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N. I. O. Computer Programs 1

by

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C O N T E N T S

			Page
INTRODUCTION			1
GENERAL REMARKS			1
N.I.O. PROGRAMS			
1	Card sorting	March 1958	3
2	Cross band spectra	September 1958	4
3	Fourier analysis, RAE 528 modified	December 1959	6
4	Card sorting, Southport	December 1959	7
5	Correlation, RAE 506 modified	September 1959	8
6	Pitch and roll buoy predominant azimuth angles	March 1960	9
7	Pitch and roll buoy filtered directional spectrum	March 1960	11
8	Smoothing	March 1960	12
9	Dover Straits	May 1960	14
10	Combining two series	March 1959	16
11	Sixth differences	October 1959	17
12	Slope correction to waveheights	November 1959	19
R.A.E. PROGRAMS			
174	Correlation		21
300	Fourier analysis		24
528	Fourier analysis		25

INTRODUCTION

This report contains a collection of programs written for Deuce between January 1958 and June 1960. Programs 5, 10 and part of 2 are due to Mrs. Pamela Edwards, who has also done a considerable amount of work on the final editing of the program cards and on preparing the descriptions.

The programs were developed (and are now used for production runs) on the two computers at the Royal Aircraft Establishment, Farnborough; we should like to thank the Mathematical Services Department for their help.

The descriptions of the programs (including a few frequently used standard ones) are complete only when read in conjunction with the general remarks under the appropriate heading; this is particularly important when preparing data and operating the computer.

The report is intended primarily as a manual for the operator, and so only brief comments on the mathematics and programming methods are given.

GENERAL REMARKS

Title — This includes the source of the program, if it is not original. Most of the original programs use standard subroutines.

Code — The present possibilities are

Deuce:
(a) Deuce basic
(b) Alphacode
(c) Programs under the control of a general interpretive program (GIP), for example, Scheme B.

Purpose — This amplifies the title.

Order of cards — This gives the complete assembly for input to Deuce.

Data — These comments apply also to the parameter cards and end cards. Unless otherwise stated, the following conventions are observed:-

(1) Unspecified columns may be left blank, and this usually includes the whole of the β-field. Anything punched on an unspecified column in the α-field is ignored, but this is not necessarily true for the β-field.

(2) All specified columns must be punched, zeros being inserted where necessary. Decimal numbers are punched from left to right in the usual way. Positive signs (that is, holes in the Y-row) need not be punched.

(3) The columns containing data are not stated explicitly when the lay-out is obvious. For example, 8 signed 4 digit numbers fit into the α-field in only one way, and the signs must be overpunched; that is, in the same column as the first digit.

(4) In general the data is in decimal form, but layout parameters, for example, are in binary.

(5) When the position of the data is specified by a layout parameter, it is not necessary to include all the digits appearing on the card, provided that the specified digits (and the sign) are consecutive.

Operation — Before starting, make sure that the stop-normal key is up - that is, at normal. Unless otherwise stated, all other keys should be in the horizontal position. The field switches on the reader and the punch must be at the 64 column setting.

After reading the program cards with the initial input key, wait until the program stops before running in the data (that is, reading with the run in key). (Large sets of data may be read in sections.) The stopper is sometimes specified, but in most cases it is

either N, 16 - 1
or N, N - 13

where $1 \leqslant N \leqslant 8$, and depends on where the read subroutine is stored.

All cards must be placed in the reader with the Y row downwards, that is, leading, and with the face of the card away from the operator.

The program may be restarted by reading the last card (if both α and β-fields are punched) or the last two cards (if only the α-field is punched) with the initial input key. This is not true for programs which only use the high speed store. Unless otherwise stated, it may be assumed that the last card restarts the program.

Output Unspecified columns are left blank. A few of the programs fit standard listing boards, but in general a board must be plugged to fit the output.

Parameters These are either set to the most commonly required values, or the position is left blank. Careful checking is essential. Parameters are often binary numbers (see Note 1).

Ideally, there should be nothing in this section, as variables are more conveniently set by a special parameter card. But it is sometimes easier to set those variables which are constant for most sets of data within the program, leaving the parameter card for more variable quantities.

Restrictions These have been specified and tested as fully as possible, but it should be remembered that some deficiency in a program might come to light only when a particular case is encountered - perhaps after many hours of production. All results should be checked as carefully as possible.

Failures Besides the specified failures, a program may not work because of the unforeseen cases mentioned above (see Restrictions). Also older programs may not work because of later modifications to the computer and/or the timing connected with the reader and the punch.

Time In general this excludes reading the program, but includes reading data, the calculation and the punching time. The approximate times given are intended as a guide, and should not be relied on too much.

Method This section contains definitions of the symbols used elsewhere in the description, together with a summary of the method. In general,

n = number of terms in a series

N = other counts.

Notes (1) A number $x = a.2^b$ is said to be in standard floating binary (s.f.b.) if

either $-1 \leqslant a < -\frac{1}{2}$,

or $\frac{1}{2} \leqslant a < 1$,

or $a = 0$,

and b is an integer. The store of Deuce also demands that a is specified to 30 bp (binary places) and that $-2^{31} \leqslant b < 2^{31}$.

N.I.O. PROGRAM 1

Title Card sorting (written for job 459, MS, RAE). RAE Program 566.

Code Deuce basic.

Purpose To separate 4 series.

Order of cards Program (cards 0-27), data.

Parameter cards None.

Data 8 positive 4 digit numbers per card in the α-field. The number of cards must be a multiple of 4, and so up to 3 extra cards filled with zeros may be necessary. Unused α-field columns on the last data card must be filled with zeros.

End card None.

Operation Read the program with the initial input key and then run in the data. Run in more data when required. The last card alone will not restart the program.

Output 8 positive 4 digit numbers per card in the α-field. Signs are punched.

Parameters None.

Restrictions The number of data cards must be a multiple of 4.

Failures None.

Time About N/36 minutes, where N is the number of data cards.

Method The program alternately reads 4 cards and punches 4 cards. The numbers are rearranged as follows:-

Input: Card 1	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
2	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}
3	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}
4	x_{24}	x_{25}	x_{26}	x_{27}	x_{28}	x_{29}	x_{30}	x_{31}
Output: Card 1	x_0	x_4	x_8	x_{12}	x_{16}	x_{20}	x_{24}	x_{28}
2	x_1	x_5	x_9	x_{13}	x_{17}	x_{21}	x_{25}	x_{29}
3	x_2	x_6	x_{10}	x_{14}	x_{18}	x_{22}	x_{26}	x_{30}
4	x_3	x_7	x_{11}	x_{15}	x_{19}	x_{23}	x_{27}	x_{31}

Notes (1) The original data came from a film containing 4 traces, which were read concurrently.

