NATIONAL INSTITUTE OF OCEANOGRAPHY

WORMLEY, GODALMING, SURREY

N.I.O. Computer Programs 8

N.I.O. Internal Report No. 8

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Errata to

N.I.O. Internal Report No. N.8

1) Cover: Line 4 should read:

N.I.O. INTERNAL REPORT No. N.8

2) N.I.O. Program 82 p.1.
Line 14 should read:

......a maximum of 14 data.......

Line 32 should read:

......of main variables (directives) required......

Last line but five should read:

......(or other data as on program tape)

3) N.I.O. Program 82 p.2.
Line 5 should read:

...... +2M+2L(int. pt. K/2) + (2M......

Last line but two should read:

......for the first set of data......

4) N.I.O. Program 82 p.3
After line 31 insert:

ψ_{xy} the lagged normalised cross-covariances for each pair of series (preceded by the value of r) for r = -L (1) L.

Line 34 should read:

......auto-spectra, in (physical units)^2......

Last line but two should read:

......numbers 1 to R.

5) N.I.O. Program 82 p.4
Line 31 should read:

......yn-1, etc.

Line 32 should read:

......the respective conversion factors......

6) N.I.O. Program 82 p.6
Line 4 should read:

Coherence: $\gamma^2 = \dfrac{I^2}{E_{xx}\,E_{yy}}$

Line 7 should read:

......(n being an integer......

Line 8 should read:

......a decimal having values between......

4th January, 1967.

N.I.O. COMPUTER PROGRAMS 8

edited by

Pamela Edwards

Internal Report N.8

National Institute of Oceanography

N.8

73	Echo sounding conversion tables
* 74	Spectral Analysis (with data selection and error detection)
* 75	Fourier Analysis
76	Rough tidal weights from harmonic constants
77) 77A)	Determination of position from Loran coordinates (Ballarin)
78	Data sorting
79	Analysis of a marine benthic community
80	Approximation to a directional wave spectrum
81	Conversion of fixed field cards to paper tape
* 82	Spectral Analysis with pre-whitening
83	Underwater trajectory
84	Five to seven track tape conversion

* These programs are available in compiled form on magnetic tape.

N.I.O. PROGRAM 73

Title Echo sounding conversion tables

Language EMA

Machine ATLAS 1

Purpose To obtain a fathom by fathom interpolation of Matthews' "Velocity of sound in sea water", with tables of corrected depths in fathoms and metres. Two sets of tables are normally required, one for each of the echo-sounder nominal working velocities of 800 and 820 fathoms/second.

Tapes All documents are normally on one tape, i.e. job description followed by program and data.

Job description The output specification should be set to (100 + 30 W) lines or (1 + W) blocks of 7 track tape, where W is the total number of lines of input data.

The store required is 15/60 blocks

Computing instructions = (250 + 100 W).

Program and data

The program starts

COMPILER EMA

and ends

CLOSE

The parameters follow and consist of

1) the nominal working velocity of the echo-sounder in fathoms/second, e.g. 800.

2) the Matthews area number

3) W, the number of lines of data for that area, each line consisting of 3 numbers.

Then follows the data, which consists of W lines of values (in fathoms) of depth at 100 fathom intervals, correction for that depth, and correction for 100 fathoms deeper.

New sets of data can follow as required, starting at parameter 2), i.e. the area number. Only one value of echo-sounding velocity is permissible in any one run.

The last set of data should end

/ and then

*** Z

Output Each page of output is headed by the Matthews area number and the nominal working velocity of the echo-sounder. The columns are headed and are:-

the sounding depth (fathoms)
corrected depth (fathoms)
corrected depth (metres)

Restrictions The corrections must be given at 100 fathom intervals.

For depths greater than 5000 fathoms the layout of the output would be incorrect, i.e. the page width would be exceeded.

Cost £ W/10 approx.

<u>Method</u> The correction is interpolated linearly at fathom intervals from that given at 100 fathom points and is then added to the observed depth. A factor converts the corrected depth from fathoms to metres. The data is taken from D.J. Matthews' "Tables of the velocity of sound in pure water and sea water for use in echo-sounding and sound-ranging" published by the Hydrographic Department, Admiralty, 1939.

<u>Programmer</u> STEWART KEMPSTER (see JAMES CREASE)

N.I.O. PROGRAM 74

Title Spectral Analysis (with data selection and error detection).

Language EMA.

Machine Atlas 1 (University of London, A.C.S.)

Purpose To compute the means, variances, normalised auto-covariances, and auto-spectra of 1-7 time series; also, for 2 or more series, to compute the normalised cross-covariances, co- and quad-spectra, phase lags, intensities, and coherences for the series taken in pairs. The program differs from N.I.O. Program 67 in that up to 8 series may be read in although only 7 or less are actually to be used. In addition, the program will detect and record errors in the data.

Tapes 1) Program tape

2) Job description and first (or only) set of data.

Other sets of data may follow, up to a maximum of 14 data tapes in all.
Long tapes may be split into two parts if required, but this will limit the total number that can be analyzed.

In this program, the job description and first set (or part of a set) of data form one document. Each succeeding tape is a separate document and so is the program itself.

Program tape

(Capitals denote actual punching)

COMPILER EMA

S001 NIO PROGRAM 74 SELECTED SPECTRA 17/7/1965 (or date of appropriate version)

MAIN q

AUXILIARY (o,p)

DUMPS 0

Then follows the program, ending with

* * * Z

Program parameters

The Directives for chapters 1 and 2 must be set as described on the program listing at the beginning of the relevant chapters; directives for chapter 0 need not be altered.

The main store requirement q should then be set to be greater than the total number of main variables (directives) required in any of the three chapters. (Note that X ⟶ 1000 allocates 1001 storage locations).

The auxiliary store requirement p should be set to be greater than $[D' + (L + M + 2)K^2]$ or $[D' + NK]$, whichever is the greater, where $D' = K(2M + 6)$, L is the number of lags required, M is the number of frequencies required, N is the number of terms/series and K is the number of series to be used in the computations.

When using the program with several sets of data, the main and auxiliary directives should be set to their largest required values.

Job description and data tapes

JOB

Job No. (see P. Edwards), and Job Title.

INPUT

0 S001 NIO PROGRAM 74 SELECTED SPECTRA 17/7/1965 (or other date as on program tape).

SELF = 1

The remaining sets of data, if any, are then listed, with the document numbers running consecutively from 2 to a maximum of 14. Note that the two parts of a single set of data must be on consecutive documents, and in the correct order.

e.g.	2	S001	RUN	32
	3	S001	RUN	74 PART 1
	4	S001	RUN	74 PART 2
	5	S001	RUN	67

Then follows:-

OUTPUT

0 LINEPRINTER m LINES

STORE s BLOCKS

COMPUTING t INSTRUCTIONS

The number of lines of output

$$m = B\left\{150+L+M+2L(\text{int.pt.}K/2) + (M+6)\left[\frac{K(K-1)}{2}\right]\right\}$$

where B is the number of sets of data.

The store s blocks $= 11 + \left(\frac{q + p + 6000}{512}\right)$

The computing t instructions $\simeq 5000 + K^2NL/80$ for one set of data and $B(5000 + K^2NL/80) - 1800(B-1)$ for B sets of data.

