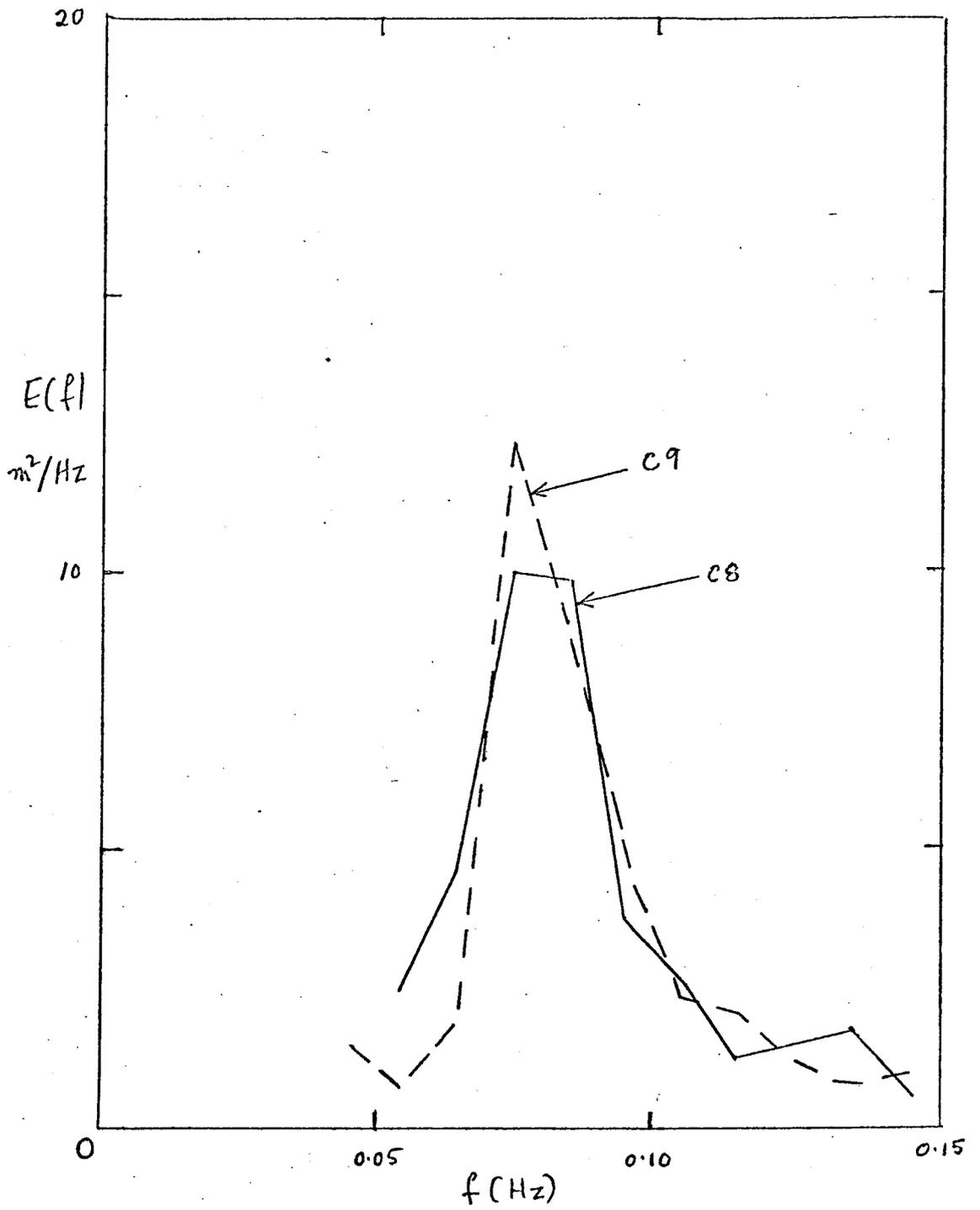