(2) The 4 packs of cards are finally separated by the Hollerith collator or by the sorter.

N.I.O. PROGRAM 2

Title Cross band spectra, for use with output of RAE 300 (periodogram). In its present form it cannot be used with output of RAE 528.

Code Deuce basic.

Purpose To condense and combine the periodogram outputs of two relate sets of data.

Order of cards Program (cards 0-52), series 1 of data, end card, series 2 of data, end card.

Parameter card None.

Data Output of periodogram program RAE 300.

Columns	17 - 20	s ;	sign in column 17
	21 - 30	I_s ;	21
	31 - 38	A_s ;	31
	39 - 46	B_s ;	39

All the numbers are integers. I may not have more than eight significant digits, and A and B may not have more than six significant digits.

End card P_1 on row 1_α ; P_1 on row 2_α .

Operation
(1) Read the program with the initial input key.
(2) Run in series 1 of data, followed by an end card.
(3) Run in series 2 of data, followed by an end card.
(4) The machine will stop on 5, 0-13, when the value $N.P_1$ should be set up on the I.D. switches in binary;
(5) Single shot. This will allow the program to continue and to punch the first output card . It stops a ain on 5,0-13.
(6) Either leave the former value of N on the I.D. switches or set up a new value of N.

Repeat (5) and (6) until all the output cards have been punched, when the machine will hoot.

To use the program with new data, read in the last card of the program with the initial input key. Then run in the data as usual.

Output

Columns	17 - 20	s_1	
	21 - 24	N	
	25 - 28	s_N	
	29 - 32	b	
	33 - 40	ΣI ;	sign in column 33
	41 - 48	Σi ;	41
	49 - 56	$10^{-2}\Sigma A\alpha$;	49
	57 - 64	$10^{-2}\Sigma B\beta$;	57
	65 - 72	$10^{-8}\Sigma I^2$	
	73 - 80	$10^{-8}\Sigma i^2$	

All these numbers are integers.

Parameters The scaling factors are chosen according to the magnitude of the input and the required accuracy of the output. Full details may be found in Computing Folder 1, which is kept by the person in charge of the N.I.O. library programs.

(1) $k_1.P_{22}$ on card number 25, row 3_β ; $k_1 = 7$ is set

(2) $k_2.P_{22}$ on card number 25, rows 4_β , 5_β ; $k_2 = 4$ is set.

These values allow I and i to have seven significant digits, and A, α, B and β to have four (and probably some terms of five) significant digits.

Restrictions (1) $n_1 = n_2 \leq 511$. If the input data exceeds this number it must be broken up into several batches; in this case the last card of the program must be run in in front of each batch of data.

(2) $b = s_N - s_1/N-1 \leq 19$. If $b > 19$ the calculation will be carried out correctly but s_N and b will be wrongly punched.

Failures

(1) 4, 7 - 24 x } s (series 1) $\neq$ s (series 2)
(2) 8, 7 - 24 x } $n_1 \neq n_2$
(3) 2, 0 - 17 x more data required
(4) 6, 7 - 24 x b is too large, i.e. $b \geq 20$. Give a single shot to continue.
(5) 5, 0 - 13 x (after hoot) $\Sigma N > n$ (see note 2)]
(6) 1, 7 - 24 x } overflow in one of ΣI, Σi, $\Sigma A\alpha$, $\Sigma B\beta$. A single shot allows the program to continue. A large overflow may not be recorded. See separate note for further details.

Time About $n\ (N + 8)/N$ seconds.

Method s = a(b)c, as defined in RAE 300.
n = total number of input cards in one series.
N = number of terms in one group.
s_1 and s_N are the first and last values of S in any one group of N.
I, A and B are I_S', A_S' and B_S' as defined in RAE 300.
i, α and β are similarly for a second series of data.
k_1 and k_2 are scaling factor constants.

Notes (1) If N is to remain constant the stopper on card number 37, row 2_β, can be removed (the instruction here is 0-13 x); in this case $N.P_1$ must be set up on the I.D. switches before the data are run in.

(2) If the last group of data has fewer than N values, N may be "counted down" on the I.D. switches until a single shot has effect, that is, until the new value of N is appropriate.

(3) The following convention may be followed for the "optional" output columns:-

Columns 5 - 7 job number
8 - 10 run and parameter numbers
11, 12 scaling factors for (1) I and i, and (2) A, α, B and β.

(4) This program is going to be rewritten.

N.I.O. PROGRAM 3

Title Fourier analysis (periodogram). RAE 528 modified. RAE Program 567.

Code Deuce basic.

Purpose To calculate the harmonic components A_s, B_s for selected s.

Order of cards Program (cards 0-31), s-parameter card, s-data, parameter card, data.

Parameter cards (1) s-parameter card: columns 17-20 N + 1

(2) s-data: the N values of s are punched as 8 positive 4 digit integers per card in the α-field. An x (minus) must be punched in the column after the last value of s. This may mean starting a fresh card.

(3) Parameter card: columns 17-20 n ,
21-24 k .

Data The n values of y_r are punched as 8 signed 4 digit integers per card, in the α-field.

End card None.

Operation (1) Read the program with the initial input key.
(2) Run in the s-parameter card followed by the s-data.
(3) Run in the parameter card followed by the data.

When a run is complete the program stops, and operation (3) may be repeated. If new s-data is required the program must be restarted.

Output

Columns 18 - 21	s ;	sign in column	17
24 - 30	A_s ;		23
33 - 39	B_s ;		32
42 - 48	I_s ;		41

There are three more decimal places in A_s, B_s and I_s than in y_r.

Parameters None

Restrictions (1) $0 \leq s \leq 9999$ (not tested for $s > 4428$)
(2) $1 \leq N \leq 95$
(3) $n^2M \leq 2{\cdot}14 \times 10^9$ where $|y_r| \leq M$, $r = 0(1)\ n-1$

Failures None

Time About nN/200 seconds

Method

$$A_0 = \frac{1}{n} \sum_{r=0}^{n-1} y_r , \qquad B_0 = 0 \quad \text{when } s = 0$$

$$\begin{matrix} A_s \\ B_s \end{matrix} = \frac{2}{n} \sum_{r=0}^{n-1} y_r \begin{matrix} \cos \\ \sin \end{matrix} \left(\frac{2\pi rs}{kn}\right) \quad \text{when } s > 0$$

$$I_s = \sqrt{A_s^2 + B_s^2} \quad \text{for all } s,$$

where s is an integer. If $0 \leq c \leq 13$ is an integer, then $k = 2^c$.