Then follows the first or only set of data preceded by its parameters:-

1) DATA
2) DATA TITLE
3) Title of the first set of data (one line only)
4) An integer (A) equal to 1 or 2 according to whether the data is in one or two parts.
5) An integer (K) equal to the number of series to be used in the spectral analysis.
6) An integer (L) equal to the number of covariance lags required.
7) An integer (M) equal to the number of frequencies required.
8) An integer (R) equal to the total number of series to be read in.
9) K integers, being the numbers of the series required for spectral analysis. (N.B.: The first series is 1 and the last is R). The numbers may be in any order.
10) The K numbers required to convert the K series to physical units (each number being a ratio of physical/computing units).
11) The K numbers being the time lags of each of the K series relative to the first, in decimal fractions of the time between successive terms of the same series. Thus the first number should be zero, and if all the series were sampled simultaneously all the other numbers will be zero also.

12) T, the sampling interval

Then follows the data, the R series of numbers being arranged sequentially, i.e., with the first terms of all the series first, the second terms next, and so on. The data must obey normal Atlas rules.

Any subsequent sets of data should begin:

1) DATA

S001 RUN 32 (or other name, exactly as used in the job description)

Then continue punching from 2) as for the first set of data i.e.

DATA TITLE etc.

If a set of data is split into two parts, the second part should begin

DATA

S001 RUN 74 PART 2 (e.g.)

followed by the data.

Each set of data should be terminated with

ø ø >> (with two exceptions listed below)

followed by 12" runout then

* * * Z

If the data is split into two parts, the first half should have)) in place of >>

and the final, or only, data tape should have // in place of >>.

<u>Output</u> [The series are referred to as numbers 1 to K in the order in which they were read in, not in the order that they were selected in item 9 above.]

Data title.

Number of terms in each series.

Time lags (decimal fractions of time between successive terms of the same series).

X, the means of the series.

X', the means converted to physical units.

V_x, the variances of the series.

V_x', the variances converted to physical units.

Ψ_{xx}, the lagged normalised auto-covariances (preceded by the value r) for r = 0(1)L, where L is the number of lags specified.

V_{xy}, V_{xy}', the un-normalised cross-covariances at zero lag, firstly in computing units and secondly in physical units.

Ψ_{xy}, the lagged normalised cross-covariances for each pair of series (preceded by the value r) for r = -L(1)L.

The frequency increment between successive spectral estimates, in cycles/time unit.

E_{xx}, auto-spectra, in (physical units)^2/cycle/time unit (preceded by s, where s = 0(1)M).

N.8 Cross-spectra for each pair of series, preceded by s and consisting of:-

E_{xy}, $E_{xy}{}^*$, co and quad-spectra (in physical units^2/cycle/time unit.)

I, intensity.

ϕ, phase (in degrees).

γ^2, coherence.

If data contains any errors the words "ERROR IN DATA" will be printed followed by a list of the series and term numbers containing the errors. Errors in any of the R series will be recorded and the series are referred to by the numbers 1 to R.

Letters and spurious characters will all be detected, but a single space will not. This means that two adjacent numbers may be read as one number resulting finally in there being one too few terms in the last series. This will be detected and the words "SERIES HAVE DIFFERENT NUMBERS OF TERMS" will be printed.

Single >,) or / will be logged as errors but >>,)) or // will be read as data terminators.

If the total number of incorrect terms exceeds 4R the logging will cease and the program will go on to the next set of data. This will also happen if the total number of individual spurious characters or letters exceeds 100 (e.g. if a spurious letter shift occurs all the succeeding characters will be taken as one term and no usef l purpose would be served by logging them all to the end of the data.

If a set of data contains any errors at all, even in the series not used, the program will not carry out any computations but will proceed to the next set of data.

Restrictions

$1 \leqslant K \leqslant 7$

$1 \leqslant R \leqslant 8$

$K \leqslant R$

$1 \leqslant M \leqslant L \leqslant N/2$

$N < 10,000$

Maximum number of data documents = 14

Cost

Examples (all for 1 set of data; price per set reduces for B > 1)

1)	For analyzing 1 series of 1200 terms, M = L = 50	£9
2)	For 2 series of 2400 terms, L = 100, M = 50	£12
3)	For 7 series of 500 terms, L = M = 50	£18
4)	For 3 series of 3000 terms, L = M = 100	£25
5)	For 2 series of 5000 terms, L = M = 500	£

Definitions

Let the first series of data x_i be $x_0, x_1, x_2, x_3 \ldots\ldots x_{N-1}$, and the second series of data y_i be

$y_0, y_1, y_2, y_3 \ldots\ldots y_{N-1}$, etc.

Let F, G, etc., be the respective conversion factors from computing to physical units.

Means:

$$X = \frac{1}{N} \sum_{i=0}^{N-1} x_i \text{ for each series of N terms.}$$

$$X' = FX$$

Variances:

$$V_x = \frac{1}{N-1} \sum_{i=0}^{N-1} x_i{}'^2$$

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where $x_i{}' = x_i - X$, then $V_x{}' = F^2 \ V_x$

Lagged Normalised auto-covariances:

$$\Psi_{xx}(r) = \left[(N-r-1)V_x\right]^{-1} \sum_{i=0}^{N-r-1} x_{i+r}' \; x_i'$$

Cross-covariances:

$$V_{xy} = \frac{1}{N-1} \sum_{i=0}^{N-1} x_i' \, y_i'$$

$$V_{xy}' = FG \; V_{xy}$$

Lagged Norm lised cross-covariances:

Negative lags:-

$$\Psi_{xy}(r) = \left[(N-r-1)\sqrt{V_x V_y}\right]^{-1} \sum_{i=0}^{N-r-1} x_{i+r}' \, y_i'$$

Positive lags:-

$$\Psi_{yx}(r) = \left[(N-r-1)\sqrt{V_x V_y}\right]^{-1} \sum_{i=0}^{N-r-1} x_i' \, y_{i+r}'$$

Frequency increment between successive spectral estimates is

$0{\cdot}5/LT$ cycles/time unit (e.g., c/sec.)

i.e., $0{\cdot}5/L$ cycles/time interval (T)

Auto-spectra:

$$E_{xx}(s) = 4TV_x' \sum_{r=0}^{L}{}'' \phi_{xx}(r) \cos\left(\frac{rs\pi}{L}\right)$$

where $\sum''$ means the sum with the first and last terms halved,

and $\phi_{xx}(r) = \Psi_{xx}(r) \cos^2\left(\frac{\pi r}{2L}\right)$

Cross-spectra:

Co-spectra:

$$E_{xy}(s) = 4TFG\sqrt{V_x V_y} \sum_{r=0}^{L}{}'' \tfrac{1}{2}\left[\phi_{yx}(r) + \phi_{xy}(r)\right] \cos\left(\frac{rs\pi}{L}\right)$$

where $\phi_{yx}(r) = \Psi_{yx}(r) \cos^2\left(\frac{\pi r}{2L}\right)$

and $\phi_{xy}(r)$ is similarly defined.

Quad-spectra:

$$E_{xy}^*(s) = 4TFG\sqrt{V_x V_y} \sum_{r=0}^{L}{}'' \tfrac{1}{2}\left[\phi_{yx}(r) - \phi_{xy}(r)\right] \sin\left(\frac{rs\pi}{L}\right)$$

The Cross-spectrum is then defined as $E_{xy} + iE_{xy}^*$

Intensity: $I = \sqrt{E_{xy}^{2} + E_{xy}^{*2}}$

Phase lag: $\phi = \tan^{-1} \dfrac{E_{xy}^{*}}{E_{xy}}$

If ϕ is positive series x leads series y in time.