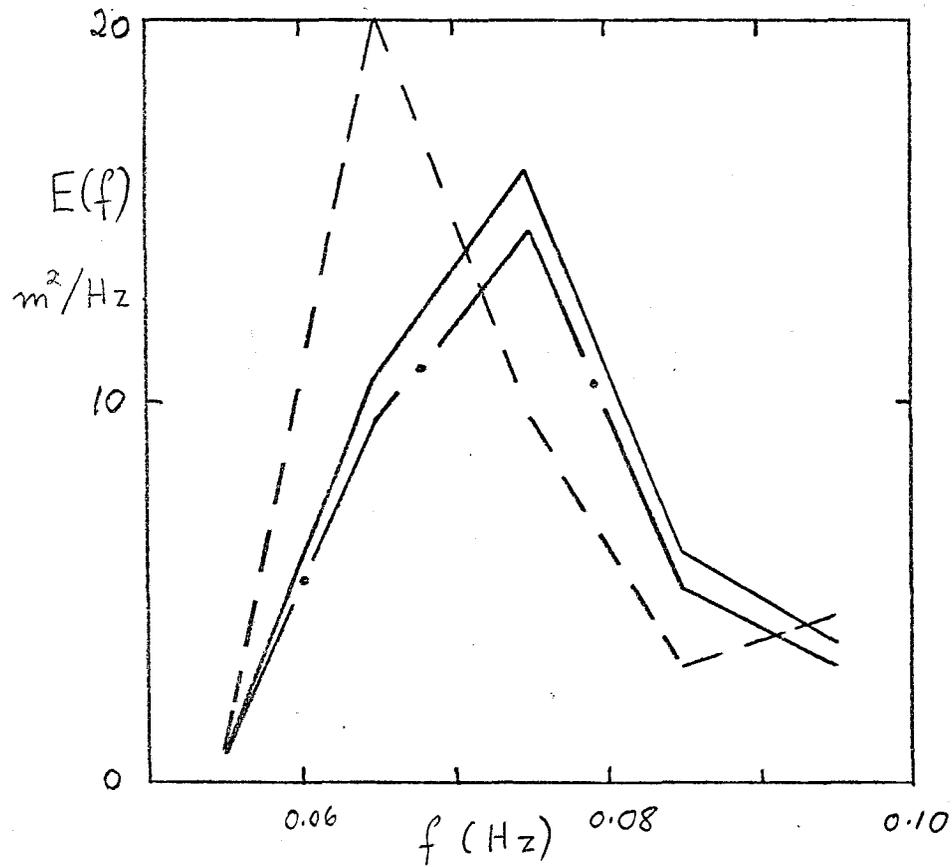