Notes (1) The method of calculation is described in The Computer Journal, 1, 4, p. 162.

N.I.O. PROGRAM 4

Title Card sorting, Southport. Written for job 852 (MS, RAE). RAE Program 568.

Code Deuce basic.

Purpose Given one value of each of N series per card, to form N packs of cards, one pack per series, and with eight values per card.

Order of cards Program (cards 0-31), parameter card, data.

Parameter card Columns 17-20 m

Data N signed 4 digit numbers in the α-field, punched consecutively, and with overpunched signs, with the first sign and digit in column 17.

End card None.

Operation
(1) Read in the program with the initial input key. The program stops on 7, 5-5.
(2) Run in the parameter card and data. The program is still at 7, 5-5.
(3) Single shot. The program reads the parameter card and data and stops on 7, 13-12.
(4) Single shot. The first set of results is punched. The program stops at 7, 13-12.
(5) Repeat (4), making N times in all, and N sets of output. The program does not stop at 7, 13-12 after the Nth time.
(6) The hooter sounds and the program stops at 7, 5-5. After clearing the hooter, operations (2) to (6) may be repeated with more data.

Output N packs of cards, with 8 signed 4 digit numbers per card.
If m/64 is an integer, each pack will contain m/64 cards. Otherwise, each pack will contain the integral part of (m/64 + 1) cards.

Parameters $(N-1),P_{17}$ on card number 5, row 9α. This is set for N = 6.

Restrictions
(1) $1 \leq N \leq 8$
(2) $8 \leq m \leq 7520$, that is, 1 ≤ number of cards ≤ 940

Failures None

Time About Nn/40 seconds.

Method The values on columns 17-20 of the input data are punched successively on the first pack of output cards, with eight values per card. Then the values from columns 21-24 form the second pack of output, and so on for the N series, giving N packs of output.

We define m as 8 x the number of data cards.

Notes (1) The stopper after stage (3) of Operation allows time for the manual setting of new series numbers, zeroising card numbers, running out cards and so on.

(2) If any column in the input is left blank the corresponding output column contains a nine.

(3) If m/64 is not an integer (that is, if the number of cards is not a multiple of 8), the last output card will be completed with zeros.

N.I.O. PROGRAM 5

Title Correlation, based on RAE 506 (unavailable) (MS, RAE). RAE Program 548.

Code Deuce basic.

Purpose To find the autocorrelation coefficients of a time series or the cross correlation coefficients of two series.

Order of cards Program (cards 0-55), parameter card, data.

Parameter card Yα row: data layout. A one in each digit (or sign) position except the last digit of a number, and zeros everywhere else.

The rest of the card is exactly as for RAE 174 except that zeros must be punched instead of blank columns. No positive signs may be punched.

Data Either 1 or 2 signed numbers per card. If 2 are specified the first is series 1 and the second series 2; if only 1 is specified it is called series 2, in which case the autocorrelation parameter is K_2. Columns other than those specified are ignored. The sign is in the column preceding the first digit, and the first sign must be in column 17.

End card None.

Operation (1) Read the program with the initial input key.
(2) Run in the parameter card, followed by the data.
When one run is complete, (2) may be repeated.

Output See RAE 174.

Parameters (1) $c.P_1$ in binary on card 8, row Y_α. Here $c = 5000$.
(2) $r.P_1$ in binary on card 7, row 4_α. Here $r = 5$.
(3) $10.P_1$ in binary on card 8, row 6_α.

Restrictions (1) $0 < n \leqslant 3488$
(2) $0 \leqslant K_1, K_2 \leqslant 159$
(3) $0 \leqslant L_1, L_2 \leqslant 127$
(4) $-5005 \leqslant x_i, y_i \leqslant 4994$, $i = 0(1)n-1$

Failures See RAE 174. There are also 2 failures peculiar to this program, namely:-

(1) 6, 6-6, with hooter; x_i and/or $y_i > 4994$
(2) 6, 7-7, with hooter; x_i and/or $y_i < -5005$.

Time About $(Nn/60) + n$ seconds, where N is any of K_1, K_2, L_1, L_2.

Method The main calculation is exactly the same as for RAE 174. Before entering the main section the data are made to satisfy certain conditions. If the original number x_i satisfies Restriction (4), then the program calculates Z_i, where

$$Z_i = \frac{x_i + c + r}{10}$$

and $0 \leqslant Z_i \leqslant 999$.

Notes (1) The checks are the same as for RAE 174.

(2) It is clear that by juggling with Parameters (1), (2) and (3) and Restriction (4), and taking account of the relation between x_i and Z_i, almost any data may be dealt with and made to satisfy $0 \leqslant Z_i \leqslant 999$. The failures depend on c rather than on the actual values named in Restriction (4).

N.I.O. PROGRAM 6

Title Pitch and roll buoy predominant azimuth angles. Written for job 832 part 1 (MS, RAE).

Code Deuce basic.

Purpose To find the two main directions of wave propagation from the first five Fourier coefficients.

Order of cards Program (cards 0-37), data.

Parameter card None.

Data

Columns			
17 - 18	10σ ;	sign in column 17	
19 - 24	$10^3 a_0$;	19	
25 - 30	$10^3 a_1$;	25	
31 - 36	$10^3 b_1$;	31	
37 - 42	$10^3 a_2$;	37	
43 - 48	$10^3 b_2$;	43	

All the numbers are integers, and 6 digits (2 for 10σ) must be punched.

End card None.

Operation Read the program with the initial input key and then run in the data. The program alternately reads 1 card and punches 1 card until no more data is left. Further data cards may then be run in.

Output

Columns			
17 - 19		10σ ;	sign in column 17
21 - 24	θ	(degrees) ;	21
25 - 26	θ	(minutes) ;	
28 - 31	ψ	(degrees) ;	28
32 - 33	ψ	(minutes) ;	
35 - 40		$10^3(a_0 - I)$;	35
42 - 47		$10^3 I$;	42

Parameters None.

Restrictions (1) $0 \leq |\sigma| \leq 9{\cdot}9$

(2) $0 < |a_0| \leq 999{\cdot}999$

(3) $0 \leq |a_1| \leq 999{\cdot}999$, and similarly for b_1, a_2, b_2.