Coherence: $\gamma^2 = \dfrac{I^2}{E_{xx} E_{yy}}$

Corrections for time lags between series

If there exists a time lag xT between corresponding terms of series n and series 1 (n being an integer with values from 2 to 7 inclusive, and x being a decimal having values between 0 and 1 inclusive), then the cross-spectral estimates $E_{xy}(s)$ and $E_{xy}^{*}(s)$ are modified as follows to $F_{xy}(s)$ and $F_{xy}^{*}(s)$ where:-

$$F_{xy}(s) = E_{xy}(s) \cos\left(\frac{\pi s x}{L}\right) - E_{xy}^{*}(s) \sin\left(\frac{\pi s x}{L}\right)$$

and $$F_{xy}^{*}(s) = - E_{xy}^{*}(s) \cos\left(\frac{\pi s x}{L}\right) - E_{xy}(s) \sin\left(\frac{\pi s x}{L}\right)$$

Then the computations of intensity, phase lag and coherence proceed using $F_{xy}(s)$ and $F_{xy}^{*}(s)$ in place of $E_{xy}(s)$ and $E_{xy}^{*}(s)$.

Notes

1) This program uses NIO Programme -1 to carry out the computations, programmer: JAMES CREASE.

2) The method of evaluating the power spectrum is identical to that in NIO Programs 60, 67 and 69 except that the terminal values of the spectra are evaluated slightly differently in NIO 60 and 69.

Programmer BRIAN HINDE

N.I.O. PROGRAM 75

Title Fourier Analysis

Language EMA

Machine ATLAS I

Purpose To compute the Fourier coefficients of a single set of data points

Tapes
1) Program Tape
2) Job description and data

Program Tape COMPILER EMA

S001 NIO PROGRAM 75 FOURIER ANALYSIS (ATLAS) 4/3/1966
(or date of appropriate version)

MAIN 480

AUXILIARY (0, p)

DUMPS 0

Then follows the program ending with

*** Z

The auxiliary store p ⩾ n (the total number of terms)

Job description and data

The output specification should extend to 3W/2 lines where W is the number of harmonics required or to the integral part of (1 + W/40) tape blocks.

The store required ⩾ ~~10~~ 15 + p/512

The computing instructions = 500 + Wn/20

The data begins with a run number (A) which must be integer followed by W, the number of harmonics required.

Then follow the n values of y_r which may be positive or negative but should not exceed 1000 for the layout as at present arranged. The tape s ould end with > on a new line followed in the usual way by *** Z

Output The output is headed with the run number and then follow the values of j, a_j, b_j, c_j and Ψ_j (see Method) with 3 digits before and 6 after the decimal point. Extra new lines are included after every 10th and 100th harmonic.

If there is a spurious character in the data the program outputs this character followed by the next ten numbers on the data tape.

Restrictions $n \leqslant 100{,}000$

$W \leqslant \frac{n-1}{2}$ for n odd and $\leqslant \frac{n}{2}$ for n even

$A \leqslant 9999$

Cost Approximately $\frac{3n}{20}$ shillings when $W \simeq n/2$

Method Given F(x) is of period 2π, its values y_r being known only at a discrete set of equally spaced points, x_r, covering a period interval, the program computes the coefficients a_j, b_j, c_j, Ψ_j when F(x) is expressed in the form:-

$$y = a_0 + a_1\cos x + a_2\cos 2x + a_3\cos 3x + \ldots\ldots\ldots$$
$$+ b_1\sin x + b_2\sin 2x + b_3\sin 3x + \ldots\ldots\ldots$$

or

$$y = c_0 + c_1\sin(x + \Psi_1) + c_2\sin(2x + \Psi_2) + \ldots\ldots$$

$$y_r = y_0\ y_1\ y_2\ \ldots\ldots\ y_{n-1}$$

$$x_r = 0\ \frac{2\pi}{n}\ \frac{2\pi.2}{n}\ \ldots\ldots\ \frac{2\pi(n-1)}{n}$$

where $r = 0(1)\ n-1$ and n = number of terms in $F(x)$.

If j refers to any coefficient

a_j = the coefficient of the j^{th} cosine harmonic

b_j = the coefficient of the j^{th} sine harmonic

c_j = the coefficient of the j^{th} $\sin F(x)$ harmonic

Ψ_j = the phase of the j^{th} $\sin F(x)$ harmonic. ($\Psi_j = x_j$ on output)

$$a_0 = \frac{1}{n}\sum_{r=0}^{n-1} y_r \ :\ b_0 = 0\ :\ c_0 = a_0\ :$$

$$a_j = \frac{2}{n}\sum_{r=0}^{n-1} y_r \cos\left(2\,\frac{rj\pi}{n}\right) = \frac{2}{n}A_j$$

$$b_j = \frac{2}{n}\sum_{r=0}^{n-1} y_r \sin\left(\frac{2rj\pi}{n}\right) = \frac{2}{n}B_j$$

$$c_j = \pm\sqrt{a_j^2 + b_j^2}\ :\ \Psi_j = \tan^{-1}\left(\frac{a_j}{b_j}\right)$$

where $j = 1(1)\ \frac{n-1}{2}$ when n is odd

$j = 1(1)\ \frac{n}{2} - 1$ and

$$a_{\frac{n}{2}} = \frac{1}{n}\sum_{r=0}^{n-1} y_r \cos(r\pi)\ :\ b_{\frac{n}{2}} = 0\ :\ c_{\frac{n}{2}} = a_{\frac{n}{2}}$$

when n is even,

In this program a_j and b_j are calculated from A_j and B_j, where

$$A_j = t_0 - t_1\cos\left(\frac{2j\pi}{n}\right)$$

$$B_j = t_1\sin\left(\frac{2j\pi}{n}\right) \text{ and}$$

$$t_n = 0$$

$$t_{n-1} = y_{n-1}$$

$$t_i = 2t_{i+1}\cos\left(\frac{2j\pi}{n}\right) - t_{i+2} + y_r \text{ where } i = n-2\ (-1)0.$$

Notes — The program is based on RAE 281/A by Miss E.V. Hartley

Programmer — BRIAN HINDE

N.I.O. PROGRAM 76

Title Rough tidal weights from harmonic constants

Language Mercury Autocode

Purpose To compute two sets of six weights for expressing roughly the diurnal and semi-diurnal tide at a port as convolutions of the complex spherical harmonic coefficients C_2^1 and C_2^2 (reference 2) with lags -2, 0, and +2 days respectively, using the principal harmonic constants.

Tapes Program. Data.

Data Ten pairs (H in feet, g in degrees) from the following constituents in the order given:

Q_1 O_1 P_1 K_1 J_1 $2N_2$ N_2 M_2 S_2 K_2

Further similar sets of ten pairs from other ports may follow.

End indication

Program and data tapes end with ——>

Operation

Read program, then data. Output for one species is punched before reading data for next species, or for next port.

Output

1) 5 rows: f H g X Y for diurnals

2) A 3 x 4 array:

p_0	p_0'	u_{-1}	v_{-1}
p_1	p_1'	u_0	v_0
q_1	q_1'	u_1	v_1

3) Weighted variances of X values before and after least squares fit, followed by similar variances of Y values.

4) 21 rows: f X Y R ϕ for f = 0·85 (·01) 1·05

5 - 8) As 1) - 4) for semi-diurnals, except that in 8) f = 1·85 (·01) 2·05.

Restrictions and failures None

Time 0·7 + 0·9p minutes, where p is the number of ports.