Fig. 4 One dimensional spectra measured during cloverleaf buoy measurements C8 and C9.

C6 at station V



C7 at station WR(I)

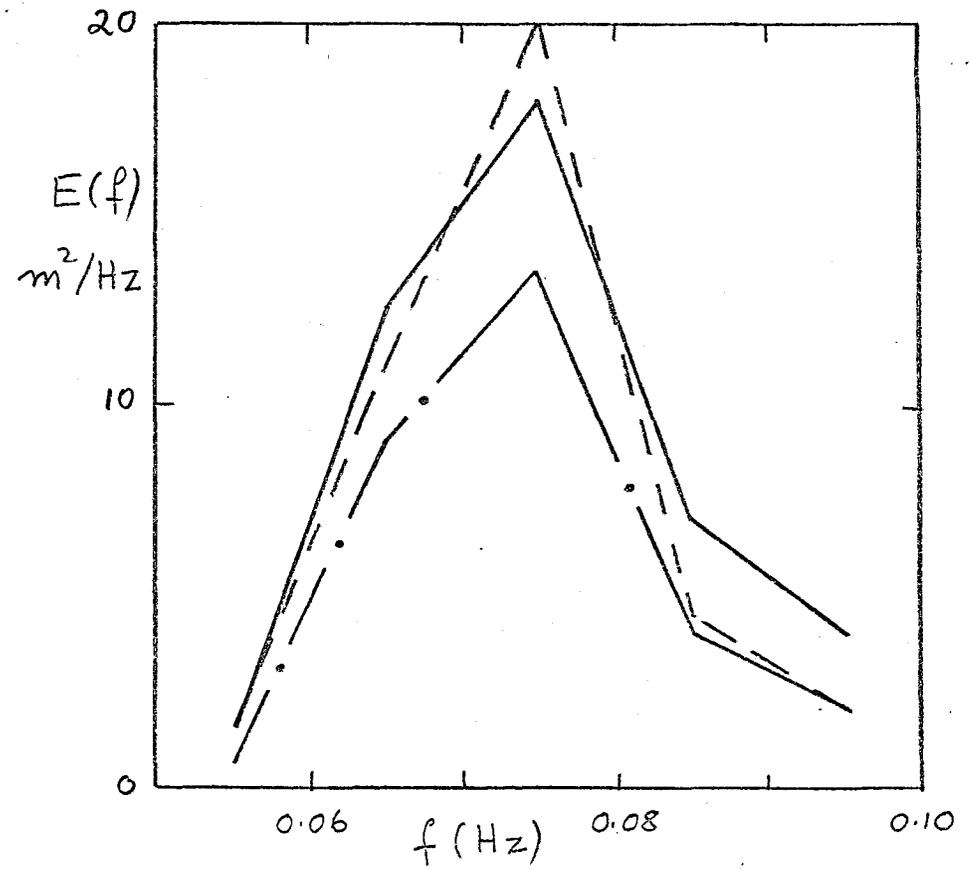


Fig. 5 Comparison of measured and predicted inshore wave spectra (Refraction calculations were made by HRS).

- Measured spectrum
- Predicted spectrum (no friction)
- • - Predicted spectrum (friction factor = 0.08).

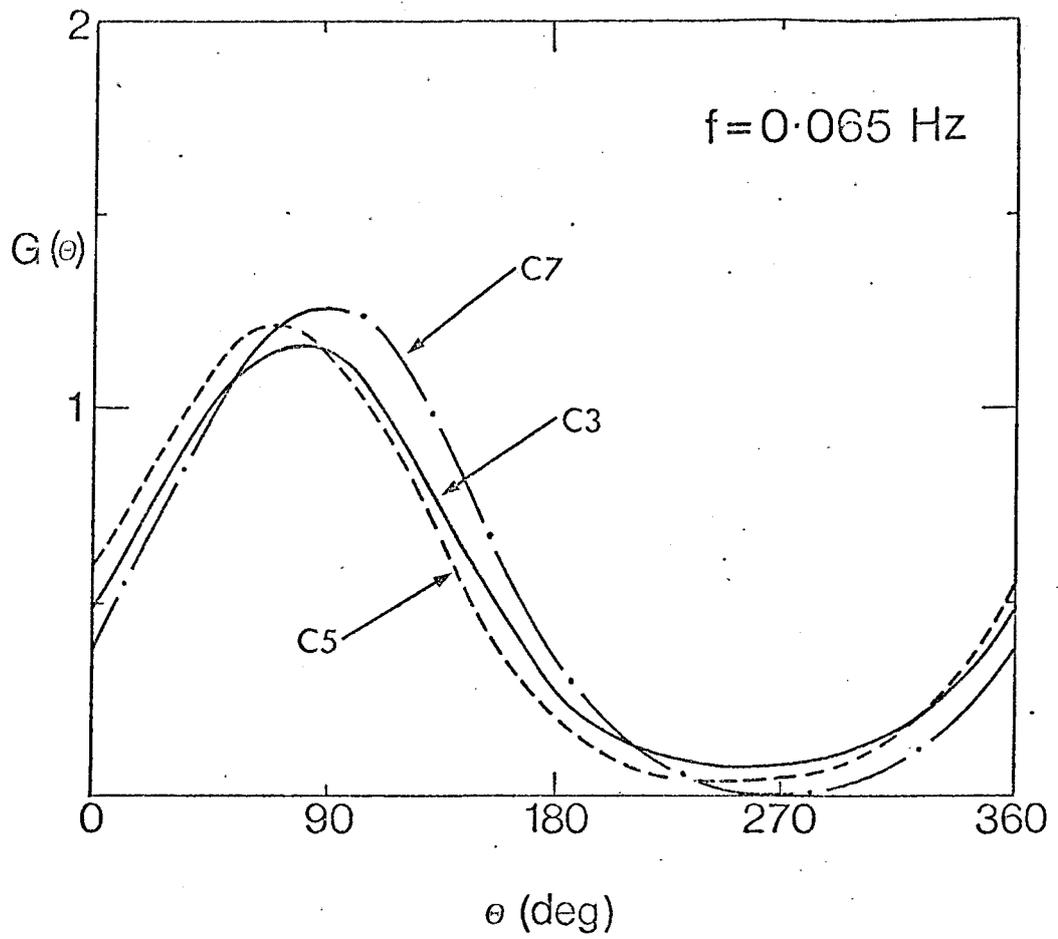


Fig. 6 Normalized directional distributions at three stations for a frequency of 0.065 Hz.