Failures If the data are incompatible (for example, if they would give $|\cos \psi| > 1$) the program will fail. It may stop on 4, 24-14 (that is, a number > 1 is about to be produced during the floating to fixed point conversion) or there may be other stoppers in the standard subroutines, or there may be no indication at all.

Time About N/14 minutes, where N is the number of data cards.

Method σ = reference number

$$\theta = \cos^{-1} x/(x^2 + y^2)^{\frac{1}{2}} \qquad 0 \leq \theta \leq 2\pi$$

$$\psi = \cos^{-1} z/2(x^2 + y^2)^{\frac{1}{2}} \qquad 0 \leq \psi \leq 2\pi$$

$$a_0 I = a_0^2 - a_1^2 - b_1^2 - (P^2 + Q^2)^{\frac{1}{2}}$$

where $x = (a_0 - I)a_1 - (a_1 a_2 + b_1 b_2)$

$y = (a_0 - I)b_1 - (a_1 b_2 - a_2 b_1)$

$z = (a_0 - I)^2 - (a_2^2 + b_2^2)$

$$P = a_0 a_2 - a_1^2 + b_1^2$$

$$Q = a_0 b_2 - 2a_1 b_1$$

Notes (1) For a theoretical account see M.S. Longuet-Higgins, Proc. Camb. Phil. Soc., 51·4, 590-603, 1955.

(2) The data is the same as in N.I.O. Program 7.

N.I.O. PROGRAM 7

Title Pitch and roll buoy filtered directional spectrum. Written for job 832 part 2 (MS, RAE).

Code Deuce basic.

Purpose To obtain a smoothed estimate of the directional energy spectrum of the waves from the first five Fourier coefficients.

Order of cards Program (cards 0-34), data.

Parameter card None.

Data

Columns 17 - 18	10σ ;	sign in column	17
19 - 24	$10^3 a_0$;		19
25 - 30	$10^3 a_1$;		25
31 - 36	$10^3 b_1$;		31
37 - 42	$10^3 a_2$;		37
43 - 48	$10^3 b_2$;		43

All the numbers are integers, and 6 digits (2 for 10σ) must be punched.

End card None.

Operation Read the program with the initial input key and then run in the data. The program alternately reads 1 card and punches 12 cards until no more data is left. Further data cards may then be run in.

Output

Columns 17 - 19	10σ ;	sign in column	17
21 - 24	ϕ (degrees) ;		21
26 - 32	10^{6-r} F ;		26

Parameters 10^{-r} in standard floating binary on card number 29, rows 6_β, 7_β, where r is defined in Method. Normally r = 1.

Restrictions
(1) $0 \leqslant |\sigma| \leqslant 9{\cdot}9$
(2) $0 < |a_0| \leqslant 999{\cdot}999$
(3) $0 \leqslant |a_1| \leqslant 999{\cdot}999$, and similarly for a_1, a_2, b_2.
(4) $10^{-r}|F| < 1 \quad r \geqslant 0$ See note (1)

Failures None.

Time About N/2 minutes, where N is the number of data cards.

Method $6\pi F = 3a_0 + 4(a_1 \cos\phi + b_1 \sin\phi) + (a_2 \cos 2\phi + b_2 \sin 2\phi)$,

where $\phi = -180^\circ(+30^\circ)\ 150^\circ$.

An upper bound for $|F|$ can be derived.

Suppose $|\max(a_0, a_1, b_1, a_2, b_2)| = M \leqslant 999{\cdot}999$.

Then $|F| \leqslant \frac{13M}{6\pi} < 0{\cdot}7\ M$.

We must choose the parameter $r \geqslant 0$ so that

$$|10^{-r} F| < 1$$

but is as large as possible subject to this restriction.

N.I.O. PROGRAM 8

Title Smoothing. Written for job 670 (MS, RAE). RAE Program 570.

Code Deuce basic.

Purpose To scale and smooth a series.

Order of cards Program (cards 0-43), parameter card, data.

Parameter card Columns 25 - 28 a
33 - 36 c
37 - 40 k
41 - 44 p

Data Columns 21 - 30 I_s ; sign in column 21.
See parameter (1).

End card None.

Operation (1) Read the program with the initial input key. It stops at 6, 9-8.
(2) Run in the parameter card, followed by the data. The program is still at 6, 9-8.
(3) Give a single shot.
(4) When one run is completed the program returns to 6, 9-8. Operations (2) and (3) may then be repeated.

Output Columns 17 - 21 s ; sign in column 17
23 - 32 $q_1 I_s'$; 23
33 - 42 $q_2 I_s''$; 33

Parameters (1) Data layout. A one in each digit (or sign) position except the last digit of a number, and zeros everywhere else. Column 17 must be zero. This is set as described under Data. The layout parameter is on card 11, row 3_α.
(2) q_1 in standard floating binary on card 6, rows 2_α, 3_α.
(3) q_2 in standard floating binary on card 6, rows 4_α, 5_α.

Restrictions (1) $0 \leqslant c - a + 1 \leqslant 1824$, $a \geqslant 0$
(2) $1 \leqslant p \leqslant 912$, but the smoothing is useful only when $0 \leqslant 2p - 1 \leqslant c - a + 1$
(3) $0 \leqslant |k| \leqslant 9999$.
(4) $|k|\, Mq_1 / p^2 < 1$, $4|k|\, Mq_2 < 1$, where $M = \max. |I_s / s^4|$, and q_1, q_2 are the largest possible multiples of 10 or $\frac{1}{10}$ subject to the above restrictions. Normally $q_1 = q_2 = 1$. See Note (2).
(5) $|I_s| \leqslant 2{\cdot}14 \times 10^9$.

Failures None.

Time About $\frac{3}{2}(c - a + 1)$ seconds.

Method The smoothed value I_s'' is defined by

$$I_s'' = I'_{s-p+1} + 2I'_{s-p+2} + \ldots + (p-1)I'_{s-1} + pI'_s + (p-1)I'_{s+1} + \ldots + 2I'_{s+p-2} + I'_{s+p-1} \qquad (1)$$

where $I_s' = \frac{k}{p^2} \frac{I_s}{s^4}$, and $s = a(1)c$. (2)

I_s is given for $s = a(1)c$ and k, p are given constant integers. We define $I_s'' \equiv 0$ for $a \leqslant s \leqslant a + p - 2$ and $c - p + 2 \leqslant s \leqslant c$. It is easy to show that

$$\nabla I_s'' = I_s'' - I_{s-1}'' = \sum_{q=s}^{s+p-1} (I_q' - I_{q-p}') \qquad (3)$$

$$\nabla^2 I_s'' = \nabla I_s'' - \nabla I_{s-1}'' = I'_{s+p-1} - 2I'_{s-1} + I'_{s-p-1} \qquad (4)$$

The program has three stages:-

Stage 1. I''_{a+p-1} and I''_{a+p} (and hence $\nabla I''_{a+p}$) are calculated from (1).