Method For each pair (H,g), a complex linear admittance is evaluated from the forumla

$$Z = X+iY = K(H/H')e^{-ig} \tag{1}$$

In (1), $H' = \frac{1}{2}c'(M/E)\ (a^2\ p^2/c^2) = 0{\cdot}5857\ c'$ feet, where c' is the appropriate coefficient given in reference 1 (e.g. 0·90812 for M_2), and

$$K = \sqrt{(5/24\pi)} \quad \text{(minus for diurnals)},$$

in accordance with the normalising factor used for spherical harmonics in reference 2. (The actual value of Kc'/H' used in the program is ± 0·4397). The admittance Z(f), (f = frequency in cycles per day), relates local sea level to the gravitation potential coefficients $a_2^1(t)$ or $a_2^2(t)$ used in reference 2. Sine waves of period 0·5 in f are fitted to the sets of X(f) or Y(f) values by the weighted least-squares formulae:

$$\sum_{n=1}^{5} H_n^2 \left[p_0 + p_1 \cos 4\pi f_n + q_1' \sin 4\pi f_n - X(f_n) \right]^2 \text{ minimum}$$

$$\sum^{5} H_n^2 \left[p_0' + p_1' \cos 4\pi f_n + q_1' \sin 4\pi f_n - Y(f_n) \right]^2 \text{ minimum}$$

where the appropriate frequencies f_n are included (as also c'_n) as program constants.

The weights u_r and v_r in the convolution form

$$\sum_{n=-1}^{1} \left[u_r a\ (t-2\cdot 0r) + v_r\ b\ (t-2\cdot 0r) \right] \qquad \text{(t in days)}$$

are derived from the sets p_r and q_r by the relations

$$u_0 = p_0, \quad v_0 = q_0,$$
$$u_{-1} = \tfrac{1}{2}(p_1 + q_1'), \quad v_{-1} = \tfrac{1}{2}(p_1' - q_1)$$
$$u_1 = \tfrac{1}{2}(p_1 - q_1'), \quad v_1 = \tfrac{1}{2}(p_1' + q_1).$$

The first two variances given are

$$\Sigma H_n^2\, X_n^2 \Big/ \Sigma H_n^2, \text{ and } \Sigma H_n^2 \left[X_n^2 - p_0 X_n - p_1 X_n \cos 4\pi f_n - q_1 X_n \sin 4\pi f_n \right] \Big/ \Sigma H_n^2$$

respectively; the second two variances relate to Y similarly.

The final lists (4) and (8) are of

$$\tilde{X}(f) = p_0 + p_1 \cos 4\pi f + q_1 \sin 4\pi f,$$

$$\tilde{Y}(f) = p_0' + p_1' \cos 4\pi f + q_1' \sin 4\pi f,$$

$$\tilde{R}(f) = \sqrt{(\tilde{X}^2 + \tilde{Y}^2)}, \text{ and } \phi = \arctan(\tilde{Y}/\tilde{X})$$

for the given ranges of values of f.

Notes

1) The weights u, v are intended, not for a serious prediction formula, but for a convenient "complex sea level" input in non-linear analysis.

2) The weights H^2 in the least-squares formulae assume a sampling variance of X and Y proportional to (noise energy/signal energy), with the noise energy level constant over the frequency band concerned.

3) The constituents M_1 and L_2 would fill gaps in the frequency scale, but have been omitted because they are relatively small, their definition is dubious, and the interpolation between the reliable pairs O_1, K_1 and M_2, S_2 is probably adequate.

4) An analysis involving lags other than ± 2 days can be obtained by replacing the instruction $C_0 = 4\pi$ at the head of Chapter 0 by $C_0 = h\pi/12$ where h is the required lag in hours.

Programmer D.E. CARTWRIGHT

References

(1) Doodson A.T. (1921) Proc. Roy. Soc. A 100, 305-329.

(2) Munk W.H. and Cartwright, D.E. (1965) Phil. Trans. Roy. Soc. (in press).

N.I.O. PROGRAM 77

Title	Determination of position from Loran coordinates (BALLARIN).
Language	EMA
Machine	ATLAS 1
Purpose	To compute the geographical position from the two Loran readings of a Loran chain having a common master and two slaves.
Inputs	0) Program 1) Job description and data

Program

The program starts

COMPILER EMA

S001 NIO 77 POSITION FROM LORAN COORDINATES (BALLARIN), 20/7/1966 (or date of appropriate version)

and ends

CLOSE

***Z

Job description and data

In the job description, the number of lines of output = (number of Loran pairs for which position is required + number of captions + 100),

the store required = 15/70 blocks,
the number of computing instructions = 700 + 8 (number of positions to be computed) + 8 (number of Loran chains).

The job description is followed by

DATA

1) DATA TITLE
2) one line of general description of the data
3) geographical position of master station in degrees and minutes (with decimals). N and W are +ve, S and E are -ve
4) geographical position of first slave
5) geographical position of second slave
6) coding delay of first slave i.e. the reading on the slave base-line extension
7) coding delay of second slave i.e. the reading on the slave base-line extension
8) / (solidus)
9) DATA TITLE
10) one line of specific comment on the data to follow
11) Loran readings of first slave
12) Loran readings of second slave

Items 11) and 12) may be repeated as often as required, i.e. any number of pairs of Loran readings may be input.

A new specific comment may be introduced by repeating items 8) and 9).

A new Loran chain may be introduced by following item 12) by * (asterisk) and returning to item 1). The last set of data should be followed by > and ***Z.

Output

General data title
Specific data title
Headed columns as follows:-

- slave 1 coordinates
- slave 2 "
- geographical position, i.e. latitude and longitude
- error - should be zero

If a second geographical position appears on the same line this means that the two hyperbolae defined by the pair of Loran readings have a second intersection within $\frac{1}{2}$ radian (about 1700 miles) of the master. The choice between the two positions then depends on the user's knowledge of approximate position.

Method

This is as described by BALLARIN in "The International Hydrographic Review" vol. XXVII, 2, p.31-50. It involves transferring coordinates on a spheroid to coordinates on a sphere. The spheroid used is HAYFORD'S.

The velocity of propagation of radio waves in the ground mode over the sea is taken to be 299·6929 m/μs.

Note

The common station of the three must be the master.

Programmer

JAMES CREASE

N.I.O. PROGRAM 77/A

This program is similar to NIO Program 77; it differs in its purpose, which is to produce a table of geographical positions from a regular sequence of pairs of Loran readings from a Loran chain. In this program (unlike NIO Program 77) the common station of the Loran chain need not be the master; indeed, it might be the slave of two separate masters at the other stations.

The only practical ways in which NIO Program 77/A differs are as follows:-

Data item 6) coding delay of first master-slave pair, i.e. the Loran reading on the slave or master base-line extension, whichever is less, followed by +1 if the master station is the common station and -1 otherwise.

item 7) same as 6) but for the second master-slave pair.

item 11) first Loran reading of the first pair followed by the increment (+ ve or - ve) for successive readings, followed by the number of increments.

item 12) first Loran reading of the second pair, and so on as in item 11).

Note that the station which acts as master for one pair is not necessarily master for the second pair.

Output The layout of the output will be the same as for NIO Program 77 but will start with the geographical position of the point of intersection of the first pair of Loran readings listed in items 6) and 7). Following positions will be those obtained by incrementing the second Loran reading by the specified increment for the given number of steps. When this cycle is complete the first reading is incremented one step and the cycle of increments on the second pair repeated, and so on.

N.I.O. PROGRAM 78

Title Data sorting.

Language EMA

Machine Atlas I (University of London, A.C.S.)

Purpose Given a sequence of three-digit integers a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2, a_3 etc. to punch out two 5-hole tapes

i) a_1, a_2, a_3 etc.
ii) c_1, c_2, c_3 etc

Program Tape (Capitals denote actual punching)

COMPILER EMA

S001 NIO PROGRAM 78 DATA SORTING 18/11/1965 (or date of appropriate version)

MAIN 5500

AUXILIARY (0,0)

DUMPS 0

Then follows the program, ending with

*** Z

Job description and data tape

JOB

Job No. (see P. Edwards), and Job Title

INPUT

0 S001 NIO PROGRAM 78 DATA SORTING 18/11/1965 (or other date as on program tape)

SELF = 1

OUTPUT

1 FIVE-HOLE PUNCH 5 BLOCKS

2 FIVE-HOLE PUNCH 5 BLOCKS

STORE 27 BLOCKS

COMPUTING 4000 INSTRUCTIONS

DATA

Then follows the data to be sorted, which must obey normal Atlas EMA rules.

The data may end with b̸ b̸ >
b̸ b̸ > >
b̸ b̸ /
or b̸ b̸ / /

Outputs i) OUTPUT 1

Job No. and Job Title

Then follows the first data term followed by the fifth, the ninth etc. punched in a suitable form for printing or inputting to NIO 74.

The tape will end with >>

END OUTPUT followed by the logging information.

ii) OUTPUT 2

Job No. and Job Title

Then follows the third data term followed by the seventh etc.

The tape will end with //

END OUTPUT followed by the logging information

Any errors in data will be indicated by the words "FAULT IN DATA" on output 1.

Restrictions Total number of terms of input $\leq$ 10,000.

Cost £7 for 10,000 terms.

Programmer BRIAN HINDE

N.I.O. PROGRAM 79

Title	Analysis of a marine benthic community
Language	EMA
Machine	ATLAS 1
Purpose	Special purpose program written for MARTIN ANGEL
Tapes	Job description, program and data all on one tape
Data	Pairs of values of numbers of readings of species and sediment grain size.
Output	For specified block sizes, values were obtained of mean, variance, covariance, correlation coefficient, and t (significance).
Method	The method followed was that of pattern analysis with covariance analysis described by GREIG-SMITH.
Programmer	JAMES CREASE

References

GREIG-SMITH, P. 1952. The use of random and contiguous quadrats in the study of the structure of plant communities. Ann. Bot. 16, 293-316.

GREIG-SMITH, P. 1961. Data on pattern analysis within plant communities.
1. The analysis of pattern. J. Ecol., 49, 695-702.

KERSHAW, K.A. 1957. The use of cover and frequency in the detection of pattern in plant communities. Ecology, 38, 291-299.

KERSHAW, K.A. 1961. Association and covariance analysis of plant communities. J. Ecol., 48, 233-242.

N.I.O. PROGRAM 80

Title Approximation to a directional wave spectrum.

Language EMA

Machine ATLAS 1

Purpose To estimate the directional spectrum of sea waves from values of the first four angular harmonics (as obtained using NIO Program 68 for example).

Tapes
1) Program tape
2) Job description and data

Program tape (Capitals denote actual punching)

COMPILER EMA

S001 NIO PROGRAM 80 APPROX DIRECTIONAL WAVE SPECTRUM 31/1/1966
(or date of appropriate version)

MAIN 2000

AUXILIARY (0, 0)

DUMPS 0

Then follows the program, ending with

*** Z

Job description and data

JOB

Job No., Job Title

INPUT

0 S001 NIO PROGRAM 80 APPROX DIRECTIONAL WAVE SPECTRUM 31/1/1966
(or other date as on program tape)

SELF = 1
OUTPUT
0 LINEPRINTER ℓ LINES

STORE 20 BLOCKS

COMPUTING t INSTRUCTIONS

The number of lines of output is given by $\ell = 100 + (10M+50)B$
where M is the number of frequencies stipulated
and B is the number of sets of data specified.

The computing instructions $t = 750 + 50BM$.

Then follows the data:-

1) DATA
2) DATA TITLE
3) Title of the first set of data (one line only)
4) An integer (M) equal to the number of frequencies at which the directional spectrum is required.

Then followsthis sequence for each of the M frequencies:-

a) An integer representing the frequency (e.g. 5 for 0·05 c/s).
b) Angular harmonics a_1 a_2 b_1 b_2.
c) The energy at that frequency (e.g. the doubly integrated acceleration spectral estimate).
d) A calibration factor to bring the resulting spectrum to the required units (e.g. 3·2808 for converting metres to feet).

The next set of data then follows beginning with DATA TITLE

The last set of data should be followed by

DATA TITLE

END OF DATA

>

x x x Z

Output

Data Title

The angular harmonics a_1 to a_4 b_1 to b_4 for each frequency and also the value of c (see method).

The angular distribution of energy (normalised) at 15° intervals and at each frequency where the angles are the directions of propagation of energy.

The directional spectrum of energy in physical units at 15° intervals and at each frequency where the angles are the directions of arrival of energy.

The last set of results is terminated by END OF DATA

Restrictions

$M \leqslant 50$

$B \leqslant 20$

Cost

e.g. For M = 6, 22/- for a single set of data, 10/6 per set of data when 4 sets are input together.

Method

At each frequency:-

$$c = \frac{S_1 + S_2}{2}$$

where $S_1 = \frac{c_1}{1-c_1}$, $S_2 = \frac{(1 + 3c_2 + \sqrt{1 + 14c_2 + c_2^2})}{2(1 - c_2)}$

and $c_1 = +\sqrt{a_1^2 + b_1^2}$

$c_2 = +\sqrt{a_2^2 + b_2^2}$

$$\begin{matrix} a_3 \\ b_3 \end{matrix} = \frac{c(c-1)(c-2)}{(c+1(c+2)(c+3)} \cdot \begin{matrix} \cos 3\bar{\theta} \\ \sin 3\bar{\theta} \end{matrix}$$

where $\bar{\theta} = \tan^{-1} \frac{b_1}{a_1}$

and

$$\begin{matrix} a_4 \\ b_4 \end{matrix} = \frac{c(c-1)(c-2)(c-3)}{(c+1)(c+2)(c+3)(c+4)} \cdot \begin{matrix} \cos 4\bar{\theta} \\ \sin 4\bar{\theta} \end{matrix}$$

The angular distribution is then evaluated:-

$$G_8(\phi) = \frac{1}{\pi}\left[0{\cdot}50 + \left\{\sum_1^4 W_n\,(a_n\cos\phi + b_n\sin\phi)\right\}\right]$$

where $W_1 = 0{\cdot}8889$

$W_2 = 0{\cdot}6222$

$W_3 = 0{\cdot}3394$

$W_4 = 0{\cdot}1414$

for $\phi = 0, 15°, 30° \ldots\ldots\ldots\ldots 345°$

Finally the directional spectrum is evaluated:-

$$F(\sigma, \phi) = E(\sigma)\ K^2\ G_8(\phi')$$

where $\phi' = \phi + 180°$
and K is the calibration factor mentioned in section 4 d) of the data specification.

Notes

The above method should only be employed when either the data were insufficient to evaluate a_{3-4}, b_{3-4} directly (e.g. from a pitch-roll-heave buoy) or when the the values of these coefficients are in doubt (e.g. when the signal-to-noise ratio in wave curvature signals was poor).