Stage 2. $\nabla^2 I''_s$ is calculated from (4) and $\nabla I''_s$ and I''_s are built up for $s = (a + p + 1)(1)(c - p + 1)$. This involves addition and subtraction only, and is far quicker than direct evaluation of (1).

Stage 3. I''_{c-p+1} is calculated from (1) and compared with the last value obtained in Stage 2.

Notes (1) The program was originally intended to accept the output from the periodogram program RAE 300; that is, s, I_s, A_s and B_s for $s = a(1)c$. The essential number I_s (regarded as an integer) is in columns 21-30, with sign in column 21.

(2) A brief derivation of restriction (4) is as follows:

The stored quantities which must not overflow are s, I_s, I'_s, I''_s, $|\nabla I''_s|$ and $|\nabla^2 I''_s|$, of which the first two are integers with upper limit $2^{31} - 1$, and the others are fractions with upper limit unity.
By definition, s and I_s are in range.

Let $M = \max. \left| \frac{I_s}{s^4} \right|$

Then $|I'_s| \leqslant \frac{|k|M}{p^2}$ (5)

From equations (1) and (2) we see that

$$|I''_s| \leqslant \frac{M|k|}{p^2} |1 + 2 + \ldots + (p - 1) + p + (p - 1) + \ldots + 2 + 1|$$
$$= |k|M$$

Also $|\nabla I''_s| \leqslant 2|I''_s|$, $|\nabla^2 I''_s| \leqslant 4|I''_s|$,

so the inequality which covers I''_s, $\nabla I''_s$ and $\nabla^2 I''_s$ is

$$4|I''_s| \leqslant 4|k|M \qquad (6)$$

Equations (5) and (6) give the inequalities of restriction (4), namely,

$$\frac{|k|M}{p^2} q_1 \leqslant 1$$

$$4|k|M\, q_2 \leqslant 1 ,$$

where q_1 and q_2 are independent constants.

(3) It is not possible to give an adequate analysis of the rounding errors here, but it is clear that there may be such errors in the two initial values of I''_s, and also that the error in the last value of I''_s will depend to some extent on the number of steps. The check value of I''_s (see Method, Stage 3) will give an indication of this error, and if it is too large to be accepted, then either the range of s must be reduced or the number of decimal places increased by increasing q_1 and/or q_2.

(4) The program could be adapted to smooth I_s in other ways, for example, by setting $s \equiv 1$ in the appropriate routine. However, it is a special purpose program, and would probably be inefficient in any other context.

N.I.O. PROGRAM 9

Title Dover Straits. Written for job 885 (MS, RAE).

Code Deuce basic.

Purpose Given the barometric pressure at Felixstowe, Tangmere (or Thorney Island) and Calais, to calculate the difference in water level between Ramsgate and Dunkirk due to the pressure difference and wind stress.

Order of cards Program (cards 0-25), data, end card.

Parameter card None.

Data (1) Tangmere.

Columns	17 - 20	reference number;	sign in column 17
	22 - 26	p_1 ;	22
	27 - 31	p_2 ;	27
	37 - 41	p_3 ;	37

(2) Thorney Island

Columns	17 - 20	reference number;	sign in column 17
	22 - 26	p_1 ;	22
	27	x (minus)	
	32 - 36	p_2' ;	32
	37 - 41	p_3 ;	37

End card x (minus) in column 17

Operation (1) Read the program with the initial input key. It stops on 6, 3-13
(2) Run in the data, followed by the end card.
(3) When the end card has been read the program stops at 8, 8-8. Clearing the hooter and giving a single shot causes the program to return to 6, 3-13. Operation (2) may then be repeated.

Output

Columns	17 - 21	reference number ;	sign in column 17
	22 - 25	d ;	22
	26 - 29	d' ;	26
	30 - 33	δ_1 ;	30
	34 - 37	δ_2 ;	34
	38 - 41	$\bar{\delta}_1$;	38
	42 - 45	$\bar{\delta}_2$;	42

All the numbers (except the reference number, which is an integer) are given to 2D.

Parameters The a, b, c are punched in binary (to 30 bp) on card 22, rows 2α-9α, and card 23, rows $Y\alpha$- 5α, as follows:

Card 22, row	2α	a_1	now set as + 1797/5960
	3α	a_2	+ 1780/5960
	4α	a_3	- 3577/5960
	5α	c_1	+ 0·0327
	6α	a_1'	+ 1995/6480
	7α	a_2'	+ 1780/6480
	8α	a_3'	- 3775/6480
	9α	c_2	- 0·0844
Card 23, row	$Y\alpha$	b_1	+ 3283/5960
	$X\alpha$	b_2	- 1660/5960

Card 23 row	0_α	b_3	now set as	$-1623/5960$
	1_α	c_3		$+0{\cdot}241922$
	2_α	b_1'		$+3531/6480$
	3_α	b_2'		$-1660/6480$
	4_α	b_3'		$-1871/6480$
	5_α	c_4		$+0{\cdot}970296$

Restrictions (1) $0 \leqslant$ reference number $\leqslant 9999$

(2) $|p_1| \leqslant 9999{\cdot}9$, and similarly for p_2, p_2' and p_3.

(3) $|d| \leqslant 9{\cdot}99$, and similarly for d', δ_1, δ_2, $\bar{\delta}_1$, and $\bar{\delta}_2$.

(4) $N \geqslant 5$, where N is the number of data cards.

Failures None.

Time N/29 minutes, where N is the number of data cards.