The above method uses a_{1-2}, b_{1-2} to estimate mean directions and width parameters to fit the form

$$G(\phi)\alpha \quad \cos^{2c}\tfrac{1}{2}(\theta-\bar{\theta}),$$

Full details of this method of analysis may be found in:-

Longuet-Higgins, M.S., Cartwright, D.E., & Smith, N.D., "Ocean Wave Spectra" pub. by Prentice-Hall (1963) pp 111-132.

and

Cartwright, D.E., & Smith, N.D., "Buoy Technology" pub. by Marine Technology Society, Washington (1964) pp 112-121.

Programmer BRIAN HINDE

N.I.O. PROGRAM 81

Title Conversion of fixed field cards to paper tape

Language EMA and ATLAS I machine code

Machine ATLAS I

Purpose To convert data punched in fixed field format on cards into data on five or seven track paper tape. Four versions of the program are available, being for one or four input decks, each deck consisting of either two or three series. The series will be interleaved in a form suitable as input for NIO Programs 67, 74, 82 etc.

Input

1) Job description and program are normally combined as one document on input stream 0.
2) For single channel version:- One deck of cards on input 1
   For four channel version:- Four decks of cards on inputs 1-4.

Job description and program tape

The titles of the data decks should be listed exactly as on input 1 (single channel version) or on inputs 1-4.

The output will appear on the output stream corresponding to the input and the specification for each channel should extend to $\frac{3D}{4000}$ blocks where D is the number of digits to be punched out. The output can be on five or seven track tape.

The store required for compilation is 70 blocks
The store required for execution is 25 blocks (single channel version) or 31 blocks (four channel version) for two series; these are increased to 30 and 36 respectively for three series per channel.

The computing instructions ≈ 750 + NSB

where N = Number of terms per series
S = Number of series per channel
B = Number of channels (1 or 4)

```
COMPILER EMA
MAIN q ( q > sum of directives in chapter 0)
AUXILIARY (0,0)
DUMPS 0
```

Then follows the program title indicating the fields to be read from the cards, the number of channels, the number of series per channel and the program date.

e.g.

```
>> NIO PROGRAM 81 FIXED FIELD CARDS (COLUMNS 21-52) TO
>> PAPER TAPE FOR FOUR CHANNEL INPUT, 2 SERIES 19/5/1966
```

At label 20 at the end of the program is given the field specification for the data to be read. e.g. If the data is in columns 17-20 and 24-27 only, then the line should read 20) INTEGERS 17, 20, 24, 27, -1.

The table must always end with -1 and the number of columns specified must be even. The number of columns per field must not be less than four.

The directives at the beginning of chapter 0 and the limits on XI, YI' and ZK in that chapter should be set as required.

The program ends with

```
*** Z
```

Data

For each input channel:-

Cards 1-3: Blank
Card 4: DATA in columns 1-4
Card 5: Title exactly as in Job description, beginning in column 1
Card 6 onwards: Series 1 punched in the fields specified in the program. There should be no blank fields and the value of any of the terms should not exceed 9998. No over punching is allowed, so a negative number must be indicated by - in a column by itself. Decimal points may, however, be indicated by punching in rows 10, 8 and 3. All fields must be at least four columns long although smaller numbers can be accommodated by punching non-significant zeros or leaving the appropriate columns blank.

After the last number of series 1, the number 9999 should be punched in every remaining field of the card or, if the series ends in the last field, a card with 9999 in every field should be inserted. Then follows the second series terminated in the same way as the first, then the third series if required. The final card is the usual terminator card with 7 & 8 punched in its first column and Z in its last column.

If two or three sets of data are to be input using the four channel program, dummy inputs must be inserted, consisting of a card with 9999 in every field, after the title cards 1-5. This is followed by the terminator card as usual.

Output

On each output stream:-

NIO PROGRAM 81 OUTPUT

Sum of series 1

Sum of series 2

Sum of series 3 (if present)

Then follows the two series arranged as below:-

x_1 y_1 x_2 y_2

x_3 y_3 x_4 y_4 etc.

or the three series

x_1 y_1 z_1 x_2 y_2 z_2

x_3 y_3 z_3 x_4 y_4 z_4 etc.

Each number is preceded by its sign (if negative) or one space and followed by two spaces.

If a spurious character (those other than + - . 0-9) occurs in the input the words "FAULT IN DATA" will be output followed by "P=" and the faulty character.

If the two (or three) series have different numbers of terms the words "SERIES HAVE DIFFERENT NUMBERS OF TERMS" will be output.

If a blank field occurs in the input the words "BLANK FIELD" will be output.

Following the fault diagnosis will be the number of the last correct term read in labelled with

"I=" for series 1

"I′=" for series 2

and "K=" for series 3

If there are more terms per series than allowed for in the program (normally 2500), "TOO MANY TERMS" will be output

Whenever a fault is located the program will immediately go on to the next input channel.

Restrictions

$N \leq 2500$ (but can be greater by altering the limits set in chapter O)

$S = 2$ or 3 only

$B = 1$ or 4 only

No. of columns per field ≥ 4 and all terms ≤ 9998.

Cost

$\left(40 + \frac{NSB}{30}\right)$ shillings approximately.

Notes

The program uses routine 101 written by the staff of the Atlas Computing Service, University of London.

Programmer

BRIAN HINDE

N.I.O. PROGRAM 82

Title Spectral Analysis with pre-whitening.

Language EMA

Machine Atlas I

Purpose To compute the means, variances, normalised auto-covariances and auto-spectra of 1-7 time series with pre-whitening; also, for 2-7 series, to compute the normalised cross-covariances, co- and quad-spectra, phase lags, intensities and coherences for the series taken in pairs. Up to 8 series may be read in and up to 7 can be selected for analysis.

Tapes

1) Program

2) Job description and first set of data

Other sets of data may follow, up to a maximum of 4 data tapes in all.

Long tapes may be split into two parts if required, but this will limit the total number that can be analysed.

In this program, the job description and first set of data normally form one document. Each succeeding tape is a separate document and so is the program itself.

Program tape Begins

COMPILER EMA

S001 NIO PROGRAM 82 SPECTRAL ANALYSIS WITH PRE-WHITENING
19/4/1966 (or date of appropriate version)

and ends

*** Z

Program parameters

The Directives for chapters 1 and 2 must be set as described on the program listing at the beginning of the relevant chapters; directives for chapter 0 need not be altered.

The main store requirement should then be set to be greater that the total number of main cariables (directives) required in any of the three chapters. (Note that X ⟶ 1000 allocates 1001 storage locations.)

The auxiliary store requirement should be set to be greater than $[D' + (L + M + 2)K^2]$ or $[D' + NK]$, whichever is the greater, where $D' = K(2M + 6)$, L is the number of lags required, M is the number of frequencies required, N is the number of terms/series and K is the number of series to be used in the computations.

When using the program with several sets of data, the main and auxiliary directives should be set to their largest required values.

Job Description

JOB

Job No., Job Title

INPUT

O S001 NIO PROGRAM 82 SPECTRAL ANALYSIS WITH PRE-WHITENING
19/4/1966 (or other data as on program tape)

SELF = 1

The remaining sets of data, if any, are then listed, with the document numbers running consecutively from 2 to a maximum of 14. The two parts of a single set of data must be on consecutive documents, and in the correct order.

e.g. 2 S001 RUN 3L

3 S001 RUN 74 PART 1

4 S001 RUN 74 PART 2

Total lines of output generated =

$$B\left\{150 + L + 2M + 2L(\text{inst. pt. } K/2)+(2M + 6)\left[\frac{K(K-1)}{2}\right]\right\}$$

where B is the number of sets of data.