Method p_1 = pressure at Felixstowe

p_2 = pressure at Tangmere

p_2' = pressure at Thorney Island

p_3 = pressure at Calais

$$d_1 = a_1p_1 + a_2p_2 + a_3p_3 = a_1'p_1 + a_2'p_2' + a_3'p_3$$

$$d_2 = b_1p_1 + b_2p_2 + b_3p_3 = b_1'p_1 + b_2'p_2' + b_3'p_3$$

$$\delta_1 = c_1d_1$$

$$\delta_2 = c_2(d_1^2 + d_2^2)^{\frac{1}{2}}\,(c_3d_1 + c_4d_2)$$

$$8\bar{\delta}_1^{(s)} = \delta_1^{(s-2)} + 2\delta_1^{(s-1)} + 2\delta_1^{(s)} + 2\delta_1^{(s+1)} + \delta_1^{(s+2)}$$

$$8\bar{\delta}_2^{(s)} = \delta_2^{(s-2)} + 2\delta_1^{(s-1)} + 2\delta_1^{(s)} + 2\delta_1^{(s+1)} + \delta_1^{(s+2)}$$

The a, b, c are given parameters. The program alternately reads one card and punches one card. The first two cards and the last two cards have $\bar{\delta}_1 = \bar{\delta}_2 \equiv 0$.

Notes (1) In practice, the pressures p_1, p_2, p_2', p_3 had a constant (900·0) subtracted before they were punched. This simplification does not affect the program provided

$$a_1 + a_2 + a_3 \equiv 0,$$

and similarly for the a', b and b'.

(2) Only 4 signs can be obtained comfortably on the Hollerith 805 lister, so in this case the signs of d_1 and d_2 (which have the same signs as δ_1 and δ_2 respectively) were not listed.

N.I.O. PROGRAM 10

Title Combining two series. Written for job 538 (MS, RAE). RAE Program 569.

Code Deuce basic.

Purpose To difference corresponding terms of two series, and to sum the resulting series.

Order of cards Program (cards 0-18), data (in pairs of cards).

Parameter card None.

Data Columns 18 - 27 x_i or y_i; sign in column 17.
There must be n pairs of cards.

End card None.

Operation (1) Read in the program with the initial input key.
(2) Run in the data, taking care not to split up pairs of data cards.
The last card alone will not restart the program.

Output Columns 18 - 27 w_i; sign in column 17.

Parameters n.P_1 on card number 01, row 9_α.

Restrictions (1) $0 < n \leqslant 499$
(2) $|x_i| \leqslant 999\,999$, $|y_i| \leqslant 999\,999$, for $i = 1(1)\ n$.
See also Note (2).

Failures None.

Time About 2n seconds.

Method If there are n pairs of data cards, denoted by (x_1, y_1), (x_2, y_2) ..... (x_i, y_i) ..... (x_n, y_n), then the program computes $W_i = \frac{1}{2}(x_i - y_i)$ for $i = 1\ (1)\ n$, and also $\sum_{i=1}^{n} W_i$

Notes (1) If $W_i' = \frac{1}{2}(x_i + y_i)$ is required instead of W_i, the destination number on card number 18, row Y_α should be changed from 26 to 25. (This has not been tested.)

(2) The program has been fully tested only for values within the limits given under Restrictions, but it may well work for larger numbers. Care must be taken to ensure that ΣW_i (in 17_2) does not overflow.

(3) Given two packs of cards, A and B, the corresponding terms of the two series can be paired by placing the A pack in front of the B pack and sorting the whole lot on the sorter (sorting on the card number and starting at the units end of the card number as usual).

N.I.O. PROGRAM 11

Title Sixth differences. RAE Program 549.

Code Deuce basic.

Purpose To obtain central differences of a given numerical function.

Order of cards Program (cards 0-31), parameter card, data, end card.

Parameter card Y_α row: a one in each digit (or sign) position except the last digit of a number, and zeros everywhere else.

Data Two integers in the α-field. Both must have signs in the same column as the first digit (see Parameters) and the first number must begin in column 17. Nothing may be punched in the Y row of column 48.

End card Y_α row: P_{32} only.

Operation
(1) Read the program with the initial input key. It stops on 3,0-16.
(2) Run in the parameter card, data and end card.
(3) When one difference table is complete, the hooter sounds and the program returns to 3,0-16. The hooter may then be cleared and operation (2) repeated.

Output 8 signed 8 digit integers per card. Non-significant zeros are suppressed. The cards are in pairs as follows:-

Card 1	x	f(x)	space	δ^2	space	δ^4	space	δ^6
Card 2	space	space	δ	space	δ^3	space	δ^5	space

Card 1	columns 17 - 24	x	;	sign in column 17
	25 - 32	f(x)	;	25
	41 - 48	δ^2	;	41
	57 - 64	δ^4	;	57
	73 - 80	δ^6	;	73
Card 2	columns 33 - 40	δ	;	sign in column 33
	49 - 56	δ^3	;	49
	65 - 72	δ^5	;	65

The missing differences at the beginning and end of the table are replaced by spaces.

Parameters If the data has signs in the column before the first digit, then the timing number on card 16, row 9_α must be changed from 0 to 8.

Restrictions
(1) $|f(x)| \leq 9999\ 9999$. If any function value or difference exceeds this limit, wrong results will be punched and values depending on the incorrect one may be affected.
(2) There must be at least 6 data cards.

Failures None.

Time About n/15 minutes, where n is the number of input cards.

Method The first integer on each data card is regarded as the argument, and the second is differenced. Except at the beginning and end of the difference table, the program alternately reads 1 card and punches 2 cards.

Notes (1) The program makes use of two apparently unpublished subroutines both due to G. Cork and J.M. Watt (MS, RAE) and originally designed for use with sterling. Both subroutines have been slightly modified, and the relevant parts of these versions are briefly described below. Together they occupy DL 6, 7, 8.

(a) Binary to 4 digit binary coded decimal conversion.
First order, entry 6_{20}.

Stores used	13	14	15	16	19	20	21
Entry	link	marker	x	divisor	-	-	-
Exit	-	-	-	-	-	-	x coded

The marker consists of many 3 digit groups, the first one to be interpreted being in P_{30-32}. These groups are interpreted as follows:-

First group:	0	Code first digit
	1	Code first digit suppressing non-significant zeros.
Other groups:	0,1	Code space
	2,3	Multiply by 10 and code next digit.

If the value of any of these groups is even, non-significant zeros will not be suppressed from that point.

The divisor = 16N-1 where N is the number of units corresponding to a most significant digit of 1. For example, if the number to be coded is x, and x = 432, then N = 100.

If x has 8 digits or less the coded result is entirely in 21_2.

(b) Punch decimal from 4-digit binary.
First order, entry 8_{26}.

Stores used	13	14	15	16	17_1	17_2	17_3	18_2	18_3	21	10_{24-31}
Entry	link	-	-	-	0_α	X_α	Y_α	X_β	Y_β	-	4 digit binary
Exit	-	-	-	-	-	--	-	-	-	-	4 digit binary

If any digit is d, then if d = 0, 1, 2, ... 9, an appropriate hole is punched in that column and if d ≥ 10, the corresponding column is left blank.

The layout of binary coded digits ready for punching in columns 17-80 is as follows:-

DL 10	P_{1-4}	P_{5-8}	P_{9-12}	P_{13-16}	P_{17-20}	P_{21-24}	P_{25-28}	P_{29-32}
mc 24	24	23	22	21	20	19	18	17
25	32	31	30	29	28	27	26	25
26	40	39	38	37	36	35	34	33
27	48	47	46	45	44	43	42	41
28	56	55	54	53	52	51	50	49
29	64	63	62	61	60	59	58	57
30	72	71	70	69	68	67	66	65
31	80	79	78	77	76	75	74	73

N.I.O. PROGRAM 12

Title Slope correction to waveheights. Written for job 596 (MS, RAE).

Code Deuce basic.

Purpose To modify a set of data, using two related sets of data and suitable constants.

Order of cards Program (cards 0-40), parameter card, data.

Parameter card

Columns 17 - 20	n
21 - 24	c_1
25 - 28	$10^7 c_2$
29 - 32	$10^7 c_3$
33 - 48	zeros

No column in the α-field may be left blank, and a zero should be punched where there is no other digit.

Data 8 positive 4 digit numbers per card in the α-field. If n is not a multiple of 8 the last card must be filled with zeros.

End card None.

Operation
(1) Read the program with the initial input key. The program stops on 3, 16-1.
(2) Run in the parameter card, followed by R, P, Q in that order.
(3) When one calculation is finished the hooter sounds and the program stops on 3, 16-1. The hooter may then be cleared and operation (2) repeated.

Output Q_r^*, $(r = 0(1)\ n-1)$, punched as 8 positive 4 digit numbers per card in the α-field. No signs are punched. If the number of data cards is not a multiple of 4, up to 3 superfluous cards containing zeros will be punched.

Parameters None.

Restrictions
(1) Each number x in the data must satisfy $0 \leqslant x \leqslant 999$.
(2) $0 \leqslant n \leqslant 2560$
(3) $Q_r - \bar{Q} + c_1 \geqslant 0$ for $0 \leqslant r \leqslant n - 1$.
(4) $0 \leqslant c_2 (P_r - \bar{P})^2 + c_3 (R_r - \bar{R})^2 \leqslant 214$ for $0 \leqslant r \leqslant n - 1$.
(5) $0 \leqslant c_1 \leqslant 9999$, and similarly for $10^7 c_2$ and $10^7 c_3$.

Failures None

Time About $3n/500$ minutes.

Method $Q_r^* = Q_r - (Q_r - \bar{Q} + c_1)\ [c_2 (P_r - \bar{P})^2 + c_3 (R_r - \bar{R})^2]$ for $0 \leqslant r \leqslant n-1$.

where n = the number of terms in each series,

and $\bar{P} = \frac{1}{n} \sum_{0}^{n-1} P_r$, with similar definitions of $\bar{Q}$ and $\bar{R}$.

Notes (1) The program was originally written for data obtained from the pitch and roll buoy. The aim was to obtain the vertical acceleration Q^* from the pitch P, roll R and acceleration Q.

(2) At the end of the program DL 12 contains the following integers, apart from constants and working space.

mc	0	n
	1	c_1

mc	2	$10^7 c_2$
	3	$10^7 c_3$
	4	number of cards (either $[\frac{n}{8}]$ or $[\frac{n}{8}] + 1$, depending on n)
	9	$\vec{Q}$
	10	$\vec{P}$
	11	$\vec{R}$

R.A.E. PROGRAM 174

Title Correlation.

Code Deuce basic.

Purpose To find the autocorrelation coefficients of a time series or the cross correlation coefficients of two series.

Order of cards Program (cards 0-49), parameter card, series 1, series 2 (if required).

Parameter card

Columns 17 - 20	n
21 - 24	K_1
25 - 28	K_2
29 - 32	L_1
33 - 36	L_2
41 - 44	p
45 - 48	must be left blank.

If any of K_1, K_2, L_1, L_2 are omitted (that is, the corresponding columns are blank), the appropriate section of the program is omitted. If both L_1 and L_2 are omitted, only one series is read and K_1 autocorrelation coefficients are found.
If p is omitted, no blank cards are inserted.

Data The n terms of a series are punched as 8 positive 4 digit integers per card in the α-field. Series 2, if present, must have the same number of terms (n) as series 1.

End card None.

Operation
(1) Read the program with the initial input key.
(2) Run in the parameter card, followed by series 1.
(3) Run in series 2, if present.
(4) When one run is complete, (2) and (3) may be repeated.

Output There are two types of output.

(1) Autocorrelation.	Card 1	Mean
	Card 2	Variance
	Card 3	Blank
	Card 4 onwards	Correlation coefficients.
(2) Cross correlation.	Card 1	Blank
	Card 2 onwards	Correlation coefficients.

The correlation coefficients are separated by a blank card after every p output cards, and there is a blank card at the end.

There are two types of card layout.

(1) Columns 18 - 26 a ; sign in column 17
34 - 36 b ; 33
47 - 48 Marker

If either the mean or the variance is denoted by c, then $c = a.10^{-b}$.

(2) Columns 18 - 26 r_j ; sign in column 17
34 - 36 j ; 33
47 - 48 Marker

Here r_j is the correlation coefficient for lag j.

Both a and r_j are punched to 8D. The marker is defined under Method.

Parameters None.

Restrictions (1) $0 < n \leq 3488$

(2) $0 \leq K_1, K_2 \leq 159$

(3) $0 \leq L_1, L_2 \leq 127$

(4) $0 \leq x_i, y_i \leq 999$, $i = 0(1)n-1$.

Failures (1) If any of Restrictions (1)-(3) are violated, the program hoots and stops on 1, 1-1. The output staticiser lights indicate the mistake, as follows:-

P_1 : $n > 3488$

P_{17} : $K_1 > 159$ and/or $K_2 > 159$

P_{1-32} : $L_1 > 127$ and/or $L_2 > 127$

(2) There are also indications of machine failures. The program hoots and stops on 1, 1-1 (excluding the above), 6,2-2 or 5,3-3. A single shot usually repeats the appropriate section, but it is safer to restart the run completely.

Time About $3n/80$ seconds to read 1 series, and $nN/210$ seconds to calculate and punch 1 set of correlation coefficients, where N is the number of lags.