The amount of tape generated, if required, =

$$B\left\{0\cdot5+0\cdot005[L+2M] +0\cdot00L[\text{int.pt.}K/2] + \cdot01K[2M+6][K-1]\right\}B \text{ blocks.}$$

$$\text{Store required} = 11 + \left(\frac{p + q + 6000}{512}\right) + B$$

Computing instructions =

$$B\ (2000 + K^2NL/80) - 1800(B-1)$$

1) DATA
2) DATA TITLE
3) Title of the first set of data (one line only)
4) An integer (A) equal to 1 or 2 according to whether the data is in one or two parts.
5) An integer (K) equal to the number of series to be used in the spectral analysis.
6) An integer (L) equal to the number of covariance lags required.
7) An integer (M) equal to the number of frequencies required.
8) An integer (R) equal to the total number of series to be read in.
9) K integers, being the numbers of the series required for spectral analysis. (N.B.: The first series is 1 and the last is R.) The numbers may be in any order.
10) The K numbers required to convert the K series to physical units (each number being a ratio of Physical/computing units),
11) The K numbers being the time lags of each of the K series relative to the first, in decimal fractions of the time between successive terms of the same series. Thus the first number should be zero, and if all the series were sampled simultaneously all the other numbers will be zero also.
12) T, the sampling interval
13) C, the pre-whitening factor defined below in "Definitions".

Then follows the data, the R series of numbers being arranged sequentially, i.e., with the first terms of all the series first, the second terms next, and so on. The data must obey normal Atlas rules.

Any subsequent sets of data should begin:

1) DATA

S001 RUN 32 (or other name, exactly as used in the job description)

Then continue punching from 2) as for the first ser of data i.e.

DATA TITLE etc.

If a set of data is split into two parts, the second part should begin

DATA

S001 RUN 74 PART 2(e.g.)

followed by the data.

Each set of data should be terminated with

$\not{b}$ $\not{b}$ > > (with two exceptions listed below)

followed by 12" runout then

* * * Z

If the data is split into two parts, the first half should have)) in place of > >

and the final, or only, data tape should have // in place of > >.

Output

[The series are referred to as numbers 1 to K in the order in which they were read in, not in the order that they were selected in item 9 above.]

Data title.

NIO Program 82 output.

Number of terms in each series.

Time lags (decimal fractions of time between successive terms of the same series).

X, the means of the series.

X', the means converted to physical units.

V_x, the variances of the series.

V_x', the variances converted to physical units.

Ψ_{xx}, the lagged normalised auto-covariances (preceded by the value r) for $r = O(1)L$, where L is the number of lags specified.

V_{xy}, V_{xy}', the un-normalised cross-covariances at zero lag, firstly in computing units and secondly in physical units.

The frequency increment between successive spectral estimates, in cycles/time unit.

E_{xx}, E'_{xx} auto-soectra, in (physical units)^2/cycles/time unit (preceded by s, where $s = O(1)M$), first for the pre-whitened data and then after correcting for the pre-whitening factor.

Cross-spectra for each pair of series, preceded by s and consisting of:-

E_{xy}, E_{xy}^*, E'_{xy}, E'^*_{xy}, co- and quad-spectra (in physical units^2/cycle/time unit) both uncorrected and corrected.

I, I', intensity, uncorrected and corrected.

ϕ, phase (in degrees)

γ^2, coherence

The corrected values will not be printed for any frequency where the reciprocal of the attenuation factor due to pre-whitening is greater than 10^{10}

If data contains any errors the words "ERROR IN DATA" will be printed followed by a list of the series and term numbers containing the errors. Errors in any of the R series will be recorded and the series are referred to by the numbers 1 ro R.

Letters and spurious characters will all be detected, but a single space will not. This means that two adjacent numbers may be read as one number resulting finally in there being one too few terms in the last series. This will be detected and the words "SERIES HAVE DIFFERENT NUMBERS OF TERMS" will be printed.

Single >,) or / will be logged as errors but > >,)) or // will be read as data terminators.

If the total number of incorrect terms exceeds 4R the logging will cease and the program will go on to the next set of data. This will also happen if the total number of individual spurious characters or letters exceeds 100 (e.g. if a spurious letter shift occurs all the succeeding characters will be taken as one term and no useful purpose would be served by logging them all to the end of the data.)

If a set of data contains any errors at all, even in the series not used, the program will not carry out any computations but will proceed to the next set of data.

Restrictions

$1 \leq K \leq 7$

$1 \leq R \leq 8$

$K \leq R$

$1 \leq M \leq L \leq N/2$

$N < 10{,}000$

Maximum number of data documents = 14

Cost

Examples (all for 1 set of data; price per set reduces for B > 1)

1) For analysing 1 series of 1200 terms, M = L = 50 £9
2) For 2 series of 2400 terms, L = 100, M = 50 £12
3) For 7 series of 500 terms, L = M = 50 £18
4) For 3 series of 3000 terms, L = M = 100 £25

Definitions

Let the first series of data x_i be $x_0, x_1, x_2, x_3 \ldots x_{N-1}$, and the second series of data y_i be $y_0, y_1, y_2, y_3 \ldots$ $\ldots y_{-1}$, etc.

Let F, G, etc. be the respective conversation factors from computing to physical units

Pre-whitening: $x_i = x_i + C\, x_{i+1}$

Means: $$X = \frac{1}{N-1} \sum_{i=0}^{N-2} x_i$$ for each series of N terms.

$$X' = FX$$

Variances: $$V_x = \frac{1}{N-2} \sum_{i=0}^{N-2} x_i'^2$$

where $x_i' = x_i - X$, then $V_x' = F^2 V_x$

Lagged Normalised auto-covariances:

$$\Psi_{xx}(r) = \Big[(N-r-2) V_x \Big]^{-1} \sum_{i=0}^{N-r-2} x_{i+r}'\, x_i'$$

Cross-covariances:

$$V_{xy} = \frac{1}{N-2} \sum_{i=o}^{N-2} x_i' \, y_i'$$

$$V_{xy}' = FG \; V_{xy}$$

Lagged normalised cross-covariances:

Negative lags:-

$$\Psi_{xy}(r) = \left[(n-r-2)\sqrt{V_x V_y} \right]^{-1} \sum_{i=o}^{N-r-2} x_{i+r}' \, y_i'$$

Positive lags:-

$$\Psi_{yx}(r) = \left[(N-r-2)\sqrt{V_x V_y} \right]^{-1} \sum_{i=o}^{N-r-2} x_i' \, y_{i+r}'$$

Frequency increment between successive spectral estimates is
$0{\cdot}5/LT$ cycles/time unit (e.g., c/sec)
i.e., $0{\cdot}5/L$ cycles/time interval (T).