Method (1) If we are given 2 series, each with n terms, denoted by

$x_0, x_1, \ldots\ldots\ldots x_{n-1}$ (series 1)

$y_0, y_1, \ldots\ldots\ldots y_{n-1}$ (series 2)

then the cross correlation coefficient is defined by

$$r_j = \frac{C^2}{AB} ,$$

where
$$(n-j)C^2 = (n-j)\sum_{1}^{n-j} x_i y_{i+j} - \left[\sum_{1}^{n-j} x_i\right]\left[\sum_{j+1}^{n} y_i\right] ,$$

$$(n-j)A^2 = (n-j)\sum_{1}^{n-j} x_i^2 - \left[\sum_{1}^{n-j} x_i\right]^2 ;$$

$$(n-j)B^2 = (n-j)\sum_{j+1}^{n} y_i^2 - \left[\sum_{j+1}^{n} y_i\right]^2$$

and where $j \geq 0$ is the lag number.

The autocorrelation coefficient is obtained when $x_i \equiv y_i$ for all i in the above formulae. In this case we may also define the mean $\bar{x}$ and the variance σ_x^2 as follows:-

$$n\bar{x} = \sum_{1}^{n} x_i$$

$$(n-1)\sigma_x^2 = \sum_{1}^{n} x_i^2 - n\bar{x}^2 .$$

(2) The numbers K and L on the parameter card denote the number of lags in the 4 possible cases,

$\psi_{xy}(R) = -1$

$\psi_{yx}(R) = +1$ $\sin \frac{R\,9\pi}{L}$

x	y
1	0
$\frac{1}{2}$	$\frac{1}{2}$
0	1
$-\frac{1}{2}$	$\frac{1}{2}$
-1	0
$-\frac{1}{2}$	$-\frac{1}{2}$
0	
$\frac{1}{2}$	

K_1 : series 1 autocorrelation, with marker 1,1

K_2 : series 2 autocorrelation, with marker 2,2.

L_1 : cross correlation, series 1 leading, with marker 1,2

L_2 : cross correlation, series 2 leading, with marker 2,1

Also p = punch spacing.

"Series 1 leading" means that x_{n-i} and y_i, i = 0(1)j-1, are excluded from the calculation of r_j.

Notes There are two simple checks on the output.

(1) The first autocorrelation coefficient (that is, the one with zero lag) is unity.

(2) If r_0 and r_0' denote the cross correlation coefficients with zero lag for series 1 leading and series 2 leading respectively, then $r_0 \equiv r_0'$.

R.A.E. PROGRAM 300

Title Fourier analysis (periodogram)

Code Deuce basic.

Purpose To calculate the harmonic components A_s, B_s for s = a(b)c.

Order of cards Program (cards 0-28), parameter card, data.

Parameter card

Columns	17 - 20	n
	21 - 24	zeros
	25 - 28	a
	29 - 32	b
	33 - 36	c

Data The n values of y_r are punched as 8 signed 4 digit integers per card, in the α-field.

End card None.

Operation (1) Read the program with the initial input key.
(2) Run in the parameter card, followed by the data.
When a run is complete, (2) may be repeated.

Output

Columns	17 - 20	s ;	sign in column 17
	21 - 30	I_s ;	21
	31 - 38	A_s ;	31
	39 - 46	B_s ;	39

All these numbers are integers.

Parameters None.

Restrictions
(1) $0 \leqslant s \leqslant 2047$
(2) $n M^2 \leqslant 1{\cdot}08 \times 10^9$, where $|y_r| \leqslant M$, $r = 0(1)n-1$
(3) $n^2 M \leqslant 2{\cdot}14 \times 10^9$

Failures
(1) s is punched incorrectly for s > 2047, but the other quantities may be correct.
(2) If Restriction (3) is violated, the program might punch the corresponding I_s as -1 and sound the hooter. But rubbish may be punched with no indication.

Time About nN/200 seconds, where N is the number of values of s.

Method

$$A_0 = \sum_{r=0}^{n-1} y_r \quad , \quad B_0 = 0 \qquad \text{when } s = 0$$

$$\begin{matrix} A_s \\ B_s \end{matrix} = \sum_{r=0}^{n-1} y_r \begin{matrix} \cos \\ \sin \end{matrix} \left(\frac{2\pi rs}{n}\right) \qquad \text{when } s > 0$$

$$I_s = \frac{2}{n}\left(A_s^2 + B_s^2\right) \qquad \text{for all } s,$$

where s is an integer.

Notes The method of calculation is described in The Computer Journal, 1, 4, p. 162.

R.A.E. PROGRAM 528

Title Fourier analysis (periodogram)

Code Deuce basic.

Purpose To calculate harmonic components A_s, B_s for $s = 0(a_1)b_1(a_2)b_2(a_3)b_3$

Order of cards Program (cards 0-28), parameter card, data.

Parameter cards

Columns	
17 - 20	n
21 - 24	k
25 - 28	a_1
29 - 32	b_1
33 - 36	a_2
37 - 40	b_2
41 - 44	a_3
45 - 48	b_3

No column may be left blank, so if a_2, b_2, a_3, b_3 are not needed, zeros should be punched in their place.

Data The n values of y_r are punched as 8 signed 4 digit integers per card, in the α-field.

End card None.

Operation
(1) Read the program with the initial input key.
(2) Run in the parameter card, followed by the data.
When a run is complete, (2) may be repeated.

Output

Columns			
18 - 21	s ;	sign in column	17
24 - 30	A_s ;		23
33 - 39	B_s ;		32
42 - 48	I_s ;		41

There are three more decimal places in A_s, B_s, and I_s than in y_r.

Parameters None.

Restrictions $n^2M \leq 2{\cdot}14 \times 10^9$, where $|y_r| \leq M$, $r = 0(1)n-1$.

Failures None

Time About $nN/200$ seconds, where N is the number of values of s.

Method

$$A_0 = \frac{1}{n}\sum_{r=0}^{n-1} y_r , \quad B_0 = 0 \quad \text{when } s = 0$$

$$\begin{matrix} A_s \\ B_s \end{matrix} = \frac{2}{n}\sum_{r=0}^{n-1} y_r \begin{matrix} \cos \\ \sin \end{matrix} \left(\frac{2\pi rs}{kn}\right) \quad \text{when } s > 0$$

$$I_s = \sqrt{A_s^2 + B_s^2} \qquad \text{for all s}$$

where s is an integer. If $0 \leq c \leq 13$ is an integer, then $k = 2^c$.

Notes The method of calculation is described in The Computer Journal, 1, 4, p. 162.