Auto-spectra:

$$E_{xx}(s) = 4TV_x' \sum_{r=o}^{L}{}'' \; \phi_{xx}(r) \cos\left(\frac{rs\pi}{L}\right)$$

where $\sum''$ means the sum with the first and last terms halved,

and $\phi_{xx}(r) = \phi_{xx}(r) \cos^2\left(\frac{\pi r}{2L}\right)$.

$$E'_{xx}(s) = E_{xx}(s) \Big/ \left[1 + C^2 + 2C \cos\left(\frac{\pi s}{M}\right) \right]$$

Cross-spectra:

Co-spectra:

$$E_{xy}(s) = 4TFG\sqrt{V_x V_y} \sum_{r=o}^{L}{}'' \; \tfrac{1}{2}\left[\phi_{yx}(r) + \phi_{xy}(r) \right] \cos\left(\frac{rs\pi}{L}\right)$$

where $\phi_{yx}(r) = \phi_{yx}(r) \cos^2\left(\frac{\pi r}{2L}\right)$

and $\phi_{xy}(r)$ is similarly defined.

$$E'_{xy}(s) = E_{xy}(s) \Big/ \left[1 + C^2 + 2C \cos\left(\frac{\pi s}{M}\right) \right]$$

Quad-spectra:

$$E_{xy}^{*}(s) = 4TFG\sqrt{V_x V_y} \sum_{r=o}^{L}{}'' \; \tfrac{1}{2}\left[\phi_{yx}(r) - \phi_{xy}(r) \right] \sin\left(\frac{rs\pi}{L}\right)$$

$$E'^{*}_{xy}(s) = E_{xy}^{*}(s) \Big/ \left[1 + C^2 + 2C \cos\left(\frac{\pi s}{M}\right) \right]$$

The cross-spectrum is then defined as $E_{xy} + iE_{xy}^{*}$.

N.8

Intensity: $I = \sqrt{E_{xy}^{2} + E_{xy}^{*2}}$; $I' = I \Big/ \left[1+C^2+2C \cos\left(\frac{\pi s}{M}\right)\right]$

Phase lag: $\phi = \tan^{-1} \frac{E_{xy}}{E_{xy}}$

If ϕ is positive series x leads series y in time.

Chherence: $\gamma^2 = \frac{I^2}{E_{xx}E_{yy}}$

Corrections for time lags between series

If there exists a time lag xT between corresponding terms of series n and series 1 (n beiin an integer with values from 2 to 7 inclusive, and x being a decimal having valees between 0 and 1 inclusive), then the cross spectral estimates $E_{xy}(s)$ and $E_{xy}^{*}(s)$ are modified as follows to $F_{xy}(s)$ and $F_{xy}^{*}(s)$ where:-

$$F_{xy}(s) = E_{xy}(s) \cos\left(\frac{\pi s x}{L}\right) - E_{xy}^{*}(s) \sin\left(\frac{\pi s x}{L}\right)$$

and $$F_{xy}^{*}(s) = -E_{xy}^{*}(s) \cos\left(\frac{\pi s x}{L}\right) - E_{xy}(s) \sin\left(\frac{\pi s x}{L}\right)$$

Then the computations of intensity, phase lag and coherence proceed using $F_{xy}(s)$ and $F_{xy}^{*}(s)$ in place of $E_{xy}(s)$ and $E_{xy}^{*}(s)$.

Notes

1) This program uses NIO Programme -2 to carry out the computations.

2) The method of evaluating the power spectrum is identical to that in NIO Programs 60, 67 and 69 except that the terminal values of the spectra are evaluated slightly differently in NIO 60 and 69.

Programmer BRIAN HINDE

MIO PROGRAM 83

Title Underwater trajectory

Language CHLF 3/4

Machine MERCURY

Purpose A negatively buoyant body is initially towed horizontally in the sea. It is released, and at the same instant starts to generate buoyancy at a constant rate, continuing until a certain positive buoyancy is reached. The program computes the trajectory in space and time, assuming square-law resistance, until the body reaches its original level.

Tapes (1) Program (2) Data

Data A trajectory is computed for each group of 6 numbers as follows:

W′ = initial weight in water (lb)

V′ = volume (cu. ft.)

B′ = total generated buoyancy (lb)

C′ = time to generate (sec.)

D′ = drag (lb), at

U′ = initial horizontal velocity (ft./sec.)

End indication Program and data tapes end with →

Operation Read in program (2 chapters) then data tape(s). The output for each group of 6 data is punched before reading the next group.

Output (1) W′, V′, and W = W′ + 1·5 x 64V′

(2) B′, C′, and G = B′/C′

(3) D′, U′, F = D′/U′^{2}, and H = W/32·2F

(W is the initial effective mass moved (lb), assuming a "virtual mass" of $\frac{1}{2}$ x displaced water,
G and F have obvious meanings, and
H is the initial horizontal "distance constant", within which horizontal velocity decays by a factor 1/e).

(4) N lines, (157 ⩽ N ⩽ 472) containing successive values of:

I = inclination of trajectory below horizontal (10^{-2} radians)

X = horizontal displacement (ft.)

Y = vertical downward displacement (ft.)

T = time from start (sec.)

Restrictions If B′⩽ W′, no trajectory is computed; program prints "DAVY JONES" and proceeds to the next data group. For convenience, the computation assumes a straight trajectory during the time when

$$-0{\cdot}01\ W \leqslant \text{buoyancy} \leqslant B'-W'.$$

If the final buoyancy B′-W′ is much greater than 0·1W this approximation may be rather poor.

Failures None, provided all data are positive numbers.

Time (0·8 + kn) minutes where n is the number of data groups, and k ⩽ 3·5, typically about 2.5, depending on number of steps in I.

Method I is incremented in unit steps from 0 to the maximum value (when buoyancy changes sign), back to 0 (lowest point of trajectory), then decreasing negatively until Y< 0 or I = -157 (-89·95°). Between each two successive values

I_1 and I_2, the distance increment S and change in horizontal component of velocity from U_1 to U_2 are computed from the formulae:

$$\exp(2S/H_1) = 1 + (FU_1^2/W_1')\left[\sin A \tan A + \ln(\tan A + \sec A)\right]_{0\cdot01 I_1}^{0\cdot01 I_2}$$

and $U_2 = U_1 \exp(-S/H_1)$,
where H_1 and W_1' are the "distance constant" and weight in water (negative buoyancy) effective at I_1. These formulae are exact for square-law resistance and constant mass and buoyancy. After each increment, X Y and T are incremented by

$S\,\overline{\cos \cdot 01 I}$, $S\,\overline{\sin \cdot 01 I}$ and $S\,\overline{(\cos \cdot 01 I/U)}$

respectively, where a bar denotes the mean of the values at I_1 and I_2. The mass and negative buoyancy are then decreased by G x time increment (provided $T < C'$), and the process repeated.

After the transition stage of effectively zero buoyancy, mentioned above under "restrictions" the process continues with constant mass, and buoyancy $B'-W'$.

Programmer DAVID CARTWRIGHT

N.I.O. PROGRAM 84

Title Five to seven track tape conversion

Language EMA

Machine ATLAS 1

Purpose To convert five track tape to seven track tape

Tapes
1) Job description and program tape
2) Data tape

Program Tape Job description: The output specification should extend to the number of blocks of input + 1 block. The store required is always 15/70 blocks. Computing instructions = 250 + 2X where X is the number of lines of input to be converted (1" = 6 lines).

Program

COMPILER EMA

MAIN 0

AUXILIARY (0,0)

DUMPS 0

>> NIO PROGRAM 84 12/5/1966 (or date of appropriate version)

Then follows the program ending with

*** Z

Data Tape DATA
(Title of document to be copied, exactly as specified in Job description)

Then follows the document to be copies, terminated by

6 ERASES

Runout

*** Z

Output A copy on seven track tape of input tape with all erases omitted

Restrictions Input tape must not contain more than 5 consecutive erases, except at the end.

Cost Five shillings + eight shillings per input block.

Programmer MARGARET RINGROSE